Magneto-thermal modeling of high temperature superconducting tapes using a mixed $h-\phi$ finite element approach with thin shells

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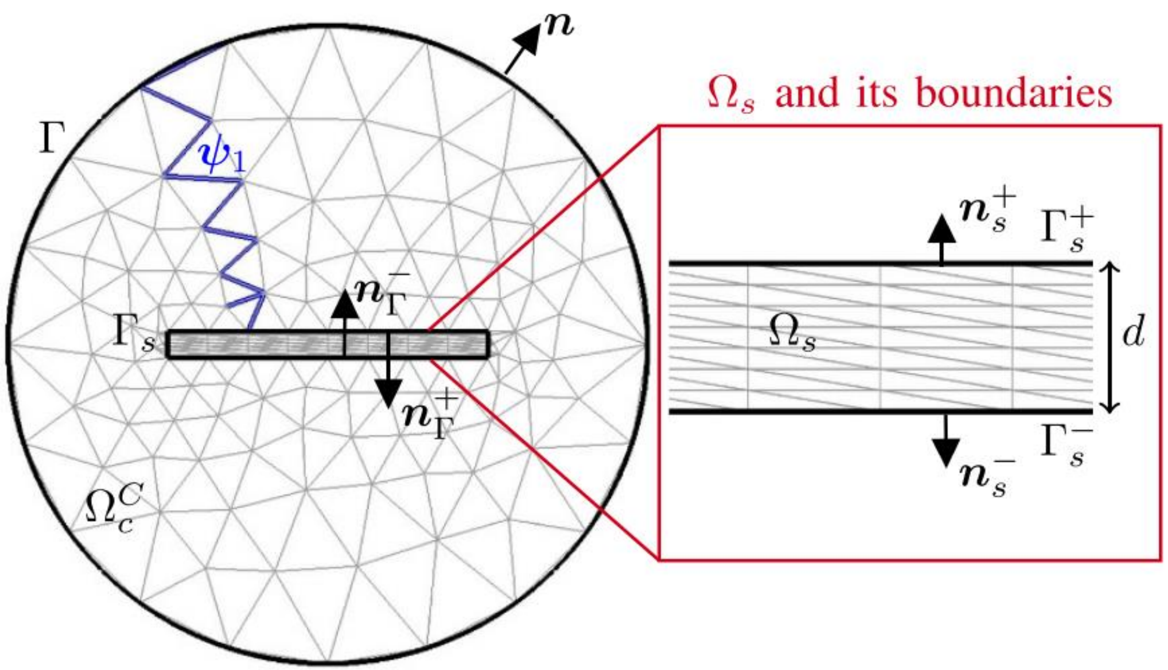
OUTLINE

- **The $h - \phi$ interface and thin-shell model**
- Belfem: where we're at
- Belfem vs. GetDP performance comparisons
- Thermal model
- Next steps

THE $h-\phi$ INTERFACE AND THIN-SHELL MODEL



B. De Sousa Alves et al.,
SuST 35 (2), 024001 (2021)

$$\left.\begin{aligned}\boldsymbol{n}\times\left(\boldsymbol{h}^+-\boldsymbol{h}^-\right)&=0\\ \boldsymbol{n}\times\left(\boldsymbol{e}^+-\boldsymbol{e}^-\right)&=0\end{aligned}\right\}\text{tangential condition}$$

$$\left.\boldsymbol{n}\cdot\left(\boldsymbol{b}^+-\boldsymbol{b}^-\right)=0\right\}\text{normal condition}$$

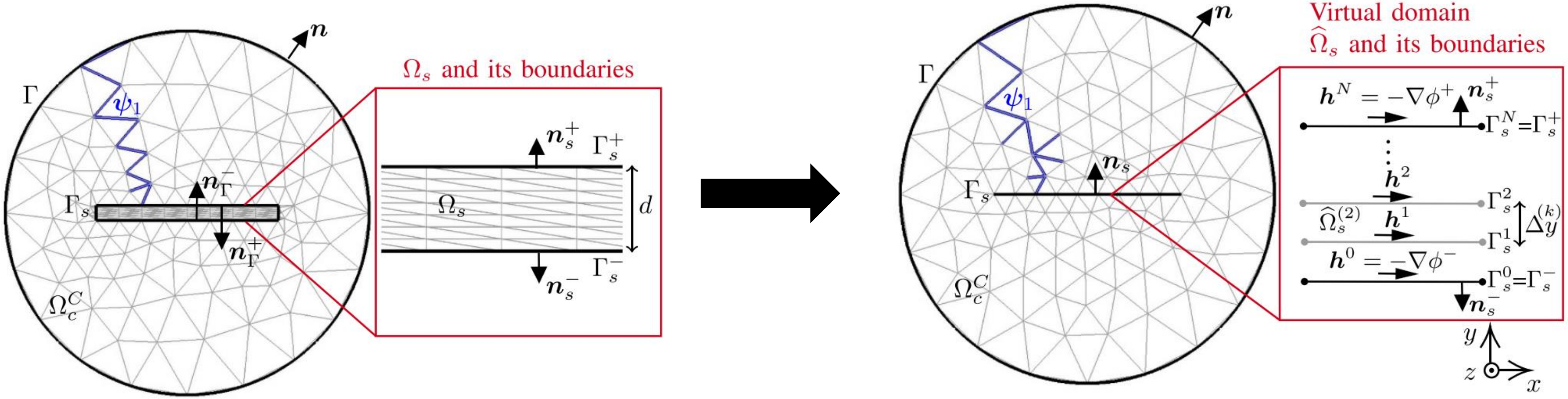
- In a finite element problem, the $\boldsymbol{h}$ formulation (Faraday’s law), leads to the weak form:

$$\left(\rho\nabla\times\boldsymbol{h},\nabla\times\boldsymbol{g}\right)_\Omega+\partial_t\left(\mu\boldsymbol{h},\boldsymbol{g}\right)_\Omega + \left\langle\boldsymbol{n}\times\boldsymbol{e},\boldsymbol{g}\right\rangle_{\Gamma_e}=0$$

- Since $\boldsymbol{j}=\boldsymbol{0} \Rightarrow \boldsymbol{h}=-\nabla\phi$ in the air, which significantly reduces the number of DoFs.
- **Different formulations = Interfacial terms**

THE $h-\phi$ INTERFACE AND THIN-SHELL MODEL

- Creation of a virtual domain with N layers to avoid the necessity of a very fine mesh, while keeping the accuracy. T-S has already proven to be very accurate and can allow to model different material types for different layers (see Alves et al. 2021)

