Overview of theoretical efforts on microscopic nuclear level densities

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Nuclear level densities (NLD)

Definition: number of levels per energy bin
Hauser-Feshbach nuclear input

- Main ingredients of statistical model calculations within the Hauser-Feshbach approach
  - Nuclear Level Densities
  - $\gamma$-strength functions ($\gamma$SF)
  - Optical model potentials
Impact of NLDs, γSF in neutron capture rates

Variations of neutron capture rates at 1.5 GK

<table>
<thead>
<tr>
<th>Nuclear Level Density</th>
<th>γ ray Strength Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Temperature matched to the Fermi Gas model (CT+BSFG)[19]</td>
<td>Kopecky-Uhl generalized Lorentzian (KU) [17]</td>
</tr>
<tr>
<td>Back-shifted Fermi Gas model (BSFG)[19],[20]</td>
<td>Hartree-Fock BCS + QRPA (HF-BCS+QRPA) [21]</td>
</tr>
<tr>
<td>Generalized Super fluid model (GSM)[22], [23]</td>
<td>Hartree-Fock-Bogolyubov + QRPA (HFB+QRPA) [24]</td>
</tr>
<tr>
<td>Hartree Fock using Skyrme force (HFS) [25]</td>
<td>Modified Lorentzian (Gor-ML)[26]</td>
</tr>
</tbody>
</table>

Hartree-Fock-Bogoliubov (Skyrme force) + combinatorial method (HFBS-C) [27]

![Diagram showing variations in neutron capture rates for different elements at 1.5 GK. The diagram uses colors to represent the strength of neutron capture rates with stable, <5, 5-10, 10-20, 20-100, >100 categories. The elements Ga, Zn, Cu, Ni, Co, Fe, Mn are shown with varying colors indicating their neutron capture rate strengths. The diagram includes a horizontal line at Z = 28 and a vertical line at N = 40, highlighting the distribution of neutron capture rates.]

S. N. Liddick et al., PRL 116 242502 (2016)
Experimental NLDs

• Low energy discrete experimental levels
• Level density from neutron resonance spacings at the neutron separation energy (available only for specific spins)
• Oslo method and β-Oslo technique (require normalization to the low energy discrete levels and to the level density at neutron separation energies or theoretical estimates)
• Particle evaporation technique
NLDs from theory

Phenomenological models

• Fermi gas model

\[
\rho(E_x, J, \pi) = \frac{1}{2} \frac{12J + 1}{2\sigma^2} \exp \left[ - \frac{(J + 1/2)^2}{\sigma^2} \right] \frac{1}{\sqrt{2\pi}\sigma} \frac{\sqrt{\pi} \exp[2\sqrt{aU}]}{12 \ a^{1/4} U^{5/4}}
\]

• Constant temperature model

\[
\rho(E_x, J, \pi) = \frac{1}{2} \frac{12J + 1}{2\sigma^2} \exp \left[ - \frac{(J + 1/2)^2}{\sigma^2} \right] \frac{1}{T} \exp \left[ \frac{E_x - E_0}{T} \right]
\]

✓ Extensively studied, available in reaction codes (e.g. TALYS)

✗ Parameters \((a, U, E_0, T)\) must be determined from the available experimental data or from empirical expressions, knowledge of the spin distribution and spin cut-off parameter \(\sigma\) is required
NLDs from theory

Microscopic models
• Based on Hartree-Fock calculations
✓ spin and parity dependent NLDs
✓ Level densities available for thousands of nuclides, high excitation energies and spins
✓ available in reaction codes (TALYS)
× many-body correlations missing

S. Goriely et al., ADNDT 77 311 (2001)  S. Hilaire et al., PRC 86 064317 (2012)
S. Goriely et al., PRC 78 064307 (2008)
NLDs from theory

SHELL MODEL APPROACHES
(spin and parity dependent NLDs)

- Configuration interaction shell model calculations using conventional diagonalization; \( \times sd\)-nuclei

- Shell Model Monte Carlo; mid-mass and heavy nuclei

Y. Alhassid et al., PRL 99 162504 (2007)
M. Bonett-Matiz et al. PRC 88 011302 (R) (2013)
NLDs from theory

SHELL MODEL APPROACHES (spin and parity dependent NLDs)

– Lanczos method, computes moments of the Hamiltonian; mid-mass nuclei (requires about 100 iterations in the full model space)

– Moments method, computes the first two moments of the Hamiltonian; does not require diagonalization in the full model space

W. E. Ormand et al., PRC 102 014315 (2020)  
R. Sen’kov et al., PRC 93 064304 (2016)
Calculation of level density – Moments method

<table>
<thead>
<tr>
<th>Partitions, $p$</th>
<th>$d_{5/2}$</th>
<th>$s_{1/2}$</th>
<th>$d_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

$E_{g.s.}$: Shell model
$\eta$: cut-off (2.8)

$$\rho(E; \alpha) = \sum_p D_{ap} G_{ap}(E)$$

$$E_{ap} = \frac{1}{D_{ap}} Tr^{ap} H$$

$$\sigma_{ap}^2 = \frac{1}{D_{ap}} Tr^{ap} H^2 - E_{ap}^2$$

$$G_{ap} = G(E - E_{ap} + (E_{g.s.} - \eta \sigma))$$

$$G(x; \sigma) = C \begin{cases} e^{-x^2/2\sigma^2}, & |x| \leq \eta \sigma, \\ 0, & |x| > \eta \sigma \end{cases}$$

R. Sen’kov et al., CPC 184, 215 (2013)
Model spaces

Tested with
- \( sd - 0d_{5/2}, 0d_{3/2}, 1s_{1/2} \)
- \( pf - 0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2} \)
- \( jj44 - 0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2} \)
- \( pf + 0g_{9/2} \)

Extensions
- \( jj55 - 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2} \)

...
MM vs Exact SM calculations NLDs

MM vs Experimental NLDs

\[ s + p + sd + pf - ^{28}\text{Si} \]

R. Sen’kov et al., PRC 93 064304 (2016)

sd – positive parity

S. Karampagia et al., ADNT 1, 120 (2017)
MM vs other models & Oslo method

\( ^{52}\text{Fe}, J^\pi = 0^+ \)
- exact SM (pf-shell, gx1a)
- moments method
- model of Goriely et al.

\( ^{52}\text{Fe}, J^\pi = 1^+ \)
- exact SM (pf-shell, gx1a)
- moments method
- model of Goriely et al.

\( \text{pf} - ^{52}\text{Fe} \)

\( \text{pfg}_{9/2} - ^{56}\text{Fe} \)

\( \text{pf & pfg}_{9/2} - ^{56}\text{Fe} \)

R. Sen’kov et al., PRC 82 024304 (2010)
S. Goriely et al., PRC 78 064307 (2008)
Challenges

• Shell model level densities have a finite excitation range (~12 MeV); need for an algorithm to continue to higher excitation energies
• Ground state is required; directly from a shell model calculation, other extrapolation techniques
• Complex process to insert moments method level density in tables of nuclear reaction codes, such as TALYS.
• Availability of reliable shell model interactions (away from stability what?)
Thank you

Collaborators
• MSU: Vladimir Zelevinsky, Alex Brown
• CMU: George Perdikakis, Mihai Horoi