



THE LOW-ENERGY FRONTIER  
OF THE STANDARD MODEL

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cipanp18.berkeley.edu

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May 29 - June 3  
Hyatt Regency Indian Wells Resort and Spa, Palm Springs, CA*

# Dispersive analysis of hadronic light-by-light contribution to $(g-2)_\mu$

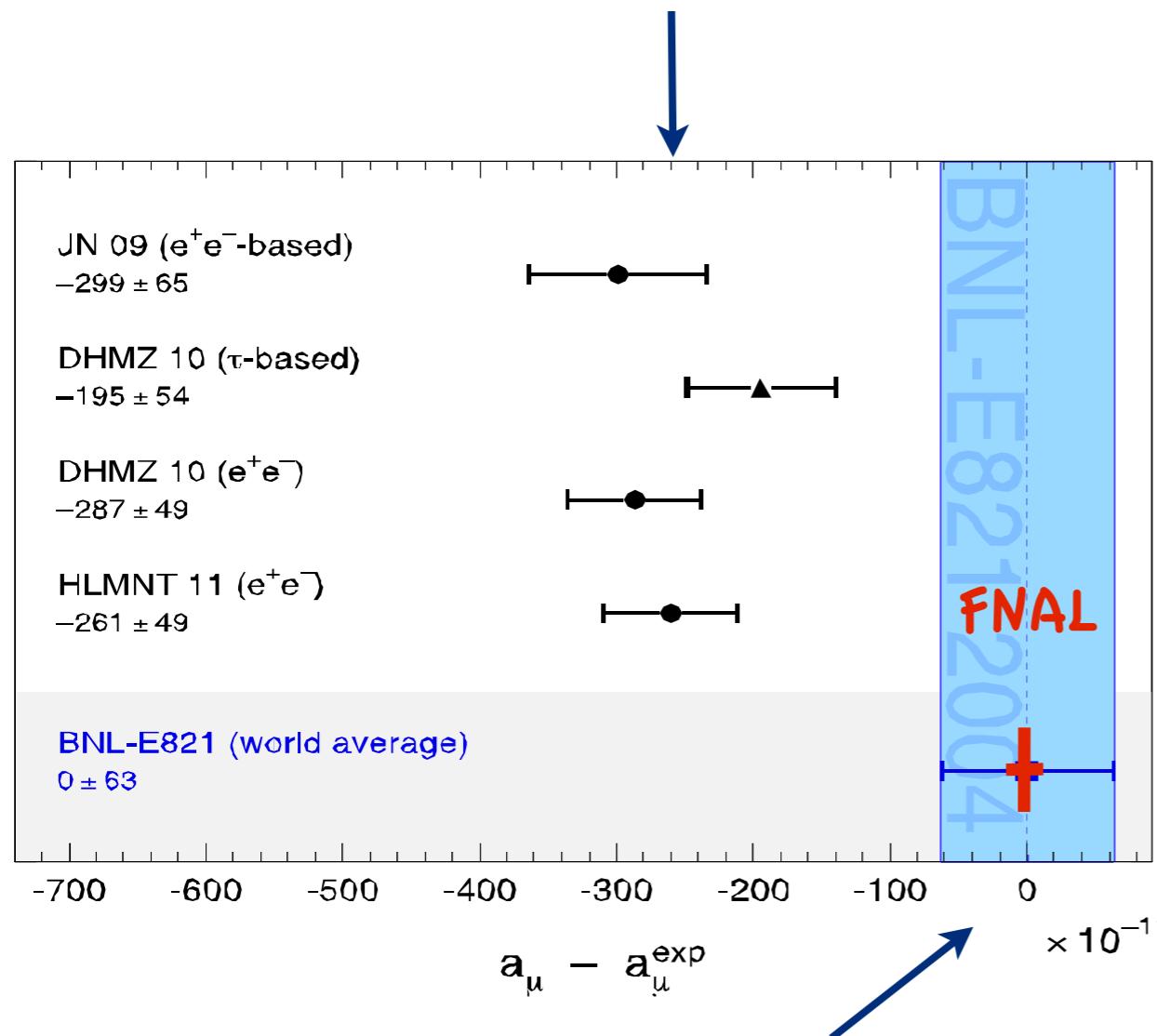
Marc Vanderhaeghen (in collaboration with Igor Danilkin)

**CIPANP 2018, May 29- June 3, 2018**

Palm Springs, CA (USA)

# $(g-2)_\mu$ : theory vs experiment

SM predictions for  $a_\mu$



$$a_\mu^{\text{exp}} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \\ (28.1 \pm 3.6_{\text{th}} \pm 6.3_{\text{exp}}) \times 10^{-10}$$

Teubner et al. (2017)

3 - 4  $\sigma$  deviation  
from SM value !

Errors or new physics ?

New FNAL, J-PARC experiments

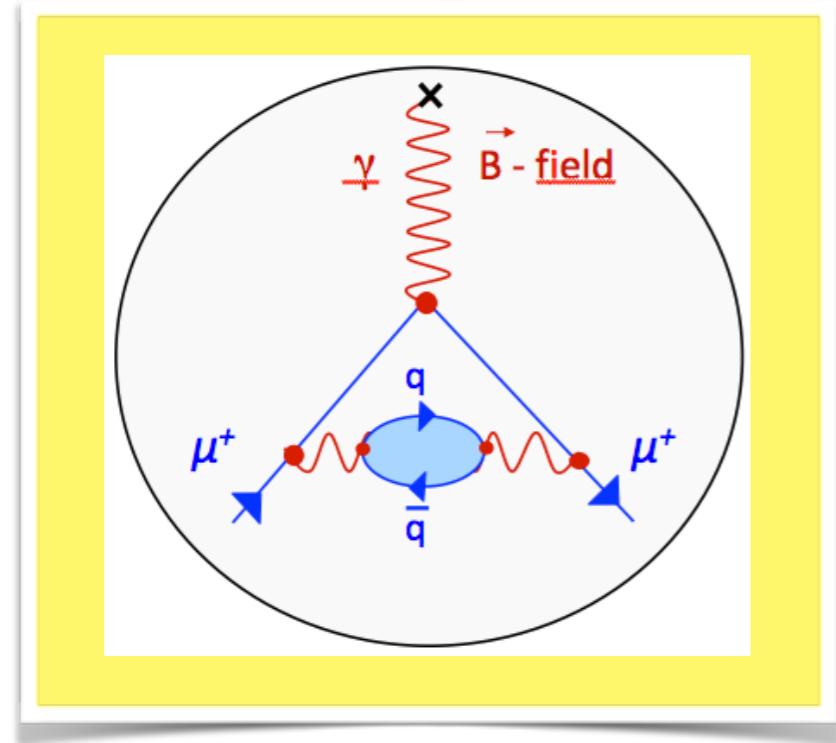
$\delta a_\mu^{\text{FNAL}} = 1.6 \times 10^{-10}$  talk R. Hong

factor 4 improvement in exp. error

-> Improve theory !

# Hadronic contributions to $(g-2)_\mu$

hadronic vacuum polarization (HVP)

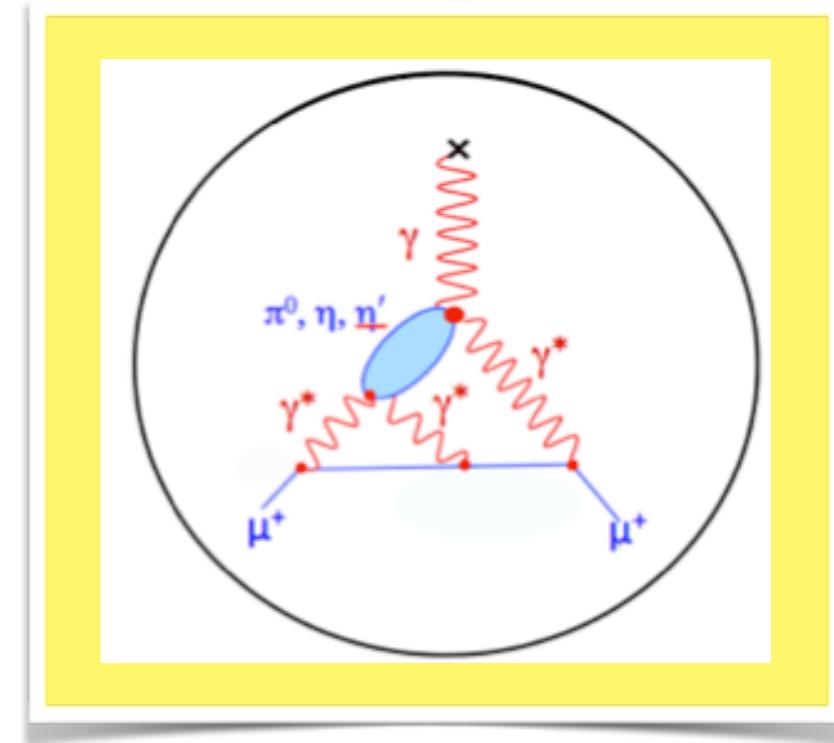


$$a_\mu^{\text{I.o. had, VP}} = (692.2 \pm 2.5) \times 10^{-10}$$

Teubner et al. (2017)

talk Ch. Redmer

hadronic light-by-light scattering (HLbL)



$$a_\mu^{\text{had, LbL}} = (10.5 \pm 2.6) \times 10^{-10} \quad (\text{I})$$

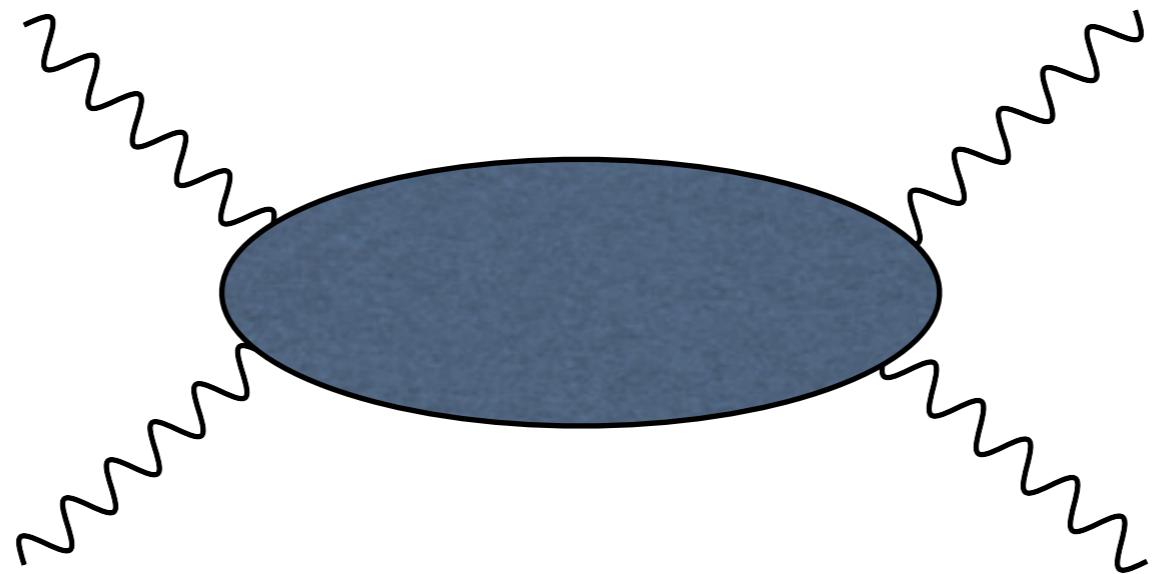
$$= (10.2 \pm 3.9) \times 10^{-10} \quad (\text{II})$$

- (I) Prades, de Rafael, Vainshtein (2009)
- (II) Jegerlehner, Nyffeler (2009) Jegerlehner (2015)

New FNAL and J-Parc  $(g-2)_\mu$  expt. :  $\delta a_\mu^{\text{exp}} = 1.6 \times 10^{-10}$

HVP determined by cross section measurements of  $e^+e^- \rightarrow \text{hadrons}$

