Hadronic Matrix Elements for Muon $g-2$ in Lattice QCD

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Outline

- Introduction
  - Hadronic Contributions to $g - 2$
  - Lattice QCD
- Hadronic Light-by-Light (HLbL)
  - Connected/Disconnected Diagrams
  - Systematics
- Hadronic Vacuum Polarization (HVP)
  - Summary of Existing Computations
- Improvements to Lattice HVP
  - Exclusive Channel Study for Long-Distance
  - Bounding Method

Introduction

Muon $g-2$

Precision measurement of the muon magnetic moment

$$\vec{\mu} = g \frac{e}{2m_\mu} \vec{S}$$

Want to know g factor, Dirac equation predicts $g = 2$
Radiative corrections alter to $g = 2(1+a) \implies a = \frac{g-2}{2}$

Muon $g-2$ especially sensitive to loop corrections
$\implies$ deviation from expectation could point to new physics

BNL and J-PARC to measure and improve upon BNL result

PDG world average	$a_\mu = 11\,659\,209.1 \pm 5.4 \pm 3.3 \times 10^{-10}$
RBC/UKQCD(2018)	$a_\mu = 11\,659\,181.6 \pm 3.7 \times 10^{-10}$
KNT(2018)	$a_\mu = 11\,659\,182.1 \pm 3.6 \times 10^{-10}$

Experiment-Theory difference is 27.4(7.3) $\implies$ 3.7σ tension!

Pieces of Muon $g-2$ Theory Prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.5	2.7
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	2.6
Total SM prediction	11 659 181.7	3.8
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		≈ 1.6

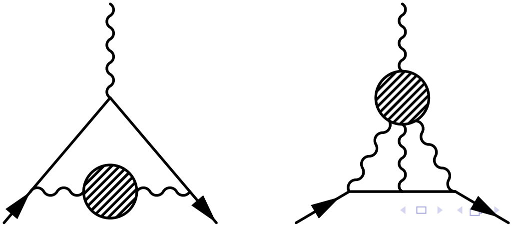
Hadronic pieces of $g-2$ are the largest contributions to theoretical uncertainty

Dispersive approaches or lattice QCD may be used to compute
Hadronic Vacuum Polarization (HVP) and Hadronic Light-by-Light (HLbL)

Difficult to assign uncertainties to models used in dispersive approach
$\Longrightarrow$ lattice QCD can give better understanding of uncertainties on hadronic contributions

HVP comes from R-ratio/Lattice QCD

Glasgow Concensus [0901.0306] for best estimate of HLbL uncertainty



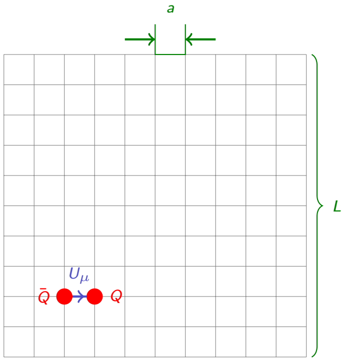
Lattice QCD: Formalism

- Lattice QCD is a technique to numerically evaluate path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

- Discretize spacetime $\implies$ #DOF finite
- Lattice spacing a provides UV cutoff
- Lattice size L provides IR cutoff

- Quark fields defined on sites $\implies$ $Q(x)$
- Gauge fields defined between sites $\implies$ $U_\mu(x)$
- Euclidean time $\implies$ correlators $\propto e^{-Et}$



Typical strategy is to construct operators at "source," allow them to propagate through time, then annihilate at "sink"

Evaluate correlation functions on fixed background gauge field, compute on many gauge fields for Monte Carlo average

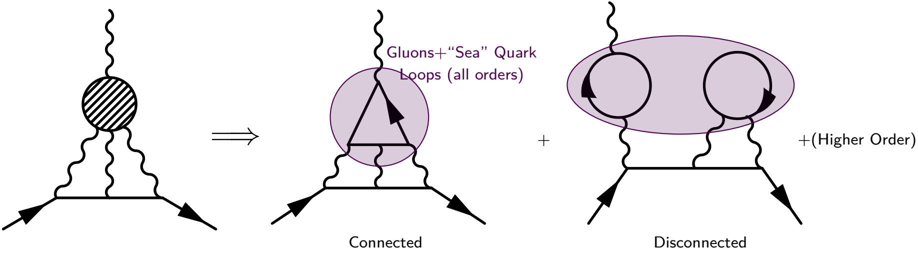
Correlation functions are products of matrix elements times exponentials, e.g.

