Current status of nuclear forces from chiral EFT
From QCD to nuclei

QCD

symmetries (especially the chiral symmetry); lost of information (LECs)

effective chiral Lagrangian $L_{\text{eff}}(\pi, N)$

integrate out $|\vec{p}| \sim \sqrt{M_\pi m_N}$ (but retain $|\vec{p}| \sim M_\pi$): Chiral Perturbation Theory

nuclear forces and currents

$5M \text{ eV}$

$\text{LET predictions}$

$\text{ab initio}$ many-body methods: lattice, FY, NCSM,…

nuclear structure and dynamics
Effective Lagrangian:

\[ \mathcal{L}_\pi = \frac{F^2}{4} \text{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots, \]

\[ \mathcal{L}_{\pi N} = \bar{N} (i v \cdot D + g_A u \cdot S)N + \ldots, \]

\[ \mathcal{L}_{NN} = -\frac{1}{2} C_S (\bar{N}N)^2 + 2 C_T (\bar{N}SN)^2 + \ldots \]
Effective Lagrangian:

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\mathcal{L}_\pi = \frac{F^2}{4} \text{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi) + \ldots,
\]

\[
\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots,
\]

\[
\mathcal{L}_{NN} = -\frac{1}{2} C_S (\bar{N}N)^2 + 2C_T (\bar{N}SN)^2 + \ldots
\]

Auxiliary quantities (not observable):
More difficult to calculate than Feynman graphs
(renormalizability, off-shell consistency...)

\[Q = \text{momenta of particles or } M_\pi \sim 140 \text{ MeV}
\text{breakdown scale } \Lambda_b\]

Chiral Perturbation Theory

GB dynamics
Weinberg, Gasser, Leutwyler, ...

πN dynamics
Bernard-Kaiser-Meißner et al.

Nuclear forces
Weinberg, van Kolck, Kaiser, EGM, ...

Nuclear currents
Park et al, Bochum-Bonn, JLab-Pisa
Chiral expansion of the nuclear forces

<table>
<thead>
<tr>
<th>Two-nucleon force</th>
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- LO (Q⁰): Two-nucleon force
  - In Weinberg '90

- NLO (Q²): Two-nucleon force
  - Ordonez, van Kolck '92

- N²LO (Q³): Two-nucleon force
  - Ordonez, van Kolck '92

- N³LO (Q⁴): Two-nucleon force
  - Kaiser '00 - '02
  - Bernard, EE, Krebs, Meißner, '08, '11

- N⁴LO (Q⁵): Two-nucleon force
  - Entem, Kaiser, Machleidt, Nosyk '15
  - EE, Krebs, Meißner '15
  - Girlanda, Kievsky, Viviani '11
  - Krebs, Gasparyan, EE '12,'13
  - still have to be worked out

---

- Much more involved than just calculating Feynman diagrams…
- A similar program is being pursued for in chiral EFT with explicit Δ(1232) DOF
**Electromagnetic currents**

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001

<table>
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<td><img src="image3" alt="Diagram" /></td>
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<tr>
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<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>$Q^{0}$</td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
<td><img src="image9" alt="Diagram" /></td>
</tr>
<tr>
<td>$Q^{1}$</td>
<td><img src="image10" alt="Diagram" /></td>
<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
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Our results differ from the ones of the JLab-Pisa group (Pastore et al., 08-11)

- parameter-free static two-pion exchange
- depend on $C_2$, $C_4$, $C_5$, $C_7 + L_1$, $L_2$; no loop corrections
- depend on $C_T$

- no $1/m$ corrections
### Chiral expansion of the axial current and charge operators

<table>
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</tr>
<tr>
<td>$Q^0$</td>
<td><img src="image7.png" alt="Diagram" /></td>
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<td><img src="image9.png" alt="Diagram" /></td>
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**Comparison with Baroni et al. (TOPT)**
- didn’t consider $1/m$-corrections at order $Q^1$
- looked only at irreducible 3N graphs
- different results for $\pi$-exchange current contributions
- differences in tree-level $1\pi$-terms

