Achievements and Open Issues in the Determination of Fragmentation Functions

Thirteenth Conference on the Intersections of Particle and Nuclear Physics

Emanuele R. Nocera

School of Physics and Astronomy - University of Edinburgh

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Outline

1 Introduction and open issues
   ▶ Hadrons in the final state, factorisation and evolution
   ▶ Three examples on why we should care about fragmentation functions

2 Achievements
   ▶ Data: global fits of $\pi^\pm$, $K^\pm$, $h^\pm$ and $D^*$ fragmentation functions
   ▶ Theory: impact of GM-VFN scheme, NNLO corrections, small-$z$ resummation
   ▶ Methodology: simultaneous fits of unpolarised/polarised PDFs and FFs

3 Conclusions

DISCLAIMER

I will focus on collinear fragmentation functions only

Emphasis on recent achievements and on topics which I’ve worked on recently

Apologies in advance for not discussing your favourite subject

For an extensive review of topics not addressed in this talk, please see

1. Introduction and open issues

\[ e^+ + e^- \rightarrow h + X \]

single-inclusive annihilation (SIA)

\[ \ell + N \rightarrow \ell' + h + X \]

semi-inclusive deep-inelastic scattering (SIDIS)

\[ N_1 + N_2 \rightarrow h + X \]

high-\(p_T\) hadron production in \(pp\) collisions (PP)

\[ \frac{d\sigma^h}{dz} = F_T^h(z, Q^2) + F_L^h(z, Q^2) = F_2^h(x, Q^2) \]

\[ F_{k=T,L,2}^h = \frac{4\pi\alpha_s^2}{Q^2} \langle e^2 \rangle \left\{ D^h_{\Sigma} \otimes C^S_{k,q} + n_f D^h_{g} \otimes C^S_{k,g} + D^h_{NS} \otimes C^NS_{k,q} \right\} \]


\[ \frac{d\sigma^h}{dx dy dz} = \frac{2\pi\alpha_s^2}{Q^2} \left[ \frac{1+(1-y)^2}{y} 2F_1^h + \frac{2(1-y)}{y} F_L^h \right] \]

2\(F_1^h = e_q^2 \left\{ q \otimes D^h_q + \frac{\alpha_s}{2\pi} \left[ q \otimes C^1_{qq} \otimes D^h_q + q \otimes C^1_{gq} \otimes D^h_g + g \otimes C^1_{qq} \otimes D^h_q \right] \right\} \]

\[ F_L^h = \frac{\alpha_s}{2\pi} \sum_{q,\bar{q}} e_q^2 \left[ q \otimes C^L_{qq} \otimes D^h_q + q \otimes C^L_{gq} \otimes D^h_g + g \otimes C^L_{qq} \otimes D^h_q \right] \]

up to NLO [NPB 160 (1979) 301; PRD 57 (1998) 5811]

partial NNLO [PRD 95 (2017) 034027]

\[ E_h \frac{d^3\sigma}{dp_{T,h}^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab} \otimes D^h_c \]

\[ \sum_{i,j,k} \int \frac{dx_a}{x_a} \int \frac{dx_b}{x_b} \int \frac{dz}{z^2} f_i/p_a(x_a) f_j/p_b(x_b) D^{h/k}(z) \hat{\sigma}^{ij} \delta(\hat{s} + \hat{t} + \hat{u}) \]

Evolution of FFs: DGLAP equations \[\text{[NPB 126 (1977) 298]}\]

A set of \((2n_f + 1)\) integro-differential equations (\(n_f=\)number of active flavours)

\[
\frac{\partial}{\partial \ln \mu^2} D_i(x, \mu^2) = \sum_{j}^{n_f} \int_{x}^{1} \frac{dz}{z} P_{ji} (z, \alpha_s(\mu^2)) D_j \left( \frac{x}{z}, \mu^2 \right)
\]

\[
P^{(0)}_{qq} \rightarrow P^{(0)}_{gq} \rightarrow P^{(0)}_{qg} \rightarrow P^{(0)}_{gg}
\]


Must be careful with fixed-order splitting functions as \(z \rightarrow 0\) (\(m = 1, \ldots, 2k + 1\))

**SPACE-LIKE CASE**

\[
P_{ji} \propto \frac{\alpha_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}
\]

**TIME-LIKE CASE**

\[
P_{ji} \propto \frac{\alpha_s^{k+1}}{z} \log^{2(k+1)-m-1} z
\]

Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as \(z\) decreases, the unresummed SGLs spoil the convergence of the FO series for \(P(z, \alpha_s)\) if \(\log \frac{1}{z} \geq O \left( \alpha_s^{-1/2} \right)\)
Example 1: The strange (polarised) parton distribution and SIDIS

If SIDIS data is used to determine $\Delta s$, $K^\pm$ FFs for different sets lead to different results. Such results may differ significantly among them and w.r.t. the results obtained from DIS.

