The nucleon axial coupling from Quantum Chromodynamics

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Art by Bart-W. van Lith
Some outstanding questions

The origin of matter
Mechanism of asymmetric matter vs. antimatter production
Recent status: Tokai 2 Kamioka (T2K)
Current effort: T2HK with Hyper-Kamiokade
DUNE @ Fermilab to Sanford

Searches for dark matter
Only 5% is regular matter
Many ongoing efforts
Recent result Xenon1T @ Grand Sasso
Stronger bound on WIMP dark matter

Properties of the neutrino
What is the mass of the neutrino?
What is the origin of the neutrino mass?
Searches for neutrino-less double beta decay
e.g. CANDLES @ Kamioka with Ca-48

Additionally, there are specific “puzzles”
**The neutron lifetime puzzle**

**Experimental measurements**

*Bottle-type experiment*
Traps neutrons in bottle and measures how many are left.

*Beam-type experiment*
Count protons emerging from a beam of neutrons.

If neutrons decay to dark matter, bottle lifetime will be **shorter** than beam lifetime!

Precision LQCD can be use to discriminate between beam and bottle measurements.

**Big Bang Nucleosynthesis**
Prediction of the Helium-4 mass fraction is sensitive to the neutron lifetime.

Current observed values of $Y_p$ have $>$1% uncertainty, but improvement in both quantities may shed light on BSM physics.
**Nucleon axial form factor**

What is it?
Axial coupling as function of momentum transfer
Fourier Transform of axial-charge density
Dictates quasi-elastic scattering of T2HK, DUNE

**T2K:** CP conservation excluded at 90% CI
Hint of mechanism for leptogenesis?
Need more precise determination at T2HK, DUNE

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**Precision axial form factor from LQCD**

- $2 \Delta \ln L$
- Allowed 90% CL
  - Normal Ordering
  - Inverted Ordering

**Experimental result:**
- Dipole is over-constraining
- Experimental result noisy at $Q^2 \neq 0$
- Use LQCD to calculate $G_A(Q^2)$

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**Technical Details:**
- $G_A(Q^2)/G_A(0)$
- $M_A$ set to 1.1 GeV
- $eN \rightarrow e\pi N$: 1.069(16)
- $\nu N \rightarrow \nu N$: 1.026(21)

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**Additional Information:**
- http://www.dunescience.org/
Proton charge radius puzzle

In 2010 the size of the proton was measured in muonic Hydrogen
Radius shrank by 4% with 5σ tension with atomic Hydrogen
Result published in Nature 466, 213-216 (08 July 2010)
New York Times ran an article four days later gaining popularity

Result challenges *lepton universality*

![Proton charge radius](image)

Lepton universality is also challenged in recent *B*-meson semileptonic decays
experimental data @ ~4σ

![Lepton universality](image)

Proton radius and multiple independent *B*
decay discrepancy -> new physics?

Lattice QCD can directly calculate the radius.

![Lattice QCD](image)
Connecting QCD to nuclear physics

Experiments require fundamental understanding of nuclear physics
DUNE uses Argon target
Dark matter detectors use Xenon
0vbb use nuclear spectroscopy

Goal: Understand how nuclei interact from first principle theory

Quantum chromodynamics (QCD)
Modern fundamental description of the strong interaction
Much of nuclear physics is in principle described by approximately 1 parameter

Problem: elegant theory but hard to evaluate
Nuclear physics emerges from non-perturbative dynamics of QCD

Solution:
Discretize theory: Lattice QCD non-perturbatively regulates the theory
Discretization allows for numerical evaluation

LQCD can determine nuclear properties difficult or impossible to measure from experiment
Lattice QCD with many-body effective field theory is the only way to understand nuclear physics from first-principles
The nucleon axial coupling

Fundamental parameter to much of nuclear physics
Benchmark calculation for Lattice QCD

Today I will present the first percent-level determination of $g_A$ from QCD

$$g_A^{\text{QCD}} = 1.2711(126)$$
$$g_A^{\text{UNCA}} = 1.2772(020)$$

Phys. Rev. C 97, 035505

(experiment is still 6 times more precise, but we are catching up!)

The nucleon axial coupling

chiral, continuum, and infinite volume

$g_A$ from LQCD in chronological order
Introduction to Lattice QCD

Lattice QCD is QCD with non-perturbative (lattice) regularization. Allows for first-principles approach to calculating hadronic observables.

