

# Experimental status of $|V_{ub}/V_{cb}|$ and $\gamma(\phi_3)$

**Abi Soffer**

Tel Aviv University

On behalf of the Belle II Collaboration

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# Outline

- Importance of  $|V_{ub}/V_{cb}|$  and  $\gamma$ 
  - Why they are related
- Introduction to the experiments
  - BABAR, Belle, CLEO-c, LHCb, Belle II
- Measurements of  $|V_{ub}/V_{cb}|$ 
  - Different methods
- Measurements of  $\gamma$ 
  - Different methods

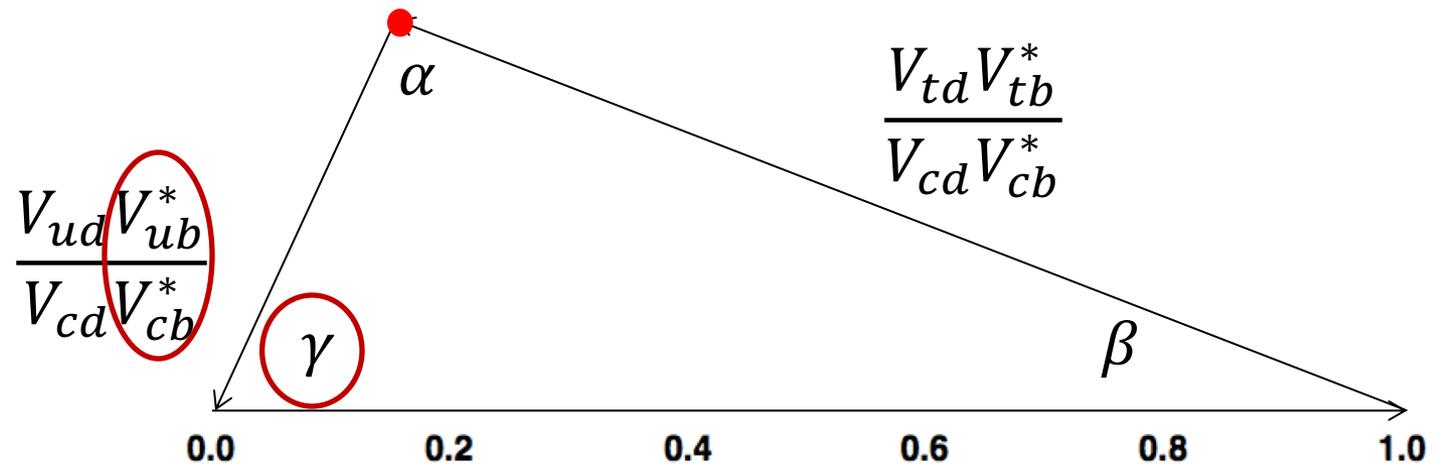
I will give only a selection of results and averages from [PDG](#) based mostly on [HFLAV](#) and [CKMFitter](#)

# Importance of $|V_{ub}/V_{cb}|$ and $\gamma$

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix contains 4 fundamental parameters of the SM.
- 2 of the 4 parameters can be parameterized as  $|V_{ub}/V_{cb}|$  and  $\gamma$
- They originate from some high-scale new physics
- Their precision measurement is critical to the definition of the SM
- It also provides a stress-test of the SM and probe of NP

# CKM unitarity triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



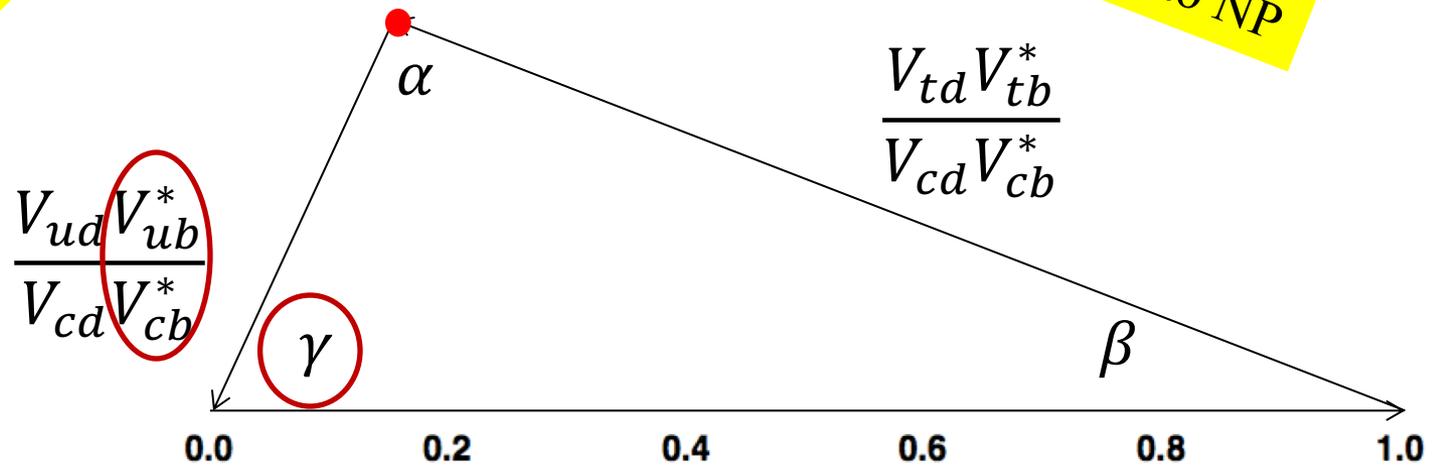
Testing the consistency of this SM picture probes NP

# Probing new physics (NP)

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Tree diagrams:  
SM (+ potentially new CC)

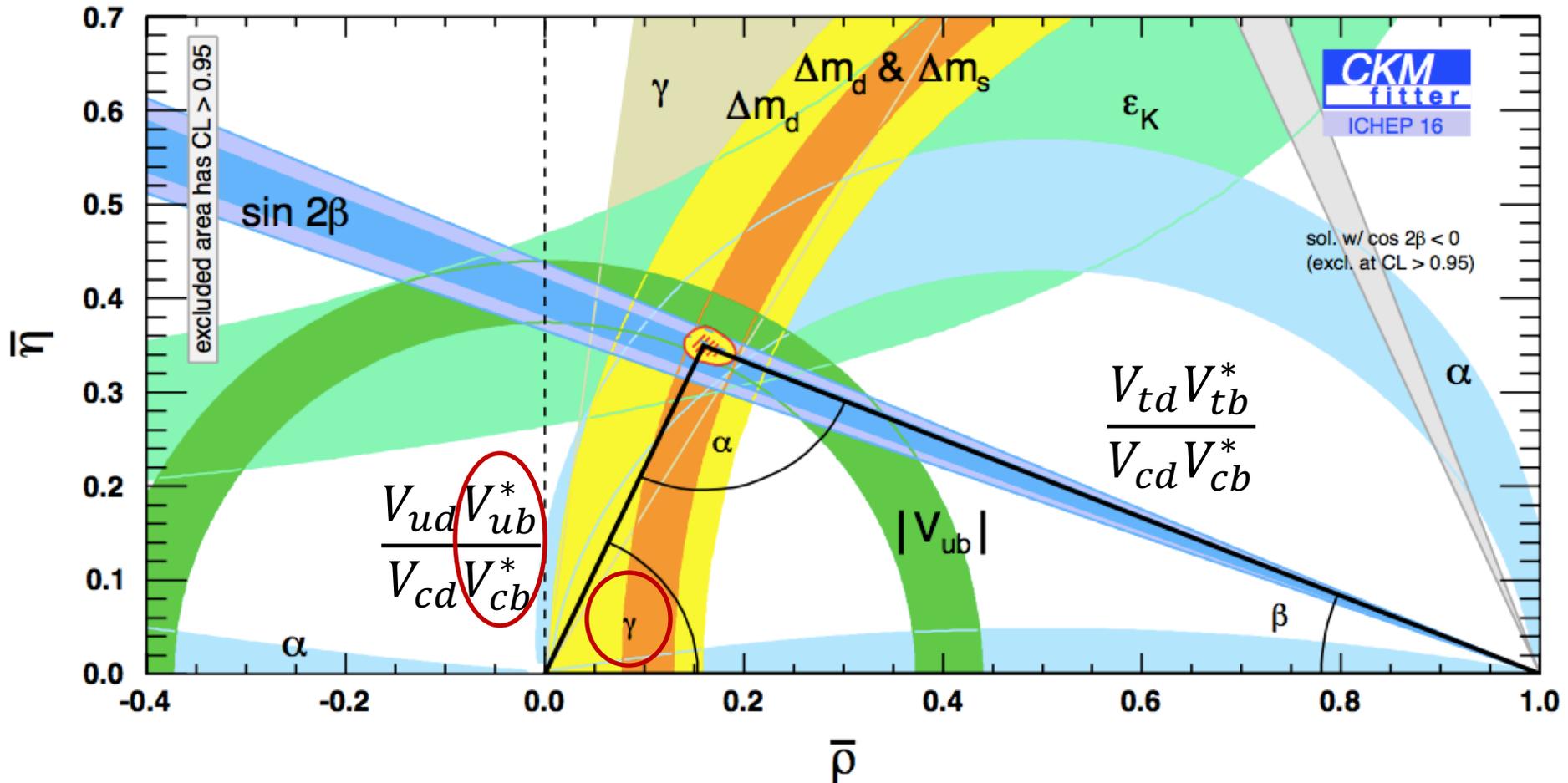
Loops:  
More sensitive to NP



Testing the consistency of this SM picture probes NP

# Unitarity triangle constraints

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



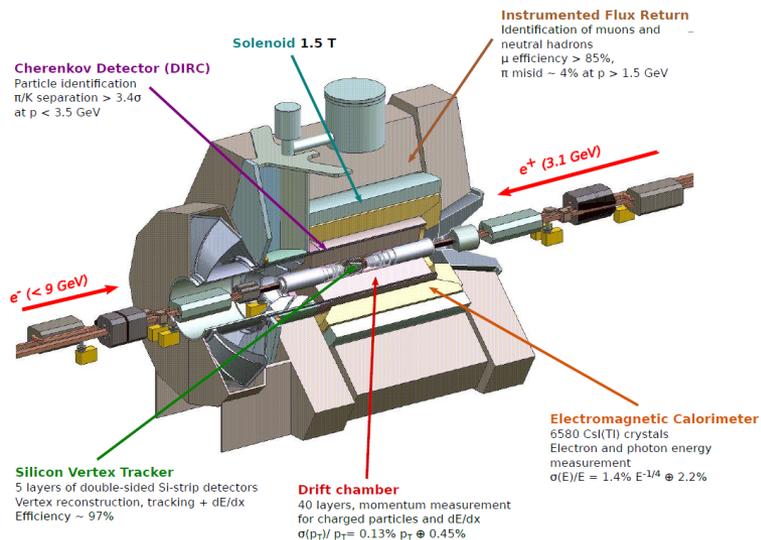
Testing the consistency of this SM picture probes NP