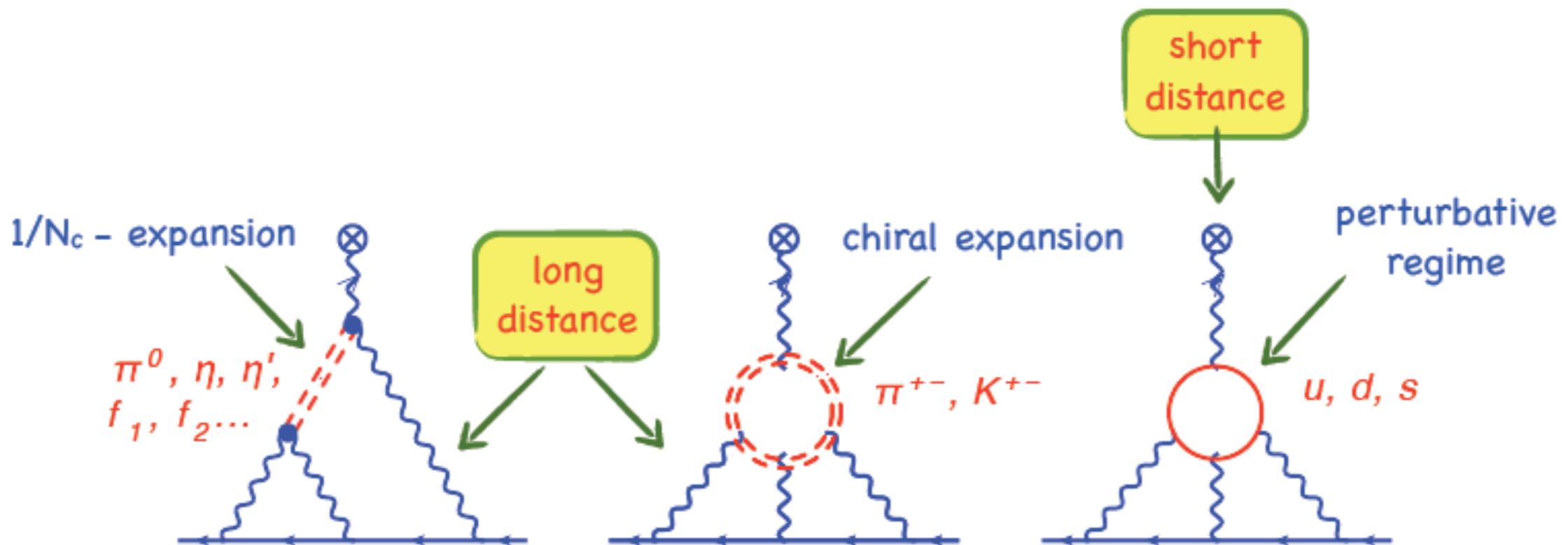
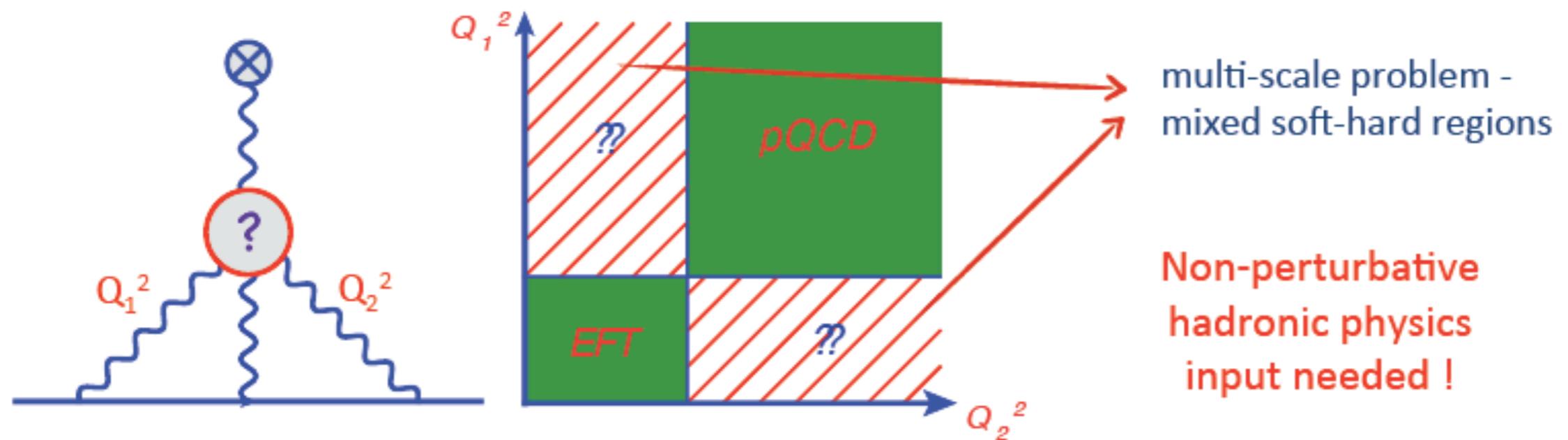
measurements of meson transition form factors required as input to reduce uncertainty



**what is known about hadronic LbL scattering ?**

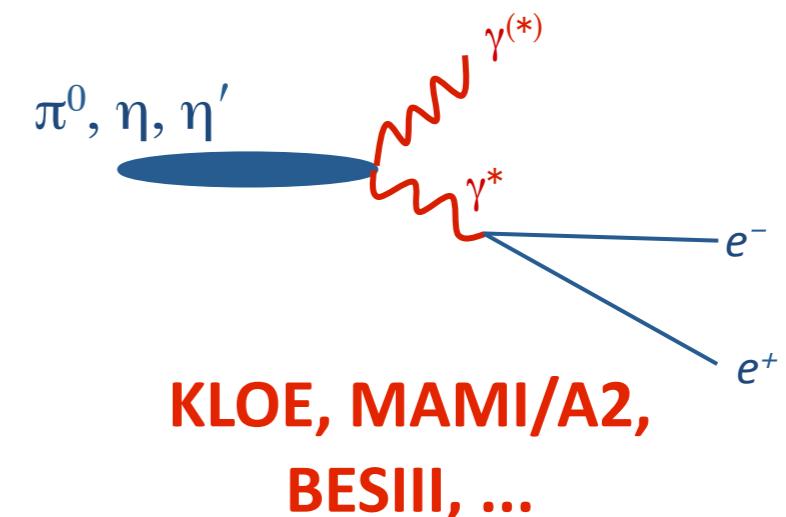
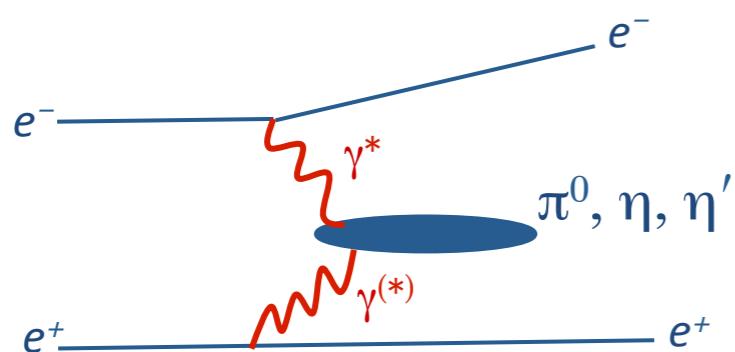


# hadronic LbL corrections to $(g-2)_\mu$ : relevant contributions

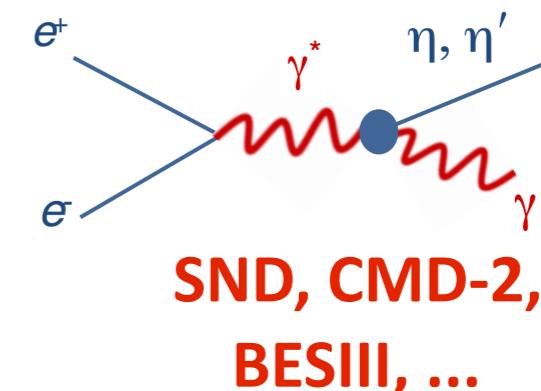


# hadronic LbL corrections to $(g-2)_\mu$

→ experimental input: meson transition FFs,  $\gamma^* \gamma^* \rightarrow$  multi-meson states, meson Dalitz decays



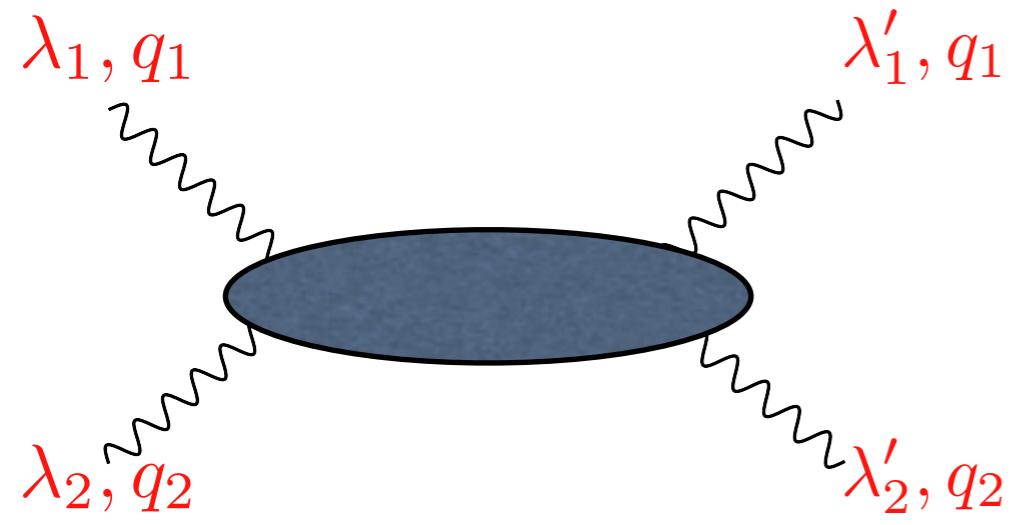
**CLEO, BaBar,  
Belle, BESIII, ...**



→ theory developments:

- sum rules, dispersion relations
- lattice QCD
- Dyson-Schwinger
- phenomenology, modeling

# Theory: sum rules for LbL scattering (I)



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \rightarrow \gamma^*(\lambda'_1, q_1) + \gamma^*(\lambda'_2, q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \quad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes:

$$M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2}(\nu, Q_1^2, Q_2^2) \quad \lambda = 0, \pm 1$$

discrete symmetries:

81



8 independent amplitudes:

$$P : \quad M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2} = M_{-\lambda'_1 - \lambda'_2, -\lambda_1 - \lambda_2}$$

$$T : \quad M_{\lambda'_1 \lambda'_2, \lambda_1 \lambda_2} = M_{\lambda_1 \lambda_2, \lambda'_1 \lambda'_2}$$

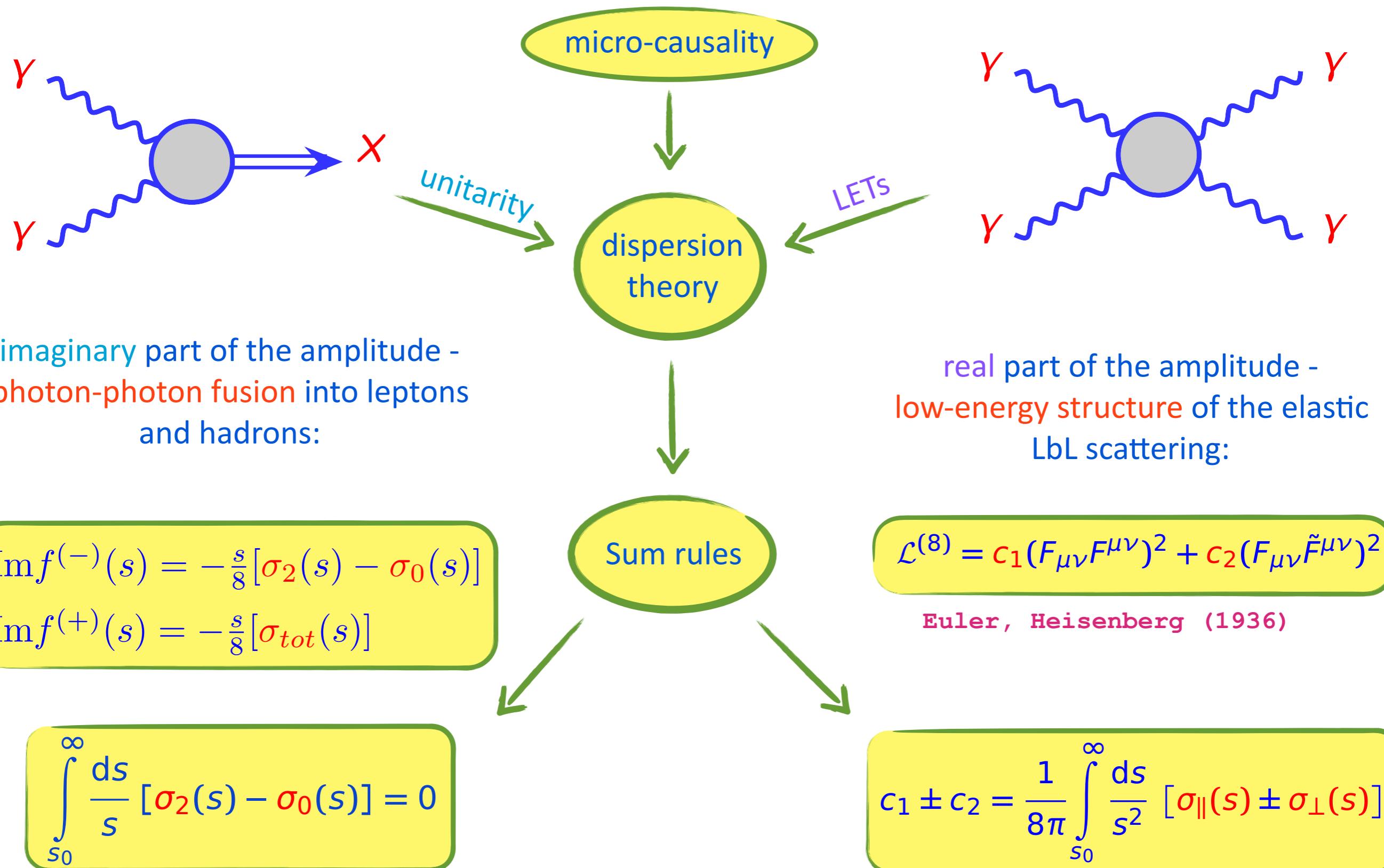
$$M_{++,++}, M_{+-,+-}, M_{++,-},$$

$$M_{00,00}, M_{+0,+0}, M_{0+,0+}, M_{++,00}, M_{0+,-0}$$

T

T and L

# sum rules for LbL scattering (III)



# sum rules for LbL scattering: 3 superconvergence relations

→ helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

for  $Q^2 = 0$ : GDH sum rule

Gerasimov, Moulin (1975),  
Brodsky, Schmidt (1995)

the I=0 channel

meson contributions to helicity  
SR for  $Q_1^2 = 0$  (in nb)

