Multi-Angle Calculations of Matter-Neutrino Resonance

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Neutrino Flavor in Astrophysics

- Astrophysical environments (early Universe, supernovae, compact object mergers) are strongly affected by neutrinos.
- Neutrinos can undergo flavor transformation.
- Electron neutrinos interact with matter differently than other flavors, so neutrino flavor matters.
- Flavor physics can therefore affect observables:
  - Elemental abundances
  - Neutrino spectra
  - Dynamics of mergers and supernovae
Flavor Transformation in Dense Environments

- Flavor can be described by $N_F \times N_F$ density matrices $f$
- For each neutrino momentum, there are two matrices: one for neutrinos and one for anti-neutrinos.
- $f$ evolve according to a Schrödinger equation:
  \[
  i \frac{\dot{f}}{f} = \left[ N + M - V, f \right]
  \]
  \[
  i \frac{\dot{f}}{f} = \left[ N + M + V, \bar{f} \right]
  \]
- $N, M$ and $V$ are the neutrino, matter and vacuum potentials, respectively.
Regimes of Flavor Transformation

Flavor transformation can occur when combinations of scales in the Hamiltonian become comparable to the vacuum term:

\[
M \approx V \quad \text{MSW Effect}
\]

\[
N \approx V \quad \text{Collective Oscillations}
\]

\[
M + N \approx V \quad \text{Matter-Neutrino Resonance}
\]

Turbulence & Fast Oscillations can also lead to flavor transformation, outside of these regimes.
Matter-Neutrino Resonance in Compact Object Mergers

- In mergers, the anti-neutrino contribution to $H$ can be larger than the neutrino contribution.
- In this case, the neutrino potential has the opposite sign to the matter potential.
- A cancellation between matter and neutrino potentials can occur. This is known as a matter-neutrino resonance (MNR).
- MNRs can lead to flavor transformation even when matter & neutrino potentials are individually $\gg V$. 
Previous Work: Single-Angle MNR
Malkus, Friedland, McLaughlin (2014)

- Single-angle approximations set the flavor to be the same for all neutrino trajectories, and follow one trajectory.

- In many models of mergers and supernovae, \( M \sim R^{-3} \) and \( N \sim R^{-4} \) for sufficiently large \( R \).

- For \( |N| \) initially > \( |M| \), with opposite sign, there is a cancellation at some value of \( R = R_{\text{MNR}} \), where \( N = -M \).
Single-Angle MNR

- In single-angle MNR, neutrinos can transform fully while anti-neutrinos return to original state.
- Matter + neutrino potential remains near zero until transformation is complete.

![Survival Probabilities and Total / Matter Potential graphs](image-url)
Multi-Angle MNR

- In a non-isotropic system, neutrino potential depends on the propagation angle.

\[
H_\mu \propto \int \tilde{d}q q_\mu \left( f_q - \bar{f}_q \right) = \left\{ H_0, \vec{H}_R \right\}
\]

\[
N = k^\mu H_\mu / E = H_0 - H_R \cos \theta
\]

In this illustration, \( N_1 < N_2 \)
Multi-Angle MNR

- The location of the MNR depends on the value of $N$.
- **Neutrinos on different trajectories cross the MNR at different locations.**
- Because the MNR is spread out over a wide region, multi-angle models with MNR can be expected to behave very differently than single-angle models.
- Example of multi-angle models:
  - Beam of neutrinos with a nonzero opening angle (Shalgar 2017)
  - Spherical bulb model (Vlasenko, McLaughlin)
  - “Realistic” merger geometry, with cylindrical or no symmetry (very computationally expensive).
The Spherical Bulb Model

- We use the simplest self-consistent model: single-energy spherical bulb with two flavors.
- This model captures the key feature of multi-angle MNR: the position of the resonance varies for different trajectories.
- We begin with a neutrino driven wind-like model: compact central source, high entropy, relatively low matter density proportional to $R^{-3}$.
- Some of these assumptions are relaxed later.
Spherical Bulb Geometry

\[ u_0 = \cos \theta_0 \]

\[ R_{NS} \]

\[ u = \cos \theta \]

\[ M = \sqrt{2} G_F Y_e n_B \propto Y_e R^{-3} \]

\[ N = \sqrt{2} G_F \left[ (n_\nu - n_\bar{\nu}) - u (\Phi_\nu - \Phi_\bar{\nu}) \right] \propto R^{-4} \]
Model Parameters