# The experiments

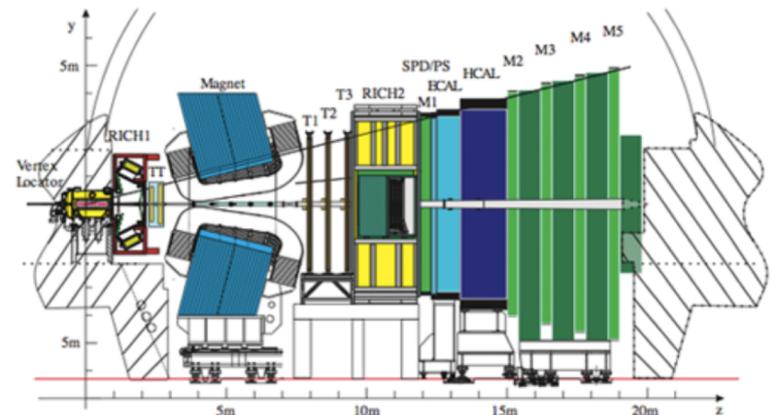
- BABAR: 1999-2008,  $\sim 470\text{M } e^+e^- \rightarrow B\bar{B}$
- Belle 1999-2010,  $\sim 770\text{M } e^+e^- \rightarrow B\bar{B}$
- LHCb 2010- ,  $pp \rightarrow b\bar{b}X$
- CLEO-c 2003-2006?,  $\sim 3\text{M } e^+e^- \rightarrow D\bar{D}$
- Belle II 2019-2026?,  $\sim 50,000\text{M } e^+e^- \rightarrow B\bar{B}$

- Silicon tracker
- Gas-based tracker
- Cherenkov hadron-ID
- EM calorimeter
- Muon system

## BABAR



## LHCb



# $V_{ub}/V_{cb}$

See Matic Lubej's talk  
this afternoon

- Measured via  $b \rightarrow u\ell\bar{\nu}$  and  $b \rightarrow c\ell\bar{\nu}$  (where  $\ell = e, \mu$ )
  - Measurements with  $\tau$  also possible in principle, but...
    - they are experimentally less precise
    - not yet clear what the  $\sim 4\sigma$  SM discrepancy in  $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}$  is telling us
- Measured via both **exclusive** and **inclusive** final states.

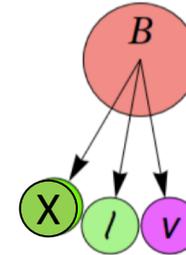


- Significant model dependence  $\rightarrow$  different values of  $|V_{ub}|$  &  $|V_{cb}|$
- Close interaction between theoretical and experimental inputs

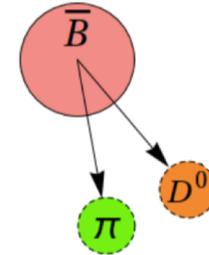
# Other- $B$ tagging at a $e^+e^-$ B factory

- Hadronic
  - Fully reconstruct in  $>1000$  hadronic decay chains
  - Reduces combinatorial background
  - $\vec{p}_\nu$  well determined
  - Efficiency  $\sim 0.5\%$

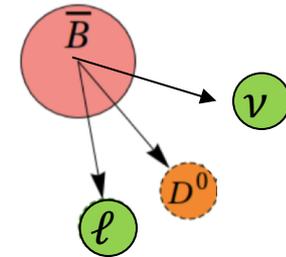
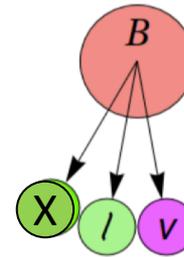
Signal-B



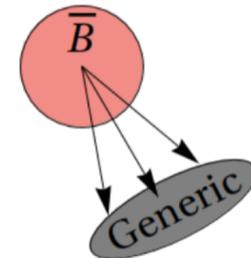
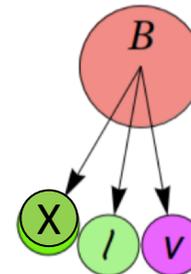
tag-B



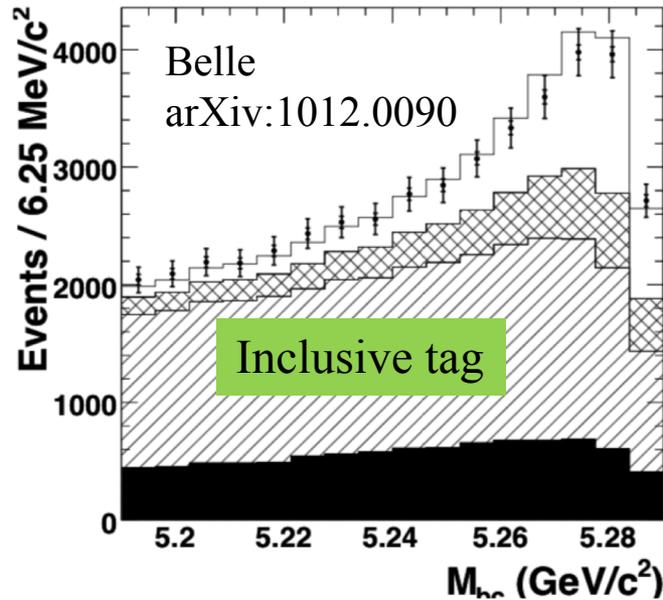
- Semileptonic
  - Reconstruct as  $B \rightarrow \bar{D}^{(*)} \ell^+ \nu$
  - Reduces combinatorial background
  - Efficiency  $\sim 0.5\%$



- Inclusive
  - Use all tracks and clusters to try and reconstruct neutrino 4-momentum
  - Efficiency  $\sim 10\%$
  - Higher background
  - $\vec{p}_\nu$  not well determined: finite B momentum, missing/fake particles

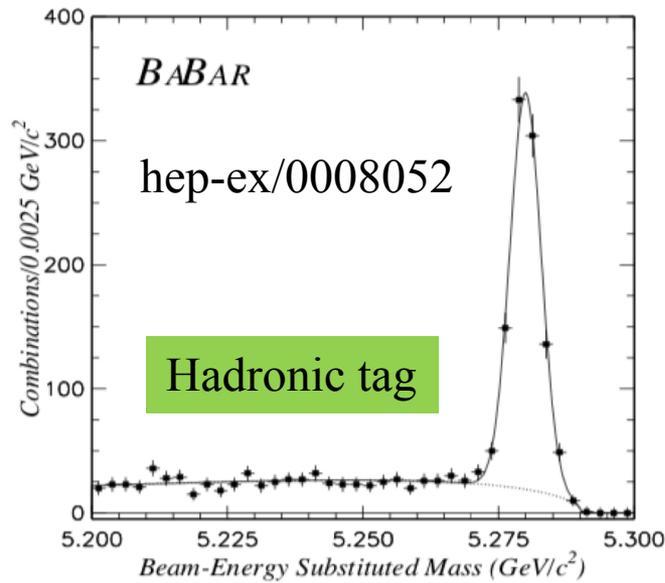


# A comparison of other-B tagging methods

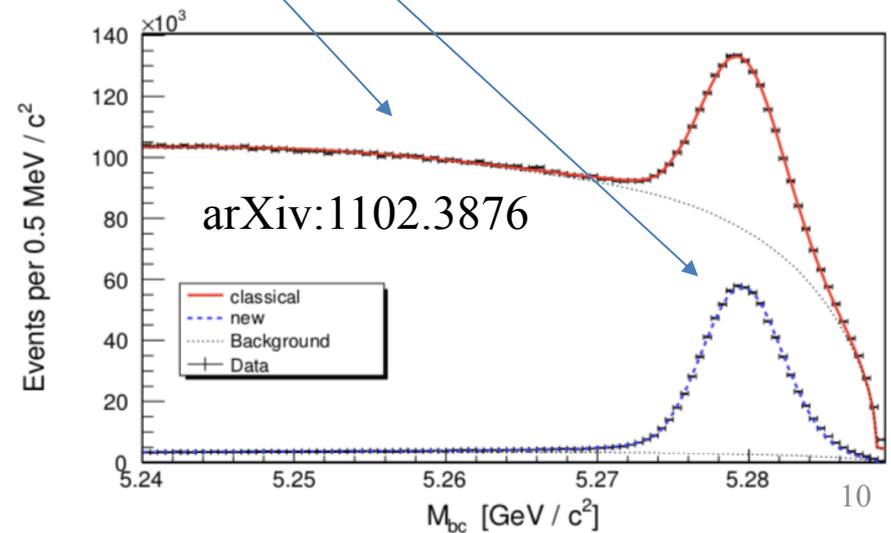


“Beam-constrained mass”

$$M_{bc} \equiv \sqrt{E_{beam}^2 - p_B^2}$$



Old and new algorithms, same efficiency



# $V_{cb}$ from exclusive $\bar{B} \rightarrow D^* \ell \bar{\nu}$

- Experiments measure differential decay rate, given by:

$$\frac{d\Gamma}{dw}(\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{\text{ew}} \mathcal{F}(w))^2$$

