Progress in the Nucleon EDM Calculations in Lattice QCD

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Outline

- $\theta_{\text{QCD}}$-induced nucleon EDM:
  - Previous calculations
  - New challenges
  - Improved techniques on a lattice
  - Outlook for $\theta_{\text{QCD}}$-nEDM

- Quark chromo-EDM-induced nucleon EDM
  - Preliminary results at the physical point
CP Violation: Electric Dipole Moments

EDMs are the most sensitive probes of CPv:
- Prerequisite for Baryogenesis
- Evidence for SM Extensions
- \( \theta_{QCD} \) in particular) Strong CP problem

\[ \vec{d}_N = d_N \frac{\vec{S}}{S} \quad \mathcal{H} = -\vec{d}_N \cdot \vec{E} \]

OR

\[ \mathcal{L}_{int} = eA_{\mu}^{em}\mathcal{V}_{\mu} \quad \text{(P,T-even)} \]

\[ + eA_{\mu}^{em}\mathcal{A}_{\mu} \quad \text{(P,T-odd)} \]
# Experimental Outlook: Neutron EDM

<table>
<thead>
<tr>
<th>CURRENT LIMIT</th>
<th>10^{-28} e cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spallation Source @ORNL</td>
<td>&lt; 300</td>
</tr>
<tr>
<td>Ultracold Neutrons @LANL</td>
<td>~30</td>
</tr>
<tr>
<td>PSI EDM</td>
<td>&lt; 50 (I), &lt;5 (II)</td>
</tr>
<tr>
<td>ILL PNPI</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>Munich FRMII</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>RCMP TRIUMF</td>
<td>&lt; 50 (I), &lt;5 (II)</td>
</tr>
<tr>
<td>JPARC</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>Standard Model (CKM)</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

**nEDM sensitivity**:

- 1–2 years: next best limit
- 3–4 years: x10 improvement
- 7-10 years: x100 improvement

[B.Filippone's talk, KITP 2016]
Nucleon EDMs: a Window into New Physics

Effective quark-gluon CPv interactions organized by dimension

\[ \mathcal{L}_{\text{eff}} = \sum_i \frac{c_i}{[\Lambda(i)]^{d_i-4}} \mathcal{O}_i^{[d_i]} \]

- \( d = 4 \): \( \theta_{QCD} \)
- \( d = 5(6) \): quark EDM, quark-gluon chromo EDM
- \( d = 6 \): 4-fermion CPv, 3-gluon (Weinberg)

\[ d_{n,p} = d_{n,p}^\theta \theta_{QCD} + d_{n,p}^{cEDM} c_{cEDM} + \ldots \]

lattice QCD calculations are needed to relate to constrain \( \theta_{QCD}, c_{cEDM}, \ldots \)

Experiments:
- Neutron EDM; Proton EDM??
- Nuclear EDMs: 199Hg, 225Ra, ...
- 2 Higgs Doublets?
- Extradimensions?
- SUSY? GUT?

CP-odd Nucleon Structure on a Lattice

**CP-broken vacuum on a lattice:**
- Linear response to CP-odd interaction (e.g., QCD $\theta$-term)

$$\langle \mathcal{O} \ldots \rangle_{\mathcal{CP}} = \langle \mathcal{O} \ldots \rangle_{\mathcal{CP-even}} - i\theta \langle Q \cdot \mathcal{O} \ldots \rangle_{\mathcal{CP-even}} + O(\theta^2)$$


- Simulation with dynamical (imaginary) $\theta'_{QCD}$

$$\langle \mathcal{O} \ldots \rangle_\theta \sim \int DU \, e^{-S-\theta'Q} \langle \mathcal{O} \ldots \rangle$$


new gauge ensembles ⇒ better sampling of $Q\neq0$ sectors

**Extraction of $d_N$**
- Nucleon spectrum in the bg. electric field

[S.Aoki et al '89; E.Shintani et al '06; E.Shintani et al, PRD75, 034507(2007)]

$$\langle N(t)\bar{N}(0) \rangle_{\theta,E} \sim e^{-(E \pm d_N \cdot E)t}$$

- $P,T$-odd Form Factor $d_N=F_3(0)/2m_N$

[E.Shintani et al '05, '15; F.Berruto et al '05; A.Shindler et al '15; C.Alexandrou et al '15]