Evaluate Feynman path integral on the lattice:

\[ \langle A \rangle = \frac{1}{Z} \int [d\psi][d\bar{\psi}][dU] A e^{-S[\bar{\psi}, \psi, U]} \]

\[ = \frac{1}{Z} \int [dU] \det (\hat{D} + m) e^{-S[U]} A \]

Importance sample gauge field \( \sim e^{-S[U] - \ln \det \hat{D}} \)

Observables from simple average:

\[ \langle A \rangle \approx \frac{1}{N} \sum_{i=1}^{N} A[U_i] \]

Major lattice uncertainties and related issues:

- continuum limit: \( t_{\text{comp.}} \propto 1/a^6 \)
- infinite volume: \( t_{\text{comp.}} \propto V^{5/4} \)
- light pion mass: exponentially bad
- condition number
- signal-to-noise
Hadron spectroscopy on the lattice

Very successful history in hadron spectroscopy

$D, D^*, D_s, D_s^*, B, B^*, B_c, B_c^*$

$B$ mesons offset by $-4000$ MeV

updated of plot in [hep-lat/1203.1204]
Flavor physics from Lattice QCD

Over-constrain CKM unitarity with great success

Why is $g_A$ different?

- Large statistical uncertainty
- Large systematic uncertainty
- $g_A$ vs. pion mass show no clear trend
Nucleon signal-to-noise problem

Exponentially larger signal-to-noise compared to mesonic systems

For an nucleon annihilation operator $N$, the time evolution of correlator (signal) is

$$\langle N \bar{N} \rangle = \sum_i \langle N | i \rangle \langle i | \bar{N} \rangle e^{-E_i t} \propto e^{-M_N t}$$

(Euclidean spacetime, long time limit)

The variance of the correlator (noise) is

$$\text{Var} \langle N \bar{N} \rangle = \langle |N \bar{N}|^2 \rangle - |\langle N \bar{N} \rangle|^2$$

$$= \langle |\pi \pi|^3 \rangle - |\langle N \bar{N} \rangle|^2 \propto e^{-3M_\pi t}$$

(long time limit only the lightest mode survives)

Signal-to-noise between mesonic and baryonic systems

light (pion/kaon) \( s/n \propto e^{-[\text{MeV}]t} / e^{-[\text{MeV}]t} \)

heavy-light (B/D) \( s/n \propto e^{-[\text{GeV}]t} / e^{-[\text{GeV}]t} \)

nucleon \( s/n \propto e^{-[\text{GeV}]t} / e^{-[\text{MeV}]t} \)
Signal-to-noise in data

\[ m_{\text{eff}} = \ln \frac{C(t)}{C(t+1)} \]

\[ C(t) \propto e^{-M_{\text{n}}t} \]

Pion
Relative uncertainty **constant** with time.

**B-meson**
Rel. uncertainty grows but controlled.

**Nucleon**
Rel. uncertainty *may* have correlated fluctuations before overwhelmed by noise.

(Light) baryons are most susceptible to systematic errors.
Overcoming noise

Get more statistics
Precision determination of $g_A$ is believed to be an exascale problem.

Use a different computational strategy
Signal-to-noise is exponentially better at small time separations. However, signal is polluted with systematics that needs to be fully controlled.

Standard computational strategy typically yields data > 1fm to suppress systematics
The Feynman-Hellmann theorem

\[
\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle
\]

can be evaluated on the lattice

\[
\frac{\partial m_{\text{eff}}}{\partial \lambda} = \langle n | \mathcal{J} | n \rangle
\]

from the definition of \( m_{\text{eff}} \)

\[
\left. \frac{\partial m_{\text{eff}}}{\partial \lambda} \right|_{\lambda=0} = \left[ \frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+1)}{C(t+1)} \right] \bigg|_{\lambda=0}
\]

Putting everything together

Good: Access small \( t \) where s/n is exp. improved.
Challenge: Control very large systematic effects.
Sensitivity analysis

Excited-states present at small $t$
Correlated fluctuations at larger $t$

Additionally:
- $p$-value > 0.05
- Gaussian bootstrap dist.
- e.s. subtract data is const. inside fit region

![Graphs showing sensitivity analysis results](image-url)
Renormalization

Purpose

- the axial and vector currents have discretization errors
- correct for differences by matching lattice to continuum vertex functions

Details

- non-perturbative Rome-Southampton renormalization procedure
- non-exceptional kinematics (RI-SMOM)

**Calculate ratio**

\[
\frac{Z_A \, \hat{g}_A}{Z_V \, \hat{g}_V}
\]

developed by definition

\[
Z_V \hat{g}_V = 1
\]

\[
\frac{Z_A}{Z_V} = 1
\]
@ one part in 10,000

Conclusion

the ratio \(g_A/g_V\) is continuum-like
HISQ gauge configurations and mixed action

