New Formulation of the $\gamma W$-box correction to neutron and nuclear $\beta$-decay

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In collaboration with
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Outline

• Beta decay in presence of RC
• Dispersive representation of the $\gamma W$-box
• Physics input to the dispersion integral
• Nuclear effects
• New formulation of RC for $V_{ud}$ extraction
• Can nuclear effects turn the inner correction inside-out?
Neutron $\beta$-decay in presence of RC

Beyond RC that enter the Fermi constant:

\[
\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_\mu^2 V_{ud}^2}{(2\pi)^5} (1 + 3\lambda^2) |\vec{\rho}_e| E_e (E_m - E_e)^2 F(\beta) \left(1 + \frac{\alpha}{\pi} \text{Re} \, e\right) \left(1 + \frac{\alpha}{2\pi} \delta^{(1)}\right) \times \\
\left[1 + \left(1 + \frac{\alpha}{2\pi} \delta^{(2)}\right) a\vec{\beta} \cdot \hat{p}_\nu + \hat{s} \cdot \left(1 + \frac{\alpha}{2\pi} \delta^{(2)}\right) A\vec{\beta} + B\hat{p}_\nu\right]
\]

Coulomb distortion - Fermi fn. 
\[F(\beta) \approx 1 + \alpha\pi/\beta\]

"Inner" correction - depends on hadron structure; independent of kinematics

"Outer" corrections - IR-sensitive; depend on kinematics; independent of hadronic structure
Exactly calculable

Sirlin '67, Marciano & Sirlin '86 …

Separation due to scale hierarchy: Q-values from <1 keV (n) to few MeV (nuclei); Hadronic scales: at least 140 MeV - on top of $\alpha/2\pi \sim 10^{-3} \rightarrow 10^{-5}$ effect <<

Wilkinson '82, Severijns et al. '17
Radiative corrections - In & Out

1-loop RC (specific for a semileptonic process)

Outer: retain only IR divergent pieces

Inner: everything else

W,Z-exchange: UV-sensitive, pQCD; model-independent
When $\gamma$ involved - possible sensitivity to long range physics
Model-dependent!
$\gamma W$-box

Consider the box at zero energy and zero momentum transfer

$$T_{\gamma W} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu(k - q + m_e)\gamma^\nu(1 - \gamma_5)v_\nu}{q^2[(k - q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\gamma W}$$

Hadronic tensor: two-current correlator

$$T_{\gamma W}^{\mu\nu} = \int dxe^{iqx} \langle p | T[J_{em}^\mu(x)J_W^\nu(0)]|n\rangle$$

General gauge-invariant decomposition (spin-independent)

$$T_{\gamma W}^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1 + \frac{1}{(p \cdot q)} \left(p - \frac{(p \cdot q)}{q^2} q\right)^\mu \left(p - \frac{(p \cdot q)}{q^2} q\right)^\nu T_2 + \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(p \cdot q)} T_3$$
\[ \gamma W\text{-box} \]

Consider the box at zero energy and zero momentum transfer

\[
T_{\gamma W} = \sqrt{2}e^2 G_F V_{ud} \int \frac{d^4q}{(2\pi)^4} \frac{\bar{u}_e \gamma^\mu (k - \slashed{q} + m_e) \gamma^\nu (1 - \gamma_5) u_\nu}{q^2[(k - q)^2 - m_e^2]} \frac{M_W^2}{q^2 - M_W^2} T_{\gamma W}^{\mu \nu}
\]

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General gauge-invariant decomposition (spin-independent)

\[
T_{\gamma W}^{\mu \nu} = \left( -g^{\mu \nu} + \frac{q^{\mu} q^{\nu}}{q^2} \right) T_1 + \frac{1}{(p \cdot q)} \left( \frac{(p \cdot q)}{q^2} - q \right)^\mu \left( p - \frac{(p \cdot q)}{q^2} q \right)^\nu T_2 + \frac{ie^{\mu \nu \alpha \beta} p_\alpha q_\beta}{2(p \cdot q)} T_3
\]