For more details: [review article by Hermann Krebs](#) (in preparation)
A new generation of accurate & precise chiral NN potentials

— semi-local, coordinate-space-regularized up to N⁴LO

— semi-local, momentum-space-regularized up to N⁴LO⁺
   Reinert, Krebs, EE, EPJA 54 (2018) 88

— nonlocal, momentum-space-regularized up to N⁴LO⁺
   Entem, Machleidt, Nosyk, PRC 96 (2017) 024004

Other chiral EFT interactions on the market:
local potentials up to N²LO [Gezerlis et al. ’14]; minimally nonlocal N³LO potential including N²LO Δ(1232) contributions [Piarulli et al.’15]; N²LO potentials tuned to heavier nuclei [Ekström, Carlsson et al.] …
The long and short of nuclear forces

Conventional picture:

- Internucleon potential (MeV) vs. Separation (fm)
- Contact interactions

Chiral EFT:

- Internucleon potential (MeV) vs. Separation (fm)
- Contact interactions
- Multiple GB exchange (ChPT)
Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:

- **LO \([Q^0]\):** 2 operators (S-waves)
- **NLO \([Q^2]\):** + 7 operators (S-, P-waves and \(\varepsilon_1\))
- **N^2LO \([Q^3]\):** no new terms
- **N^3LO \([Q^4]\):** + 12 operators (S-, P-, D-waves and \(\varepsilon_1, \varepsilon_2\))
- **N^4LO \([Q^5]\):** no new terms
The long and short of nuclear forces

- Short-range interactions have to be tuned to experimental data. In the isospin limit, one has according to NDA:

  \[
  \begin{align*}
  \text{LO } [Q^0] & : \quad 2 \text{ operators (S-waves)} \\
  \text{NLO } [Q^2] & : \quad +7 \text{ operators (S-, P-waves and } \varepsilon_1) \\
  \text{N}^2\text{LO } [Q^3] & : \quad \text{no new terms} \\
  \text{N}^3\text{LO } [Q^4] & : \quad +12 \text{ operators (S-, P-, D-waves and } \varepsilon_1, \varepsilon_2) \\
  \text{N}^4\text{LO } [Q^5] & : \quad \text{no new terms}
  \end{align*}
  \]

- The long-range part of nuclear forces and currents is \textit{completely determined} by the chiral symmetry of QCD + experimental information on \( \pi N \) scattering

\[
\text{predicted in a parameter-free way}
\]
Determination of $\pi N$ LECs

Pion-nucleon scattering up to $Q^4$ in heavy-baryon ChPT

Fettes, Meißner ’00; Krebs, Gasparyan, EE ’12

Order $Q$:  

Order $Q^2$:  

Order $Q^3$:  

Order $Q^4$:  

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Matching ChPT to $\pi N$ Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- $\chi$ expansion of the $\pi N$ amplitude expected to converge best within the Mandelstam triangle

- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT
  \[ \bar{X} = \sum_{m,n} x_{mn} \nu^{2m+k}t^n, \quad X = \{ A^\pm, B^\pm \} \]

- Closer to the kinematics relevant for nuclear forces...
Determination of $\pi N$ LECs

Pion-nucleon scattering up to $Q^4$ in heavy-baryon ChPT

Fettes, Meißner ’00; Krebs, Gasparyan, EE ’12

Relevant LECs (in GeV$^{-n}$) extracted from $\pi N$ scattering

<table>
<thead>
<tr>
<th>LEC Type</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$\bar{d}_1 + \bar{d}_2$</th>
<th>$\bar{d}_3$</th>
<th>$\bar{d}_5$</th>
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<tr>
<td>$[Q^4]_{\text{HB, NN, GW PWA}}$</td>
<td>-1.13</td>
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<td>6.18</td>
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Notice:

- some LECs show sizable correlations (especially $c_1$ and $c_3$)...
- KH PWA and Roy-Steiner LECs lead to comparable results in the NN sector
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Notice:
- some LECs show sizable correlations (especially $c_1$ and $c_3$)...
- KH PWA and Roy-Steiner LECs lead to comparable results in the NN sector

