Sensitivity study for the $^{12}C(\alpha,\gamma)^{16}O$ astrophysical reaction rate

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Outline

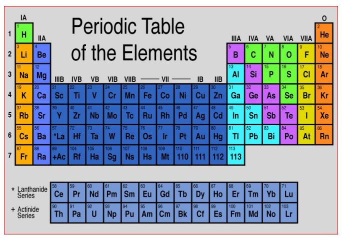
What is the origin of the chemical elements and how did they evolve?

Key reactions in stellar nucleosynthesis

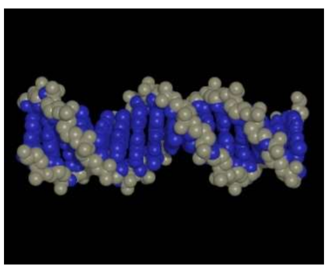
- **Importance of the $^{12}C(\alpha,\gamma)^{16}O$ reaction**
- **How do we predict the impact of new experiments?**
- **R-matrix approach**
- **Example: bubble chamber experiment at JLab**
- **Results & Summary**

$^{12}C(\alpha,\gamma)^{16}O$ Reaction

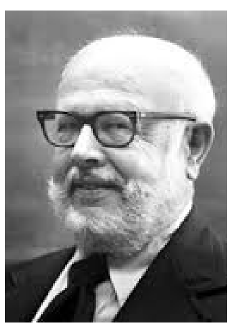
The *holy grail* of nuclear astrophysics

