Interactions between Decuplet Baryons from Lattice QCD

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HAL(Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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ΩΩ interaction with J=0 at almost physical point

“di-Omega” Most Strange dibaryon

May 29, 2018@CIPANP2018
Introduction

Baryon (B=1)

Proton, Neutron, Lambda,…

Dibaryon (B=2)

Deuteron (1930s)

Dibaryon = Bound (or resonance) two baryon states
Introduction: SU(3) classification for Baryon (B=1)

All octet baryons are stable under strong decay
Introduction: SU(3) classification for Baryon (B=1)

Octet (S=1/2)

- n
- p
- udd
- uud
- uus
- dds
- sus
- Σ-
- Σ^0, Λ
- Σ^+

Decuplet (S=3/2)

- Δ-
- Δ^0
- Δ^+
- Δ^{++}
- udd
- dud
- uud
- uuu
- Σ-
- Σ^0
- Σ^+
- Σ^{*-}
- uds
- sus
- Ξ-
- Ξ^0
- Ξ^{*-}
- Σ^{*-}
- sss
- Ω^- 

All octet baryons are stable under strong decay.
In Decuplet baryons, only Ω-baryon is stable.
Introduction: SU(3) classification for Dibaryon candidates (B=2)

1) octet-octet system

\[ 8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus 10 \oplus 10 \oplus 8_a \]

H-dibaryon (J=0) Jaffe (1977)

Deuteron (J=1)

2) decuplet-octet system

\[ 10 \otimes 8 = 35 \oplus 8 \oplus 10 \oplus 27 \]

N\(\Omega\) system (J=2) Goldman et al (1987)

3) decuplet-decuplet system

\[ 10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus 10 \]

\(\Omega\Omega\) system (J=0) Zhang et al (1997)

\(\Delta\Delta\) system (J=3) Dyson, Xuong (1964)

Oka, Yazaki (1980)

found as a “resonance” by CELSIUS/WASA, 2009
Previous model works on $\Omega\Omega$ in J=0

SU(3) chiral quark model

$\Delta M_{\Omega\Omega} = -166\text{MeV}$

Quark Disloc/Color-screen model
F. Wang et al (1992)

$\Delta M_{\Omega\Omega} = 43 \pm 18\text{MeV}$

$\Delta M_{\Omega\Omega} \equiv E_{\Omega\Omega} - 2M_{\Omega}$

- Bound/unbound problem highly depends on models and their parameters.
- To clarify $\Omega\Omega$ interaction in our world, first-principle calculations are needed.
Baryon-Baryon interaction from lattice QCD
-HAL method-
Aoki, Hatsuda, Ishii, PTP123, 89 (2010)

c.f. another method: Luscher’s direct method

Nambu-Bethe-Salpeter (NBS) w.f.
\[ \Psi_n (\vec{r}) e^{-E_n t} = \sum_{\vec{x}} \langle 0 | B_1 (t, \vec{r} + \vec{x}) B_2 (t, \vec{x}) | E_n \rangle \]

Local operators \( B_1 \) and \( B_2 \) for \( \Omega \) baryon
\[ \Omega_{\alpha, k} (x) = \epsilon^{abc} \left[ s^T_a (x) C \gamma_k s_b (x) \right] s_{c, \alpha} (x) \]

In asymptotic region \((r >> R)\)
Helmholtz eq. is satisfied:
\[ (\nabla^2 + k^2) \Psi (\vec{r}) = 0 \]
\[ \Psi(\vec{r}) \sim A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr} \]
Baryon-Baryon interaction from lattice QCD
-HAL method-

\[ \Omega \]

\[ \Omega \]

\[ \begin{align*}
\Psi_n(r) &= e^{-E_n t} \\
&= \sum \langle 0 | B_1(t, \vec{r} + \vec{x}) B_2(t, \vec{x}) | E_n \rangle
\end{align*} \]

Nambu-Bethe-Salpeter (NBS) w.f.

Local operators \( B_1 \) and \( B_2 \) for \( \Omega \) baryon

\[ \Omega_{\alpha,k}(x) = \epsilon^{abc} \left[ T_a(x) C\gamma_k S_b(x) \right] s_{c,\alpha}(x) \]

In interacting region,

Schroedinger type equation is satisfied

\[ \left( \vec{p}_n^2 + \nabla^2 \right) \Psi_n(r) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \Psi_n(\vec{r}') \]
Nonlocal potential $U(r,r')$

$$(\hat{p}_n^2 + \nabla^2) \Psi_n (\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}') \Psi_n (\vec{r}')$$