$$C(t) = \sum_n |\langle \Omega | \mathcal{O} | n \rangle|^2 \, e^{-E_n t}$$

Hadronic Light-by-Light

HLbL Diagrams

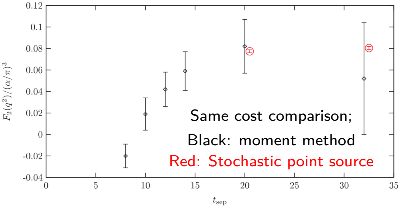
Pieces computed only existing HLbL computation:



Connected contribution $\rightarrow$ PRD 93 (2015) 014503

Disconnected contribution $\rightarrow$ PRL 118 (2016) 022005

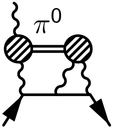
For connected diagram, improved computation method:

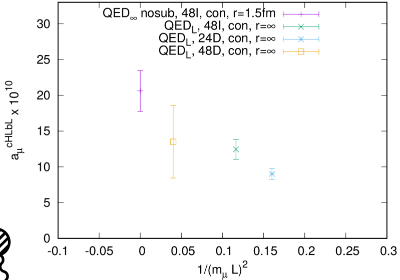


Systematics

Many systematic corrections to account for:

- QCD/QED discretization effects ($a \to 0$ limit)
- QCD/QED finite volume effects ($L \to \infty$ limit)
- Sub-leading disconnected diagrams
- Long-distance π^0, $\pi^+\pi^-$ contributions





[C.Lehner, priv.comm.]

PRL 118 (2016) 022005:

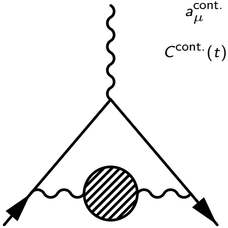
$$a_\mu^{\text{HLbL}} = 5.35(1.35) \times 10^{-10}$$

Disclaimer: Potentially large finite-volume systematics

Future work will add missing systematic corrections, combine with dispersive approach

Hadronic Vacuum Polarization

HVP in LQCD



$$a_\mu^{\text{cont.}} = \int_0^\infty dt\, K(t) C^{\text{cont.}}(t) \qquad a_\mu^{\text{latt.}} = \sum_t w_t C^{\text{latt.}}(t)$$

$$C^{\text{cont.}}(t) = \int_0^\infty d\sqrt{s}\, s\, R(s)\, e^{-\sqrt{s}t} \qquad C^{\text{latt.}}(t) = \sum_{i,n} |\langle \Omega | J_i | n \rangle|^2 e^{-E_n t}$$

$$= \frac{1}{3} \sum_{i,\vec{x}} \langle J_i(\vec{x}, t) J_i(0) \rangle$$

w_t from Bernecker, H. Meyer: 1107.4388 [hep-lat]

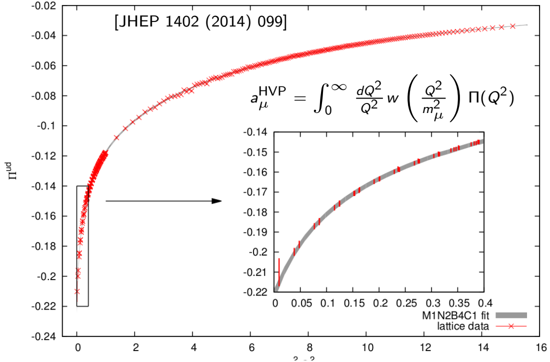
HVP formulated in momentum space:

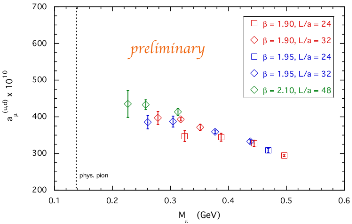
$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{m_\mu^2} w\left(\frac{Q^2}{m_\mu^2}\right) \hat{\Pi}(Q^2)\,, \qquad \hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$$

