Two-photon effects in electromagnetic processes on nucleon

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Electroweak probe: precision tool of hadron structure

time honored tool: electroweak probe

how accurate do we know the proton size and its spatial structure?
Proton radius puzzle

\[ R_E = 0.8409 \pm 0.0004 \text{ fm} \]

\[ R_E = 0.8775 \pm 0.0051 \text{ fm} \]

5.6 \sigma difference !?

**μH data:**

- Pohl et al. (2010)
- Antognini et al. (2013)

**ep data:**

- CODATA (2012)
Lamb shift: status of known corrections

μH Lamb shift: summary of corrections

1-loop eVP
proton size
2-loop eVP
μSE and μVP
discrepancy
1-loop eVP in 2 Coul.
recoil
2-photon exchange
hadronic VP
proton SE
3-loop eVP
light-by-light

Two-photon exchange: largest theoretical uncertainty
Lamb shift: hadronic corrections

\[ T^{\mu \nu}(p, q) = \frac{i}{8\pi M} \int d^4x \, e^{iqx} \langle p | T j^\mu(x) j^\nu(0) | p \rangle \]
\[ = \left( -g^{\mu \nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, Q^2) \]

Lower blob contains both elastic (nucleon) and inelastic states

Information contained in **forward, double virtual Compton scattering**

- Described by two amplitudes \( T_1 \) and \( T_2 \): function of energy \( \nu \) and virtuality \( Q^2 \)

- Imaginary parts of \( T_1 \), \( T_2 \): unpolarized structure functions of proton

\[ \Delta E \text{ evaluated through an integral over } Q^2 \text{ and } \nu \]

\[ \Delta E = \Delta E^{el} + \Delta E^{subtr} + \Delta E^{inel} \]

Elastic state: involves **nucleon form factors**

Subtraction: involves **nucleon polarizabilities**

Inelastic, dispersion integrals: involves **structure functions** \( F_1, F_2 \)

**Hadron physics input required**
Lamb shift: subtraction function

Low-energy expansion of forward, doubly virtual Compton scattering contains a subtraction term $T_1(0, Q^2)$

effective Hamiltonian:

$$\mathcal{H} = -\frac{1}{2} 4\pi \alpha_E \vec{E}^2 - \frac{1}{2} 4\pi \beta_M \vec{B}^2$$

electric and magnetic polarizabilities

subtraction term

$$T_{\text{non-Born}}^{1}(0, Q^2) = Q^2 \beta_M + \mathcal{O}(Q^4)$$

Theory analyses:

**BChPT**
Lensky, Pascalutsa (2010)

**HBChPT**
Griesshammer, McGovern, Phillips (2013)

**PDG ’14 values:**

$$\alpha_E = (11.2 \pm 0.2) \times 10^{-4} \text{ fm}^3$$

$$\beta_M = (2.5 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

**HBChPT**
Birse, McGovern (2012)

**BChPT**
Birse, McGovern (2012)

Empirical result (based on HERA data)
Tomalak, Vdh (2016)
Lamb shift: hadronic corrections summary

Polarizability correction on 2S level in μH in μeV

<table>
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<tr>
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<tbody>
<tr>
<td>ΔE^{(subt)}_{2S}</td>
<td>1.8</td>
<td>-2.3</td>
<td>-</td>
<td>5.3 (1.9)</td>
<td>4.2 (1.0)</td>
<td>-2.3 (4.6)^a</td>
<td>-3.0</td>
</tr>
<tr>
<td>ΔE^{(inel)}_{2S}</td>
<td>-13.9</td>
<td>-13.8</td>
<td>-</td>
<td>-12.7 (5)</td>
<td>-12.7 (5)^b</td>
<td>-13.0 (6)</td>
<td>-5.2</td>
</tr>
<tr>
<td>ΔE^{(pol)}_{2S}</td>
<td>-12 (2)</td>
<td>-11.5</td>
<td>-18.5</td>
<td>-7.4 (2.4)</td>
<td>-8.5 (1.1)</td>
<td>-15.3 (5.6)</td>
<td>-8.2(1.2)^{+1.2}_{-2.3}</td>
</tr>
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^a Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the ‘elastic’ and ‘polarizability’ contributions