→ sum rules involving L photons

$$0 = \int_{s_0}^{\infty} ds \left[ \frac{\tau_{TL}}{Q_1 Q_2} \right]_{Q_2^2=0}$$

State	m (MeV)	$\Gamma_{\gamma\gamma}$ (keV)	SR <sub>1</sub> ( $Q^2 = 0$ )
$\eta$	$547.862 \pm 0.017$	$0.516 \pm 0.020$	$-193 \pm 7$
$\eta'$	$957 \pm 0.06$	$4.35 \pm 0.25$	$-304 \pm 17$
$f_2(1270)$	$1275.5 \pm 0.8$	$2.93 \pm 0.40$	$(\Lambda=2) 434 \pm 60$ $(\Lambda=0) \approx 0$
$f_2(1565)$	$1562 \pm 13$	$0.70 \pm 0.14$	$56 \pm 11$
.....			
sum			$-7 \pm 64$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q_1^2)^2} \left[ \sigma_{\parallel} + \sigma_{LT} + \frac{(s + Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

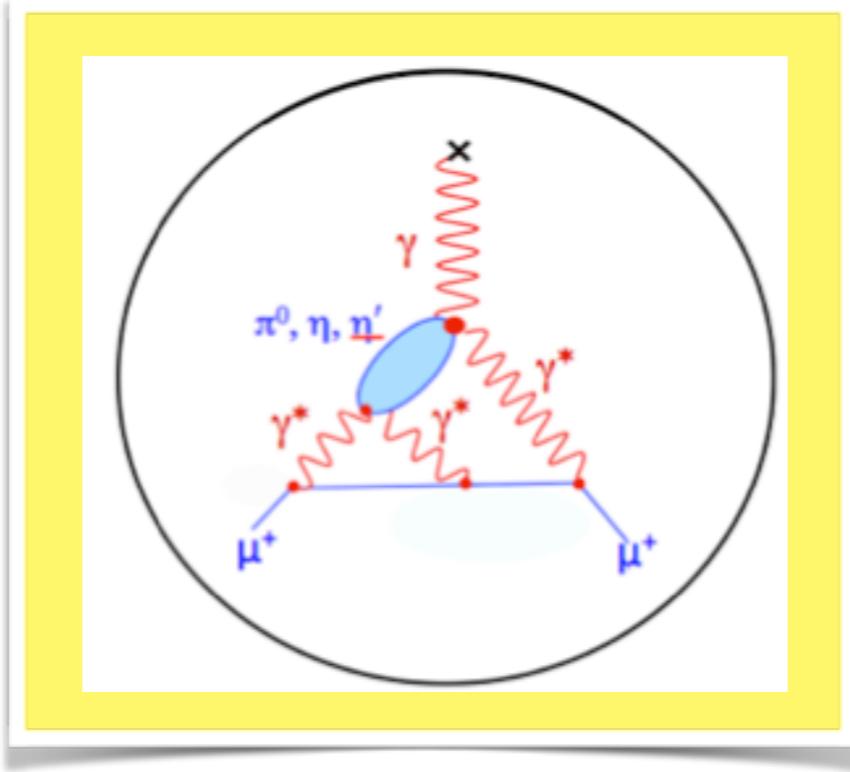
lowest few meson states saturate sum rules

Pascalutsa, Vdh (2010)

Pascalutsa, Pauk, Vdh (2012, 2014)

→ Comparison lattice calculation for forward  $\gamma^* \gamma^*$  scattering  
with dispersive estimates

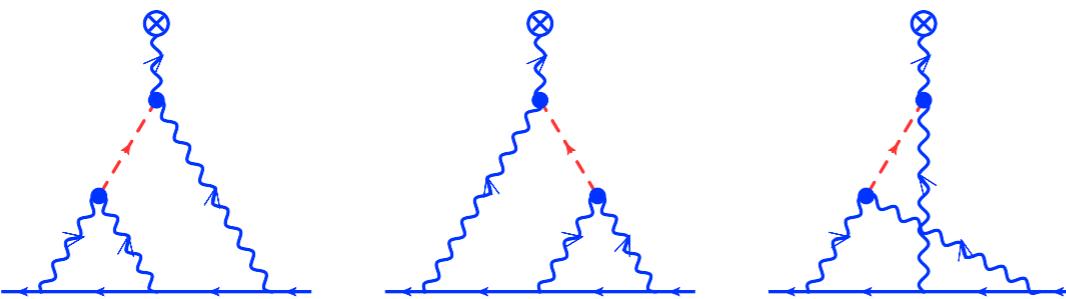
talk V. Pascalutsa



how to estimate the HLbL contribution to  $a_\mu$  ?

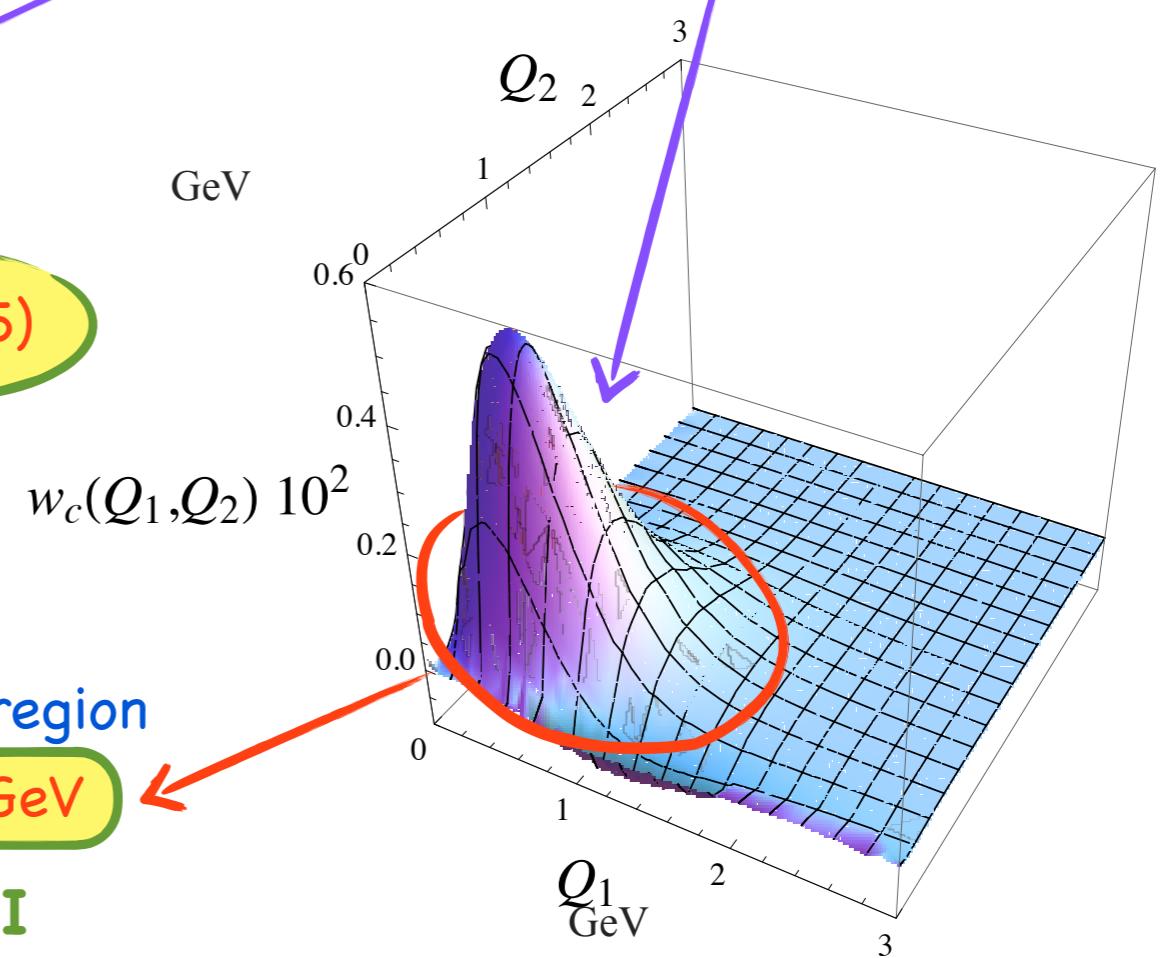
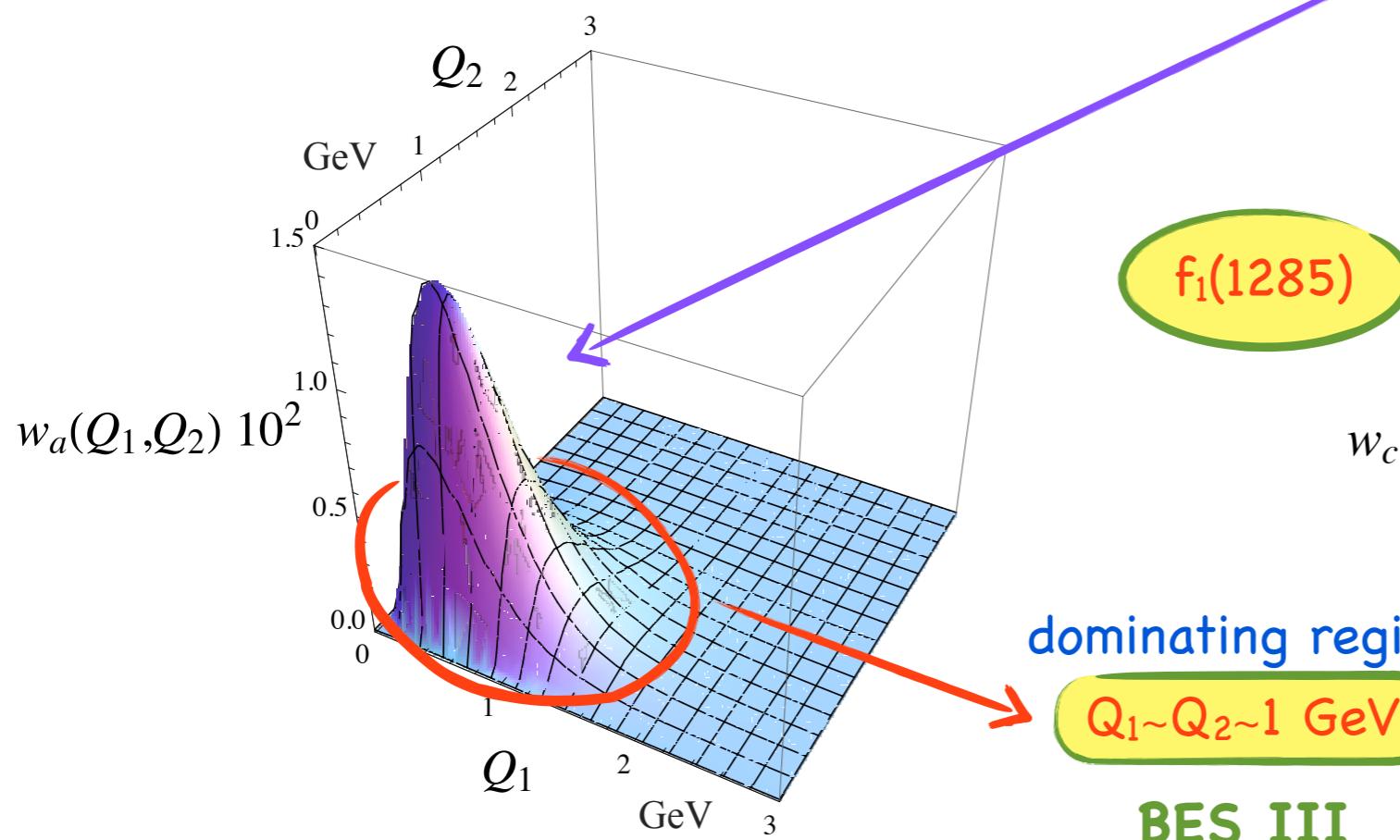