$$w \equiv v_B \cdot v_{D^*}$$

Phase-space

$\sim 1$ , EW correction

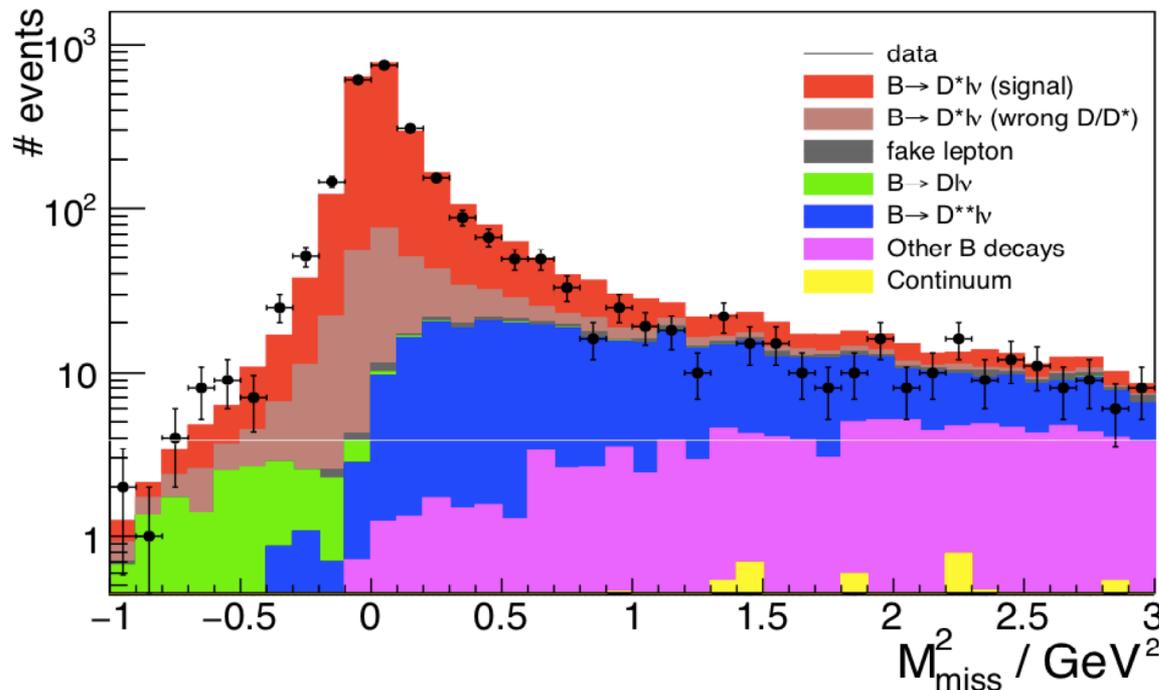
Form factor (lattice or lightcone sum rules)

- Measurements performed by Belle, BABAR, CLEO, LEP.
- Most use a form-factor polynomial expansion (CLN) around  $w = 1$ , considered by PDG to be too constraining given current high precision
- PDG prefers the more flexible BGL form-factor parameterization (Boyd, Grinstein, Lebed, PRL 74, 4603)
- $\sim 10\%$  differences seen b/w CLN and BGL parameterizations
- Only one analysis (Belle, arXiv:1702.01521) uses BGL...

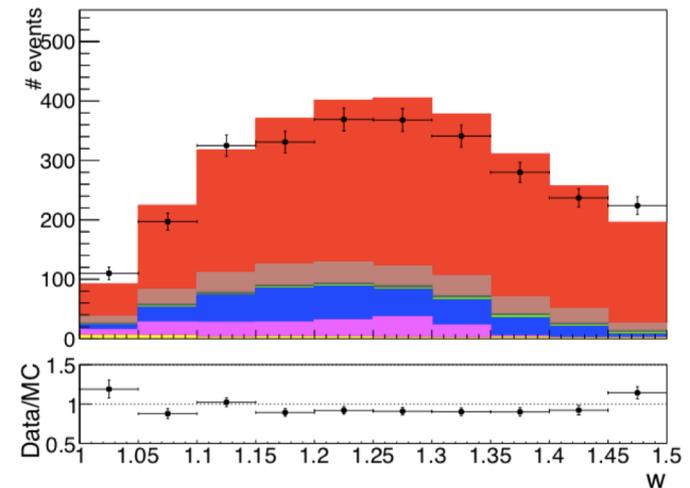
# Recent Belle had-tag $\bar{B} \rightarrow D^* \ell \bar{\nu}$ analysis

arXiv:1702.01521

$$p_{\text{miss}} = p_{\nu} = p_{e^+e^-} - p_{\text{tag}} - p_{D^*} - p_{\ell}$$



One of the distributions:



From these inputs, PDG quotes  $|V_{cb}| = (41.9^{+2.0}_{-1.9}) \times 10^{-3}$

# $V_{cb}$ from inclusive $\bar{B} \rightarrow X_c \ell \bar{\nu}$

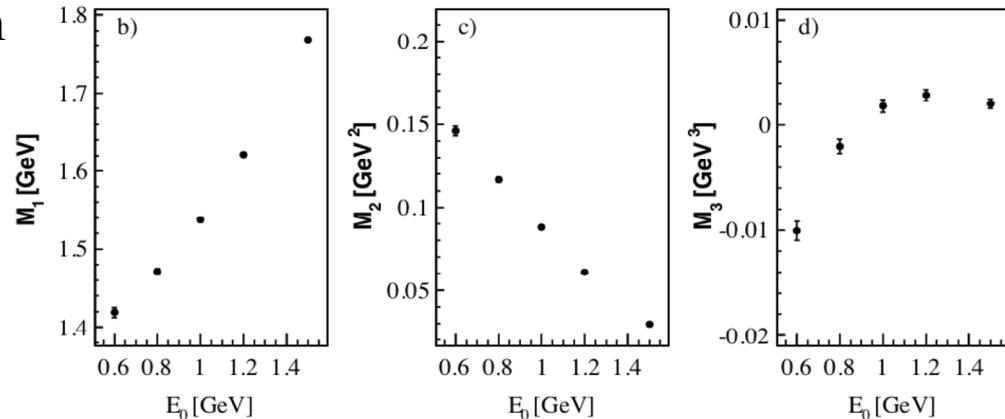
- Differential decay rate is given as a heavy-quark expansion (HQE) in terms of lepton-energy moments,

$$\langle E_e^n \rangle_{E_e > E_{\text{cut}}} = \int_{E_{\text{cut}}}^{E_{\text{max}}} \frac{d\Gamma}{dE_e} E_e^n dE_e \bigg/ \int_{E_{\text{cut}}}^{E_{\text{max}}} \frac{d\Gamma}{dE_e} dE_e$$

for different value of the minimal lepton energy  $E_{\text{cut}}$ ,  
and similarly for the hadronic invariant mass and hadronic energy.

- E.g., Lepton-energy moments from BABAR arXiv:0908.0415:  $\longrightarrow$

- Global Fits to these moments are used to obtain  $V_{cb}$  in various  $b$ -mass schemes



- PDG choose the kinetic scheme result  $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$

# $V_{ub}$ from inclusive $\bar{B} \rightarrow X_u \ell \bar{\nu}$

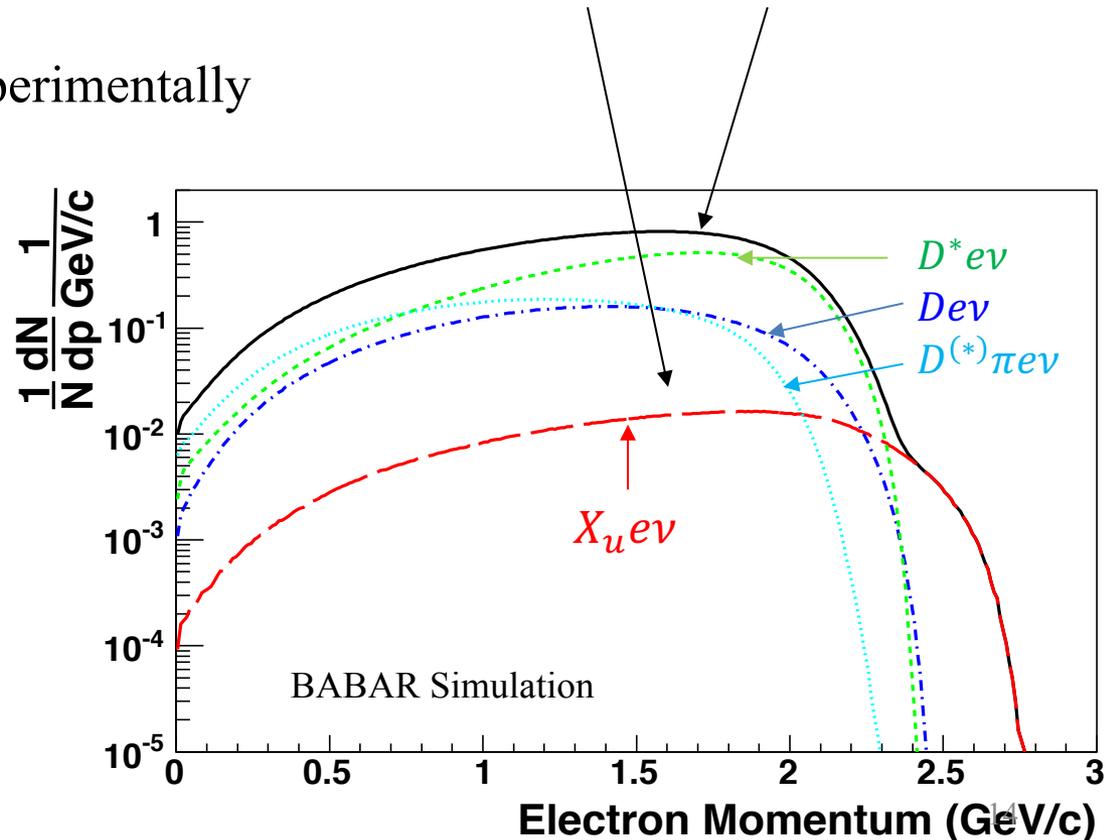
- The **total**  $B \rightarrow X_u \ell \nu$  decay rate is calculated based on the operator product expansion (OPE) in  $\alpha_s$  and  $\Lambda_{\text{QCD}}/m_b$ , with a  $\sim 5\%$  uncertainty
  - But the **total rate** is hard to measure, due to large  $B \rightarrow X_c \ell \nu$  background
  - $V_{ub}$  measurement requires model for distributions of  $B \rightarrow X_u \ell \nu$  and  $B \rightarrow X_c \ell \nu$

- The **high- $E_\ell$**  region favored experimentally

- Projecting to full spectrum incurs large corrections from a nonperturbative "shape function" (SF) that accounts for the Fermi motion of the  $b$  inside the  $B$ .

