Experimental status of $|V_{ub}/V_{cb}|$ and $\gamma(\phi_3)$

Abi Soffer

Tel Aviv University
On behalf of the Belle II Collaboration
Outline

• Importance of $|V_{ub}/V_{cb}|$ and $\gamma$
  – Why they are related

• Introduction to the experiments
  – BABAR, Belle, CLEO-c, LHCb, Belle II

• Measurements of $|V_{ub}/V_{cb}|$
  – Different methods

• Measurements of $\gamma$
  – Different methods

I will give only a selection of results and averages from PDG based mostly on HFLAV and CKMFitte
Importance of $|V_{ub}/V_{cb}|$ and $\gamma$

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix contains 4 fundamental parameters of the SM.
- 2 of the 4 parameters can be parameterized as $|V_{ub}/V_{cb}|$ and $\gamma$
- They originate from some high-scale new physics
- Their precision measurement is critical to the definition of the SM
- It also provides a stress-test of the SM and probe of NP
CKM unitarity triangle

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

Testing the consistency of this SM picture probes NP
Testing the consistency of this SM picture probes NP
Unitarity triangle constraints

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

Testing the consistency of this SM picture probes NP.
The experiments

- BABAR: 1999-2008, \( \sim 470\text{M} \ e^+e^- \rightarrow B\bar{B} \)
- Belle: 1999-2010, \( \sim 770\text{M} \ e^+e^- \rightarrow B\bar{B} \)
- LHCb: 2010-, \( pp \rightarrow b\bar{b}X \)
- CLEO-c: 2003-2006?, \( \sim 3\text{M} \ e^+e^- \rightarrow D\bar{D} \)
- Belle II: 2019-2026?, \( \sim 50,000\text{M} \ e^+e^- \rightarrow B\bar{B} \)

- Silicon tracker
- Gas-based tracker
- Cherenkov hadron-ID
- EM calorimeter
- Muon system
\[ \frac{V_{ub}}{V_{cb}} \]

- Measured via \( b \to u\ell\bar{\nu} \) and \( b \to c\ell\bar{\nu} \) (where \( \ell = e, \mu \))
  
  -- Measurements with \( \tau \) also possible in principle, but...
  - they are experimentally less precise
  - not yet clear what the \( \sim 4\sigma \) SM discrepancy in \( \bar{B} \to D^{(*)}\tau\bar{\nu} \) is telling us

- Measured via both exclusive and inclusive final states.

\[
\begin{align*}
\bar{B} \to \pi\ell\bar{\nu} \\
\bar{B} \to D^{(*)}\ell\bar{\nu}
\end{align*}
\]

\[
\begin{align*}
\bar{B} \to X_u\ell\bar{\nu} \\
\bar{B} \to X_c\ell\bar{\nu}
\end{align*}
\]

- Significant model dependence \( \rightarrow \) different values of \( |V_{ub}| \) & \( |V_{ub}| \)

- Close interaction between theoretical and experimental inputs
Other-$B$ tagging at a $e^+e^-$ B factory

- **Hadronic**
  - Fully reconstruct in $>1000$ hadronic decay chains
  - Reduces combinatorial background
  - $\vec{p}_\nu$ well determined
  - Efficiency $\sim 0.5\%$

- **Semileptonic**
  - Reconstruct as $B \rightarrow \bar{D}(^*) \ell^+\nu$
  - Reduces combinatorial background
  - Efficiency $\sim 0.5\%$

- **Inclusive**
  - Use all tracks and clusters to try and reconstruct neutrino 4-momentum
  - Efficiency $\sim 10s\%$
  - Higher background
  - $\vec{p}_\nu$ not well determined: finite B momentum, missing/fake particles
A comparison of other-B tagging methods

“Beam-constrained mass”

$$M_{bc} \equiv \sqrt{E_{beam}^2 - p_B^2}$$

Old and new algorithms, same efficiency

Belle
arXiv:1012.0090

Inclusive tag

BABAR

hep-ex/0008052

Hadronic tag

arXiv:1102.3876
\( V_{cb} \) from exclusive \( \bar{B} \to D^* \ell \bar{\nu} \)

- Experiments measure differential decay rate, given by:

\[
\frac{d\Gamma}{dw}(\bar{B} \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{\text{ew}} F(w))^2
\]

\( w \equiv \nu_B \cdot \nu_{D^*} \)

- Phase-space

\( \sim 1, \text{EW correction} \)

- Form factor (lattice or lightcone sum rules)

- Measurements performed by Belle, BABAR, CLEO, LEP.