# single meson contributions to $a_\mu$



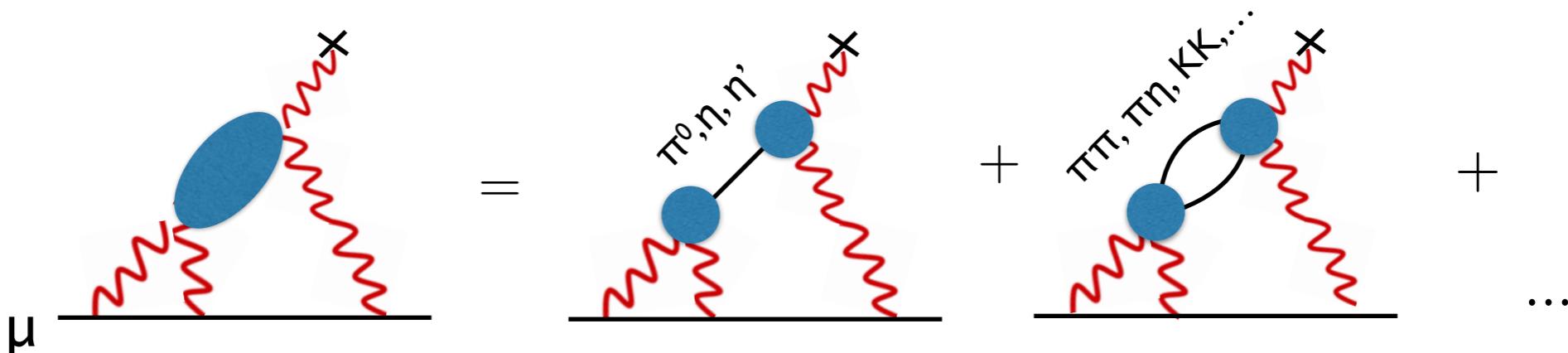
for  $\pi^0$ : Knecht, Nyffeler (2002)  
extended in many works

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m m_A^5} \int dQ_1 \int dQ_2 [2w_a(Q_1, Q_2) + w_c(Q_1, Q_2)]$$



Pauk, vdh (2013)

# HLbL to $a_\mu$ : present status



→ Total HLbL

$[a_\mu \text{ in units } 10^{-10}]$

Authors	$\pi^0, \eta, \eta'$	$\pi\pi, KK$	scalars	axial vectors	quark loops	Total
BPaP(96)	8.5(1.3)	-1.9(1.3)	-0.68(0.20)	0.25(0.10)	2.1(03)	8.3(3.2)
HKS(96)	8.3(0.6)	-0.5(0.8)	—	0.17(0.17)	1.0(1.1)	9.0(1.5)
KnN(02)	8.3(1.2)	—	—	—	—	8.0(4.0)
MV(04)	11.4(1.0)	—	—	2.2(0.5)	—	13.6(2.5)
PdRV(09)	11.4(1.3)	-1.9(1.9)	-0.7(0.7)	1.5(1.0)	0.23	<b>10.5(2.6)</b>
N/JN(09)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	2.2(0.5)	2.1(0.3)	11.6(3.9)
J(15)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	<b>0.75(0.27)</b>	2.1(0.3)	<b>10.2(3.9)</b>

B=Bijnens, Pa=Pallante, P=Prades, H=Hayakawa, K=Kinoshita, S=Sanda, Kn=Knecht, N=Nyffeler,  
M=Melnikov, V=Vainshtein, dR=de Rafael, J=Jegerlehner

→

Tensor meson contribution:  $\sim 0.1 \times 10^{-10}$  (small relative to  $1.6 \times 10^{-10}$ )

Pauk, vdh (2013)

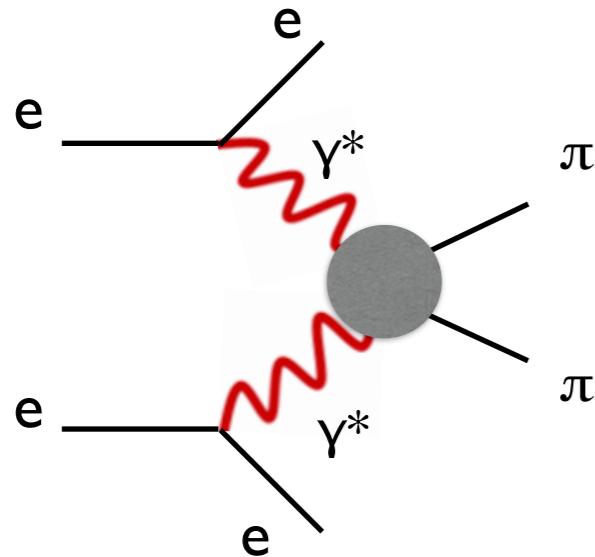
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Improvements: include multi-meson channels in a data-driven / dispersive approach

Danilkin, vdh (2016)

# Improvements: multi-meson production

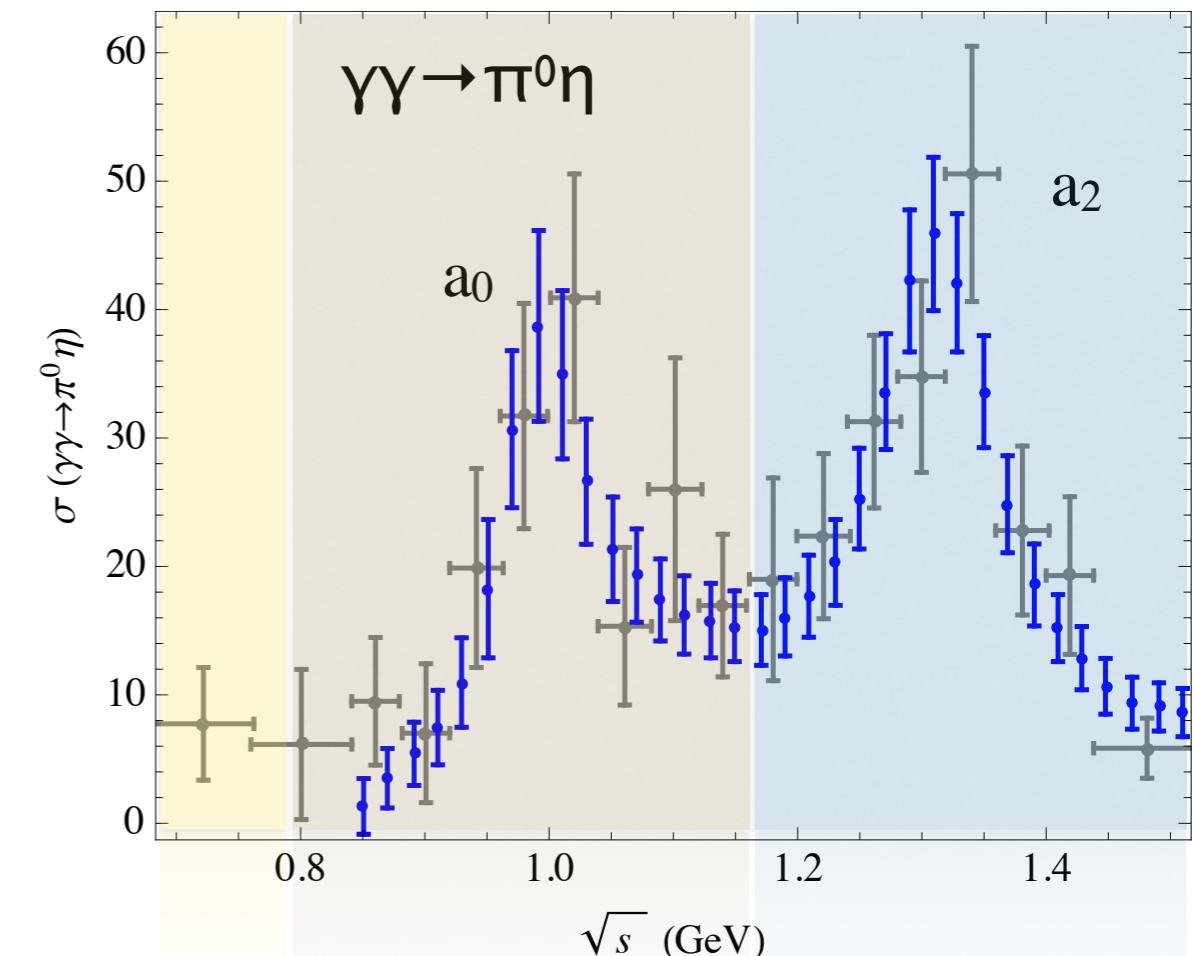
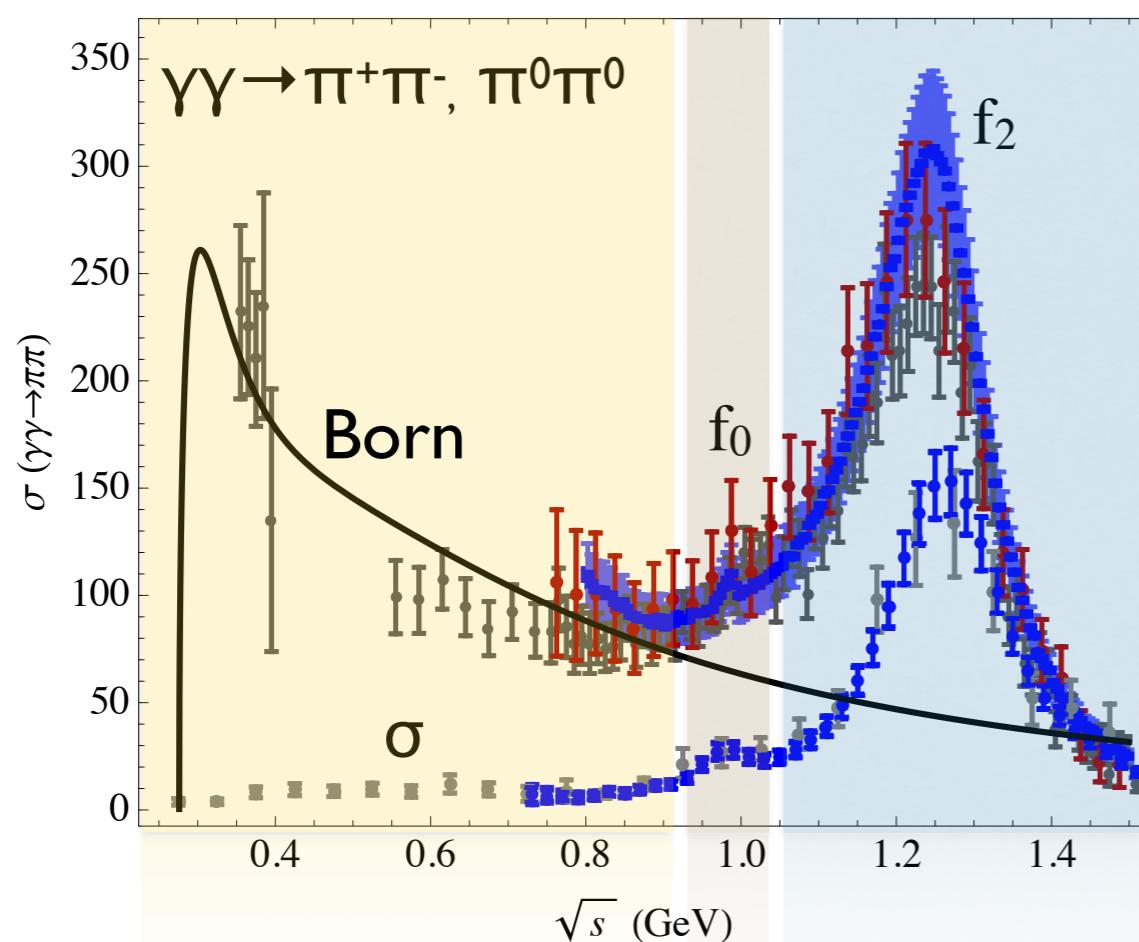
Observables in experiment  $e^+e^- \rightarrow e^+e^-\pi\pi$