With the LECs taken from $\pi N$, the long-range NN force is completely fixed (parameter-free)
The cutoff $\Lambda$ has to be kept finite, $\Lambda \sim \Lambda_b$ (unless all counterterms are taken into account in the calculations) [Lepage '97; EE, Gegelia '09]. In practice, low values of $\Lambda$ are preferred:

- many-body methods require soft interactions,
- spurious deeply-bound states for $\Lambda > \Lambda_{\text{crit}}$ make calculations for $\Lambda > 3$ unfeasible…

→ it is crucial to employ a regulator that minimizes finite-$\Lambda$ artifacts!
Regularization

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→ it is crucial to employ a regulator that minimizes finite-$\Lambda$ artifacts!

Nonlocal: $V_{1\pi}^{\text{reg}} \propto \frac{e^{-\frac{p'^4 + p^4}{\Lambda^4}}}{\tilde{q}^2 + M_{\pi}^2} \rightarrow \frac{1}{\tilde{q}^2 + M_{\pi}^2} \left(1 - \frac{p'^4 + p^4}{\Lambda^4} + \mathcal{O}(\Lambda^{-8})\right)$

\textit{affect long-range interactions…}

EE, Glöckle, Meißner '04;
Entem, Machleidt '03;
Entem, Machleidt, Nosyk '17; …
Regularization

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\[ \rightarrow \text{it is crucial to employ a regulator that minimizes finite-}\Lambda \text{artifacts!} \]

**Nonlocal:**

\[ V_{1\pi}^{\text{reg}} \propto \frac{e^{-\frac{p^4+p^4}{\Lambda^4}}}{q^2 + M^2_\pi} \rightarrow \frac{1}{q^2 + M^2_\pi} \left( 1 - \frac{p^4 + p^4}{\Lambda^4} + \mathcal{O}(\Lambda^{-8}) \right) \]

*affect long-range interactions…*

**Local:**

\[ V_{1\pi}^{\text{reg}} \propto \frac{e^{-\frac{q^2+M^2_\pi}{\Lambda^2}}}{q^2 + M^2_\pi} \rightarrow \frac{1}{q^2 + M^2_\pi} (1 + \text{short-range terms}) \]

\[ \rightarrow \text{does not affect long-range physics at any order in } 1/\Lambda^2\text{-expansion} \]

- Application to \( 2\pi \) exchange does not require re-calculating the corresponding diagrams:

\[ V(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2 + \mu^2} + \ldots \overset{\text{reg}}{\rightarrow} V_\Lambda(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2 + \mu^2} e^{-\frac{q^2}{2\Lambda^2}} + \ldots \]

- Convention: choose polynomial terms such that \( \Delta^n V_{\Lambda, \text{long}}(r^2) \big|_{r=0} = 0 \)
Regularization

Regularized $2\pi$-exchange potential:

$$W_{C,\Lambda}(q) = e^{-\frac{q^2}{2\Lambda^2}} \frac{2}{\pi} \int_{2M_\pi^2}^{\infty} \mu \, d\mu \, \frac{\rho(\mu)}{q^2 + \mu^2} e^{-\frac{\mu^2}{2\Lambda^2}}$$

Various regularization approaches

$\Lambda = 500$ MeV

Does it matter in practice?
To fix NN contact interactions, use scattering data together with $B_d = 2.224575(9)$ MeV and $b_{np} = 3.7405(9)$ fm.

Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured.

However, certain data are mutually incompatible within errors and have to be rejected. The 2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp: 2158 proton-proton + 2697 neutron-proton data below $E_{lab} = 300$ MeV.
- Significant correlations within the $^1S_0$ and $^3S_1$-$^3D_1$ channels but little correlations otherwise. Still, all LECs can be accurately determined...

- Natural units for the LECs according to NDA:

$$|\tilde{C}_i| \sim \frac{4\pi}{F^2}, \quad |C_i| \sim \frac{4\pi}{F^2\Lambda^2}, \quad |D_i| \sim \frac{4\pi}{F^2\Lambda^4}, \quad |E_i| \sim \frac{4\pi}{F^2\Lambda^6}$$

Assuming $\Lambda_b = 600$ MeV [EE, Krebs, Meißner EPJA 51 (15) 53; Furnstahl, Klco, Phillips, Wesolowski, PRC 92 (15) 024005], the LECs come out of a natural size.