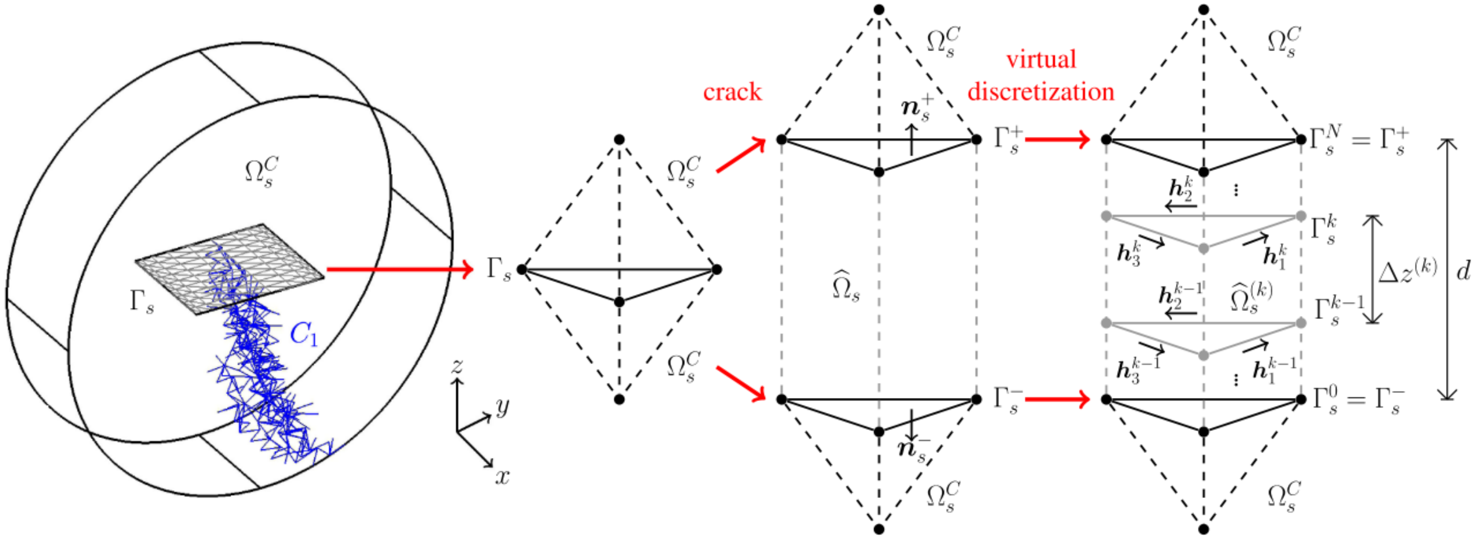


B. De Sousa Alves et al., SuST 35 (2), 024001 (2021)

$$(\rho\nabla\times\boldsymbol{h},\nabla\times\boldsymbol{g})_{\Omega_S}+\partial_t(\mu\boldsymbol{h},\boldsymbol{g})\rightarrow\sum_{k=1}^{N}(\rho\nabla\times\boldsymbol{h},\nabla\times\boldsymbol{g})_{\widehat{\Omega}_S^k}+\sum_{k=1}^{N}\partial_t(\mu\boldsymbol{h},\boldsymbol{g})_{\widehat{\Omega}_S^k}$$

THE $h-\phi$ INTERFACE AND THIN-SHELL MODEL

- And the same idea can be applied in 3-D, and we realized recently that it has even more advantages



B. De Sousa Alves et al., IEEE Trans. Appl. Supercond. 32 (5), 7500411 (2022)

$$\boldsymbol{j} = \nabla \times \boldsymbol{h} = \begin{bmatrix} \boxed{\begin{matrix} -\partial_z h_y \\ \partial_z h_x \end{matrix}} \\ \partial_x h_y - \partial_y h_x \end{bmatrix}$$

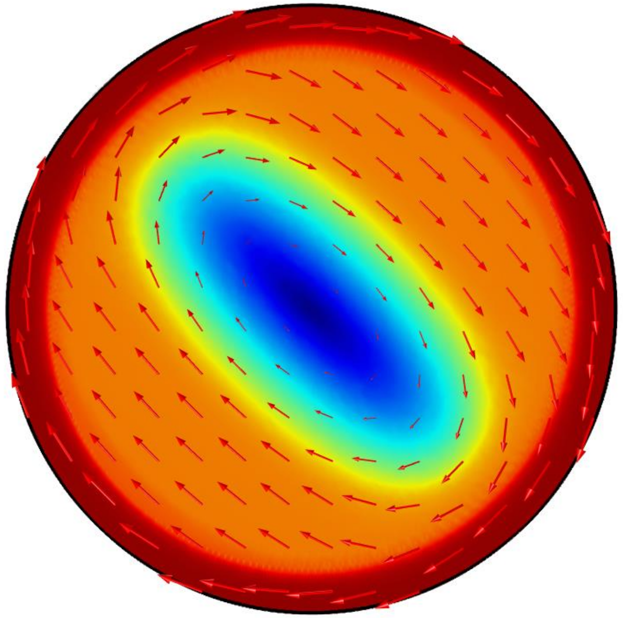
In-plane current

Out-of-plane current (intra-tape current sharing)

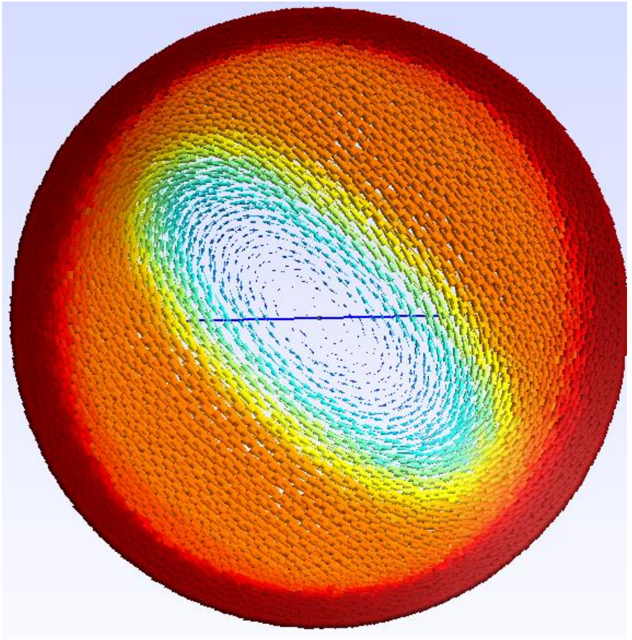
THE $h - \phi$ INTERFACE AND THIN-SHELL MODEL

- Simple test case to verify this out-of-plane current sharing: thin 3-D SC disc ($N = 5$) in a background AC field at an angle ($b_x = b_y = b_z$)

“Thick”-shell (COMSOL)

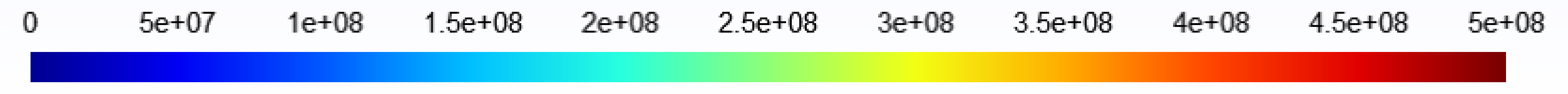


Thin-shell (GetDP, Alves’ implementation)



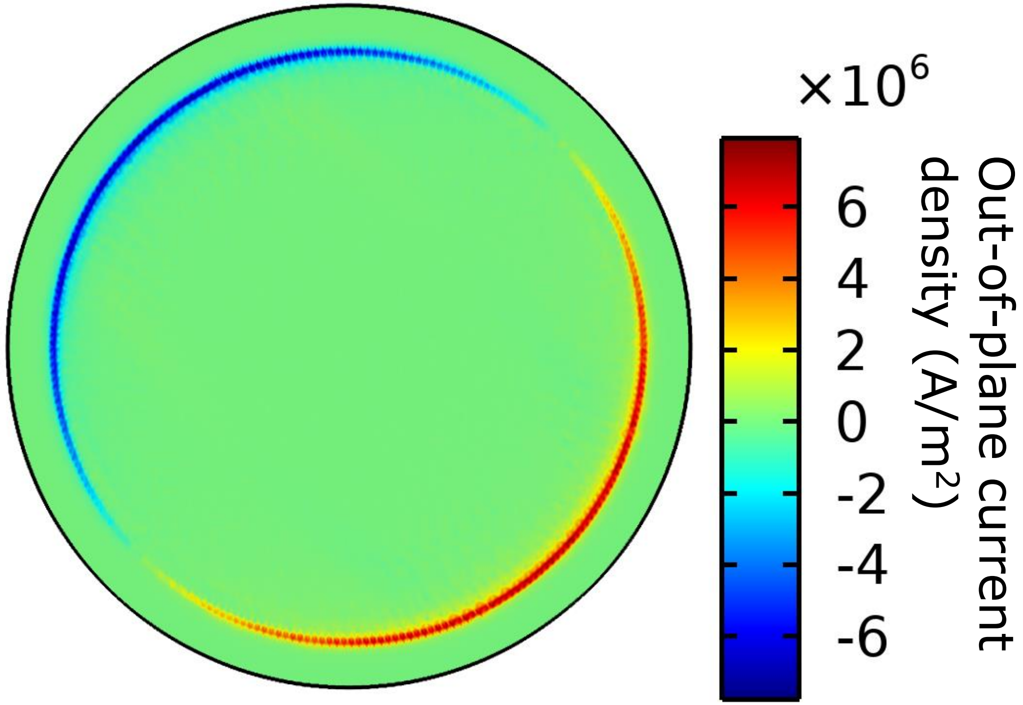
(Slice in the middle at $t = T/4$)

In-plane current density (A/m^2)



THE $h-\phi$ INTERFACE AND THIN-SHELL MODEL

- And the out-of-plane current is indeed non-zero (in COMSOL, not possible in GetDP, will be implemented in BELFEM)



(Slice in the middle at $t = T/4$)