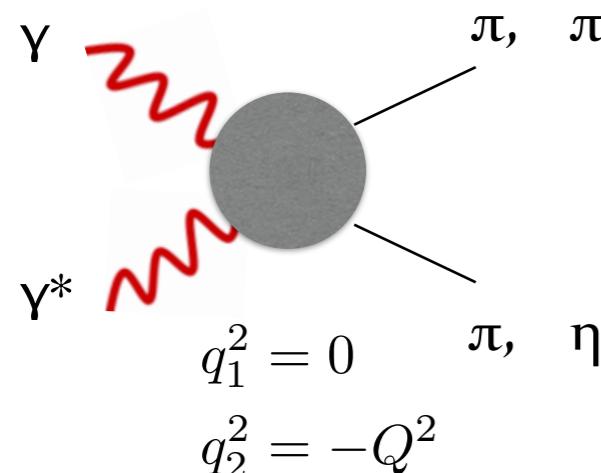
$$d\sigma = \frac{\alpha^2}{16\pi^4 Q_1^2 Q_2^2} \frac{2\sqrt{X}}{s(1-4m^2/s)^{1/2}} \cdot \frac{d^3 \vec{p}'_1}{E'_1} \cdot \frac{d^3 \vec{p}'_2}{E'_2}$$

$$\times \left\{ 4\rho_1^{++}\rho_2^{++} \sigma_{TT} + \rho_1^{00}\rho_2^{00} \sigma_{LL} + 2\rho_1^{++}\rho_2^{00} \sigma_{TL} + \dots \right\},$$

$\gamma\gamma \rightarrow \pi\pi, KK, \eta\eta, \pi\eta$  (Belle: 07,08,09,10,..)  
 $\gamma\gamma^* \rightarrow \pi\pi, \pi\eta$  (BESIII in progress)



# Cross section



$C=+1: J^{PC}=0^{++}, 2^{++}, \dots$

Helicity amplitudes

$$\langle \pi(p_1)\pi(p_2) | T | \gamma(q_1, \lambda_1)\gamma(q_2, \lambda_2) \rangle = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - q_1 - q_2) H_{\lambda_1 \lambda_2}$$

$$H_{\lambda_1 \lambda_2} = H^{\mu\nu} \epsilon_\mu(\lambda_1) \epsilon_\nu(\lambda_2), \quad \lambda_1 = \pm 1, \lambda_2 = \pm 1, 0$$

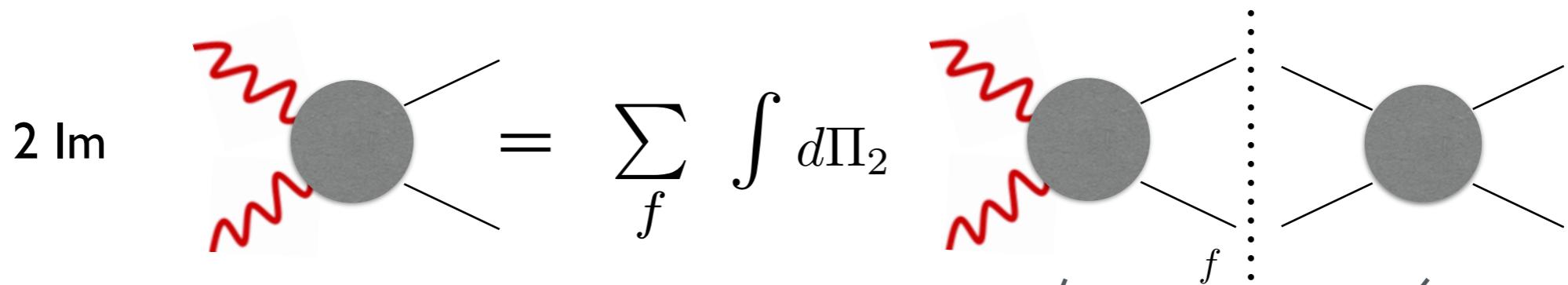
P symmetry: **6** → **3** independent amplitudes  $H_{++}, H_{+-}, H_{+0}$

Cross sections

$$\sigma_{TT} = \pi \alpha^2 \frac{\rho(s)}{4(s + Q^2)} \int d\cos\theta (|H_{++}|^2 + |H_{+-}|^2)$$

$$\sigma_{TL} = \pi \alpha^2 \frac{\rho(s)}{2(s + Q^2)} \int d\cos\theta |H_{+0}|^2$$

# Unitarity



Partial wave expansion

$$H_{\lambda_1 \lambda_2}(s, t) = \sum_{J=0}^{\cancel{\infty}} (2J+1) h_{J, \lambda_1 \lambda_2}(s) d_{\lambda_1 - \lambda_2, 0}^J(\theta)$$

$J_{max} = 2$

$$T(s, t) = \sum_{J=0}^{\cancel{\infty}} (2J+1) t_J(s) P_J(\theta)$$

$J_{max} = 2$

These “diagonalise unitarity” and contain resonance information

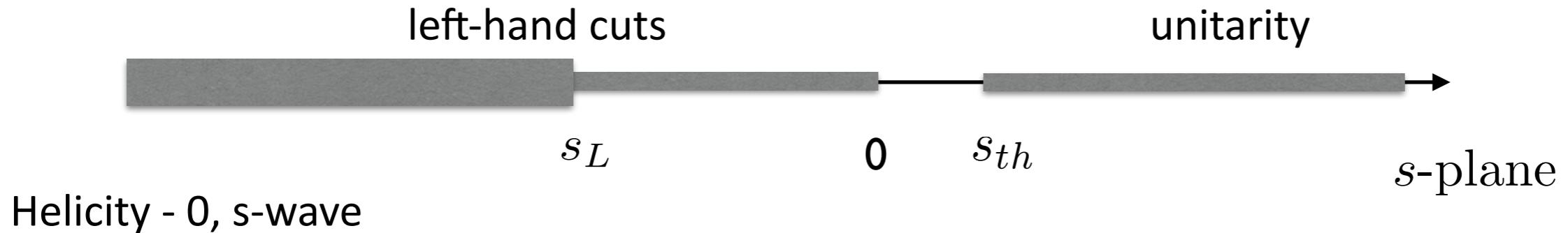
definite:  $J, \lambda_1, \lambda_2$

$$\text{Im } h_{\gamma\gamma^* \rightarrow \pi\pi}(s) = \rho_{\pi\pi} h_{\gamma\gamma^* \rightarrow \pi\pi} t_{\pi\pi \rightarrow \pi\pi}^* + \rho_{KK} h_{\gamma\gamma^* \rightarrow KK} t_{KK \rightarrow \pi\pi}^* + \dots$$

# Dispersion relation

Morgan et al. (1998)  
 Garcia-Martin et al. (2010)  
 Moussallam (2013)

Write a dispersive representation for  $\Omega^{-1}(s)(h(s) - h^{Born}(s))$  definite:  $J, \lambda_1, \lambda_2$



$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s) \left[ \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'^2} \frac{\Omega(s')^{-1}}{s' - s} \begin{pmatrix} \text{Im } \bar{h}_{++}(s') \\ \text{Im } \bar{k}_{++}(s') \end{pmatrix} - \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'^2} \frac{\text{Im } \Omega(s')^{-1}}{s' - s} \begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix} \right]$$



Coupled-channel Omnes  
function

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi \rightarrow \pi\pi} & \Omega_{\pi\pi \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\pi} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$

Unitarity  $s \geq s_{th}$

$$\begin{aligned} \text{Im } h(s) &= h(s) \rho(s) t^*(s) \\ \text{Im } \Omega(s) &= \Omega(s) \rho(s) t^*(s) \end{aligned}$$

# What has been done so far?

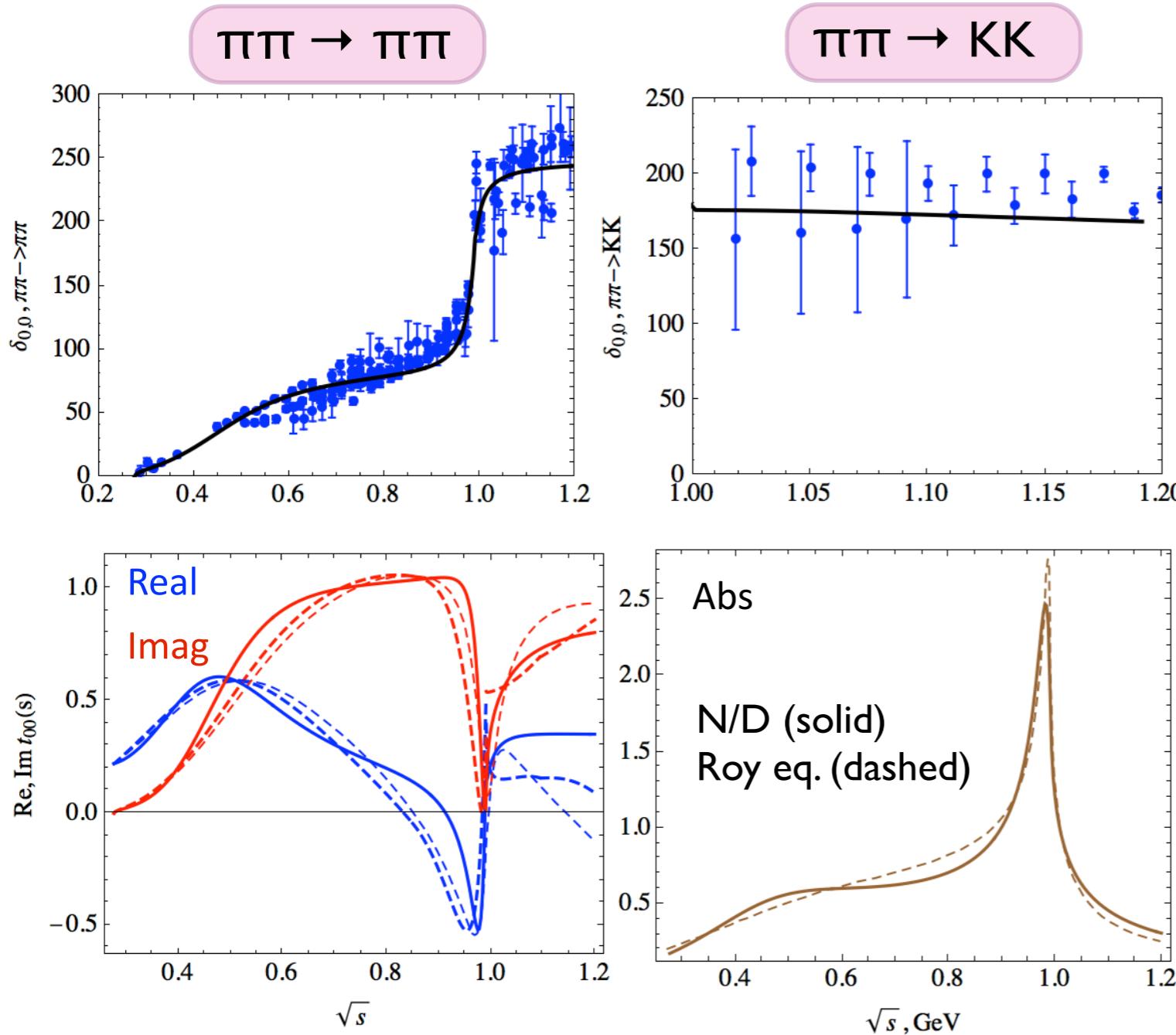
$Q^2 = 0$	Approach	Inelasticity	Number of fitted parameters to $\sigma_{\gamma\gamma \rightarrow \text{MM}}$	Range of applicability
[Hoferichter et. al. 2011]	Roy-Steiner	$\pi\pi$	0	$\sqrt{s} < 0.98 \text{ GeV}$
[Morgan et. al. 1998]	Disp, Omnes	$\pi\pi$	0	$\sqrt{s} \lesssim 0.6 \text{ GeV}$
[Dai et. al. 2014]	Amplitude anal.	$\pi\pi, KK$	>20	$\sqrt{s} < 1.5 \text{ GeV}$
[Garcia-Martin et.al. 2010]	Disp, Omnes	$\pi\pi, KK$	6	$\sqrt{s} < 1.3 \text{ GeV}$
[Current work]	Disp, Omnes	$\pi\pi, KK$ $\pi\eta, KK$	0	$\sqrt{s} < 1.4 \text{ GeV}$
<hr/>				
$Q^2 \neq 0$				
[Moussallam 2013]	Disp, Omnes	$\pi\pi, J=0$	0	$\sqrt{s} \lesssim 0.8 \text{ GeV}$
[Colangelo et.al. 2017]	Roy-Steiner	$\pi\pi, J=0$	0	$\sqrt{s} \lesssim 0.8 \text{ GeV}$
[Current work]	Disp, Omnes	$\pi\pi, KK, J=0,2$ $\pi\eta, KK$	0	$\sqrt{s} < 1.4 \text{ GeV}$