- So measure  $V_{ub}$  with different

- kinematic regions
- analysis methods
- calculation schemes



# Some inclusive $V_{ub}$ results ( $10^{-5}$ ), RPP 2016/7

Ref.	cut (GeV)	BLNP	GGOU	DGE
CLEO [108]	$E_e > 2.1$	$428 \pm 50 \begin{smallmatrix} +31 \\ -36 \end{smallmatrix}$	$421 \pm 49 \begin{smallmatrix} +23 \\ -33 \end{smallmatrix}$	$390 \pm 45 \begin{smallmatrix} +26 \\ -28 \end{smallmatrix}$
BABAR [111]	$E_e - q^2$	$453 \pm 22 \begin{smallmatrix} +33 \\ -38 \end{smallmatrix}$	not available	$417 \pm 20 \begin{smallmatrix} +28 \\ -29 \end{smallmatrix}$
BABAR [110]	$E_e > 2.0$	$454 \pm 26 \begin{smallmatrix} +27 \\ -33 \end{smallmatrix}$	$450 \pm 26 \begin{smallmatrix} +18 \\ -25 \end{smallmatrix}$	$434 \pm 25 \begin{smallmatrix} +23 \\ -25 \end{smallmatrix}$
Belle [109]	$E_e > 1.9$	$493 \pm 46 \begin{smallmatrix} +27 \\ -29 \end{smallmatrix}$	$493 \pm 46 \begin{smallmatrix} +17 \\ -22 \end{smallmatrix}$	$485 \pm 45 \begin{smallmatrix} +21 \\ -25 \end{smallmatrix}$
BABAR [113]	$q^2 > 8$ $m_X < 1.7$	$430 \pm 23 \begin{smallmatrix} +26 \\ -28 \end{smallmatrix}$	$432 \pm 23 \begin{smallmatrix} +27 \\ -30 \end{smallmatrix}$	$427 \pm 22 \begin{smallmatrix} +20 \\ -20 \end{smallmatrix}$
[113]	$P_+ < 0.66$	$415 \pm 25 \begin{smallmatrix} +28 \\ -27 \end{smallmatrix}$	$424 \pm 26 \begin{smallmatrix} +32 \\ -32 \end{smallmatrix}$	$424 \pm 26 \begin{smallmatrix} +37 \\ -32 \end{smallmatrix}$
[113]	$m_X < 1.55$	$430 \pm 20 \begin{smallmatrix} +28 \\ -27 \end{smallmatrix}$	$429 \pm 20 \begin{smallmatrix} +21 \\ -22 \end{smallmatrix}$	$453 \pm 21 \begin{smallmatrix} +24 \\ -22 \end{smallmatrix}$
[113]	$E_\ell > 1$	$432 \pm 24 \begin{smallmatrix} +19 \\ -21 \end{smallmatrix}$	$442 \pm 24 \begin{smallmatrix} +9 \\ -11 \end{smallmatrix}$	$446 \pm 24 \begin{smallmatrix} +13 \\ -13 \end{smallmatrix}$
Belle [115]	$E_\ell > 1$	$449 \pm 27 \begin{smallmatrix} +20 \\ -22 \end{smallmatrix}$	$460 \pm 27 \begin{smallmatrix} +10 \\ -11 \end{smallmatrix}$	$463 \pm 28 \begin{smallmatrix} +13 \\ -13 \end{smallmatrix}$
BABAR [121]	$E_\ell > 0.8$	$456 \pm 13 \begin{smallmatrix} +28 \\ -26 \end{smallmatrix}$	$396 \pm 10 \pm 17$	supersedes [110]

Inclusive tagging

$p_\ell$

Some  $V_{ub}$  values are recalculations by HFLAV

Hadronic tagging

$p_\ell$

BLNP: B.O. Lange, M. Neubert, and G. Paz, Phys. Rev. **D72**, 073006 (2005).  
 GGOU: P. Gambino *et al.*, JHEP **0710**, 058 (2007).  
 DGE: J.R. Andersen and E. Gardi, JHEP **0601**, 097 (2006).

- Note that results with different cuts are correlated
- PDG averages over methods with additional error:

$$|V_{ub}| = (4.49 \pm 0.15_{\text{exp}} \begin{smallmatrix} +0.16 \\ -0.17 \end{smallmatrix}_{\text{theo}} \pm 0.17_{\Delta\text{BF}}) \times 10^{-3}$$

108. A. Bornheim *et al.* (CLEO Collab.), Phys. Rev. Lett. **88**, 231803 (2002).  
 109. A. Limosani *et al.* (Belle Collab.), Phys. Lett. **B621**, 28 (2005).  
 110. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. **D73**, 012006 (2006).  
 111. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. Lett. **95**, 111801 (2005), Erratum: Phys. Rev. Lett. **97**, 019903 (2006).  
 112. R. Kowalewski and S. Menke, Phys. Lett. **B541**, 29 (2002).  
 113. J.P. Lees *et al.* (BABAR Collab.), arXiv:1112.0702.  
 114. I. Bizjak *et al.* (Belle Collab.), Phys. Rev. Lett. **95**, 241801 (2005).  
 115. P. Urquijo *et al.* (Belle Collab.), Phys. Rev. Lett. **104**, 021801 (2010).  
 121. J. P. Lees *et al.* (BABAR Collab.) Phys. Rev. **D95**, 072001 (2017)

# $V_{ub}$ from exclusive $\bar{B} \rightarrow \pi \ell \bar{\nu}$ (PDG17)

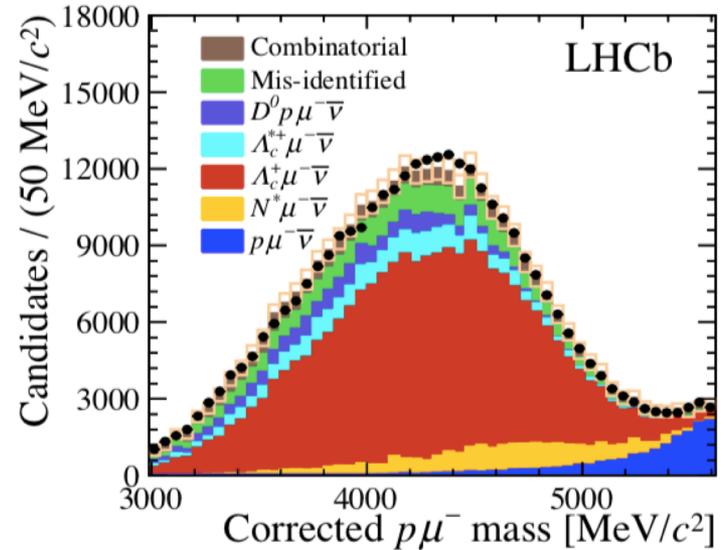
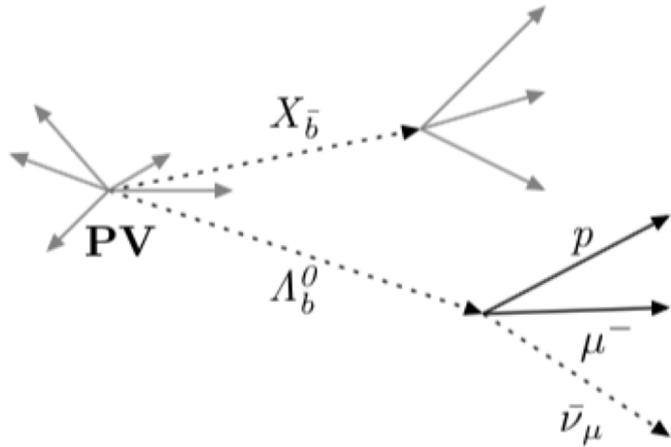
- As with exclusive  $V_{cb}$ , relate partial Br to  $V_{ub}$  using lattice or sum rules

	$\mathcal{B} \times 10^4$	$\mathcal{B}(q^2 > 16) \times 10^4 \text{ GeV}^2$
CLEO $\pi^+, \pi^0$ [129]	$1.38 \pm 0.15 \pm 0.11$	$0.41 \pm 0.08 \pm 0.04$
● BABAR $\pi^+, \pi^0$ [130]	$1.41 \pm 0.05 \pm 0.08$	$0.32 \pm 0.02 \pm 0.03$
● BABAR $\pi^+$ [131]	$1.44 \pm 0.04 \pm 0.06$	$0.37 \pm 0.02 \pm 0.02$
● Belle $\pi^+, \pi^0$ [143]	$1.48 \pm 0.04 \pm 0.07$	$0.40 \pm 0.02 \pm 0.02$
Belle SL $\pi^+$ [144]	$1.41 \pm 0.19 \pm 0.15$	$0.37 \pm 0.10 \pm 0.04$
Belle SL $\pi^0$ [144]	$1.41 \pm 0.26 \pm 0.15$	$0.37 \pm 0.15 \pm 0.04$
● Belle Had $\pi^+$ [124]	$1.49 \pm 0.09 \pm 0.07$	$0.45 \pm 0.05 \pm 0.02$
● Belle Had $\pi^0$ [124]	$1.48 \pm 0.15 \pm 0.08$	$0.36 \pm 0.07 \pm 0.02$
BABAR SL $\pi^+$ [145]	$1.38 \pm 0.21 \pm 0.08$	$0.46 \pm 0.13 \pm 0.03$
BABAR SL $\pi^0$ [145]	$1.78 \pm 0.28 \pm 0.15$	$0.44 \pm 0.17 \pm 0.06$
BABAR Had $\pi^+$ [146]	$1.07 \pm 0.27 \pm 0.19$	$0.65 \pm 0.20 \pm 0.13$
BABAR Had $\pi^0$ [146]	$1.52 \pm 0.41 \pm 0.30$	$0.48 \pm 0.22 \pm 0.12$
Average [147]	$1.45 \pm 0.02 \pm 0.04$	$0.38 \pm 0.01 \pm 0.01$

$$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$$

# $|V_{ub}/V_{cb}|$ with $\Lambda_b$ decays at LHCb

arXiv:1504.01568



Similar topology for  $\Lambda_b \rightarrow \Lambda_c^+ \mu \bar{\nu}$   
with  $\Lambda_c^+ \rightarrow p K^- \pi^+$

Measure  $\hat{p}_{\Lambda_b}$  and  $q^2 = (p_\mu + p_\nu)^2$  from  
vertex position with 1 (4)  $\text{GeV}^2$   
resolution for right (wrong) solution. Measure:

$$m_{corr} = \sqrt{m_{p\mu}^2 + p_\perp^2 + p_\perp}$$

$p_\perp = p\mu$  momentum  $\perp$   $\Lambda_b$  momentum

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu\bar{\nu})_{q^2 > 15\text{GeV}^2}}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu\bar{\nu})_{q^2 > 7\text{GeV}^2}} = (0.95 \pm 0.04 \pm 0.07) \times 10^{-2}$$

