# Overview of theoretical efforts on microscopic nuclear level densities

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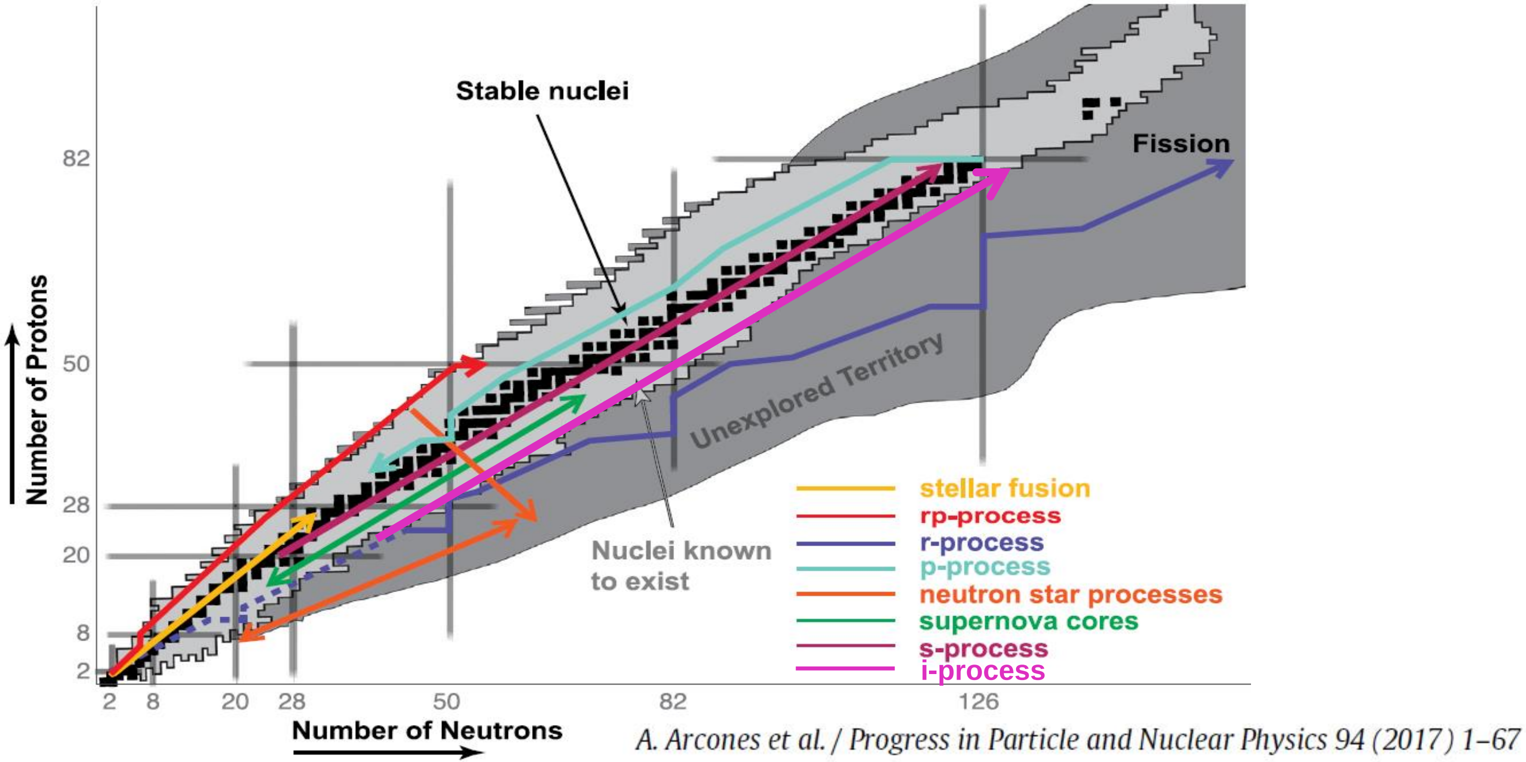
# Nuclear level densities (NLD)

