Nuclear-Matter Equation of State from Chiral Effective Field Theory

Homogeneous nuclear matter

- theoretical testbed for benchmarking nuclear forces
  - saturation point \((n_0, a_v)\)
  - incompressibility \((K)\)
  - symmetry energy \((S_v)\) and its slope \((L)\) at saturation density

- many-body perturbation theory, but also in QMC, CC, SCGF, …

for a recent review see: Hebeler et al., Annu. Rev. Nucl. Part. Sci. 65, 457

Bethe–Weizsäcker formula

\[
\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\text{sym}}(n)
\]
Nuclear matter interacts via the strong interaction (disregard Coulomb)

- QCD is non-perturbative at low energies of interest
- modern approach: chiral EFT
  - relevant degrees of freedom instead of quarks/gluons
  - use nucleons and pions

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  - use nucleons and pions
  - pion exchanges and short-range contact interactions
  - systematic expansion of nuclear forces:
    \[ Q = \max \left( \frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b} \right) \sim \frac{1}{3} \]

Steven Weinberg

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Hierarchy of nuclear forces in chiral EFT

May 29, 2018 | CIPANP 2018: Palm Springs | Christian Drischler | 5

Many-body forces

Expansion

2N force | 3N force | 4N force

\( Q^0 \) \_ LO | | 
\( Q^2 \) \_ NLO | | 
\( Q^3 \) \_ N2LO | \( C_D \) | 
\( Q^4 \) \_ N3LO | \( C_E \) | 

... and ongoing work at \( N^4 \text{LO} \) and even \( N^5 \text{LO} \)...

see talk by E. Epelbaum on Friday

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...
The number of diagrams increases rapidly!

<table>
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<tr>
<th>$n$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>3</td>
<td>39</td>
<td>840</td>
<td>27 300</td>
<td>1 232 280</td>
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Integer sequence A064732:
Number of labeled Hugenholtz diagrams with $n$ nodes.
Significant challenges remain!

Higher orders: particle-hole contributions
Coraggio et al., PRC 89, 044321; Holt, Kaiser, PRC 95, 034326

Approximated normal-ordering
Holt et al., PRC 81, 024002; Hebeler, Schwenk, PRC 82, 014314

Neglected residual 3N diagrams
Hagen et al., PRC 89, 014319; Kaiser, EPJ A 48, 58

Higher many-body forces
Hebeler et al., PRC 91, 044001

development of a novel Monte-Carlo framework
Nuclear-Matter Equation of State from Chiral Effective Field Theory

Efficient Monte-Carlo framework

represent interactions as matrices in spin-isospin space
- based on analytic expressions, incl. NN, 3N, and 4N forces
- no need for partial-wave decompositions

efficient evaluation of diagrams in MBPT (single-particle basis)
- implementing diagrams has become straightforward (also ph)
- spin-isospin traces are fully automated; multidim. momentum integrals
- rapid increase of number of diagrams: 3 (3rd), 39 (4th), 840 (5th)

analytic form of the forces & diagrams ➔ automatic code generation ➔ optimized computation
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Nuclear saturation

include contributions from up to

- NN (4\textsuperscript{th}), NN plus 3N (3\textsuperscript{rd}),
- residual 3N–3N term (2\textsuperscript{nd})

good many-body convergence

Hebeler \textit{et al.}, PRC \textbf{83}, 031301
Carlsson \textit{et al.}, PRX \textbf{6}, 011019

interactions are perturbative for these densities

Coester-like linear correlation

Coester \textit{et al.}, PRC \textbf{1}, 769

\begin{equation}
E_{\text{sym}} = 31.1 - 32.5 \text{ MeV} \\
L = 44.8 - 56.2 \text{ MeV}
\end{equation}
Nuclear-Matter Equation of State from Chiral Effective Field Theory

Guiding finite nuclei

Ekström et al., Phys. Rev. C 91, 551301

Infinite Matter

\[ \Delta r_{\text{ch}} = r_{\text{ch}}^{\text{theo}} - r_{\text{ch}}^{\text{exp}} \]

Finite Nuclei

Ab initio calculations overbind medium-mass and heavy nuclei, underestimate charge radii
Fits to saturation region

use the Monte-Carlo framework to constrain 3N LECs

- $N^2$LO / $N^3$LO EMN potentials with $\Lambda = 450$ MeV & $\Lambda = 500$ MeV
  Entem, Machleidt, Nosyk, PRC 96, 024004

- fit to $^3$H binding energy: $c_E(c_D)$ consistently at $N^2$LO / $N^3$LO

- study saturation properties: 3rd order contribution important!

reasonable fits to saturation at $N^2$LO & $N^3$LO identified
Fits to saturation region

Neutron and symmetric matter with consistent NN + 3N forces

- 4N HF energy $\sim 150$ keV @ $n_0$
- narrow ranges for $E_{\text{sym}}$ and $L$
- uncertainties from chiral EFT

Symmetric matter @ $N^3$LO:

- reduced cutoff dependence
- reduced theo. uncertainties

Epelbaum et al., EPJ A 51, 53
Outlook

1. Apply the new Hamiltonians to finite nuclei calculations will provide additional insights …

2. Perform calculations in asymmetric matter extract astrophysical quantities, mass-radius relations, …

3. Extend framework to finite temperatures study thermal properties, tabulate equation of state, …

4. Study dilute Fermi gas at fourth order with Wellenhofer, Hebeler, Schwenk, in preparation

Thank you for your attention!