Light-Cone Physics and Large-Momentum Effective Theory

Yong Zhao
Massachusetts Institute of Technology

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Outline

- **Hadron structure and the parton model**
  - Infinite momentum frame and the light-cone
  - First-principle calculation of parton physics
- **Large-momentum effective theory**
  - A novel approach to calculate light-cone observables
- **Parton physics from lattice QCD**
  - Gluon helicity
  - Parton distribution function

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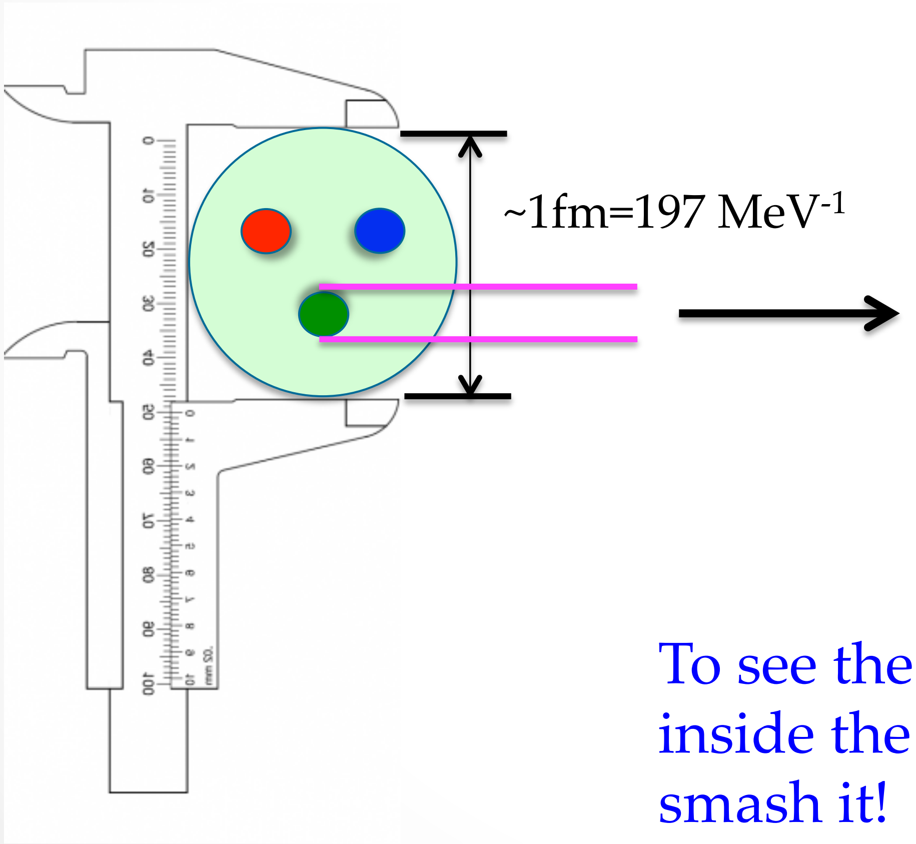
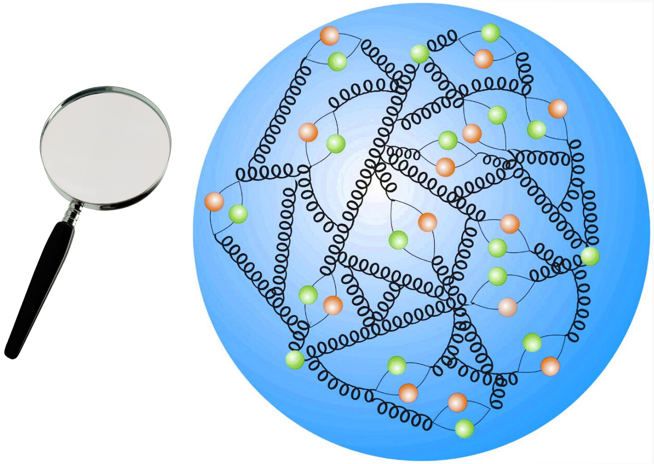
One of the most important goals of high-energy and nuclear physics today is to understand the inner structure of the hadrons:

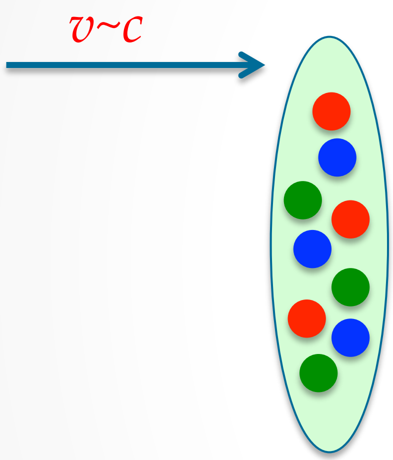


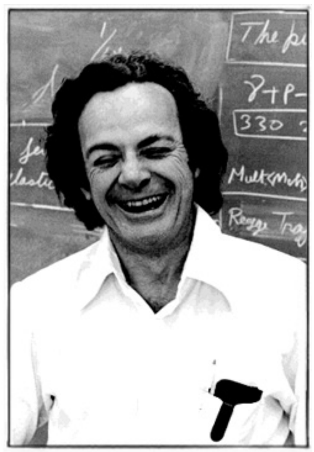


To see the quarks and gluons inside the proton you have to smash it!

Feynman's Parton Model

- Hadrons in high-energy scattering



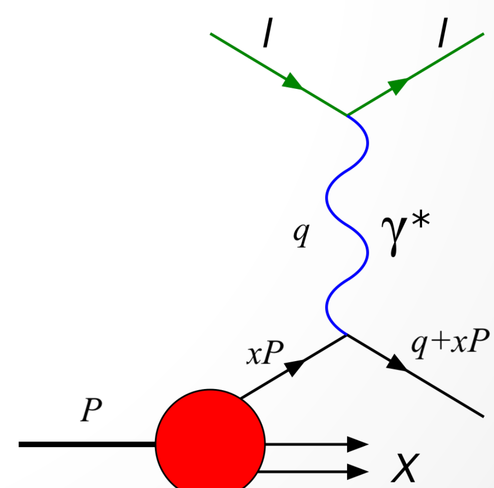


VERY HIGH-ENERGY COLLISIONS OF HADRONS

Richard P. Feynman
California Institute of Technology, Pasadena, California

(Received 20 October 1969)

Proposals are made predicting the character of longitudinal-momentum distributions in hadron collisions of extreme energies.