- Neutrinosphere radius $R_{\text{NS}}$. Neutrino flux is proportional to $R_{\text{NS}}^2$.
- MNR radius $R_{\text{MNR}}$. Here, defined as $R$ at which $M = N$ for radially emitted neutrinos ($u_0 = 1$).
- Ratio of anti-neutrino to neutrino contribution to potential, $\alpha$. For MNR models, $\alpha > 1$.
- Vacuum mixing angle, mass hierarchy, etc.
- For benchmark model, choose $R_{\text{NS}} = 15$ km, $R_{\text{MNR}} = 60$ km, $\alpha = 1.4$, normal hierarchy, $\theta_{13}$.
Single-Angle vs. Spherical Bulb: Survival Probabilities

**Single-Angle**

- Neutrinos (Blue line)
- Anti-Neutrinos (Red line)

**Multi-Angle**

- Neutrinos (Blue line)
- Anti-Neutrinos (Red line)
Single-Angle vs. Spherical Bulb: Total Potential

Single-Angle

Multi-Angle

\( \frac{(N+M)}{M} \) vs. \( R, \text{ km} \)

\( \frac{(N+M)}{M} \) vs. \( R, \text{ km} \)

- Blue line: \( u0 = 1 \)
- Orange line: \( u0 = 0 \)
- Dashed line: Untransformed
Survival Probabilities as Function of Emission Angle

Survival Probabilities, $R = 120$ km

Survival Probabilities, $R = 240$ km
Effect on Matter Composition

Assumptions: neutrons & protons in equilibrium with neutrinos, neglect electron capture

- MNR results in a modest $Y_e$ decrease (~ few percent).
- Not as much of an effect as in single-angle calculations.
- However, even a modest decrease of $Y_e$ can have large effects on nucleosynthesis, particularly near $Y_e \sim 0.4$ and high entropy.
Sensitivity to Model Parameters

Anti-neutrino to neutrino ratio ($\alpha$)

$R_{NS} = 15$ km, $R_{MNR} = 60$ km

Larger values of $\alpha$ lead to greater difference between neutrino and anti-neutrino flavor transformation.
Sensitivity to Model Parameters

Neutrinosphere radius
\[ \alpha = 1.4, \ R_{\text{MNR}} = 60 \ \text{km} \]

Large neutrinosphere radius (and large neutrino flux) slightly suppresses flavor transformation
Sensitivity to Model Parameters

MNR radius

$\alpha = 1.4$, $R_{NS} = 15$ km

$Larger MNR radius enhances flavor transformation
Sensitivity to Model Parameters

Shallow Density Profile ($M \sim R^{-1}$ instead of $R^{-3}$)

$\alpha = 1.4$, $R_{NS} = 15$ km

Shallow matter density profile suppresses MNR, but flavor transformation is restored for larger $R_{MNR}$. 
Extended Neutrinosphere Model

- In a compact object merger, a significant fraction of neutrinos (~50%) come from sources outside the neutrinosphere (the accretion disk and the scattered halo).
- These extra neutrinos contribute disproportionately to $N$.
- We can model this by extending the initial radius from $R_{\text{NS}}$ to $R_{E} > R_{\text{NS}}$, and adding neutrinos on shallower trajectories:

![Diagram showing the extended neutrinosphere model]
Extended Neutrinosphere Model

- Pattern of flavor transformation very similar to bulb model, but MNR takes longer (note the horizontal scale).
- This is because the “tail” of the neutrino distribution at large emission angles takes a long time to pass through resonance.
Conclusion

We examined multi-angle MNR in a spherical bulb model and found that:

- A new type of neutrino flavor transformation occurs with MNR in multi-flavor calculations.
- This type of flavor transformation is qualitatively different from that seen in single-angle calculations.
- Neutrinos and anti-neutrinos transform differently, altering proton-neutron ratio and possibly affecting nucleosynthesis.
- The results are robust, remaining qualitatively similar under a wide variety of physical conditions.
- Therefore, neutrino flavor transformation due to MNR is likely to be important in compact object mergers.