Nucleon Polarizabilities:

Status Report
and
First Partial-Wave Analysis of Compton Scattering Data

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***absent because of US visa delay**

in Collaboration with
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@ CIPANP
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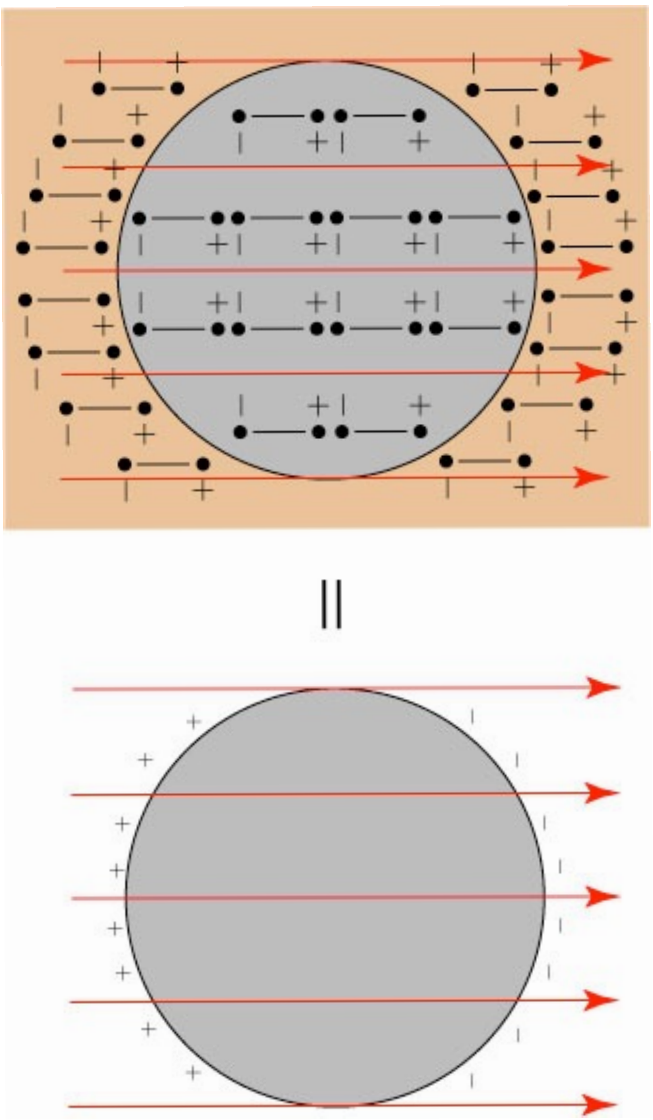
JG|U

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Concept of polarizabilities

- A dielectric system in external e.m. field is polarized, e.g. in a uniform electric field:



- induced electric dipole polarization:

$$\vec{P} = \alpha_{E1}\,\vec{E} \qquad \text{(linear dielectric)}$$

electric polarizability

- for polarization induced by magnetic field:

$$\vec{P} = \beta_{M1}\,\vec{B}$$

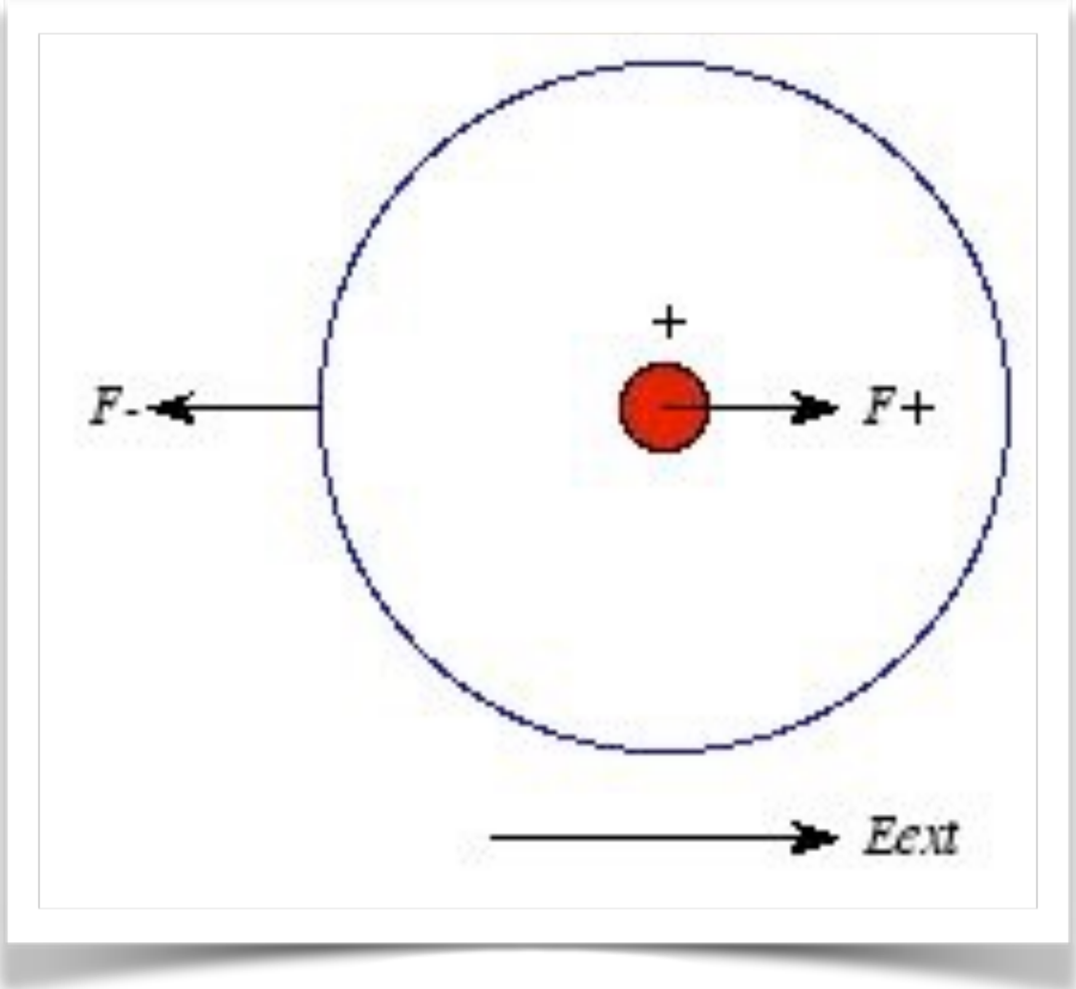
magnetic polarizability

"Classical atom."

The external field displaces the nucleus w.r.t. the electron cloud until the forces are equal:

$$\vec{F}_{ext} = \vec{F}_{cloud}$$

$$e\,\vec{E}_{ext} = \tfrac{1}{3} e\rho\,\vec{d} = \frac{e^2}{3V}\vec{d}$$



The induced polarization,

$$\vec{P} = e\vec{d} \equiv \alpha_{E1}\vec{E}_{ext}$$

yields:

$$\alpha_{E1} = 3V$$ proportional to the volume

Quantum atom

Include the external field as perturbation:

$$H_{pert} = e\,\vec{r}\cdot\vec{E}_{ext} = e\,r\,E_{ext}\cos\theta$$

1st order yields the Stark effect.

2nd order, the polarizability effect:

$$\Delta E^{(2)} = \sum_{n=2} \frac{\langle 1s|H_{pert}|n\rangle^2}{E_1 - E_n} = \frac{1}{2}\alpha_{E1} E_{ext}^2$$

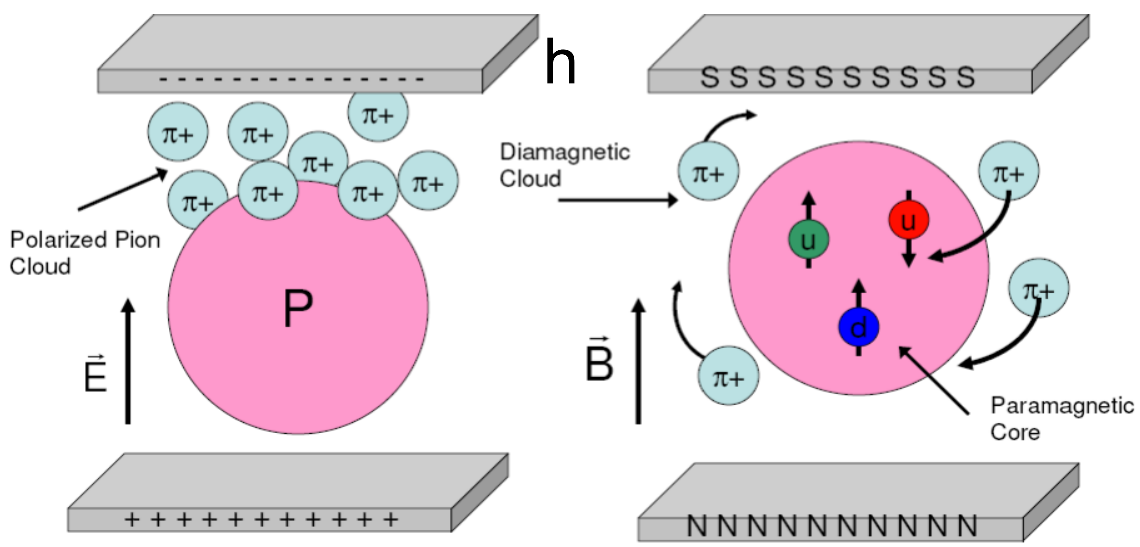
$$\alpha_{E1} = -2e^2 \sum_{n=2} \frac{\langle 1s|r\cos\theta|n\rangle^2}{E_1 - E_n} \approx 1.7 \times 4\pi a_{Bohr}^3 = 5V$$

probes the excitation spectrum!

Nucleon is different

Proton: $V \sim \langle r_p \rangle^3 \approx 0.6 \text{ fm}^3$, cf. experiment: $\alpha_{E1p}^{(exp.)} = (11 \pm 1) \times 10^{-4} \text{ fm}^3$

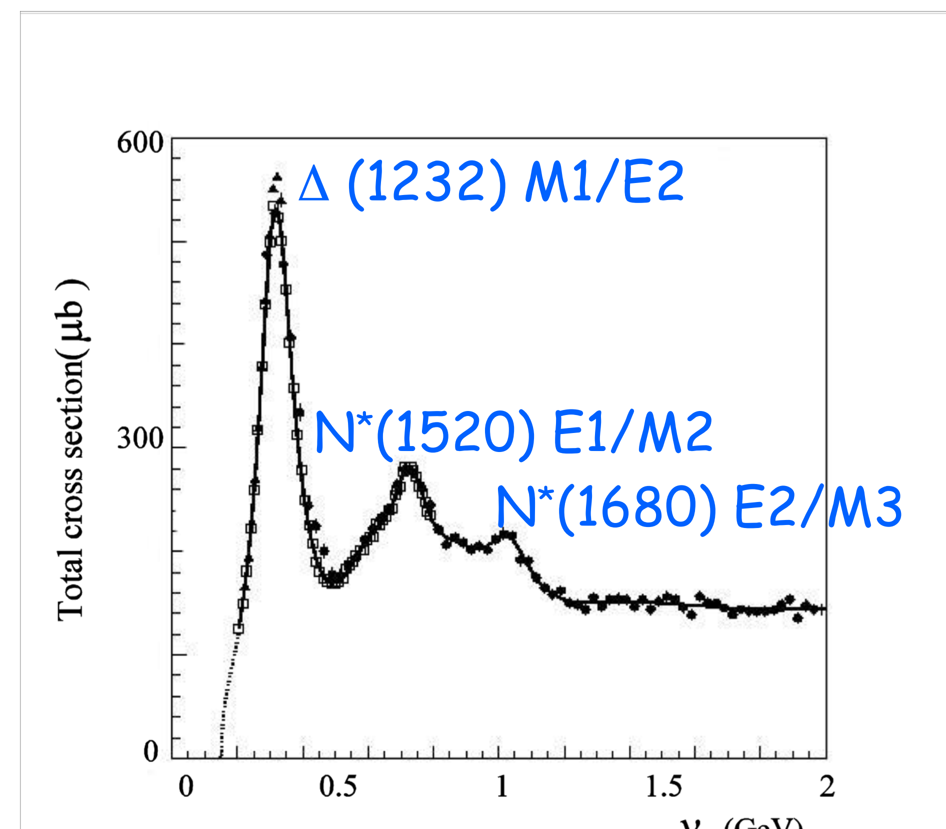
1000 times "stiffer" than hydrogen!