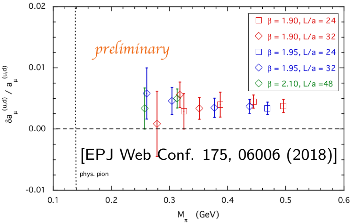
$$= \Pi_{\mu\nu}(Q) = \int d^4x e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi(Q^2)$$

$w_t \rightarrow K(t)$ relate momentum space to position space

ETMC



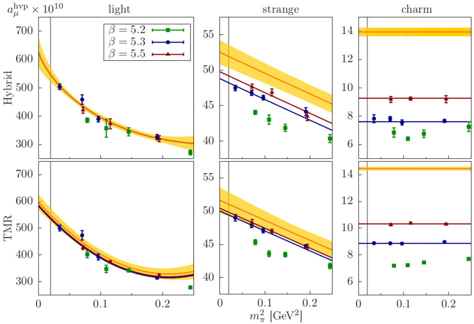




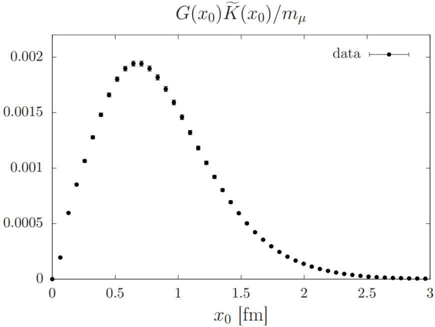
Good control over low-momentum region in 2013 HVP computation (top)

Change of strategy for 2018, preliminary results in conference proceedings (bottom)

Mainz



[JHEP 1710 (2017) 020]

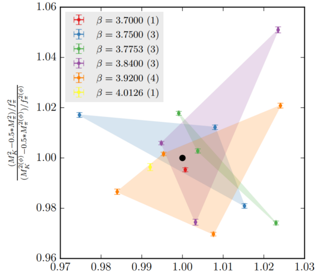


[von Hippel, KEK HVP workshop (2018)]

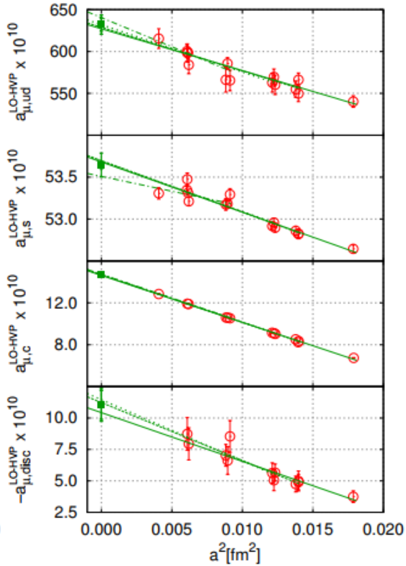
Computation of HVP in momentum space and position space (left)

Integrand of a_μ for data at physical M_π (right)

BMW



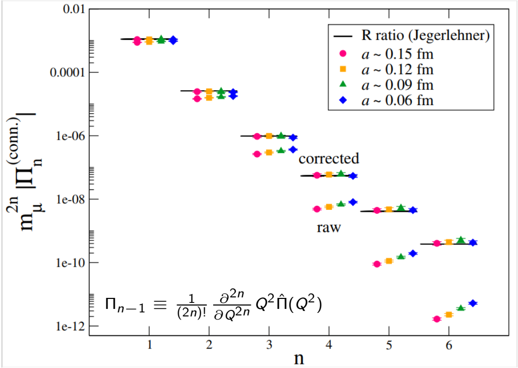
[1711.04980 [hep-lat]]



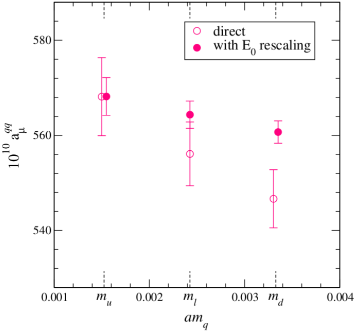
Many physical mass ensembles, 4 quark flavors in sea

Excellent control over continuum limit

Fermilab Lattice, HPQCD, MILC



[Van de Water, KEK HVP workshop (2018)]