Definition: number of levels per energy bin




A. Arcones et al. / Progress in Particle and Nuclear Physics 94 (2017) 1–67

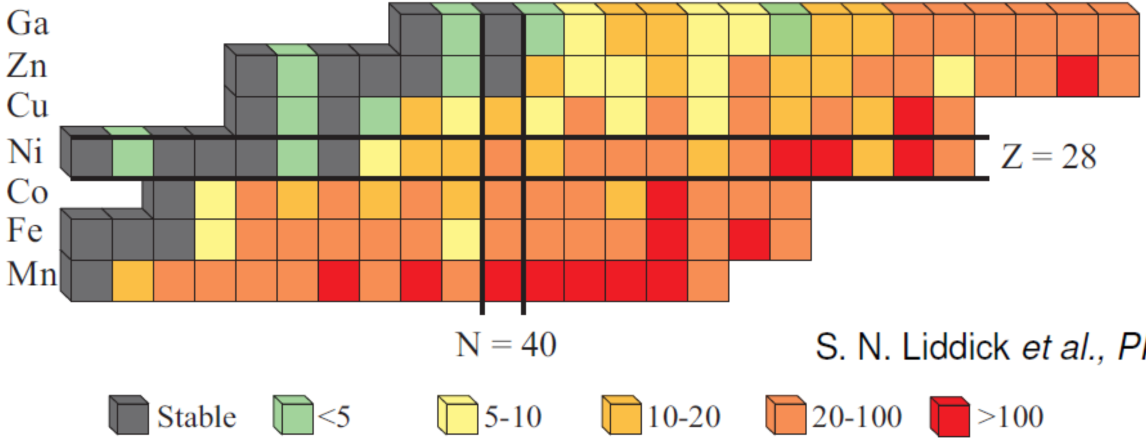
# Hauser-Feshbach nuclear input

- Main ingredients of statistical model calculations within the Hauser-Feshbach approach
  - Nuclear Level Densities
  - γ-strength functions (γSF)
  - Optical model potentials

# Impact of NLDs, γSF in neutron capture rates

## Variations of neutron capture rates at 1.5 GK

| Nuclear Level Density | γ ray Strength Function |
|---|---|
| Constant Temperature matched to the Fermi Gas model (CT+BSFG)[19] | Kopecky-Uhl generalized Lorentzian (KU) [17] |
| Back-shifted Fermi Gas model (BSFG)[19],[20] | Hartree-Fock BCS + QRPA (HF-BCS+QRPA) [21] |
| Generalized Super fluid model (GSM)[22], [23] | Hartree-Fock-Bogolyubov + QRPA (HFB+QRPA) [24] |
| Hartree Fock using Skyrme force (HFS) [25] | Modified Lorentzian (Gor-ML)[26] |
| Hartree-Fock-Bogoliubov (Skyrme force) + combinatorial method (HFBS-C) [27] | |




S. N. Liddick *et al., PRL 116 242502 (2016)*

# Experimental NLDs

- Low energy discrete experimental levels
- Level density from neutron resonance spacings at the neutron separation energy (available only for specific spins)
- Oslo method and β-Oslo technique (require normalization to the low energy discrete levels and to the level density at neutron separation energies or theoretical estimates)
- Particle evaporation technique

# NLDs from theory

Phenomenological models

- Fermi gas model

$$\rho(E_x, J, \pi) = \frac{1}{2}\frac{2J+1}{2\sigma^2}\exp\left[-\frac{(J+1/2)^2}{\sigma^2}\right]\frac{1}{\sqrt{2\pi}\sigma}\frac{\sqrt{\pi}}{12}\frac{exp\left[2\sqrt{aU}\right]}{a^{1/4}U^{5/4}}$$

- Constant temperature model

$$\rho(E_x, J, \pi) = \underbrace{\frac{1}{2}\frac{2J+1}{2\sigma^2}\exp\left[-\frac{(J+1/2)^2}{\sigma^2}\right]}_{\text{spin distribution}}\frac{1}{T}exp\left[\frac{E_x - E_0}{T}\right]$$

✓ Extensively studied, available in reaction codes (e.g. TALYS)

× Parameters ($a, U, E_0, T$) must be determined from the available experimental data or from empirical expressions, knowledge of the spin distribution and spin cut-off parameter $\sigma$ is required

