Achievements and Open Issues in the Determination of Fragmentation Functions

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on the Intersections of Particle and Nuclear Physics

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Outline

1. Introduction and open issues
   - Hadrons in the final state, factorisation and evolution
   - Three examples on why we should care about fragmentation functions
2. Achievements
   - Data: global fits of $\pi^{\pm}$, $K^{\pm}$, $h^{\pm}$ and D^{*} fragmentation functions
   - Theory: impact of GM-VFN scheme, NNLO corrections, small-z resummation
   - Methodology: simultaneous fits of unpolarised/polarised PDFs and FFs
3. Conclusions

DISCLAIMER

I will focus on collinear fragmentation functions only

Emphasis on recent achievements and on topics which I've worked on recently

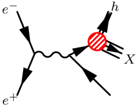
Apologies in advance for not discussing your favourite subject

For an extensive review of topics not addressed in this talk, please see

[Prog.Part.Nucl.Phys. 91 (2016) 136]

1. Introduction and open issues

Factorisation of physical observables [Adv.Ser.Direct.HEP 5 (1988) 1]

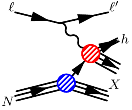


$e^+ + e^- \to h + X$
single-inclusive annihilation (SIA)

$$\frac{d\sigma^h}{dz} = F_T^h(z,Q^2) + F_L^h(z,Q^2) = F_2^h(x,Q^2)$$

$$F_{k=T,L,2}^h = \frac{4\pi\alpha_{\rm em}^2}{Q^2}\langle e^2\rangle \left\{ D_\Sigma^h \otimes C_{k,q}^{\rm S} + n_f D_g^h \otimes C_{k,g}^{\rm S} + D_{\rm NS}^h \otimes C_{k,q}^{\rm NS} \right\}$$

up to NNLO [PLB 386 (1996) 422; NPB 487 (1997) 233; PLB 392 (1997) 207]



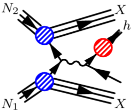
$\ell + N \to \ell' + h + X$
semi-inclusive deep-inelastic scattering (SIDIS)

$$\frac{d\sigma^h}{dx\,dy\,dz} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left[\frac{1+(1-y)^2}{y} 2F_1^h + \frac{2(1-y)}{y} F_L^h\right]$$

$$2F_1^h = e_q^2 \left\{ q\otimes D_q^h + \frac{\alpha_s}{2\pi}\left[q\otimes C_{qq}^1 \otimes D_q^h + q \otimes C_{gq}^1 \otimes D_g^h + g \otimes C_{qg}^1 \otimes D_q^h \right]\right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2 \left[q\otimes C_{qq}^L \otimes D_q^h + q \otimes C_{gq}^L \otimes D_g^h + g \otimes C_{qg}^L \otimes D_q^h \right]$$

up to NLO [NPB 160 (1979) 301; PRD 57 (1998) 5811]
partial NNLO [PRD 95 (2017) 034027]



$N_1 + N_2 \to h + X$
high-p_T hadron production in pp collisions (PP)

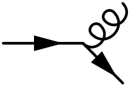
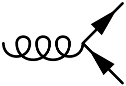
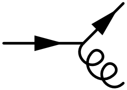
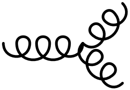
$$E_h \frac{d^3\sigma}{dp_{T,h}^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$
$$=$$
$$\sum_{i,j,k}\int \frac{dx_a}{x_a}\int\frac{dx_b}{x_b}\int\frac{dz}{z^2} f^{i/p_a}(x_a) f^{j/p_b}(x_b) D^{h/k}(z)\hat{\sigma}^{ij\to k}\delta(\hat{s}+\hat{t}+\hat{u})$$

up to NLO [PRD 67 (2003) 054004; PRD 67 (2003) 054005]

Evolution of FFs: DGLAP equations [NPB 126 (1977) 298]

A set of $(2n_f + 1)$ integro-differential equations (n_f=number of active flavours)

$$\frac{\partial}{\partial \ln \mu^2} D_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z, \alpha_s(\mu^2)\right) D_j\left(\frac{x}{z}, \mu^2\right)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

