Isolating Neutrino Cross Section Uncertainties with Theory

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May 31, 2018

CIPANP 2018

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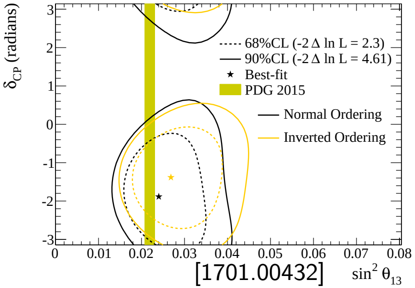
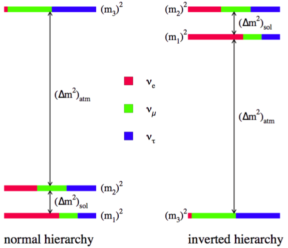
  - Lattice QCD Introduction
  - (Short) Summary of Recent Computations
  - Staggered lattice QCD computation of g_A
- Conclusions

Introduction

Neutrino Oscillation Experiment Goals



[1701.00432]



[1307.5487]

Neutrino oscillation parameters are experimental targets of the upcoming decades

Experiments have several goals:

- precision measurements of $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and θ_{ij}
- determine value of δ_{CP}
- determine sign of Δm_{31}^2; i.e. mass hierarchy

To maximize potential for physics discoveries in oscillation experiments, need precise supporting theoretical predictions

Near/Far Detector Paradigm

Ratio of near/far detector cancels out many experiment systematics

$$\frac{\Phi_{\text{near}}(E_\nu)}{\Phi_{\text{far}}(E_\nu)} = \frac{N_{\text{near}}(E_\nu)}{\sigma_A(E_\nu)} \frac{\sigma_A(E_\nu)}{N_{\text{far}}(E_\nu)} P_{\nu_i \to \nu_j}(E_\nu) \implies \frac{N_{\text{near}}(E_\nu)}{N_{\text{far}}(E_\nu)} P_{\nu_i \to \nu_j}(E_\nu)$$

- Flux normalization errors cancel, ratios can be determined precisely

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Insufficient for next-generation precision measurements

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Insufficient for next-generation precision measurements

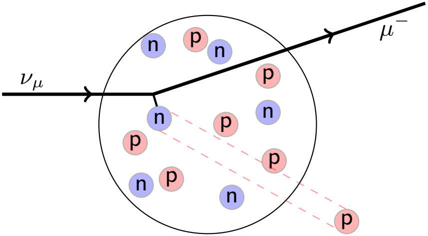
- Assumes that energies can be determined precisely
- Assumes near/far detectors are the same
- Assumes near/far detectors see the same beam

Without these, nuclear cross sections σ_A must still be known precisely

Nuclear Cross Sections

- σ_A determined from some nuclear model, from MC generator
- Most MC generators assume weakly-interacting gas of free nucleons
- Quasielastic scattering off single nucleon allows exact determination of E_ν

$$E_\nu^{QE} = \frac{2(M_n - E_b)E_\ell - ((M_n - E_b)^2 - M_p^2 + m_\ell^2)}{2(M_n - E_b - E_\ell + p_\ell \cos\theta_\ell)}$$



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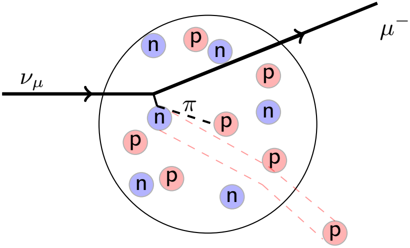

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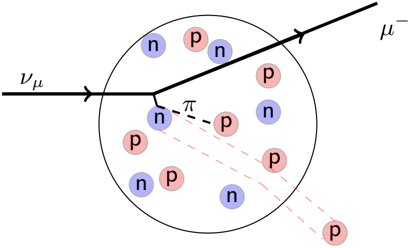
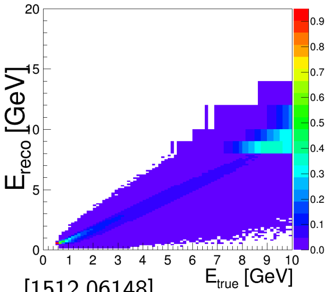
- Quasielastic scattering off single nucleon allows exact determination of E_ν
- Nuclear rescattering can change observed spectrum, topologies
  $\Longrightarrow$ energies cannot be determined event by event!
- Extraction of neutrino energies can only be done as a statistical average including all FSI
- "Reconstruction" to remove nuclear effects, determined from MC studies