V-V correlator is cancelled exactly
By the 3-current correlator - Sirlin ’67
Reason: conserved vector-isovector current

Axial current not conserved \(\rightarrow\) only A-V correlator contributes
Consider the box at zero energy and zero momentum transfer

Define the box contribution as

\[ T_W + T^{VA}_W = -\sqrt{2} G_F V_{ud} (1 + \Box^{VA}_\gamma) \bar{u} e p (1 - \gamma_5) v \nu \]

Loop integral with \( T_3 \)

\[ \Box^{VA}_\gamma = 4\pi \alpha \Re \int \frac{d^4 q}{(2\pi)^4} \frac{M^2_W}{M^2_W + Q^2} \frac{Q^2 + v^2}{Q^4} \frac{T_3(\nu, Q^2)}{M\nu} \]

Forward amplitude \( T_3 \) – unknown; Its absorptive part could be related to production of on-shell intermediate states a \( \gamma W \)-analog of the SF \( F_3 \)

\[ \text{Im} T^\gamma_W (\nu, Q^2) = 2\pi F^\gamma_W (\nu, Q^2) \]
\( \mathcal{W} \)-box in dispersion representation

\[ T_3 - \text{analytic function inside the contour } C \text{ in the complex } \nu\text{-plane determined by its singularities on the real axis - poles + cuts} \]

\[ T_3(\nu, Q^2) = \frac{1}{2\pi i} \int_C \frac{T_3(z, Q^2)dz}{z - \nu} \quad \nu \in C \]
\( \gamma W \)-box in dispersion representation

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Crossing behavior: photon is isoscalar or isovector

\[ T_{3\gamma W} = T_{3}^{(0)} + T_{3}^{(3)} \]

Different isospin channels behave differently under crossing

\[ T_{3}^{(0)}(\nu, Q^2) = -T_{3}^{(0)}(-\nu, Q^2), \quad T_{3}^{(3)}(\nu, Q^2) = +T_{3}^{(3)}(-\nu, Q^2) \]
$\gamma W$-box in dispersion representation

$T_3$ – analytic function inside the contour $C$ in the complex $\nu$-plane determined by its singularities on the real axis – poles + cuts

\[
T_3(\nu, Q^2) = \frac{1}{2\pi i} \oint_C \frac{T_3(z, Q^2)dz}{z - \nu} \quad \nu \in C
\]

Crossing behavior: photon is isoscalar or isovector

Different isospin channels behave differently under crossing

\[
T_3^{(0)}(-\nu, Q^2) = -T_3^{(0)}(\nu, Q^2), \quad T_3^{(3)}(-\nu, Q^2) = +T_3^{(3)}(\nu, Q^2)
\]

Dispersion representation of the $\gamma W$-box correction at zero energy

\[
\Box^{VA(0)}_{\gamma W} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2),
\]

\[
\Box^{VA(3)}_{\gamma W} = 0,
\]

\[
q = \sqrt{\nu^2 + Q^2}
\]
Input into dispersion integral

\[ \square_{\gamma W}^{V A (0)} = \frac{\alpha}{\pi M} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2) \]

Dispersion in energy:
scanning hadronic intermediate states

Relation to the old Marciano & Sirlin's result:

\[ \square_{\gamma W}^{V A} = \frac{\alpha}{8\pi} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F(Q^2) \]

Dispersion representation of M&S loop function F:

\[ F(Q^2) = \int_0^\infty d\nu \frac{8(\nu + 2q)}{M\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2) \]
Input into dispersion integral

Dispersion in $Q^2$: scanning dominant physics pictures

For each value of $Q^2$ we can relate $F$ to particular hadronic processes

$F(Q^2) = \int_0^{\infty} d\nu \frac{8(\nu + 2q)}{M\nu(\nu + q)^2} F_3^{(0)}(\nu, Q^2)$

$W^2 = (p + q)^2$

Boundaries not well-defined

DIS

Regge

Born

Res. + B.G

$N\pi$

For each value of $Q^2$ we can relate $F$ to particular hadronic processes

$\text{Im} [W^{++} + n \rightarrow \gamma^* + p] \leftrightarrow \begin{cases} \sigma(\gamma^* + p \rightarrow X) \\ \sigma(W^{++} + n \rightarrow X) \end{cases}$
M&S notation of the $\gamma W$-box correction:

$$\Box_{\gamma W}^{VA} = \frac{\alpha}{2\pi} [c_B + c_{int} + c_{DIS}]$$

What can be improved?