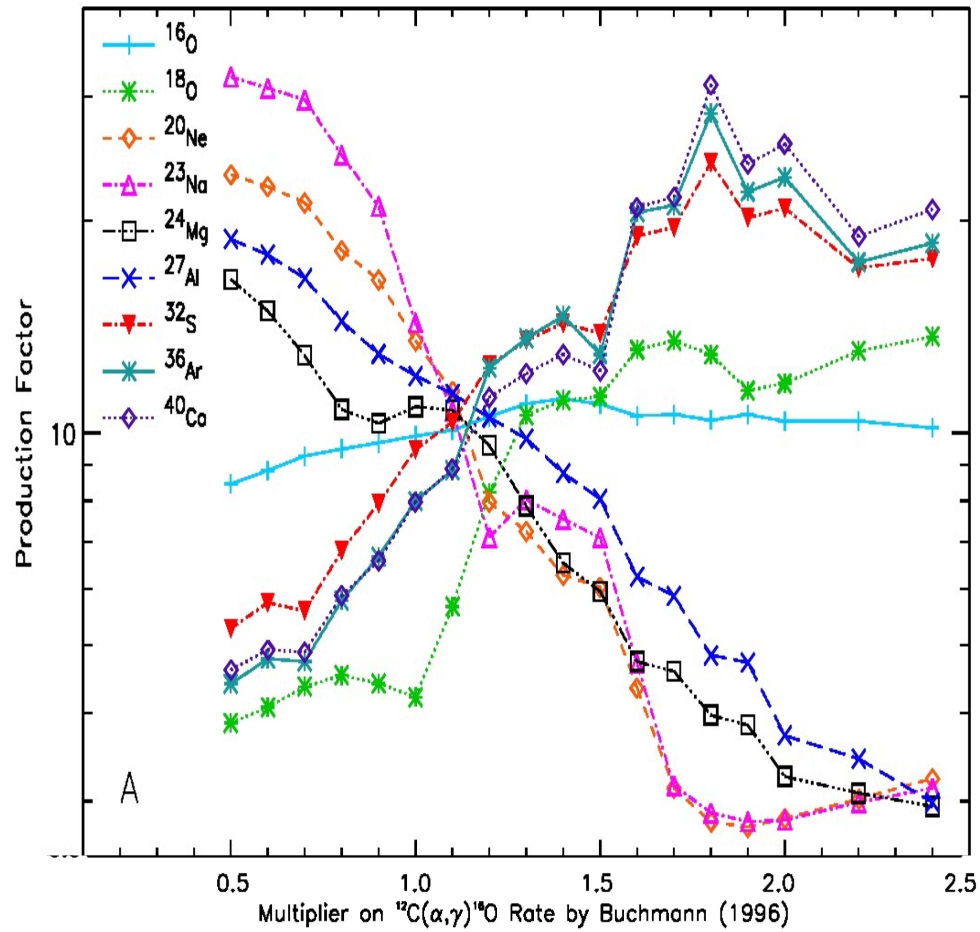


Affects the synthesis of most of the elements



Sets the $N(^{12}C)/N(^{16}O)$ ratio in the universe





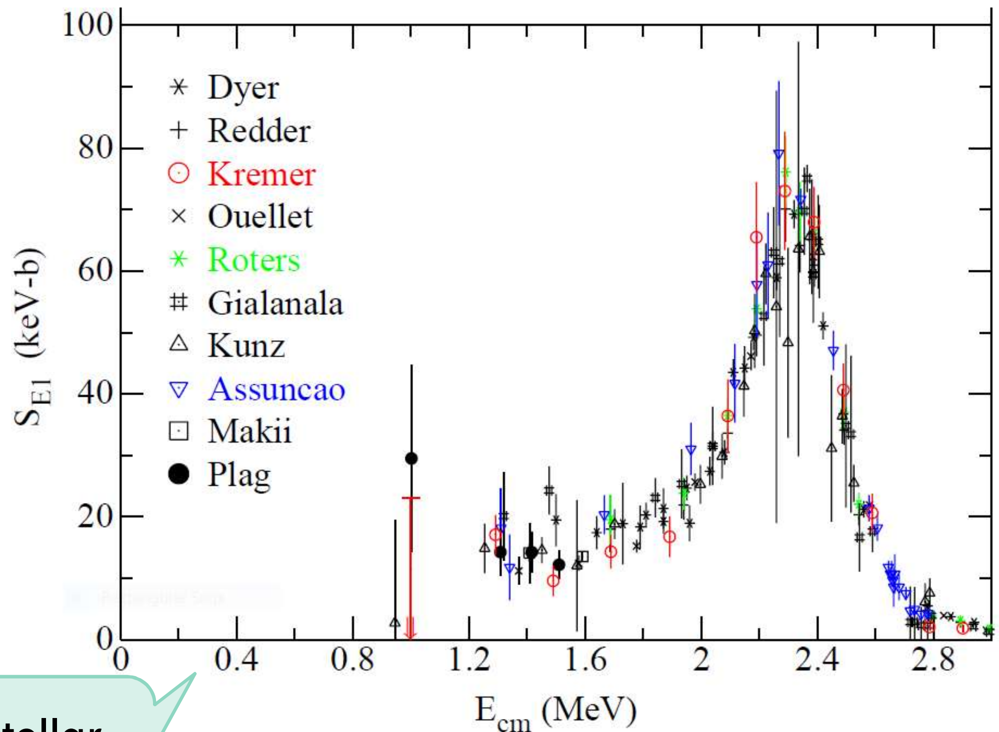
S. Woosley, A. Heger, Phys. Rep. **442** (2007) 269

ASTROPHYSICAL S-FACTOR $^{12}C(\alpha,\gamma)^{16}O$

S-Factor removes 1/E dependence and Coulomb barrier

$$S \equiv E_{CM}\sigma(\alpha,\gamma)e^{2\pi\eta}$$

$$\eta = \frac{1}{137} Z_\alpha Z_{^{12}C} \sqrt{\frac{m_{^{12}C\alpha}}{2E_{CM}}}$$



R-matrix Extrapolation to stellar helium burning at E = 300 keV

10 E1 and 6 E2 data sets used in this analysis

R-matrix approach