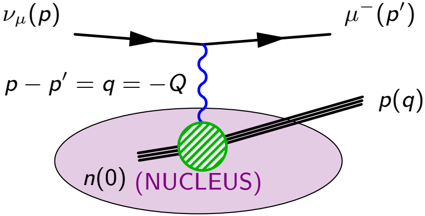
[1512.06148]

Quasielastic scattering

Nucleon amplitudes used as building block to construct sophisticated nuclear models

QE scattering is relatively easy measurement,
relatively theoretically clean:
ν interacts with nearly-free nucleon

QE is primary signal measurement process
for neutrino oscillation experiments



Assumed to be single nucleon interaction, can obtain free nucleon amplitudes

$\implies$ studies of QE scattering enable construction of better nuclear models

CCQE Cross section

$$\frac{d\sigma_{CCQE}}{dQ^2}(E_\nu, Q^2) \propto \frac{1}{E_\nu^2}\left(A(Q^2) \mp \left(\frac{s-u}{M_N^2}\right) B(Q^2) + \left(\frac{s-u}{M_N^2}\right)^2 C(Q^2)\right)$$

$$s - u = 4M_N E_\nu - Q^2 - m_\ell^2 \qquad \eta \equiv \frac{Q^2}{4M_N^2}$$

$$\begin{aligned} A(Q^2) = \frac{m_\ell^2 + Q^2}{M_N^2} \times \\ &\Big[(1+\eta)F_A^2 - (1-\eta)(F_1^2 + \eta F_2^2) + 4\eta F_1 F_2 \\ &- \frac{m_\ell^2}{4M_N^2}\left((F_1+F_2)^2 + (F_A + 2F_P)^2 - 4(1+\eta)F_P^2\right)\Big] \end{aligned}$$

$$B(Q^2) = 4\eta F_A (F_1 + F_2) \qquad C(Q^2) = \frac{1}{4}\left(F_A^2 + F_1^2 + \eta F_2^2\right)$$

- F_1, F_2 from high-statistics monoenergetic e^- scattering on proton target
- F_P suppressed by lepton mass corrections, constrained by PCAC

$\Longrightarrow$ F_A largest contributor to systematic errors

Focus

Want to inform nuclear models with theoretically clean/robust form factors

Take two approaches to constrain nucleon form factor:

- Reanalysis of deuterium bubble chamber data
  - use model-independent z expansion parametrization to study systematic uncertainties
- Lattice QCD calculation
  - compute the axial matrix element from first principles

First step is to compute axial charge: $g_A = F_A(Q^2)\big|_{Q^2=0}$

Future extensions of this work will compute Q^2 dependence, fit to z expansion parametrization

Deuterium Bubble Chamber Reanalysis

Dipole Form Factor

Most analyses assume the Dipole axial form factor (Llewellyn-Smith, 1972):

$$F_A^{\text{dipole}}(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{m_A^2}\right)^2}$$

[Phys.Rept.3 (1972),261]

Dipole is an ansatz:

- inconsistent with QCD
- unmotivated in interesting energy region

$\Longrightarrow$ uncontrolled systematics and therefore underestimated uncertainties

Large variation in m_A over many experiments
(dubbed the "axial mass problem"):