^b Taken from Ref. [12]


Elastic contribution on 2S level: ΔE_{2S} = -23 μeV


Total hadronic correction on Lamb shift

ΔE_{TPE}(2P - 2S) = (33 ± 2) μeV

...or about 10% of needed correction
Two-photon exchange in lepton-nucleon scattering
e-p scattering: unpolarized cross sections

\[ G_M = F_1 + F_2 \]
\[ G_E = F_1 - \tau F_2 \]

\[ \tau \equiv \frac{Q^2}{4M^2} \]
\[ \frac{1}{\varepsilon} \equiv 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \]

\[ \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 \]

Rosenbluth separation technique

Andivahis et al. (1994)

Bernauer et al. (2010, 2013)
e⁻p scattering: double polarization

\[ \vec{e} + p \rightarrow e + \vec{p} \]

Akhiezer, Rekalo (1974)

\[ d\sigma_{pol} = d\sigma_{unpol}(1 + h S_x P_t + h S_z P_t) \]

\[
\begin{align*}
P_t &= -\sqrt{\frac{2\varepsilon(1 - \varepsilon)}{\tau} \frac{G_E G_M}{\sigma_R}} \\
P_l &= \sqrt{1 - \varepsilon^2} \frac{G_M^2}{\tau \sigma_R}
\end{align*}
\]

\[
\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1 + \varepsilon)} \frac{G_E}{G_M}}
\]
Rosenbluth vs polarization transfer measurements of $G_E/G_M$ of proton

Two methods: two different results
most likely: $2\gamma$-exchange correction
2γ-exchange in e⁻ scattering: general

\[
P = \frac{p + p'}{2}, \quad K = \frac{k + k'}{2}
\]

\[
te(k, h) \leftrightarrow e(k', h')
\]

\[
N(p, \lambda) \leftrightarrow N(p', \lambda')
\]

\[
t = (k - k')^2, \quad u = (k - p')^2, \quad s = (p + k)^2, \quad \nu = \frac{s - u}{4}
\]

discrete symmetries \[\rightarrow\] \(m_e = 0\) \[\rightarrow\] 3 structure amplitudes

\[
T = \frac{e^2}{Q^2} \bar{e}(k', h') \gamma_\mu e(k, h).\bar{N}(p', \lambda') \left[ G_M(\nu, t) \gamma^\mu - F_2(\nu, t) \frac{P^\mu}{M} + F_3(\nu, t) \frac{K P^\mu}{M^2} \right] N(p, \lambda)
\]


Leading contribution to cross section - interference term

1 photon diagram

\[
\delta_{TPE} \sim \mathcal{R}G_M, \mathcal{R}F_2, \mathcal{R}F_3
\]

2 photon exchange diagram
for real part:

3 independent observables

\[ \sigma_R \frac{G_M^2}{G_M^2} = 1 + \frac{\varepsilon}{\tau G_M^2} Y_{2\gamma}^M + 2 \varepsilon \frac{G_E}{\tau G_M} Y_{2\gamma}^E + 2 \varepsilon \left(1 + \frac{G_E}{\tau G_M}\right) Y_{2\gamma}^3 + O(\varepsilon^4) \]

\[ -\sqrt{\frac{\tau (1 + \varepsilon)}{2 \varepsilon}} \frac{P_l}{P_l} = \frac{G_E}{G_M} \]

\[ + Y_{2\gamma}^E - \frac{G_E}{G_M} Y_{2\gamma}^M + \left(1 - \frac{2 \varepsilon}{1 + \varepsilon G_M}\right) Y_{2\gamma}^3 + O(\varepsilon^4) \]

