# Symmetries and Interactions from Lattice QCD

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NORTH

LVX

**IBERTA** 

HAPEL H

CIPANP18, Palm Springs, CA May 30, 2018



# The Standard Model and Beyond



Dark matter





# The Standard Model and Beyond

#### Matter-antimatter asymmetry

Dark matter



UX

nEDM at PS

# Lattice QCD

- Numerical solution to QCD:
  - Non-perturbative formulation of QCD in discretized, finite spacetime
  - Currently our only reliable technique for solving QCD at low energies
  - All uncertainties are quantifiable and may be systematically removed
    - Extrapolations to continuum, infinite volume, physical pion mass





#### How can a solution of QCD teach us about new physics?

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I. Look for discrepancies between the SM and experiment: g<sub>A</sub>, proton radius (see talk by S. Syritsyn, Sat. 17:50), muon g-2 (see talk by A. Meyer, Weds. 15:00)



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I. Look for discrepancies between the SM and experiment: g<sub>A</sub>, proton radius (see talk by S. Syritsyn, Sat. 17:50), muon g-2 (see talk by A. Meyer, Weds. 15:00)



2. Match new physics model at high energies to nuclear experiments: 0vββ, nucleon/ nuclear EDM (see talk by S. Syritsyn, Tues. 17:50), DM searches (see talk by E. Rinaldi, Tues. 15:00)





### What depends on $g_A$ ?



• Free neutron lifetime



 $\bar{\nu}_e$ 

- Free neutron lifetime
- Nuclear beta decay

10

• Free neutron lifetime

 $\pi$ 

- Nuclear beta decay
- Nuclear force

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  Big Bang nucleosynthesis
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Big Bang nucleosynthesis

• Stellar processes

- Free neutron lifetime
- Nuclear beta decay
- Nuclear force

Big Bang nucleosynthesis

Stellar processes

#### Very precisely measured experimentally





Figure: A. P. Serebrov, E. A. Kolomensky, A. K. Fomin, I. A. Krasnoschekova, A. V. Vassiljev, D. M. Prudnikov, I. V. Shoka, A. V. Chechkin, M. E. Chaikovskiy, V. E. Varlamov, S. N. Ivanov, A. N. Pirozhkov, P. Geltenbort, O. Zimmer, T. Jenke, M. Van der Grinten, M. Tucker, arXiv:1712.05663



NIST

Figure: A. P. Serebrov, E. A. Kolomensky, A. K. Fomin, I. A. Krasnoschekova, A. V. Vassiljev, D. M. Prudnikov, I. V. Shoka, A. V. Chechkin, M. E. Chaikovskiy, V. E. Varlamov, S. N. Ivanov, A. N. Pirozhkov, P. Geltenbort, O. Zimmer, T. Jenke, M. Van der Grinten, M. Tucker, arXiv: 1712.05663



NIST

Pirozhkov, P. Geltenbort, O. Zimmer, T. Jenke, M. Van der Grinten, M. Tucker, arXiv:1712.05663



NIST

• Look for new physics/resolve lifetime puzzle

- Look for new physics/resolve lifetime puzzle
- in-medium effects/axial form factors (see talk by R. Gupta, Thurs. 16:40)
  - neutrinoless double beta decay (see recent work by NPLQCD), long baseline neutrino experiments



- Look for new physics/resolve lifetime puzzle
- in-medium effects/axial form factors (see talk by R. Gupta, Thurs. 16:40)
  - neutrinoless double beta decay (see recent work by NPLQCD), long baseline neutrino experiments
- Build quantitative connection between QCD & nuclear physics
  - g<sub>A</sub> should be a benchmark
    - one of the simplest hadron structure matrix elements