# NLDs from theory

Microscopic models

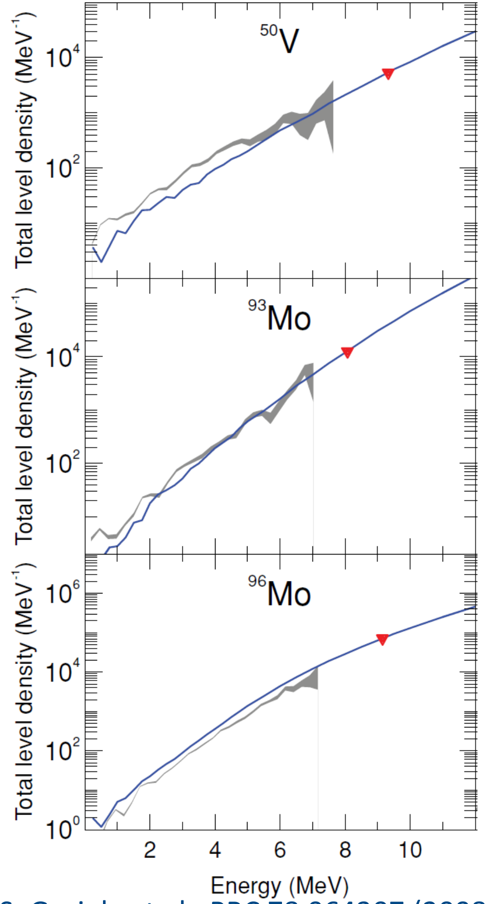
- Based on Hartree-Fock calculations

✓ spin and parity dependent NLDs

✓ Level densities available for thousands of nuclides, high excitation energies and spins

✓ available in reaction codes (TALYS)

× many-body correlations missing



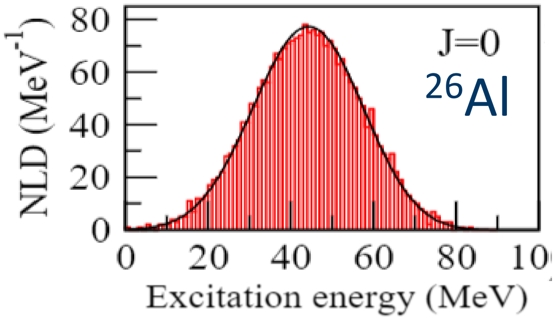

S. Goriely et al., ADNDT 77 311 (2001)    S. Hilaire et al., PRC 86 064317 (2012)

S. Goriely et al., PRC 78 064307 (2008)

# NLDs from theory

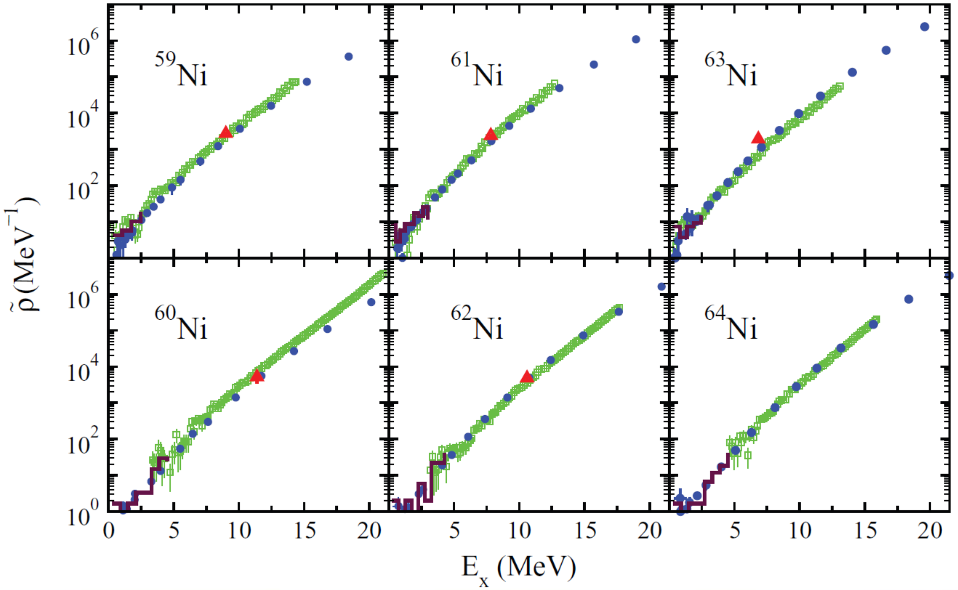
SHELL MODEL APPROACHES
(spin and parity dependent NLDs)

– Configuration interaction shell model calculations using conventional diagonalization; × *sd*-nuclei