- Most use a form-factor polynomial expansion (CLN) around \( w = 1 \), considered by PDG to be too constraining given current high precision

- PDG prefers the more flexible BGL form-factor parameterization (Boyd, Grinstein, Lebed, PRL 74, 4603)

- \( \sim 10\% \) differences seen b/w CLN and BGL parameterizations

- Only one analysis (Belle, arXiv:1702.01521) uses BGL…
Recent Belle had-tag $\bar{B} \to D^* \ell \bar{\nu}$ analysis

arXiv:1702.01521

$p_{\text{miss}} = p_{\nu} = p_{e^+e^-} - p_{\text{tag}} - p_{D^*} - p_{\ell}$

One of the distributions:

From these inputs, PDG quotes $|V_{cb}| = (41.9 \pm 2.0) \times 10^{-3}$
\( V_{cb} \) from inclusive \( \bar{B} \to X_c \ell \bar{\nu} \)

- Differential decay rate is given as a heavy-quark expansion (HQE) in terms of lepton-energy moments,

\[
\langle E_{e}^{n} \rangle_{E_{e}>E_{\text{cut}}} = \int_{E_{\text{cut}}}^{E_{\text{max}}} \frac{d\Gamma}{dE_{e}} E_{e}^{n} dE_{e} \left/ \int_{E_{\text{cut}}}^{E_{\text{max}}} \frac{d\Gamma}{dE_{e}} dE_{e} \right.
\]

for different value of the minimal lepton energy \( E_{\text{cut}} \), and similarly for the hadronic invariant mass and hadronic energy.

- E.g., Lepton-energy moments from BABAR arXiv:0908.0415:

- Global Fits to these moments are used to obtain \( V_{cb} \) in various \( b \)-mass schemes

- PDG choose the kinetic scheme result \( |V_{cb}| = (42.2 \pm 0.8) \times 10^{-3} \)
$V_{ub}$ from inclusive $\bar{B} \rightarrow X_u \ell \bar{\nu}$

- The total $B \rightarrow X_u \ell \nu$ decay rate is calculated based on the operator product expansion (OPE) in $\alpha_s$ and $\Lambda_{QCD}/m_b$, with a $\sim 5\%$ uncertainty
  - But the total rate is hard to measure, due to large $B \rightarrow X_c \ell \nu$ background
  - $V_{ub}$ measurement requires model for distributions of $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_c \ell \nu$

- The high-$E_\ell$ region favored experimentally
  - Projecting to full spectrum incurs large corrections from a nonperturbative ”shape function” (SF) that accounts for the Fermi motion of the $b$ inside the $B$.

- So measure $V_{ub}$ with different
  - kinematic regions
  - analysis methods
  - calculation schemes
Some inclusive $V_{ub}$ results ($10^{-5}$), RPP 2016/7

<table>
<thead>
<tr>
<th>Ref.</th>
<th>cut (GeV)</th>
<th>BLNP</th>
<th>GGOU</th>
<th>DGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO</td>
<td>$E_e &gt; 2.1$</td>
<td>$428 \pm 50$</td>
<td>$421 \pm 49$</td>
<td>$390 \pm 45$</td>
</tr>
<tr>
<td>BABAR</td>
<td>$E_e - q^2$</td>
<td>$453 \pm 22$</td>
<td>not available</td>
<td>$417 \pm 20$</td>
</tr>
<tr>
<td>BABAR</td>
<td>$E_e &gt; 2.0$</td>
<td>$454 \pm 26$</td>
<td>$450 \pm 26$</td>
<td>$434 \pm 25$</td>
</tr>
<tr>
<td>Belle</td>
<td>$E_e &gt; 1.9$</td>
<td>$493 \pm 46$</td>
<td>$493 \pm 46$</td>
<td>$485 \pm 45$</td>
</tr>
<tr>
<td>BABAR</td>
<td>$q^2 &gt; 8$</td>
<td>$430 \pm 23$</td>
<td>$432 \pm 23$</td>
<td>$427 \pm 22$</td>
</tr>
<tr>
<td></td>
<td>$m_X &lt; 1.7$</td>
<td>$430 \pm 24$</td>
<td>$422 \pm 21$</td>
<td>$453 \pm 23$</td>
</tr>
<tr>
<td></td>
<td>$P_+ &lt; 0.66$</td>
<td>$415 \pm 25$</td>
<td>$424 \pm 26$</td>
<td>$424 \pm 26$</td>
</tr>
<tr>
<td></td>
<td>$m_X &lt; 1.55$</td>
<td>$430 \pm 24$</td>
<td>$442 \pm 24$</td>
<td>$446 \pm 24$</td>
</tr>
<tr>
<td>Belle</td>
<td>$E_\ell &gt; 1$</td>
<td>$449 \pm 27$</td>
<td>$460 \pm 27$</td>
<td>$463 \pm 28$</td>
</tr>
<tr>
<td>BABAR</td>
<td>$E_\ell &gt; 0.8$</td>
<td>$456 \pm 13$</td>
<td>$396 \pm 10$</td>
<td>$396 \pm 10$</td>
</tr>
</tbody>
</table>