- The potential is energy-independent but non-local.
- The local leading potential can be obtained by its derivative expansion (c.f. Okubo-Marshak expansion):

$$U(\vec{r},\vec{r}') = V_c(r) + V_\sigma(r)(\vec{S}_1 \cdot \vec{S}_2) + S_{12} V_{T_1}(r)$$

$$+ O(\nabla^2)$$

$$= V_{C,\text{eff}}(r) + O(\nabla^2)$$

- The convergence of the expansion can be checked.
- The NLO term is explicitly determined by utilizing two source functions (Iritani et. al, arXiv:1805.02365)
Time-dependent HAL method

- original (t-indep) HAL method \(\Rightarrow\) applicable for each NBS w.f.

\[
G_{BB}(\vec{r}, t) = \langle 0 | B(\vec{y}, t) B(\vec{x}, t) \bar{J}(t_0; J^P) | 0 \rangle
\]

\[
\mathcal{R}(\vec{r}, t; J^P) = \frac{G_{BB}(\vec{r}, t)}{G_B(t)} = \sum A_n \psi_n(\vec{r}) e^{-(W_i - 2m_B)t}
\]

\[
\int dr' U(r, r') \psi_{W_0}(r') = (E_{W_0} - H_0) \psi_{W_0}(r)
\]

\[
\int dr' U(r, r') \psi_{W_1}(r') = (E_{W_1} - H_0) \psi_{W_1}(r)
\]

- Many states contribute to the R-correlator
- As lattice size increases, the extraction of g.s. becomes difficult

\[
E_n \sim 2\sqrt{m_B^2 + (2\pi n/L)^2} \sim E_0 \quad (L \gg 1)
\]

The same problem appears for the direct method (Iritani et al. JHEP(2016), PRD(2017))
Time-dependent HAL method

- new (t-dep) HAL method $\Rightarrow$ directly applicable for $R$-correlator

\[
\int dr' U(r, r') \psi_{W_0}(r') = (E_{W_0} - H_0) \psi_{W_0}(r)
\]
\[
\int dr' U(r, r') \psi_{W_1}(r') = (E_{W_1} - H_0) \psi_{W_1}(r)
\]

\[\Delta E_n = \frac{k_n^2}{m_B} - \frac{\Delta E_n^2}{4m_N}\]

All equations are combined as

\[
\left( \frac{1}{4m_B} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{m_B} \right) R = \int U(\vec{r}, \vec{r}') R d^3 r'
\]

G.S. saturation is not required.

$\Rightarrow$ “Elastic state saturation” is required

Weaker condition as $L \rightarrow \infty$
Experiment

- rich data for less strange quarks
- More strange quarks, more difficult experiment due to short life time

Lattice QCD

- better S/N for more strange quarks
- Less strange quarks, more difficult numerical simulation due to increasing statistical noise

ΩΩ system is the best S/N ratio calculation on lattice
Interactions in $\Omega\Omega$ (J=0) system

1) Nf = 2+1, L = 1.93fm, $m_\pi=1015$MeV, SU(3) limit
2) Nf=2+1, L = 3fm, $m_\pi=700$MeV, SU(3) breaking
3) Nf=2+1, L = 8.1fm, $m_\pi=146$MeV, almost physical mass
1) $N_f=2+1$ full QCD with $L = 1.93\,\text{fm}$ $m_\pi = 1015\,\text{MeV}$, SU(3) limit $\Omega \Omega$ in $J=0$

$m_\Omega = 2220 \,\text{MeV}$

- Short range repulsive core and attractive pocket are found
- Phase shift shows the system is in the unitary limit

*let's consider the lighter quark masses with SU(3) breaking*
2) $N_f=2+1$ full QCD with $L = 3\text{fm}$, $\Omega\Omega$ in $J = 0$

\[ m_\pi = 700\text{MeV} \text{ w. SU}(3) \text{ breaking} \]

\[ m_\Omega = 1970\text{MeV} \]

- Short range repulsive core and attractive pocket are found
- Potential is nearly independent on “t” within error
- Phase shift shows rapid changes depending on “t”
- *The system may appear close to the unitary limit*

c.f. Direct method by Buchoff et al., PRD(2012): L=4fm, $m_\pi = 390\text{MeV}$

$a = 0.16 \pm 0.22 \text{ fm} \leq \text{ unitary limit} $
Numerical Setup at (almost) physical mass

2+1 flavor gauge configurations
- Iwasaki gauge action & O(a) improved Wilson quark action
- $a = 0.0846$ [fm], $a^{-1} = 2333$ [MeV]
- $96^3 \times 96$ lattice, $L = 8.1$ [fm]
- 400 confs x 48 source positions x 4 rotations