Y. Alhassid et al., PRL 99 162504 (2007)
M. Bonett-Matiz et al. PRC 88 011302 (R) (2013)

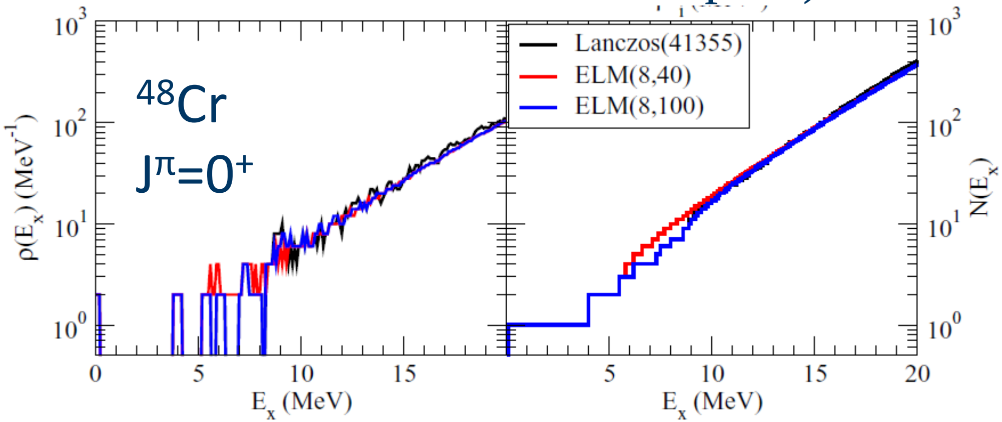
– Shell Model Monte Carlo; mid-mass and heavy nuclei



# NLDs from theory

## SHELL MODEL APPROACHES (spin and parity dependent NLDs)

- Lanczos method, computes moments of the Hamiltonian; mid-mass nuclei (requires about 100 iterations in the full model space)



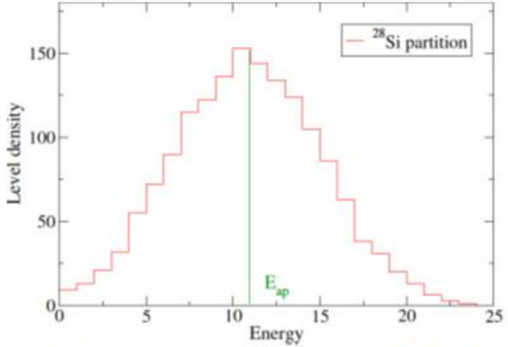

- Moments method, computes the first two moments of the Hamiltonian; does not require diagonalization in the full model space

W. E. Ormand et al., PRC 102 014315 (2020) R. Sen’kov et al., PRC 93 064304 (2016)

# Calculation of level density – Moments method

Partitions, p

| p | $d_{5/2}$ | $s_{1/2}$ | $d_{3/2}$ |
|---|---|---|---|
| 1 | 6 | 0 | 0 |
| 2 | 5 | 1 | 0 |
| 3 | 5 | 0 | 1 |
| 4 | 4 | 2 | 0 |
| ... | ... | ... | ... |
| 15 | 0 | 2 | 4 |




Each partition p, gives a gaussian distribution

$$E_{ap} = \frac{1}{D_{ap}} Tr^{ap} H$$

$$\sigma_{ap}^2 = \frac{1}{D_{ap}} Tr^{ap} H^2 - E_{ap}^2$$

$E_{g.s.}$ : Shell model

$\eta$ : cut-off (2.8)

$$G_{ap} = G(E - E_{ap} + E_{g.s}; \sigma_{ap})$$

$$G(x;\sigma) = C \begin{cases} e^{-x^2/2\sigma^2}, & |x| \le \eta\sigma, \\ 0, & |x| > \eta\sigma \end{cases}$$