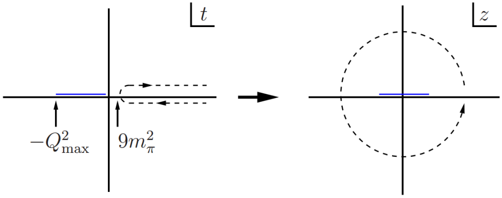
- $m_A = 1.026 \pm 0.021$ (Bernard *et al.*, [arXiv:00107088])
- $m_A^{\text{eff}} = 1.35 \pm 0.17$ (MiniBooNE, [arXiv:1002.2680])

Essential to use model-independent parameterization of F_A instead

z Expansion

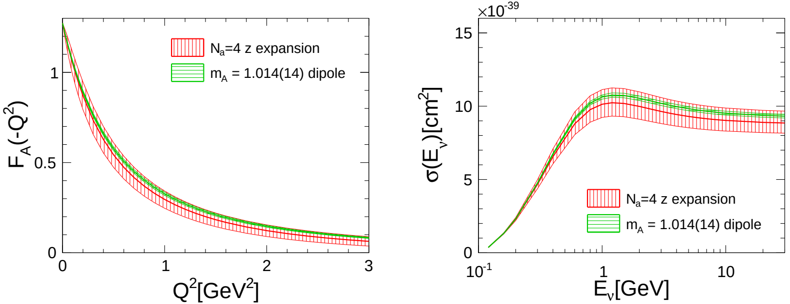
The z Expansion [arXiv:1108.0423] is a conformal mapping which takes kinematically allowed region ($t = -Q^2 \leq 0$) to within $|z| < 1$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \qquad t_c = 9m_\pi^2$$



- Model independent: motivated by analyticity arguments from QCD
- Only few parameters needed: unitarity bounds
- Sum rules regulate large-Q^2 behavior

Reanalysis Results Summary [1603.03048 [hep-ph]]



$$\frac{1}{F_A(0)} \frac{dF_A}{dQ^2}\bigg|_{Q^2=0} \equiv -\frac{1}{6} r_A^2$$

$$r_A^2 = 0.46(22)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n \to \mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.1(0.9) \times 10^{-39} \mathrm{cm}^2$$

compared to [Eur. Phys. J. C 53, 349]:

$$r_A^2 = 0.453(13)\ \mathrm{fm}^2\,, \quad \sigma_{\nu n \to \mu p}(E_\nu = 1\ \mathrm{GeV}) = 10.63(0.14) \times 10^{-39} \mathrm{cm}^2$$

Dipole model significantly underestimates error from nucleon form factor

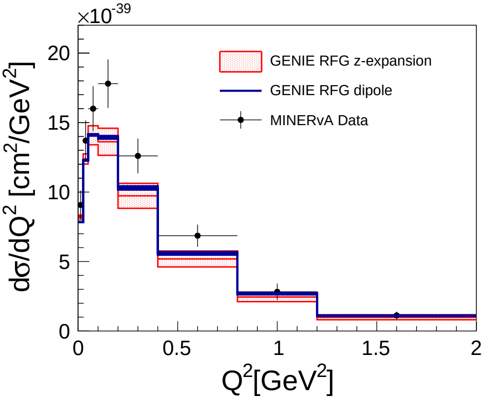
Most theoretically clean data do not constrain form factor precisely

z Expansion in GENIE

z expansion coded into GENIE - may be turned on with configuration switch

Officially released in production version 2.12

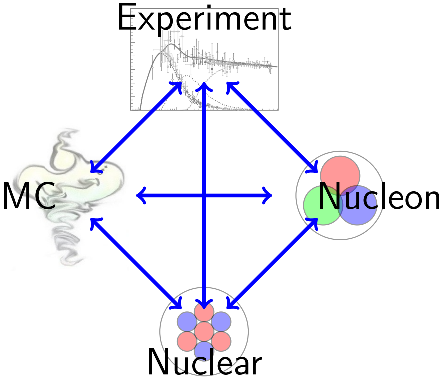
Uncertainties on free-nucleon cross section as large as data-theory discrepancy
$\implies$ need to improve F_A determination to make headway on nuclear effects



See tutorial: https://indico.fnal.gov/event/12824/

Lattice QCD

Why Lattice QCD?



