recent progress on hadron spectroscopy from lattice QCD

Jozef Dudek
this presentation is dedicated to the memory of Mike Pennington,

scientist,
mentor,
friend
While QCD may be a solid part of the standard model, and hadrons are ubiquitous in HEP experiments, there remain significant mysteries in how hadrons are built from quarks and gluons.
QCD, hadrons and the standard model

While QCD may be a solid part of the standard model, and hadrons are ubiquitous in HEP experiments, there remain significant mysteries in how hadrons are built from quarks and gluons.

unexpected?
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**light scalar meson resonances**

<table>
<thead>
<tr>
<th>Resonance</th>
<th>$I^G(J^P)$</th>
<th>Mass $m$</th>
<th>Full width $\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0(500)$ or $\sigma$</td>
<td>$0^+(0++)$</td>
<td>$(400-550)$ MeV</td>
<td>$(400-700)$ MeV</td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>$0^+(0++)$</td>
<td>$990 \pm 20$ MeV</td>
<td>$10$ to $100$ MeV</td>
</tr>
<tr>
<td>$K_0^*(800)$ or $\kappa$</td>
<td>$\frac{1}{2}(0^+)$</td>
<td>$980 \pm 20$ MeV</td>
<td>$50$ to $100$ MeV</td>
</tr>
</tbody>
</table>
While QCD may be a solid part of the standard model, and hadrons are ubiquitous in HEP experiments, there remain significant mysteries in how hadrons are built from quarks and gluons.

**QCD, hadrons and the standard model**

**unobserved?**

**pure YM glueballs**

<table>
<thead>
<tr>
<th>4000 -</th>
<th>3000 -</th>
<th>2000 -</th>
<th>1000 -</th>
</tr>
</thead>
<tbody>
<tr>
<td>3++</td>
<td>0++</td>
<td>2++</td>
<td>0++</td>
</tr>
</tbody>
</table>

Morningstar 
& Peardon

**lqcd hybrid hadrons**

<table>
<thead>
<tr>
<th>2000 -</th>
<th>1500 -</th>
<th>1000 -</th>
</tr>
</thead>
<tbody>
<tr>
<td>q̅q+g+g+...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m−m_p</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| qqq+g+g+... |
| m−m_N   |

N_{3/2}^+  
N_{1/2}^+  
Δ_{1/2}^+  
Δ_{3/2}^+  

**hadspec**

Morningstar 
& Peardon
the lattice as a tool for QCD

quark & gluon fields on a **finite space-time grid** (in Euclidean time)

introduce: **lattice spacing**, **lattice volume**, often \( m_q > m_q^{\text{phys}} \)

Monte Carlo sample field configurations

**hadron spectrum** from two-point correlation functions

\[
\langle 0 | \mathcal{O}'(t) \mathcal{O}(0) | 0 \rangle = \int D\psi D\bar{\psi} DA \left[ \mathcal{O}'(t) \mathcal{O}(0) \right] e^{-S[\psi, \bar{\psi}, A]}
\]

\[
\langle 0 | \mathcal{O}'(t) \mathcal{O}(0) | 0 \rangle = \sum_n A'_n A_n e^{-E_n t}
\]

*field configuration probability*

spectrum of QCD eigenstates
precise spectroscopy of stable hadrons

lattice QCD light hadron spectrum

summary compiled by Andreas Kronfeld


QCD+QED mass shifts

BMW Collaboration

Science 347 1452 (2015)
but much of the excitement in hadron spectroscopy is in **heavier states**

and they are **resonances** observed through their decays

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**Excited spectroscopy**

same non-perturbative dynamics **binds** and causes the **decay** — can’t be separated within QCD ...  

a faithful QCD calculation should give all the **scattering physics** at once ...
resonances on the lattice?