Only dispersive analyses are shown

# Omnes function $|l|=0, \{\pi\pi, K\bar{K}\}$

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi \rightarrow \pi\pi} & \Omega_{\pi\pi \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\pi} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$



Danilkin, Gil, Lutz (2011, 2013)

**Bounded p.w. amplitudes and  
Omnes at large energies**

$$T(s) = \Omega(s) N(s)$$

$$N(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s') (U(s) - U(s'))}{s' - s}$$

$$\Omega^{-1}(s) = 1 - \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s')}{s' - s}$$

$$U(s) = \sum_k C_k \xi(s)^k$$

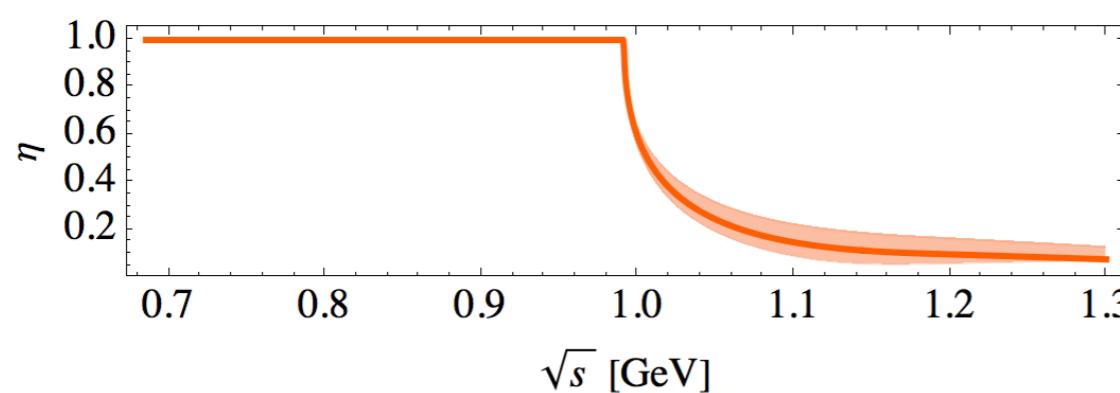
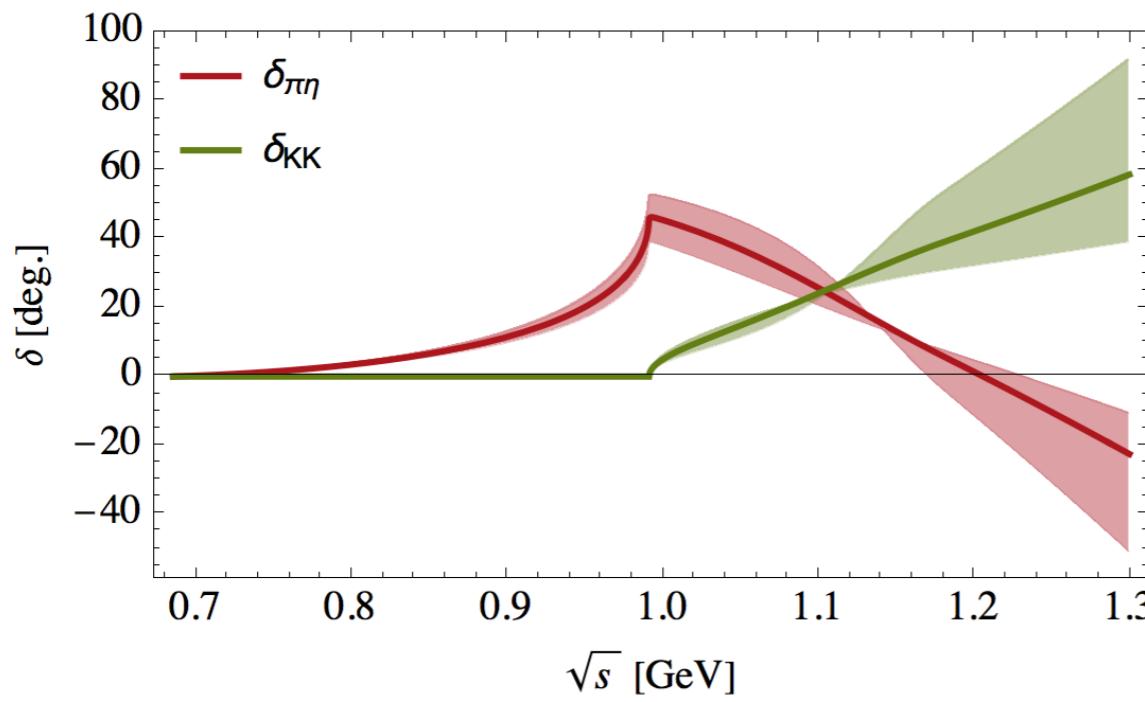
$C_k$  fitted to exp. data  
and Roy Eq. solutions

# Omnes function $|=1, \{\pi\eta, K\bar{K}\}$

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta \rightarrow \pi\eta} & \Omega_{\pi\eta \rightarrow K\bar{K}} \\ \Omega_{K\bar{K} \rightarrow \pi\eta} & \Omega_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}$$

$\pi^0\eta \rightarrow \pi^0\eta$



**Bounded p.w. amplitudes and  
Omnes at large energies**

$$T(s) = \Omega(s) N(s)$$

$$N(s) = U(s) + \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s') (U(s) - U(s'))}{s' - s}$$

$$\Omega^{-1}(s) = 1 - \frac{s}{\pi} \int_R \frac{ds'}{s'} \frac{\rho(s') N(s')}{s' - s}$$

$$U(s) = \sum_k C_k \xi(s)^k$$

**$C_k$  matched to SU(3)  
ChPT at threshold**

Danilkin, Gil, Lutz (2011, 2013)

# Left-hand cuts

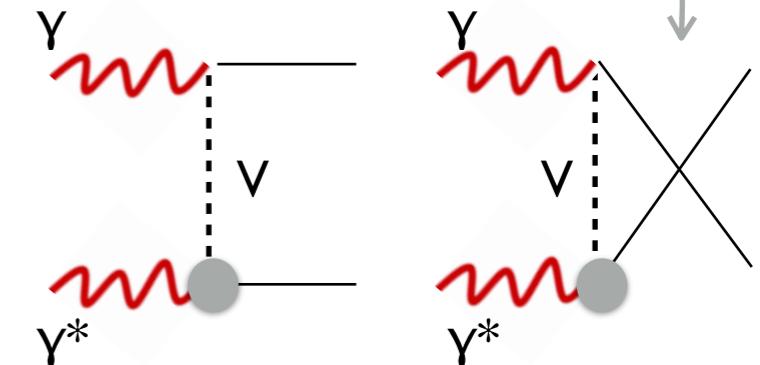
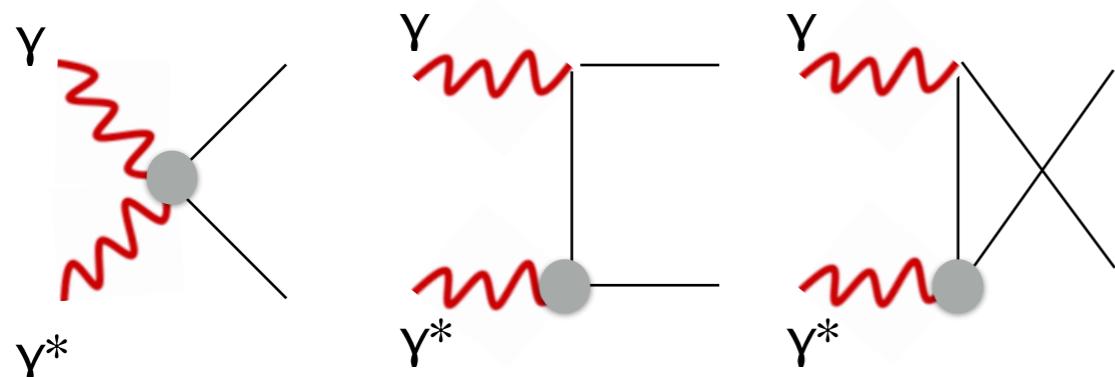
Morgan et al. (1998)

Dispersive integral for  $J=0$

Moussallam (2013)

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s) \left[ \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'^2} \frac{\Omega(s')^{-1}}{s' - s} \begin{pmatrix} \text{Im } \bar{h}_{++}(s') \\ \text{Im } \bar{k}_{++}(s') \end{pmatrix} \right. \\ \left. - \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'^2} \frac{\text{Im } \Omega(s')^{-1}}{s' - s} \begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix} \right]$$

Scalar QED (pion pole contribution)



Fearing, Scherer (1998)

Colangelo et al. (2015)

Vertex  $\pi\pi\gamma\gamma^*$

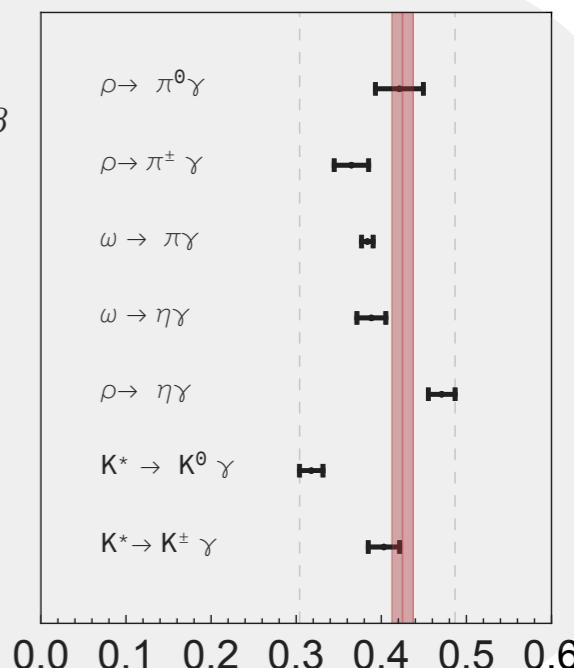
$$\langle \pi^+ | j_\mu(0) | \pi^+(p') \rangle = (p + p')_\mu F_\pi(Q^2)$$

$$F_\pi(Q^2) = \frac{1}{1 + Q^2/M_\rho^2}$$

$$\mathcal{L} = e C_V \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} \partial^\alpha \phi V^\beta$$

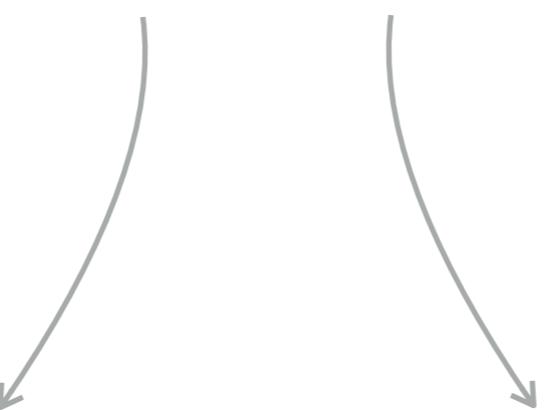
$$F_{\pi\omega}(Q^2) = \frac{1}{1 + Q^2/M_\rho^2}$$

$$F_{\pi\rho}(Q^2) = \frac{1}{1 + Q^2/M_\omega^2}$$



# Subtraction constants

Dispersive integral for J=0

$$\begin{pmatrix} h_{++}(s) \\ k_{++}(s) \end{pmatrix} = \begin{pmatrix} h_{++}^{Born}(s) \\ k_{++}^{Born}(s) \end{pmatrix} + \Omega(s) \left[ \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} b \\ d \end{pmatrix} s + \frac{s^2}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'^2} \frac{\Omega(s')^{-1}}{s' - s} \begin{pmatrix} \text{Im } \bar{h}_{++}(s') \\ \text{Im } \bar{k}_{++}(s') \end{pmatrix} \right. \\ \left. - \frac{s^2}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'^2} \frac{\text{Im } \Omega(s')^{-1}}{s' - s} \begin{pmatrix} h_{++}^{Born}(s') \\ k_{++}^{Born}(s') \end{pmatrix} \right]$$