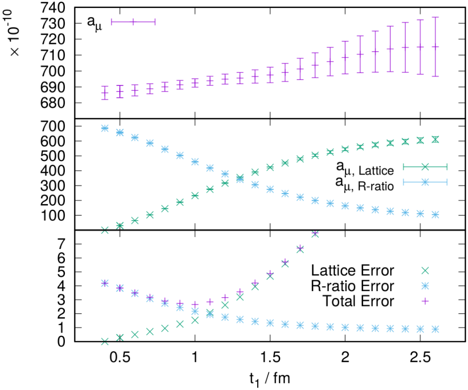


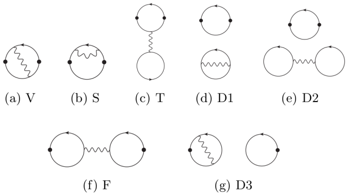
[PRL 120 (2018) 152001]

Agreement with moments from R-ratio for physical mass ensembles

First collaboration to include strong isospin breaking ($m_u \neq m_d$) effects in sea quarks

RBC/UKQCD



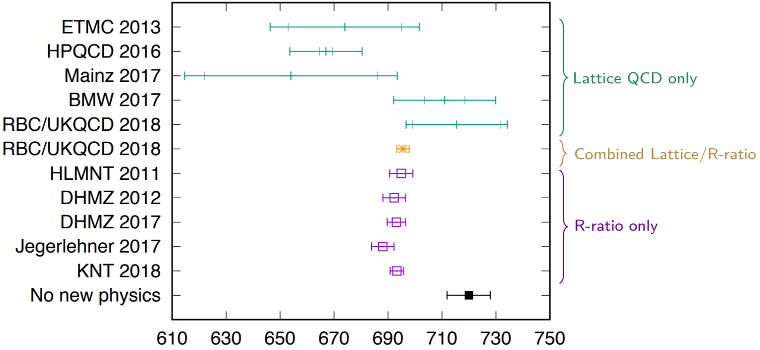


[1801.07224 [hep-lat]]

Computation of QED corrections at physical pion mass

Combined analysis of R-ratio+Lattice QCD results to get improved precision on HVP

Summary of Results

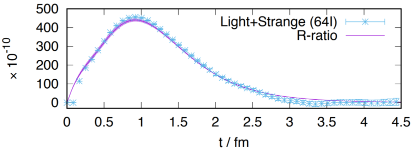


[1801.07224 [hep-lat]] $a_\mu \times 10^{10}$

Lattice calculations are less precise than R-ratio at present

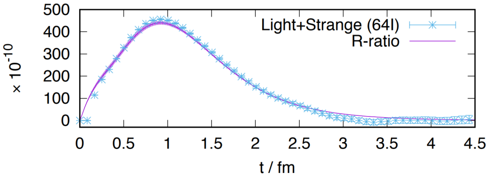
Lattice QCD/R-ratio are precise in complementary regions
$\Longrightarrow$ Combined Lattice/R-ratio for most precise estimate of HVP

Can improve lattice-only estimate of HVP by controlling statistical noise in long-distance correlation function



HVP Long-Distance Reconstruction

Long-Distance Reconstruction



Long-distance correlation function from Lattice QCD is noisy
BUT: this region well-described by only a few exponentials

Replace correlation function by an explicit sum of exponentials (N finite):

$$C(t)\Big|_{t>t_{\max}} = \sum_{n}^{N} |a_n|^2 e^{-E_n t}$$

Replaces statistical error of correlation function with systematic error of reconstruction

Good control over systematics associated with reconstruction
$\Longrightarrow$ improved statistical precision of HVP

Computation Details

All results shown for one coarse ensemble:

- $a \approx 0.20 \text{ fm} \approx 1.0 \text{ GeV}^{-1}$,
- $24^3 \times 64$ (4.8 fm box)
- physical pion mass ensemble