Require extrapolation $F_3(Q^2\rightarrow0)$
$\theta_{QCD}$-induced Nucleon EDM

Summary of nEDM from LQCD circa 2015
[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Phenomenology: $|d_n| \approx \theta_{QCD} \times (0.4 \ldots 2.5) \cdot 10^{-3}$ e fm
- Lattice [Guo et al 2015]: $|d_n| \approx \theta_{QCD} \times (4 \cdot 10^{-3}$ e fm)
  $\implies$ tighter constraint on $\theta_{QCD}$?

Unfortunately, there was a problem...
**Nucleon "Parity Mixing"**

**CPv interaction induces a chiral phase in fermion fields:**

\[
\langle \text{vac} | N | p, \sigma \rangle \propto e^{i\alpha \gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma} \\
\text{u} \left[ u^T C \gamma_5 d \right] \\
\text{(P-even lattice nucleon field)}
\]

\[
(\tilde{\phi} + m_N e^{-2i\alpha \gamma_5}) \tilde{u}_p = 0 \\
\sum_{\sigma} \tilde{u}_{p,\sigma} \tilde{u}_{p,\sigma} \sim (-i\phi^c + m_N e^{2i\alpha \gamma_5})
\]

Vector current M.E. has to be defined with positive-parity spinors to define \( F_{2,3} \)


\[
\langle N_{p'} | \bar{q} \gamma^\mu q | N_p \rangle \propto \bar{u}_{p'} \left[ F_1 \gamma^\mu + \left( F_2 + i F_3 \right)^\gamma_5 \frac{i\sigma^{\mu\nu}(p' - p)_\nu}{2m_N} \right] u_p \]

\[
\Gamma^\mu \frac{\gamma_4 u = +u}{u \gamma_4 = +u}
\]

... otherwise, \( F_{2,3} \) mix under chiral rotation and lead to fake EDM/EDFF signal

\[
e^{i\alpha \gamma_5} \Gamma^\mu e^{i\alpha \gamma_5} \leftrightarrow \Gamma^\mu
\]

\[
e^{2i\alpha} \left( \text{"}F_2\text{"} + i \text{"}F_3\text{"} \right) = (F_2 + iF_3)_{\text{true}}
\]

\[
\text{"}F_3\text{"} \approx [F_3]_{\text{true}} - 2\alpha [F_2]_{\text{true}}
\]

\[
\text{"}d_{n,p}\text{"} \approx [d_{n,p}]_{\text{true}} - 2\alpha \frac{\kappa_{n,p}}{2m_N}
\]

**The same issue is addressed correctly in EFT (ChPT) calculations**
Nucleon "Parity Mixing" (2)


- coupling of E,B to spin in the forward limit
  \[
  \langle H_{\text{int}} \rangle = eA_\mu \langle J^\mu \rangle = -\frac{eG_M(0)}{2m_N} \vec{\Sigma} \cdot \vec{H} - \frac{eF_3(0)}{2m_N} \vec{\Sigma} \cdot \vec{E}
  \]

- poles of the Dirac operator in bg. electric & magnetic fields
  \[
  \mathcal{L}_N = \bar{N} \left[ i\phi - me^{-2i\alpha\gamma_5} - Q\gamma_\mu A^\mu - (\kappa + i\tilde{\zeta}\gamma_5) \frac{1}{2} F_{\mu\nu} \sigma^{\mu\nu} \right] N
  \]
  \[
  E_N(p = 0) - m_N = -\frac{\kappa}{2m_N} \vec{\Sigma} \cdot \vec{H} - \frac{\zeta}{2m_N} \vec{\Sigma} \cdot \vec{E} + O(\kappa^2, \zeta^2)
  \]
  with $\kappa + i\zeta = e^{2i\alpha\gamma_5}(\tilde{\kappa} + i\tilde{\zeta})$