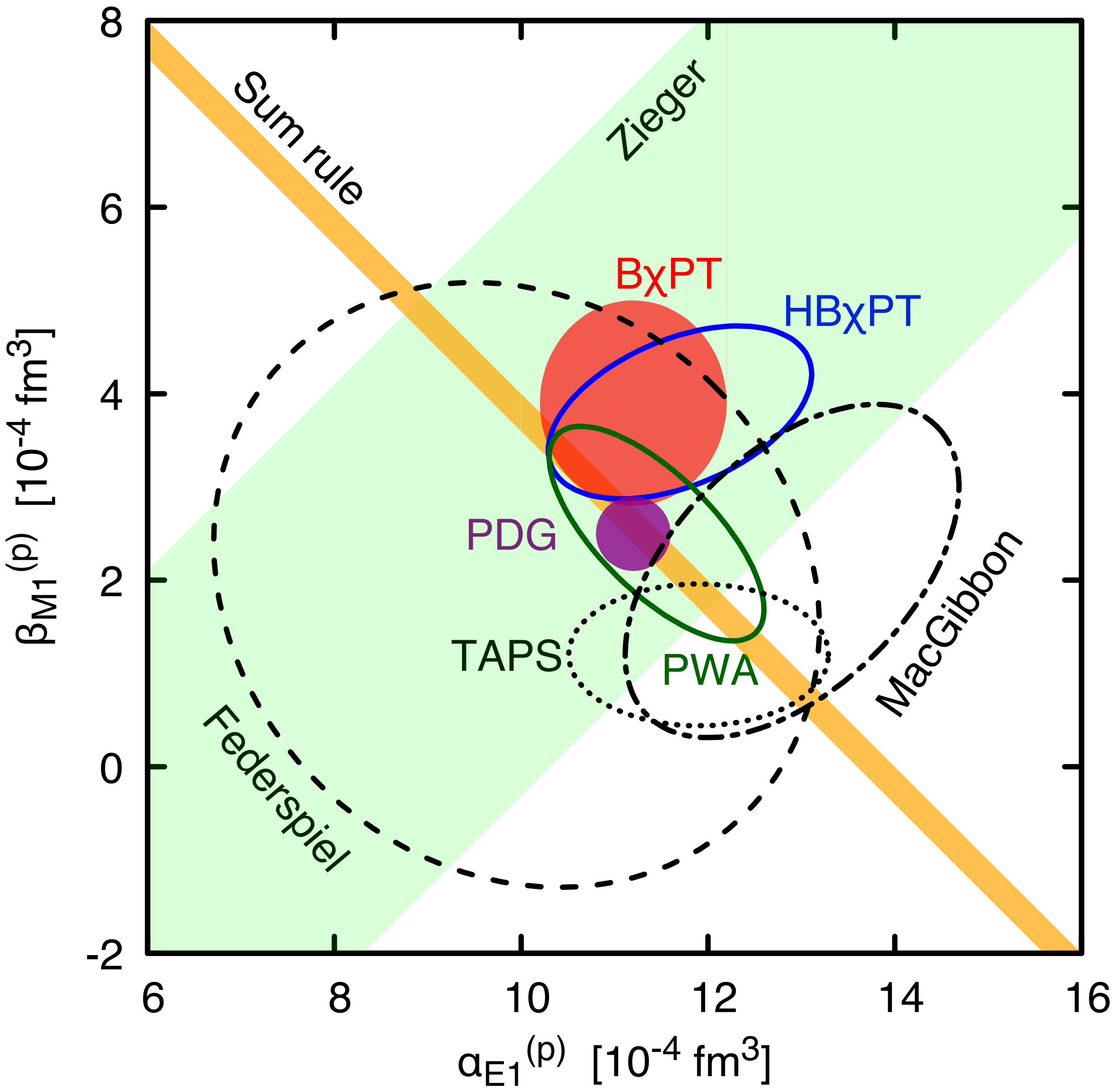
$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \text{fm}^3$$

[Baldin sum rule (1960)]

diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$



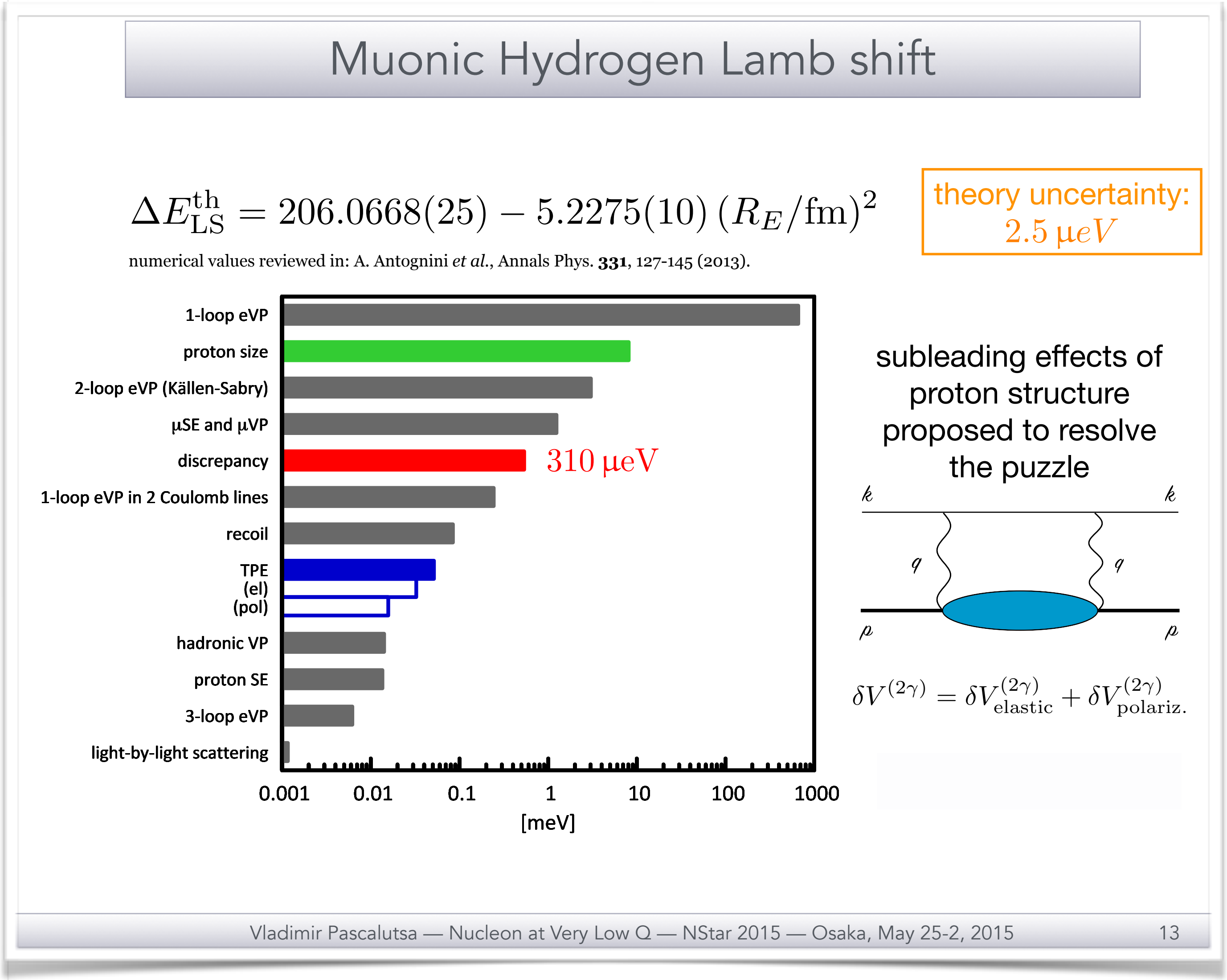
Static polarizabilities of the proton

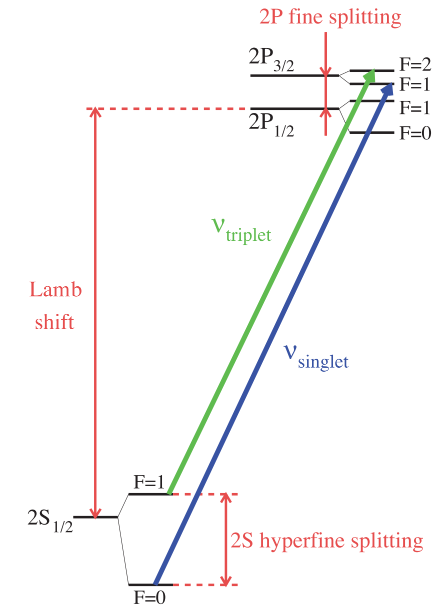


- **TAPS:** fit to TAPS/MAMI data based on fixed-*t* DRs of L'vov et al.
  Olmos de Leon et al., *EPJA (2001)*
- **BChPT:** "postdiction"
  Lensky & VP, *EPJC (2010)*
  Lensky, McGovern & VP, *EPJC (2015)*
- **HBChPT:** fit to world data
  Grieβhammer, McGovern & Phillips, *EPJA (2013)*
- **PWA:** fit to world data
  Krupina, Lensky & VP, *PLB (2018)*

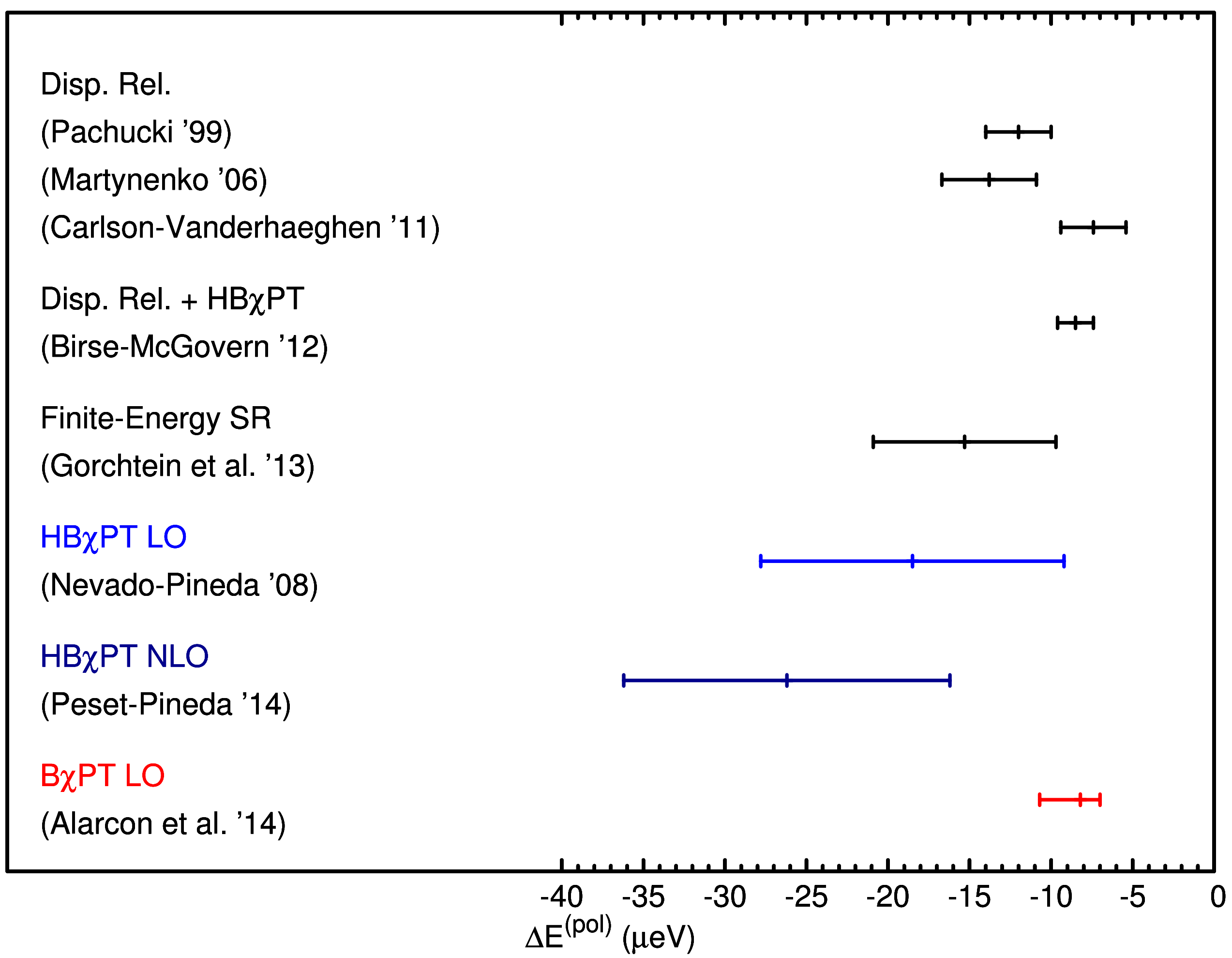
Partial-Wave Analysis (PWA):
differences between DR and ChPT extractions are due to database inconsistencies, improvements — new experiments — are needed!