$\rightarrow$ How well do we know kaon FFs?

Can SIDIS data be used to determine $s$?
What is the bias induced by FFs onto PDFs?

$\rightarrow$ How well do we know kaon FFs?
Fragmentation functions: why should we bother?

Example 2: Heavy quark fragmentation: the $D^*$ case
Constrain the low-$x$ (gluon) PDFs through charm production in the forward region
[EPJ C75 (2015) 396; JHEP 1602 (2016) 130]

Compute the prompt atmospheric neutrino flux

Extract information on the medium in heavy ion collisions
[JHEP 1703 (2017) 146]
Fragmentation functions: why should we bother?

Example 3: Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties.

How well do we know the gluon FF?
2. Achievements
**Available fragmentation function sets (status 2018)**

<table>
<thead>
<tr>
<th>DATA</th>
<th>DEHSS</th>
<th>HKNS</th>
<th>JAM</th>
<th>NNFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIA</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>SIDIS</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>PP</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
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</table>

<table>
<thead>
<tr>
<th>METH.</th>
<th>DEHSS</th>
<th>HKNS</th>
<th>JAM</th>
<th>NNFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistical treatment</td>
<td>Iterative Hessian</td>
<td>Hessian</td>
<td>Monte Carlo</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>68% - 90%</td>
<td>$\Delta \chi^2 = 15.94$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parametrisation</td>
<td>standard</td>
<td>standard</td>
<td>standard</td>
<td>neural network</td>
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</table>

<table>
<thead>
<tr>
<th>THEORY</th>
<th>DEHSS</th>
<th>HKNS</th>
<th>JAM</th>
<th>NNFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>pert. order</td>
<td>(N)NLO</td>
<td>NLO</td>
<td>NLO</td>
<td>LO, NLO, NNLO</td>
</tr>
<tr>
<td>HF scheme</td>
<td>ZM(GM)-VFN</td>
<td>ZM-VFN</td>
<td>ZM-VFN</td>
<td>ZM-VFN</td>
</tr>
<tr>
<td>hadron species</td>
<td>$\pi^\pm, K^\pm, p/\bar{p}, h^\pm$</td>
<td>$\pi^\pm, K^\pm, p/\bar{p}$</td>
<td>$\pi^\pm, K^\pm$</td>
<td>$\pi^\pm, K^\pm, p/\bar{p}$</td>
</tr>
</tbody>
</table>

- many others (including analyses for specific hadrons)

**Focus on $\pi$ and $K$ which constitute the largest fraction in measured yields**

- **BKK96** [PRD 53 (1996) 3553]
- **DSV97** [PRD 57 (1998) 5811]
- **BFGW00** [EPJ C19 (2001) 89]
- **K**
- **$\Lambda^0$**
- **$h^\pm$**
- **AESS11** [PRD 83 (2011) 034002]
- **AKSRV17** [PRD 96 (2017) 034028]
- **LSS15** [PRD 96 (2016) 074026]
- **$\eta$**
- **$D^*$**

Focus on $\pi$ and $K$ which constitute the largest fraction in measured yields.

Emanuele R. Nocera (Edinburgh) | FFs: achievements and open issues | 1st June 2018
Comparison at NLO (pions): NNFF1.0 - JAM - DEHSS

Differences due to data set, kinematic cuts and fitting methodology

Larger NNFF1.0 uncertainties where less or no data (flexibility of NN parametrisation)

Expect larger uncertainty on $D_{g}^{\pi\pm}$ than $D_{\Sigma}^{\pi\pm}$ visible in NNFF1.0, but not in DEHSS (bound from $pp$ data?) nor in JAM (functional form?)
Global fit of pion fragmentation functions [PRD 91 (2015) 014035]