The ideal situation: lots of redundancy and checks between elements of analysis

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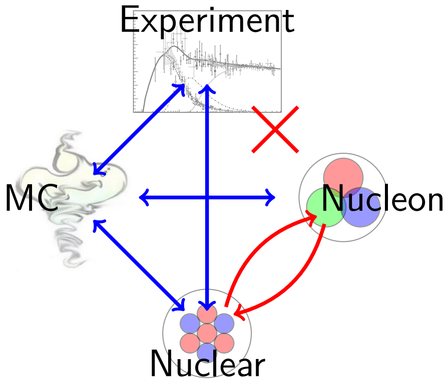

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In reality: F_A not well determined by experiment

$\implies$ nucleon amplitudes constrained by/used to constrain nuclear models

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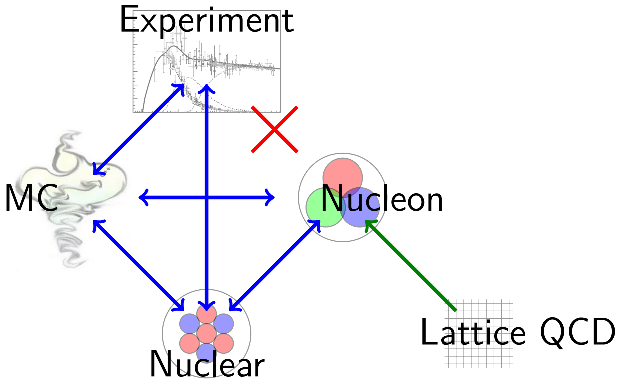

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In reality: F_A not well determined by experiment
$\implies$ nucleon amplitudes constrained by/used to constrain nuclear models

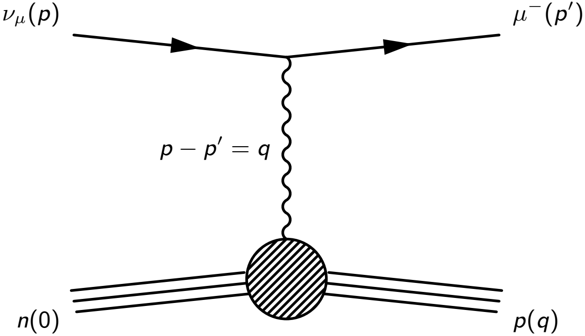
Lattice QCD acts as a disruptive technology to break degeneracy

How Does Lattice Help?

Lattice is well suited to compute matrix elements:

$$\mathcal{M}_{\nu_\mu n \to \mu p}(p, p') = \langle \mu(p')| \left(V_\mu - A_\mu\right) |\nu(p)\rangle \, \langle p(q)| \left(V_\mu - A_\mu\right) |n(0)\rangle$$

Systematically improvable: more computing power $\implies$ more precision



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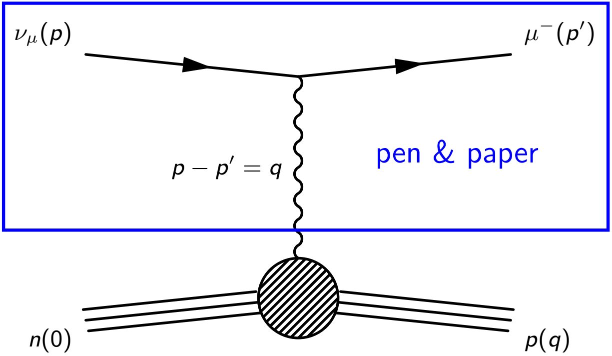

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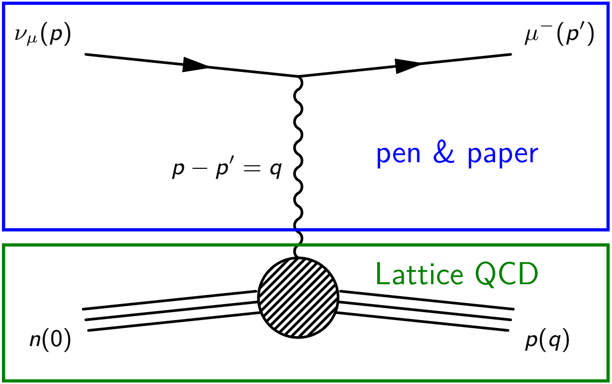