The approach can be illustrated within **one-dimensional quantum mechanics**

Imagine two identical bosons separated by a distance $z$ interacting through a finite-range potential $V(z)$.

$$\psi(\mathbf{z} > R) \sim \cos(p|z| + \delta(p))$$
‘scattering’ in a finite-volume

now put the system in a ‘box’ — periodic boundary condition at \( z = \pm L/2 \)

\[
\psi(|z| > R) \sim \cos \left( p |z| + \delta(p) \right)
\]

\[
\psi(L/2) = \psi(-L/2)
\]

\[
\frac{d\psi}{dz}(L/2) = \frac{d\psi}{dz}(-L/2)
\]

momentum quantization condition

\[
p = \frac{2\pi}{L} n - \frac{2}{L} \delta(p)
\]

reversing the logic:

if you can compute the **discrete finite-volume spectrum** in a quantum theory, you can find the **scattering amplitude**
an elastic resonance — the $\rho$ in $\pi\pi$

canonical resonance ‘bump’
described by a rapidly rising phase-shift
an elastic resonance — the $\rho$ in $\pi\pi$ — lattice QCD

$E / \text{MeV}$

$\delta_{\pi}^\rho$

$E / \text{MeV}$

PRD87 034505 (2013) $m_\pi \sim 391 \text{ MeV}$
an elastic resonance — the $\rho$ in $\pi\pi$ — lattice QCD

$E / \text{MeV}$

$m_\pi \sim 391 \text{ MeV}$

$E / \text{MeV}$

$m_\pi \sim 236 \text{ MeV}$

scattering phase-shift

scattering phase-shift

other irreps

other irreps
an elastic resonance — the $\rho$ in $\pi\pi$ — lattice QCD
most resonances decay into more than one final state

e.g. two-channel scattering described by a $t$-matrix

$$t(E) = \begin{pmatrix} t_{11}(E) & t_{12}(E) \\ t_{21}(E) & t_{22}(E) \end{pmatrix}$$

finite-volume spectrum as a function of scattering becomes more complicated

$$\text{coupled-channel spectrum}$$

solutions $E_n(L)$ of

$$\det \left[ t^{-1}(E) - \tilde{M}(E, L) \right] = 0$$

no longer a one-to-one mapping from energy to scattering ...

... can parameterize the energy dependence of the scattering $t$-matrix

first lattice QCD calculations of coupled meson-meson scattering have appeared in the last four years ...
a(0(980)) in $\pi\eta$

a narrow resonance seen in the $\pi\eta$ final state

e.g.

$$pp \rightarrow p\,\eta\pi^0\,p$$

WA102

$$\pi^-p \rightarrow \eta\pi^+\pi^-\,n$$

E852

$$\gamma\gamma \rightarrow \eta\pi^0$$

Belle
a(980) in $\pi\eta$  

a narrow resonance seen in the $\pi\eta$ final state

e.g.

$pp \rightarrow p \, \eta \pi^0 \, p$

right at the $K\bar{K}$ threshold

$\pi^- p \rightarrow \eta \pi^+ \pi^- \, n$

$\gamma \gamma \rightarrow \eta \pi^0$

will need a coupled $\pi\eta$, $K\bar{K}$ approach ...
πη/KK scattering in lattice QCD

first calculation — unphysically heavy u,d quarks

\[ m_\pi \approx 391 \text{ MeV} \]
\[ m_K \approx 549 \text{ MeV} \]
\[ m_\eta \approx 587 \text{ MeV} \]

**E**

- **[000]**
- **[100]**
- **[110]**
- **[111]**
- **[200]**
πη/KK scattering in lattice QCD

$\pi\eta$ to $\pi\eta$ coupled-channel scattering amplitudes

$\rho_i\rho_j |t_{ij}|^2$

$E / \text{MeV}$

$K\bar{K} \rightarrow K\bar{K}$

$\pi\eta \rightarrow \pi\eta$

$\pi\eta \rightarrow K\bar{K}$

$\pi\eta$ to $KK$ coupled-channel scattering amplitudes

$E / \text{MeV}$

$\rho_i\rho_j |t_{ij}|^2$

$\pi\eta$ to $KK$ coupled-channel scattering amplitudes

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$\pi\eta$ to $KK$ coupled-channel scattering amplitudes