$$\rho(E;a) = \sum_p D_{ap} G_{ap}(E)$$

R. Sen'kov et al., CPC 184, 215 (2013)

# Model spaces

•Any model space for which **an effective shell model Hamiltonian is available**

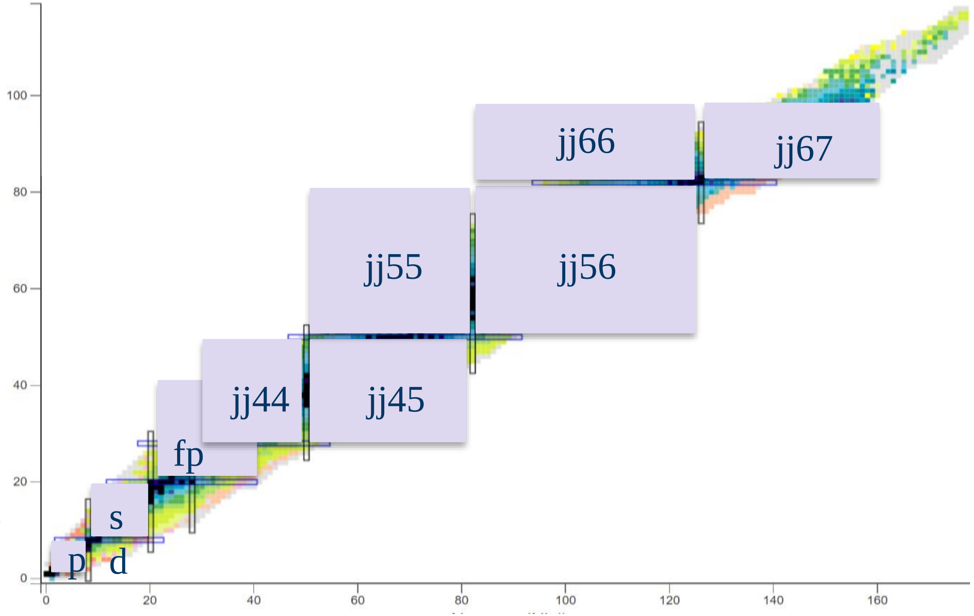
Tested with

- *sd* – $0d_{5/2}, 0d_{3/2}, 1s_{1/2}$
- *pf* – $0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}$
- *jj44* – $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- *pf* + $0g_{9/2}$