LO [Sov. J. Nucl. Phys. 15 (1973) 438; NPB 126 (1977) 298; NPB 136 (1978) 445]
NLO [NPB 175 (1980) 27, PLB 97 (1980) 497, PRD 48 (1993) 116]
NNLO [PLB 638 (2006) 61, PLB 659 (2008) 290, NPB 854 (2012) 133]

Must be careful with fixed-order splitting functions as $z \to 0$ $(m = 1, \ldots, 2k+1)$

SPACE-LIKE CASE

$$P_{ji} \propto \frac{\alpha_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$$

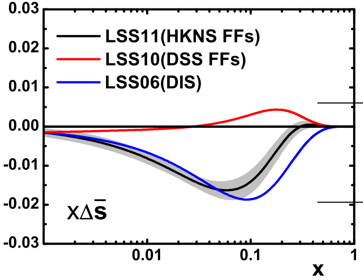
TIME-LIKE CASE

$$P_{ji} \propto \frac{\alpha_s^{k+1}}{z} \log^{2(k+1)-m-1} z$$

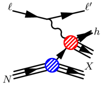
Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as z decreases, the unresummed SGLs spoil the convergence of the FO series for $P(z,\alpha_s)$ if $\log\frac{1}{z} \geq \mathcal{O}\left(\alpha_s^{-1/2}\right)$

Fragmentation functions: why should we bother?

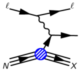
Example 1: The strange (polarised) parton distribution and SIDIS

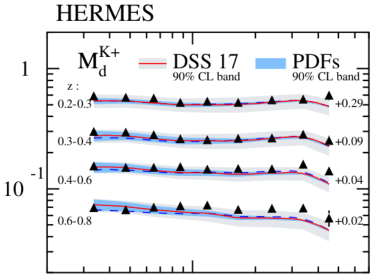


directly from SIDIS Kaon data



indirectly from DIS + SU(3)





If SIDIS data is used to determine Δs, $K^{\pm}$ FFs for different sets lead to different results
Such results may differ significantly among them and w.r.t. the results obtained from DIS
$\longrightarrow$ How well do we know kaon FFs?

Can SIDIS data be used to determine s?
What is the bias induced by FFs onto PDFs?
$\longrightarrow$ How well do we know kaon FFs?

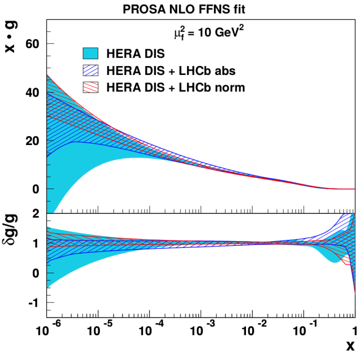
Fragmentation functions: why should we bother?

Example 2: Heavy quark fragmentation: the D^* case

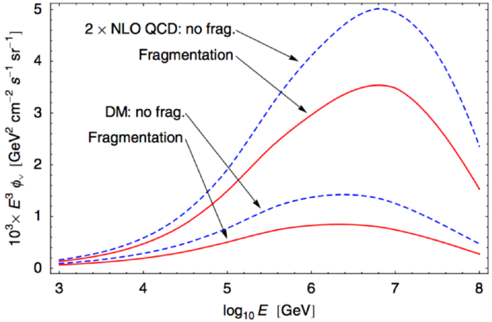
Constrain the low-x (gluon) PDFs through charm production in the forward region
[EPJ C75 (2015) 396; JHEP 1602 (2016) 130]

Compute the prompt atmospheric neutrino flux
[PRD 78 (2008) 043005; JHEP 1506 (2015) 110]

Extract information on the medium in heavy ion collisions
[JHEP 1703 (2017) 146]

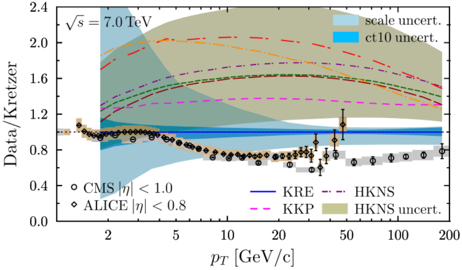


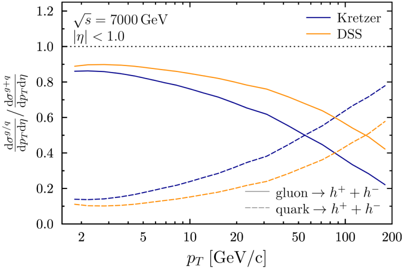
w or w/o considering $\frac{d\sigma(pp\to hX)}{dE_h} = \int_{E_h}^{\infty}\frac{dE_c}{E_c}\frac{d\sigma(pp\to cX)}{dE_c}D_c^h(E_h/E_c)$.