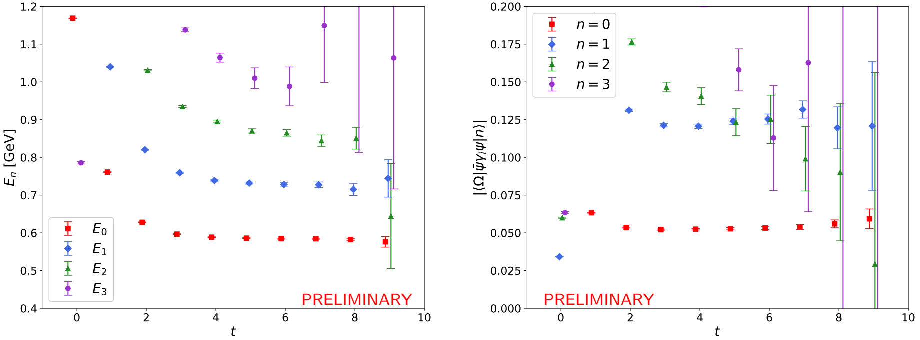
Correlation functions computed both with local currents and distillation
4×4 basis of correlation functions used to improve uncertainties

Have data on two other ensembles, 32^3 and 48^3 lattice volumes
Analysis is underway but incomplete

Will include at least one more lattice ensemble to control systematics:

- continuum limit
- volume dependence

GEVP Results



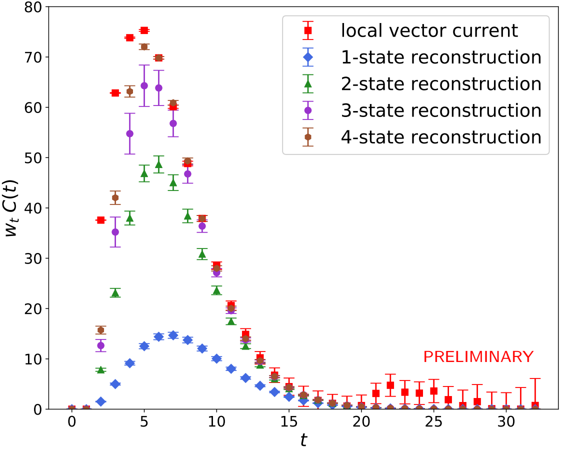
Spectrum/overlap (matrix element) on left/right

4×4 operator basis, 4 state reconstruction

Each choice of n asymptotes to single-state energy/overlap as $t \to \infty$

Precise determinations of overlap factors are necessary for good reconstruction

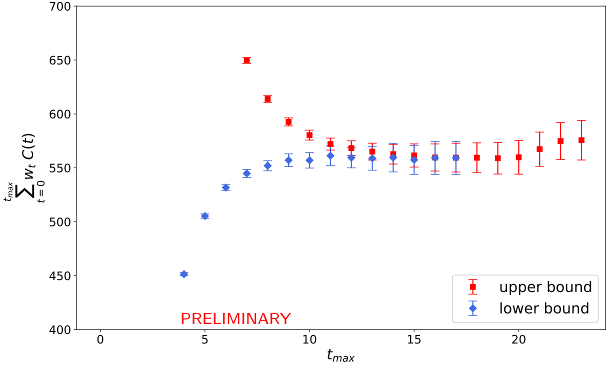
a_μ Integrand Reconstruction



GEVP used to reconstruct long-distance behavior of the local vector current correlation function

More states $\Longrightarrow$ more timeslices described well by reconstruction

Bounding Method



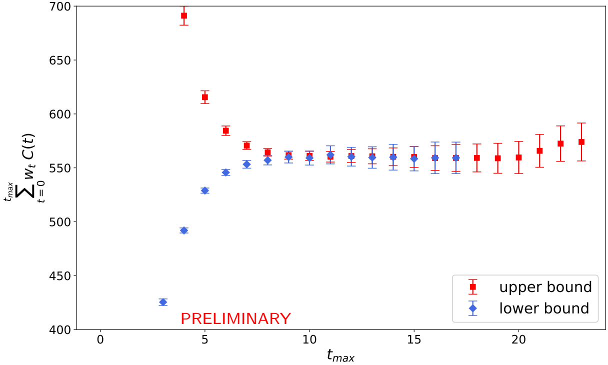
No bounding method: $a_\mu^{HVP} = 516(51)$

