Model-Independent Constraints on $R_{J/\Psi}$

18xx.xxxxx
Within the Standard Model, *lepton universality* is broken only by the Higgs interaction.
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...but $m_\nu$ implies this isn’t the end of the story
...so let’s do some precision physics!

$$B_c^+ \rightarrow \bar{b}\bar{c}\bar{\nu} = L\mu H\mu q^2 + M_2 W + O(\alpha_{em}, GF) \equiv R(hb \rightarrow h\tau\bar{\nu}\tau) \equiv B(hb \rightarrow h\ell\bar{\nu}\ell) = ??$$
...so let’s do some precision physics!

$$M_{b \rightarrow cl\bar{\nu}} = \frac{L^\mu H_\mu}{q^2 + M_W^2} + O(\alpha_{em}, G_F)$$
...so let’s do some precision physics!

\[
M_{\bar{b} \to \bar{c} l \bar{\nu}} = \frac{L^\mu H_\mu}{q^2 + M_W^2} + O(\alpha_{em}, G_F)
\]

\[
R(h_b \to h_c) \equiv \frac{B(h_b \to h_c \tau \bar{\nu}_\tau)}{B(h_b \to h_c l \bar{\nu}_l)} = ???
\]
Ratios of semileptonic $b$–quark decays, they persisted...

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<tr>
<th>Ratio</th>
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<th>$R_{theory}$</th>
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<td>0.641(17)</td>
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\[1\] Aaij:2017tyk.

Constraints on $R_{J/\Psi}$
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Call it... er... $0-0.55$

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Constraints on $R_{J/\Psi}$
Only model-dependent predictions exist

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Taking the largest/smallest $B(B^+ \rightarrow J/\psi \tau^+ \bar{\nu}_\tau)$ and compute a worst-case scenario $R_{J/\psi} = [0, 3]$. 
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Taking the largest/smallest $\mathcal{B}(B_c^+ \rightarrow J/\psi \tau^+ \bar{\nu}_\tau)$ and $\mathcal{B}(B_c^+ \rightarrow J/\psi l^+ \bar{\nu}_l)$ and compute a **worst-case** scenario $R_{J/\psi} = [0, 3]$
What’s the worst that can happen?

The structure of the **Standard Model** puts **restrictions** on how the hadronic matrix element can vary.
What’s the worst that can happen?

The structure of the Standard Model puts restrictions on how the hadronic matrix element can vary

\[
\langle V(p', \epsilon) | V^\mu - A^\mu | P(p) \rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M + m} \epsilon^*_\nu p'_\rho p_\sigma V(q^2) - (M + m)\epsilon^{*\mu} A_1(q^2) \\
+ \frac{\epsilon^* \cdot q}{M + m} (p + p')^\mu A_2(q^2) + 2m \frac{\epsilon^* \cdot q}{q^2} q^\mu A_3(q^2) - 2m \frac{\epsilon^* \cdot q}{q^2} q^\mu A_0(q^2) \tag{1}
\]

\[
A_3(q^2) = \frac{M + m}{2m} A_1(q^2) - \frac{M - m}{2m} A_2(q^2) \tag{2}
\]

where \( A_3(0) = A_0(0) \) and the masses are given by \( M = m_P, m = m_V \).
Any problems?

Lattice data for \( V(q^2) \), \( A_1(q^2) \) aren't wildly off.

Semi-positive definiteness of form factor:
\[
F_i(q^2_{\text{max}}), F_i(0) \geq 0
\]

Upper limit from state overlap:
\[
F_i(q^2_{\text{max}}), F_i(0) \leq N \Gamma(M,m) \times 1
\]

Coefficient bounds from dispersive relations:
\[
\sum_{i,n=0} a_{2in} \leq 1
\]

Strict prediction would require additional assumptions about priors, but min/max values are independent of this.
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So what can the Standard Model allow?