* What is the physics content of the interpolating function?
* Are the M&S constraints on $F_{int}$ justified?
Physics input

Elastic (Born) contribution

\[ \Box_{\gamma W}^{VA, \text{Born}} = -\frac{\alpha}{\pi} \int_0^\infty dQ \frac{2\sqrt{4M^2 + Q^2 + Q}}{(\sqrt{4M^2 + Q^2 + Q})^2} G_A(Q^2) G_M^s(Q^2) \]

\[ M \ & \ S \]

\[ \Box_{\gamma W}^{VA, \text{Born}} \bigg|_{\text{MS}} = \frac{\alpha}{2\pi} (0.829 \pm 0.083) \]

\[ \text{Central value:} \]
\[ \text{two dipoles integrated to (0.823 GeV)}^2 \]
\[ \text{Uncertainty: vary axial dipole masses between 1 and 1.4 GeV} \]

\[ \text{New evaluation} \]

\[ \Box_{\gamma W}^{VA, \text{Born}} = \frac{\alpha}{2\pi} (0.908 \pm 0.049) \]

\[ \text{Central value: full } Q^2 \text{ integral} \]
\[ \text{with most recent FF parametrization} \]
\[ \text{Uncertainty: from recent analyses} \]
\[ \text{Magnetic FF: Ye, Arrington, Hill, Lee ‘18} \]
\[ \text{Axial FF: Bhachatarya, Hill, Paz ‘11} \]
Inelastic contributions

\[ \Box_{\gamma W}^{Inel.} = \frac{\alpha}{\pi} \int_0^\infty \frac{dQ^2}{1 + \frac{Q^2}{M_W^2}} \int_{\nu_\pi}^\infty \frac{d\nu}{(\nu + q)^2} \frac{\nu + 2q}{M\nu} F_3^{(0),inel.} \]

Split the \( Q^2 \) integral: above (1.5 GeV)\(^2 \) - DIS; below - hadronic stuff
Physics input

Inelastic contributions

\[ \Box_{\gamma W}^{Inel.} = \frac{\alpha}{\pi} \int_0^\infty \frac{dQ^2}{1 + \frac{Q^2}{M_W^2}} \int_\nu^\infty \frac{d\nu}{(\nu + q)^2} \frac{\nu + 2q}{M_\nu} F_3^{(0),inel.} \]

Split the \( Q^2 \) integral: above \((1.5 \text{ GeV})^2\) - DIS; below - hadronic stuff

DIS contribution

\[ \Box_{\gamma W}^{DIS} = \frac{2\alpha}{\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{Q^2(Q^2 + M_W^2)} \int_0^{x\pi} dx \frac{1 + 2\sqrt{1 + 4M^2x^2/Q^2}}{(1 + \sqrt{1 + 4M^2x^2/Q^2})^2} F_3^{(0)}(x, Q^2) \]
Physics input

Inelastic contributions

\[ \Box_{\gamma W}^{\text{Inel.}} = \frac{\alpha}{\pi} \int_0^\infty \frac{dQ^2}{1 + \frac{Q^2}{M_W^2}} \int_{\nu_\pi}^\infty \frac{d\nu}{(\nu + q)^2} \nu + 2q F_3^{(0),\text{inel.}}(x, Q^2) \]

Split the \( Q^2 \) integral: above \((1.5 \text{ GeV})^2 \) – DIS; below – hadronic stuff