# $|V_{ub}/V_{cb}|$ summary (PDG 2017)

$$|V_{ub}|/|V_{cb}| = 0.107 \pm 0.007 \quad (\text{inclusive})$$

$$|V_{ub}|/|V_{cb}| = 0.088 \pm 0.006 \quad (\text{exclusive})$$

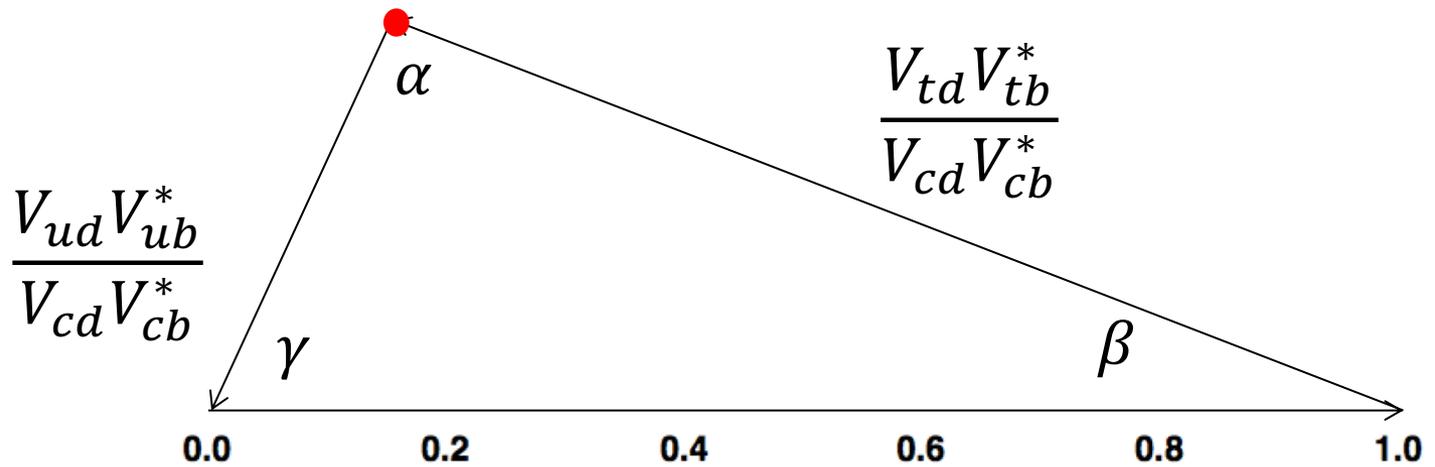
$2\sigma$  difference (mostly due to  $2.6\sigma$  difference in  $V_{ub}$  results)

$$|V_{ub}|/|V_{cb}| = 0.080 \pm 0.004 \pm 0.004 \quad (\text{LHCb})$$

LHCb result is based on 1 precise lattice calculation.

Errors on B-factory results include differences b/w theory inputs, more crosschecks

# Measuring $\gamma$



- To leading order in  $\lambda = \sin \theta_C \approx 0.22$ , can take

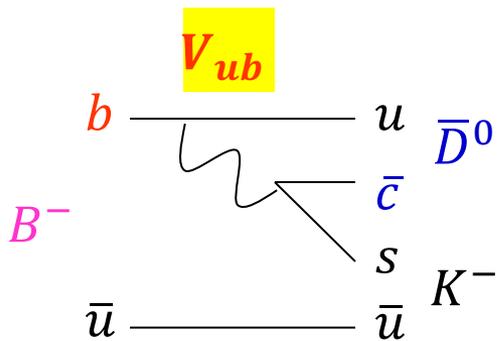
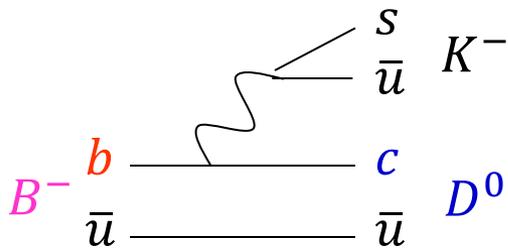
$$\gamma = \arg(-V_{ub}^*)$$

- $\gamma$  is a CP-odd complex phase
- Its measurement requires
  1. Interference between two amplitudes, one with  $V_{ub}$
  2. Comparison of between CP-conjugate processes

# The GLW method

Gronau-London-Wyler (PLB 253, 483; PLB 265, 172)

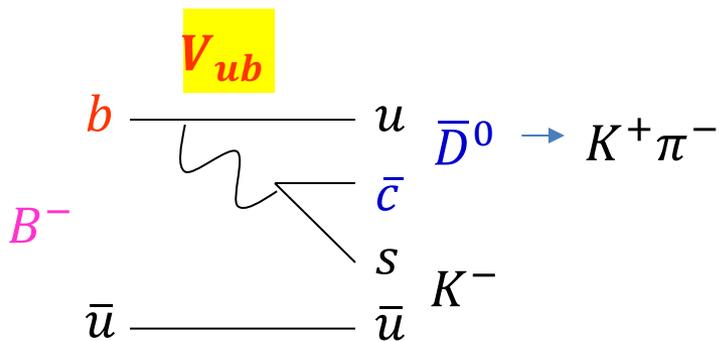
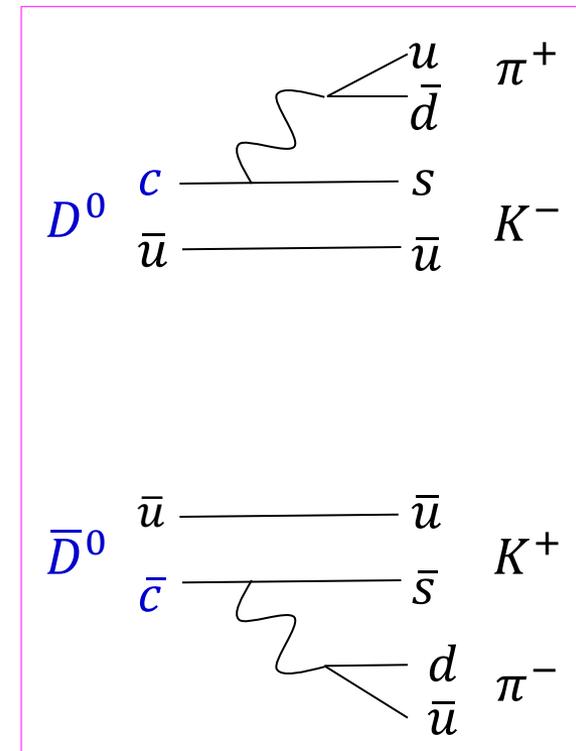
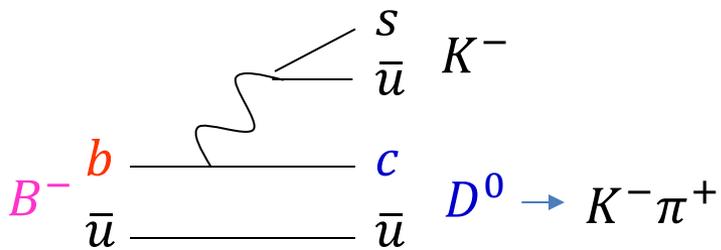
- Consider the processes  $B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$



# The GLW method

Gronau-London-Wyler (PLB 253, 483; PLB 265, 172)

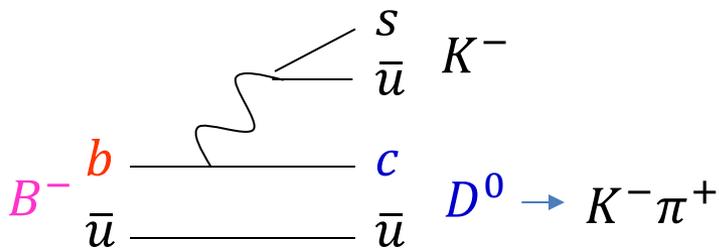
- Consider the processes  $B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$
- Their amplitudes are measured via Cabibbo-allowed charm decays



# The GLW method

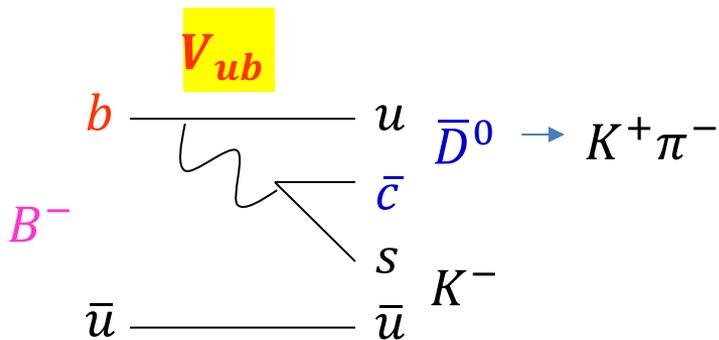
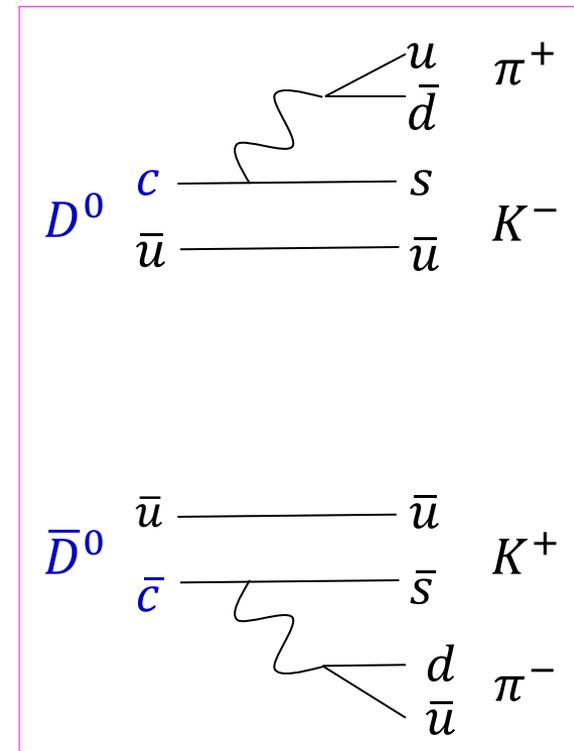
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Amplitude