\[ \frac{P_l}{P_l^{Born}} = 1 \]

\[ -2 \varepsilon \left(1 + \frac{\varepsilon}{\tau G_M^2}\right)^{-1} \left\{ \varepsilon \left[1 + \frac{1 - G_E^2}{\tau G_M^2} + \frac{G_E}{\tau G_M}\right] Y_{2\gamma}^3 \right. \]

\[ + O(\varepsilon^4) \]

\[ Y_{2\gamma}^M(\nu, Q^2) = R \left( \frac{\delta G_M}{G_M} \right) \]

\[ Y_{2\gamma}^E(\nu, Q^2) = R \left( \frac{\delta G_E}{G_M} \right) \]

\[ Y_{2\gamma}^3(\nu, Q^2) = \frac{\nu}{M^2} R \left( \frac{\delta G_M}{G_M} \right) \]

\[ \tilde{G}_M(\nu, Q^2) = G_M(\nu, Q^2) + \delta \tilde{G}_M \]

\[ \tilde{F}_2(\nu, Q^2) = F_2(\nu, Q^2) + \delta \tilde{F}_2 \]

\[ \tilde{F}_3(\nu, Q^2) = 0 + \delta \tilde{F}_3 \]

\[ \tilde{G}_E \equiv \tilde{G}_M - (1 + \tau) \tilde{F}_2 \]

\[ \tilde{G}_E(\nu, Q^2) = G_E(\nu, Q^2) + \delta \tilde{G}_E \]
extraction of $2\gamma$-amplitudes: data

Rosenbluth data: JLab (Hall A)

$Q^2 = 2.64 \text{ GeV}^2$

Polarization data: JLab (Hall C)

$Q^2 = 2.5 \text{ GeV}^2$

Qattan et al. (2005)

Meziane et al. (2011)
extraction of $2\gamma$-amplitudes: fit

$Q^2 = 2.64$ GeV$^2$

extracted $2\gamma$ amplitudes are in the (expected) 2-3 % range
status of $2\gamma$-exchange corrections

- Tsai (1961), Mo & Tsai (1968)
  box diagram calculated using only nucleon intermediate state
  and using $q_1 \approx 0$ or $q_2 \approx 0$ in both numerator and denominator
  -> gives correct IR divergent terms

- Maximon & Tjon (2000)
  same as above but make above approximation only in the numerator (calculate 4-point fct)
  + use on-shell nucleon form factors in loop integral

- Blunden, Melnitchouk, Tjan (2003), Kondratyuk & Blunden (2007)
  further improvement by keeping full numerator, insert higher resonances

  same as previous from dispersive approach

- Bystritsky, Kuraev, Tomasi-Gustaffson (2006)
  assumption that dominant region comes from $q_1 \approx q_2 \approx q/2$ (obtained TPE is very small)
non-forward scattering proton state

\[ l \rightarrow \gamma \rightarrow p \]

\[ l' \rightarrow \gamma \rightarrow p' \]

Dirac and Pauli form factors

box diagram

assumption about the vertex

Blunden, Melnitchouk, Tjon (2003)

dispersion relations

based on on-shell information

Borisyuk, Kobushkin (2008)
Tomalak, Vdh (2014)
forward scattering

near-forward scattering
account for all inelastic $2\gamma$
2\gamma-exchange at low $Q^2$

2\gamma blob: near-forward virtual Compton scattering

\[ \delta_{2\gamma} \sim a \sqrt{Q^2} + b Q^2 \ln Q^2 + c Q^2 \ln^2 Q^2 \]

\[ \delta_{2\gamma} = \int d\nu_\gamma d\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2)) \]

\[ Q^2 = 0.05 \text{ GeV}^2 \]

\[ Q^2 = 0.25 \text{ GeV}^2 \]

2\gamma at large $\varepsilon$ agrees with empirical fit

Feshbach inelastic elastic

McKinley, Feshbach (1948)
M. Gorchtein (2013)