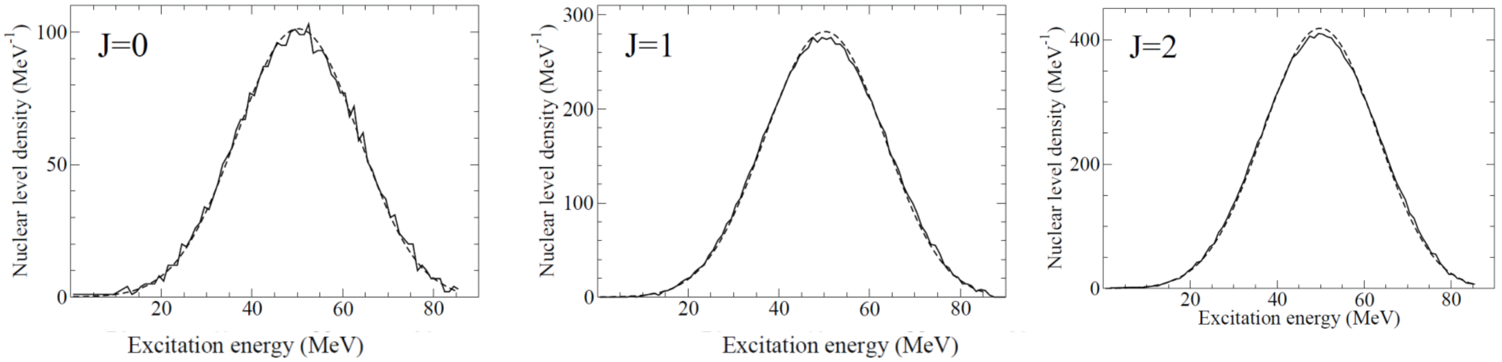
*Extensions*

– *jj55* – $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

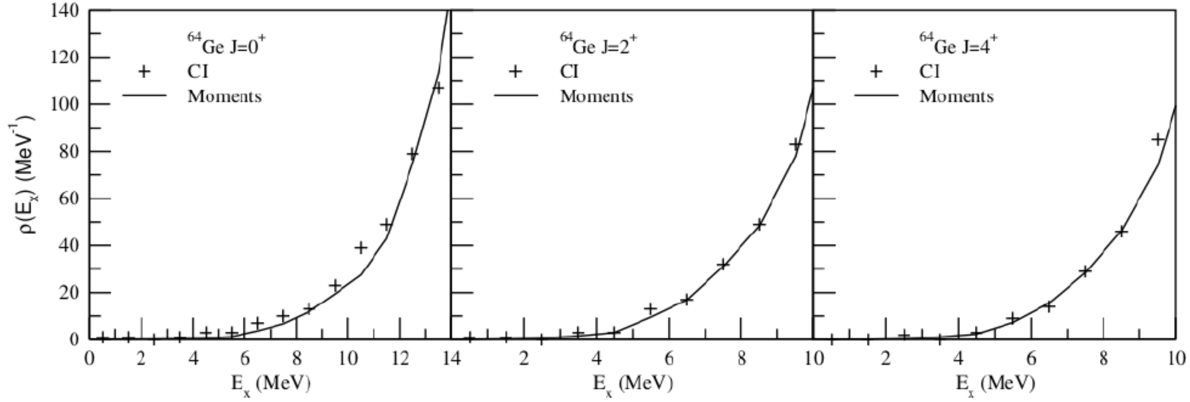
…



# MM vs Exact SM calculations NLDs



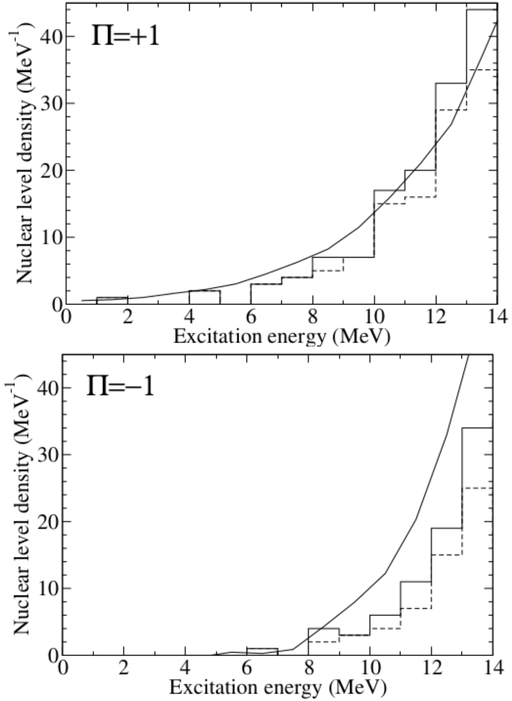

sd – $^{28}$Si




jj44 – $^{64}$Ge

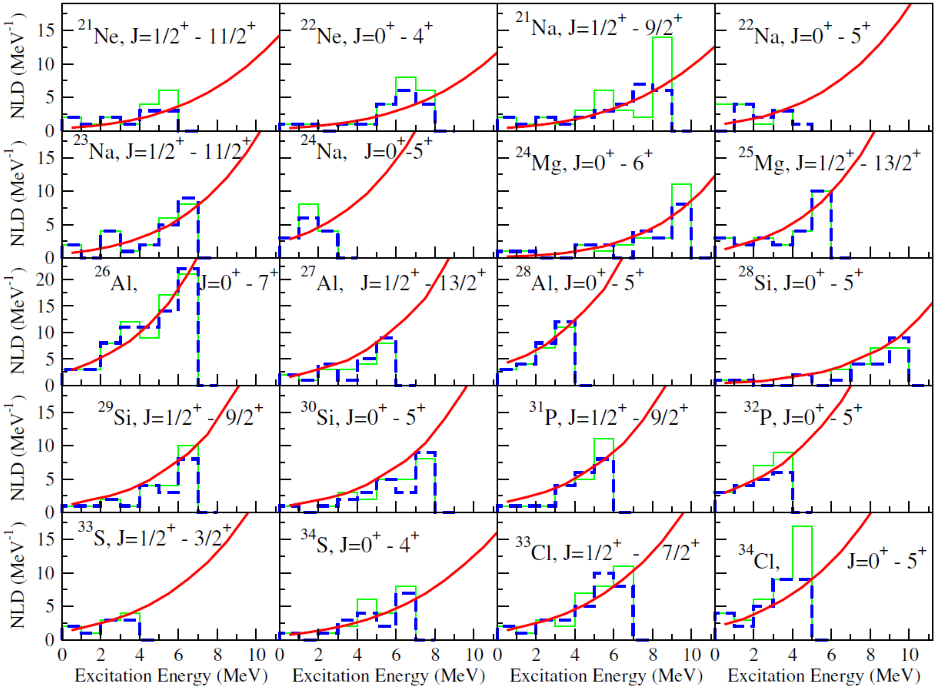
R. Sen'kov et al., PRC 93 064304 (2016), M. Scott et al., PoS (2008)