- Numerical test: compare EDFF with mass shift in uniform bg. electric field

\[\begin{array}{c}
\text{bg. electric} \\
\text{field result} \\
\end{array}\]

\[
F_{3_0}^{D} = \left[ F_{3_0}^{D} \right]_{\text{true}} - 2\alpha_D F_{2n}
\]

\[
\alpha_D \approx 30(0.2)
\]

Large $F_{2n}$ contribution to "$F_{3_0}$"
Recent Lattice Results on $\theta_{QCD}$-induced nEDM

Correction to previous results:

$[F_3]_{\text{true}} = "F_3" + 2\alpha F_2$

- [F. Guo et al (QCDSF), PRL115:062001 (2015)]
  dynamical calculations with finite imag. $\theta'$ angle

- [C.Alexandrou et al (ETMC), PRD93:074503 (2016)]
  $d_n=-0.045(06)\ e\ fm\ (\sim 7.5\sigma) \rightarrow +0.008(6)\ e\ fm\ (1.3\sigma)$

**Uniform bg.electric field method is not affected by "parity mixing"**

**Precision in Ref. [E.Shintani et al, D78:014503 (2008)] is insufficient for comparison**

<table>
<thead>
<tr>
<th>$m_\pi$ [MeV]</th>
<th>$m_N$ [GeV]</th>
<th>$F_2$</th>
<th>$\alpha$</th>
<th>$F_3$</th>
<th>$F_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>373</td>
<td>1.216(4)</td>
<td>$-1.50(16)^a$</td>
<td>$-0.217(18)$</td>
<td>$-0.555(74)$</td>
</tr>
<tr>
<td>p</td>
<td>530</td>
<td>1.334(8)</td>
<td>$-0.560(40)$</td>
<td>$-0.247(17)^b$</td>
<td>$-0.325(68)$</td>
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<tr>
<td>n</td>
<td>690</td>
<td>1.575(9)</td>
<td>$-1.715(46)$</td>
<td>$-0.070(20)$</td>
<td>$-1.39(152)$</td>
</tr>
<tr>
<td>n</td>
<td>605</td>
<td>1.470(9)</td>
<td>$-1.698(68)$</td>
<td>$-0.160(20)$</td>
<td>$0.60(2.98)$</td>
</tr>
<tr>
<td>n</td>
<td>465</td>
<td>1.246(7)</td>
<td>$-1.491(22)^c$</td>
<td>$-0.079(27)^d$</td>
<td>$-0.375(48)$</td>
</tr>
<tr>
<td>n</td>
<td>360</td>
<td>1.138(13)</td>
<td>$-1.473(37)^c$</td>
<td>$-0.092(14)^d$</td>
<td>$-0.248(29)$</td>
</tr>
</tbody>
</table>

After removing spurious contributions,

- no lattice signal for $\theta_{QCD}$-induced nEDM $\Rightarrow d_N$ is very small
- no conflict with phenomenology values or $m_q$ scaling

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Sergey N. Syritsyn

Nucleon EDMs on a Lattice

CIPANP 2018, Palm Springs, CA
\(\theta\)-Term Noise Reduction for EDM

Lattice signal for \(\theta\text{-nEDM}\) 
\[d_N \sim \langle Q \cdot (N(x) J_\mu \bar{N}(0)) \rangle_{CP\text{-even}}\]

Top. charge \(Q\) is global 
\[Q \sim \int_{V_4} (G \tilde{G}) \quad \text{with} \quad \langle |Q|^2 \rangle \sim V_4\]

\(\Rightarrow\) Variance of correlator \(\sim V_4\)