**Table:**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Data Type</th>
<th>Norm.</th>
<th># Data</th>
<th>χ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPC [48]</td>
<td>incl.</td>
<td>1.043</td>
<td>17</td>
<td>17.3</td>
</tr>
<tr>
<td></td>
<td>uds tag</td>
<td>1.043</td>
<td>9</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>c tag</td>
<td>1.043</td>
<td>9</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>b tag</td>
<td>1.043</td>
<td>9</td>
<td>9.2</td>
</tr>
<tr>
<td>TASSO [49]</td>
<td>34 GeV incl.</td>
<td>1.043</td>
<td>11</td>
<td>30.2</td>
</tr>
<tr>
<td></td>
<td>44 GeV incl.</td>
<td>1.043</td>
<td>7</td>
<td>22.2</td>
</tr>
<tr>
<td>SLD [19]</td>
<td>incl.</td>
<td>0.986</td>
<td>28</td>
<td>15.3</td>
</tr>
<tr>
<td></td>
<td>uds tag</td>
<td>0.986</td>
<td>17</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>c tag</td>
<td>0.986</td>
<td>17</td>
<td>16.1</td>
</tr>
<tr>
<td></td>
<td>b tag</td>
<td>0.986</td>
<td>17</td>
<td>5.8</td>
</tr>
<tr>
<td>ALEPH [16]</td>
<td>incl.</td>
<td>1.020</td>
<td>22</td>
<td>22.9</td>
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<tr>
<td>DELPHI [17]</td>
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<td>17</td>
<td>28.3</td>
</tr>
<tr>
<td></td>
<td>uds tag</td>
<td>1.000</td>
<td>17</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>b tag</td>
<td>1.000</td>
<td>17</td>
<td>10.6</td>
</tr>
<tr>
<td>OPAL [18, 20]</td>
<td>incl.</td>
<td>1.000</td>
<td>21</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>u tag</td>
<td>0.786</td>
<td>5</td>
<td>31.6</td>
</tr>
<tr>
<td></td>
<td>d tag</td>
<td>0.786</td>
<td>5</td>
<td>33.0</td>
</tr>
<tr>
<td></td>
<td>s tag</td>
<td>0.786</td>
<td>5</td>
<td>51.3</td>
</tr>
<tr>
<td></td>
<td>c tag</td>
<td>0.786</td>
<td>5</td>
<td>30.4</td>
</tr>
<tr>
<td></td>
<td>b tag</td>
<td>0.786</td>
<td>5</td>
<td>14.6</td>
</tr>
<tr>
<td>BaBar [28]</td>
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<td>45</td>
<td>46.4</td>
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<tr>
<td>Belle [29]</td>
<td>incl.</td>
<td>1.044</td>
<td>78</td>
<td>44.0</td>
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<tr>
<td>HERMES [30]</td>
<td>π⁺ (p)</td>
<td>0.980</td>
<td>32</td>
<td>27.8</td>
</tr>
<tr>
<td></td>
<td>π⁻ (p)</td>
<td>0.980</td>
<td>32</td>
<td>47.8</td>
</tr>
<tr>
<td></td>
<td>π⁺ (d)</td>
<td>0.981</td>
<td>32</td>
<td>40.3</td>
</tr>
<tr>
<td></td>
<td>π⁻ (d)</td>
<td>0.981</td>
<td>32</td>
<td>59.1</td>
</tr>
<tr>
<td>COMPASS [31]</td>
<td>π⁺ (d)</td>
<td>0.946</td>
<td>199</td>
<td>174.2</td>
</tr>
<tr>
<td></td>
<td>π⁻ (d)</td>
<td>0.946</td>
<td>199</td>
<td>229.0</td>
</tr>
<tr>
<td>PHENIX [21]</td>
<td>π⁰</td>
<td>1.112</td>
<td>15</td>
<td>15.8</td>
</tr>
<tr>
<td>STAR [33, 36]</td>
<td>0 ≤ η ≤ 1</td>
<td>π⁰</td>
<td>1.161</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.954</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALICE [32]</td>
<td>7 TeV π⁰/π⁺</td>
<td>1.006</td>
<td>16</td>
<td>17.2</td>
</tr>
</tbody>
</table>

**Plots:**

- $D_{u+\bar{u}}$: most precise ($B$-factory SIA data)
- Very little or no charge symmetry breaking (SIDIS)
- $D_g$: significant shift of the central value ($pp$ data)
Good flavour separation (SIDS data)