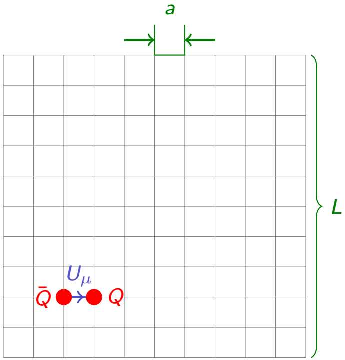
Lattice QCD: Formalism

- Lattice QCD is a technique to numerically evaluate path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

- Discretize spacetime $\implies$ #DOF finite
- Lattice spacing a provides UV cutoff
- Lattice size L provides IR cutoff

- Quark fields on sites $\implies$ $Q(x)$
- Gauge fields between sites $\implies$ $U_\mu(x)$
- Euclidean time $\implies$ correlators $\propto e^{-Et}$



Typical strategy is to construct operators at "source," allow them to propagate through time, then annihilate at "sink"

Evaluate correlation functions on fixed background gauge field, compute on many gauge fields for Monte Carlo average

Correlation functions are products of matrix elements times exponentials, e.g.

$$C(t) = \sum_n |\langle \Omega | \mathcal{O} | n \rangle|^2 \, e^{-E_n t}$$

Nucleon axial form factor $G_A(Q^2)$

Previously, [Lin,0802.0863], [Yamazaki,0904.2039], [Bratt,1001.3620], [Bali,1412.7336]

Needed for neutrino oscillation experiments:

Charged current quasielastic (CCQE) neutrino-nucleus interaction must be known to high precision.

Connecting quark - nucleon level: $G_A(Q^2)$ form factor.
nucleon - nucleus level: nuclear model.

Traditionally: information on $G_A(Q^2)$ extracted from expt. using dipole fit:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{{M_A}^2})^2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

World average (pre 1990) from ν scattering $M_A = 1.026(21)$ GeV.

Overconstrained form: different measurements, different M_A.

Lower energy expts: e.g. MiniBooNE: $M_A = 1.35(17)$ GeV

[Aguilar-Arevalo,1002.2680]

Systematics being explored including new analysis of old expt data:
$\langle r_A^2 \rangle = 0.46(22)\ \text{fm}^2 \rightarrow M_A = 1.01(24)$ GeV from z-expansion [Meyer,1603.03048].

Several computations of $F_A(Q^2)$ appeared in response:

LHPC 1703.06703 [hep-lat]
ETMC 1705.03399 [hep-lat]
CLS 1705.06186 [hep-lat]
PNDME 1705.06834,1801.01635,1801.03130 [hep-lat]

Additional g_A computations: (CalLat) 1704.01114,1710.06523
(JLQCD) 1805.10507

Ref.	g_A	$\langle r_A^2 \rangle$ [fm^2]
LHPC	1.208(6)(16)(1)(10)	0.213(6)(13)(3)(0)
ETMC	1.212(33)(22)	0.267(9)(11)
CLS	$1.278(68)(^{+00}_{-87})$	$0.360(36)(^{+80}_{-88})$
PNDME	1.20(3)	0.25(6)
CalLat	1.285(17)	–
JLQCD	1.123(28)(29)(90)	–

Systematics being explored including new analysis of old expt data:
$\langle r_A^2 \rangle = 0.46(22)$ fm^2 $\rightarrow$ $M_A = 1.01(24)$ GeV from z-expansion [Meyer,1603.03048].