The nuclear wave function, $\Psi_{(E(J)}$, can be expanded in terms of a complete set of states, $X_{\lambda(J)}$

$$\Psi_{E(J)} = i\hbar^{1/2} e^{-i\phi_c} \Sigma_{\lambda\mu} A_{\lambda\mu} \Gamma^{1/2}_{\lambda\mu} X_{\lambda(J)}$$

$A_{\lambda\mu}$ relates the internal wave function and the observed resonances

$$\left(A^{-1}\right)_{\lambda\mu} = (E_\lambda - E)\,\delta_{\lambda\mu} - \xi_{\lambda\mu}$$

where E_λ are the level energies and ξ are given in terms of the shift factors, S_c the boundary condition constants, b_c and the penetration factors, P_c

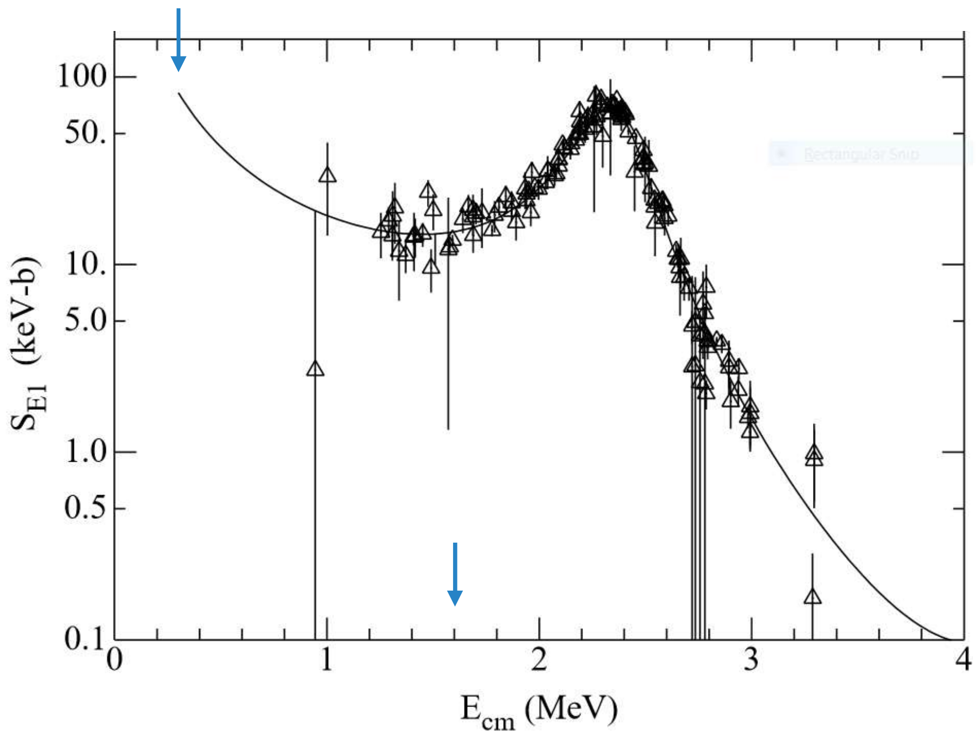
$$\xi_{\lambda\mu} = \Sigma_c[(S_c - b_c) + iP_c]\gamma_{\lambda c}\gamma_{\mu c}$$