Bounding method $T = 2.4$ fm, no reconstruction: $a_\mu^{HVP} = 563.9(9.1)$

Disclaimer: very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects

Could expect $10 - 20\%$ systematic errors

Bounding Method

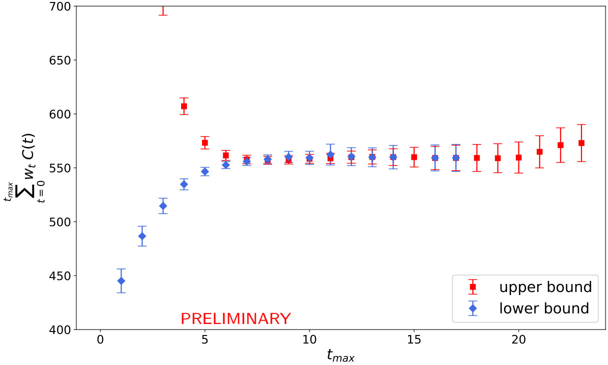


No bounding method:	$a_\mu^{HVP} = 516(51)$
Bounding method $T = 2.4\ \mathrm{fm}$, no reconstruction:	$a_\mu^{HVP} = 563.9(9.1)$
Bounding method $T = 1.8\ \mathrm{fm}$, 1 state reconstruction:	$a_\mu^{HVP} = 560.9(4.5)$

Disclaimer: very large lattice spacing: $a^{-1} = 1.015\ \mathrm{GeV}$, finite volume effects

Could expect $10 - 20\%$ systematic errors

Bounding Method



No bounding method:	$a_\mu^{HVP} = 516(51)$
Bounding method $T = 2.4$ fm, no reconstruction:	$a_\mu^{HVP} = 563.9(9.1)$
Bounding method $T = 1.8$ fm, 1 state reconstruction:	$a_\mu^{HVP} = 560.9(4.5)$
Bounding method $T = 1.4$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 556.8(3.8)$

Disclaimer: very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects

Could expect $10 - 20\%$ systematic errors

Conclusions

- $g-2$ is an interesting and exciting topic to work on
- Hadronic contributions to $g-2$ are difficult to estimate with theory
- Lattice QCD is a first principles method capable of accessing necessary matrix elements

Hadronic Light-by-light:

- First estimate including fully connected and leading disconnected diagrams available
- Work toward addressing neglected systematics

Hadronic Vacuum Polarization:

- Lots of activity from many collaborations
- Movement toward addressing all systematic corrections
- Study of exclusive channels able to reduce uncertainty on all-lattice computation of muon HVP
- Part of ongoing lattice study to address all lattice systematics in HVP computation

Lots of data to analyze, lots of work ahead of us!

RBC & UKQCD Collaborations

BNL & RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Yong-Chull Jang
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
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Tobias Tsang
Oliver Witzel
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Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

Thank you for your attention!