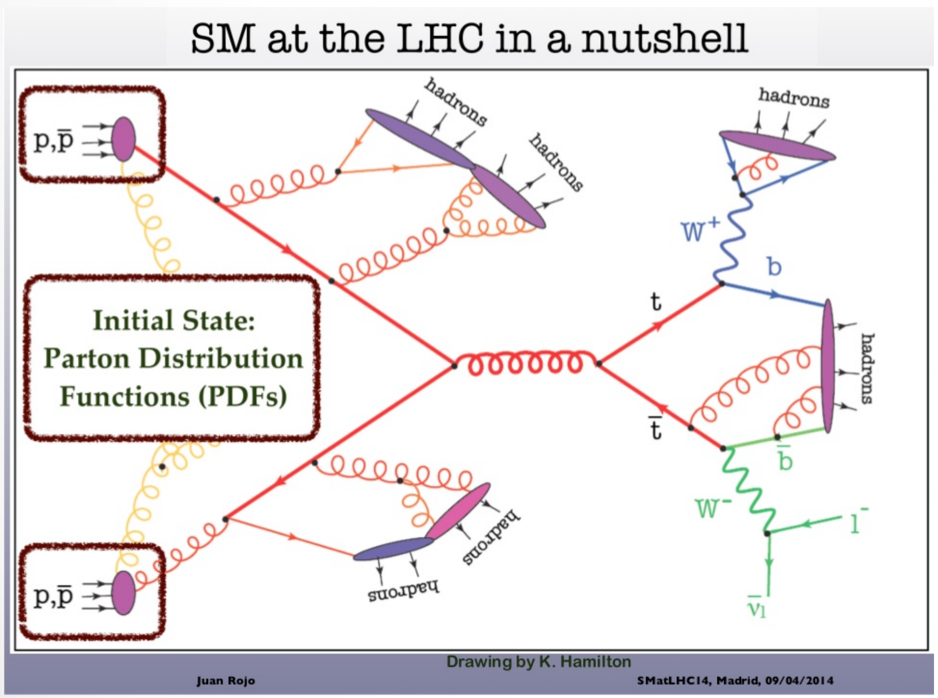


Infinite momentum frame (IMF): Quarks and gluons "frozen" in the transverse plane due to Lorentz dilation effect.

Parton model: hadron as a beam of free particles with certain momentum density.

Parton Distribution Functions (PDFs)

- PDFs are the basic inputs for making standard-model predictions in high-energy scattering.

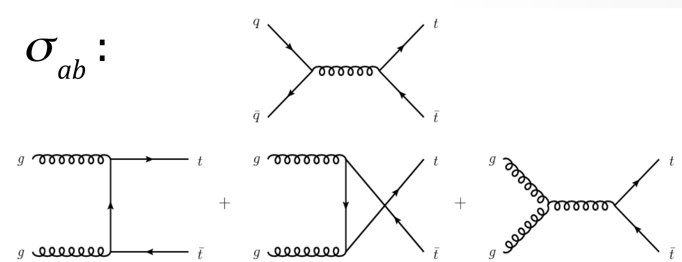


Picture Credit: Juan Rojo

QCD Factorization theorem:

$$\sigma = \sum_{a,b} f_a(x_1) \otimes f_b(x_2) \otimes \sigma_{ab}$$

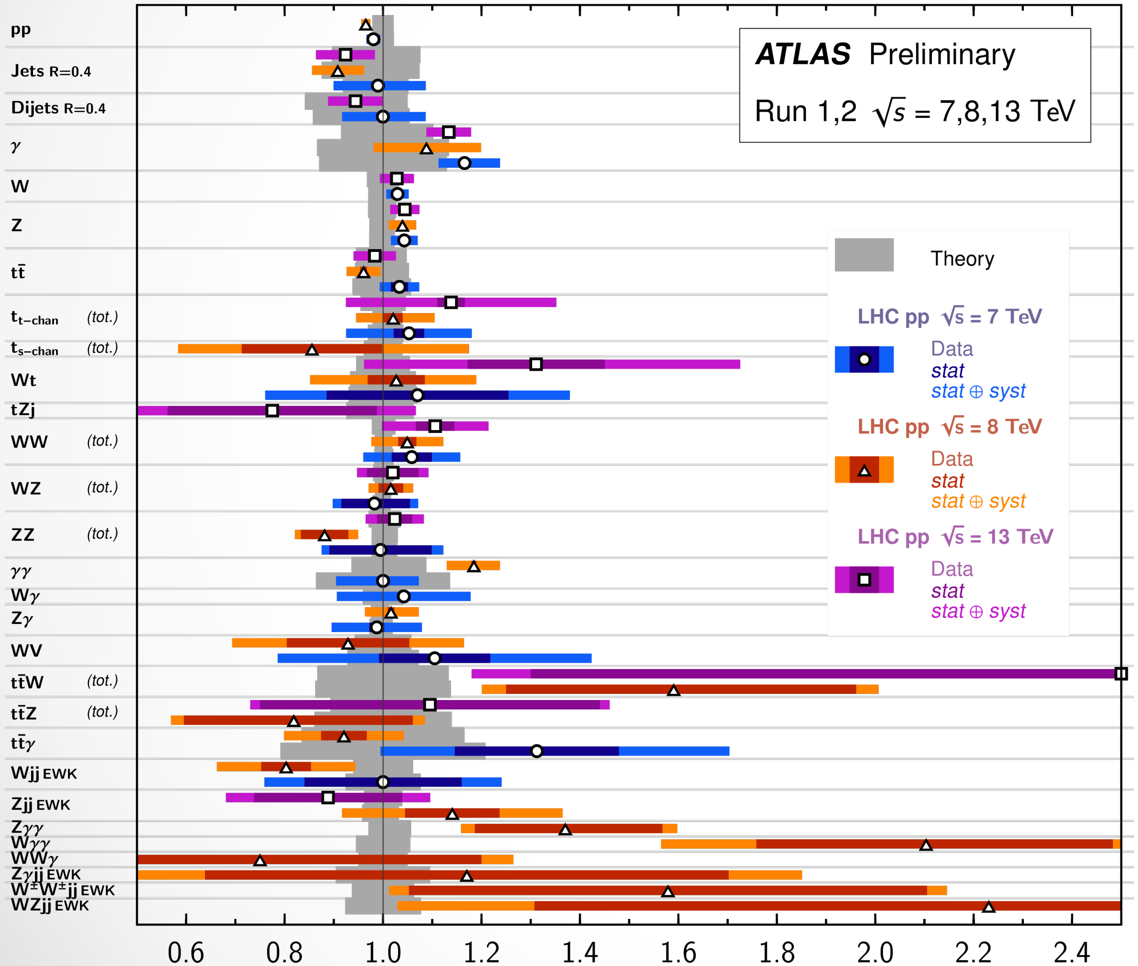
σ_{ab} :