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- The $h - \phi$ interface and thin-shell model
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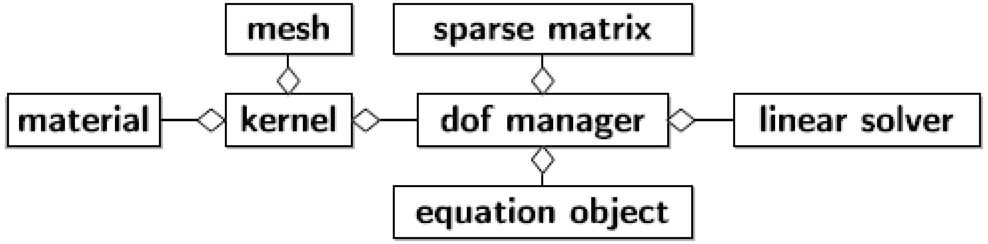
BELFEM: WHERE WE'RE AT

- **BELFEM:** BErkeley Lab Finite EleMent framework
  - Integrated platform gathering all state-of-the-art features required to study quench in REBCO tapes, cables and magnets

- **Includes (2-D):**
  - h, ϕ, a formulations, and the couplings
  - Thin-shell model
  - Coupling with thermal diffusion model (lumped-mass)
  - High-performance linear algebra solvers
  - Parallel computing (OpenMP, MPI)

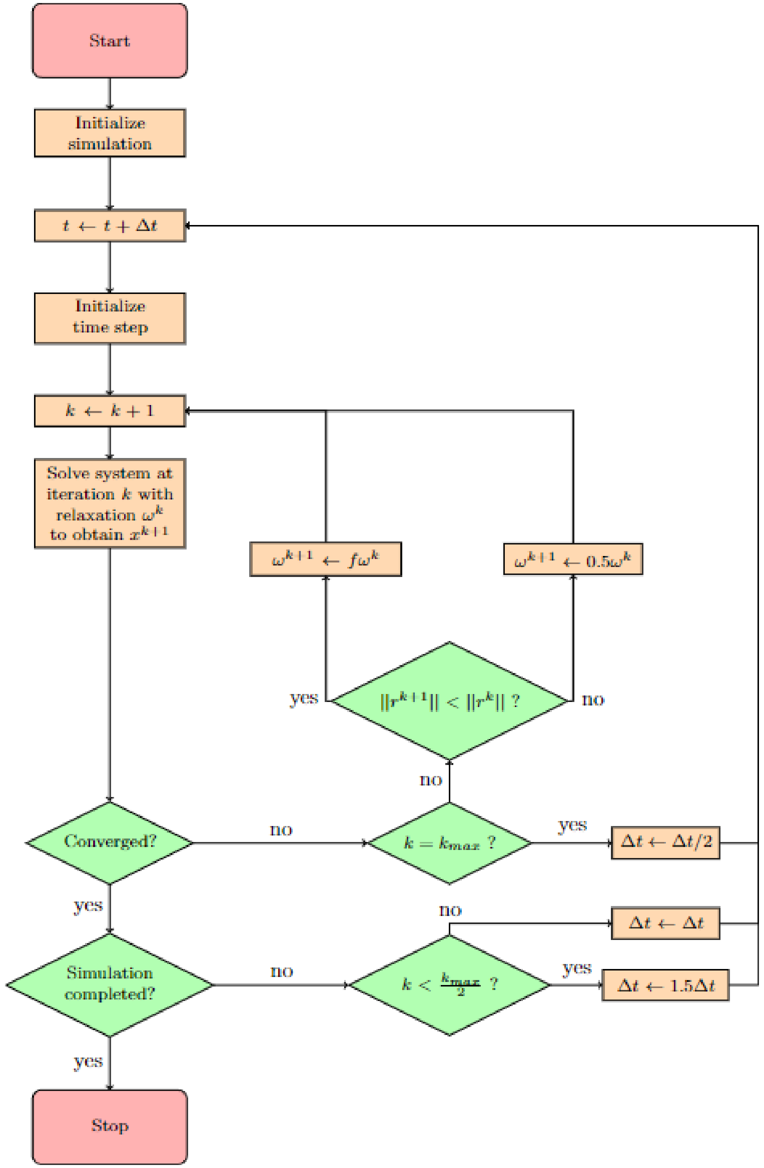


- **To be implemented**
  - 3-D (!!)

BELFEM: WHERE WE'RE AT

- Major issue recently solved: checkerboarding effect when calculating the current density (example of a bulk SC with a background $\boldsymbol{b}$ field in the y direction)

BELFEM: WHERE WE’RE AT


Start

Initialize simulation

$t \leftarrow t + \Delta t$

Initialize time step

$k \leftarrow k + 1$

Solve system at iteration k with relaxation ω^k to obtain x^{k+1}

$\omega^{k+1} \leftarrow f\omega^k$

$\omega^{k+1} \leftarrow 0.5\omega^k$

yes

$\|r^{k+1}\| < \|r^k\|$?

no

no

Converged?

no

$k = k_{max}$?

yes

$\Delta t \leftarrow \Delta t/2$

yes

no

$\Delta t \leftarrow \Delta t$

Simulation completed?

no

$k < \frac{k_{max}}{2}$?