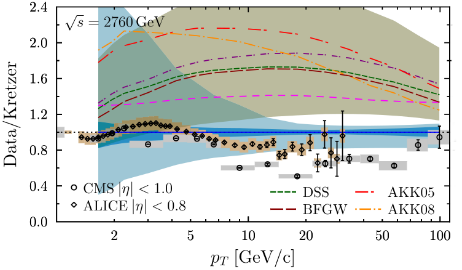


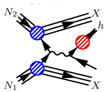
Fragmentation functions: why should we bother?

Example 3: Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE









$$E\frac{d^3\sigma}{dp_T^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}^c_{ab} \otimes D^h_c$$

Figures taken from [NPB 883 (2014) 615]

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties

$\longrightarrow$ How well do we know the gluon FF?

2. Achievements

Available fragmentation function sets (status 2018)

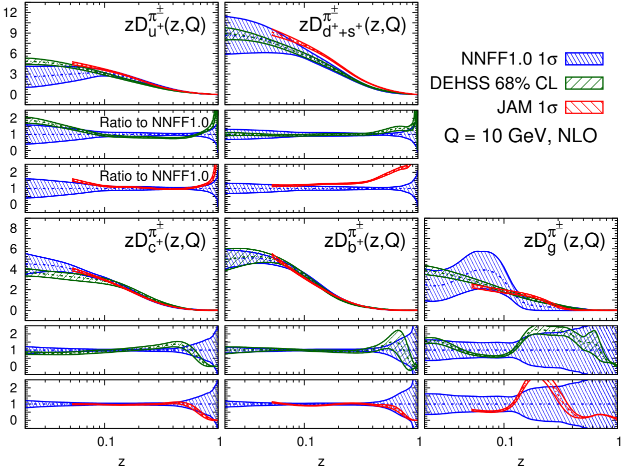
		DEHSS	HKNS	JAM	NNFF
DATA	SIA	☑	☑	☑	☑
	SIDIS	☑	☒	☒	☒
	PP	☑	☒	☒	☒
METH.	statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Monte Carlo	Monte Carlo
	parametrisation	standard	standard	standard	neural network
THEORY	pert. order	(N)NLO	NLO	NLO	LO, NLO, NNLO
	HF scheme	ZM(GM)-VFN	ZM-VFN	ZM-VFN	ZM-VFN
	hadron species	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$
	latest update	PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004	EPJ C77 (2017) 516

$+$ many others (including analyses for specific hadrons)

Focus on π and K which constitute the largest fraction in measured yields

BKK96 [PRD 53 (1996) 3553]	K^0	AESS11 [PRD 83 (2011) 034002]	η
DSV97 [PRD 57 (1998) 5811]	Λ^0	AKSRV17 [PRD 96 (2017) 034028]	D^*
BFGW00 [EPJ C19 (2001) 89]	$h^\pm$	LSS15 [PRD 96 (2016) 074026]	SIDIS only

Comparison at NLO (pions): NNFF1.0 - JAM - DEHSS



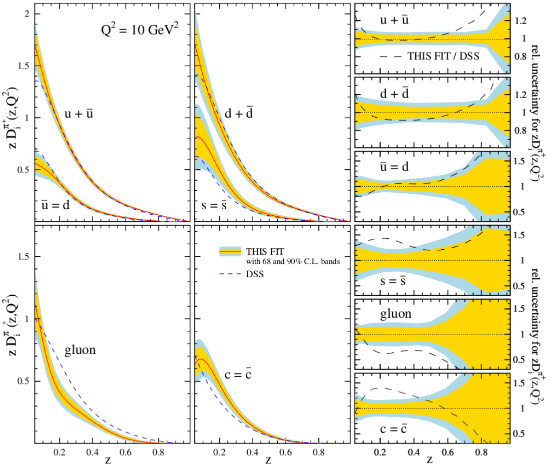
Differences due to data set, kinematic cuts and fitting methodology

Larger NNFF1.0 uncertainties where less or no data (flexibility of NN parametrisation)

Expect larger uncertainty on $D_g^{\pi^\pm}$ than $D_\Sigma^{\pi^\pm}$
visible in NNFF1.0, but not in DEHSS (bound from pp data?) nor in JAM (functional form?)