- Note that results with different cuts are correlated
- PDG averages over methods with additional error:

$$|V_{ub}| = (4.49 \pm 0.15_{\text{exp}} ^{+0.16}_{-0.17_{\text{theo}}} \pm 0.17_{\Delta BF}) \times 10^{-3}$$

Some $V_{ub}$ values are recalculation by HFLAV

Inclusive tagging

Hadronic tagging

$P_\ell$
$V_{ub}$ from exclusive $\bar{B} \rightarrow \pi \ell \bar{\nu}$ (PDG17)

- As with exclusive $V_{cb}$, relate partial Br to $V_{ub}$ using lattice or sum rules

<table>
<thead>
<tr>
<th></th>
<th>$B \times 10^4$</th>
<th>$B(q^2 &gt; 16) \times 10^4$ GeV$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO $\pi^+, \pi^0$ [129]</td>
<td>1.38 ± 0.15 ± 0.11</td>
<td>0.41 ± 0.08 ± 0.04</td>
</tr>
<tr>
<td>BABAR $\pi^+, \pi^0$ [130]</td>
<td>1.41 ± 0.05 ± 0.08</td>
<td>0.32 ± 0.02 ± 0.03</td>
</tr>
<tr>
<td>BABAR $\pi^+$ [131]</td>
<td>1.44 ± 0.04 ± 0.06</td>
<td>0.37 ± 0.02 ± 0.02</td>
</tr>
<tr>
<td>Belle $\pi^+, \pi^0$ [143]</td>
<td>1.48 ± 0.04 ± 0.07</td>
<td>0.40 ± 0.02 ± 0.02</td>
</tr>
<tr>
<td>Belle SL $\pi^+$ [144]</td>
<td>1.41 ± 0.19 ± 0.15</td>
<td>0.37 ± 0.10 ± 0.04</td>
</tr>
<tr>
<td>Belle SL $\pi^0$ [144]</td>
<td>1.41 ± 0.26 ± 0.15</td>
<td>0.37 ± 0.15 ± 0.04</td>
</tr>
<tr>
<td>Belle Had $\pi^+$ [124]</td>
<td>1.49 ± 0.09 ± 0.07</td>
<td>0.45 ± 0.05 ± 0.02</td>
</tr>
<tr>
<td>Belle Had $\pi^0$ [124]</td>
<td>1.48 ± 0.15 ± 0.08</td>
<td>0.36 ± 0.07 ± 0.02</td>
</tr>
<tr>
<td>BABAR SL $\pi^+$ [145]</td>
<td>1.38 ± 0.21 ± 0.08</td>
<td>0.46 ± 0.13 ± 0.03</td>
</tr>
<tr>
<td>BABAR SL $\pi^0$ [145]</td>
<td>1.78 ± 0.28 ± 0.15</td>
<td>0.44 ± 0.17 ± 0.06</td>
</tr>
<tr>
<td>BABAR Had $\pi^+$ [146]</td>
<td>1.07 ± 0.27 ± 0.19</td>
<td>0.65 ± 0.20 ± 0.13</td>
</tr>
<tr>
<td>BABAR Had $\pi^0$ [146]</td>
<td>1.52 ± 0.41 ± 0.30</td>
<td>0.48 ± 0.22 ± 0.12</td>
</tr>
<tr>
<td>Average [147]</td>
<td>1.45 ± 0.02 ± 0.04</td>
<td>0.38 ± 0.01 ± 0.01</td>
</tr>
</tbody>
</table>

$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$
$|V_{ub}/V_{cb}|$ with $\Lambda_b$ decays at LHCb

arXiv:1504.01568

Similar topology for $\Lambda_b \rightarrow \Lambda_c^+ \mu \bar{\nu}$
with $\Lambda_c^+ \rightarrow pK^-\pi^+$