Effect on muonic-hydrogen Lamb shift





Compiled by: Hagelstein, Miskimen & VP, *Prog. Part. Nucl. Phys. (2016)*



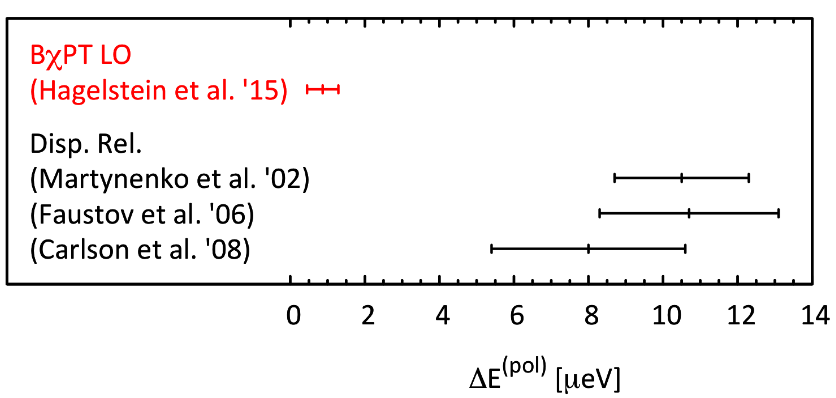
Hyperfine splitting in muonic hydrogen

HFS theory status

$$\Delta E_{\mathrm{HFS}}(1S) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak+hVP}} + \underbrace{\Delta_{\mathrm{Zemach}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}}_{\Delta_{\mathrm{TPE}}}]\Delta E_0^{\mathrm{HFS}}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

	μp		$\mu\,^3\mathrm{He}^+$		
	Magnitude	Uncertainty	Magnitude	Uncertainty	
$\Delta E_0^{\mathrm{HFS}}$	182.443 meV	0.1×10^{-6}	1370.725 meV	0.1×10^{-6}	
Δ_{QED}	1.1×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}	
$\Delta_{\mathrm{weak+hVP}}$	2×10^{-5}	2×10^{-6}			
Δ_{Zemach}	7.5×10^{-3}	7.5×10^{-5}	3.5×10^{-2}	2.2×10^{-4}	$\leftarrow G_E(Q^2), G_M(Q^2)$
Δ_{recoil}	1.7×10^{-3}	10^{-6}	2×10^{-4}		$\leftarrow G_E, G_M, F_1, F_2$
Δ_{pol}	4.6×10^{-4}	8×10^{-5}	$(3.5 \times 10^{-3})^*$	$(2.5 \times 10^{-4})^*$	$\leftarrow g_1(x,Q^2), g_2(x,Q^2)$



A. Antognini BVR47, PSI 09.02.2016 – p. 5

First experiments are planned at PSI and JPARC!

ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Castello di Trento ("Trint"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum,

Nucleon Spin Structure at Low Q: A Hyperfine View

Trento, July 2 - 6, 2018

Main Topics

- New measurements of spin structure functions, polarizabilities and form factors
- Sum rules, dispersion relations and empirical parametrizations
- Chiral perturbation theory of nucleon spin polarizabilities
- Progress in lattice QCD of the nucleon spin structure
- Hyperfine structure of muonic hydrogen

Confirmed Speakers

M.W. Ahmed (*Duke University*, USA), J. M. Alarcon (*JLab*, USA), C. Alexandrou (*University of Cyprus, Nicosia*, Cyprus), F. Hagelstein (*Universität Bern*, Switzerland), C. Carlson (*College of William and Mary*, USA), S. Kanda (*Riken*, Japan), S. Kuhn (*Old Dominion University*, USA), V. Lensky (*Universität Mainz*, Germany), P. Martel (*Universität Mainz*, Germany), H.W. Lin (*Michigan State University*, USA), K. Ottnad (*Universität Mainz*, Germany), E. Pace (*University of Rome Tor Vergata and INFN*, Italy), K. Pachucki (*University of Warsaw*, Poland), A. Pineda (*IFAE, Barcelona*, Spain), Jan Rijneveen (*University of Bochum*, Germany), Nora Rijneveen (*University of Bochum*, Germany), M. Ripani (*INFN Genoa*, Italy), S. Sconfietti (*INFN Pavia*, Italy), K. Slifer (*University of New Hampshire*, USA), N. Sparveris (*Temple University*, USA), L. Tiator (*Universität Mainz*, Germany), A. Vacchi (*INFN Trieste*, Italy).

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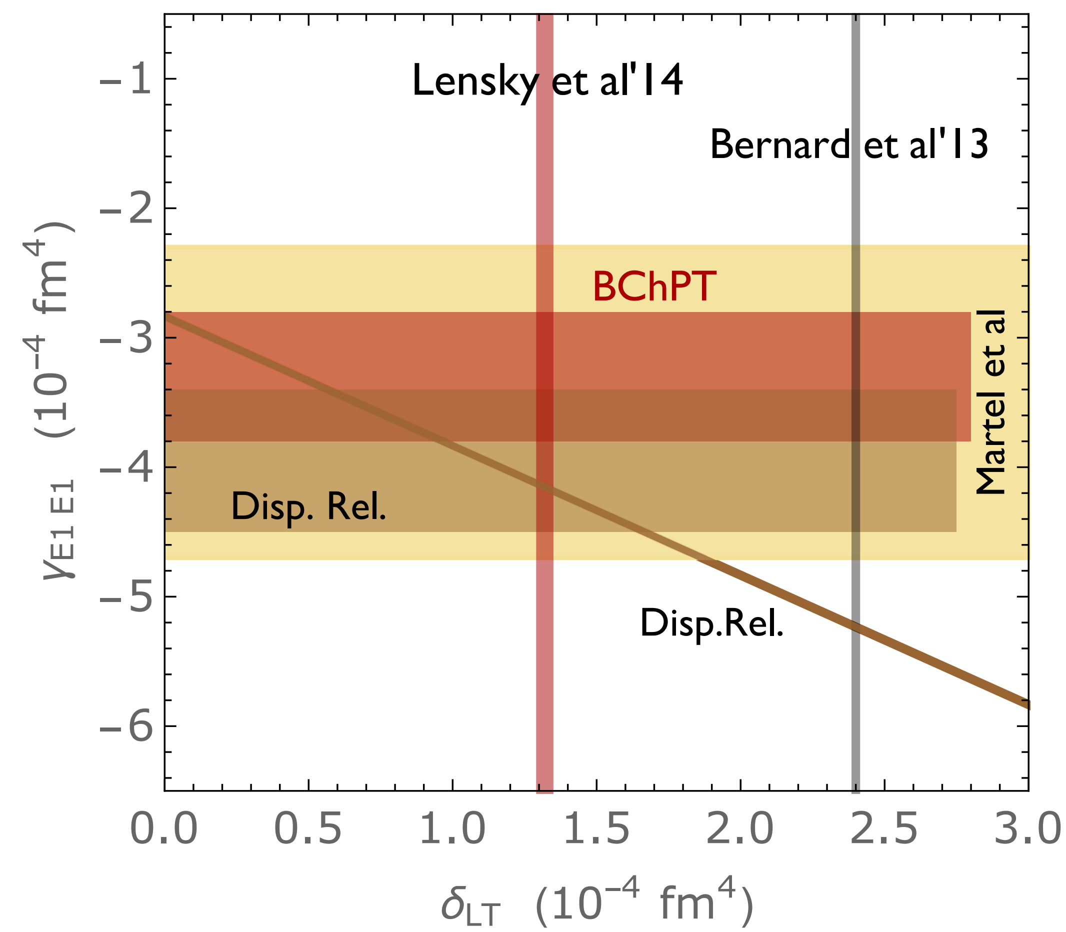
Spin structure at low Q

One of the problems is **"deltaLT puzzle"**: where two ChPT calculations disagree by about a factor of 2

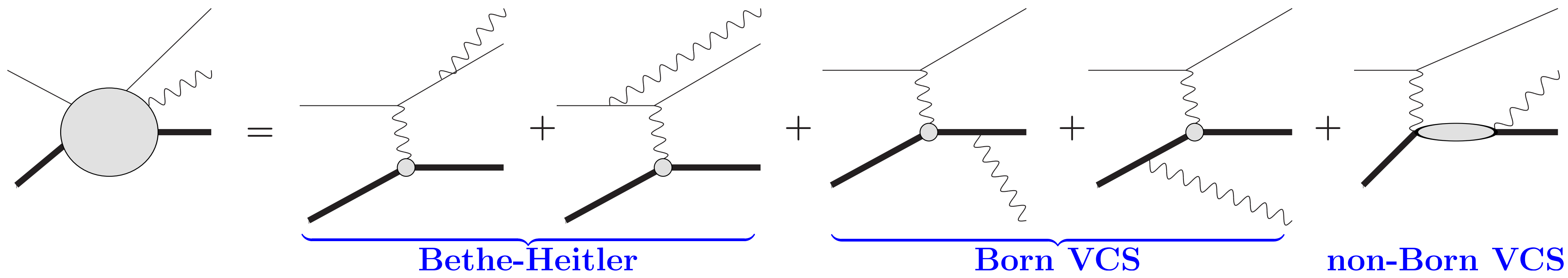


New relations among polarizabilities, e.g.:

$$\delta_{LT} = -\gamma_{E1E1} + \text{VCS spin GPs}$$

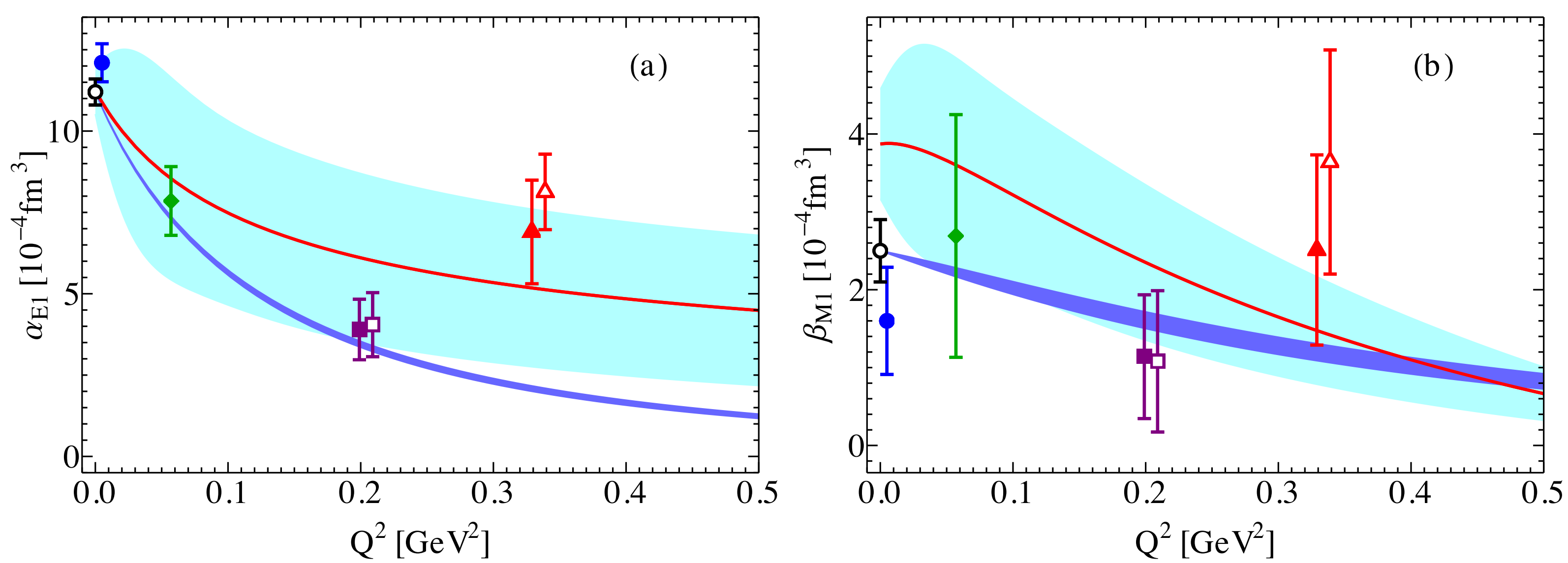
VP & Vanderhaeghen, *PRD (2015)*
Lensky, VP, Vanderhaeghen & Kao, *PRD (2017)*
Lensky, Hagelstein, VP & Vanderhaeghen, *PRD (2018)*