Global fit of pion fragmentation functions [PRD 91 (2015) 014035]

experiment		data type	norm. N_i	# data in fit	χ^2
TPC [48]		incl.	1.043	17	17.3
		uds tag	1.043	9	2.1
		c tag	1.043	9	5.9
		b tag	1.043	9	9.2
TASSO [49]	34 GeV	incl.	1.043	11	30.2
	44 GeV	incl.	1.043	7	22.2
SLD [19]		incl.	0.986	28	15.3
		uds tag	0.986	17	18.5
		c tag	0.986	17	16.1
		b tag	0.986	17	5.8
ALEPH [16]		incl.	1.020	22	22.9
DELPHI [17]		incl.	1.000	17	28.3
		uds tag	1.000	17	33.3
		b tag	1.000	17	10.6
OPAL [18, 20]		incl.	1.000	21	14.0
		u tag	0.786	5	31.6
		d tag	0.786	5	33.0
		s tag	0.786	5	51.3
		c tag	0.786	5	30.4
		b tag	0.786	5	14.6
BABAR [28]		incl.	1.031	45	46.4
BELLE [29]		incl.	1.044	78	44.0
HERMES [30]		π^+ (p)	0.980	32	27.8
		π^- (p)	0.980	32	47.8
		π^+ (d)	0.981	32	40.3
		π^- (d)	0.981	32	59.1
COMPASS [31] prel.		π^+ (d)	0.946	199	174.2
		π^- (d)	0.946	199	229.0
PHENIX [21]		π^0	1.112	15	15.8
STAR [33–36]	$0 \leq \eta \leq 1$	π^0	1.161	7	5.7
	$0.8 \leq \eta \leq 2.0$	π^0	0.954	7	2.7
	$\lvert\eta\rvert < 0.5$	$\pi^\pm$	1.071	8	4.3
	$\lvert\eta\rvert < 0.5$	$\pi^+,\pi^-/\pi^+$	1.006	16	17.2
ALICE [32]	7 TeV	π^0	0.766	11	27.7
TOTAL:				973	1154.6



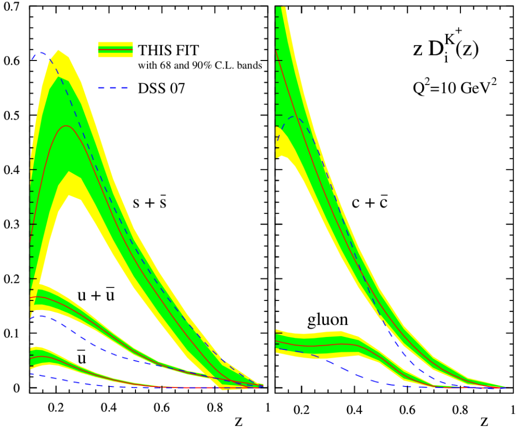
$D_{u+\bar{u}}$: most precise (B-factory SIA data)

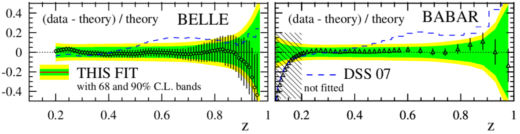
Very little or no charge symmetry breaking (SIDIS)

D_g: significant shift of the central value (pp data)