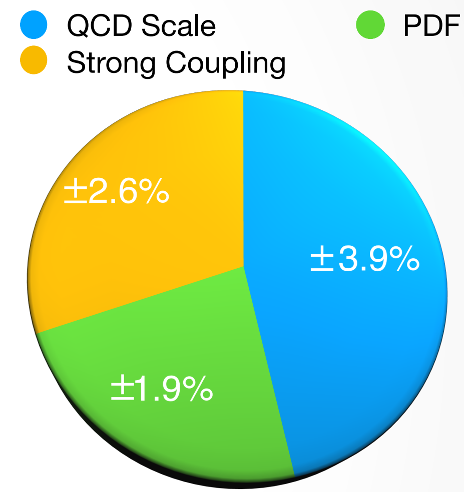
$f_a(x)$, $f_b(x)$ are universal properties of the hadron:

- Contributions from low energy, or, infrared (IR) degrees of freedom;
- Intrinsically nonperturbative.

PDFs and the Standard Model



https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/SM/

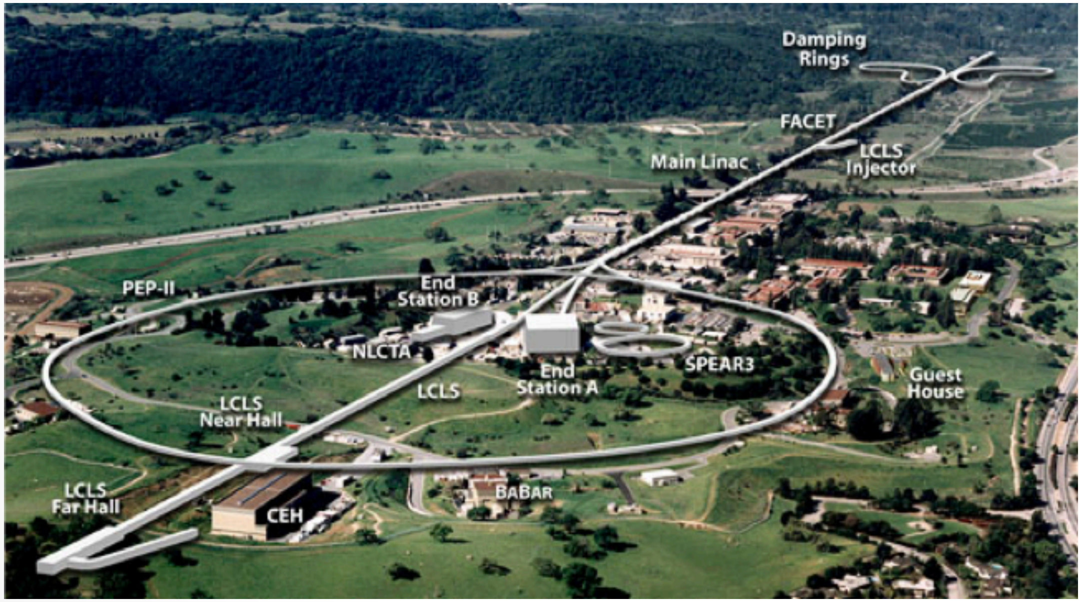


Theoretical uncertainties in the gluon-gluon fusion Higgs production cross section (M_H=125.09 GeV) at $\sqrt{s}$=13 TeV at N^{3}LO QCD and NLO electroweak accuracies.

https://twiki.cern.ch/twiki/bin/view/LHCPhysics, Calculated by the Zürich Group.

PDFs from Phenomenology

- PDFs have been measured extensively at the state-of-the-art high-energy physics programs since the late 1960s.



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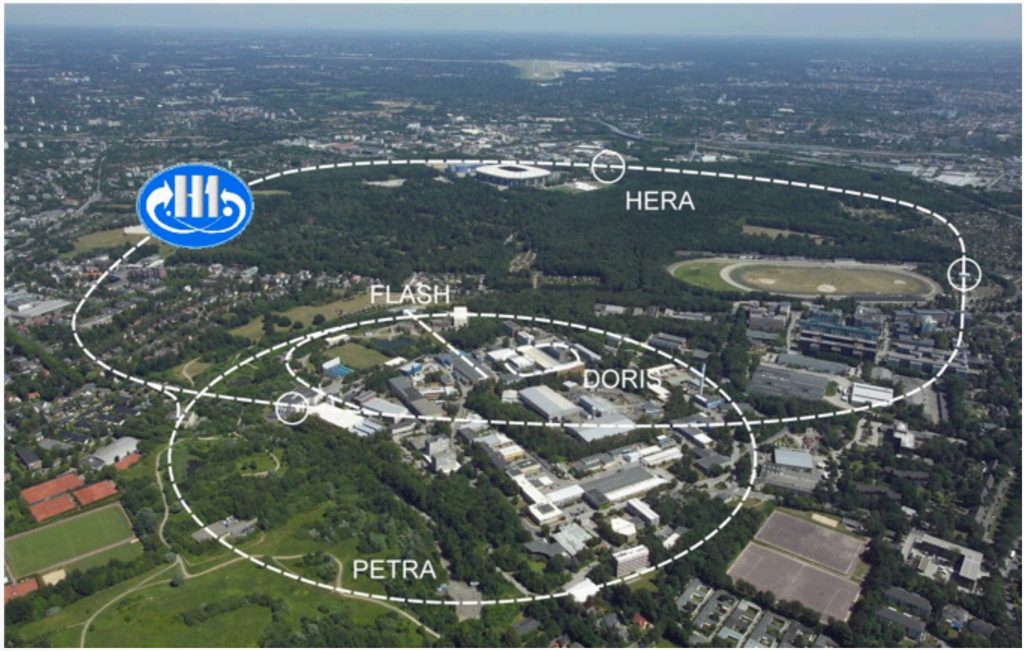