$$|A|$$

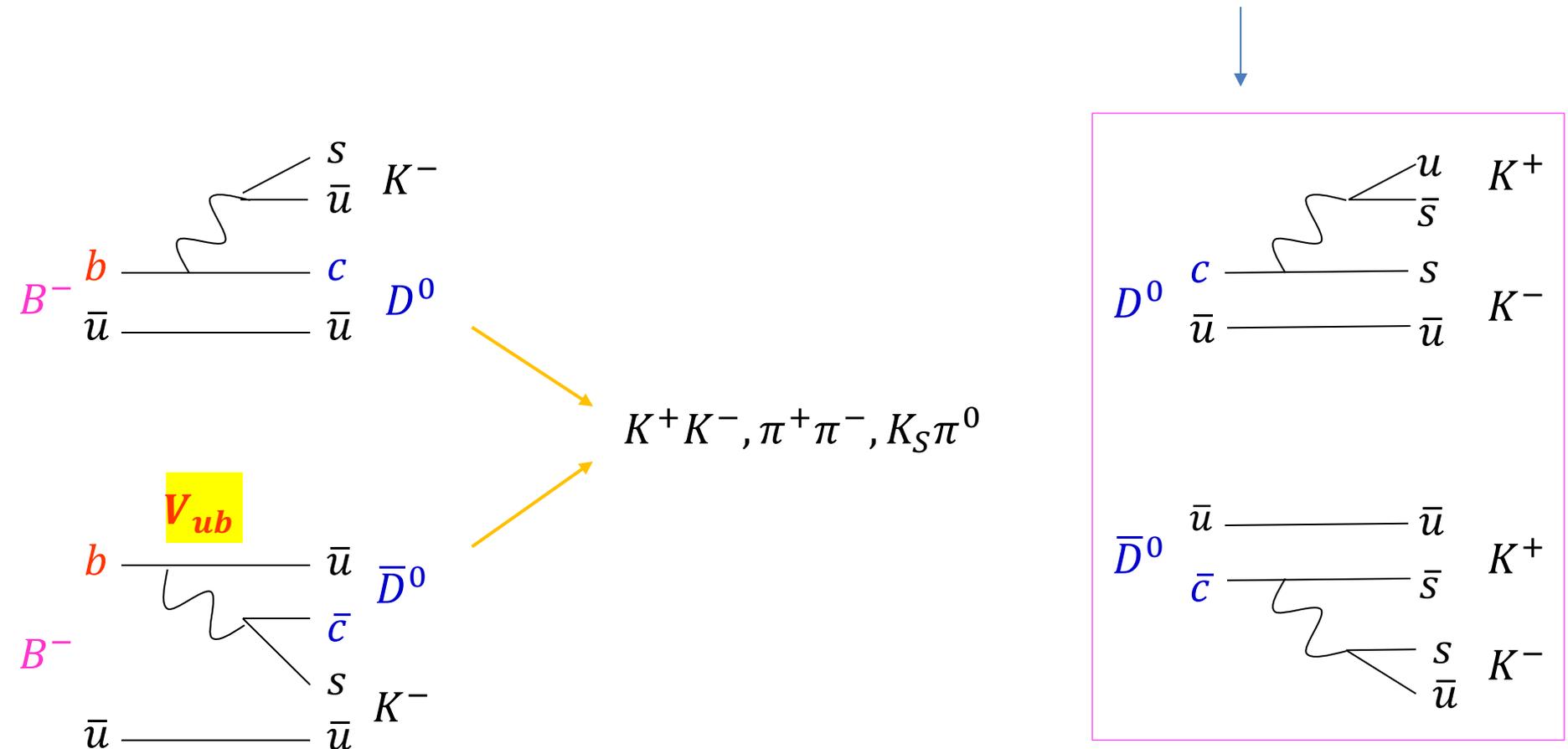


$$|A| r_B e^{i(-\gamma + \delta_B)}$$

Some CP-even phase  
due to strong interaction

# The GLW method

- Interference when both  $D^0$  and  $\bar{D}^0$  decay to a common CP eigenstate



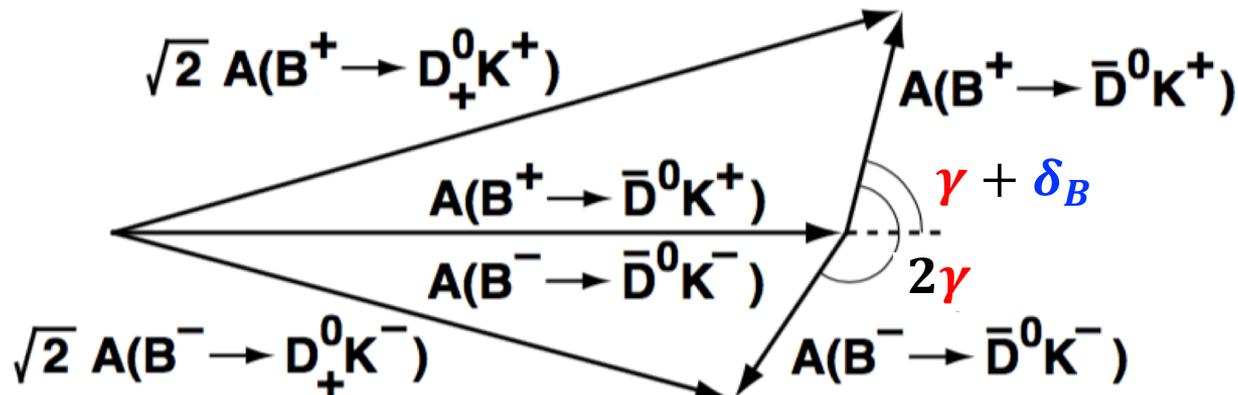
- Write the CP eigenstates as

$$D_{\pm}^0 = \frac{1}{\sqrt{2}} (D^0 \pm \bar{D}^0)$$

- Then the decay amplitudes satisfy

$$\begin{aligned} A(B^- \rightarrow D_{\pm}^0 K^-) &= \frac{1}{\sqrt{2}} (A(B^- \rightarrow D^0 K^-) \pm A(B^- \rightarrow \bar{D}^0 K^-)) \\ &= \frac{1}{\sqrt{2}} |A| (1 \pm r_B e^{i(-\gamma + \delta_B)}) \end{aligned}$$

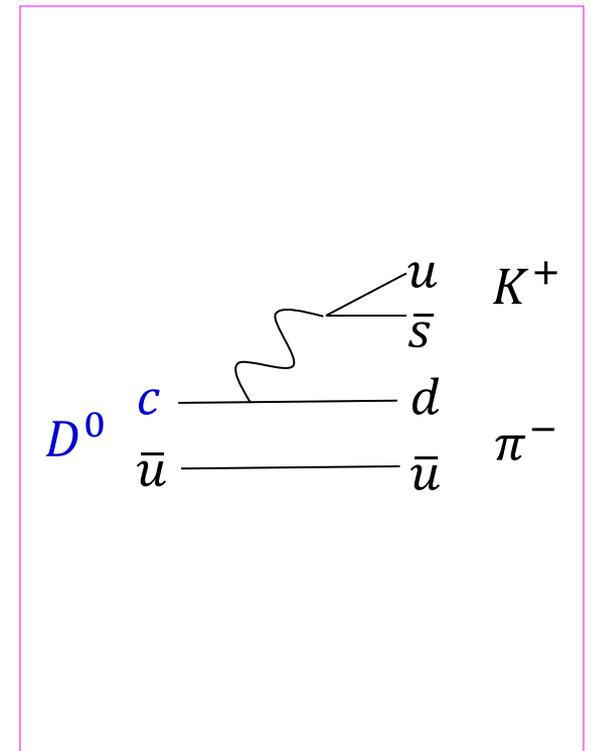
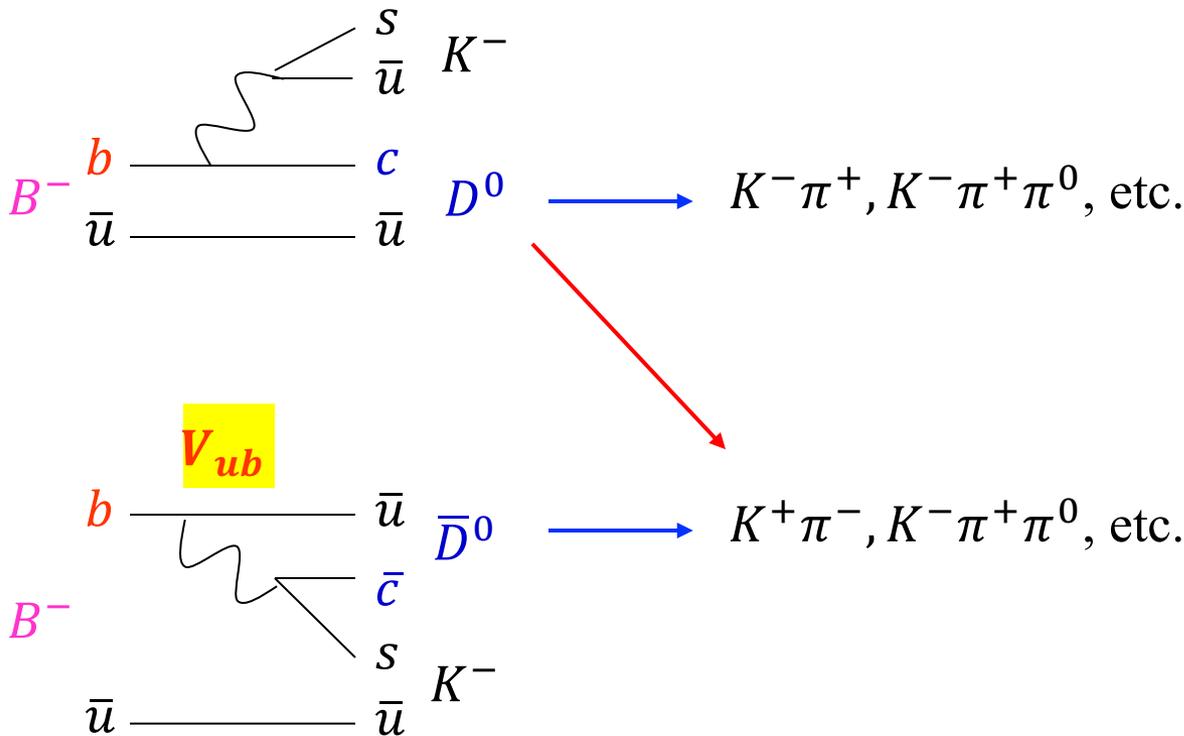
- With  $-\gamma$  becoming  $+\gamma$  for the  $B^+$  decay
- This gives two triangles from which  $\gamma$  is extracted, with  $r_B$  and  $\delta_B$ :