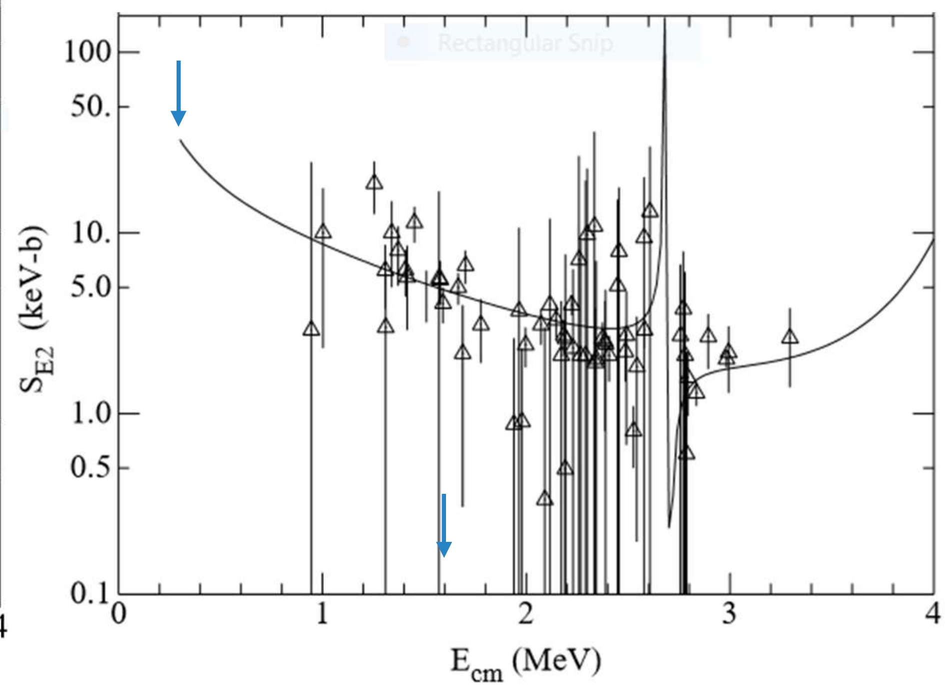
and $\gamma_{\lambda c}$ are the reduced width amplitudes. The collision matrix is given by

$$U^{lJ\mathcal{L}}_{\gamma\alpha} = ie^{-i\phi_l} \Sigma_{\lambda\mu} A_{\lambda\mu} \Gamma^{1/2}_{\lambda lJ} \Gamma^{1/2}_{\mu\gamma f lJ} + \left(\frac{8\pi(\mathcal{L}+1)}{(2J+1)\mathcal{L}\hbar v_\alpha}\right)^{1/2} \frac{k_\gamma^{\mathcal{L}+1/2}}{(2\mathcal{L}+1)!!} \left\langle \Psi_{f(J_f)} || H^{\mathcal{L}} || \psi_i \right\rangle$$

R. J. deBoer et al, RMP (2017); A. M. Lane, R. G. Thomas, RMP (1958)

E1 and E2 ground state S-factors





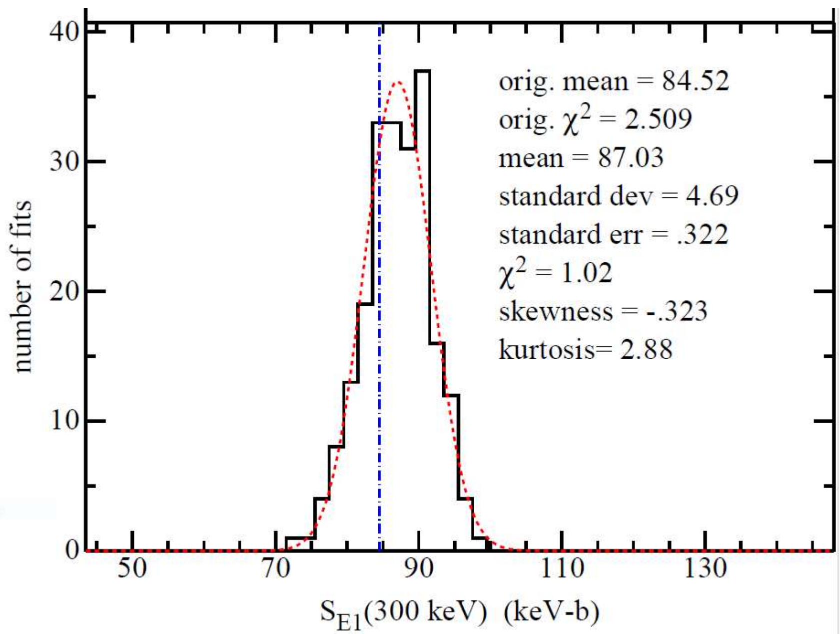
Method:

1. Fit the data, extrapolate to 300 keV
2. Generate pseudo-data from fit that is randomized according to a normal distribution within the statistical errors of data
3. Re-fit the pseudo data, extrapolate to 300 keV
4. Repeat step 2 and 3 about 100-250 times

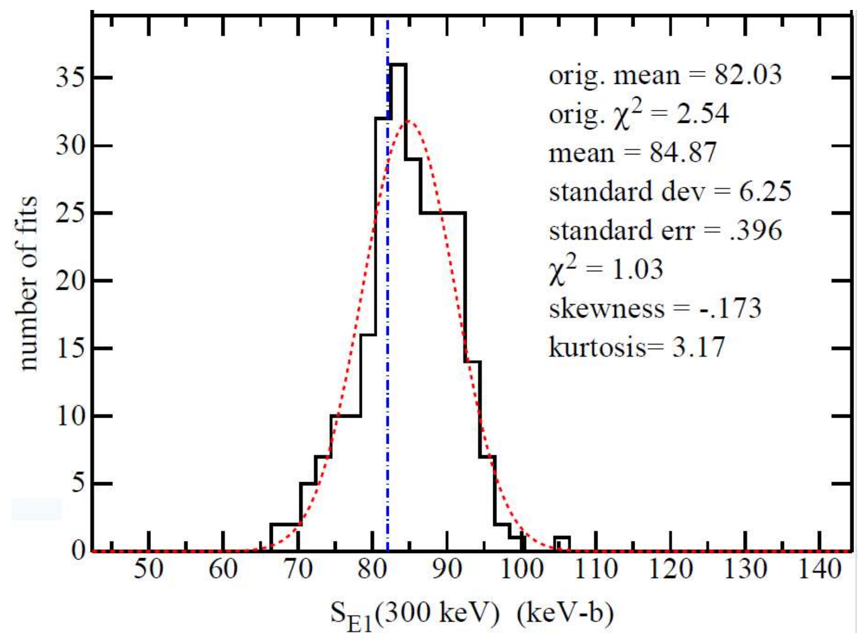
- E2 projection is about ½ that of E1.
- Better E2 data necessary
- Or, measure total cross sections