Error Budget

$a_\mu^{\text{ud, conn, isospin}}$	$202.9(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.1)_{\mathrm{V}}(0.2)_{\mathrm{A}}(0.2)_{\mathrm{Z}}$	$649.7\boxed{(14.2)_{\mathrm{S}}}\boxed{(2.8)_{\mathrm{C}}(3.7)_{\mathrm{V}}}\boxed{(1.5)_{\mathrm{A}}}(0.4)_{\mathrm{Z}}(0.1)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}$
$a_\mu^{\text{s, conn, isospin}}$	$27.0(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$	$53.2(0.4)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.3)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$
$a_\mu^{\text{c, conn, isospin}}$	$3.0(0.0)_{\mathrm{S}}(0.1)_{\mathrm{C}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$	$14.3(0.0)_{\mathrm{S}}(0.7)_{\mathrm{C}}(0.1)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$
$a_\mu^{\text{uds, disc, isospin}}$	$-1.0(0.1)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}$	$-11.2(3.3)_{\mathrm{S}}(0.4)_{\mathrm{V}}(2.3)_{\mathrm{L}}$
$a_\mu^{\text{QED, conn}}$	$0.2(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}$	$5.9\boxed{(5.7)_{\mathrm{S}}}(0.3)_{\mathrm{C}}(1.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.1)_{\mathrm{E}}$
$a_\mu^{\text{QED, disc}}$	$-0.2(0.1)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.0)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}$	$-6.9\boxed{(2.1)_{\mathrm{S}}}(0.4)_{\mathrm{C}}(1.4)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.3)_{\mathrm{E}}$
a_μ^{SIB}	$0.1(0.2)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E48}}$	$10.6(4.3)_{\mathrm{S}}(0.6)_{\mathrm{C}}(6.6)_{\mathrm{V}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.3)_{\mathrm{E48}}$
$a_\mu^{\text{udsc, isospin}}$	$231.9(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.1)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{M}}$	$705.9(14.6)_{\mathrm{S}}(2.9)_{\mathrm{C}}(3.7)_{\mathrm{V}}(1.8)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(2.3)_{\mathrm{L}}(0.1)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}(0.0)_{\mathrm{M}}$
$a_\mu^{\text{QED, SIB}}$	$0.1(0.3)_{\mathrm{S}}(0.0)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.0)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}$	$9.5(7.4)_{\mathrm{S}}(0.7)_{\mathrm{C}}(6.9)_{\mathrm{V}}(0.1)_{\mathrm{A}}(0.0)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(1.3)_{\mathrm{E48}}$
$a_\mu^{\text{R-ratio}}$	$460.4(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$	
a_μ	$692.5(1.4)_{\mathrm{S}}(0.2)_{\mathrm{C}}(0.2)_{\mathrm{V}}(0.3)_{\mathrm{A}}(0.2)_{\mathrm{Z}}(0.0)_{\mathrm{E}}(0.0)_{\mathrm{E48}}(0.0)_{\mathrm{b}}(0.1)_{\mathrm{c}}(0.0)_{\overline{\mathrm{S}}}(0.0)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}(0.7)_{\mathrm{RST}}(2.1)_{\mathrm{RSY}}$	$715.4(16.3)_{\mathrm{S}}(3.0)_{\mathrm{C}}(7.8)_{\mathrm{V}}(1.9)_{\mathrm{A}}(0.4)_{\mathrm{Z}}(1.7)_{\mathrm{E}}(2.3)_{\mathrm{L}}(1.5)_{\mathrm{E48}}(0.1)_{\mathrm{E64}}(0.3)_{\mathrm{b}}(0.2)_{\mathrm{c}}(1.1)_{\overline{\mathrm{S}}}(0.3)_{\overline{\mathrm{Q}}}(0.0)_{\mathrm{M}}$

TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

Full program of computations to reduce uncertainties:

- Reduce statistical uncertainties on light connected contribution
- Compute QED contribution
- Improve lattice spacing determination
- Finite volume and continuum extrapolation study

Generalized EigenValue Problem

Two vector current operators:

- Local $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu \psi(x)$
- Smeared $\mathcal{O}_1 = \sum_{xyz} \bar{\psi}(x) f(x-z)\gamma_\mu f(z-y)\psi(y)$

Two 2π operators with different momenta

$$\mathcal{O}_n = \left| \sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z-y)\psi(y) \right|^2:$$

- $\mathcal{O}_2 : \frac{L}{2\pi}\vec{p}_\pi = (1,0,0)$
- $\mathcal{O}_3 : \frac{L}{2\pi}\vec{p}_\pi = (1,1,0)$

Correlators arranged in a 4×4 symmetric matrix:

	$\mathcal{O}_0$	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$
$\mathcal{O}_0$	$C_\rho^{(2)}$	$C_\rho^{(2)}$	$C_{\rho\to\pi\pi}^{(3)}$	$C_{\rho\to\pi\pi}^{(3)}$
$\mathcal{O}_1$		$C_\rho^{(2)}$	$C_{\rho\to\pi\pi}^{(3)}$	$C_{\rho\to\pi\pi}^{(3)}$
$\mathcal{O}_2$			$C_{\pi\pi\to\pi\pi}^{(4)}$	$C_{\pi\pi\to\pi\pi}^{(4)}$
$\mathcal{O}_3$				$C_{\pi\pi\to\pi\pi}^{(4)}$

Analyze with Generalized EigenValue Problem (GEVP) method:

$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on a_μ^{HVP}

$$\widetilde{C}(t;T,E) = \begin{cases} C(t) & t < T \\ C(T)e^{-E(t-T)} & t \geq T \end{cases}$$

Upper bound: $E = E_0$, ground state of spectrum

Lower bound: $E = \log[\frac{C(T)}{C(T+1)}]$

With exclusive state reconstruction, get improved bounding method precision by replacing
$C(t) \to C(t) - \sum_n |c_n|^2 e^{-E_n t}$

Add back contribution from reconstruction after applying bounding method