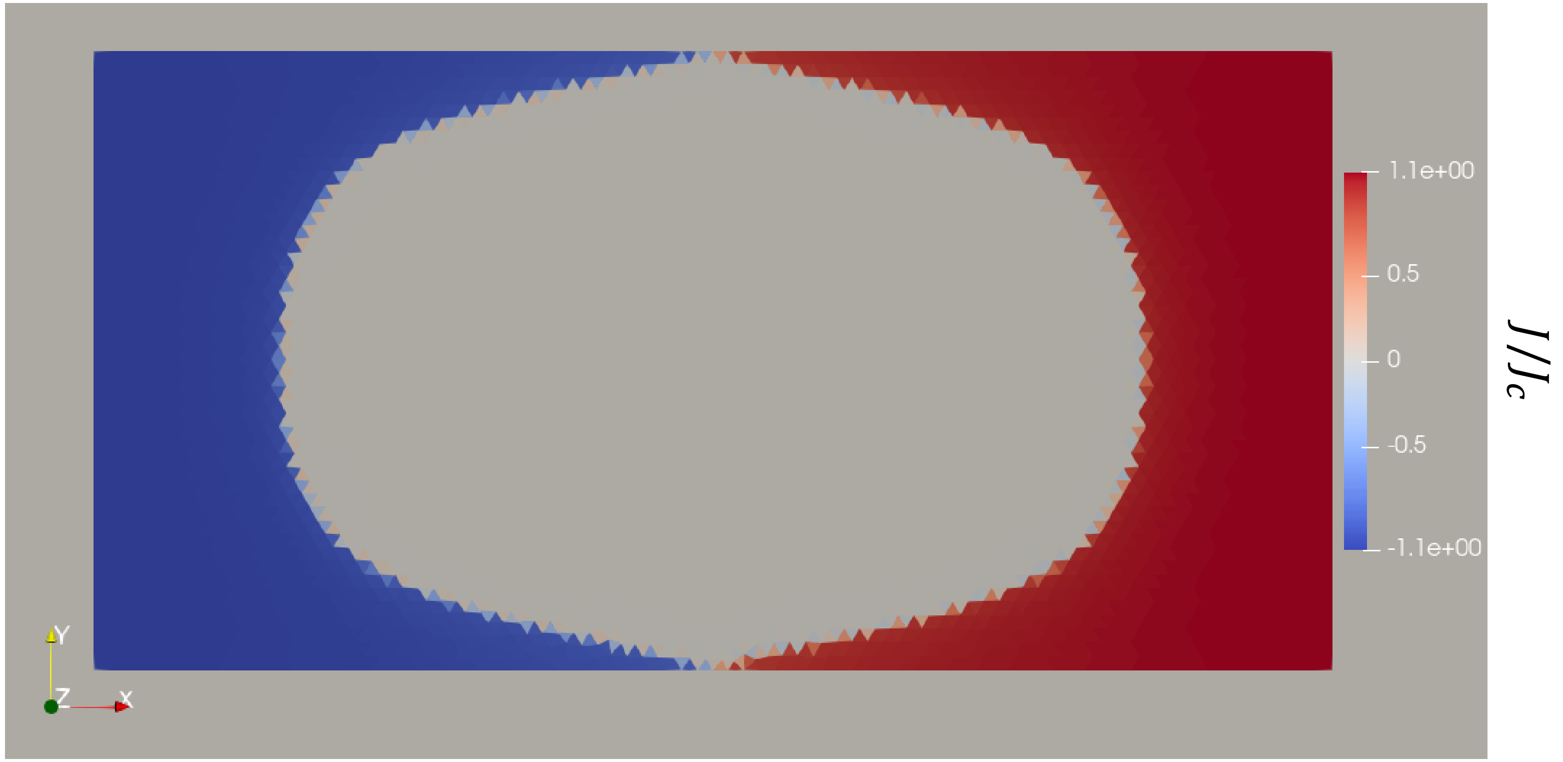
yes

$\Delta t \leftarrow 1.5\Delta t$

yes

Stop

- Solution: adaptive relaxation factor **and** time step



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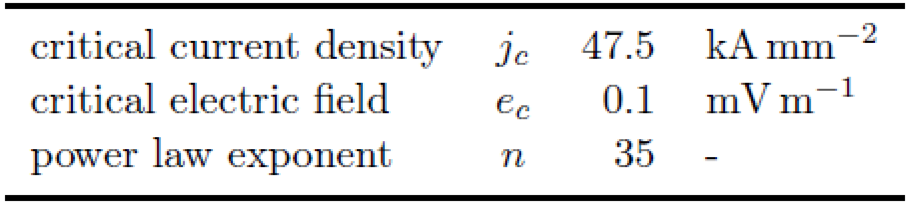
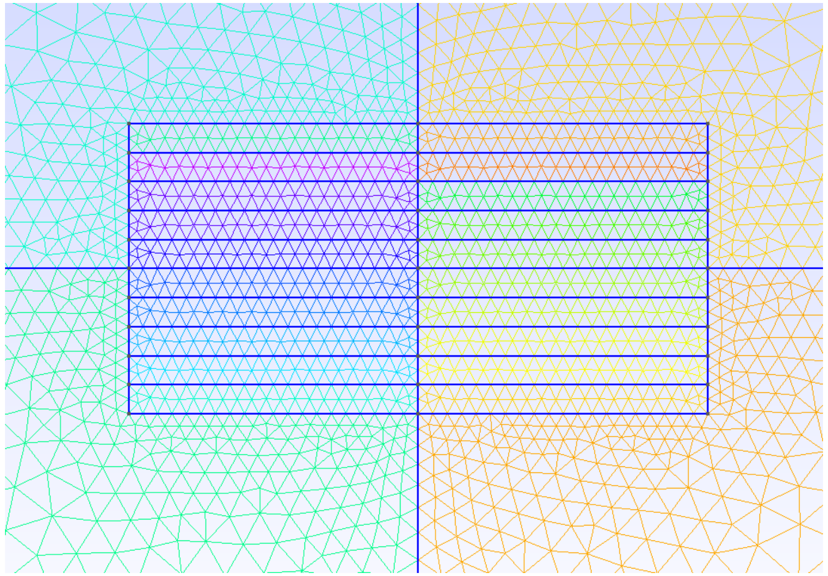
BELFEM VS. GETDP PERFORMANCE COMPARISONS

- To illustrate the relevance of BELFEM, we compared it against GetDP (current software in which T-S are implemented)

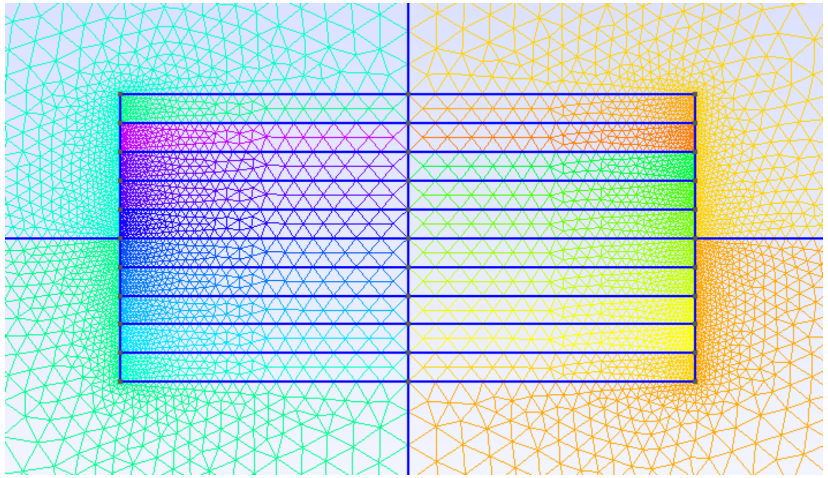
- Stack: 11 tapes, $N = 1$, $w = 4$ mm, $t = 1$ µm, $I = 90$ A/tape, $f = 50$ Hz



critical current density	j_c	47.5	$\mathrm{kA\,mm^{-2}}$
critical electric field	e_c	0.1	$\mathrm{mV\,m^{-1}}$
power law exponent	n	35	-



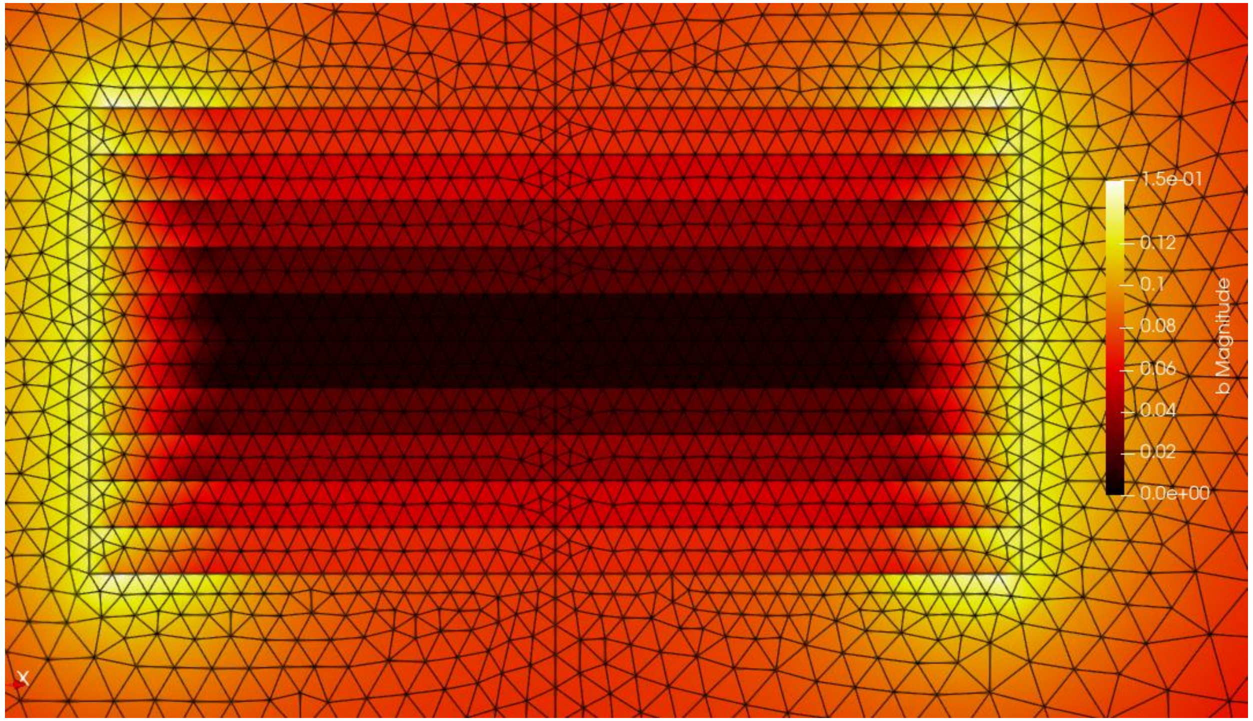
(Coarse mesh)