Virtual Compton scattering (VCS) and generalized polarizabilities (GPs)

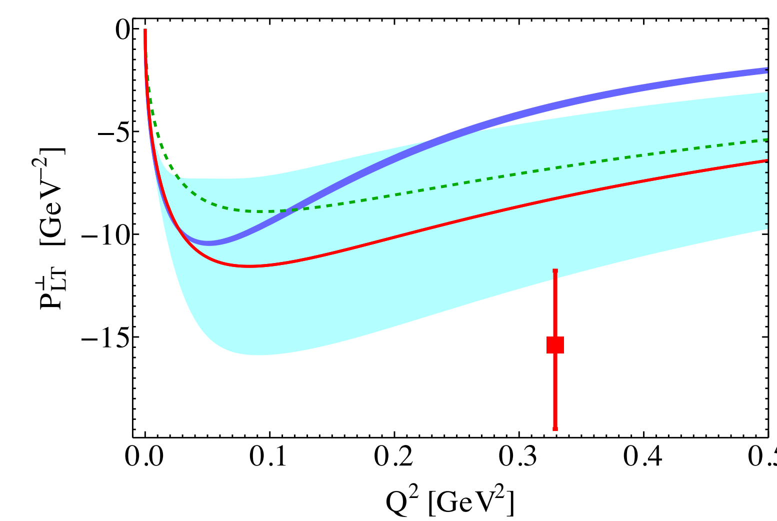


NLO-BChPT: Lensky, VP & Vanderhaeghen, *EPJC (2017) [1612.08626]*

Fixed-*t* DR: Pasquini et al., *PRC (2000); EPJA (2001)*



open circle, PDG 2014 [61]; blue circle, Olmos de León et al [62]; green diamond, MIT-Bates (DR) [7, 8]; green open diamond, MIT-Bates (LEX) [7, 8]; purple solid square, MAMI (DR) [13]; purple open square, MAMI (LEX) [13]; red solid triangle, MAMI1 (LEX) [9]; red solid inverted triangle, MAMI1 (DR) [11]; red open triangle, MAMI2 (LEX) [10]. Some of the data points are shifted to the right in order to enhance their visibility; namely, Olmos de León, MIT-Bates (LEX), MAMI LEX, MAMI1 DR and MAMI2 LEX sets have the same values of Q^2 as PDG, MIT-Bates (DR), MAMI DR, and MAMI1 LEX, respectively.



preliminary MAMI data:
L. Corea, H. Fonvieille, H. Merkel et al. [A1 Coll.]

Partial-wave analysis (PWA) of Compton scattering data below pion production threshold

Krupina, Lensky & VP, *Phys. Lett. B782 (2018) 34.*

Sum rule determination of forward Compton scattering

PHYSICAL REVIEW D **92,** 074031 (2015)

Evaluation of the forward Compton scattering off protons: Spin-independent amplitude

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(Received 2 September 2015; published 21 October 2015)

PHYSICAL REVIEW D **94,** 034043 (2016)

Evaluation of the forward Compton scattering off protons. II. Spin-dependent amplitude and observables

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(Received 7 April 2016; published 31 August 2016)

Review

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Review

Nucleon polarizabilities: From Compton scattering to hydrogen atom

CrossMark

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Basic Introduction

IOP Concise Physics

Causality Rules

A light treatise on dispersion relations and sum rules

Vladimir Pascalutsa

Compton scattering specifics

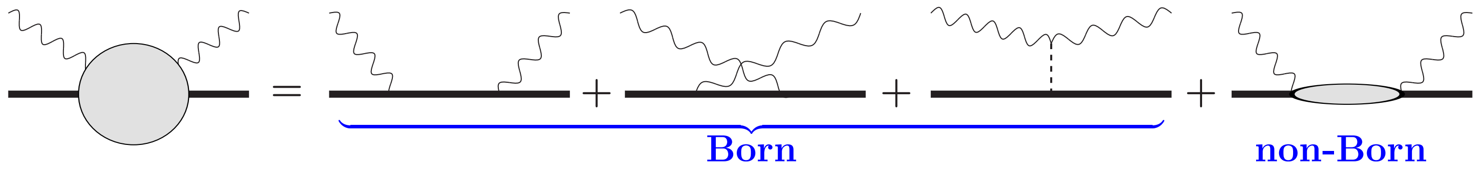


FIG. 1: Mechanisms contributing to real CS: Born and non-Born terms.

- No resonances (below pion production threshold)
- Multipoles are real, neglecting radiative corrections
- Forward-scattering is determined, via the sum rules (photoabsorption cross sections):

  yields linear relations on the multipoles, rather than bilinear

- Not much data (about 100 data points, many from old experiments)

Multipole Expansion

W. Pfeil, H. Rollnik and S. Stankowski, "A partial-wave analysis for proton Compton scattering in the delta(1232) energy region," Nucl. Phys. B **73**, 166 (1974).
and references therein

$$T_{\sigma'\lambda',\sigma\lambda} = \sum_{J=1/2}^{\infty} (2J+1)\, T^{J}_{\sigma'\lambda',\sigma\lambda}(\omega)\, d^{J}_{\sigma'-\lambda',\sigma-\lambda}(\theta)$$

Helicity amplitudes

PW amplitudes

Wigner d-functions

Multipole amplitudes $f^{l\pm}_{\rho\rho'}(\omega)$, with $\rho, \rho' = E$(lectric), or M(agnetic)
angular momentum l

Unitary relation to pi-photoproduction multipoles (between 1 and 2 pion thresholds):

$$\mathrm{Im}\, f^{\ell\pm}_{EE} = k \sum_c \left|E^{(c)}_{(\ell\pm1)\mp}\right|^2, \qquad \mathrm{Im}\, f^{\ell\pm}_{MM} = k \sum_c \left|M^{(c)}_{\ell\pm}\right|^2,$$

$$\mathrm{Im}\, f^{(\ell\pm1)\mp}_{EM} = \mathrm{Im}\, f^{\ell\pm}_{ME} = \mp k \sum_c \mathrm{Re}\big(E^{(c)}_{\ell\pm} M^{(c)*}_{\ell\pm}\big),$$

where the sum is over the charged πN states, i.e: $c = \pi^0 p,\ \pi^+ n$

We expand the non-Born piece only, truncated at J=3/2 (only J=1/2, 3/2 are taken into account):

$$f = f^{\mathrm{Born}} + \bar{f}$$

$$\bar{f} = (\bar{f}^{1+}_{EE}, \bar{f}^{1-}_{EE}, \bar{f}^{1+}_{MM}, \bar{f}^{1-}_{MM}, \bar{f}^{1+}_{EM}, \bar{f}^{1+}_{ME}, \bar{f}^{2+}_{EE}, \bar{f}^{2-}_{EE}, \bar{f}^{2+}_{MM}, \bar{f}^{2-}_{MM})$$

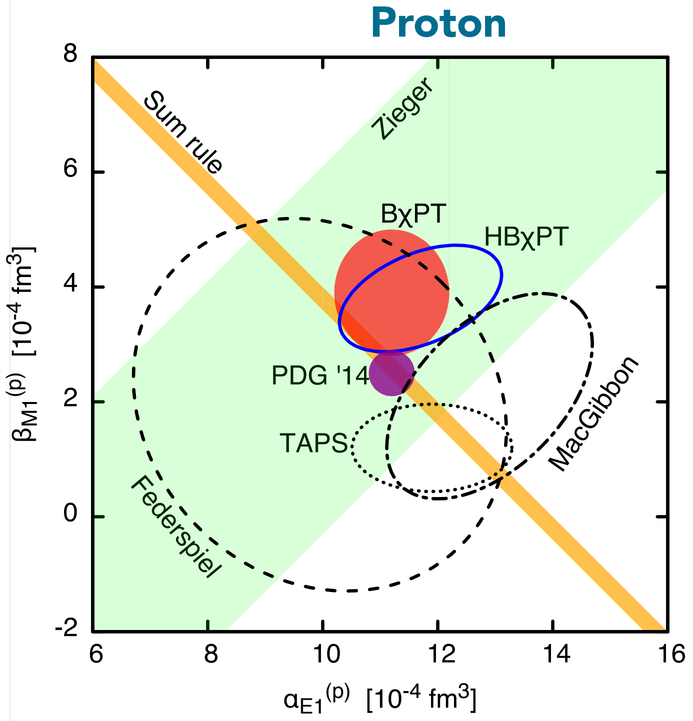
Dynamic to Static Polarizabilities

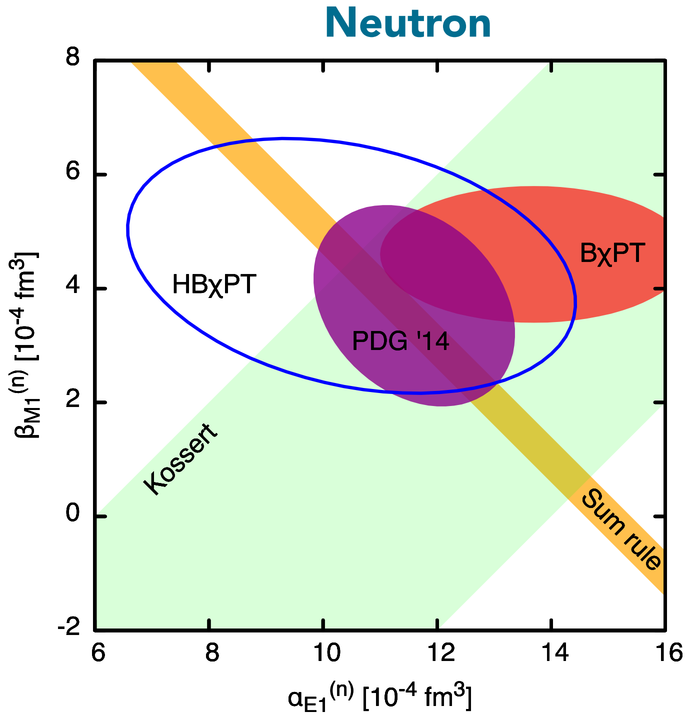
$$\begin{pmatrix}\alpha_{E\ell}(\omega)\\ \beta_{M\ell}(\omega)\end{pmatrix} = \frac{[\ell(2\ell-1)!!]^2}{\omega^{2\ell}}\left[(\ell+1)\bar{f}^{\,\ell+}_{\substack{EE\\MM}}(\omega) + \ell\bar{f}^{\,\ell-}_{\substack{EE\\MM}}(\omega)\right]$$

$$\gamma_{\substack{E\ell E\ell\\M\ell M\ell}}(\omega) = \frac{2\ell-1}{\omega^{2\ell+1}}\left[\bar{f}^{\,\ell+}_{\substack{EE\\MM}}(\omega) - \bar{f}^{\,\ell-}_{\substack{EE\\MM}}(\omega)\right], \qquad \bar{f}^{\,\ell\pm}_{\substack{EE\\MM}} \sim \omega^{2\ell}, \qquad \bar{f}^{\,\ell+}_{\substack{EM\\ME}} \sim \omega^{2\ell+1}$$

$$\gamma_{\substack{E\ell M(\ell+1)\\M\ell E(\ell+1)}}(\omega) = 2^{2-\ell}\frac{(2\ell+1)!!}{\omega^{2\ell+1}}\bar{f}^{\,\ell+}_{\substack{EM\\ME}}(\omega).$$

Static Limit, energy=0:





Forward spin polarizability

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

Backward spin polarizability

$$\gamma_\pi = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

Observables: bilinear relations

Angular distribution

$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s}\sum_{\sigma'\lambda'\sigma\lambda}\left|T_{\sigma'\lambda',\sigma\lambda}\right|^2$$

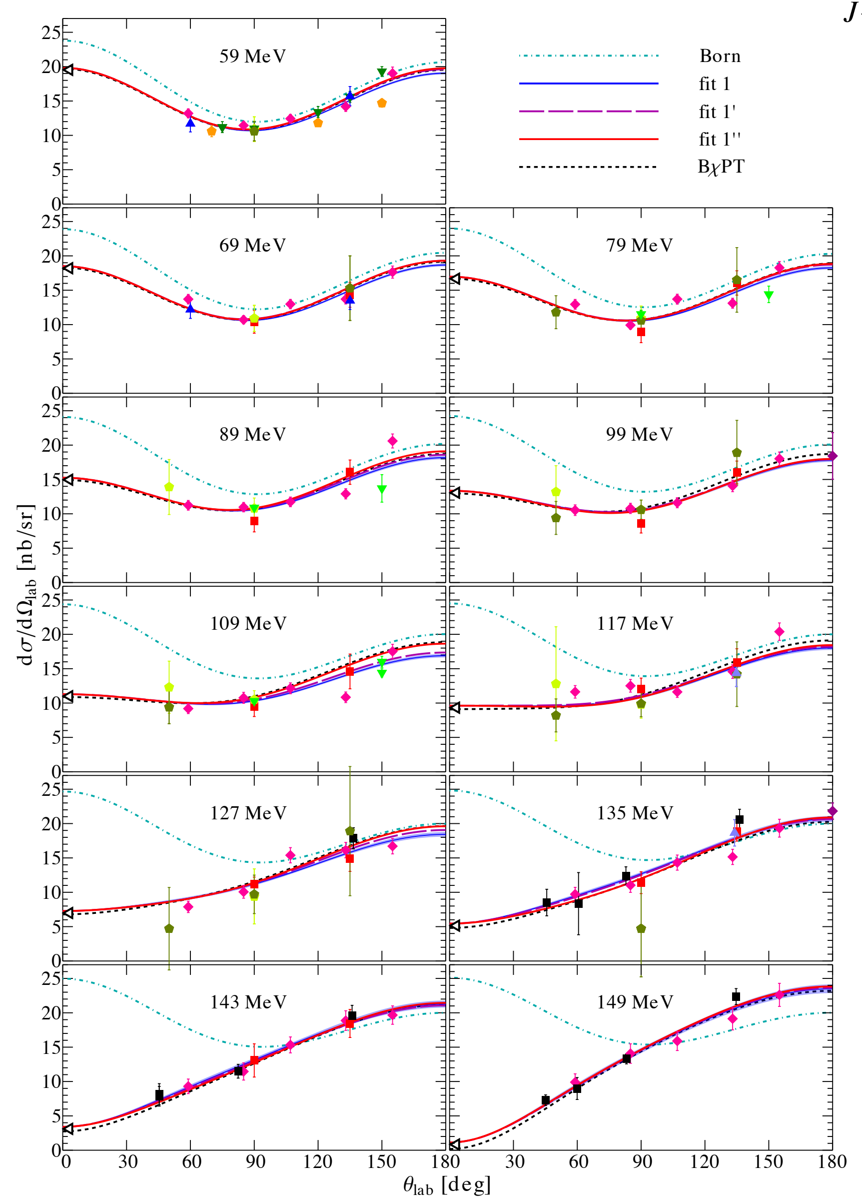
$$\frac{d\sigma}{d\Omega} = \sum_{n=0}^{4} c_n \cos n\theta \qquad \text{for } J < 5/2$$

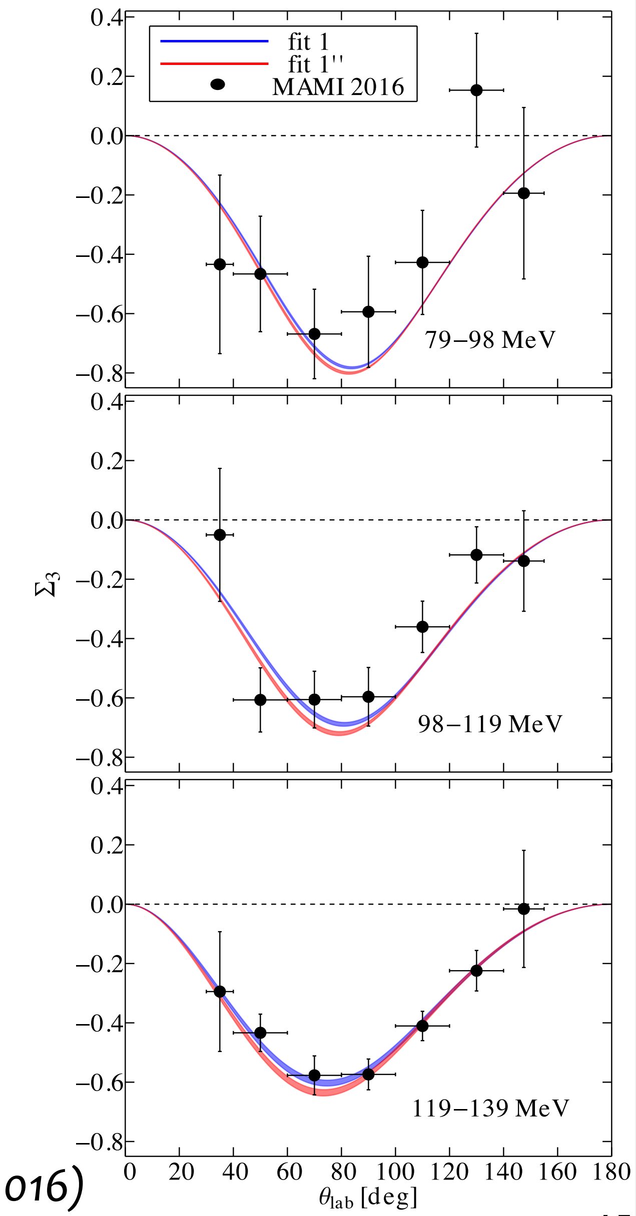
Beam asymmetry

$$\Sigma_3 = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

$$\frac{d\sigma}{d\Omega}\Sigma_3 = \frac{1}{128\pi^2 s}\sum_{\sigma'\lambda'\lambda}\mathrm{Re}(T^*_{\sigma'\lambda',-1\lambda}T_{\sigma'\lambda',1\lambda})$$

$$\overset{J<5/2}{=} \sin^2\theta\sum_{n=0}^{2} d_n \cos n\theta,$$





MAMI 2016:
Sokhoyan et al, *EPJA (2016)*

Forward-scattering Sum Rules: linear relations

$$T_{\sigma'\lambda'\sigma\lambda} \overset{t=0}{=} \chi^\dagger_{\lambda'} \left\{ f(\nu)\, \vec{\varepsilon}^{\,*}_{\sigma'} \cdot \vec{\varepsilon}_\sigma + g(\nu)\, i\, (\vec{\varepsilon}^{\,*}_{\sigma'} \times \vec{\varepsilon}_\sigma) \cdot \vec{\sigma} \right\} \chi_\lambda$$

Spin-independent amplitude ($f(\nu)$)

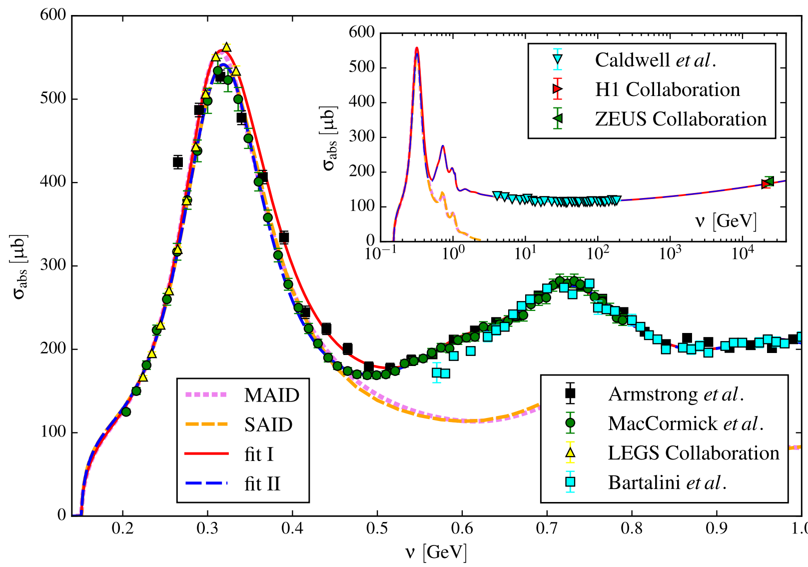
Spin-dependent amplitude ($g(\nu)$)

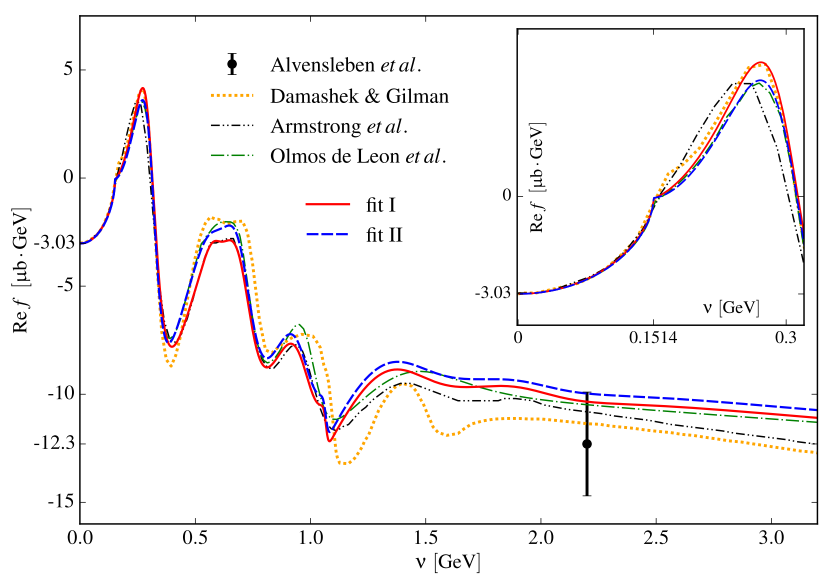
$$\begin{aligned}
f(\nu) &= -\frac{\alpha}{M} + \frac{\nu^2}{4\pi^2} \int_0^\infty \frac{d\nu'}{\nu'^2 - \nu^2 - i0^+} \left[\sigma^{\rm abs}_{1/2}(\nu') + \sigma^{\rm abs}_{3/2}(\nu')\right] \\
&= \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1)^2 \left\{ (L+2) \left(f_{EE}^{(L+1)-} + f_{MM}^{(L+1)-} \right) + L \left(f_{EE}^{L+} + f_{MM}^{L+} \right) \right\} \\
&\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f_{EE}^{1-} + 2 f_{EE}^{1+} + f_{MM}^{1-} + 2 f_{MM}^{1+} + 6 f_{EE}^{2-} + 9 f_{EE}^{2+} + 6 f_{MM}^{2-} + 9 f_{MM}^{2+} \right),
\end{aligned}$$

$$\begin{aligned}
g(\nu) &= -\frac{\alpha \varkappa^2 \nu}{2M^2} + \frac{\nu^3}{4\pi^2} \int_0^\infty \frac{d\nu'}{\nu'} \frac{\sigma^{\rm abs}_{1/2}(\nu') - \sigma^{\rm abs}_{3/2}(\nu')}{\nu'^2 - \nu^2 - i0^+} \\
&= \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1) \left\{ (L+2) \left(f_{EE}^{(L+1)-} + f_{MM}^{(L+1)-} \right) - L \left(f_{EE}^{L+} + f_{MM}^{L+} \right) - 2L(L+2) \left(f_{EM}^{L+} + f_{ME}^{L+} \right) \right\} \\
&\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f_{EE}^{1-} - f_{EE}^{1+} - 6 f_{EM}^{1+} - 6 f_{ME}^{1+} + f_{MM}^{1-} - f_{MM}^{1+} + 3 f_{EE}^{2-} - 3 f_{EE}^{2+} + 3 f_{MM}^{2-} - 3 f_{MM}^{2+} \right).
\end{aligned}$$