Tomalak, Vdh (2016)

unpolarized proton structure

\[ r_E \text{ extraction } \checkmark \]
near-forward scattering
(large $\varepsilon$)

$p + \text{all inelastic}$

non-forward scattering dispersion relations
(arbitrary $\varepsilon$)

$X = p + \pi N$
Mandelstam plane: ep scattering @ low $Q^2$

$s = (M + m_\pi)^2$

$s = M^2$

elastic threshold  inelastic threshold

unitarity relations predictive in physical region

proton intermediate state is outside physical region for $Q^2 > 0$

$\pi N$ intermediate state is outside physical region for $Q^2 > 0.064\,\text{GeV}^2$
analytical continuation: proton state

contour deformation method:

$$\int d\Omega \rightarrow \text{angular integration to integration on curve in complex plane} \rightarrow \text{deform integration contour keeping poles inside going to unphysical region}$$

analytical continuation reproduces results in unphysical region

$$Q^2 = 0.1 \text{ GeV}^2$$

Tomalak, Vdh (2014)
Blunden, Melnitchouk (2017)
πN state contribution in dispersive framework

πN is dominant inelastic 2γ

$Q^2 = 0.005 \text{ GeV}^2$

Tomalak, Pasquini, Vdh (2017)

$Q^2 = 0.05 \text{ GeV}^2$

dispersion relations agree with near-forward at large $\varepsilon$
Mandelstam plane: ep scattering @ larger $Q^2$

\[ s = (M + m_\pi)^2 \]

$\nu$, GeV$^2$

$Q^2$, GeV$^2$

\[ s = 1.6 \text{ GeV}^2 \]

\[ s = (M + m_\pi)^2 \]

unitarity relations predictive in physical region

\[ \pi N \text{ intermediate state is outside physical region for } Q^2 > 0.064 \text{ GeV}^2 \]
analytical continuation: $\pi N$ states

- pion electroproduction amplitudes: MAID2007
- analytical continuation: fit of low-$Q^2$ expansion in physical region

$$G_{1,2}(s, Q^2), \quad Q^2 F_3(s, Q^2) \sim a_1 Q^2 \ln Q^2 + a_2 Q^2 + a_3 Q^4 \ln Q^2 + \ldots$$

Tomalak, Pasquini, Vdh (2017)
2γ-exchange: comparison with data

\[ R_{2\gamma} = \frac{\sigma(e^+ p)}{\sigma(e^- p)} \approx 1 - 2\delta_{2\gamma} \]

- OLYMPUS (2016)
  - elastic
  - elastic + Δ
  - elastic + πN

k = 2.01 GeV
uncorr. + corr. uncertainties
Maximon and Tjon IR prescription

weighted Δ is similar to narrow one of Blunden et al. (2017)
πN contribution is closer to data than Δ only

Tomalak, Pasquini, Vdh (2017)
$R_{2\gamma} = \frac{\sigma(e^+ p)}{\sigma(e^- p)} \approx 1 - 2\delta_{2\gamma}$

$R_{2\gamma}$ agree with data

multi-particle $2\chi$, e.g. $\pi\pi N$, is important

near-forward $2\chi$ agree with data

Tomalak, Pasquini, Vdh (2017)
2γ-exchange: comparison with data

TPE calculation agrees with CLAS data
our best $2\gamma$-exchange amplitudes at low $Q^2$

small $Q^2$: near-forward at large $\varepsilon$, all inelastic states
$Q^2 \leq 1$ GeV$^2$: elastic+$\pi$N within dispersion relations
intermediate range: interpolation
**μp scattering**

**MUSE@PSI** (2018/19), simultaneous measurement of e⁻/e⁺ and μ⁻/μ⁺ scattering on proton beam momenta 115, 153, 210 MeV/c: \( R_E \) difference to 0.005 fm, determination of TPE effects