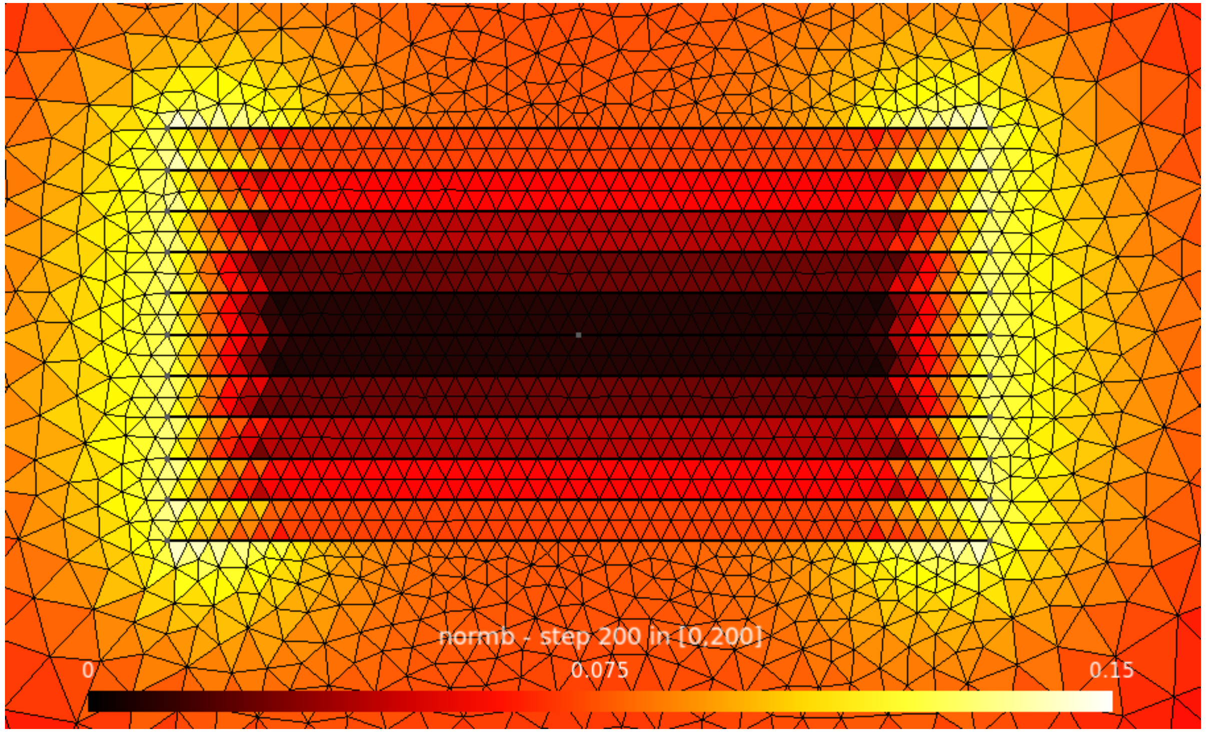
(Fine mesh)

BELFEM VS. GETDP PERFORMANCE COMPARISONS

$$|\boldsymbol{b}|\ (t = T/4)$$



BELFEM (visualisation in Paraview)



GetDP (visualisation in Gmsh)

BELFEM VS. GETDP PERFORMANCE COMPARISONS

- Performance comparisons in terms of simulation times (1 period simulated)

			GetDP (Mumps)	Belfem (Mumps)	Belfem (Strumpack)
Constant time step	Coarse mesh	Saving	8:09	3:16	2:09
		Not saving	5:13	1:56	0:55
	Fine mesh	Saving	16:26	9:09	4:45
		Not saving	10:57	7:15	2:54
Adaptive time step	Coarse mesh	Saving	4:02	0:28	0:16
		Not saving	2:36	0:23	0:11
	Fine mesh	Saving	10:41	1:15	0:32
		Not saving	7:58	1:09	0:25

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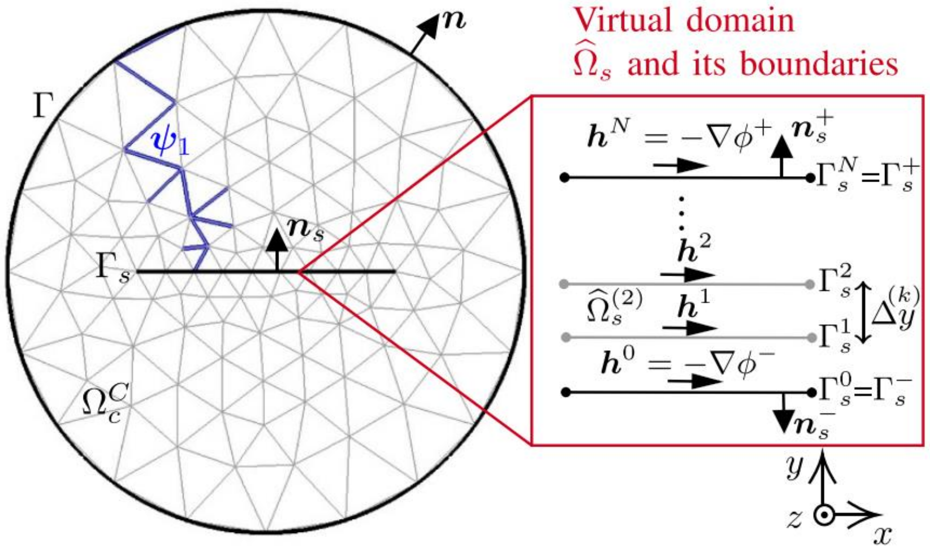
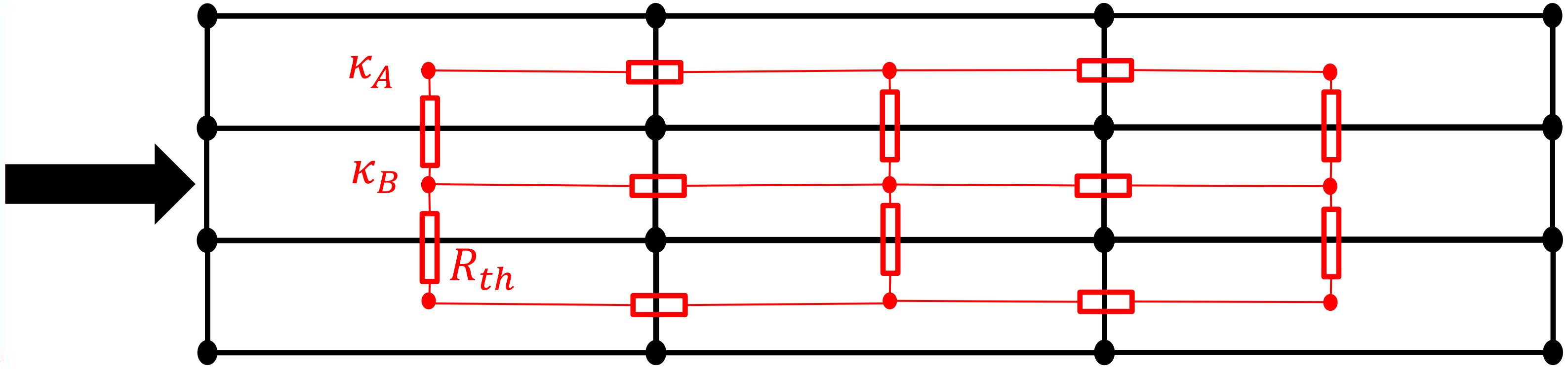
THERMAL MODEL

- For temperature diffusion calculations, the electromagnetic problem is coupled through a lumped-mass model on the virtual mesh