Empirical Evaluation of Sum Rules

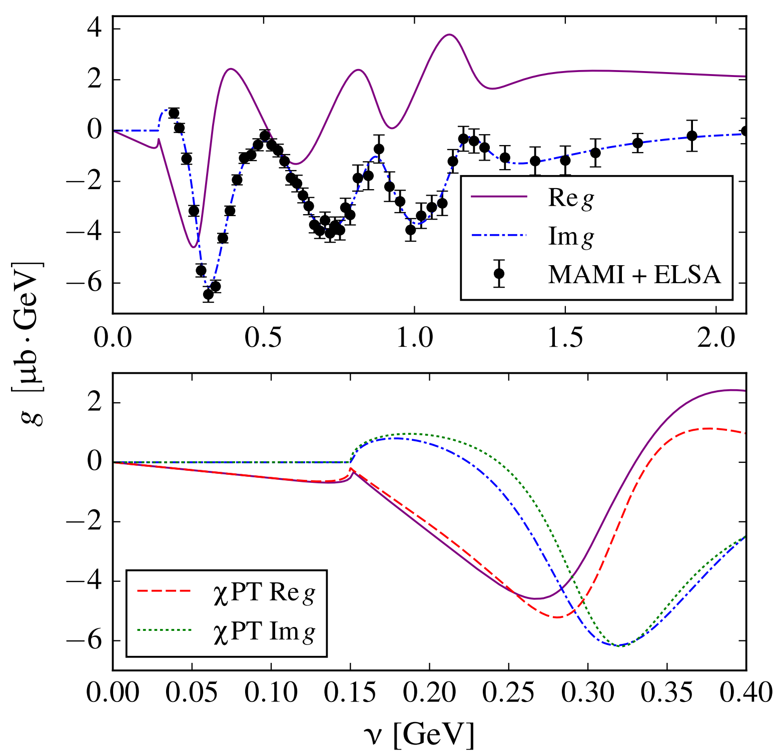
EVALUATION OF THE FORWARD COMPTON SCATTERING … PHYSICAL REVIEW D **92,** 074031 (2015)





$$f(\nu) = -\frac{Z^2\alpha}{M} + \frac{\nu^2}{2\pi^2}\int_0^\infty \mathrm{d}\nu' \, \frac{\sigma_T(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

$$g(\nu) = \frac{\nu}{2\pi^2}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu'\sigma_{TT}(\nu')}{\nu'^2 - \nu^2 - i0^+}.$$



	$O(\nu)$	$O(\nu^3)$	$O(\nu^5)$
	$I_{\rm GDH}$ (μb)	γ_0 (10^{-6} fm^4)	$\bar{\gamma}_0$ (10^{-6} fm^6)
GDH & A2 [9, 11]	≈ 212	≈ −86	
Helbing [21]	212 ± 6 ± 12		
Bianchi-Thomas [24]	207 ± 23		
Pasquini *et al.* [12]	210 ± 6 ± 14	−90 ± 8 ± 11	60 ± 7 ± 7
This work	204.5 ± 21.4	−92.9 ± 10.5	48.4 ± 8.2
GDH sum rule	204.784481(4)[a]		
BχPT [15]		−90 ± 140	110 ± 50
HBχPT [17]		−260 ± 190	

Comparison with a prediction from Chiral Perturbation Theory

Eur. Phys. J. C (2015) 75:604
DOI 10.1140/epjc/s10052-015-3791-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Predictions of covariant chiral perturbation theory for nucleon polarisabilities and polarised Compton scattering

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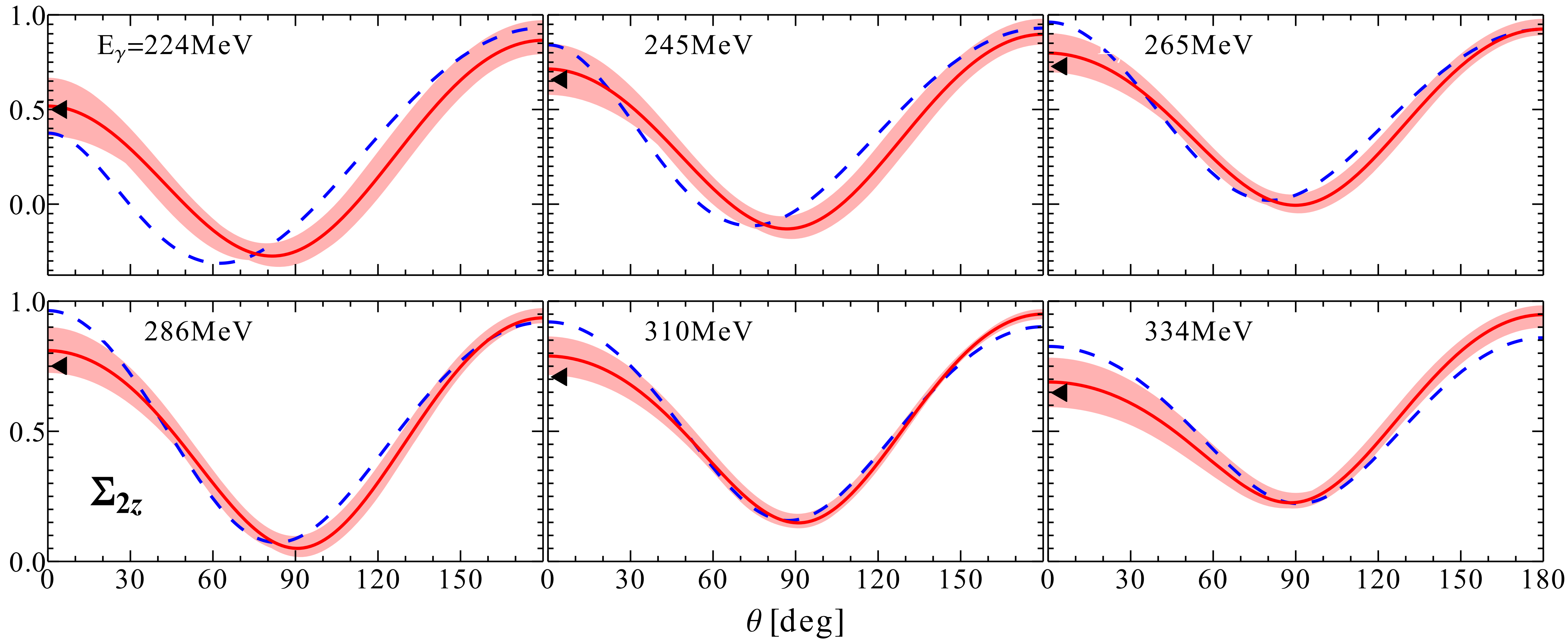


Our PWA Anzatz

1. Determine $l=1$ multipoles in the following **model-independent** form:

$$\bar{f}^{1+}_{EE}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[\frac{\alpha_{E1}}{3} + \frac{E_\gamma}{3} \left(\frac{-\alpha_{E1} + \beta_{M1}}{M} + \gamma_{E1E1} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_1^R(E_\gamma) \right],$$

$$\bar{f}^{1-}_{EE}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[\frac{\alpha_{E1}}{3} + \frac{E_\gamma}{3} \left(\frac{-\alpha_{E1} + \beta_{M1}}{M} - 2\,\gamma_{E1E1} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_2^R(E_\gamma) \right],$$

$$\bar{f}^{1+}_{MM}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[\frac{\beta_{M1}}{3} + \frac{E_\gamma}{3} \left(\frac{-\beta_{M1} + \alpha_{E1}}{M} + \gamma_{M1M1} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_3^R(E_\gamma) \right],$$

$$\bar{f}^{1-}_{MM}(E_\gamma) = E_\gamma^2 \frac{M}{\sqrt{s}} \left[\frac{\beta_{M1}}{3} + \frac{E_\gamma}{3} \left(\frac{-\beta_{M1} + \alpha_{E1}}{M} - 2\,\gamma_{M1M1} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_4^R(E_\gamma) \right],$$

$$\bar{f}^{1+}_{EM}(E_\gamma) = E_\gamma^3 \frac{M}{\sqrt{s}} \left[\frac{\gamma_{E1M2}}{6} + \frac{E_\gamma}{6} \left(\frac{-6\,\gamma_{E1M2} + 3\,\gamma_{M1E2} + 3\,\gamma_{M1M1}}{4M} - \frac{\beta_{M1}}{8M^2} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_5^R(E_\gamma) \right]$$

$$\bar{f}^{1+}_{ME}(E_\gamma) = E_\gamma^3 \frac{M}{\sqrt{s}} \left[\frac{\gamma_{M1E2}}{6} + \frac{E_\gamma}{6} \left(\frac{-6\,\gamma_{M1E2} + 3\,\gamma_{E1M2} + 3\,\gamma_{E1E1}}{4M} - \frac{\alpha_{E1}}{8M^2} \right) + \left(\frac{E_\gamma}{M} \right)^2 f_6^R(E_\gamma) \right]$$

After using sum rules,
4 global parameters (polarizabilities) and 4 energy-dependent (residual functions)

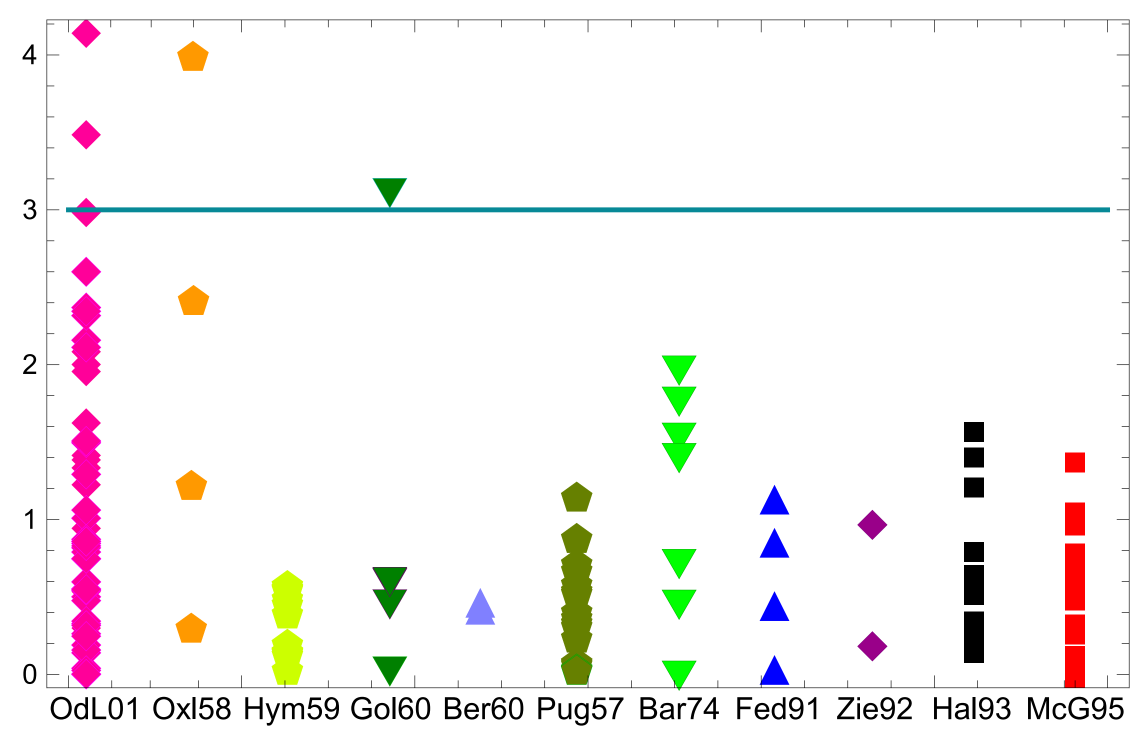
2. The $l=2$ multipoles are small and are either neglected or taken from ChPT

Fitted database of unpolarized cross section

Author	Ref.	E_γ [MeV]	ϑ [deg]	N_{data}	Symbol
Oxley et al.	[29]	60	70-150	4	⬟
Hyman et al.	[30]	60-128	50, 90	12	⬟
Goldansky et al.	[31]	55	75-150	5	▼
Bernardini et al.	[32]	120, 139	133	2	▲
Pugh et al.	[33]	59-135	50, 90, 135	16	⬟
Baranov et al.	[34]	79, 89, 109	90, 150	7	▼
Federspiel et al.	[35]	59, 70	60, 135	4	▲
Zieger et al.	[36]	98, 132	180	2	◆
Hallin et al.	[37]	130-150	45, 60, 82, 135	13	■
MacGibbon et al.	[38]	73-145	90-135	18	■
Olmos de León et al.	[15]	59-149	59-155	55	◆