DIS contribution

\[ \Box_{\gamma W}^{\text{DIS}} = \frac{2\alpha}{\pi} \int_\Lambda^2 \frac{dQ^2 M_W^2}{Q^2(Q^2 + M_W^2)} \int_0^x dx \frac{1 + 2\sqrt{1 + 4M^2x^2/Q^2}}{(1 + \sqrt{1 + 4M^2x^2/Q^2})^2} F_3^{(0)}(x, Q^2) \]

Parton model:

\[ F_3^{(0)}(x) = \frac{e_u + e_d}{8}(d(x) - \bar{u}(x)) \]

\[ \int_0^1 dx d\nu(x) = 2 \]

\( \Lambda/Q \to 0 \); loop function becomes

\[ F_{\text{DIS}}(Q^2) = \frac{1}{Q^2} \]

Large log:

\[ \Box_{\gamma W}^{\text{DIS}} = \frac{\alpha}{8\pi} \int_\Lambda^2 \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F_{\text{DIS}}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda} \]
Inelastic contributions

\[ \square_{\gamma W}^{Inel.} = \frac{\alpha}{\pi} \int_0^\infty \frac{dQ^2}{1 + \frac{Q^2}{M_W^2}} \int_0^\infty d\nu \frac{\nu + 2q}{M\nu} F_3^{(0),inel.} \]

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\[ \square_{\gamma W}^{DIS} = \frac{2\alpha}{\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{Q^2 (Q^2 + M_W^2)} \int_0^x dx \frac{1 + 2\sqrt{1 + 4M^2 x^2/Q^2}}{(1 + \sqrt{1 + 4M^2 x^2/Q^2})^2} F_3^{(0)}(x, Q^2) \]

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\[ F^{DIS}(Q^2) = \frac{1}{Q^2} \]

Large log:

\[ \square_{\gamma W}^{DIS} = \frac{\alpha}{8\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{M_W^2 + Q^2} F^{DIS}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda} \]

pQCD corrections:

cf. GLS and Bjorken SR

\[ F^{DIS} = \frac{1}{Q^2} \to \frac{1}{Q^2} \left[ 1 - \frac{\alpha_{MS}}{\pi} - C_2 \left( \frac{\alpha_{MS}}{\pi} \right)^2 - C_3 \left( \frac{\alpha_{MS}}{\pi} \right)^3 \right] \]

M&S '06; Larin, Vermaseren '97

\[ \square_{\gamma W}^{DIS} = \frac{\alpha}{4\pi} [4.11 - 0.34] = \frac{\alpha}{2\pi} 1.84(0) \]

Uncertainty: virtually zero
Physics input

The DIS contribution can be validated by data
Use the Gross-Llewellyn-Smith sum rule (GLS SR) in nu/anti-nu scattering

\[
\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi} \left[ \frac{xy^2 F_1}{1 - y - \frac{Mxy}{2E}} F_2 \pm x \left( y - \frac{y^2}{2} \right) F_3 \right]
\]

\[
\sigma^{\nu p} - \sigma^{\bar{\nu} p} \sim F_3^{\nu p} + F_3^{\bar{\nu} p} = u_v(x) + d_v(x)
\]

\[
\int_0^1 dx (u_v(x) + d_v(x)) = 3
\]

Plot vs. data derived from nu-DIS

Including pQCD corrections

\[
\text{GLS SR} = 3 \left[ 1 - \frac{\alpha_s^{MS}}{\pi} - C_2 \left( \frac{\alpha_s^{MS}}{\pi} \right)^2 - C_3 \left( \frac{\alpha_s^{MS}}{\pi} \right)^3 \right]
\]
Physics input

Inelastic contributions beyond DIS

\[
\Delta_{W}^{\text{low } Q^2} = \frac{\alpha}{\pi} \int_0^{\Lambda^2} dQ^2 \int_{\nu_\pi}^{\infty} \frac{d\nu}{(\nu + q)^2} \frac{\nu + 2q}{M\nu} F_3^{(0)}
\]
Inelastic contributions beyond DIS

\[ \square^{\text{low } Q^2}_{\gamma W} = \frac{\alpha}{\pi} \int_0^{\Lambda^2} dQ^2 \int_{\nu\pi}^{\infty} \frac{d\nu}{(\nu + q)^2} \frac{\nu + 2q}{M\nu} F_3^{(0)} \]