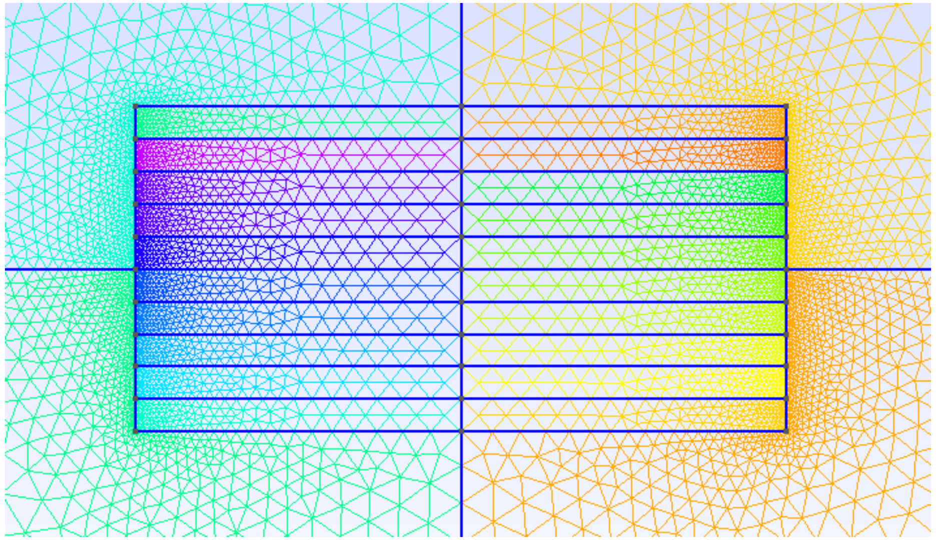


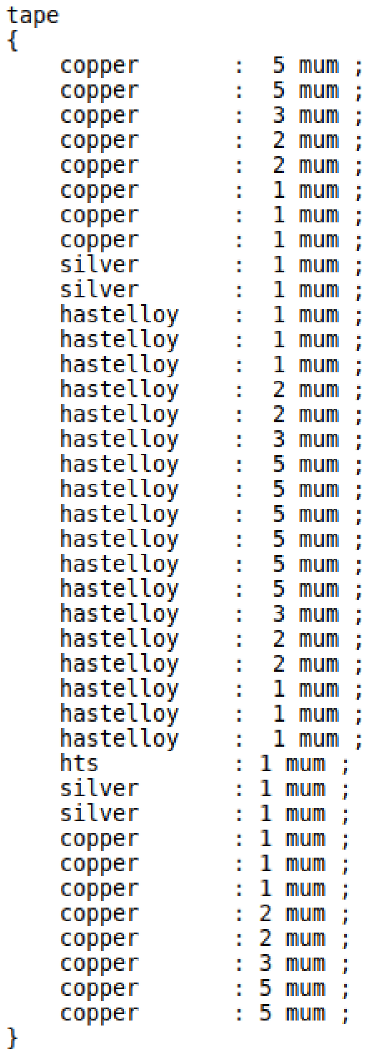


- The EM model feeds the thermal model via the **AC loss and the magnetic field**
- The thermal model feeds the EM model via the **temperature**

THERMAL MODEL

- Implementation in Belfem: the convergence is still very robust (39 layers, fine mesh)
- Could not model as many layers in GetDP…





```
tape
{
    copper        :  5 mum ;
    copper        :  5 mum ;
    copper        :  3 mum ;
    copper        :  2 mum ;
    copper        :  2 mum ;
    copper        :  1 mum ;
    copper        :  1 mum ;
    copper        :  1 mum ;
    silver        :  1 mum ;
    silver        :  1 mum ;
    hastelloy     :  1 mum ;
    hastelloy     :  1 mum ;
    hastelloy     :  1 mum ;
    hastelloy     :  2 mum ;
    hastelloy     :  2 mum ;
    hastelloy     :  3 mum ;
    hastelloy     :  5 mum ;
    hastelloy     :  5 mum ;
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    hastelloy     :  5 mum ;
    hastelloy     :  5 mum ;
    hastelloy     :  5 mum ;
    hastelloy     :  3 mum ;
    hastelloy     :  2 mum ;
    hastelloy     :  2 mum ;
    hastelloy     :  1 mum ;
    hastelloy     :  1 mum ;
    hastelloy     :  1 mum ;
    hts           : 1 mum ;
    silver        : 1 mum ;
    silver        : 1 mum ;
    copper        : 1 mum ;
    copper        : 1 mum ;
    copper        : 1 mum ;
    copper        : 2 mum ;
    copper        : 2 mum ;
    copper        : 3 mum ;
    copper        : 5 mum ;
    copper        : 5 mum ;
}
```
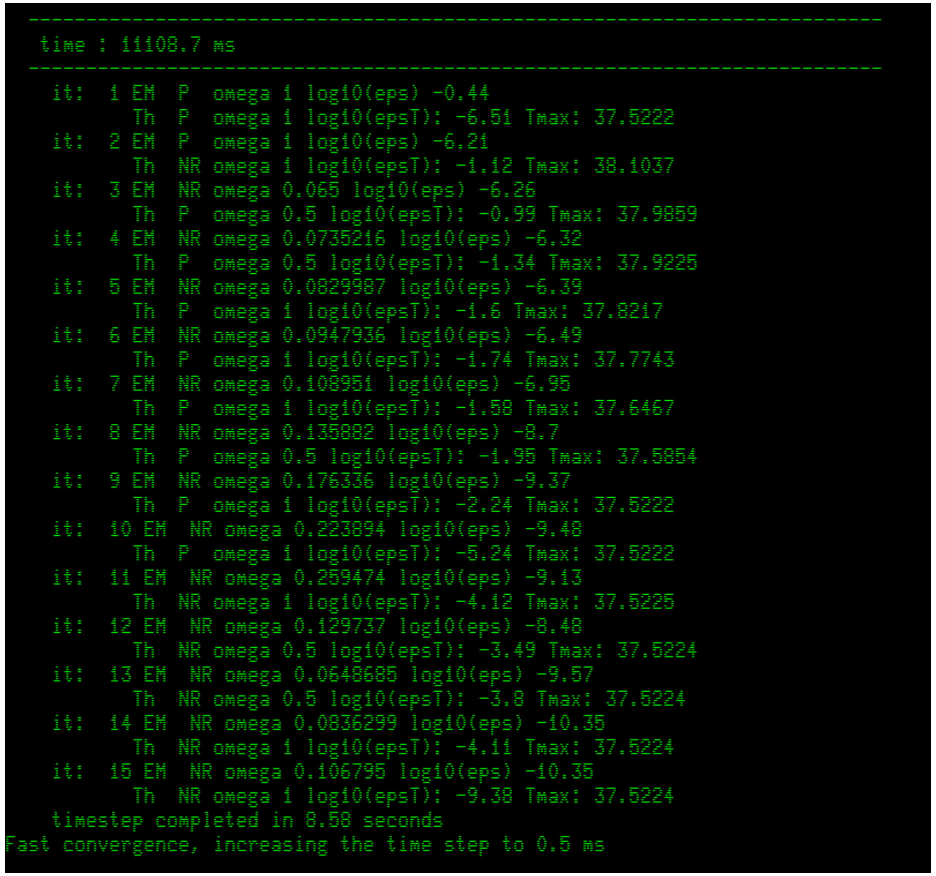
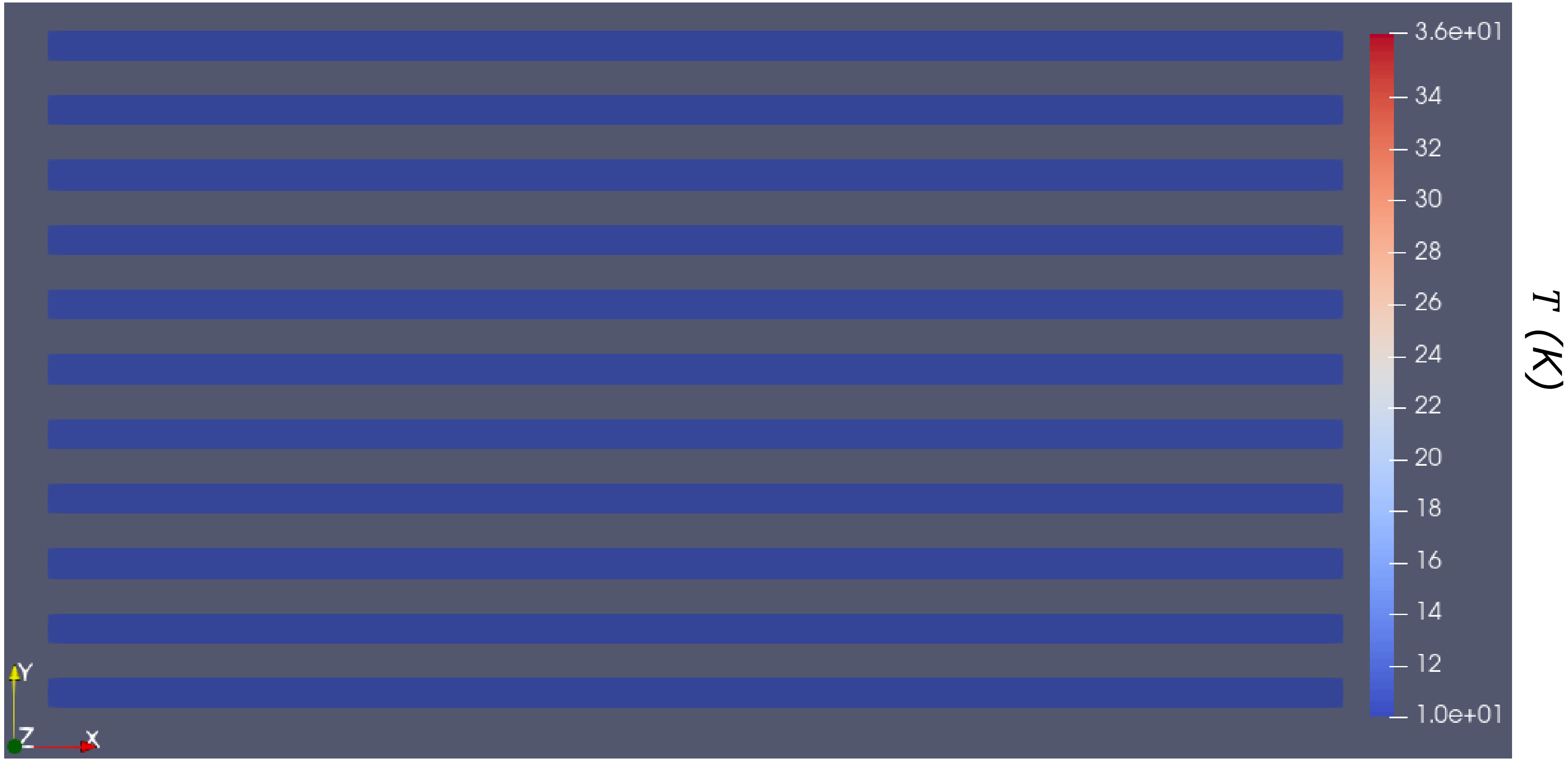



```
  ------------------------------------------------------------------------
   time : 11108.7 ms
  ------------------------------------------------------------------------
    it:  1 EM  P  omega 1 log10(eps) -0.44
               Th  P  omega 1 log10(epsT): -6.51 Tmax: 37.5222
    it:  2 EM  P  omega 1 log10(eps) -6.21
               Th  NR omega 1 log10(epsT): -1.12 Tmax: 38.1037
    it:  3 EM  NR omega 0.065 log10(eps) -6.26
               Th  P  omega 0.5 log10(epsT): -0.99 Tmax: 37.9859
    it:  4 EM  NR omega 0.0735216 log10(eps) -6.32
               Th  P  omega 0.5 log10(epsT): -1.34 Tmax: 37.9225
    it:  5 EM  NR omega 0.0829987 log10(eps) -6.39
               Th  P  omega 1 log10(epsT): -1.6 Tmax: 37.8217
    it:  6 EM  NR omega 0.0947936 log10(eps) -6.49
               Th  P  omega 1 log10(epsT): -1.74 Tmax: 37.7743
    it:  7 EM  NR omega 0.108951 log10(eps) -6.95
               Th  P  omega 1 log10(epsT): -1.58 Tmax: 37.6467
    it:  8 EM  NR omega 0.135882 log10(eps) -8.7
               Th  P  omega 0.5 log10(epsT): -1.95 Tmax: 37.5854
    it:  9 EM  NR omega 0.176336 log10(eps) -9.37
               Th  P  omega 1 log10(epsT): -2.24 Tmax: 37.5222
    it:  10 EM  NR omega 0.223894 log10(eps) -9.48
               Th  P  omega 1 log10(epsT): -5.24 Tmax: 37.5222
    it:  11 EM  NR omega 0.259474 log10(eps) -9.13
               Th  NR omega 1 log10(epsT): -4.12 Tmax: 37.5225
    it:  12 EM  NR omega 0.129737 log10(eps) -8.48
               Th  NR omega 0.5 log10(epsT): -3.49 Tmax: 37.5224
    it:  13 EM  NR omega 0.0648685 log10(eps) -9.57
               Th  NR omega 0.5 log10(epsT): -3.8 Tmax: 37.5224
    it:  14 EM  NR omega 0.0836299 log10(eps) -10.35
               Th  NR omega 1 log10(epsT): -4.11 Tmax: 37.5224
    it:  15 EM  NR omega 0.106795 log10(eps) -10.35
               Th  NR omega 1 log10(epsT): -9.38 Tmax: 37.5224
    timestep completed in 8.58 seconds
Fast convergence, increasing the time step to 0.5 ms
```