**Absolute values of the LECs in natural units**
• Significant correlations within the $^1S_0$ and $^3S_1$-$^3D_1$ channels but little correlations otherwise. Still, all LECs can be accurately determined…

• Natural units for the LECs according to NDA:

$$|\tilde{C}_i| \sim \frac{4\pi}{F^2_i}, \quad |C_i| \sim \frac{4\pi}{F^2_i \Lambda_b^2}, \quad |D_i| \sim \frac{4\pi}{F^2_i \Lambda_b^4}, \quad |E_i| \sim \frac{4\pi}{F^2_i \Lambda_b^6}$$

Assuming $\Lambda_b = 600$ MeV [EE, Krebs, Meißner EPJA 51 (15) 53; Furnstahl, Klco, Phillips, Wesolowski, PRC 92 (15) 024005], the LECs come out of a natural size.

**Absolute values of the LECs in natural units**

signals that $\Lambda_b$ gets affected by the too soft choice of $\Lambda$…
— N⁴LO⁺ yields currently the best description of the 2013 Granada database
— 40% less parameters (27+1) compared to high-precision potentials
— Clear evidence of the parameter-free chiral 2π exchange
**State-of-the-art NN potentials**

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

**neutron-proton data**

- $E_{\text{lab}} = 0 - 100 \text{ MeV}$
- $E_{\text{lab}} = 0 - 200 \text{ MeV}$
- $E_{\text{lab}} = 0 - 300 \text{ MeV}$

**proton-proton data**

- $E_{\text{lab}} = 0 - 100 \text{ MeV}$
- $E_{\text{lab}} = 0 - 200 \text{ MeV}$
- $E_{\text{lab}} = 0 - 300 \text{ MeV}$

- $N^4\text{LO}^+$, Entem, Machleidt, Nosyk, PRC 96 (2017) 024004
- (3 more LECs)

- $N^4\text{LO}^+$, this work

- $\chi^2/\text{datum}$

- $\Lambda$ [MeV]
Careful error analysis: (i) truncation error [EE, Krebs, Meißner EPJ A51 (15)], (ii) statistical uncertainty (NN LECs), (iii) uncertainty due to $\pi N$ LECs and (iv) choice of the energy range in the fits.
Careful error analysis: (i) truncation error [EE, Krebs, Mei\ss ner EPJ A51 (15)], (ii) statistical uncertainty (NN LECs), (iii) uncertainty due to $\pi N$ LECs and (iv) choice of the energy range in the fits.

**Example: deuteron asymptotic normalizations** (relevant for nuclear astrophysics)

Our determination:

\[
A_S = 0.8847_{-3}^{(+3)}(3)(5)(1) \text{fm}^{-1/2}
\]

\[
\eta \equiv \frac{A_D}{A_S} = 0.0255_{-1}^{(+1)}(1)(4)(1)
\]

Exp: \( A_S = 0.8781(44) \text{fm}^{-1/2}, \ \eta = 0.0256(4) \)

Borbely et al. ’85 

Rodning, Knutson ’90

Nijmegen PWA [errors are „educated guesses‟] Stoks et al. ’95

\( A_S = 0.8845(8) \text{fm}^{-1/2}, \ \eta = 0.0256(4) \)

Granada PWA [errors purely statistical] Navarro Perez et al. ’13

\( A_S = 0.8829(4) \text{fm}^{-1/2}, \ \eta = 0.0249(1) \)
Three-nucleon forces

N²LO: tree-level graphs, 2 new LECs
  van Kolck '94; EE et al '02

N³LO: leading 1 loop, parameter-free
  Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08, '11

N⁴LO: full 1 loop, almost completely worked out, several new LECs
  Girlanda, Kievski, Viviani '11; Krebs, Gasparyan, EE '12,'13; EE, Gasparyan, Krebs, Schat '14