# The ADS method

Atwood, Dunietz, Soni, PRL 78, 3257

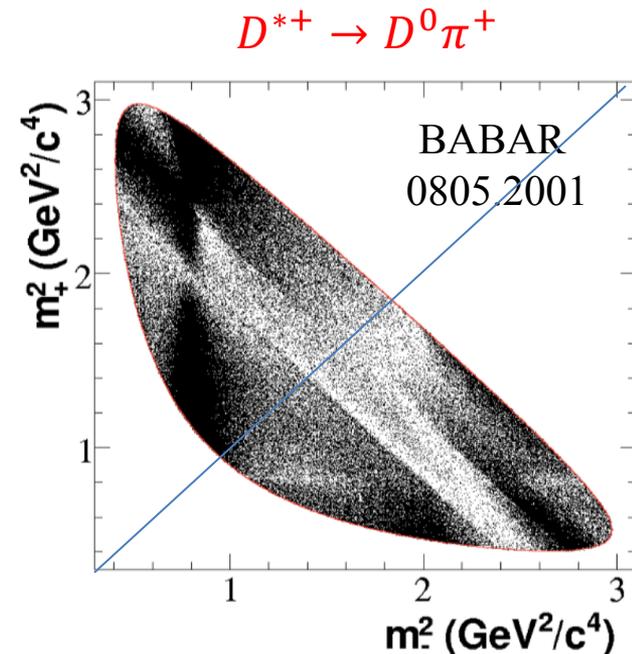
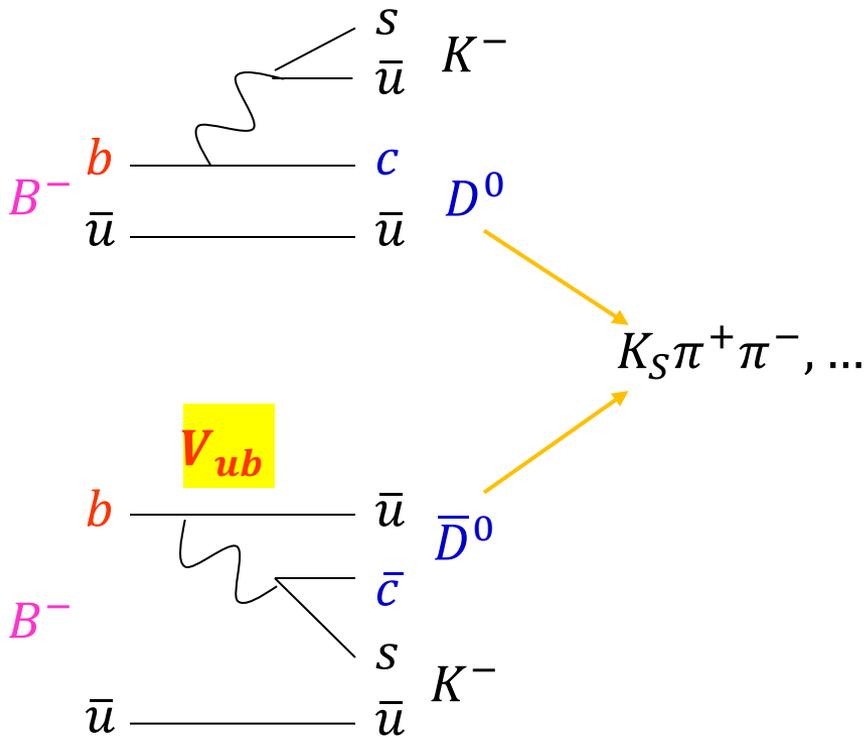
- Ratio between interfering  $B$  decays is only  $r_B \sim 0.1$ : small interference
- Exploit doubly Cabibbo-suppressed decay  $\bar{D}^0 \rightarrow K^- (n\pi)^+$ :



# The GGSZ method

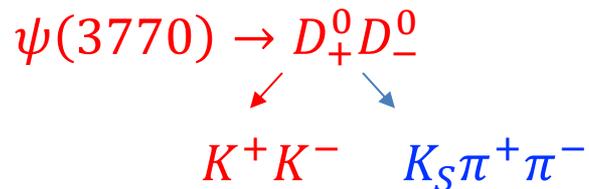
Giri, Grossman, Soffer, Zupan, PRD 68, 054018

- Exploit **multibody**  $D^0$  decays
- Help resolve trig ambiguities in  $\gamma$
- CP content depends on phase-space point
- **Complex**  $D^0$  decay amplitude can be modeled (isobar+K-matrix, etc.) with a **flavor-tagged**  $D^0$  sample:



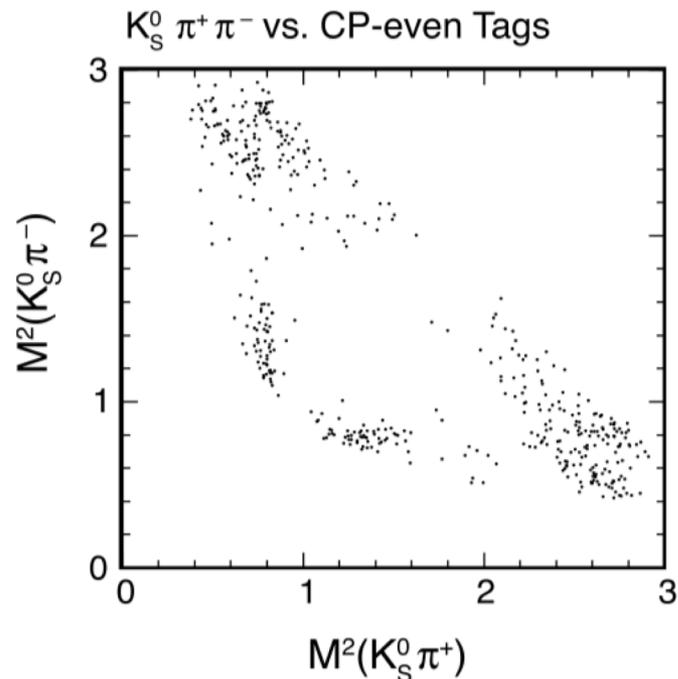
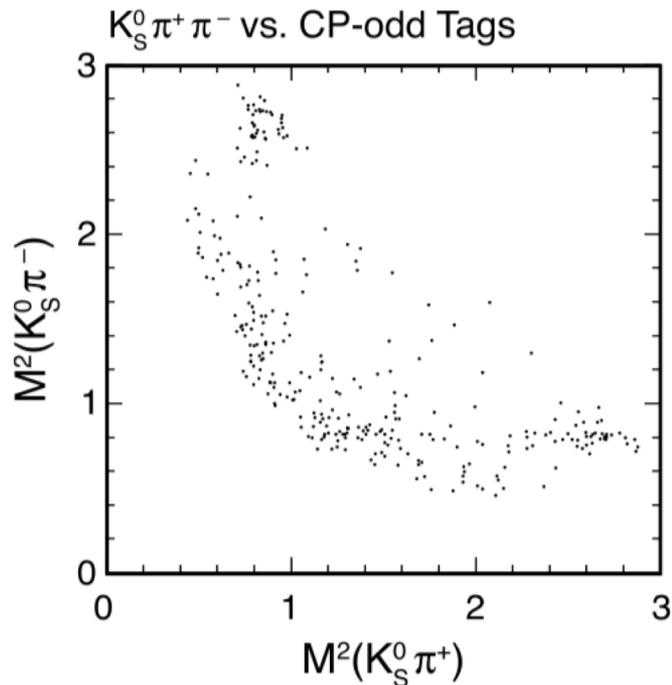
Or:

Complex  $D^0$  decay amplitude can be obtained model-independently with a CP-tagged  $D$  sample:



CLEO-c (0903.1681)

- Belle (1509.01098) & LHCb (1408.2748) measured  $\gamma$  in this model-independent method
- Stat error > model-dependent case, but no assumptions re: strong phases



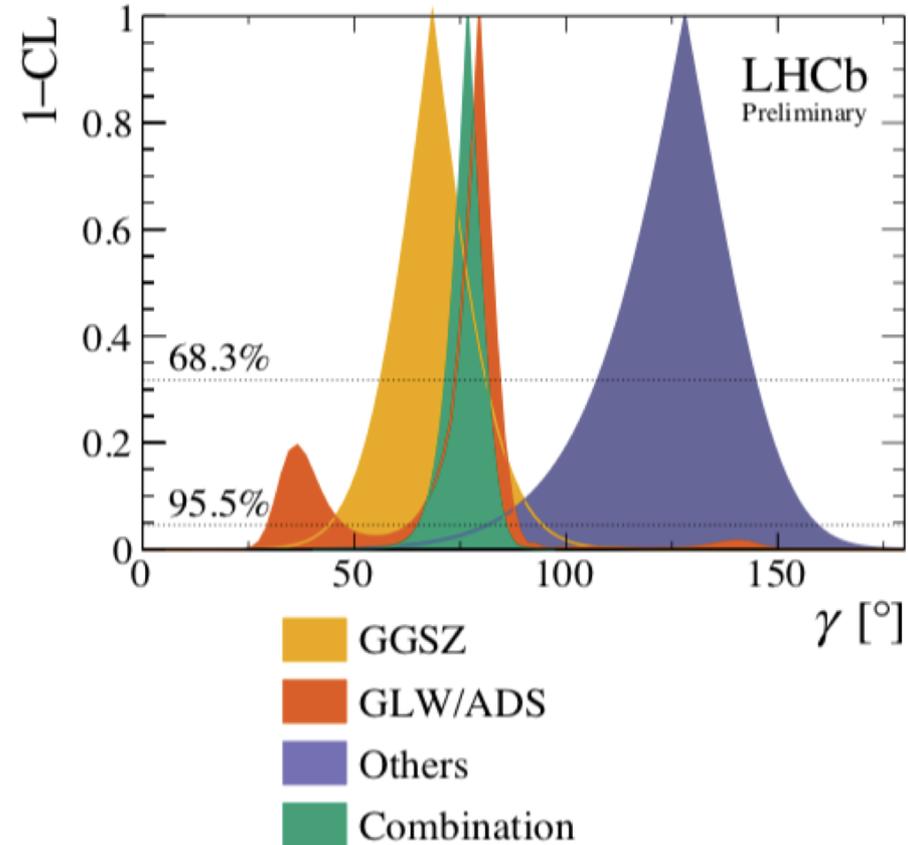
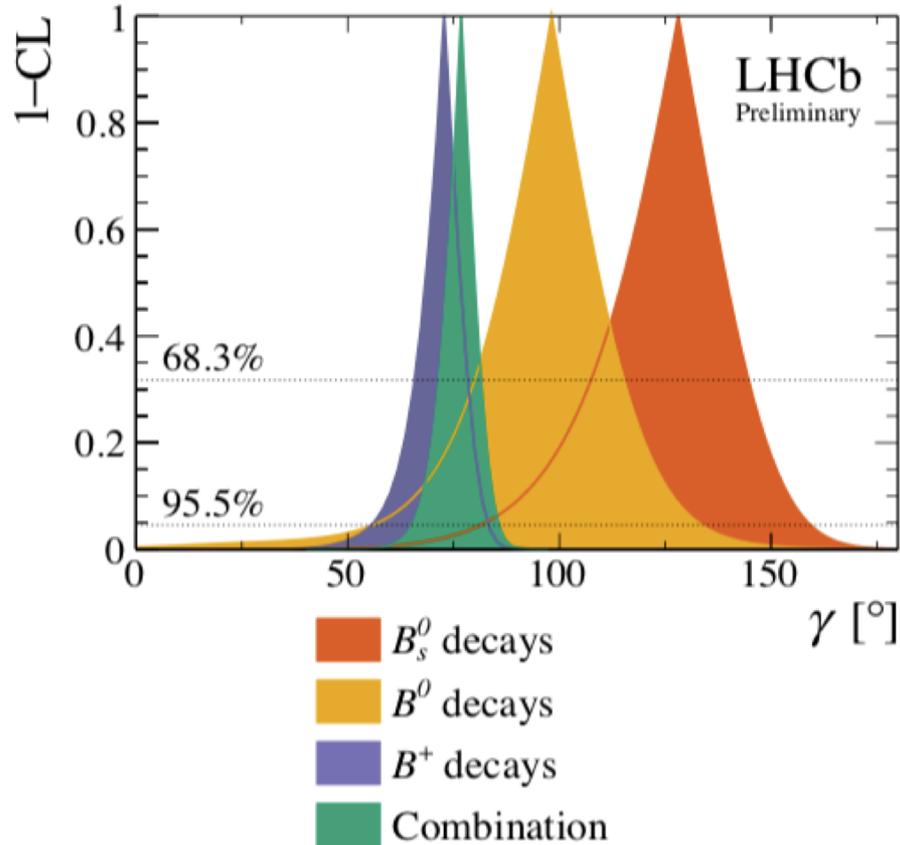
# LHCb $\gamma$ combination