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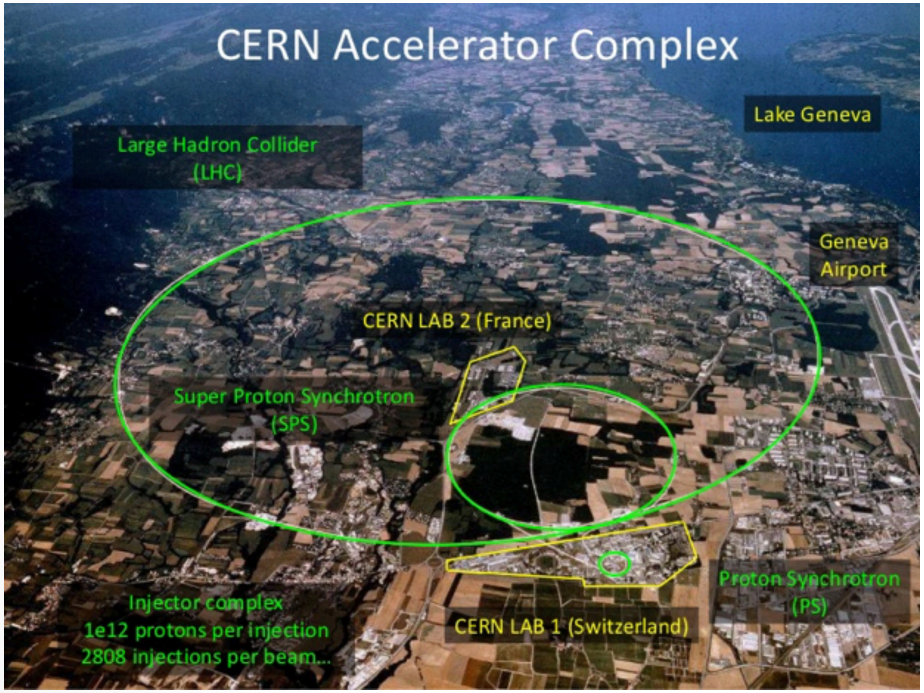

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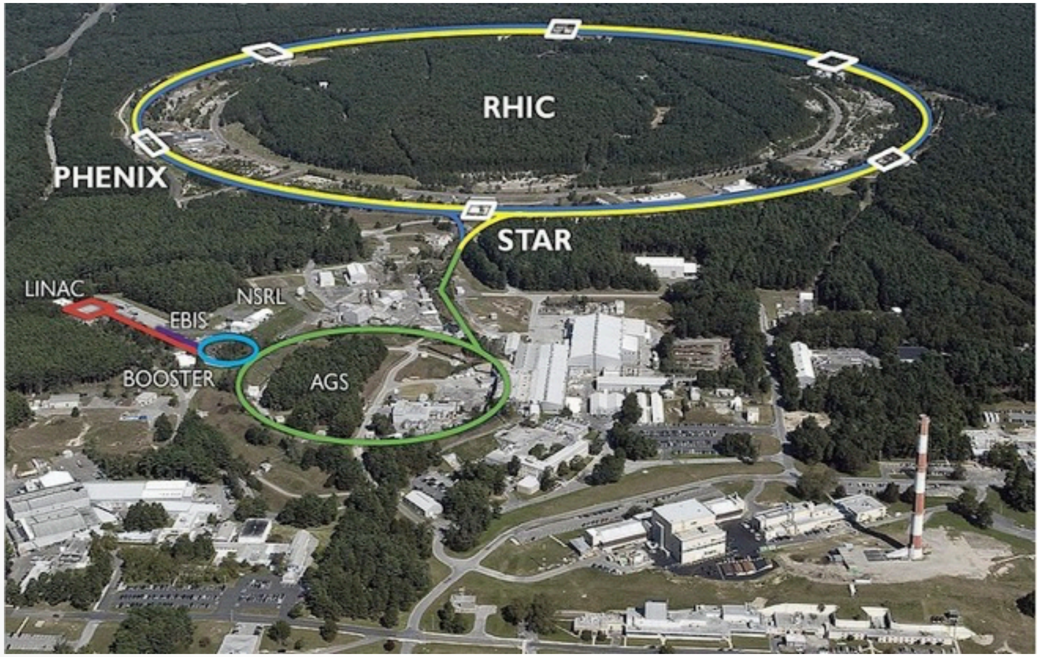

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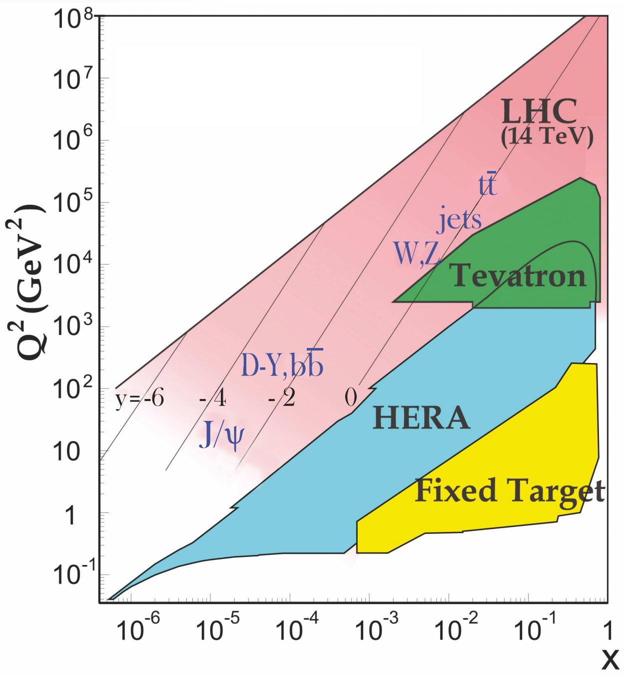
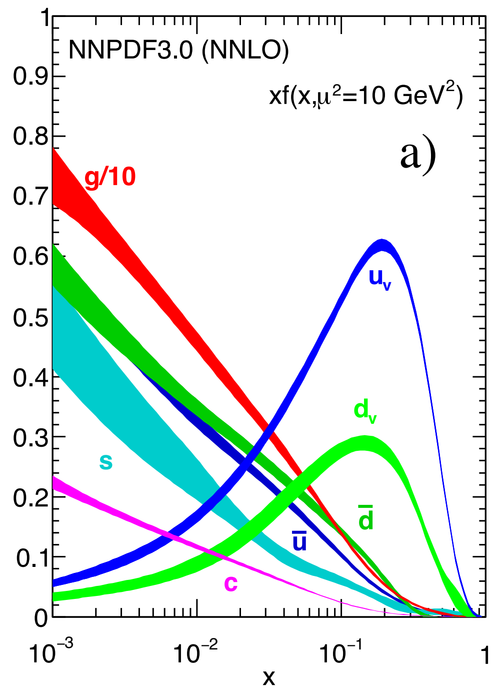
PDFs from Phenomenology

- So far our best knowledge of the PDFs comes from the global analysis of high-energy scattering data, but a first-principle calculation is still behind our time .