Fermilab Lattice/MILC Effort

We are calculating the axial charge $g_A = F_A(Q^2)\big|_{Q^2=0}$ using staggered quarks on the MILC HISQ 2+1+1 gauge ensembles

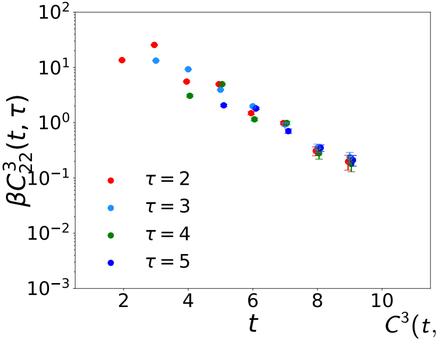
- no explicit chiral symmetry breaking in $m \to 0$ limit
- physical pion mass for multiple lattice spacings
- large volumes
- absolutely normalized
- high-statistics (computationally fast)

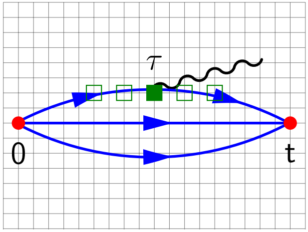
Effort is needed to handle:

- Formalism $\implies$ complicated group theory, difficult fitting

After completing the charge, we will continue to $F_A(Q^2)$ for $Q^2 > 0$

Three-Point Correlation Function





$$C^3(t,\tau) \sim \langle N|\, A_\mu \,|N\rangle \, |a|^2 e^{-M_N \tau} e^{-E_N (t-\tau)}$$

Change of strategy from previous iterations
$\rightarrow$ significant improvement in statistical precision

Current statistical errors at $t = 7$ is about 10%

Conservative estimates predict $< 5\%$ uncertainty on g_A in final analysis

Conclusions

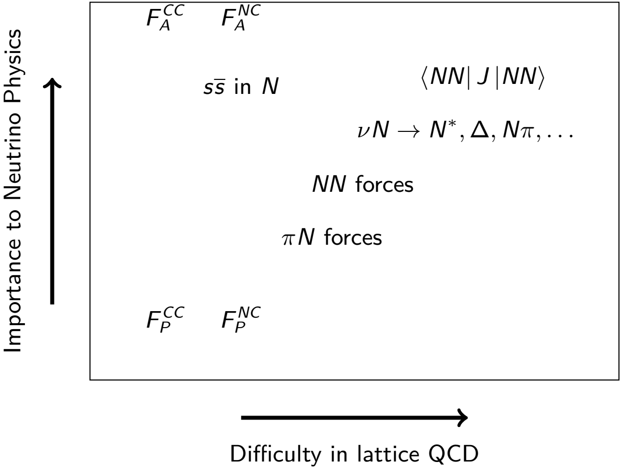
- Precise determinations of nucleon form factors are an essential part of the long-baseline neutrino oscillation program
- Dipole shape underestimates uncertainties in free-nucleon cross sections
- Need robust determination of nucleon amplitudes with realistic errors to determine impact on future neutrino oscillation experiments

- z Expansion parameterization is consistent with QCD and sufficiently general to give realistic uncertainty estimates
- Lattice QCD can access nucleon form factors from first principles
- Growing interest in neutrino physics in lattice community, can expect many new results in upcoming years
- Improvements in methodology for our own lattice computation of g_A, more precise computation expected in near future

Thanks for listening!

Backup

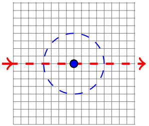
Calculations of Interest



g_A Problem

Why is g_A so difficult?

- Signal-to-Noise Grows Exponentially (Lepage, TASI 1989)
  - Signal $\propto \langle$  $\rangle \sim e^{-M_n t}$, noise$^2 \propto \langle \big|$  $\big|^2 \rangle = \langle$  $\rangle \sim e^{-3 m_\pi t}$
  - Noise gets contribution from 3-pion term
- Finite size effects
  - self-interaction via πs which wrap around periodic BC



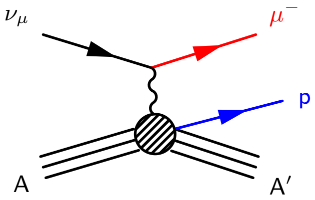
- Excited state contamination
  - Operators couple to ground state + excited states
  - Requires fitting $\sum_n e^{-E_n t}$ for many n
  - Rotation/tranlation symmetry broken by lattice