HISQ action
Errors starting at $O(\alpha_s a^2, a^4)$

Lüscher-Weisz action
Errors starting at $O(\alpha_s^2 a^2, a^4)$

Möbius domain-wall tune $m_{\text{res}} < 0.1 m_l$
Errors effectively start at $O(a^2, \alpha_s a^2)$

MILC configurations are the only publicly available dataset capable of
- chiral extrapolation to physical pion mass
- continuum extrapolation
- infinite volume extrapolation
- all ensembles have add. 4D gradient-flow smearing

unofficial MILC cow
MILC = MIMD Lattice Computation (the acronym has an acronym in the acronym)

Extrapolation to the physical point

Model average

What goes in here?
- 5 pion masses
- 3 lattice spacing
- 3 to 7 fm box ($3.2 < m_\pi L < 5.8$)
- weighted average of different models

Strategy
- the final result is insensitive to a wide array of variations
- stability of the result is enhanced though a weighted average of different models

The final result for the nucleon axial coupling is $g_A = 1.271(13)$
Bayesian model averaging

Model averaging accounts for uncertainty from different physical-point extrapolations

- in general provides better out-of-sample forecast
- naturally expressed under the Bayesian framework

Marginalize over set of models

\[ P(g_A|D) = \sum_k P(g_A|M_k, D) P(M_k|D) \]

Bayes’ Theorem gives

\[ P(M_k|D) = \frac{P(D|M_k)P(M_k)}{\sum_l P(D|M_l)P(M_l)} \]

where \( P(D|M_k) \) is marginalized over params

\[ P(D|M_k) = \int P(D|\theta_k, M_k)P(\theta_k|M_k)d\theta_k \]

Model averaged posterior distribution is

\[ E[g_A] = \sum_k E[g_A|M_k]P(M_k|D) \]

\[ \text{Var}[g_A] = \sum_k \text{Var}[g_A|M_k]P(M_k|D) \]

\[ + \left\{ \sum_k E^2[g_A|M_k]P(M_k|D) \right\} - E^2[g_A|D] \]

Kass and Raftery Bayes Factors (1995)
Model set

Taylor expansions around $m_\pi = 0$

- NLO Taylor $\epsilon_\pi$: $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$
- NNLO Taylor $\epsilon_\pi$: $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

- NLO Taylor $\epsilon_\pi^2$: $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$
- NNLO Taylor $\epsilon_\pi^2$: $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

Infinite volume extrapolation

leading order

$\delta_L = 8/3 \left( \epsilon_\pi^2 \left[ g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L) \right] \right)$

approximate NLO

$\delta_{L3} \equiv f_3\epsilon_\pi^3 F_1(m_\pi L)$

Baryon chiral perturbation theory

- NNLO $\chi$PT: $g_A^{\chi PT} + \delta_a + \delta_L$
- NNLO+ct $\chi$PT: $g_A^{\chi PT} + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

$g_A^{\chi PT} = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + g_0c_3\epsilon_\pi^3$

Continuum extrapolation

$\delta_a = a_2\epsilon_a^2 + b_4\epsilon_a^2\epsilon_\pi^2 + a_4\epsilon_a^4$

Dimensionless parameters

$\epsilon_\pi = m_\pi / 4\pi F_\pi \quad \epsilon_a = a / \sqrt{4\pi w_0^2}$

Summary of results

| Fit               | $\chi^2$/dof | $\mathcal{L}(D|M_k)$ | $P(M_k|D)$ | $P(g_A|M_k)$ |
|-------------------|--------------|----------------------|------------|--------------|
| NNLO $\chi$PT     | 0.727        | 22.734               | 0.033      | 1.273(19)    |
| NNLO+ct $\chi$PT  | 0.726        | 22.729               | 0.033      | 1.273(19)    |
| NLO Taylor $\epsilon_\pi^2$ | 0.792       | 24.887               | 0.287      | 1.266(09)    |
| NNLO Taylor $\epsilon_\pi^2$ | 0.787       | 24.897               | 0.284      | 1.267(10)    |
| NLO Taylor $\epsilon_\pi$  | 0.700       | 24.855               | 0.191      | 1.276(10)    |
| NNLO Taylor $\epsilon_\pi$  | 0.674       | 24.848               | 0.172      | 1.280(14)    |

average $1.271(11)(06)$
Chiral extrapolation models

Taylor in $m_\pi$

Taylor in $(m_\pi)^2$

χPT
Convergence of chiral expansion

\[ g_A = g_0 + \epsilon_\pi^2 \left[ (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \right] \]

\[ + g_0c_3 \epsilon_\pi^3 \]

\[ + c_4 \epsilon_\pi^4 \]

\[ + \epsilon_\pi^4 \ln(\epsilon_\pi^2) \]

\[ + \left( \frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5 \right) \ln^2(\epsilon_\pi^2) \]