Particle Data Group, 2017



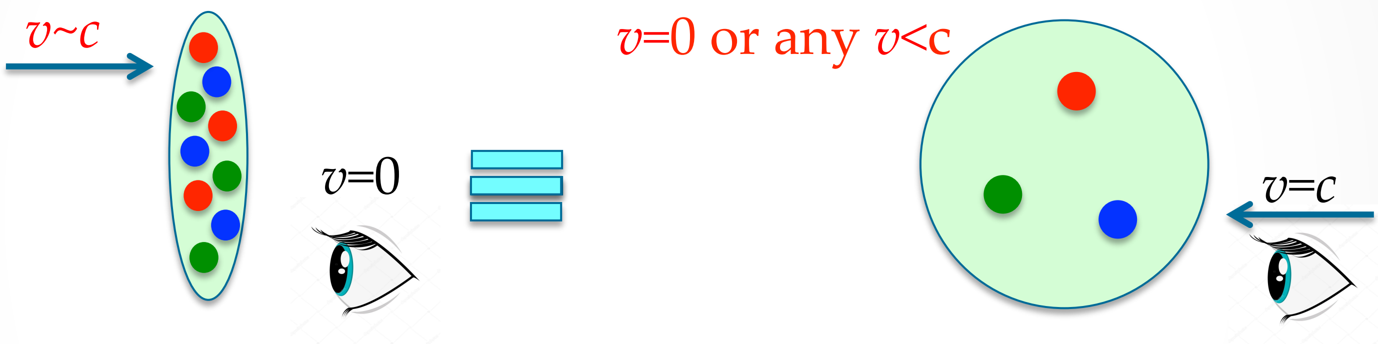
NNPDF 3.0, JPG 2015.

1. First-principle calculation can cover kinematic regions and flavor structures where experiments cannot constrain so well;

2. The cost of improving calculations can be much smaller than building larger experiments.

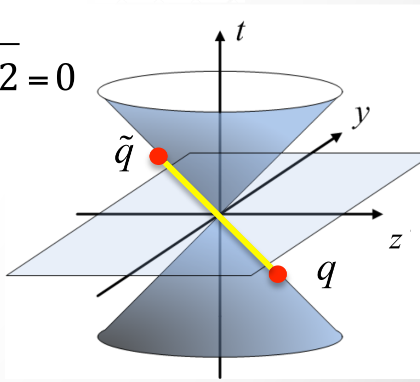
Parton Physics and the light-cone

- Instead of using the IMF, PDFs are often equivalently defined from correlation functions on the light-cone.



- Definition of PDF:

$$\xi^{\pm}=(t\pm z)/\sqrt{2}=0$$



$$\boxed{q(x,\mu)=\int\frac{d\xi^-}{4\pi}\,e^{-ixP^+\xi^-}\left\langle P\left|\bar{\psi}(\xi^-)\gamma^+U(\xi^-,0)\psi(0)\right|P\right\rangle}$$

$$U(\xi^-,0)=P\exp\left[-ig\int_0^{\xi^-}d\eta^-A^+(\eta^-)\right]$$

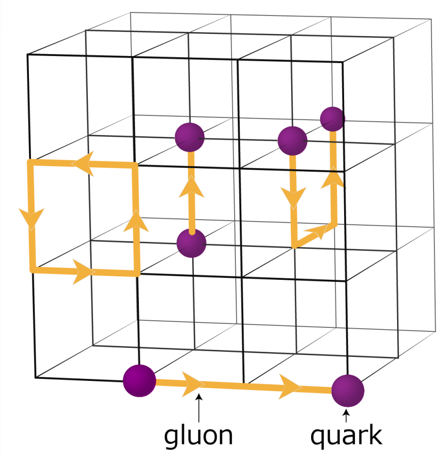
$$P^{\mu}=(P^z,0,0,P^z)$$

  - Gauge-invariant and boost invariant (independent of P^z);
  - In light-cone quantization ($A^+=0$), clear density interpretation.

$$\boxed{q(x,\mu)\sim\int dk^+d^2k_\perp\delta(k^+-xP^+)\left\langle P\left|\hat{n}(k^+,k_\perp)\right|P\right\rangle}$$

Lattice QCD Calculation of parton physics?

- Lattice gauge theory (K. G. Wilson, 1974)



- Monte Carlo simulation of QCD on a discretized Euclidean lattice
- Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.

See talks by J. Dudek and A. Nicholson.

Lattice QCD Calculation of parton physics?

However, parton physics usually cannot be directly calculated from the lattice:

- **Parton physics**
  - Minkowski space, real time;
  - Emerges in the IMF, or equivalently, on the light-cone.

$$e^{iS} \to e^{-S}$$
$$\langle O \rangle = \int D\psi D\bar{\psi} DA\ O(x) e^{-S}$$

- **Lattice QCD**
  - Euclidean space, imaginary time (*t=iτ*);
  - General difficulty of analytical continuation in time or fixing the light-cone.

$$x_E^\mu x_E^\mu = 0 \to x^\mu = (0,0,0,0)$$

Early attempts of lattice calculations

Computation of the Mellin moments of the PDF:

$$\int_0^1 dx\, x^n q(x,\mu)dx = a_n(\mu) \propto \left\langle P \middle| \bar{\psi}(0)\gamma^+ \overbrace{i\overleftrightarrow{D}^+ \cdots i\overleftrightarrow{D}^+}^{n} \psi(0) \middle| P \right\rangle$$

W. Detmold et al., EPJ 2001, PRD 2002; D. Dolgov et al. (LHPC, TXL), PRD 2002;

- Local operators are calculable on the lattice;
- Fitting the PDF with a parametrized form from the moments;
- Due to broken Lorentz symmetry, operator mixing limits the number of calculable moments to no more than 3.

$$\left(\bar{\psi}\,\Gamma DDD\,\psi\right)^B = Z_1\left(\bar{\psi}\,\Gamma DDD\,\psi\right)^R + \frac{Z_2}{a^2}\left(\bar{\psi}\,\Gamma D\,\psi\right)^R + \ldots$$

Can we directly compute the x-dependence?