χ^2 Minimization vs L Maximization

χ^2 minimization



L maximization



$R_i=(f(x_i)-d_i)^2/\sigma_i^2$

$\chi^2=\Sigma R_i$

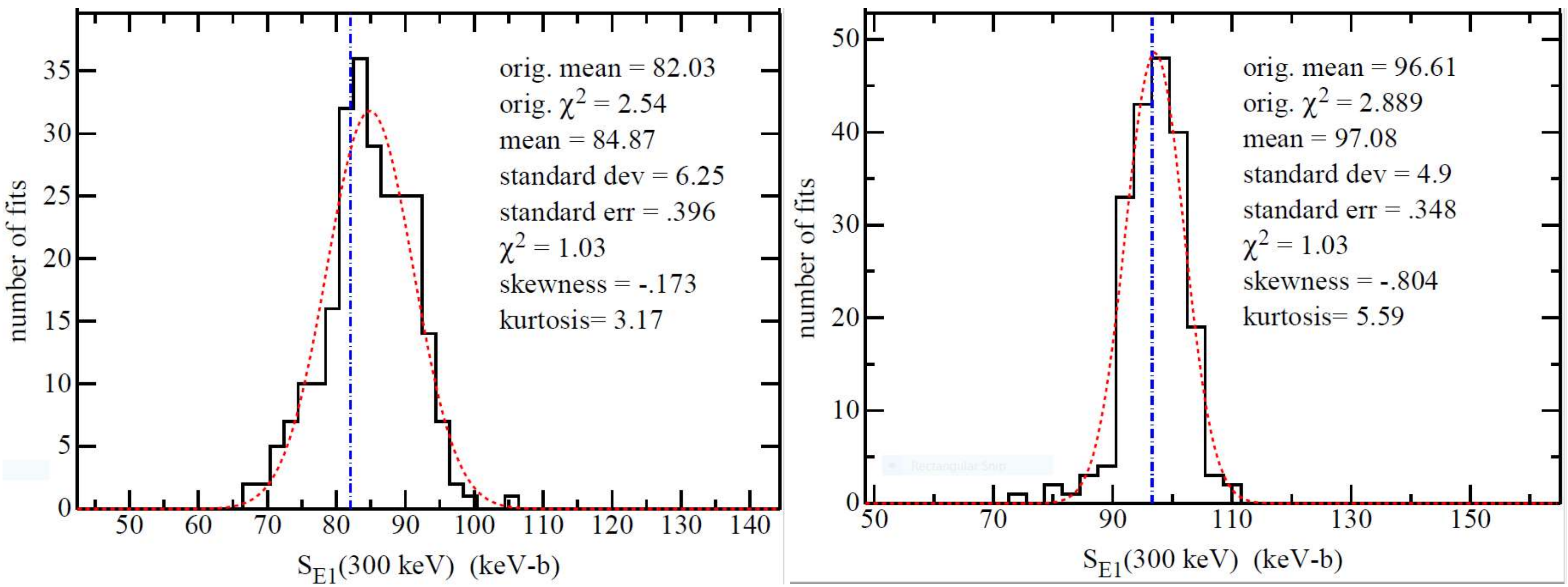
$L=\Sigma \ln((1-\exp(-R_i/2))/R_i)$

Sivia, Skilling, Data Analysis: A Bayesian Approach (2006)

Impact of low energy data

Existing E1 data

E > 1.6 MeV

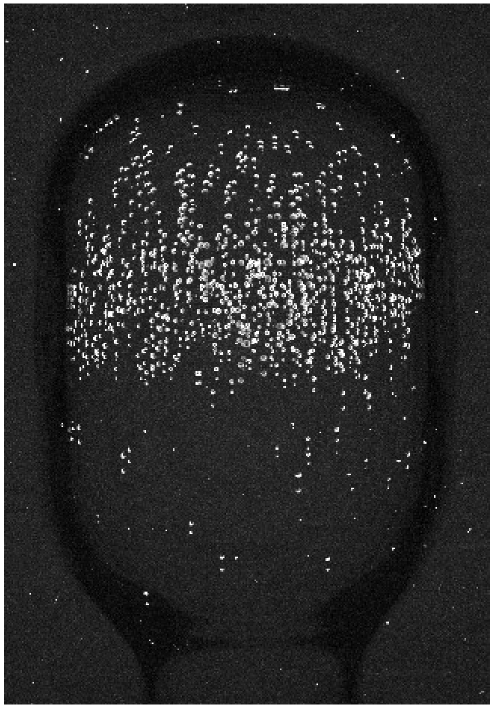


- Low energy data shift extrapolated value
- Can't rely totally on resonance data

JLab: Inverse Reaction + Bubble Chamber + Bremsstrahlung

$$\gamma + {}^{16}O \rightarrow {}^{12}C + \alpha$$

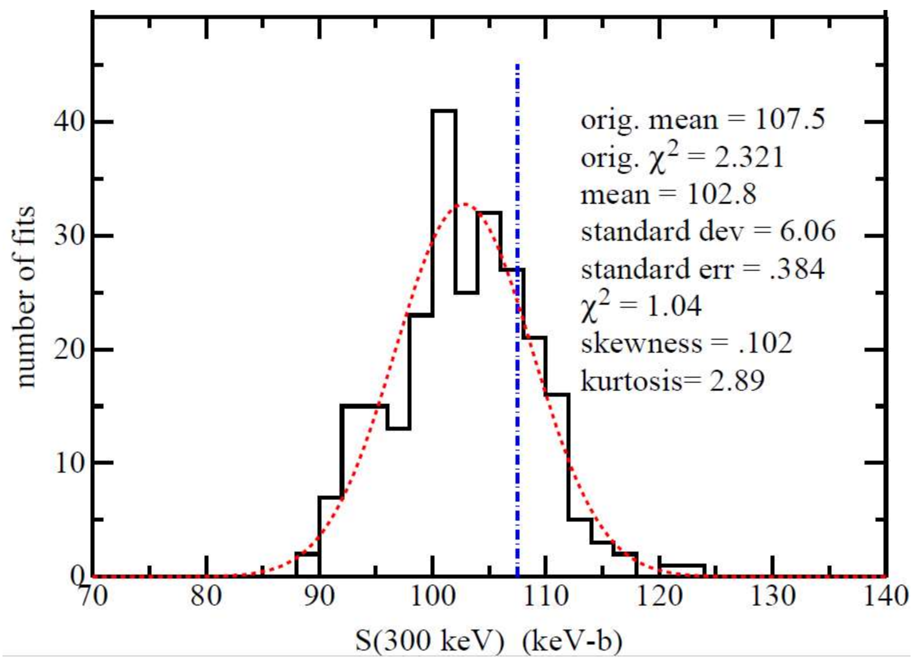
- Extra gain (>50) from inverse reaction
- Large target thickness ~ $x10^4$
- Solid Angle and Detector Efficiency = 100%
- High intensity bremsstrahlung beam
- Measures total ground state cross section