Wall source is employed. Only S-wave state is produced.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass [MeV]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>146</td>
<td>8</td>
</tr>
<tr>
<td>$K$</td>
<td>525</td>
<td>6</td>
</tr>
<tr>
<td>$N$</td>
<td>964</td>
<td>3</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>1712</td>
<td>2</td>
</tr>
</tbody>
</table>

![Graph](image)
\(\Omega\Omega\) in \(J=0\)

Nf=2+1 full QCD with \(L = 8.1\text{fm}, m_\pi = 146\text{MeV}\)

\[a_0^{(\Omega\Omega)} = 4.6(6)(+1.2\text{fm}),\]
\[r_{\text{eff}}^{(\Omega\Omega)} = 1.27(3)(+0.06\text{fm}).\]

- Short range repulsive core and attractive pocket are found
- Phase shift shows the presence of a bound state
- The state is very close to the unitary region \((r/a<1)\)

SG and K. Sasaki et.al.(HAL), PRL(2018)

“most strange dibaryon”
$$\Omega \Omega \text{ in } J = 0$$

**Binding energy and the Coulomb effect**

“most strange dibaryon”

Q = -1

\[
\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\text{LQCD}}(r) + \frac{\alpha}{r}
\]

\[
\begin{align*}
\langle B_{\Omega\Omega}^{(\text{QCD})}, B_{\Omega\Omega}^{(\text{QCD+Coulomb})} \rangle &= (1.6(6) \text{MeV}, 0.7(5) \text{MeV})
\end{align*}
\]
Conservative estimate at exact phys. pt.

$m_\pi = 146 \text{ MeV} \rightarrow 135 \text{ MeV}, \quad m_\Omega = 1712 \text{ MeV} \rightarrow 1672 \text{ MeV}$

**Figure Description:**
- Graph showing the behavior of $V(r)$ as a function of $r$.
- The graph indicates changes in the attractive pockets, becoming deeper.
- This results in an increase in B.E.

**Equations:**
- $H = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\text{LQCD}}(r)$
- Conservative estimate:
  - Only change the mass of kinetic term
  - $(B_{\Omega\Omega}^{(\text{QCD})}, B_{\Omega\Omega}^{(\text{QCD+Coulomb})}) = (1.6(6) \text{ MeV}, 0.7(5) \text{ MeV})$
  - $\rightarrow (1.3(5) \text{ MeV}, 0.5(5) \text{ MeV})$
- These changes are within errors.
Measuring Pair Correlation
→ Constrain Pairwise Interaction

\[ C_{AB}(Q) = \frac{N_{AB}^{\text{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \text{No Correlation} \\ \text{others} & \text{Interaction, Interference etc} \end{cases} \]

Deviation from “1”, tells us the behavior of the interaction.
**ΩΩ Correlation@LHC**

**The Small-Large Ratio $C_{SL}(Q)$**

Response to system size change

$$C_{SL}(Q) = \frac{C_R(Q)}{C_{R'}(Q)}$$

QS (HBT) Correlation suppresses the ratio

Nevertheless FSI dominates at low $Q$
To have 100 pairs at low Q:

Acceptance $\times$ Efficiency : 0.01

$10^{11}$ events : unreachable at LHC

Not impossible in Future J-PARC ? (int. rate $10^8$ Hz)
Summary

- We have investigated \( \Omega \Omega \) interaction \((J=0)\) from lattice QCD

- (almost) physical pion masses:
  \( \Omega \Omega \) interaction in \( ^1S_0 \)
  - short range repulsive and attractive pocket
  - a very shallow bound state

[Most strange dibaryon, di-Omega]

Dibaryon

Deuteron + di-Omega\((\Omega \Omega)\)

found in 1930s

will be found by J-PARC or FAIR?
Back Slides
Estimate NLO contribution for ΩΩ system at almost physical pt.

\[ U(r, r') = V_0(r)\delta(r - r') + \sum_{n=1} V_{2n}(r)\nabla^{2n}\delta(r - r') \]

- Determining the higher order potentials explicitly by utilizing multiple quark sources is the best way to estimate their contributions.
  
  \[ V(r) = R^{-1}(r, t) \left( \nabla^2_{m_\Omega} - \frac{\partial}{\partial t} + \frac{1}{4m_\Omega} \frac{\partial^2}{\partial t^2} \right) R(r, t) \]
  
  \[ = V_0(r) + \sum_{n=1} V_{2n}(r)R^{-1}(r, t)\nabla^{2n}R(r, t) \]

- Instead, we have estimated in two alternative ways:
  1. their contributions are estimated from its t-dependence
  2. pertubative estimate on the binding energy

  \[ |V_2/m_{2\pi}| \sim |V_0| + \text{several functional forms such as square-well form...} \]

  => B.E. changes less than 20% in all cases

  (within systematic errors from the t-dependence)