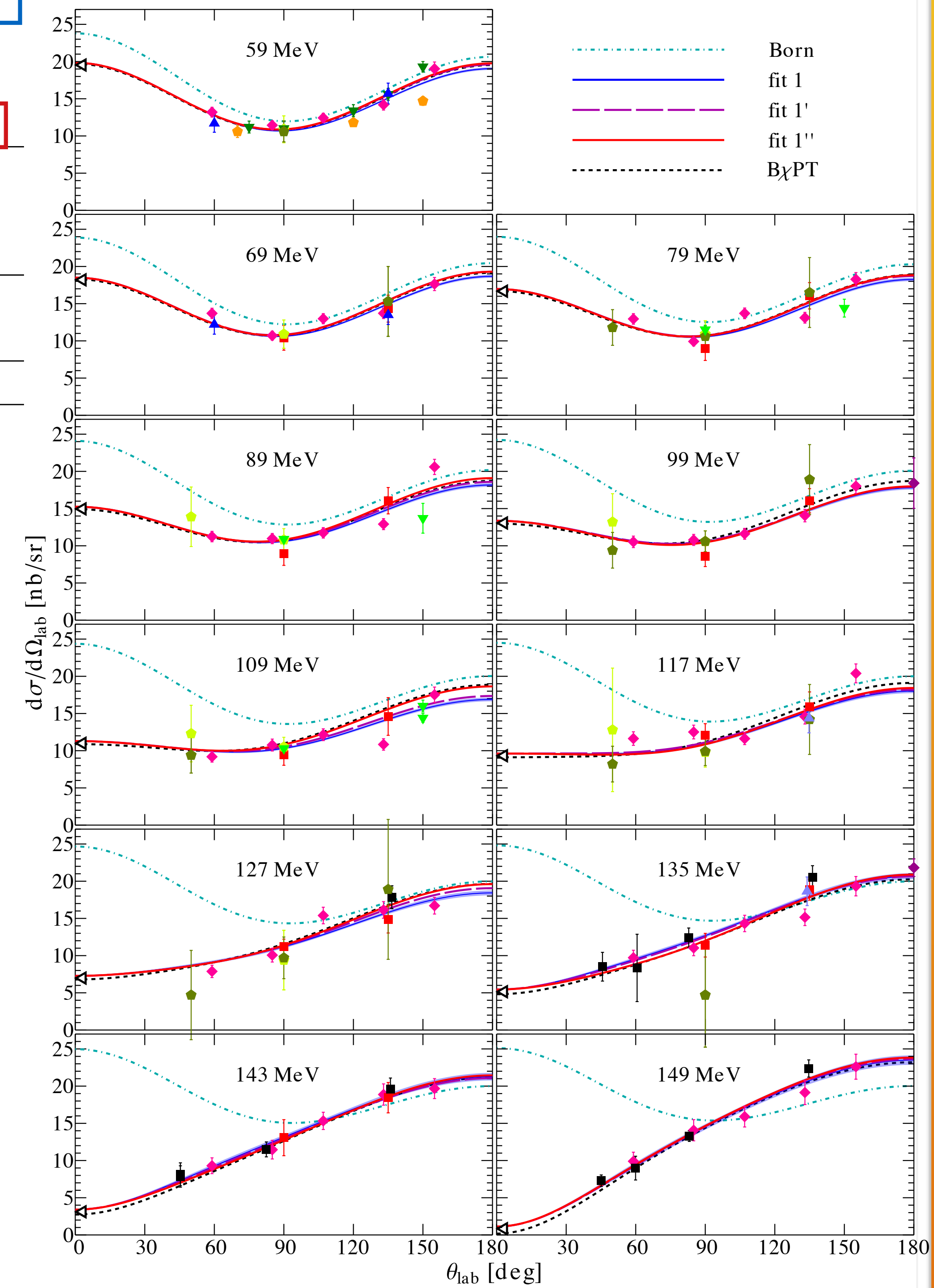
split into $N_{bins} = 11$ energy bins, 59, 69, 79, 89, 99, 109, 117, 127, 135, 143, 150 MeV

detecting outliers



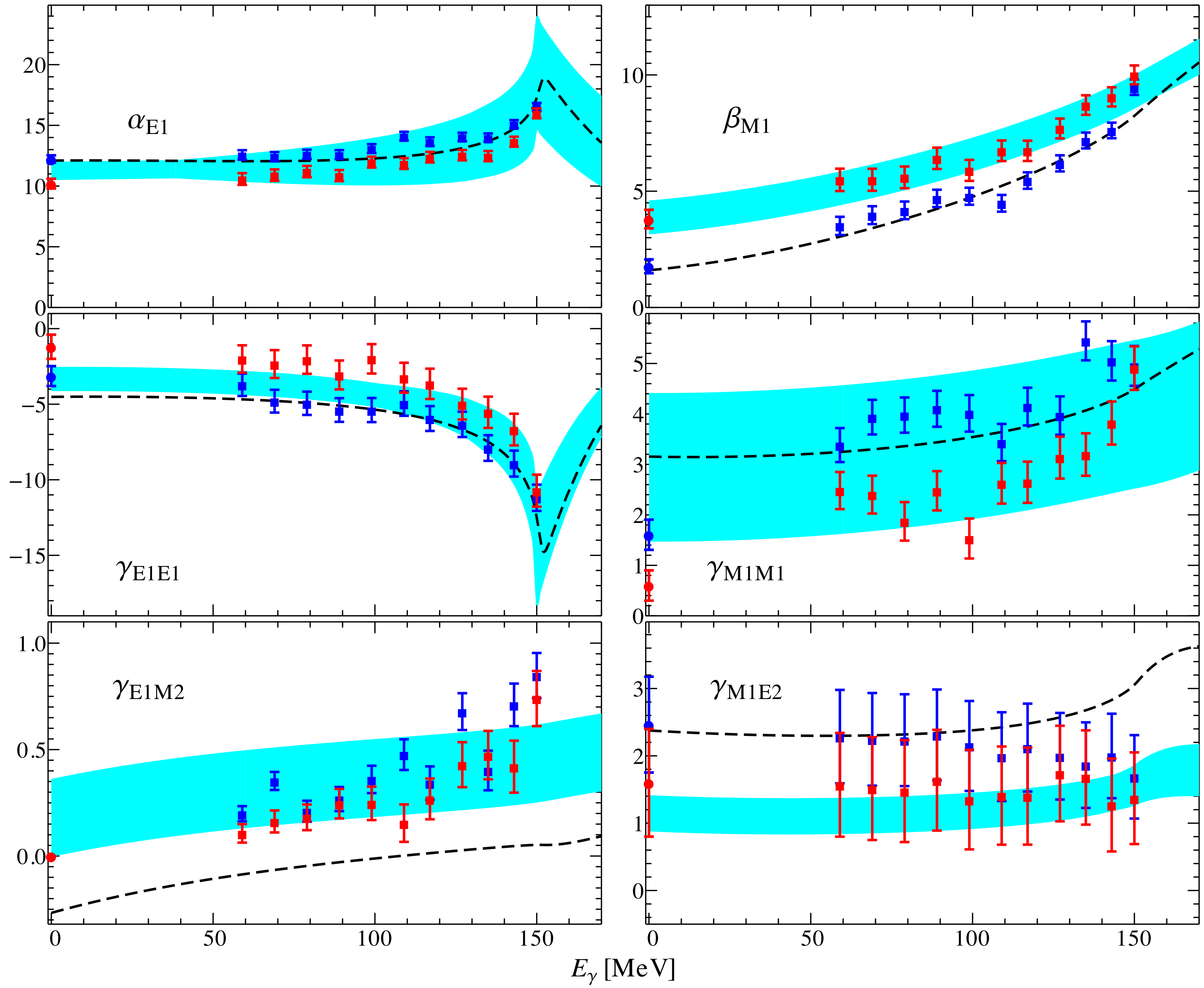
Fits and Solutions

Source	α_{E1}	β_{M1}	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}	χ^2/point
Fit 0	12.2 ± 0.3	1.8 ∓ 0.3	-1.6 ± 2.6	1.8 ± 1.1	-1.3 ± 3.7	2.0 ± 0.7	1.35
Fit 1	12.2 ± 0.3	1.8 ∓ 0.3	-3.1 ± 0.7	1.6 ± 0.3	**0.0**	2.5 ± 0.7	1.35
Fit $1_{3\sigma}$	11.8 ± 0.3	2.2 ∓ 0.3	-2.7 ± 0.6	1.5 ± 0.3	**0.0**	2.2 ± 0.7	0.97
Fit $1'$	10.6 ± 0.3	3.4 ∓ 0.3	-1.0 ± 0.8	1.0 ± 0.3	**0.0**	1.0 ± 0.7	0.99
Fit $1''$	10.2 ± 0.4	3.8 ∓ 0.4	-1.2 ± 0.8	0.6 ± 0.3	**0.0**	1.6 ± 0.8	0.62
no $l=2$							
Fit 2	11.7 ± 0.3	2.3 ∓ 0.3	-2.6 ± 0.6	1.1 ± 0.3	**0.0**	2.4 ± 0.7	1.35
Fit $2''$	10.8 ± 0.4	3.2 ∓ 0.4	-1.9 ± 0.8	0.7 ± 0.3	**0.0**	2.2 ± 0.8	0.69
BχPT	11.2 ± 0.7	3.9 ∓ 0.7	-3.3 ± 0.8	2.9 ± 1.5	0.2 ± 0.2	1.1 ± 0.3	
DR	12.1	1.6	-3.4	2.7	0.3	1.9	
MAMI 2015			-3.5 ± 1.2	3.16 ± 0.85	-0.7 ± 1.2	1.99 ± 0.29	