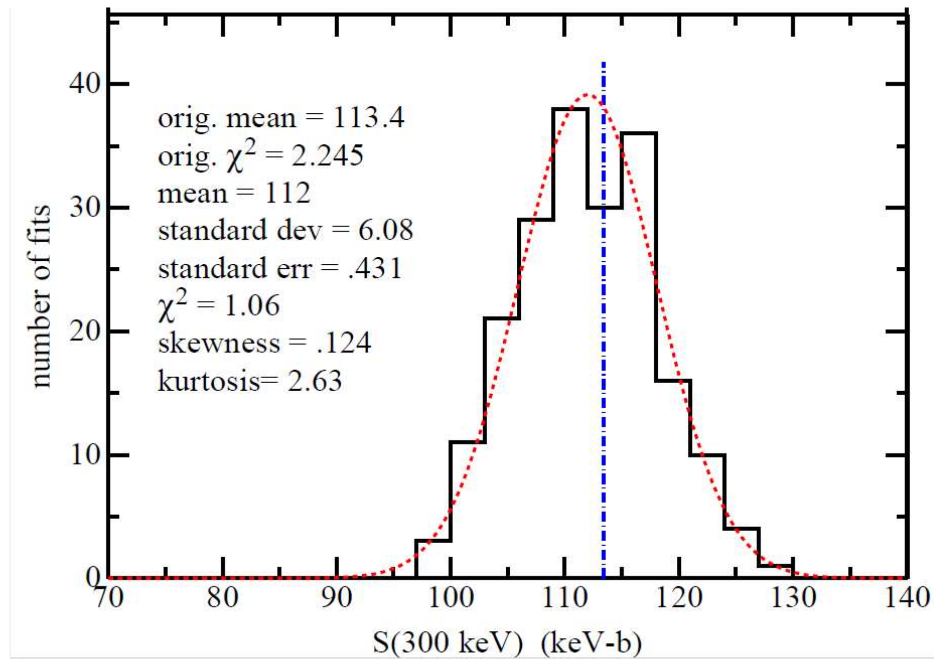
JLab experiment: R. Suleiman, E. Rehm, C. Ugalde *et al.*

Projections with and without expected JLab data

E1, E2 data



E1, E2 data + projected JLab



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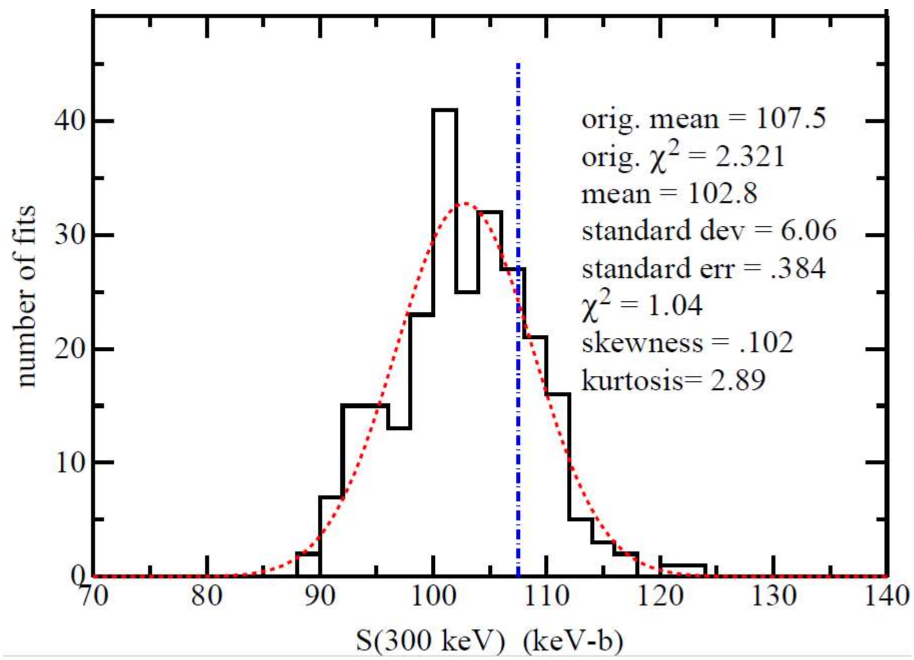
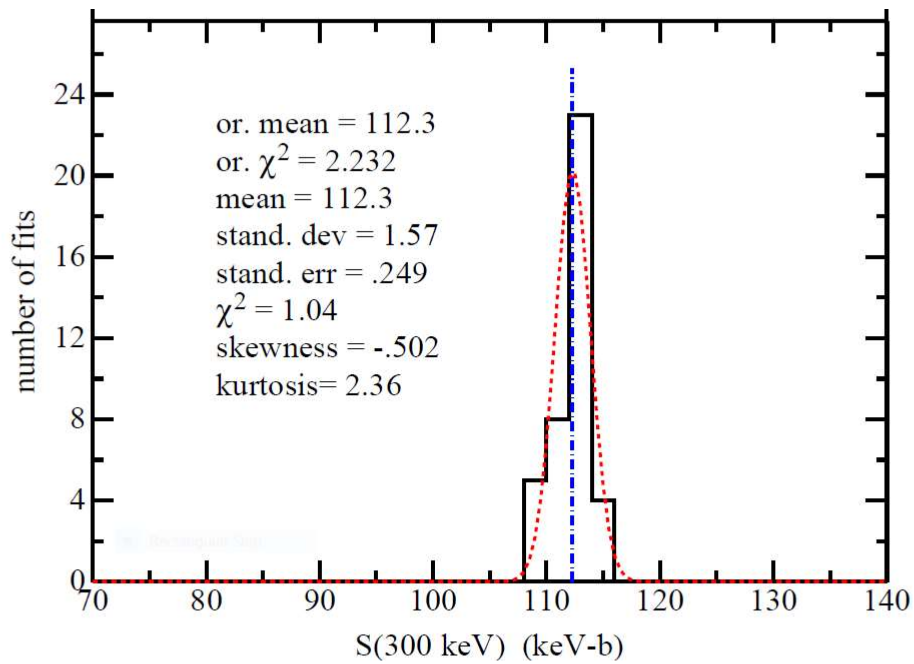
What if JLab uncertainties were 10x smaller?