M&S interpolating contribution

Constraints:

I. \( F^{\text{INT}}(\Lambda^2) = F^{\text{DIS}}(\Lambda^2) \)

II. \( F^{\text{INT}}((0.823 \text{ GeV})^2) = F^{\text{Born}}((0.823 \text{ GeV})^2) \)

III. \( F^{\text{INT}}(0) = 0. \)

\[ \square^{V_A(0)}_{\gamma W} = \frac{\alpha}{8\pi} \int_{Q_2^2}^{\Lambda^2} dQ^2 F^{\text{INT}}(Q^2) \]

3 Eqs. \( \rightarrow \) 3 free parameters

\[ F^{\text{INT}}(Q^2) = -\frac{1.490}{Q^2 + m_\rho^2} + \frac{6.855}{Q^2 + m_A^2} - \frac{4.414}{Q^2 + m_{\rho'}^2} \]
**Inelastic contributions beyond DIS**

\[
\square_{\gamma W} = \frac{\alpha}{\pi} \int_0^2 dQ^2 \int_\nu_\pi d\nu \frac{\nu + 2q}{M\nu} F_3^{(0)}
\]

\[
\square_{\gamma W}^{0} = \frac{\alpha}{8\pi} \int_{Q^2}^2 dQ^2 F^{\text{INT}}(Q^2)
\]

**M&S interpolating contribution**

**Constraints:**

I. \( F^{\text{INT}}(\Lambda^2) = F^{\text{DIS}}(\Lambda^2) \)

II. \( F^{\text{INT}}((0.823 \text{ GeV})^2) = F^{\text{Born}}((0.823 \text{ GeV})^2) \)

III. \( F^{\text{INT}}(0) = 0 \).

**3 Eqs. -> 3 free parameters**

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F^{\text{INT}}(Q^2) = -\frac{1.490}{Q^2 + m_\rho^2} + \frac{6.855}{Q^2 + m_A^2} - \frac{4.414}{Q^2 + m_{\rho'}^2}
\]

**Only constraint I is justified!**

II: no reason Born is the whole story below some arbitrary \( Q^2 \)

III: M&S claim it is required by chiral symmetry

Check in ChPT at one-loop

\[
\int_{\nu_\pi}^{\infty} \frac{d\nu}{\nu^2} F_3^{(0)}(\nu, Q^2 = 0) = \frac{2M}{Q^2} \int_0^{x_\pi} F_3^{(0)}(x, Q^2) \bigg|_{Q^2 \to 0} = 0
\]
ChPT - check if $F(0)=0$

Representative Feynman graphs

To visualize: change integration variable to $z = \nu_\pi / \nu$

$$\int_{\nu_\pi}^\infty \frac{d\nu_\pi}{\nu^2} F_3^{(0)}(\nu, Q^2 = 0) = \int_0^1 dz F_3^{(0)}(\nu_\pi / z, Q^2 = 0)$$

The surface below and above the curve is not the same

The claim of M&S about the superconvergence relation is not supported by ChPT
Inelastic states beyond DIS

Our approach
- saturate $F^{\text{INT}}$ by hadronic states:
  low-energy pi-N continuum
I=1/2 resonances
  dominant Regge exchange ($\rho$)

$\pi N$ contribution to the box:

$$\Box_{\gamma W}^{\pi N} = \frac{\alpha}{2\pi} 0.044(4)$$

I=1/2 resonances: tiny contribution!

$$\Box_{\gamma W}^{\text{Res}} \leq \frac{\alpha}{2\pi} 0.01$$
Inelastic states beyond DIS

Regge exchange

\[ F_{3}^{(0), \text{Regge}}(\nu, Q^2) = C_{R}(Q^2) \left( \frac{\nu}{\nu_0} \right)^{\alpha_{\rho}} \]

\[ W \begin{array}{c} a_1 \end{array} \begin{array}{c} \omega \end{array} \begin{array}{c} \gamma \end{array} \]

Vector/axial vector dominance:

Stodolsky, Piketty '70

Guidance from GLS sum rule:

CC process, A–V interference

Only overall normalization is changed!