THERMAL MODEL

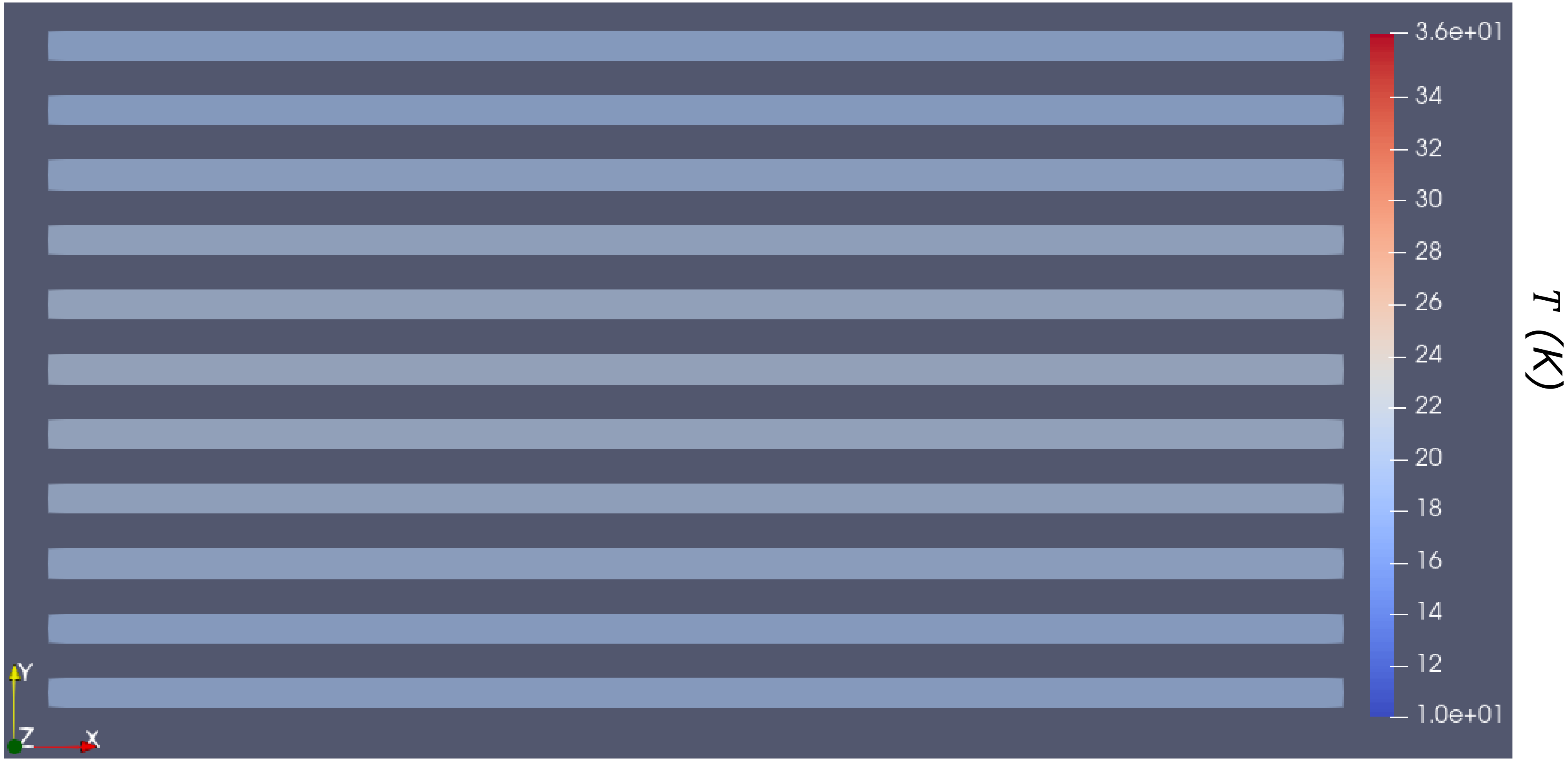
- First heating simulation (only intra-tape heating for now)