Continuum and infinite volume extrapolation

Continuum limit constant within $>1\sigma$
Volume extrap. constant within $>1\sigma$

Unknown coefficient for NLO FV is determined by Empirical Bayes method

Bayes Factor relative to optimal width
Chiral-continuum sensitivity analysis

model avg

- NNLO $\chi$PT
- NNLO+ct $\chi$PT
- NLO Taylor $\epsilon_\pi^2$
- NNLO Taylor $\epsilon_\pi^2$
- NLO Taylor $\epsilon_\pi$
- NNLO Taylor $\epsilon_\pi$

+ $O(\alpha_s a^2)$ disc.
+ $O(a)$ disc.

- omit FV
- NLO FV

- 2×LO width
- 2× all widths

- $m_\pi \leq 350$ MeV
- $m_\pi \leq 310$ MeV
- $m_\pi \geq 220$ MeV

- $a \leq 0.12$ fm
- $a \geq 0.12$ fm

- N3LO $\chi$PT
- NLO $\chi$PT($\Delta$)

final result

fits that enter the average

discretization

finite volume

prior widths

pion mass cuts

lattice spacing cuts

additional results

$m_\pi \leq 350$ MeV

$m_\pi \leq 310$ MeV

$m_\pi \geq 220$ MeV

$a \leq 0.12$ fm

$a \geq 0.12$ fm

N3LO $\chi$PT

NLO $\chi$PT($\Delta$)

1:
24
1:
28
1:
32

$g_A$

$x^2_{\text{aug}/\text{dof}}$

Bayes factor

1.24

1.28

1.32

0.0

0.5

1.0

0.0

0.5

1.0

1.28
Systematic error budget

Sources of uncertainty

statistical
\( g_A, g_V, m_\pi, \text{PDG } m_\pi \text{ & } F_\pi \)

chiral extrapolation
weighted \( \varepsilon_\pi \) coefficient uncertainties

continuum extrapolation
weighted \( \varepsilon_a \) coefficient uncertainties

finite volume
weighted \( f_3 \) coefficient uncertainties

isospin breaking
Largest uncertainty comes from
\[
\frac{|g_A(\varepsilon_{\pi^0}) - g_A(\varepsilon_{\pi^\pm})|}{2}
\]

Summary of uncertainties

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<th>Value</th>
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<tr>
<td>chiral extrapolation</td>
<td>0.15%</td>
</tr>
<tr>
<td>continuum extrapolation</td>
<td>0.12%</td>
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<tr>
<td>infinite volume extrap.</td>
<td>0.15%</td>
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<tr>
<td>isospin breaking</td>
<td>0.03%</td>
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<tr>
<td>model selection</td>
<td>0.43%</td>
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<tr>
<td>total (in quadrature)</td>
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Final result
\( g_A = 1.271(13) \)

Recap summary

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<th>Values</th>
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<td>CLS12</td>
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<tr>
<td>PDG17</td>
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</table>
The lifetime of a free neutron from LQCD

\[ \tau_n = \frac{4908.7(1.9)\text{sec}}{|V_{ud}|^2(1 + 3g_A^2)} \]

Numerator from one-loop electroweak contributions

\[ V_{ud} \text{ from FNAL/MILC 14} \]
\[ V_{ud} = 0.97438(12) \]

free neutron lifetime

880(14) seconds

\~14 minutes 40 seconds

PDG lifetime

880.2(1.0) seconds

consistent at 1.6%
Outlook

This work
First 1% determination of $g_A$ from Lattice QCD.

Neutron lifetime puzzle
The current determination of the axial coupling is statistics limited.
Neutron lifetime differ by 1% between beam vs. bottle.
A 0.3% determination of $g_A$ can discriminate two results at 1σ ($\tau \sim 0.5\%$).
New 100+ PFLOP supercomputers will help achieve this goal.

Applications to other single nucleon observables (in no particular order)

Proton radius puzzle
Atomic and muonic Hydrogen radii differ by 4%.
Goal to directly determine the radius at 1%.
Development of new methods may let us achieve this goal.

Nucleon axial form factor
Long baseline neutrino experiments may uncover large sources of leptonic CP-violation.
Precise experiments with precise prediction of the entire axial form factor is needed.
The Feynman-Hellmann method may be applied to non-zero momentum (and other ideas).

Charm content of the nucleon
WIMP-N cross section is particularly sensitive to the charm content.
Need ~10% precision to motivate detector size. [PRL 112, 211602]
Collaborators

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Pavlos Vranas
André Walker-Loud

These calculations are made possible by

[Logos of various institutions]