Global fit of kaon fragmentation functions [PRD 95 (2017) 094019]

experiment	data type	norm. N_i	# data in fit	χ^2
TPC [37]	incl.	1.003	12	13.4
SLD [33]	incl.	1.014	18	17.2
	uds tag	1.014	10	31.5
	c tag	1.014	10	21.3
	b tag	1.014	10	11.9
ALEPH [30]	incl.	1.026	13	29.7
DELPHI [31]	incl.	1.000	12	6.9
	uds tag	1.000	12	13.1
	b tag	1.000	12	11.0
OPAL [34]	*u* tag	0.778	5	9.6
	d tag	0.778	5	7.7
	s tag	0.778	5	23.4
	c tag	0.778	5	42.5
	b tag	0.778	5	16.9
BABAR [17]	incl.	1.077	45	30.6
BELLE [18]	incl.	0.996	78	15.6
HERMES [19]	K^+ (p) Q^2	0.843	36	61.9
	K^- (p) Q^2	0.843	36	29.6
	K^+ (p) x	1.135	36	75.8
	K^- (p) x	1.135	36	42.1
	K^+ (d) Q^2	0.845	36	44.7
	K^- (d) Q^2	0.845	36	41.9
	K^+ (d) x	1.095	36	48.9
	K^- (d) x	1.095	36	44.4
COMPASS [22]	K^+ (d)	0.996	309	285.8
	K^- (d)	0.996	309	265.1
STAR [24]	K^+,K^-/K^+	1.088	16	7.6
ALICE [23] 2.76 TeV	K/π	0.985	15	21.6
TOTAL:			1194	1271.7





Good flavour separation (SIDS data)

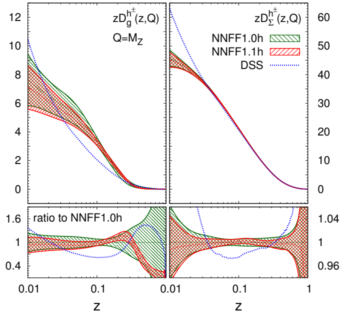
D_g: significant shift (pp data)

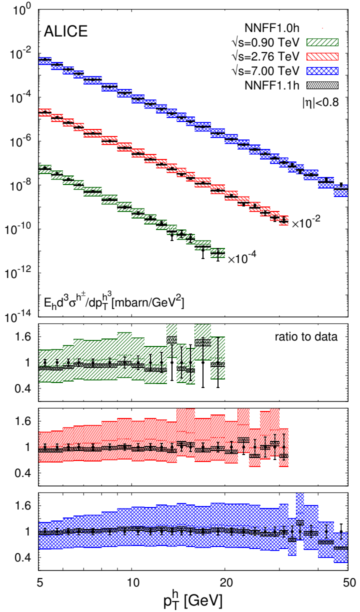
Caution with mass corrections

$D_{u+\bar{u}}$: most precise (B-factory SIA data)

Global fit of unidentified charged hadron FFs [NNPDF, in preparation]

Experiment	$\sqrt{s}$ [TeV]	$N_{\rm dat}$	$\chi^2_{\rm b}/N_{\rm dat}$	$\chi^2_{\rm a}/N_{\rm dat}$
e^+e^-	various	471 (527)	0.83	0.83
CDF	1.80	7 (49)	2.93	1.36
	1.96	60 (230)	3.45	1.23
CMS	0.90	10 (20)	3.78	1.18
	2.76	11 (22)	9.31	1.13
	7.00	17 (27)	10.5	0.98
ALICE	0.90	15 (54)	4.90	1.05
	2.76	21 (60)	11.8	0.96
	7.00	26 (65)	5.21	0.91
Total data set		638 (1054)	2.18	0.90





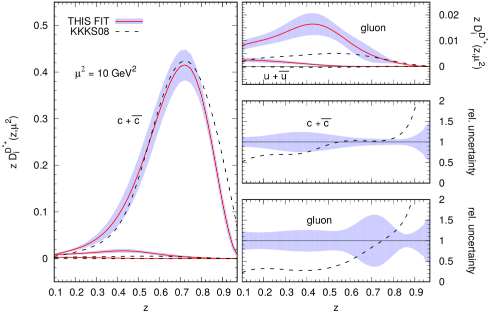
First global fit of D^* fragmentation functions [PRD 96 (2017) 034028]