Soft photon limit ( $q_1=0$ )

$$H_{\lambda_1 \lambda_2} \rightarrow H_{\lambda_1 \lambda_2}^{Born}$$

$$s = -Q^2, t = u = m_\pi^2$$

For space like photons: generalized polarizabilities

$$\pm \frac{2\alpha}{m_\pi} \frac{H_{+\pm}^n}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \dots$$

$$\pm \frac{2\alpha}{m_\pi} \frac{(H_{+\pm}^c - H_{+\pm}^{Born})}{s + Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} +$$

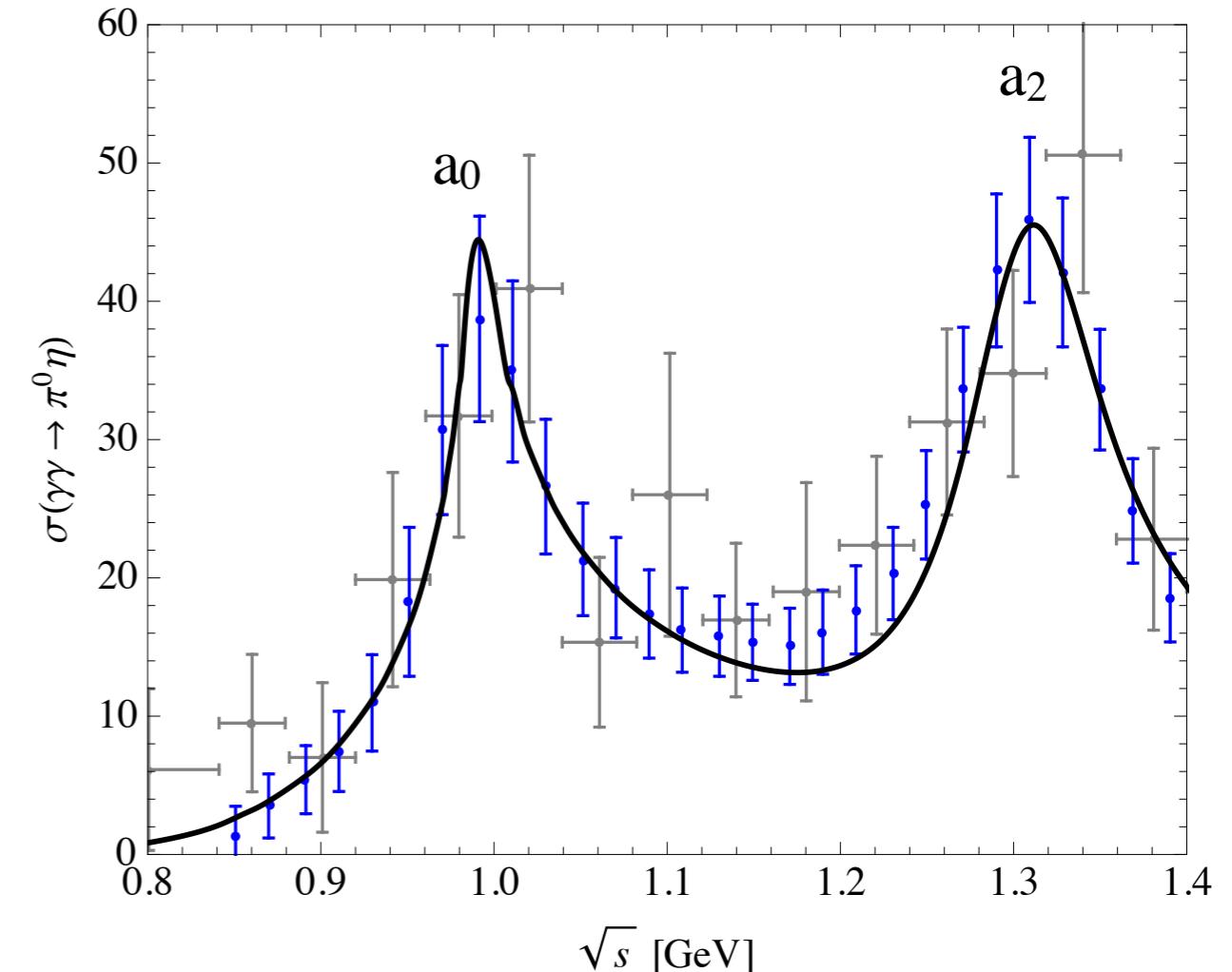
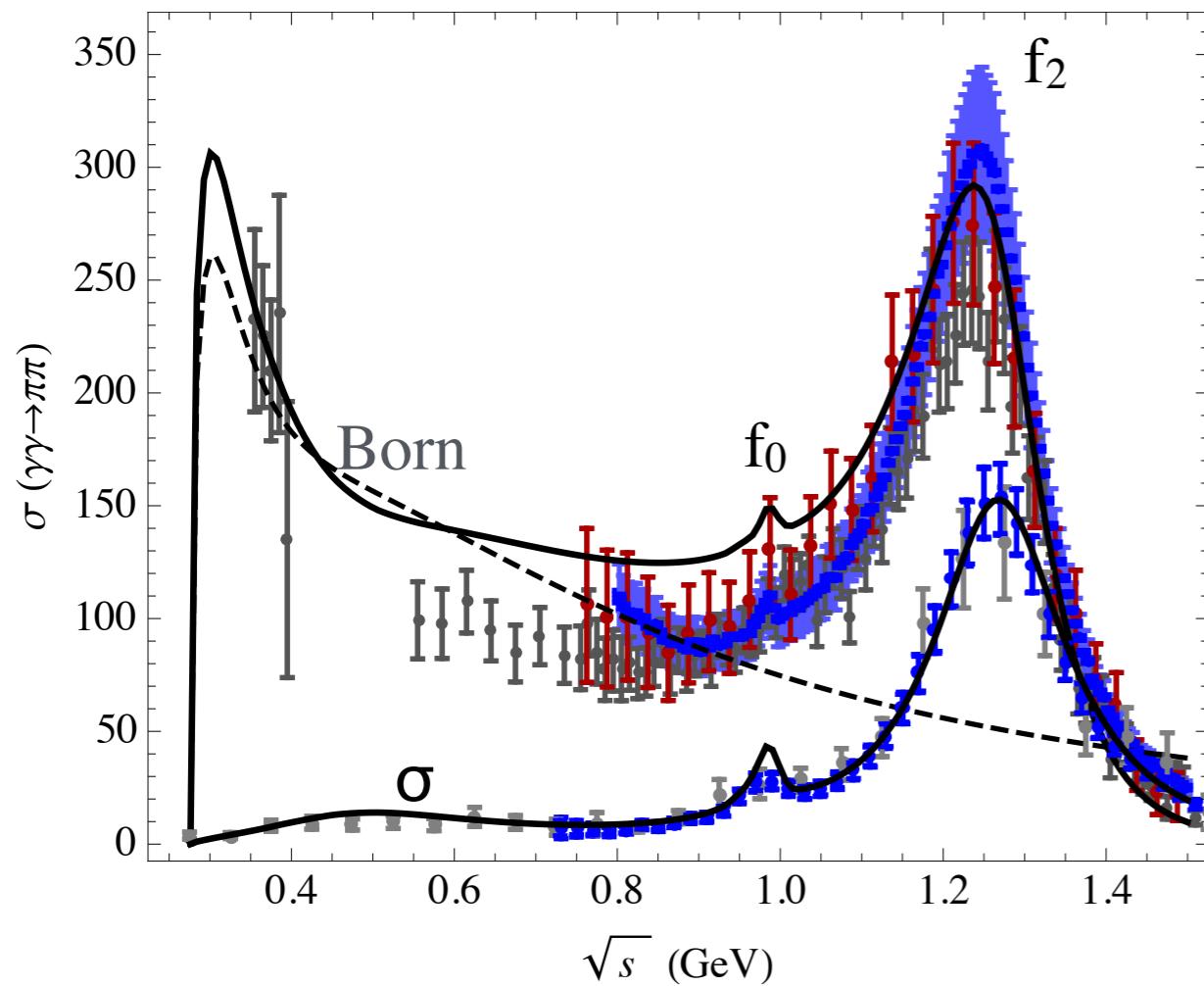
COMPASS data on  $(\alpha_1 - \beta_1)_{\pi^+}$   
(future Hall D (JLab) experiment)

# multi-meson production in $\gamma\gamma$ collisions

$\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$

$Q^2 = 0 \text{ GeV}^2$

$\gamma\gamma \rightarrow \pi^0\eta$



**Coupled-channel dispersive treatment of  $f_0(980)$  and  $a_0(980)$  is crucial**

$f_2(1270)$  described dispersively through Omnes function

$a_2(1320)$  described as a Breit Wigner resonance

$\pi^0\eta$ : [Danilkin, Deineka, Vdh \(2017\)](#)

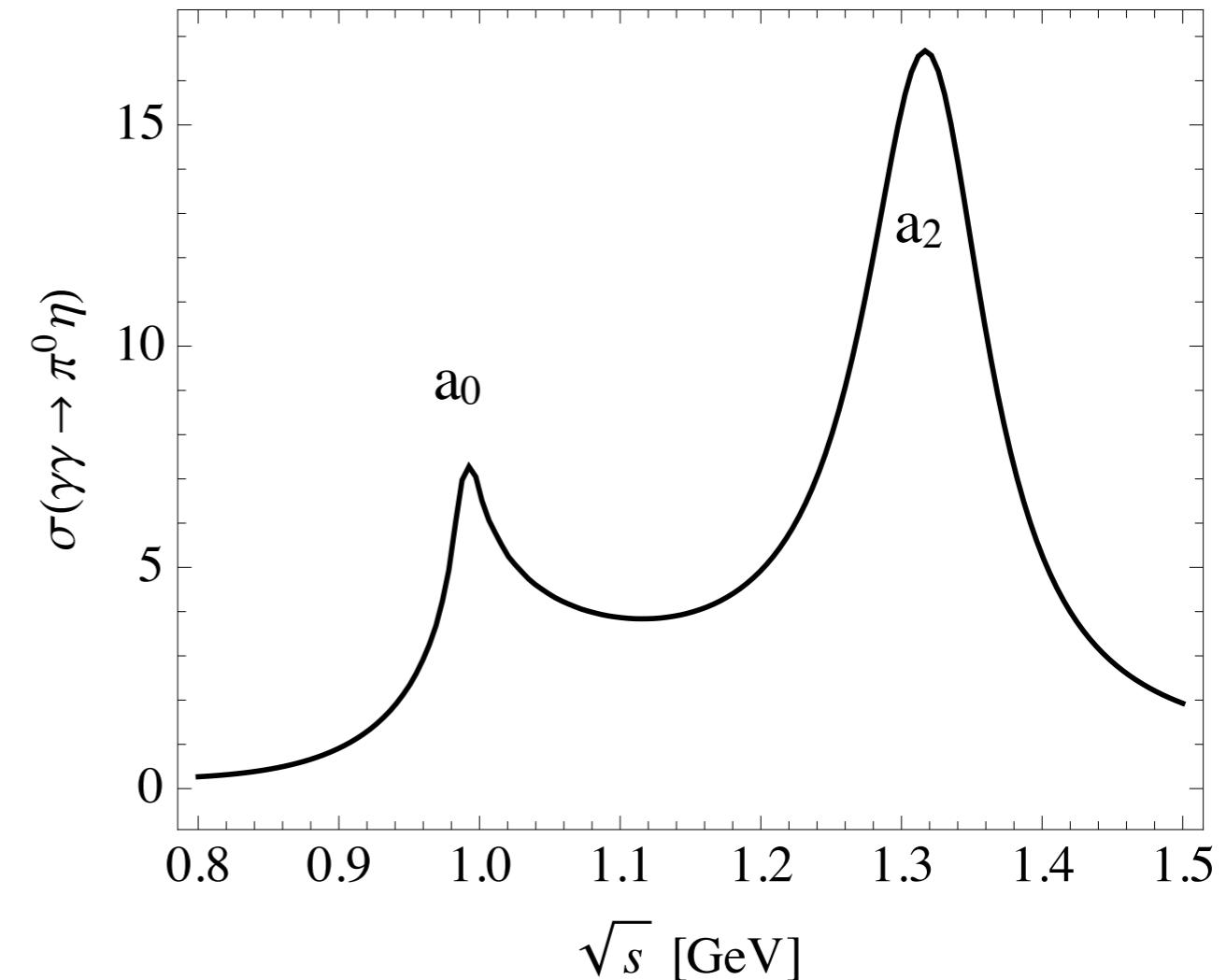
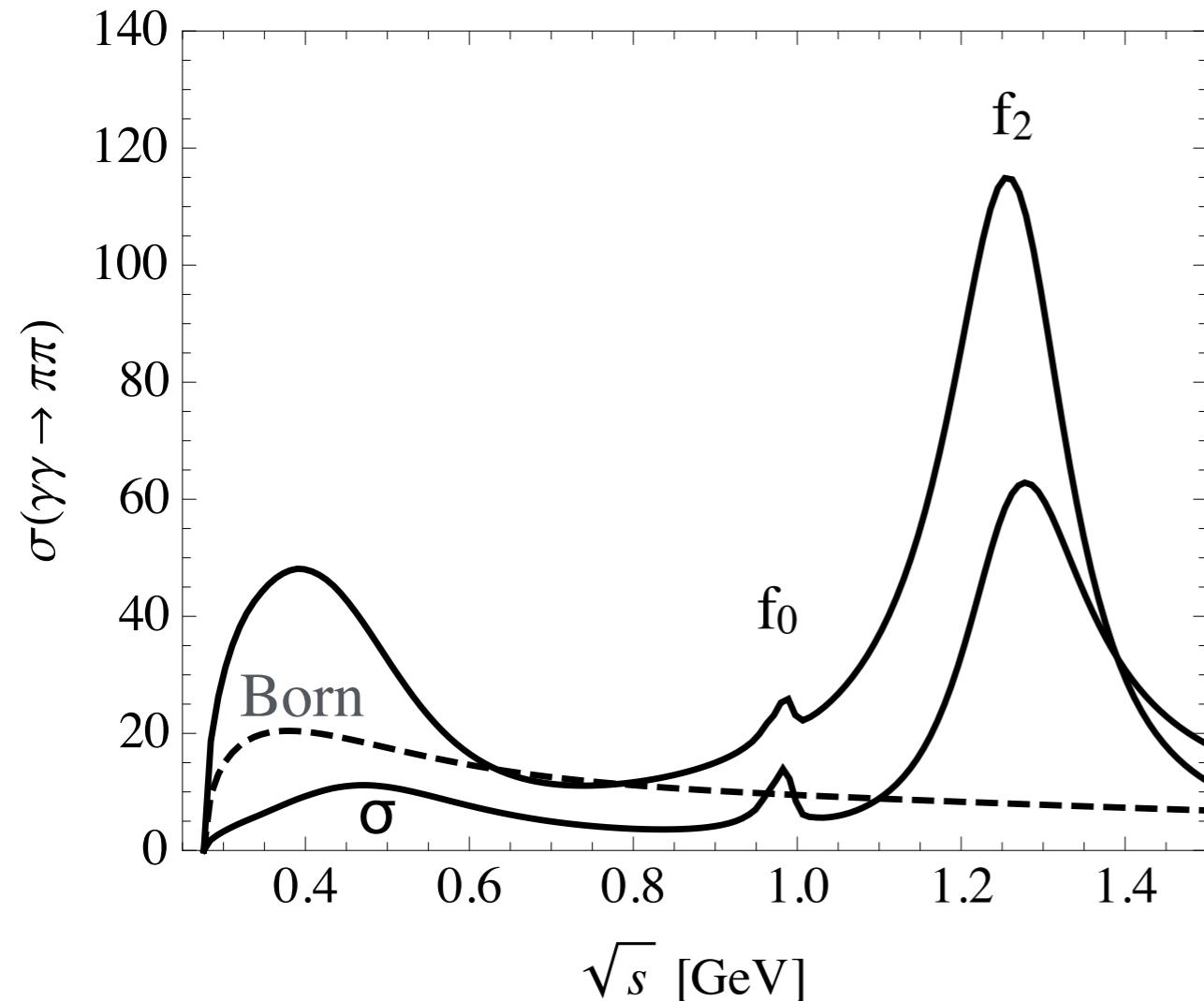
$\pi\pi$ : [Danilkin, Vdh \(in progress\)](#)