# g<sub>A</sub>:LQCD results



# g<sub>A</sub>: LQCD results

#### PHYSICAL REVIEW D

overing particles, fields, gravitation, and cosmology

LHPC0	ighlights Recent Accepted Authors Referees Search Press About a	<u> </u>
CLS1	Access by Law	-
<sup>†</sup> QCDSF1	Axial, scalar, and tensor charges of the nucleon from $2+1+1$ -	- 1
QCDSF1	Tanmov Bhattacharva, Vincenzo Cirigliano, Saul D. Cohen, Raian Gupta, Huev-Wen Lin, and Boram Yoon	-
<sup>†</sup> RQCD1	(Precision Neutron Decay Matrix Elements (PNDME) Collaboration) Phys. Rev. D <b>94</b> , 054508 – Published 19 September 2016	
ETMC1	atic effects have been grossly underestimated. To gain	-
PNDME1	a better understanding of how the various sources of er- rors contribute and to reduce the overall uncertainty to	
ETMC1	O(2%) will require at least $O(200,000)$ measurements	-
CLS1	on the seven ensembles at different $a$ and $M_{\pi}$ used in	-
	this study and the analysis of one additional ensemble at $a = 0.06$ fm and $M_{\odot} = 1.35$ MeV. Increasing the statistics	
PDG1	by a factor of four will reduce the errors in the data with	-
	the largest $t_{sep}$ we have analyzed and thus improve the	ı —
	$t_{\rm sep} \to \infty$ estimates. Adding the point at the physical	35
	quark mass and the smallest lattice spacing $a = 0.06$ fm, will further constrain the chiral fit. This level of precision	
	is achievable with the next generation of leadership-class	
	computing resources.	



• Monte Carlo noise/sign problem (nucleons) signal/noise ~  $e^{-A(m_N-3/2m_\pi)t}$ 



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- Pion mass extrapolation
- Discretization
- Finite volume
- Excited state contamination



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All the more difficult with noisy data!

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Challenges

Excited state contamination



with noisy data!

similar techniques used in: A.J. Chambers et al. (2014,2015) M.J. Savage et al. (2016)



New calculation technique based on Feynman-Hellman theorem (C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, A. Walker-Loud 2016)



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  - easier to analyze (improved systematics)
  - lower computational cost
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  - can be reused for matrix elements between different states (g<sub>A</sub> quenching)
  - New mixed action: DWF on HISQ (E. Berkowitz, et al 2017)
    - smaller discretization effects, better chiral symmetry
    - gradient flow: improved statistics

similar techniques used in: A.J. Chambers et al. (2014,2015) M.J. Savage et al. (2016)

#### Improvements





#### $g_A^{LQCD} = 1.271 \pm 0.013$ Nature, May 30, 2018

C.C. Chang, A.N., E. Rinaldi, E. Berkowitz, N. Garron, D. Brantley, H. Monge-Camacho, C. Monahan, C. Bouchard, M.A. Clark, B. Joo, T. Kurth, K. Orginos, P. Vranas, A. Walker-Loud



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Can already place stronger constraints on right-handed BSM currents than collider experiments

0.04

Alioli, S., Cirigliano, V., Dekens, W., de Vries, J., and Mereghetti, E. JHEP 05, 086 (2017)

# Neutrinoless double beta decay



• Majorana:  $v = \overline{v}$ 



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- Could be verified through observation of simultaneous double beta decay with no neutrino emission
  - Lepton number violating process
  - Lepton number asymmetry (in early Universe) can be converted to baryon number asymmetry





- Anything not forbidden by symmetry should occur in nature
- Why are neutrinos so light?



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### Seesaw Mechanism

 $\left(\begin{array}{cc} M_L & M_D \\ M_D & M_R \end{array}\right)$ 

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### Seesaw Mechanism

 $\left(\begin{array}{cc} 0 & M_D \\ M_D & M_R \end{array}\right)$ 

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### Seesaw Mechanism

 $\left(\begin{array}{cc} 0 & M_D \\ M_D & M_R \end{array}\right)$ 

 $m_l \sim M_D^2 / M_R$ 

- Anything not forbidden by symmetry should occur in nature
- Why are neutrinos so light?





### Seesaw Mechanism

 $\left(\begin{array}{cc} 0 & M_D \\ M_D & M_R \end{array}\right)$ 

 $m_l \sim M_D^2/M_R$ 

 $m_h \sim M_R$ 



## Experiment

Majorana <sup>76</sup>Ge Gerda

76**Ge** 

nEXO

136Xe







From NSAC Long Range Plan 2015

D

D



Capozzi, Valentino, Lisi, Marrone, Melchiorri, Palazzo Phys.Rev. D95 (2017) no.9, 096014

































## Relating Theory to Experiment



# Relating Theory to Experiment



# Relating Theory to Experiment


#### Relating Theory to Experiment





Long-range





Long-range





























$$Prezeau, Ramsey-Musolf, Vogel (2003)$$

$$O(M_R, \theta, \cdots)$$

$$O(M_R, \theta,$$



![](_page_81_Picture_1.jpeg)

Prezeau, Ramsey-Musolf, Vogel (2003)

![](_page_82_Picture_0.jpeg)

![](_page_82_Picture_1.jpeg)

![](_page_83_Picture_0.jpeg)

![](_page_83_Picture_1.jpeg)

![](_page_83_Picture_2.jpeg)

Unknown!