LHCb-CONF-2017-004.pdf

$B$ decay	$D$ decay	Method
$B^+ \rightarrow DK^+$	$D \rightarrow h^+h^-$	GLW
$B^+ \rightarrow DK^+$	$D \rightarrow h^+h^-$	ADS
$B^+ \rightarrow DK^+$	<u><math>D \rightarrow h^+\pi^-\pi^+\pi^-</math></u>	GLW/ADS
$B^+ \rightarrow DK^+$	<u><math>D \rightarrow h^+h^-\pi^0</math></u>	GLW/ADS
$B^+ \rightarrow DK^+$	$D \rightarrow K_s^0 h^+h^-$	GGSZ
$B^+ \rightarrow DK^+$	<u><math>D \rightarrow K_s^0 K^+\pi^-</math></u>	<u>GLS</u>
<u><math>B^+ \rightarrow D^*K^+</math></u>	$D \rightarrow h^+h^-$	GLW
<u><math>B^+ \rightarrow DK^{*+}</math></u>	$D \rightarrow h^+h^-$	GLW/ADS
<u><math>B^+ \rightarrow DK^+\pi^+\pi^-</math></u>	$D \rightarrow h^+h^-$	GLW/ADS
<u><math>B^0 \rightarrow DK^{*0}</math></u>	$D \rightarrow K^+\pi^-$	ADS
<u><math>B^0 \rightarrow DK^+\pi^-</math></u>	$D \rightarrow h^+h^-$	GLW-Dalitz
<u><math>B^0 \rightarrow DK^{*0}</math></u>	$D \rightarrow K_s^0 \pi^+\pi^-$	GGSZ
<u><math>B_s^0 \rightarrow D_s^\mp K^\pm</math></u>	<u><math>D_s^+ \rightarrow h^+h^-\pi^+</math></u>	<u>TD</u>

Underlined:  
modifications of above methods

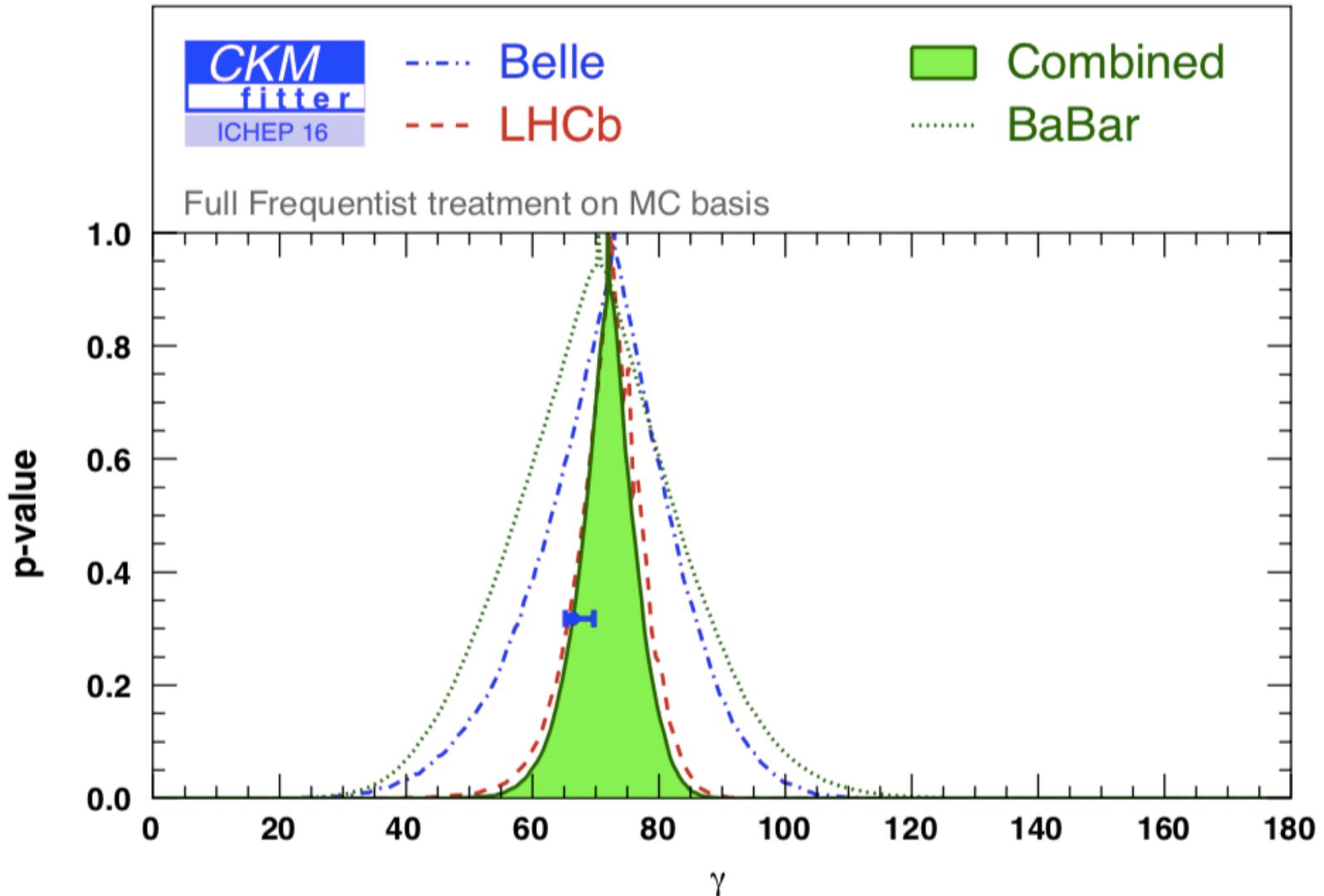
# LHCb $\gamma$ combination



$$\gamma = (76.8^{+5.1}_{-5.7})^\circ \quad (\text{Up to a } 180^\circ \text{ ambiguity})$$

# BABAR+Belle+LHCb combination

(Somewhat older LHCb results)



# Summary & comments

- $|V_{ub}/V_{cb}|$  and  $\gamma(\phi_3)$  are important SM parameters
- Crucial inputs to NP probing via unitarity triangle
- No significant inconsistency with the SM so far
- LHCb now competitive with BABAR+Belle in  $|V_{ub}/V_{cb}|$ 
  - But relies on only one lattice calculation
- LHCb dominates measurement of  $\gamma$
- Belle II will have  $\sim 35$  times more data than BABAR+Belle
- See M. Lubej's talk for expected Belle II impact on  $|V_{ub}/V_{cb}|$ 
  - Theory calculations will need to improve as well
- Belle II + LHCb should reach  $\sigma_\gamma \sim 1^\circ$  through combination of different methods

# Backup slides

- CLN expansion of form factor:

$$F(w) = F(1) - \rho^2(w - 1) + c(w - 1)^2 + \dots$$

- BGN expansion:

$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{n=0}^{\infty} a_n z^n \quad (94.6)$$

where the sum  $\sum |a_n|^2$  is bounded. Furthermore, the function  $P(z)$  takes into account the resonances in the  $(\bar{c}b)$  system below the  $\bar{D}B$  threshold, and the weighting functions  $\phi_F(z)$  are derived from the unitarity constraint on the corresponding form factor. The values of  $z$  relevant to the decay are  $0 \leq z \leq 0.06$ , hence the series in  $z$  converges rapidly and only very few terms are needed. Eq. (94.6) will be referred to as the “BGL” expansion.

# $V_{ub}$ and $V_{cb}$ summary (PDG 2017)

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3} \quad (\text{inclusive})$$

$$|V_{cb}| = (41.9 \pm 2.0) \times 10^{-3} \quad (\text{exclusive})$$

Comparable experiment & theory uncertainties

1% common systematic uncertainty.

Good agreement between the two results

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3} \quad (\text{average})$$

$$|V_{ub}| = (4.49 \pm 0.15 \pm_{-0.17}^{+0.16} \pm 0.17) \times 10^{-3} \quad (\text{inclusive})$$

$$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3} \quad (\text{exclusive})$$

Experiment

Theory

Extrapolation of model to full kinematic range