# MM vs Experimental NLDs




s + p + sd +pf – $^{28}$Si

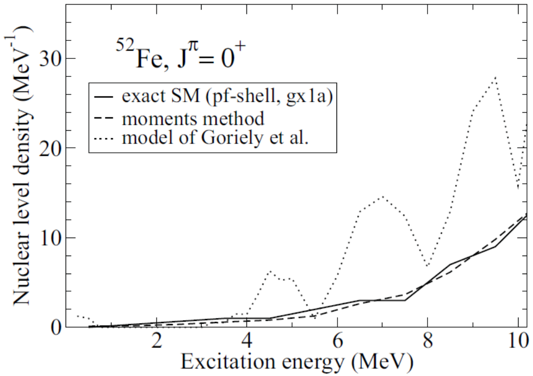
R. Sen'kov et al., PRC 93 064304 (2016)

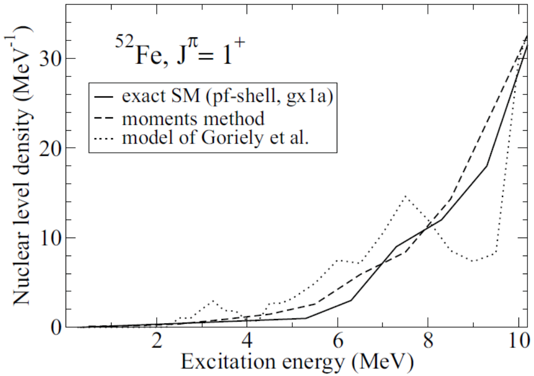



sd – positive parity

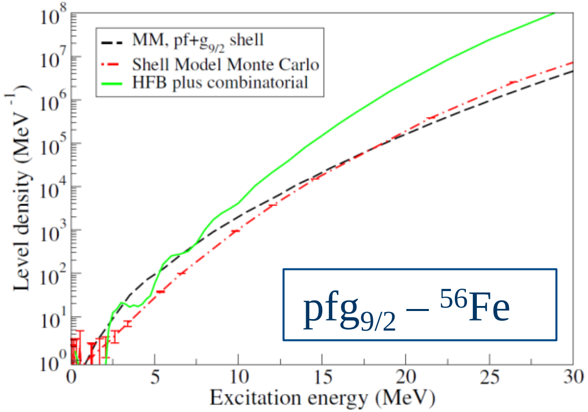
S. Karampagia et al., ADNT 1, 120 (2017)

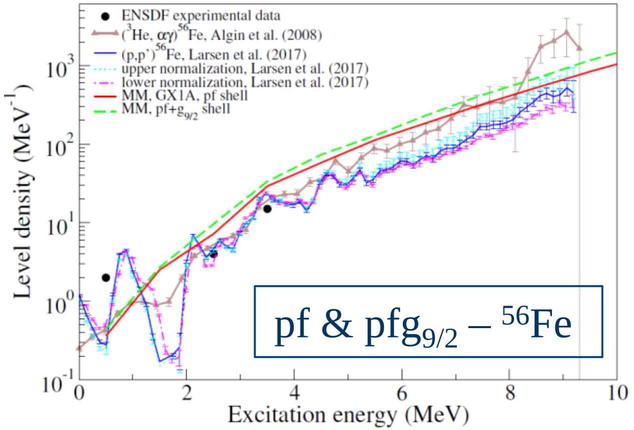
# MM vs other models & Oslo method




pf – $^{52}$Fe




R. Sen'kov et al., PRC 82 024304 (2010)
S. Goriely et al., PRC 78 064307 (2008)

# Challenges

- Shell model level densities have a finite excitation range (~12 MeV); need for an algorithm to continue to higher excitation energies
- Ground state is required; directly from a shell model calculation, other extrapolation techniques
- Complex process to insert moments method level density in tables of nuclear reaction codes, such as TALYS.
- Availability of reliable shell model interactions (away from stability what?)

# Thank you

Collaborators

- MSU: Vladimir Zelevinsky, Alex Brown
- CMU: George Perdikakis, Mihai Horoi