Nuclear Effects

Nuclear effects not well understood

$\rightarrow$ Models which are best for one measurement
are worst for another

Need to break F_A/nuclear model entanglement

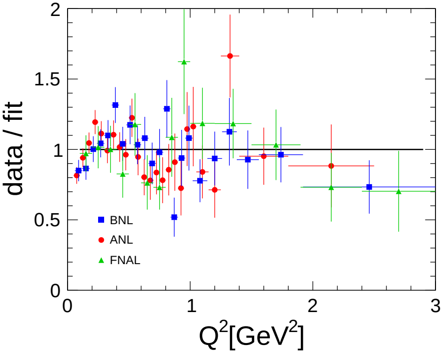


(assumed $m_A = 0.99$ GeV)

NuWro Model (χ^2/DOF)	RFG [GENIE]	RFG+ TEM	assorted others
leptonic(rate)	3.5	2.4	2.8-3.7
leptonic(shape)	4.1	1.7	2.1-3.8
hadronic(rate)	1.7[1.2]	3.9	1.9-3.7
hadronic(shape)	3.3[1.8]	5.8	3.6-4.8

(Minerva collaboration, 1305.2243,1409.4497[hep-ph])

Residuals



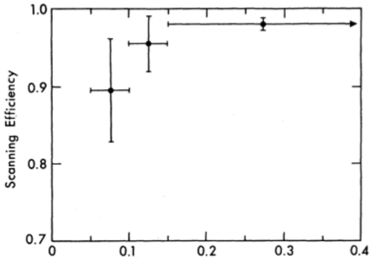
Neither z expansion, nor dipole can properly explain shape of data

Difficult to extract form factor from scattering data,
uncontrolled systematics introduced in process

Acceptance Corrections

Acceptance correction for fixing errors from hand scanning
Q^2 dependent correction, correlated between bins:

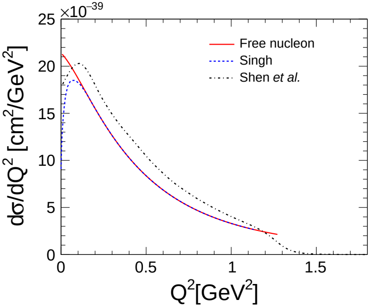
$$\frac{dN}{e(Q^2)} \to \frac{dN}{e(Q^2) + \eta\, de(Q^2)}\,, \quad \eta = 0 \pm 1$$



For ANL, BNL, FNAL respectively, $\eta = -1.9, -1.0, +0.01$; minimal improvement of goodness of fit

Deuterium Corrections

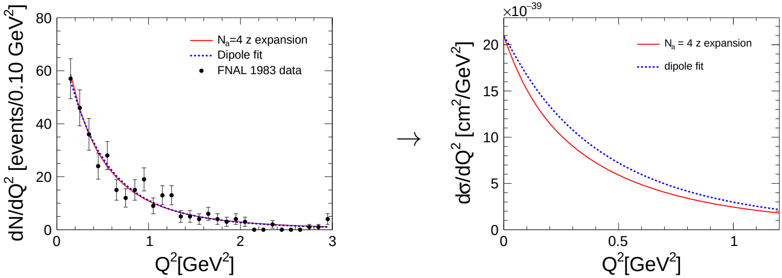
Corrections assumed to be E_ν independent
Two corrections tested:
Singh Nucl. Phys. B 36, 419,
Shen 1205.4337 [nucl-th]



Central values of Shen, Singh are consistent with each other
Final fit done with Singh, inflated error bars

Normalization Degeneracy

Despite apparent similarity of dipole/z expansion cross sections, form factors quite different



Consequence of self-consistency: cross section prediction

$$\frac{dN}{dE} \propto \frac{1}{\sigma}\frac{d\sigma}{dQ^2}$$

Cut of low-Q^2 data & floating normalization hide cross section differences