![Histogram with labeled bounds]

95% CL Upper and Lower Bounds on $R_{J/\psi}$

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<thead>
<tr>
<th>% flat</th>
<th>$n = 1$</th>
<th>$n = 2$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>[0.257, 0.314]</td>
<td>[0.2495, 0.3256]</td>
</tr>
<tr>
<td>5</td>
<td>[0.252, 0.317]</td>
<td>[0.2442, 0.3294]</td>
</tr>
<tr>
<td>20</td>
<td>[0.229, 0.333]</td>
<td><strong>[0.2191, 0.345]</strong></td>
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n > 2 unlikely to strongly affect bound, because $\frac{a_{n+1}}{a_n} \geq z_{\text{max}} = 0.027$ and $\sum a_{ni}^2 \leq 1$ heavily penalize larger n
Updated $R_{J/\psi}$ Plot
Constraints on $R_{J/\psi}$

29 May, 2018
Lattice NRQCD results provide limited input\(^2\)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Graph showing the functions \(V(q^2)\) and \(A_1(q^2)\) as a function of \(q^2\) in GeV\(^2\).}
\end{figure}

2+1+1 HISQ, \(a = 0.09\) fm, \(m_s/m_l \approx 5\) from MILC with NRQCD for \(b\)

\(^2\)Colquhoun:2016osw.
Let’s talk about analytic structure

Consider a

\[ J \equiv \bar{c} \Gamma \mu b \]

The Green's function, \( \Pi_{\mu \nu} J \), is split into spin-1 (\( \Pi_T J \)) and spin-0 (\( \Pi_L J \)) and (after subtractions) give

\[ \chi_L J (q^2) \equiv \partial \Pi_L J / \partial q^2 = 1 / \pi \int_0^\infty dt \text{Im} \Pi_L J (t) (t - q^2)^2 \]

where \( \text{Im} \Pi_{T,L} J (q^2) = 1 / 2 \sum \sigma_4 \delta_4 (q - p \sigma_2) |\langle 0 | J | \sigma_2 \rangle|^2 \) are spectral functions.

We need \( \chi_{L,T} J (q^2) \) computable in pQCD at \( q^2 = 0 \).
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$$\chi^L_J(q^2) \equiv \frac{\partial \Pi^L_J}{\partial q^2} = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \, \Pi^L_J(t)}{(t - q^2)^2}$$

(3)

where $\text{Im} \, \Pi^{T,L}_J(q^2) = \frac{1}{2} \sum_X (2\pi)^4 \delta^4(q - p_X) |\langle 0 |J|X \rangle|^2$ are spectral functions.
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We need $\chi^{L,T}_J(q^2)$ computable in pQCD at $q^2 = 0$.
Mapping $t \rightarrow z$

Use a **conformal** variable transformation
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$$z(t; t_0) \equiv \frac{\sqrt{t_{bc} - t} - \sqrt{t_{bc} - t_0}}{\sqrt{t_{bc} - t} + \sqrt{t_{bc} - t_0}},$$  \hspace{1cm} (4)$$

$t_{bc}$ is production threshold of lightest states in channel, $BD(*)$, $t_0$ defined to improve convergence. $z$ is real for $t \leq t_{bc}$ and a pure phase for $t \geq t_{bc}$. 
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Now that it's analytic, so what?

Intuition: Fraction of the $\Pi(t)$ given by subset, implying 1 is a very conservative bound.

Take an expansion around $z \approx 0$ ($z_{\text{max}} = 0.027$).

$F_i(t) = 1 |P_i(t)| \phi_i(t; t_0)$

$\sum_{n=0}^{\infty} a_{2n} z(t; t_0)$, (6)

with the bound now expressed as $\sum_{i; n=0}^{\infty} a_{2n}^2 \leq 1$. (7)

Form factors cannot change arbitrarily fast!
Now that its analytic, so what?

\[ \frac{1}{2\pi i} \sum_{i} \oint_{C} \frac{dz}{z} |\phi_{i}(z) P_{i}(z) F_{i}(z)|^{2} \leq 1, \]
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A rigorous Standard Model bound now exists

With dispersive analysis, lattice data, and physical constraints, a bound on the SM $R_{J/\Psi}$ can be made without any recourse to models.

Improvement in existing lattice form factors, or any information about the remaining two can substantially shrink bounds. Including other channels could reduce bounds, since typical $\sum a_n \approx 1$.

Questions?
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