Match to pQCD at \( Q^2 = 2 \text{GeV}^2 \)

VDM + fit to (few) low-\( Q^2 \) data below

\[ C_{R}(Q^2) = C_{R}^{\text{VDM}}(Q^2) \times h(Q^2) \]

Deduce \( h(Q^2) \), \( \Delta h(Q^2) \) from data!

\[ \square_{\gamma W}^{\text{Regge}} = \frac{\alpha}{2\pi} 0.238(14) \]
\section*{γW-box on free neutron}

Marciano & Sirlin ‘06

\[
\Box^{V A}_{\gamma W} = \frac{\alpha}{2\pi} [c_B + c_{int} + c_{DIS}] = \frac{\alpha}{2\pi} [0.83(8) + 0.14(14) + 1.84(0)]
\]

\[
\Box^{MS}_{\gamma W} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20) \times 10^{-3}
\]

\section*{New evaluation}

\[
\Box^{V A}_{\gamma W} = \frac{\alpha}{2\pi} [c_B + c_{piN} + c_{Res} + c_{Regge} + c_{DIS}] = \frac{\alpha}{2\pi} [0.91(5) + 0.044(5) + 0.01(1) + 0.238(14) + 1.84(0)]
\]

\[
\Box^{New}_{\gamma W} = \frac{\alpha}{2\pi} 3.03(5) = 3.51(6) \times 10^{-3}
\]

\(V_{ud}\) from free \(n\): about 1 sigma smaller

Numbers are preliminary but all crucial ingredients are in place.
Central value shifted by 1 sigma; uncertainty is likely to be reduced by factor 3

Currently the uncertainty for neutron decay is dominated by the experiment
Nuclear β-decay

General structure of RC for nuclear decay (see John’s talk)

\[ 1 + \text{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta). \]
Nuclear $\beta$-decay

General structure of RC for nuclear decay (see John's talk)

$$1 + \text{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$

Universal (no NS)
Nuclear $\beta$-decay

General structure of RC for nuclear decay (see John's talk)

$$1 + \text{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta).$$
Nuclear $\beta$-decay

General structure of RC for nuclear decay (see John's talk)

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Nuclear β-decay

General structure of RC for nuclear decay (see John’s talk)

\[ 1 + \text{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta). \]

Nuclear Green fn: only with 2 active N

Isospin breaking

Nuclear structure (NS)

Universal (no NS)
Nuclear β-decay

General structure of RC for nuclear decay (see John’s talk)

\[ 1 + \text{RC} = (1 + \delta_R)(1 - \delta_C)(1 + \Delta). \]

Nuclear Green fn: only with 2 active N

But data tell us differently: prominent broad QE peak - mostly 1N knock-out

\[ \Xi_{\gamma W}^{\text{Nucl.}} = \frac{\alpha}{\pi} \int_0^\Lambda^2 dQ^2 \int_{\nu_N}^\infty \frac{d\nu}{(\nu + q)^2} \frac{\nu + 2q}{M\nu} F_3(0). \]

QE peak is common for all nuclei Modify the universal correction to account for bulk QE effect
QE contribution to $\gamma W$-box

Bulk nuclear properties: Fermi momentum and break-up threshold

20 decays: $^{10}\text{C} \rightarrow ^{10}\text{B}$ through $^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$ (Towner&Hardy ’14 review)

$$\epsilon_1 = M_{A-p} + M_n - M_A$$

$$\epsilon_2 = M_{A'-n} + M_n - M_A$$

$$A - p = A' - n$$

Effective removal energies - all in a small range

$$\bar{\epsilon} = 7.68 \pm 1.32 \text{ MeV}$$

Fermi momentum also not too different for all A

$$k_F(A = 10) = 228 \text{ MeV}, \quad k_F(A = 74) = 245 \text{ MeV}$$

Can define a universal correction that correctly represents bulk nuclear effect!