# multi-meson production in $\gamma^*\gamma$ collisions

$\gamma^*\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$

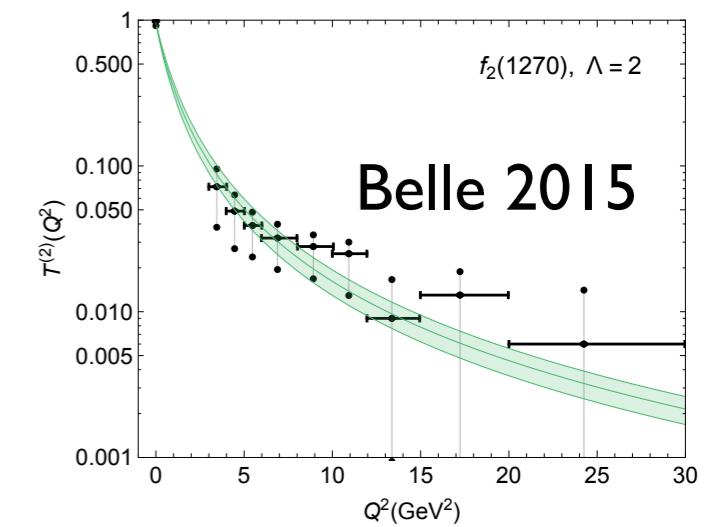
$Q^2 = 0.5 \text{ GeV}^2$

$\gamma^*\gamma \rightarrow \pi^0\eta$



**Coupled-channel dispersive treatment for  $f_0(980)$  and  $a_0(980)$**   
 $f_2(1270), a_2(1320)$  as Breit Wigner resonance, TFF taken from Belle data

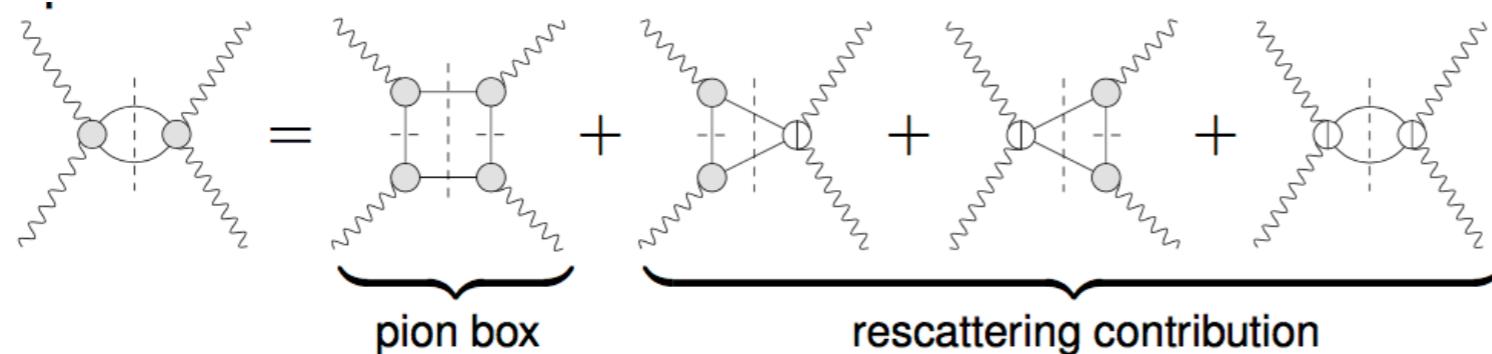
Single tagged BES-III data for  $\pi^+\pi^-$ ,  $\pi^0\pi^0$   
in range  $0.1 \text{ GeV}^2 < Q^2 < 3 \text{ GeV}^2$  under analysis



# multi-meson contributions $a_\mu$

→ pioneering dispersive analyses for  $\pi\pi$  loop contribution to  $a_\mu$

Colangelo, Hoferichter, Procura, Stoffer (2014, 2015, 2017)

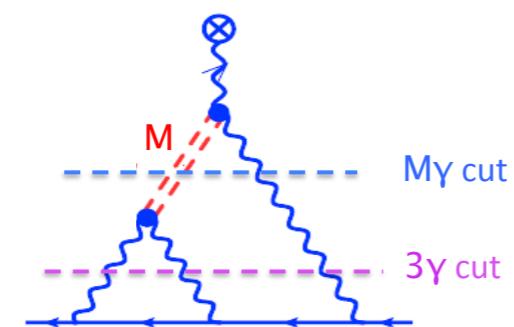


$$a_\mu^{\pi\text{-box}} = (-1.59 \pm 0.02) \times 10^{-10}$$

$$a_\mu^{\text{s-wave } \pi\pi} = (-0.8 \pm 0.1) \times 10^{-10}$$

contribution so far only for pion-pole left hand cut

→ dispersive analysis for muon Pauli FF  $\rightarrow a_\mu$

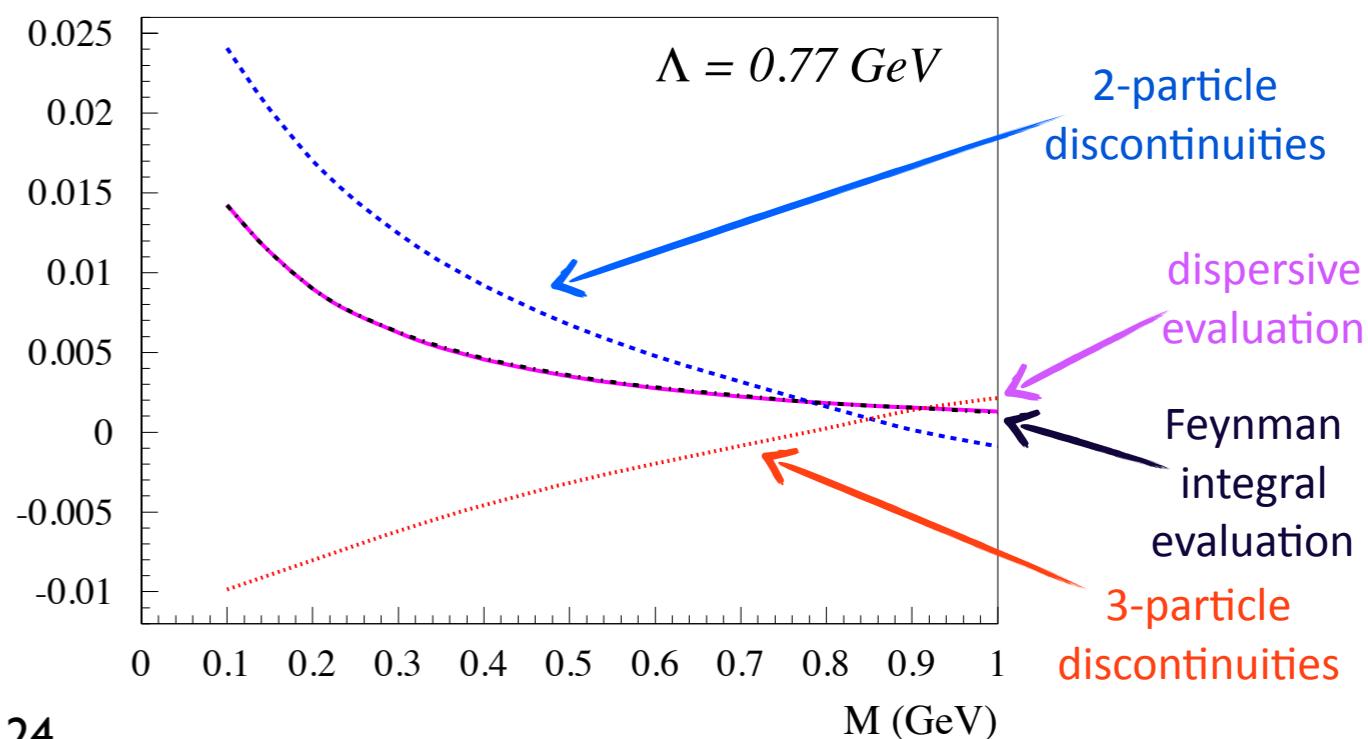


Proof of principle calculation for pseudo-scalar meson ( $M$ ) pole contributions to  $a_\mu$ :

$$\text{e.g. } a_\mu^{\pi^0} = 5.7 \times 10^{-10}$$

Pauk, vdh (2014)

$$a_\mu * M^3 / (\alpha \Gamma_{\gamma\gamma}) \text{ (in GeV}^2\text{): diagram a}$$



# Summary and outlook

- new  $a_\mu$  Fermilab and J-Parc experiments ongoing:  
aim: factor 4 improvement in experimental value
- complementary experimental program (BESIII, Belle II) ongoing as input for the hadronic contributions to the HVP and HLbL contributions to  $a_\mu$  talk Ch. Redmer
- new dispersion relation frameworks for HLbL to  $a_\mu$ :  
-> require close collaboration with experiment / validation  
(spacelike, timelike, meson decays)
- aim: data driven approach also in HLbL
- Theory goal: realistic error estimate on  $a_\mu$  reduce to  $2 \times 10^{-10}$  (20 % of HLbL) to match accuracy of forthcoming experiments  
-> Muon (g-2) Theory Initiative

Aim of concerted effort is to allow for a conclusive statement on the present  $4\sigma$  deviation in  $a_\mu$  between experiment and SM prediction !