E1, E2 data

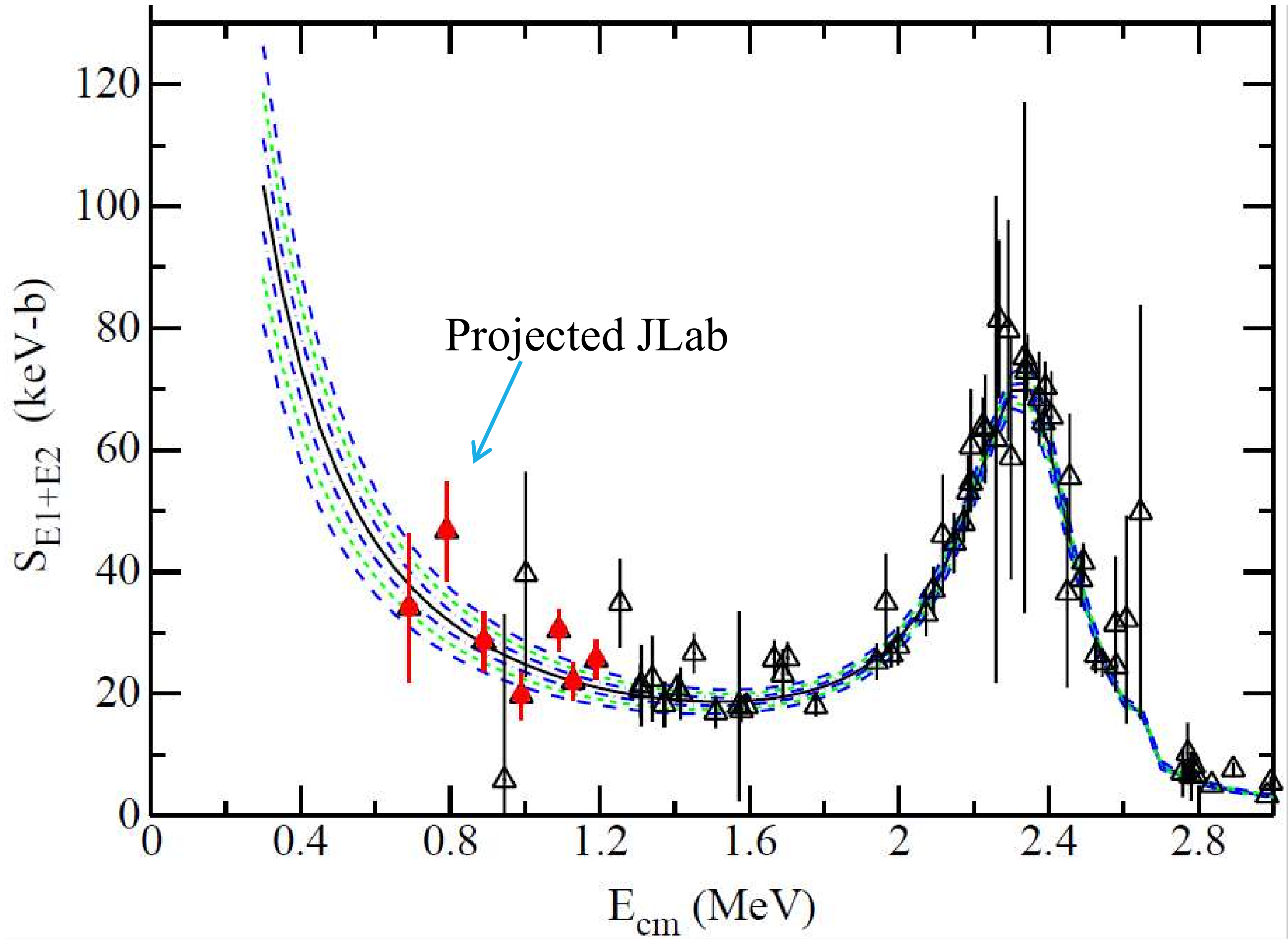


E1, E2 data + projected Jlab/10



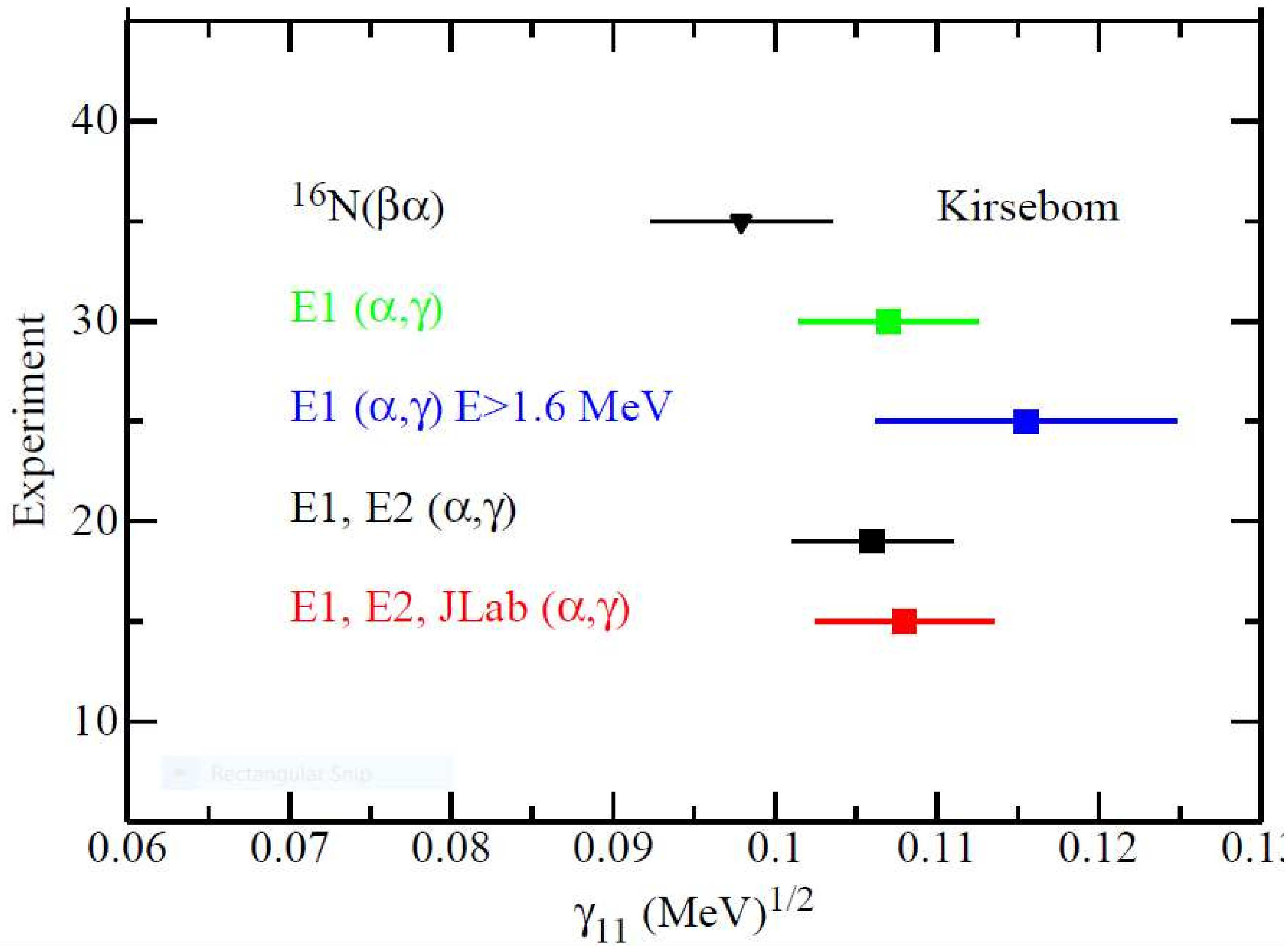
Seems to work!

1, 2, 3 sigma bands



JLab data likely will not impact statistical precision of extrapolation, but could impact extrapolated value

E1 bound state reduced width



O. Kirsebom et al arXiv 1804.02040
^{16}N β delayed α decay ISOLDE

a =6.5 fm

Model Dependence (?)

Channel radius

S(300 keV)	5.5 fm	6.5 fm
E1	82.0(6.3)	92.2(6.9)
E2	34.1(2.5)	36.8(4.4)
E1, E2	107.5(6.1)	131.6(6.3)

Summary

(α,γ) and ^{16}N decay → consistent E1 bound state reduced widths

Statistical precision remarkably small, but large χ^2 and model dependence -> more work

Low energy data impact S(300 keV) and γ_{11}, and are important for setting phase

JLab experiment likely will not impact statistical precision, but it provides new systematics, total cross section and lower energy data down to 690 keV

Similar approach could be applied to other experiments