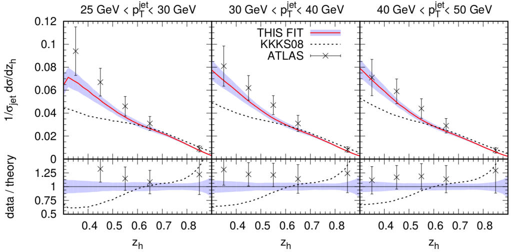
Only g, c and b FFs parametrised

Use of ZM-VFN scheme

Kinematic cut $p_T^h > 10$ GeV

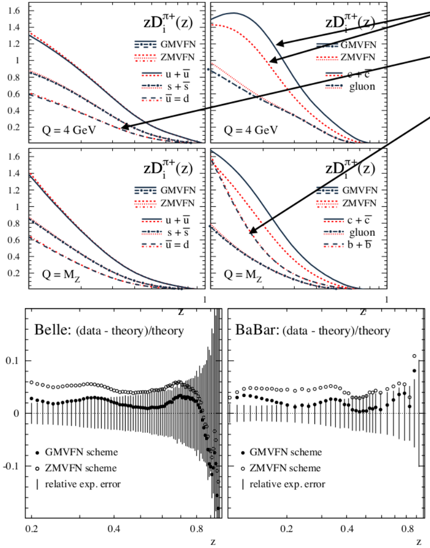
experiment		data type	#data in fit	χ^2
ALEPH [80] *Eur. Phys. J. C 16, 597 (2000)*		incl.	17	33.738
OPAL [81] *Z. Phys. C 67, 27 (1995)*		incl.	9	6.999
		c tag	9	8.388
		b tag	9	5.342
ATLAS [94] *Nucl. Phys. B 907, 717 (2016).*		$D^{*\pm}$	5	3.598
ALICE [60, 61]	$\sqrt{S} = 7$ TeV	D^{*+}	3	0.126
JHEP 1201, 128 (2012) *JHEP 1207, 191 (2012)*	$\sqrt{S} = 2.76$ TeV	D^{*+}	1	0.007
CDF [62] *Phys. Rev. Lett.91, 241804 (2003)*		D^{*+}	2	1.289
LHCb [64]	$2 \leq \eta \leq 2.5$	$D^{*\pm}$	5	10.984
JHEP 1609, 013 (2016)	$2.5 \leq \eta \leq 3$	$D^{*\pm}$	5	2.607
	$3 \leq \eta \leq 3.5$	$D^{*\pm}$	5	8.229
	$3.5 \leq \eta \leq 4$	$D^{*\pm}$	2	10.411
ATLAS [68]	$25 \leq \frac{p_T^{jet}}{\text{GeV}} \leq 30$	(jet $D^{*\pm}$)	5	4.146
Phys. Rev. D 85,	$30 \leq \frac{p_T^{jet}}{\text{GeV}} \leq 40$	(jet $D^{*\pm}$)	5	1.977
052005 (2012)	$40 \leq \frac{p_T^{jet}}{\text{GeV}} \leq 50$	(jet $D^{*\pm}$)	5	0.659
	$50 \leq \frac{p_T^{jet}}{\text{GeV}} \leq 60$	(jet $D^{*\pm}$)	5	0.791
	$60 \leq \frac{p_T^{jet}}{\text{GeV}} \leq 70$	(jet $D^{*\pm}$)	5	1.333
TOTAL:			97	100.980





See also JHEP 1605 (2016) 125 and F. Ringer's talk

Pion fragmentation functions in the GM-VFNs [PR D94 (2016) 034037]