![](_page_84_Picture_0.jpeg)

![](_page_84_Figure_1.jpeg)

![](_page_85_Picture_0.jpeg)

![](_page_85_Figure_1.jpeg)

![](_page_86_Picture_0.jpeg)

![](_page_86_Picture_1.jpeg)

![](_page_86_Figure_2.jpeg)

![](_page_87_Picture_0.jpeg)

![](_page_87_Figure_1.jpeg)

![](_page_88_Figure_0.jpeg)

![](_page_89_Figure_0.jpeg)

![](_page_90_Figure_0.jpeg)

![](_page_91_Picture_0.jpeg)

![](_page_92_Figure_0.jpeg)

## Leading order short-range:

Don't need to calculate full nn  $\rightarrow$  pp transition from LQCD (difficult)!

- I. With LQCD, calculate  $\pi \rightarrow \pi^+$  transition
- 2. Use EFT to determine  $nn \rightarrow pp$  matrix element

![](_page_93_Figure_0.jpeg)

![](_page_94_Figure_0.jpeg)

![](_page_95_Figure_0.jpeg)

![](_page_96_Figure_0.jpeg)

Agrees to 2σ with: V. Cirigliano, W. Dekens, M. Graesser, E. Mereghetti Phys.Lett. B769 (2017) 460-464

### Summary

- LQCD is a necessary step toward reliably connecting experimental signals to the SM/BSM
- Nucleon axial charge
  - Finally achieved 1% precision with LQCD!
  - Statistics dominated: can we resolve neutron lifetime puzzle?
- Leading short-range contribution to  $0\nu\beta\beta$ 
  - Complete LQCD calculation at the physical point
  - To do: Plug the results into your favorite many-body calculation!
  - Future: full NNLO calculation including two-nucleon contact

#### Very similar to calculation of hadronic parity violation (see talk by A.Walker-Loud Sat. 17:30)

NPDGamma

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1 +

 $\mathcal{N}$ 

- RIKEN/LBL: C.C. Chang
- RIKEN/BNL: E. Rinaldi
- NERSC: T. Kurth
- Liverpool: N. Garron
- UW/INT C. Monahan
- nVidia: M.A. Clark
- JLab: B. Joo
- WM/JLab: K. Orginos
- CCNY: B. Tiburzi
- LBL/UCB: A. Walker-Loud
- Glasgow: C. Bouchard
- LLNL: P. Vranas

- Jülich: E. Berkowitz
- WM/LBL: D. Brantley, H. Monge-Camacho

![](_page_99_Picture_14.jpeg)

#### Lattice Ensembles

HISQ ensembles									
<i>a</i> [ <i>fm</i> ] :	m <sub>π</sub> [MeV] <b>310</b>	220	135						
0.15	$16^3 \times 48, m_{\pi}L \sim 3.78$	$24^3 \times 48, m_{\pi}L \sim 3.99$	$32^3 \times 48, m_{\pi}L \sim 3.25$						
0.12		$24^3 \times 64, m_{\pi}L \sim 3.22$							
0.12	$24^3 \times 64, m_{\pi}L \sim 4.54$	$32^3 \times 64, m_{\pi}L \sim 4.29$	$48^3 \times 64, m_{\pi}L \sim 3.91$						
0.12		$40^3 \times 64, m_{\pi}L \sim 5.36$							
0.09	$32^3 \times 96, m_{\pi}L \sim 4.50$	$48^3 \times 96, m_{\pi}L \sim 4.73$							
0.00									

- DWF on HISQ
- Gradient flow method for smearing configs
  - $m_{res} < 0.1 m_{\ell}$  for moderate  $L_5$
  - dampens unphysical oscillations
  - noise reduction

MILC Collaboration Phys. Rev. D87 (2013) 054505

Narayanan, Neuberger (2006), Luscher (2010)

Callat arXiv:1701.07559

# Future: two-nucleon contact

- Why calculate it?
  - Formally NNLO (Weinberg counting)
  - Weinberg often doesn't converge well, particularly in the <sup>1</sup>S<sub>0</sub> channel (see talk by V. Cirigliano)
  - LO contribution vanishes for some BSM models
- Two nucleon LQCD calculations much more difficult (see talks by S. Beane, J. Bulava?)