LENPIC: Low Energy Nuclear Physics International Collaboration
Three-nucleon forces

$^2\text{LO}$: tree-level graphs, 2 new LECs
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LENPIC: Low Energy Nuclear Physics International Collaboration
\textbf{Three-nucleon forces} \\

\textbf{N}²\textbf{LO}: tree-level graphs, 2 new LECs \\
\textit{van Kolck '94; EE et al '02} \\

\textbf{Determination of the LECs }c_D, c_E \\
– Triton BE (c_D-c_E correlation) \\
– Explore various possibilities and let theory and/or data decide… \\

\begin{itemize}
  \item pd minimum of d\alpha/d\theta at 135 MeV [Sekiguchi et al.'02]
  \item nd \sigma_{tot} at 135 MeV [Abfalterer et al.'01]
  \item pd minimum of d\alpha/d\theta at 108 MeV [Ermisch et al.'03]
  \item nd \sigma_{tot} at 108 MeV [Abfalterer et al.'01]
  \item pd minimum of d\alpha/d\theta at 70 MeV [Sekiguchi et al.'02]
  \item nd \sigma_{tot} at 70 MeV [Abfalterer et al.'01]
  \item nd scattering length $a$ [Schoen et al.'03]
\end{itemize}

LENPIC, to appear \\
[based on the EKM potential, $R = 0.9$ fm]
Three-nucleon forces

\[ \text{N}^2\text{LO}: \text{tree-level graphs, 2 new LECs} \]
van Kolck '94; EE et al '02

Determination of the LECs \( c_D, c_E \)

- Triton BE (\( c_D-c_E \) correlation)
- Explore various possibilities and let theory and/or data decide...

\[ \begin{align*}
\text{pd minimum of } d\sigma/d\theta \text{ at } 135 \text{ MeV} & \quad [\text{Sekiguchi et al.'02}] \\
\text{nd } \sigma_{\text{tot}} \text{ at } 135 \text{ MeV} & \quad [\text{Abfalder et al.'01}] \\
\text{pd minimum of } d\sigma/d\theta \text{ at } 108 \text{ MeV} & \quad [\text{Ermisch et al.'03}] \\
\text{nd } \sigma_{\text{tot}} \text{ at } 108 \text{ MeV} & \quad [\text{Abfalder et al.'01}] \\
\text{pd minimum of } d\sigma/d\theta \text{ at } 70 \text{ MeV} & \quad [\text{Sekiguchi et al.'02}] \\
\text{nd } \sigma_{\text{tot}} \text{ at } 70 \text{ MeV} & \quad [\text{Abfalder et al.'01}] \\
\text{nd scattering length } a & \quad [\text{Schoen et al.'03}] \\
\end{align*} \]

LENPIC, to appear
[based on the EKM potential, \( R = 0.9 \text{ fm} \)]

yields the strongest constraint...

LENPIC: Low Energy Nuclear Physics International Collaboration
Nd total cross section at 70 MeV (preliminary)

LENPIC: Low Energy Nuclear Physics International Collaboration
Light nuclei (preliminary)
Summary and outlook

**Nuclear Hamiltonian:**
- derivation of contributions up to $N^3$LO completed already in 2011; derivation of $N^4$LO corrections done for $V_{2N}$ and almost done for $V_{3N}$ (new LECs…) and $V_{4N}$
- accurate & precise $2N$ potentials at $N^4$LO$^+$ are available,
- promising results for few-$N$ systems based on $2NF + 3NF@N^2$LO [LENPIC]

**Electroweak current operators:**
- have been worked out completely to $N^3$LO
- some $\pi N$ LECs in $1\pi$ axial charge at $N^3$LO are unknown…
  [lattice QCD? $\nu$-induced $\pi$-production? resonance saturation? large-$N_c$?…]

**Work in progress:**
- regularization of $3NF$ & currents beyond $N^2$LO (nontrivial to maintain $\chi$-symm!)

**Next steps:**
- Precision tests of the theory for $^3H$ $\beta$ decay & $\mu$ capture (validation)
- Extension to other processes, heavier nuclei, $N^4$LO, explicit $\Delta$’s, …