Further ingredients:
Free Fermi gas model (or superscaling)
+ Pauli blocking

<table>
<thead>
<tr>
<th>Decay</th>
<th>$\epsilon_2$ (MeV)</th>
<th>$\epsilon_1$ (MeV)</th>
<th>$\bar{\epsilon}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}\text{C} \rightarrow ^{10}\text{B}$</td>
<td>8.44</td>
<td>4.79</td>
<td>6.36</td>
</tr>
<tr>
<td>$^{14}\text{O} \rightarrow ^{14}\text{N}$</td>
<td>10.55</td>
<td>5.41</td>
<td>7.55</td>
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<tr>
<td>$^{18}\text{Ne} \rightarrow ^{18}\text{F}$</td>
<td>9.15</td>
<td>4.71</td>
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<td>$^{22}\text{Mg} \rightarrow ^{22}\text{Na}$</td>
<td>11.07</td>
<td>6.28</td>
<td>8.34</td>
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<tr>
<td>$^{26}\text{Si} \rightarrow ^{26}\text{Al}$</td>
<td>11.36</td>
<td>6.30</td>
<td>8.46</td>
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<td>$^{30}\text{S} \rightarrow ^{30}\text{P}$</td>
<td>11.32</td>
<td>5.18</td>
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<tr>
<td>$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$</td>
<td>11.51</td>
<td>5.44</td>
<td>7.91</td>
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<tr>
<td>$^{38}\text{Ca} \rightarrow ^{38}\text{K}$</td>
<td>12.07</td>
<td>5.33</td>
<td>8.02</td>
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<tr>
<td>$^{42}\text{Ti} \rightarrow ^{42}\text{Sc}$</td>
<td>11.55</td>
<td>4.55</td>
<td>7.25</td>
</tr>
<tr>
<td>$^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$</td>
<td>11.09</td>
<td>6.86</td>
<td>8.72</td>
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<tr>
<td>$^{34}\text{Cl} \rightarrow ^{34}\text{S}$</td>
<td>11.42</td>
<td>5.92</td>
<td>8.22</td>
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<tr>
<td>$^{38m}\text{K} \rightarrow ^{38}\text{Ar}$</td>
<td>11.84</td>
<td>5.79</td>
<td>8.28</td>
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<tr>
<td>$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$</td>
<td>11.48</td>
<td>5.05</td>
<td>7.61</td>
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<tr>
<td>$^{46}\text{Va} \rightarrow ^{46}\text{Ti}$</td>
<td>13.19</td>
<td>6.14</td>
<td>9.00</td>
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<tr>
<td>$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$</td>
<td>13.00</td>
<td>5.37</td>
<td>8.35</td>
</tr>
<tr>
<td>$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$</td>
<td>13.38</td>
<td>5.13</td>
<td>8.28</td>
</tr>
<tr>
<td>$^{62}\text{Ga} \rightarrow ^{62}\text{Zn}$</td>
<td>12.90</td>
<td>3.72</td>
<td>6.94</td>
</tr>
<tr>
<td>$^{66}\text{As} \rightarrow ^{66}\text{Ge}$</td>
<td>13.29</td>
<td>3.16</td>
<td>6.48</td>
</tr>
<tr>
<td>$^{70}\text{Br} \rightarrow ^{70}\text{Se}$</td>
<td>13.82</td>
<td>3.20</td>
<td>6.65</td>
</tr>
<tr>
<td>$^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$</td>
<td>13.85</td>
<td>3.44</td>
<td>6.90</td>
</tr>
</tbody>
</table>
QE contribution to γW-box

\[ \Box_{\gamma W}^{\text{bound neutron}} = \frac{\alpha}{2\pi} 0.91(5) \rightarrow \Box_{\gamma W}^{\text{QE}} = \frac{\alpha}{2\pi} 0.44(4) \]