charm changes significantly

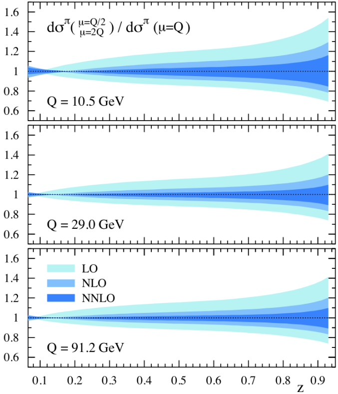
light flavors constrained by sidis

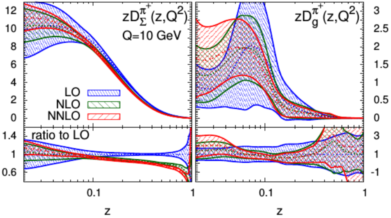
bottom constrained by high Q

experiment	data type	# data in fit	ZMVFN N_i	ZMVFN χ^2	GMVFN N_i	GMVFN χ^2
ALEPH [23]	incl.	22	0.968	21.6	0.994	23.3
BABAR [13]	incl.	39	1.019	76.7	1.002	58.2
BELLE [14]	incl.	78	1.044	19.5	1.019	11.0
DELPHI [24]	incl.	17	0.978	6.7	1.003	9.3
	uds tag	17	0.978	20.8	1.003	9.5
	b tag	17	0.978	10.5	1.003	7.8
OPAL [25]	incl.	21	0.946	27.9	0.970	15.9
SLD [26]	incl.	28	0.938	28.0	0.963	9.5
	uds tag	17	0.938	21.3	0.963	11.3
	c tag	17	0.938	34.0	0.963	19.8
	b tag	17	0.938	11.1	0.963	9.9
TPC [27]	incl.	17	0.997	31.7	1.006	27.9
	uds tag	9	0.997	2.0	1.006	2.0
	c tag	9	0.997	5.9	1.006	4.3
	b tag	9	0.997	9.6	1.006	10.9
COMPASS [28]	$\pi^\pm$ (d)	398	1.003	378.7	1.008	382.9
HERMES [29]	$\pi^\pm$ (p)	64	0.981	74.0	0.986	69.9
	$\pi^\pm$ (d)	64	0.980	107.3	0.985	103.7
PHENIX [30]	π^0	15	1.174	14.3	1.167	14.4
STAR [31]	$\pi^\pm$, π^0	38	1.205	31.2	1.202	33.8
ALICE [32]	π^0	11	0.696	33.3	0.700	31.2
TOTAL:		924		966.4		875.8

Slide: courtesy of R. Sassot

Fragmentation functions at NNLO [PR D92 (2015) 114017; EPJ C77 (2017) 516]





Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.60	0.11	0.09
BABAR	40	1.91	1.77	0.78
TASSO12	4	0.70	0.85	0.87
TASSO14	9	1.55	1.67	1.70
TASSO22	8	1.64	1.91	1.91
TPC	13	0.46	0.65	0.85
TPC-UDS	6	0.78	0.55	0.49
TPC-C	6	0.55	0.53	0.52
TPC-B	6	1.44	1.43	1.43
TASSO34	9	1.16	0.98	1.00
TASSO44	6	2.01	2.24	2.34
TOPAZ	5	1.04	0.82	0.80
ALEPH	23	1.68	0.90	0.78
DELPHI	21	1.44	1.79	1.86
DELPHI-UDS	21	1.30	1.48	1.54
DELPHI-B	21	1.21	0.99	0.95
OPAL	24	2.29	1.88	1.84
SLD	34	2.33	1.14	0.83
SLD-UDS	34	0.95	0.65	0.52
SLD-C	34	3.33	1.33	1.06
SLD-B	34	0.45	0.38	0.36
TOTAL	428	1.44	1.02	0.87

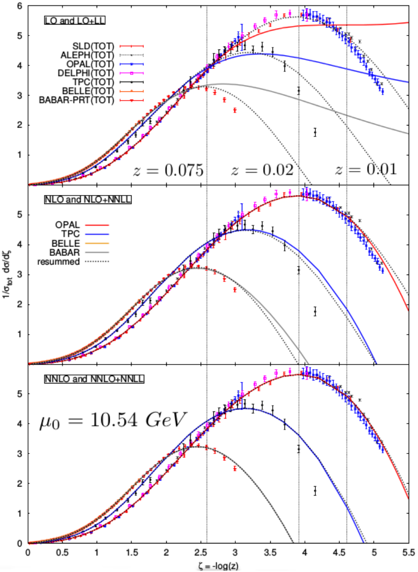
Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO
Effects less evident for $K^\pm$ and $p/\bar{p}$

Small-z resummed fragmentation functions [PRD 95 (2017) 054003]

436 Total data Points:

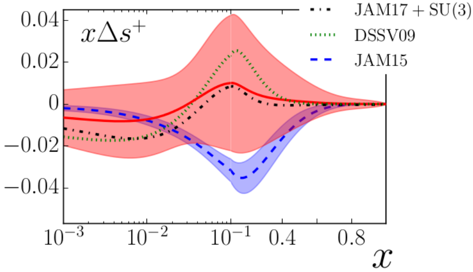
- LEP cut (z = 0.01) due to inconsistency between OPAL and ALEPH
- TPC lower cut (z = 0.02) based on difference of energy fraction $z = 2\,E_h/Q$ and three momentum fraction $x_p = z - 2m_h^2/(zQ^2) + \mathcal{O}(1/Q^4)$ in c.m.s being less than at least 15%

accuracy	χ^2	χ^2/dof
LO	1260.78	2.89
NLO	354.10	0.81
NNLO	330.08	0.76
LO+LL	405.54	0.93
NLO+NNLL	352.28	0.81
NNLO+NNLL	329.96	0.76