$E / \text{MeV}$

$\rho_i\rho_j |t_{ij}|^2$
πη/KK scattering in lattice QCD

something significant at KK threshold

is there a resonance causing this?

m_π ~ 391 MeV

PRD93 094506 (2016)
a resonance can be rigorously defined to be a pole singularity at a complex energy

\[ t_{ij}(E) \sim \frac{c_i c_j}{E_0^2 - E^2} \]

with pole position

\[ E_0 = m_R \pm i \frac{1}{2} \Gamma_R \]
a resonance can be rigorously defined to be a pole singularity at a complex energy

\[ t_{ij}(E) \sim \frac{c_i c_j}{E_0^2 - E^2} \]

with pole position \[ E_0 = m_R \pm i\frac{1}{2}\Gamma_R \]

\[ m_{a_0} = 1177(27) \text{ MeV} \]
\[ \Gamma_{a_0} = 49(33) \text{ MeV} \]
a resonance can be rigorously defined to be a pole singularity at a complex energy

\[ t_{ij}(E) \sim \frac{c_i c_j}{E_0^2 - E^2} \]

with pole position

\[ E_0 = m_R \pm \frac{i}{2} \Gamma_R \]

### Resonance Couplings

\[ \left| \frac{c_{KK}}{c_{\pi\eta}} \right|^2 = 1.7(6) \]

### Pole Position

\[ m_{a_0} = 1177(27) \text{ MeV} \]

\[ \Gamma_{a_0} = 49(33) \text{ MeV} \]
ππ, KK, ηη scattering

Experimental amplitudes

combination of broad σ resonance and narrow f0(980) at KK threshold
ππ, KK, ηη scattering in lattice QCD

considerable elastic region

three-channel almost immediate

m_π ~ 391 MeV
m_K ~ 549 MeV
m_η ~ 587 MeV
$\pi\pi, K\bar{K}, \eta\eta$ scattering in lattice QCD

$m_\pi \sim 391$ MeV
$m_K \sim 549$ MeV
$m_\eta \sim 587$ MeV

**Graph:**
- $\pi\pi / K\bar{K} / \eta\eta$ coupled-channel scattering amplitudes
- $\rho_1 \rho_2 |t_{ij}|^2$
- $E / \text{MeV}$

- $\pi\pi \rightarrow \pi\pi$
- $K\bar{K} \rightarrow K\bar{K}$
- $\pi\pi \rightarrow K\bar{K}$
- $\eta\eta \rightarrow K\bar{K}$
- $\eta\eta \rightarrow \pi\pi$
- $\eta\eta \rightarrow \eta\eta$

nothing much happening in $\eta\eta$
\( \pi \pi, K\bar{K}, \eta \eta \) scattering in lattice QCD

\( m_\pi \sim 391 \text{ MeV} \)
\( m_K \sim 549 \text{ MeV} \)
\( m_\eta \sim 587 \text{ MeV} \)

**Graph:**

- \( \pi \pi \) to \( \pi \pi \)
- \( K\bar{K} \) to \( K\bar{K} \)
- \( \pi \pi \) to \( K\bar{K} \)
- \( \eta \eta \) to \( K\bar{K} \)
- \( \eta \eta \) to \( \pi \pi \)
- \( \eta \eta \) to \( \eta \eta \)

**Strong activity at** \( K\bar{K} \) threshold
$\pi\pi$, $K\bar{K}$, $\eta\eta$ scattering in lattice QCD

$\pi\pi/K\bar{K}/\eta\eta$ coupled-channel scattering amplitudes

$m_\pi \sim 391$ MeV
$m_K \sim 549$ MeV
$m_\eta \sim 587$ MeV
ππ, K̅K, ηη scattering in lattice QCD