Constrain \(Q\) integral to the relevant volume

- constrain \(Q\) in time, 
  \[|t_Q - t_J| \leq \Delta t\]

  \([E.\text{Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)}]\)

- "cluster decomposition" \([K.-F.\text{Liu et al, 1705.06358}\): 
  constrain \(Q\) in 4-d around "sink" within \(|r|<R\)

\textit{Proper account of nucleon parity mixing is critical for correct determination of} \(F_3\)

\(\Rightarrow\) nucleon states must "settle" in the new vacuum

\[\begin{align*}
N^+ & \to \tilde{N}^+ \approx N^+ + i\alpha N^- \\
N^- & \to \tilde{N}^- \approx N^- - i\alpha N^+ 
\end{align*}\]

\(\Rightarrow\) treat time differently from space:

4d "cylinder" 
\[V_Q : |\vec{z}| < r_Q, \quad -\Delta t_Q < z_0 < T + \Delta t_Q\]

\[Q \approx \int_{V_Q} d^4 z q(z)\]

[FIG. 14. (Top) The nucleon EDM form factors from local time slice reweighting, as described in the text, for the lowest non-trivial momentum. Proton (squares) and neutron (circles). The point on the right corresponds to reweighting with the topological charge \(Q^2 = 0\). (Bottom) CP-odd mixing angle from local time slice reweighting, as described in the text, on the same ensemble.]

\[E.\text{Shintani et al (2015)}\]
Noise Reduction: $\theta$-induced Parity-mixing

Parity-mixing angle from constrained $Q$ sum

Reassuring results for noise reduction at the physical point
- required time region is small, $\Delta t_Q \geq 8a \approx 1.2$ fm
- spatial region must be large, $r_Q \geq 20a \approx 2.3$ fm
θ-nEDM Feasible at the Physical Point?

Best guess for neutron EDM $d_n$: extrapolation in $m_q \sim (m_\pi)^2$

- chiral fermions, $m_\pi = 330$ MeV
  \[ \rightarrow \text{phys.point } |F_3(0)| \approx 0.020, \ |d_n| \approx 0.002 \text{ e fm} \]

- Wilson fermions, $m_\pi = 360$ MeV [Guo et al 2015]
  \[ \rightarrow \text{phys.point } |F_3(0)| \leq 0.012, \ |d_n| \leq 0.001 \text{ e fm} \]

\[ |F_{3n}^{\text{phys}}(0)| \sim O(10^{-2}) \theta, \quad |d_n| \sim O(10^{-3}) \text{ e fm} \theta \]
Noise Reduction: $\theta$-induced EDFF $F_3$

EDFF $F_3$ from constrained $Q$ sum: \textit{the most aggressive} $Q$ cuts
- 33k lattice samples, $\sim$ 30 M core-hours on Argonne BlueGene/Q
- connected diagrams only
- result compatible with zero, $|F_{3n}| \leq 0.05$

\textit{Need to constrain} $|F_{3n}| \approx 0.01..0.02 : \theta$-nEDM remains difficult at the physical point...
Outlook for $\theta$-nEDM

Resort to simpler calculations
- heavier pion masses + EFT for extrapolations
- quenched calculations (see e.g. recent [J.Dragos et al, 1711.04730])

*Physical point calculations of $\theta$-nEDM will be necessary to renormalize effects from other CPv sources of higher-dim. [T.Bhattacharya et at (2015)]*

New lattice simulations at the physical point with dynamical $\theta^I$-term
- coarse ($a=0.2$ fm) physical-point lattice $\implies$ reduced cost due to lattice volume
- chiral lattice fermions allow independent $a\to0$, $m_q\to0$ limits
- enhance $d_N$ signal with $\langle Q \rangle \neq 0$ – more critical at light quark masses

$\implies 2018$ ALCC award for 50 M BG/Q core-hours

Ensembles with dynamical $\theta^I$-term will be also useful for CPv $\pi N$ coupling
Another Source of CPv: Quark Chromo-EDM

$$\mathcal{L}_{cEDM} = \sum_{q=u,d} \frac{\delta q}{2} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \right] q$$