![](_page_101_Picture_6.jpeg)

![](_page_101_Picture_7.jpeg)

Cirigliano, V., Dekens, W., de Vries, J., Mereghetti, E., Graesser, M., Pastore, S., van Kolck, U arXiv: 1802.10097

![](_page_101_Picture_9.jpeg)

$$\begin{split} \langle \pi^+ | \mathcal{O}^{V+} | \pi^- \rangle &= \frac{m_\pi^2}{f^2} C^{V+} \left[ 1 - \frac{16}{3} \frac{m_\pi^2}{(4\pi f)^2} \left( -\frac{1}{4} \log \frac{m_\pi^2}{\mu^2} + \frac{3}{4} \frac{m_{vs}^2}{m_\pi^2} \log \frac{m_{vs}^2}{\mu^2} + c^{V^+}(\mu) \right) \right], \\ \langle \pi^+ | \mathcal{O}^{LR} | \pi^- \rangle &= C^{LR} \left[ 1 - \frac{10}{3} \frac{m_\pi^2}{(4\pi f)^2} \left( -\frac{1}{5} \log \frac{m_\pi^2}{\mu^2} + \frac{6}{5} \frac{m_{vs}^2}{m_\pi^2} \log \frac{m_{vs}^2}{\mu^2} + c^{LR}(\mu) \right) \right], \\ \langle \pi^+ | \mathcal{O}^{S+} | \pi^- \rangle &= C^{S+} \left[ 1 - \frac{10}{3} \frac{m_\pi^2}{(4\pi f)^2} \left( -\frac{1}{5} \log \frac{m_\pi^2}{\mu^2} + \frac{6}{5} \frac{m_{vs}^2}{m_\pi^2} \log \frac{m_{vs}^2}{\mu^2} + \frac{6}{5} \frac{a^2 \Delta_I}{m_\pi^2} \left[ \log \frac{m_\pi^2}{\mu^2} + 1 \right] + c^{S+}(\mu) \right) \right] \end{split}$$

e hairpin only seems to infect the last matrix element. There is a corresponding enhancement of the finite vo ect, which can be obtained by the replacement

$$a^{2}\Delta_{I}\left[\log\frac{m_{\pi}^{2}}{\mu^{2}}+1\right] \longrightarrow a^{2}\Delta_{I}\frac{\partial}{\partial m_{\pi}^{2}}\left[4m_{\pi}^{2}\sum_{\vec{\nu}\neq\vec{0}}\frac{\mathsf{K}_{1}(m_{\pi}L|\vec{\nu}|)}{m_{\pi}L|\vec{\nu}|}\right] = -2a^{2}\Delta_{I}\sum_{\vec{\nu}\neq\vec{0}}\mathsf{K}_{0}(m_{\pi}L|\vec{\nu}|).$$

#### Contractions

• QCD interactions can mix colors below

the electroweak scale

• Must add color mixed versions of

Prezeau, Ramsey-Musolf, Vogel ops 1&2

$$\mathcal{O}_{1+}^{++} = \left(\bar{q}_{L}\tau^{-}\gamma^{\mu}q_{L}\right)\left[\bar{q}_{R}\tau^{-}\gamma_{\mu}q_{R}\right]$$
$$\mathcal{O}_{1+}^{++} = \left(\bar{q}_{L}\tau^{-}\gamma^{\mu}q_{L}\right)\left[\bar{q}_{R}\tau^{-}\gamma_{\mu}q_{R}\right)$$
$$\mathcal{O}_{2+}^{++} = \left(\bar{q}_{R}\tau^{-}q_{L}\right)\left[\bar{q}_{R}\tau^{-}q_{L}\right] + \left(\bar{q}_{L}\tau^{-}q_{R}\right)\left[\bar{q}_{L}\tau^{-}q_{R}\right]$$
$$\mathcal{O}_{2+}^{'++} = \left(\bar{q}_{R}\tau^{-}q_{L}\right)\left[\bar{q}_{R}\tau^{-}q_{L}\right) + \left(\bar{q}_{L}\tau^{-}q_{R}\right)\left[\bar{q}_{L}\tau^{-}q_{R}\right)$$
$$\mathcal{O}_{3+}^{++} = \left(\bar{q}_{L}\tau^{-}\gamma^{\mu}q_{L}\right)\left[\bar{q}_{L}\tau^{-}\gamma_{\mu}q_{L}\right] + \left(\bar{q}_{R}\tau^{-}\gamma^{\mu}q_{R}\right)\left[\bar{q}_{R}\tau^{-}\gamma_{\mu}q_{R}\right]$$

![](_page_103_Picture_6.jpeg)

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$$\mathcal{O}_{1+}^{++} = \left(\bar{q}_L \tau^- \gamma^\mu q_L\right) \left[\bar{q}_R \tau^- \gamma_\mu q_R\right]$$
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![](_page_104_Picture_6.jpeg)