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A novel approach to calculate light-cone observables

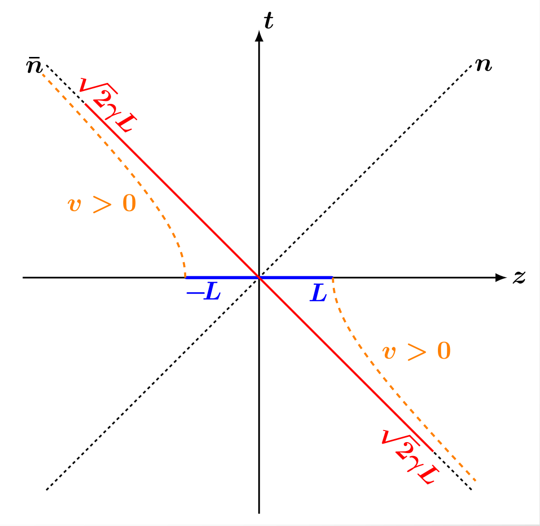
Large-momentum effective theory (LaMET) expands in powers of $1/P^z$, where P^z is the hadron momentum (Ji, PRL 2013, Sci. China Phys. Mech. Astro., 2014):

1. Construct a Euclidean quasi-observable $\tilde{O}$ which can be calculated in lattice QCD;
2. The IMF limit of $\tilde{O}$ is constructed to be a light-cone observable O at the operator level;

For example, the quasi-PDF is defined from equal-time correlation along the *z*-direction:

$$\boxed{\tilde{q}(x, P^z, \tilde{\mu}) = \int \frac{dz}{4\pi} e^{ixP^z z} \left\langle P \middle| \bar{\psi}(z) \gamma^z U(z,0) \psi(0) \middle| P \right\rangle}$$

$$U(z,0) = P\exp\left[-ig\int_0^z dz' A^z(z')\right] \qquad z^\mu = (0,0,0,z)$$



3. The matrix element of $\tilde{O}$ generally depends on P^z because it is not boost invariant, whereas that of O is independent of P^z;

$$\tilde{O}(P^z,\ \tilde{\mu}) = \left\langle P = P^z \middle| \tilde{O} \middle| P = P^z \right\rangle,$$
$$O(\mu) = \left\langle P = \text{any} \middle| O \middle| P = \text{any} \right\rangle$$

4. Taking the $P^z \longrightarrow \infty$ limit of $\tilde{O}(P^z,\ \tilde{\mu})$ is non-analytical due to the singularities in quantum field theory;

$$\lim_{P^z \to \infty} \tilde{O}(P^z, \tilde{\mu}) = ?$$

5. Nevertheless, at large P^z, $\tilde{O}(P^z, \tilde{\mu})$ can be related to $O(\mu)$ through a factorization theorem:

$$\boxed{\tilde{O}(P^z, \tilde{\mu}) = Z(\frac{\tilde{\mu}}{\mu}, \frac{\mu}{P^z}) \otimes O(\mu) + \mathrm{O}(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \frac{M^2}{P_z^2})}$$

$$O(\mu) \cong Z^{-1} \otimes \tilde{O}(P^z, \tilde{\mu})$$

- $O(\mu)$ and $\tilde{O}(P^z, \tilde{\mu})$ have the same IR physics (nonperturbative), but different ultraviolet (UV) physics (perturbative);
- $Z(\tilde{\mu}/\mu, \mu/P^z)$ matches the UV difference, and can be calculated precisely in perturbation theory.

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

Other proposals

- **Restoration of rotational symmetry to calculate higher moments**
  Z. Davoudi and M. Savage, 2012.
- **OPE of flavor-changing current-current correlation**
  D. Lin and W. Detmold, 2006.
- **OPE of the Compton amplitude** A. J. Chambers et al. (QCDSF), 2017.
- **Direct computation of the physical hadronic tensor**
  K.F. Liu (et al.), 1994, 1999, 1998, 2000, 2017.
- **Smeared Quasi-PDF with Gradient flow** C. Monahan and K. Orginos, 2017.
- **Pseudo-PDF (alternative to quasi-PDF)** A. Radyushkin, 2017; K. Orginos et al., 2017.
- **Lattice cross section** Y.-Q. Ma and J. Qiu, 2014, 2017.
- **Factorization of Euclidean correlations in coordinate space**
  V. M. Braun and D. Mueller, 2008;G. S. Bali, V. M. Braun, A. Schaefer, et al., 2017.

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Lattice collaborations working with LaMET approach

- Gluon polarization:
  - χQCD Collaboration

  A. Alexandru (George Washington U.), T. Drapper (U. Kentucky), M. Glatzmaier (U. Kentucky), K.F. Liu (U. Kentucky), R.S. Suffian (Jefferson Lab), Y.-B. Yang (Michigan State U.), Y.Z. (MIT), et al.

- PDFs:
  - Lattice Parton Physics Project (LP^3) Collaboration

    J.W. Chen (National Taiwan U.), T. Ishikawa (T.-D. Lee Institute), L. Jin (U. Connecticut and BNL), R.-Z. Li (Michigan State U.), H.-W. Lin (Michigan State U.), Y.-S. Liu (Shanghai Jiaotong U. and U. Maryland), A. Schaefer (U. Regensburg), Y.-B. Yang (Michigan State U.), J.-H. Zhang (U. Regensburg), R. Zhang (Beijing Inst. Theory), and Y.Z. (MIT), et al.
  - European Twisted Mass Collaboration (ETMC)

    C. Alexandrou (U. Cyprus) , M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.
  - DESY, Zeuthen

    J. Green, et al.
  - + Other groups whose work is in progress

Accomplishment of lattice calculations with LaMET so far

- **Gluon helicity contribution to proton spin**
  - Gluon spin provides about half of the proton spin

χQCD Collaboration, 2017

- **Unpolarized iso-vector quark distribution**
  - Gottfried sum rule violation, i.e., $\bar{d}(x) > \bar{u}(x)$

H.-W. Lin et al. (LP3), 2014, 2016, 2017,2018;
ETMC Collaboration, 2015, 2016, 2018.

- **Polarized iso-vector quark distribution**
  - Primitive results show $\Delta\bar{u}(x)>\Delta\bar{d}(x)$

H.-W. Lin et al. (LP3), 2014, 2016;
ETMC Collaboration, 2016, 2017, 2018.

- **Transversity iso-vector quark distribution**
  - Primitive results show $\delta\bar{d}(x) >\delta\bar{u}(x)$

H.-W. Lin et al. (LP3), 2016;
ETMC Collaboration, 2016.