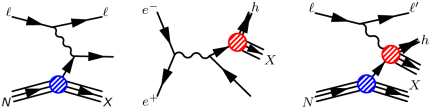


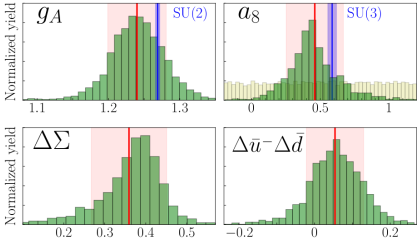
Slide: courtesy of D. P. Anderle

Simultaneous fits of (pol.) PDFs and FFs [PRL 119 (2017) 132001]



process	target	$N_{\rm dat}$	χ^2
DIS	$p, d, {}^3$He	854	854.8
SIA ($\pi^\pm, K^\pm$)		850	997.1
SIDIS ($\pi^\pm$)			
HERMES	d	18	28.1
HERMES	p	18	14.2
COMPASS	d	20	8.0
COMPASS	p	24	18.2
SIDIS ($K^\pm$)			
HERMES	d	27	18.3
COMPASS	d	20	18.7
COMPASS	p	24	12.3
Total:		**1855**	**1969.7**





$g_A = 1.24 \pm 0.04$ $\quad$ $a_8 = 0.46 \pm 0.21$

confirmation of SU(2) symmetry to $\sim 2\%$

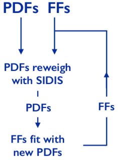
$\sim 20\%$ SU(3) breaking $\pm 20\%$

$$\Delta s^+ = -0.03 \pm 0.09$$

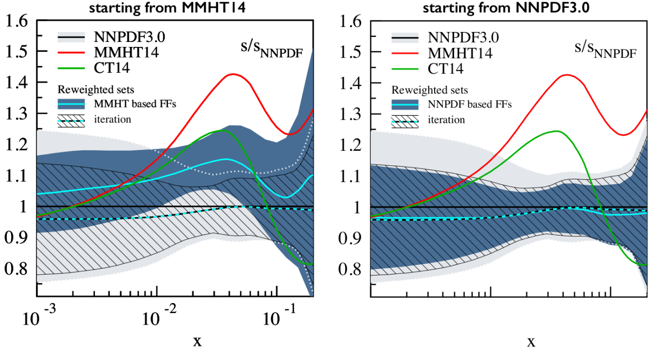
$\Delta\Sigma = 0.36 \pm 0.09$ $\quad$ $\Delta u - \Delta d = 0.05 \pm 0.08$

Simultaneous fits of (unp.) PDFs and FFs [PRD 96 (2017) 094020]

IDEA:
iterative reweighting of PDFs and fit of FFs
with kaon SIDIS data ($N_{\rm dat} = 906$)
HERMES [PRD 87 (2013) 074029]
COMPASS [PLB 767 (2017) 133]



convergence?
and very fast!
robust!



χ^2_{FF} = 1271.7 | 1041.3 | 1002.3   1017.2 | 1005.3 | 1000.6

similar results with CT14 replicas

3. Conclusions

Summary

1. A number of hard-scattering processes require an appropriate knowledge of FFs
    - probing nucleon momentum, spin and flavour
    - studying the prompt atmospheric neutrino flux
    - understanding spatial distributions and the dynamics of nuclear matter
2. Significant role of new data, including LHC data
    - increased accuracy of fragmentation functions
    - increased precision of fragmentation functions
3. Increasing sophistication of the QCD theory
    - needed to catch most of the features of the data
    - includes NNLO, heavy quark mass schemes, resummation
4. Exploit the full potential of SIDIS to improve our knowledge of PDFs
    - simultaneous fits feasible, but challenging
    - combine simultaneous and global fits to make the most from the data

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Thank you

Dependence on α_s