4	decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}^{\pm\pm,\mu}_{5+}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
V	$\pi\pi ee \text{ LO}$	<ul> <li>✓</li> </ul>	✓	X	X	X	X	X	X	X
	$\pi\pi ee$ NNLO	<b>√</b>	<b>√</b>	X	<b>√</b>	X	X	X	X	X
	$NN\pi ee$ LO	X	X	$\checkmark$	X	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	$NN\pi ee$ NLO	X	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	NNNNee LO	$\checkmark$	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

![](_page_105_Figure_1.jpeg)

- Nine operators:
  - $\pi \rightarrow \pi$ : only need parity even
  - Vector operators suppressed
     by m<sub>e</sub>
  - QCD interactions can mix colors below the electroweak scale: +2 ops

$$\begin{split} \mathcal{O}_{1+}^{ab} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}})(\bar{q}_{\mathrm{R}}\tau^{b}\gamma_{\mu}q_{\mathrm{R}}), \\ \mathcal{O}_{2\pm}^{ab} &= (\bar{q}_{\mathrm{R}}\tau^{a}q_{\mathrm{L}})(\bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}) \pm (\bar{q}_{\mathrm{L}}\tau^{a}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}}), \\ \mathcal{O}_{3\pm}^{ab} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}})(\bar{q}_{\mathrm{L}}\tau^{b}\gamma_{\mu}q_{\mathrm{L}}) \pm (\bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{R}}\tau^{b}\gamma_{\mu}q_{\mathrm{R}}), \\ \mathcal{O}_{4\pm}^{ab,\mu} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}} \mp \bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}} - \bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}), \\ \mathcal{O}_{5\pm}^{ab,\mu} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}} \pm \bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}} + \bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}). \end{split}$$

4	decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}^{\pm\pm,\mu}_{5+}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
Ľ	$\pi\pi ee \text{ LO}$	<b>\</b>	<b>√</b>	X	X	X	X	X	X	X
	$\pi\pi ee$ NNLO	<b>√</b>	<ul> <li>✓</li> </ul>	X	<b>√</b>	X	X	X	X	X
	$NN\pi ee$ LO	X	X	$\checkmark$	X	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	$NN\pi ee$ NLO	X	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	NNNNee LO	$\checkmark$	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

![](_page_106_Figure_1.jpeg)

- Nine operators:
  - $\pi \rightarrow \pi$ : only need parity even
  - Vector operators suppressed
     by m<sub>e</sub>
  - QCD interactions can mix colors below the electroweak scale: +2 ops

$$\begin{split} \mathcal{O}_{1+}^{ab} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}})(\bar{q}_{\mathrm{R}}\tau^{b}\gamma_{\mu}q_{\mathrm{R}}),\\ \mathcal{O}_{2\pm}^{ab} &= (\bar{q}_{\mathrm{R}}\tau^{a}q_{\mathrm{L}})(\bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}) \pm (\bar{q}_{\mathrm{L}}\tau^{a}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}}),\\ \mathcal{O}_{3\pm}^{ab} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}})(\bar{q}_{\mathrm{L}}\tau^{b}\gamma_{\mu}q_{\mathrm{L}}) \pm (\bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{R}}\tau^{b}\gamma_{\mu}q_{\mathrm{R}}),\\ \mathcal{O}_{4\pm}^{ab,\mu} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}} \mp \bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}} - \bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}),\\ \mathcal{O}_{5\pm}^{ab,\mu} &= (\bar{q}_{\mathrm{L}}\tau^{a}\gamma^{\mu}q_{\mathrm{L}} \pm \bar{q}_{\mathrm{R}}\tau^{a}\gamma^{\mu}q_{\mathrm{R}})(\bar{q}_{\mathrm{L}}\tau^{b}q_{\mathrm{R}} + \bar{q}_{\mathrm{R}}\tau^{b}q_{\mathrm{L}}). \end{split}$$

Ĭ	decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
V	$\pi\pi ee \text{ LO}$	<ul> <li>Image: A start of the start of</li></ul>		X	X	X	X	X	X	X
	$\pi\pi ee$ NNLO	<ul> <li>✓</li> </ul>	<ul> <li>✓</li> </ul>	X	<ul> <li>✓</li> </ul>	X	X	X	X	X
	$NN\pi ee$ LO	X	X	$\checkmark$	X	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	$NN\pi ee$ NLO	X	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
	NNNNee LO	$\checkmark$	$\checkmark$	X	$\checkmark$	X	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

![](_page_107_Figure_1.jpeg)

Left-right symmetric models

![](_page_107_Figure_3.jpeg)

![](_page_107_Figure_4.jpeg)

Prezeau, Ramsey-Musolf, Vogel (2003), Savage (1999)