- **Meson distribution amplitudes (PDA)**
  - Single-hump structure of pion PDA; Asymmetry of kaon PDA with respect to $x=1/2$

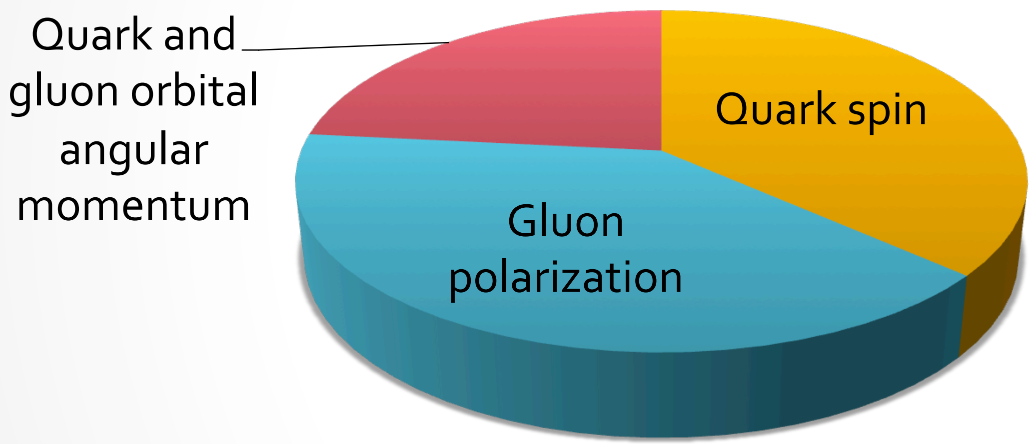
J.-W. Chen, J.-H. Zhang, H.-W. Lin et al. (LP3), 2017.

- **Pion PDF**

LP3 collaboration, 2018;

Nucleon spin crisis

Longitudinal proton spin structure



$\Delta\Sigma(Q^2\!=\!10\ \mathrm{GeV}^2) \sim 0.25$,
de Florian et al., PRD 2009

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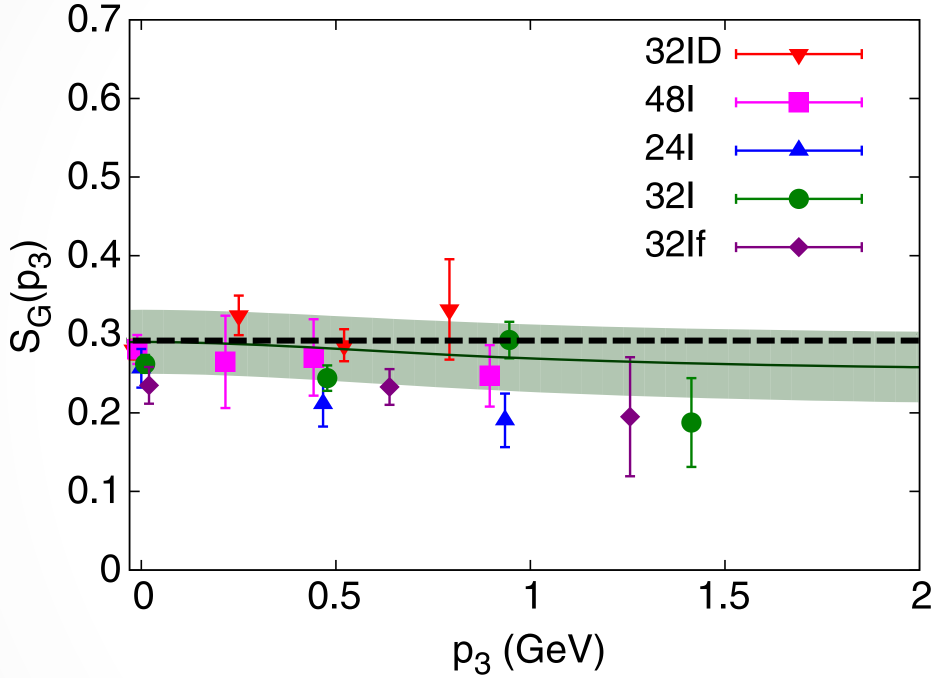
Jaffe-Manohar spin sum rule:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + l_q^z + l_g^z$$

$\Delta G(Q^2\!=\!10\ \mathrm{GeV}^2) \sim 0.2$,
de Florian et al., PRL 2014;
NNPDF collaboration, NPB 2014

First calculation of gluon helicity from lattice QCD



Also see K.-F. Liu's talk.

Extrapolation to $P^z \longrightarrow \infty$ after 1-loop corrections

X. Ji, J.-H. Zhang, and Y.Z., PRL 2013, PLB 2015; Y. Hatta, X. Ji, and Y.Z., PRD 2014; Y.-B. Yang, R. S. Sufian, Y.Z., et al (χQCD collaboration)., PRL 2017

$$\boxed{\Delta G(\mu^2 = 10\mathrm{GeV}^2) \approx S_G(\infty, \mu^2 = 10\mathrm{GeV}^2) = 0.251(47)(16)}$$

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Highlights of the Year

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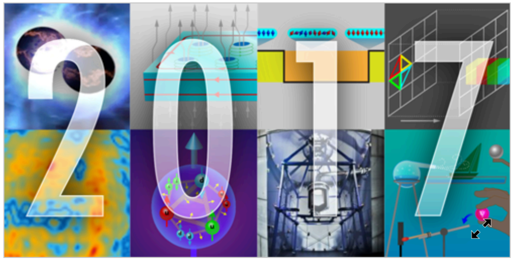
Physics looks back at its favorite stories from 2017.



Features

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Research News: Editors' Choice
Electric tears, topological ocean waves, cosmic rays, and more in our monthly recap of papers from the literature.

Q&A: Inside a Cell's Engine Room
George Shubeita describes his studies of the tiny motors that transport vital cargoes within biological cells.

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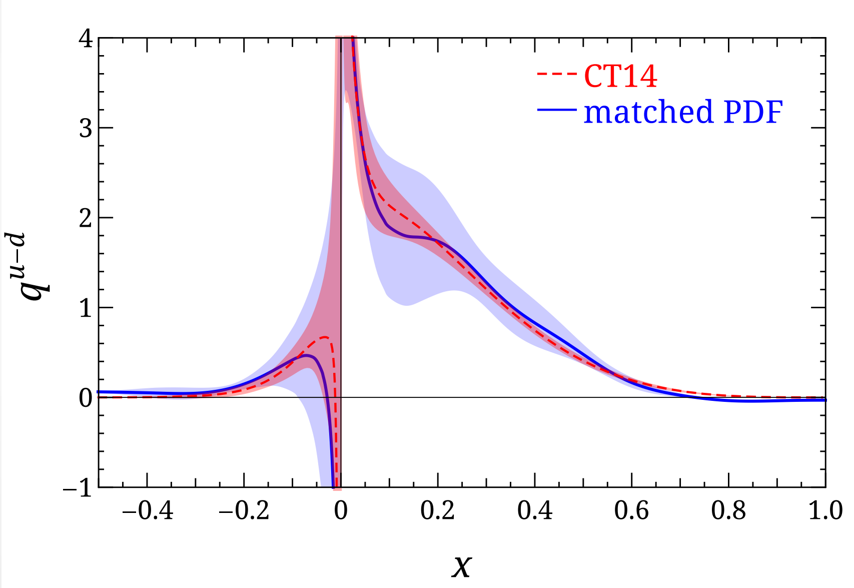
Gluons Provide Half of the Proton's Spin