Reduction: finite breakup threshold

\[ \int \frac{d\nu}{\nu^2} F_3^n \rightarrow \int \frac{d\nu}{\nu^2} F_3^{\text{Nucl}} \]

New formulation of the γW-box:

\[ \Box_{\gamma W}^{MS} = \frac{\alpha}{2\pi} 2.79(17) = 3.24(20) \times 10^{-3} \]

\[ \Box_{\gamma W}^{\text{Nucl. New}} = \frac{\alpha}{2\pi} 2.56(4) = 2.97(5) \times 10^{-3} \]

A mere shift by 1 sigma; uncertainty significantly reduced.

Nuclear Structure corrections should be revisited and possibly redefined

\[ V_{ud} \text{ from superallowed } \beta: 1 \text{ sigma larger} \]
Summary

• New dispersive representation of the $\gamma W$-box

• Data driven uncertainties

• Crucial input: GLS sum rule

• New formulation of RC for $V_{ud}$ extraction: overall small effect; uncertainty significantly reduced

• Nuclear Structure corrections may need to be reformulated

• Backup: can nuclear structure effects lead to additional energy dependence?
Turn “inner” correction inside-out?

\[\gamma W\text{-box correction at zero energy}\]

\[\Box^{VA(0)}_{\gamma W} = \frac{\alpha}{\pi M} \int_0^\infty dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_0^\infty d\nu \frac{(\nu + 2q)}{\nu(\nu + q)^2} F_3'(0)(\nu, Q^2),\]

\[\Box^{VA(3)}_{\gamma W} = 0,\]

\[\gamma W\text{-box correction with linear E-dependence}\]

\[\text{Re } \Box^{even}_{\gamma W} = \frac{\alpha_{em}}{\pi} \int_{\nu_{thr}}^\infty d\nu \int_0^{\nu_{thr}} dQ^2 \frac{F_3'(0)}{2M\nu} \left( \frac{1}{E_{min}} - \frac{\nu}{4E_{min}^2} \right),\]

\[\text{Re } \Box^{odd}_{\gamma W} = \frac{\alpha_{em}}{\pi} E \int_{\nu_{thr}}^\infty d\nu \int_0^{\nu_{thr}} dQ^2 \left[ \frac{F_1'(0)}{6M E_{min}^3} + \left( \frac{\sqrt{\nu^2 + Q^2}}{2E_{min}^3 \nu Q^2} - \frac{1}{12E_{min}^3 \nu} \right) F_2'(0) + \frac{F_3'(-)}{2M\nu} \left( \frac{1}{2E_{min}^2} - \frac{\nu}{6E_{min}^3} \right) \right]\]

Common wisdom: E-dep. negligible because should come as \((\alpha/2\pi) E/m_\pi < 10^{-5}\)

But nuclear excitations live at few MeV \(\rightarrow\) large nuclear polarizabilities

\[\alpha_E + \beta_M = \frac{2\alpha_{em}}{M} \int \frac{d\omega}{\omega^3} F_1(\omega, Q^2 = 0) = 2\alpha_{em} \int \frac{d\omega}{\omega^2} \left. \frac{F_2(\omega, Q^2)}{Q^2} \right|_{Q^2=0}\]

New energy scale: polarizability/radius\(^2\)

\[\text{Re } \Box^{odd}_{\gamma W} \sim \frac{2}{\pi} E \frac{\alpha_E + \beta_M}{R_{Ch}^2}\]

\[R_{Ch} \sim 1.2\text{fm} A^{1/3}\]

\[\alpha_E \sim (2.2 \times 10^{-3}\text{ fm}) A^{5/3}\]

Expect \[\text{Re } \Box^{odd}_{\gamma W} \sim 1 \times 10^{-3} \left( \frac{E}{5\text{ MeV}} \right) \left( \frac{A}{30} \right)\]