$t = 0$ s

THERMAL MODEL

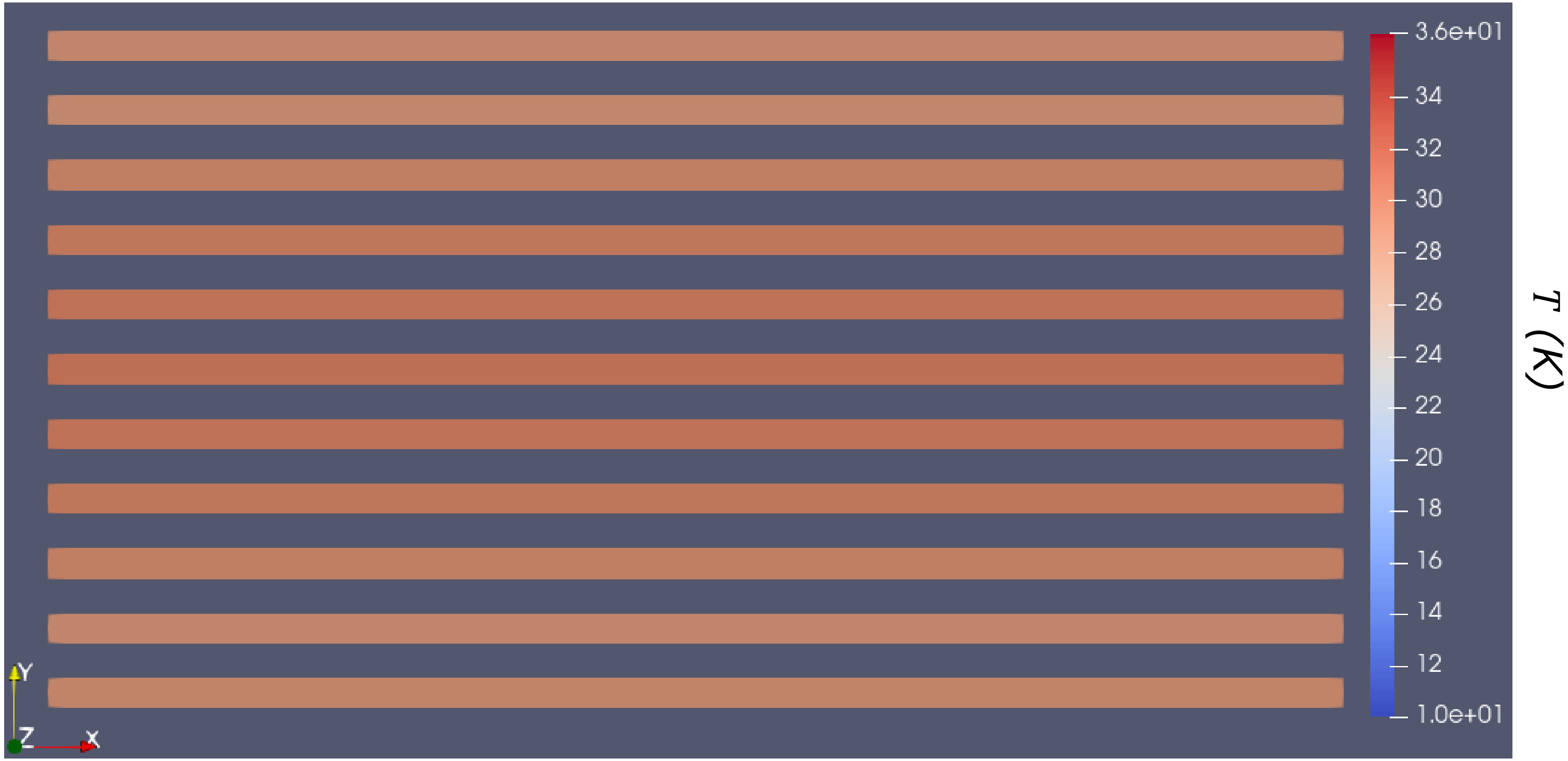
- First heating simulation (only intra-tape heating for now)



$t = 1$ s

THERMAL MODEL

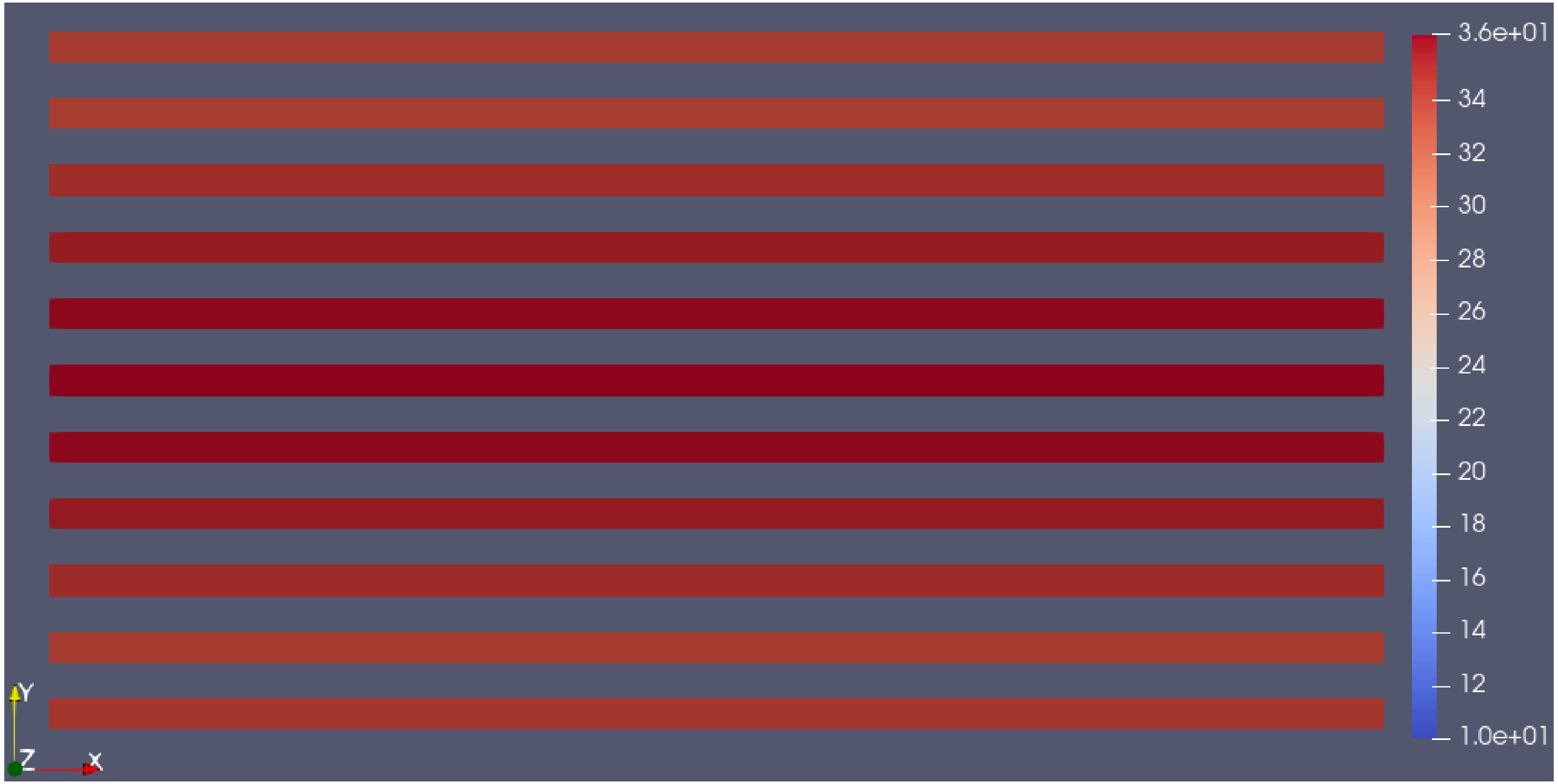
- First heating simulation (only intra-tape heating for now)



$t = 5\ \text{s}$

THERMAL MODEL

- First heating simulation (only intra-tape heating for now)



$T\ (K)$

$t = 10\ \mathrm{s}$

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NEXT STEPS

- 2-D axisym
- Model ferromagnetic materials with ϕ (Faraday)
- 3-D
- Inter-tape current sharing and interface resistance with thin-shell (current testing in GetDP and COMSOL)
- Modeling of thermal interaction with exterior
- Down the road (end of this year maybe): Simulate quench and current transfer in a CORC cable