ππ/K̅K/ηη coupled-channel scattering amplitudes

\[ |\rho_i \rho_j t_{ij}|^2 \]

\[ \sigma \]

bound-state

\[ \Gamma_R \]

resonance

\[ \frac{|c_{K\bar{K}}|}{|c_{\pi\pi}|}^2 = 1.4(3) \]

\[ m_\pi \sim 391 \text{ MeV} \]

\[ m_K \sim 549 \text{ MeV} \]

\[ m_\eta \sim 587 \text{ MeV} \]
ππ, K̅K, ηη scattering

$m_\pi \sim 391$ MeV

Experimental amplitudes

\[ \left| \frac{c_{K\bar{K}}}{c_{\pi\pi}} \right|^2 = 1.4(3) \]

$\sigma$ vs. $m_R$

$\Gamma_R$ vs. $m_R$

$\pi\pi \rightarrow \pi\pi$

$\pi\pi \rightarrow K\bar{K}$

$\pi\pi \rightarrow \eta\eta$

$\delta(500)$ or $\delta(1400)$

$\delta(980)$

Mass $m = (400-550)$ MeV
Full width $\Gamma = (400-700)$ MeV
Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV
coupled-channel resonances — status

have demonstrated presence of coupled-channel resonances in (lattice) QCD can determine pole positions (mass, width) and couplings to decay channels

would like to know if there’re simple ways to ‘understand’ them
e.g. big differences between scalar, vector, tensor mesons long-standing ideas of $q\bar{q}$ versus $qq\bar{q}\bar{q}$ versus meson-meson molecules

one possible approach to this — consider their couplings to external currents ...
coupling resonances to currents

\[ \gamma^* \rightarrow \pi\pi \]

\[ F_\pi(E_{\pi\pi}) \]

Feng et al | u,d,s | \( m_\pi = 290 \text{ MeV} \)

PRD97 054513 (2018)

\[ \gamma^* \pi \rightarrow \pi\pi \]

\[ \mathcal{A}(E_{\pi\pi}, Q^2) \]

Briceno et al | u,d,s | \( m_\pi = 391 \text{ MeV} \)

PRL115 242001 (2015)

\[ Q^2 = 0 \]
\[ Q^2 = 0.803 \text{ GeV}^2 \]

\[ m_\pi \cdot |\mathcal{A}_{\pi_\pi}\pi|^2 \]

\[ 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \]

\[ 2.0 \quad 2.1 \quad 2.2 \quad 2.3 \quad 2.4 \quad 2.5 \]

\[ E_{\pi\pi} / m_\pi \]

\[ 2.0 \quad 2.1 \quad 2.2 \quad 2.3 \quad 2.4 \quad 2.5 \]

\[ E_{\pi\pi} / \text{GeV} \]

Bulava et al | u,d,s | \( m_\pi = 280 \text{ MeV} \)

conf. proc. LAT '15

\[ \rho \rightarrow \pi\gamma^* \]

\[ m_\rho = 770 \text{ MeV} \]
\[ m_\rho = 400 \text{ MeV} \]
opportunities, challenges

addressing the new observations in charmonium (XYZ)
  – challenging, often lie above several thresholds ⇒ multiple coupled channels

predicting resonant properties of hybrid hadrons
  – preferred decay modes, couplings to photons (relevant to GlueX, see next talk)

(finally) understanding the scalar mesons ?
  – studying their behaviour with changing quark mass, evaluating their form-factors ...

  
  
  

a big current challenge is the importance of three-body final states
  – lack of a complete finite-volume formalism so far
Scattering processes and resonances from lattice QCD

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The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a complete approach to QCD is required. Presently, lattice QCD is the only available tool that provides the required nonperturbative evaluation of hadron observables. This article reviews progress in the study of few-hadron reactions in which resonances and bound states appear using lattice QCD techniques. The leading approach is described that takes advantage of the periodic finite spatial volume used in lattice QCD calculations to extract scattering amplitudes from the discrete spectrum of QCD eigenstates in a box. An explanation is given of how from explicit lattice QCD calculations one can rigorously garner information about a variety of resonance properties, including their masses, widths, decay couplings, and form factors. The challenges which currently limit the field are discussed along with the steps being taken to resolve them.

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