$D_g$: significant shift ($pp$ data)

Caution with mass corrections

$D_{u+\bar{u}}$: most precise ($B$-factory SIA data)
Global fit of unidentified charged hadron FFs [NNPDF, in preparation]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\sqrt{s}$ [TeV]</th>
<th>$N_{\text{dat}}$</th>
<th>$\chi^2_b/N_{\text{dat}}$</th>
<th>$\chi^2_a/N_{\text{dat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ e^-$</td>
<td>various</td>
<td>471 (527)</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>CDF</td>
<td>1.80</td>
<td>7 (49)</td>
<td>2.93</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>1.96</td>
<td>60 (230)</td>
<td>3.45</td>
<td>1.23</td>
</tr>
<tr>
<td>CMS</td>
<td>0.90</td>
<td>10 (20)</td>
<td>3.78</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>2.76</td>
<td>11 (22)</td>
<td>9.31</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>7.00</td>
<td>17 (27)</td>
<td>10.5</td>
<td>0.98</td>
</tr>
<tr>
<td>ALICE</td>
<td>0.90</td>
<td>15 (54)</td>
<td>4.90</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>2.76</td>
<td>21 (60)</td>
<td>11.8</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>7.00</td>
<td>26 (65)</td>
<td>5.21</td>
<td>0.91</td>
</tr>
<tr>
<td>Total data set</td>
<td>638 (1054)</td>
<td></td>
<td>2.18</td>
<td>0.90</td>
</tr>
</tbody>
</table>
First global fit of $D^*$ fragmentation functions

Only $g$, $c$ and $b$ FFs parametrised

Use of ZM-VFN scheme

Kinematic cut $p_T^h > 10 \text{ GeV}$

### Table: Experiment Data

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Data Type</th>
<th># Data in Fit</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH [80]</td>
<td>incl.</td>
<td>17</td>
<td>33.738</td>
</tr>
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<td></td>
<td>incl.</td>
<td>9</td>
<td>6.999</td>
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<td></td>
<td>c tag</td>
<td>9</td>
<td>8.388</td>
</tr>
<tr>
<td></td>
<td>b tag</td>
<td>9</td>
<td>5.342</td>
</tr>
<tr>
<td>ATLAS [94]</td>
<td>incl.</td>
<td>5</td>
<td>3.598</td>
</tr>
<tr>
<td>ALICE [60, 61]</td>
<td>$\sqrt{s} = 7 \text{ TeV}$</td>
<td>3</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{s} = 2.76 \text{ TeV}$</td>
<td>1</td>
<td>0.007</td>
</tr>
<tr>
<td>CDF [62]</td>
<td>incl.</td>
<td>2</td>
<td>1.289</td>
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<td>LHCb [64]</td>
<td>$2 \leq \eta \leq 2.5$</td>
<td>5</td>
<td>10.984</td>
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<tr>
<td></td>
<td>$2.5 \leq \eta \leq 3$</td>
<td>5</td>
<td>2.607</td>
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<tr>
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<td>$3 \leq \eta \leq 3.5$</td>
<td>5</td>
<td>8.229</td>
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<td></td>
<td>$3.5 \leq \eta \leq 4$</td>
<td>2</td>
<td>10.411</td>
</tr>
<tr>
<td>TOTAL:</td>
<td></td>
<td>97</td>
<td>100.980</td>
</tr>
</tbody>
</table>

See also JHEP 1605 (2016) 125 and F. Ringer’s talk
Pion fragmentation functions in the GM-VFNs

charm changes significantly
light flavors constrained by sidis
bottom constrained by high Q

Slide: courtesy of R. Sassot
Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO
Effects less evident for $K^\pm$ and $p/\bar{p}$
436 Total data Points:

- LEP cut \( z = 0.01 \) due to inconsistency between OPAL and ALEPH

- TPC lower cut \( z = 0.02 \) based on difference of energy fraction \( z = 2 \frac{E_h}{Q} \) and three momentum fraction \( x_p = z - 2m_h^2/(zQ^2) + \mathcal{O}(1/Q^4) \) in c.m.s being less than at least 15%