- O($a^2$) mixing with dim-3 pseudoscalar density
  ⇒ need non-perturbative subtractions

- Non-chiral (e.g. Wilson) fermions have a O($a$) clover term ("chromo-magnetic DM")

$$\mathcal{L}^{\text{clover}} = a \frac{c}{4} \bar{q} \left[ G_{\mu\nu} \sigma^{\mu\nu} \right] q$$

Condensate realignment in presence of CPv

$$q \to e^{i\gamma_5\Omega} q$$

assuring

$$\langle \text{vac} | \mathcal{L}_m + \mathcal{L}_{CP} | \pi^a \rangle = 0$$

mixes (chromo)EDM and (chromo)MDM:

$$\delta \mathcal{L}_{cEDM} = \delta (\bar{q} \left[ \tilde{D}_q G_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \right] q) = \bar{q} \left[ \{\Omega, \tilde{D}_q\} G_{\mu\nu} \sigma^{\mu\nu} \right] q \sim \delta \mathcal{L}_{cMDM}$$

⇒ Chirally-symmetric actions avoid these cMDM contributions
Quark-Gluon EDM: Insertions of dim-5 Operators

\[ \mathcal{L}^{(5)} = \sum_q \bar{d}_q \bar{\psi}(G \cdot \sigma) \gamma_5 q \]

\[ \langle N(y) \bar{N}(0) \int d^4 x \bar{q}(G \cdot \sigma) \gamma_5 q \rangle \]

First calculations: [T.Bhattacharya et al (LANL, LATTICE’15,’16)]

- This work: Only quark-connected insertions

- In future: Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with \( \theta \)-term)
Nucleon Sachs Form Factors

\[ G_E = F_1 - \frac{Q^2}{4m_N^2} F_2 \]
\[ G_M = F_1 + F_2 \]

- \((5.5 \text{ fm})^3 \times (11 \text{ fm})\) box, \(m_\pi=140 \text{ MeV}\)
- connected-only contractions
Parity Mixing (Proton)

\[ N_\delta = \epsilon^{abc} u_\delta^a \left( u^{aT} C \gamma_5 d^c \right) \]

\[ \langle N(t) \bar{N}(0) \rangle_{CP} = \frac{-i\psi + m_N e^{2i\alpha_5 \gamma_5}}{2m_N} e^{-E_N t} \]

\[ \hat{\alpha}_5 = \frac{\alpha_5}{d} = -\frac{\text{ReTr}[T^+ \gamma_5 \cdot C^{CP}_{2pt}(t)]}{\text{ReTr}[T^+ \cdot C^{CP}_{2pt}(t)]}, \quad t \to \infty \]

(flavors labeled for the proton)

similarity effect on nucleon likely due to mixing between cEDM and PS
Proton & Neutron EDFF Form Factors (bare)

- $(5.5 \text{ fm})^3 \times (11 \text{ fm})$ box
- $m_\pi = 140 \text{ MeV}$
- connected-only
- no renormalization

Proton, u-cEDM
Proton, d-cEDM
Proton, u-PS
Proton, d-PS
Neutron, u-cEDM
Neutron, d-cEDM
Neutron, u-PS
Neutron, d-PS
Previously reported lattice results for $\theta_{QCD}$-induced nEDM contain spurious contributions from mixing with the anomalous mag. moment

Corrected $\theta_{QCD}$-nEDM lattice values are small, consistent with zero
Disagreement with phenomenology/EFT is eliminated
Much higher lattice statistics are required to constrain of $\theta_{QCD}$

Based on preliminary analysis at a heavier pion mass (330 MeV),
at the physical point expect $|d_n| \approx (1..2) \times 10^{-3}$ e fm
Even with variance-reduction techniques, O(300) M core*hours may be required

Promising results for quark cEDM-induced EDFF
Renormalization & mixing subtractions are underway