The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

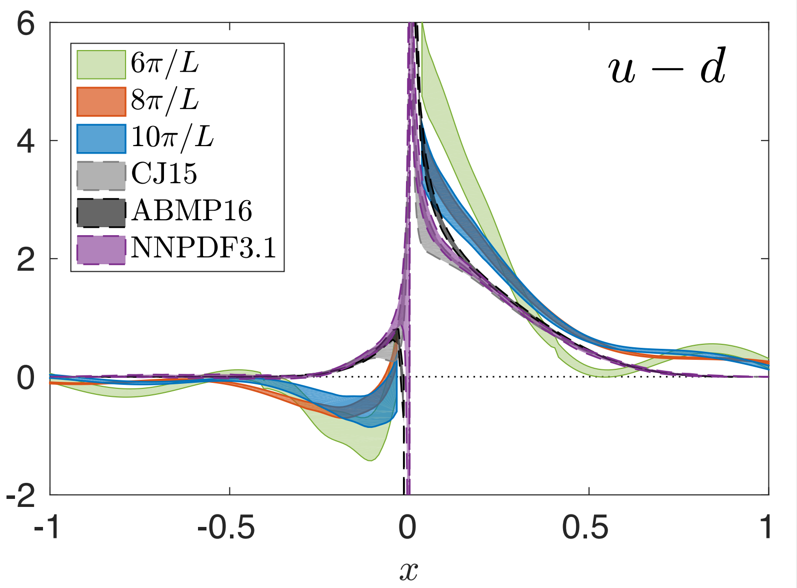
Lattice calculation of the iso-vector PDF $u(x)-d(x)-\bar{u}(-x)+\bar{d}(-x)$

J.W. Chen, L. Jin, H.-W. Lin, Y.-S. Liu, Y.-B. Yang, J.-H. Zhang, and Y.Z., (LP3), 2018



Clover valence fermions on N_f=2+1+1 flavors of HISQ generated by MILC, a=0.09fm, L=5.8fm, m_π~135MeV, N_{cf}=310, P^z=3 GeV.

C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, and F. Steffens, (ETMC), 2018



Dynamical N_f=2+1+1 twisted mass fermions by ETMC, a=0.09fm, L=4.8fm, m_π~130MeV, P^z=1.4 GeV.

A long way to here after over 3 years of dedicated efforts!

Summary and outlook

- A systematic lattice QCD calculation of parton physics from LaMET is making vigorous progress.

- In 5~10 years, expect:
  - Lattice calculation of quark PDFs to be within 10% accuracy or even better;
  - Constraints on the flavor structure of PDF at x~1 and the sea quark distributions be better than experiments;
  - Reaching smaller x region with larger nucleon momentum;
  - Lattice calculation of other distributions including TMD, GPD, gluon distributions, etc...

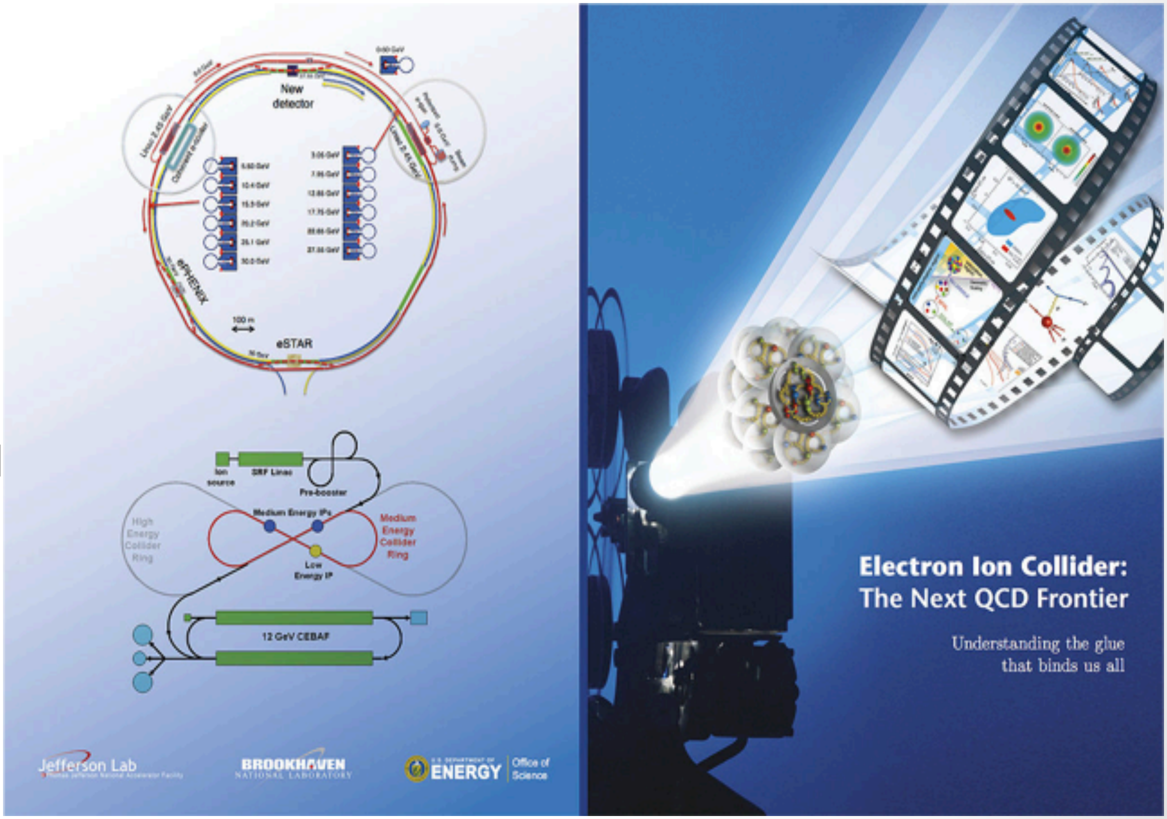
The next frontier of QCD

Electron-Ion Collider!

- **Highly Polarized Beams**
- **Large Kinematic Range**
- **High Collision Luminosity**

3-D tomography of the nucleon:

- More precise PDFs
- Sea quark distributions
- Gluonic structure of nucleon and nuclei
- Small-x physics
- Gluon spin and parton orbital angular momentum
- TMDPDFs, GPDs, Wigner distributions
- ……



A. Accardi et al., Eur.Phys.J. A52 (2016) no.9, 268.