<table>
<thead>
<tr>
<th>accuracy</th>
<th>( \chi^2 )</th>
<th>( \chi^2/dof )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>1260.78</td>
<td>2.89</td>
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<tr>
<td>NLO</td>
<td>354.10</td>
<td>0.81</td>
</tr>
<tr>
<td>NNLO</td>
<td>330.08</td>
<td>0.76</td>
</tr>
<tr>
<td>LO+LL</td>
<td>405.54</td>
<td>0.93</td>
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<tr>
<td>NLO+NNLL</td>
<td>352.28</td>
<td>0.81</td>
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<tr>
<td>NNLO+NNLL</td>
<td>329.96</td>
<td>0.76</td>
</tr>
</tbody>
</table>

\( \mu_0 = 10.54 \text{ GeV} \)
Simultaneous fits of (pol.) PDFs and FFs [PRL 119 (2017) 132001]

\[
x \Delta s^+ \\
\begin{array}{c}
\hline
\text{process} & \text{target} & N_{\text{dat}} & \chi^2 \\
\hline
\text{DIS} & p, d, ^3\text{He} & 854 & 854.8 \\
\text{SIA (} \pi^\pm, K^\pm \text{)} & & 850 & 997.1 \\
\text{SIDIS (} \pi^\pm \text{)} & \text{HERMES} & d & 18 & 28.1 \\
 & & p & 18 & 14.2 \\
 & \text{COMPASS} & d & 20 & 8.0 \\
 & & p & 24 & 18.2 \\
\text{SIDIS (} K^\pm \text{)} & \text{HERMES} & d & 27 & 18.3 \\
 & & d & 20 & 18.7 \\
 & \text{COMPASS} & p & 24 & 12.3 \\
\hline
\text{Total} & & 1855 & 1969.7 \\
\end{array}
\]

\[ g_A = 1.24 \pm 0.04 \quad a_8 = 0.46 \pm 0.21 \]

confirmation of SU(2) symmetry to \( \sim 2\%\)

\( \sim 20\% \) SU(3) breaking \( \pm 20\% \)

\[ \Delta s^+ = -0.03 \pm 0.09 \]

\[ \Delta \Sigma = 0.36 \pm 0.09 \quad \Delta u - \Delta d = 0.05 \pm 0.08 \]
IDEA:
iterative reweighting of PDFs and fit of FFs
with kaon SIDIS data ($N_{\text{dat}} = 906$)

HERMES [PRD 87 (2013) 074029]
COMPASS [PLB 767 (2017) 133]

starting from MMHT14

\[
\chi^2_{FF} = 1271.7 \quad 1041.3 \quad 1002.3
\]

starting from NNPDF3.0

\[
\chi^2_{FF} = 1017.2 \quad 1005.3 \quad 1000.6
\]

similar results with CT14 replicas
3. Conclusions
Summary

1. A number of hard-scattering processes require an appropriate knowledge of FFs
   - probing nucleon momentum, spin and flavour
   - studying the prompt atmospheric neutrino flux
   - understanding spatial distributions and the dynamics of nuclear matter

2. Significant role of new data, including LHC data
   - increased accuracy of fragmentation functions
   - increased precision of fragmentation functions

3. Increasing sophistication of the QCD theory
   - needed to catch most of the features of the data
   - includes NNLO, heavy quark mass schemes, resummation

4. Exploit the full potential of SIDIS to improve our knowledge of PDFs
   - simultaneous fits feasible, but challenging
   - combine simultaneous and global fits to make the most from the data
Summary

1. A number of hard-scattering processes require an appropriate knowledge of FFs
   - probing nucleon momentum, spin and flavour
   - studying the prompt atmospheric neutrino flux
   - understanding spatial distributions and the dynamics of nuclear matter

2. Significant role of new data, including LHC data
   - increased accuracy of fragmentation functions
   - increased precision of fragmentation functions

3. Increasing sophistication of the QCD theory
   - needed to catch most of the features of the data
   - includes NNLO, heavy quark mass schemes, resummation

4. Exploit the full potential of SIDIS to improve our knowledge of PDFs
   - simultaneous fits feasible, but challenging
   - combine simultaneous and global fits to make the most from the data

Thank you
Dependence on $\alpha_s$

$\chi^2_{\alpha_s(M_Z)} = 0.117998 \pm 0.000853$ ($\Delta\chi^2 = 1$)

$e^+e^- \rightarrow \pi^\pm X$

parabolic fit

$\alpha_s(M_Z) = 0.117998 \pm 0.000853$ ($\Delta\chi^2 = 1$)