**talk P. Reimer** (Fri, 5:30pm)

μ⁺ scattering @**COMPASS** (M2 beam line), first test 2018, aim: \( R_E \) to 0.01 fm

e⁻e⁺ vs μ⁻μ⁺ photoproduction @**MAMI** (LOI): test lepton universality test through ratio measurement
$\mu^-$ scattering: 2γ-exchange correction

\[
T^{\text{non-flip}} = \frac{e^2}{Q^2} \tilde{l}(k', h') \gamma_{\mu} l(k, h) \tilde{N}(p', \lambda')[G_M(\nu, t) \gamma^\mu - F_2(\nu, t) \frac{P^\mu}{M} + F_3(\nu, t) \frac{\hat{K} P^\mu}{M^2}]N(p, \lambda)
\]

$\delta_{2\gamma} = \frac{2}{G_M^2 + \frac{\nu}{\tau} G_E^2} \left\{ G_M \mathcal{R} G_1 + \frac{\epsilon}{\tau} G_E \mathcal{R} G_2 + \frac{1 - \epsilon}{1 - \epsilon_0} \left( \frac{\epsilon_0}{\tau} G_E \mathcal{R} G_4 - G_M \mathcal{R} G_3 \right) \right\}$


\[
\epsilon = \frac{16 \nu^2 - Q^2(Q^2 + 4M^2)}{16 \nu^2 - Q^2(Q^2 + 4M^2) + 2(Q^2 + 4M^2)(Q^2 - 2m_l^2)}
\]

$\epsilon_0 = \frac{2m_l^2}{Q^2}$

Tomalak, Vdh (2014)
μ⁻p experiment (MUSE) estimates

proton box diagram model

+ inelastic $2\gamma$ (near forward structure function calculation)

In MUSE kinematics: small inelastic $2\gamma$ $\rightarrow$ small $2\gamma$ uncertainty

Tomalak, Vdh (2014, 2016)
spin-off: TPE in normal spin asymmetries

Beam or target normal spin asymmetries:

\[
A_n \sim \alpha_{em} \sim 10^{-2}
\]

\[
B_n \sim \alpha_{em} \frac{m_e}{E_e} \sim 10^{-6} - 10^{-5}
\]

\[B_n\text{ for } ep \rightarrow e\Delta\text{ accesses } \Delta \text{ e.m. FFs}\]

Carlson, Pasquini, Pauk, Vdh (2017)

Results for QWeak kinematics:
- Nucleon = dash-dot red line
- \(\Delta\) = dashed blue line
- \(S_{11} + D_{13}\) = dotted purple line
- Total = solid black line
Summary and outlook

- **$2\gamma$-exchange corrections** are the largest **hadronic corrections** to Lamb shift in **muonic atoms**
  - $\mu H$: present TPE accuracy (2 $\mu$eV) is comparable with present Lamb shift accuracy (2.3 $\mu$eV)
  - $\mu D$, $\mu^3$He+: present TPE accuracy (15 $\mu$eV) is 5 times worse than Lamb shift accuracy (3.4 $\mu$eV)
  - crucial input for hyperfine splitting experiments ($G_M$, polarized structure functions)

- **Electron scattering** has reached level of precision where TPE effects are clearly visible
  - cross section (Rosenbluth separation) vs $\Pt/\Pt$
  - epsilon dependence of $\Pt$ (needed for a full separation of TPE effects)
  - beam and target normal spin asymmetries (are zero in absence of TPE)
  - $e^-/e^+$ cross section ratios
  - high precision data forthcoming: PRad@JLab, MAGIX@MESA

- **Muon scattering** experiments started/planned: MUSE@PSI will quantify TPE

- **Theoretical understanding**: dispersive calculations based on empirical input
  - Low $Q^2$: good quantitative understanding emerging
    - may be used to provide an improved extraction of $R_M$
  - Larger $Q^2$: a quantitative understanding is still a challenge

- TPE effects in normal spin asymmetries -> spin offs: tool to access $\Delta$ e.m. FFs