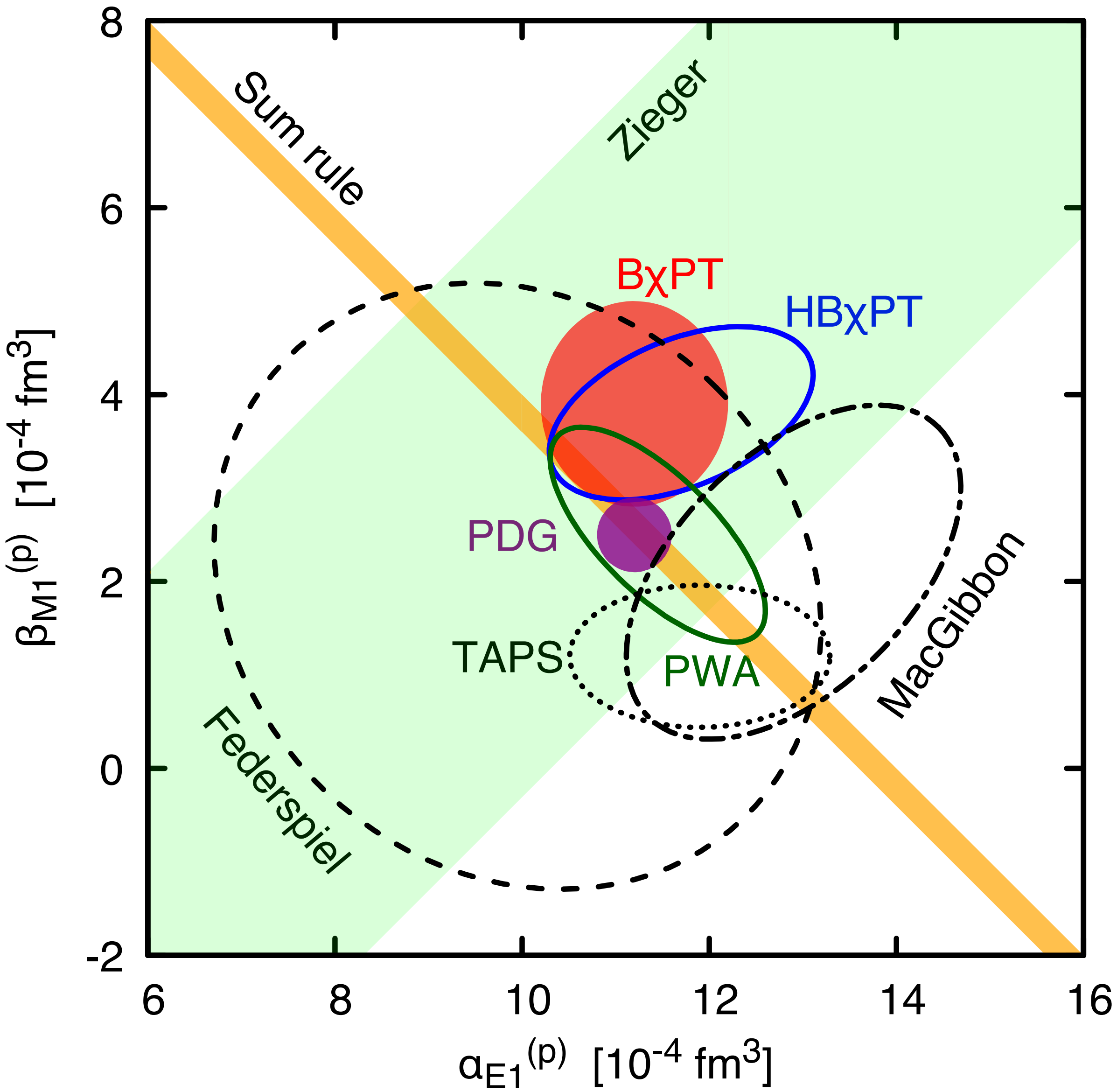


Solutions for 2 different databases vs. ChPT

	$\alpha_{E1}+\beta_{M1}$	γ_0	$\alpha_{E1}-\beta_{M1}$	γ_π
Fit 1	14.0	-0.93	10.5 ± 0.4	7.2 ± 1.0
Fit 1′	14.0	-0.93	7.2 ± 0.6	3.0 ± 1.1
Fit 1″	14.0	-0.93	6.4 ± 0.6	3.5 ± 1.2
BχPT	15.1 ± 1.0	-0.9 ± 1.4	7.3 ± 1.0	7.2 ± 1.7
DR	13.7	-1.5	10.5	7.8



Static polarizabilities of the proton



- **TAPS:** fit to TAPS/MAMI data based on fixed-*t* DRs of L'vov et al.
  Olmos de Leon et al., *EPJA (2001)*

- **BChPT:** "postdiction"
  Lensky & VP, *EPJC (2010)*
  Lensky, McGovern & VP, *EPJC (2015)*

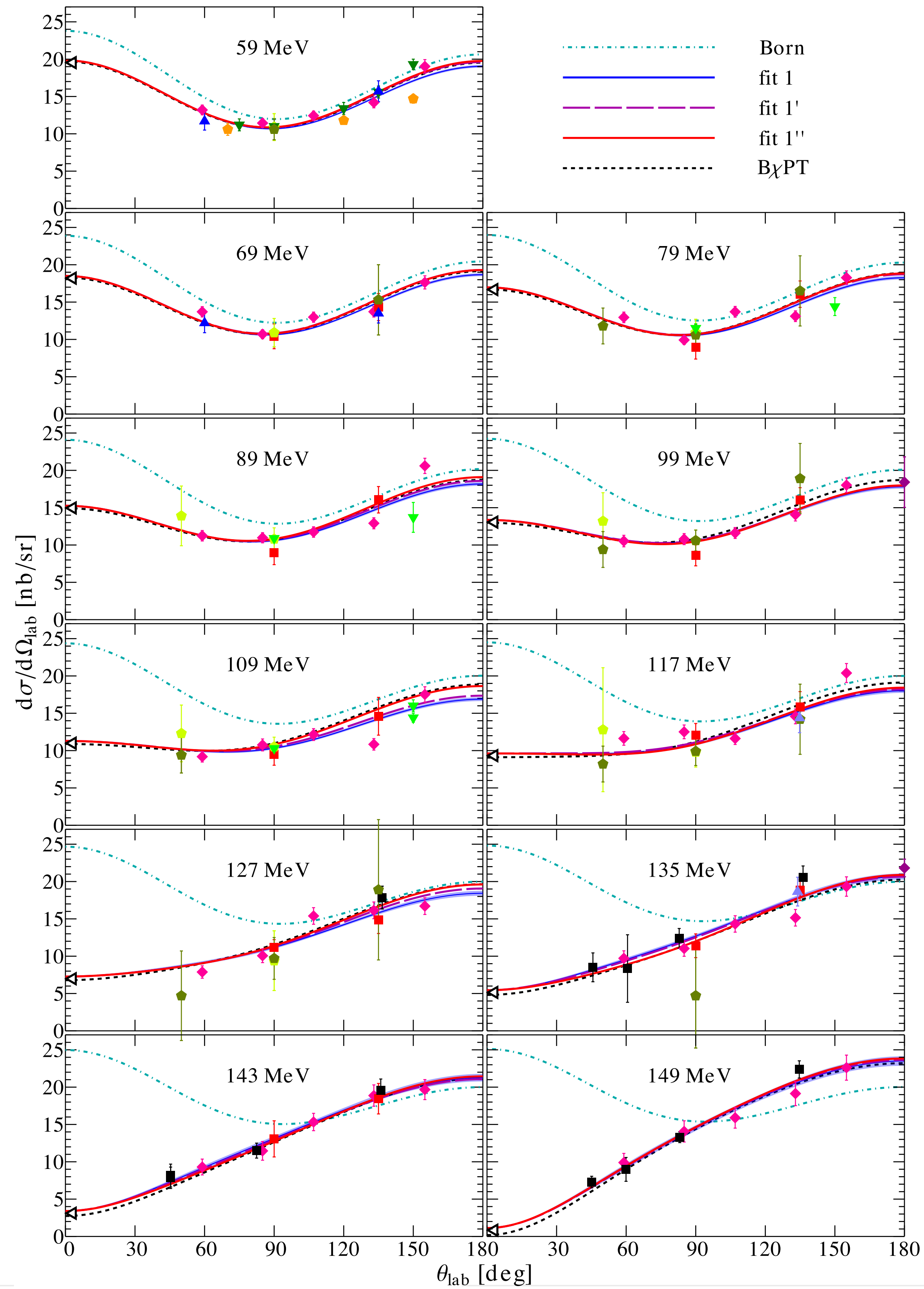
- **HBChPT:** fit to world data
  Grieβhammer, McGovern & Phillips, *EPJA (2013)*

- **PWA:** fit to world data
  Krupina, Lensky & VP, *PLB (2018)*

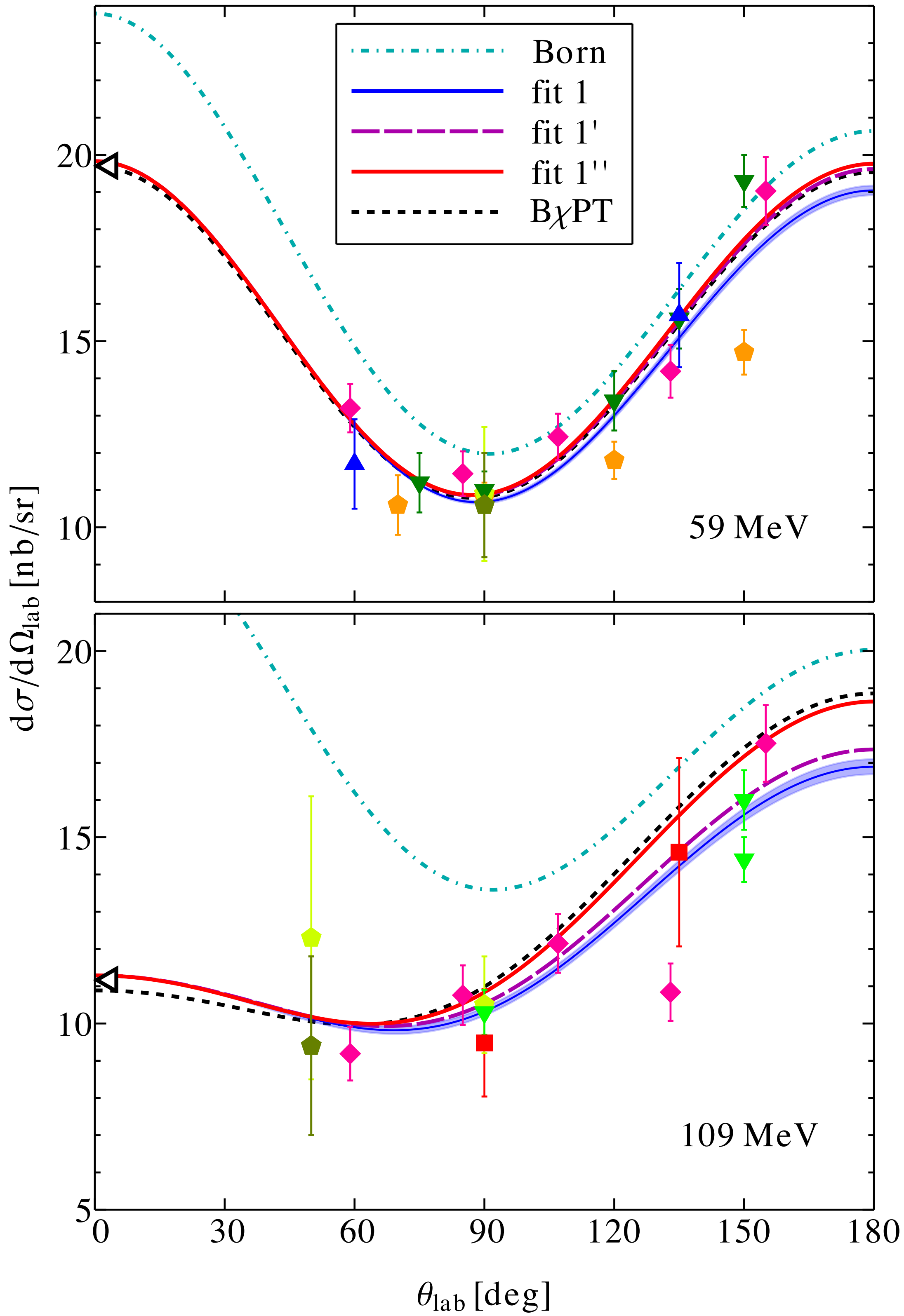
Partial-Wave Analysis (PWA):
differences between DR and ChPT extractions are due to database inconsistencies, improvements — new experiments — are needed!

How to improve on database consistency

	$\alpha_{E1}+\beta_{M1}$	γ_0	$\alpha_{E1}-\beta_{M1}$	γ_π
Fit 1	14.0	-0.93	10.5 ± 0.4	7.2 ± 1.0
Fit 1′	14.0	-0.93	7.2 ± 0.6	3.0 ± 1.1
Fit 1″	14.0	-0.93	6.4 ± 0.6	3.5 ± 1.2
BχPT	15.1 ± 1.0	-0.9 ± 1.4	7.3 ± 1.0	7.2 ± 1.7
DR	13.7	-1.5	10.5	7.8



Zooming in ···



	$\alpha_{E1}+\beta_{M1}$	γ_0	$\alpha_{E1}-\beta_{M1}$	γ_π
Fit 1	14.0	−0.93	10.5 ± 0.4	7.2 ± 1.0
Fit 1′	14.0	−0.93	7.2 ± 0.6	3.0 ± 1.1
Fit 1″	14.0	−0.93	6.4 ± 0.6	3.5 ± 1.2
BχPT	15.1 ± 1.0	-0.9 ± 1.4	7.3 ± 1.0	7.2 ± 1.7
DR	13.7	−1.5	10.5	7.8

109(10) MeV

“sweet spot” for unpolarized cross section,
because of the interplay
of scalar and spin polarizabilities

Conclusions

1. Accurate model-independent Compton PWA solutions found

Thanks to:

- No resonances (below pion production threshold)
- Multipoles are real, neglecting radiative corrections
- Forward-scattering is determined, via the sum rules (photoabsorption cross sections):

  yields linear relations on the multipoles, rather than bilinear

and despite:

- Not much data (about 100 data points, many from old experiments)

2. Discrepancies of DR vs. ChPT extractions of polarizabilities from data are due to the differences in the database

3. Database improvements needed, preferably by new precise data — coming soon from MAMI !..