Measure $\hat{p}_{\Lambda_b}$ and $q^2 = (p_\mu + p_\nu)^2$ from
vertex position with 1 (4) GeV$^2$
resolution for right (wrong) solution. Measure:

$$m_{corr} = \sqrt{m_{p\mu}^2 + p_{\perp}^2 + p_{\perp}}$$

$p_{\perp} = p_\mu$ momentum $\perp \Lambda_b$ momentum

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu\bar{\nu})_{q^2>15\text{GeV}^2}}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu\bar{\nu})_{q^2>7\text{GeV}^2}} = (0.95 \pm 0.04 \pm 0.07) \times 10^{-2}$$
\[ |V_{ub}/V_{cb}| = 0.107 \pm 0.007 \quad \text{(inclusive)} \]

\[ |V_{ub}/V_{cb}| = 0.088 \pm 0.006 \quad \text{(exclusive)} \]

2\(\sigma\) difference (mostly due to 2.6\(\sigma\) difference in \(V_{ub}\) results)

\[ |V_{ub}/V_{cb}| = 0.080 \pm 0.004 \pm 0.004 \quad \text{(LHCb)} \]

LHCb result is based on 1 precise lattice calculation.
Errors on B-factory results include differences b/w theory inputs, more crosschecks
Measuring $\gamma$

- To leading order in $\lambda = \sin \theta_C \approx 0.22$, can take
  $$\gamma = \arg(-V_{ub}^*)$$

- $\gamma$ is a CP-odd complex phase
- Its measurement requires
  1. Interference between two amplitudes, one with $V_{ub}$
  2. Comparison of between CP-conjugate processes
The GLW method

Gronau-London-Wyler (PLB 253, 483; PLB 265, 172)

- Consider the processes $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$

\[
B^- \quad b \quad \bar{u} \quad D^0 \\
\bar{u} \quad c \quad \bar{u}
\]

\[
B^- \quad b \quad V_{ub} \quad u \quad \bar{D}^0 \\
\bar{u} \quad \bar{c} \quad \bar{u} \quad \bar{u}
\]
The GLW method

Gronau-London-Wyler (PLB 253, 483; PLB 265, 172)

• Consider the processes \( B^- \rightarrow D^0K^- \) and \( B^- \rightarrow \bar{D}^0K^- \)

• Their amplitudes are measured via Cabibbo-allowed charm decays

\[
\begin{align*}
\text{B}^- & : b \rightarrow u, \bar{c} \rightarrow \bar{u}, s \rightarrow \bar{s}, K^- \\
\text{D}^0 & : c \rightarrow \bar{d}, s \rightarrow \bar{s}, K^- \\
\text{D}^{-} & : \bar{u} \rightarrow d, \bar{c} \rightarrow \bar{s}, \pi^+ \\
\end{align*}
\]
The GLW method

Gronau-London-Wyler (PLB 253, 483; PLB 265, 172)

- Consider the processes $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$
- Their amplitudes are measured via Cabibbo-allowed charm decays

Amplitude

$|A| r_B e^{i(-\gamma + \delta_B)}$

Some CP-even phase due to strong interaction
The GLW method

• Interference when both $D^0$ and $\overline{D}^0$ decay to a common CP eigenstate

$$K^+K^-, \pi^+\pi^-, K_S\pi^0$$
• Write the CP eigenstates as
  \[ D_{\pm}^0 = \frac{1}{\sqrt{2}} (D^0 \pm \bar{D}^0) \]

• Then the decay amplitudes satisfy
  \[ A(B^- \to D_{\pm}^0 K^-) = \frac{1}{\sqrt{2}} \left( A(B^- \to D^0 K^-) \pm A(B^- \to \bar{D}^0 K^-) \right) \]
  \[ = \frac{1}{\sqrt{2}} |A| \left( 1 \pm r_B e^{i(-\gamma + \delta_B)} \right) \]

• With \(-\gamma\) becoming \(+\gamma\) for the \(B^+\) decay
• This gives two triangles from which \(\gamma\) is extracted, with \(r_B\) and \(\delta_B\):
The ADS method
Atwood, Dunietz, Soni, PRL 78, 3257

- Ratio between interfering $B$ decays is only $r_B \sim 0.1$: small interference
- Exploit doubly Cabbibo-suppressed decay $\bar{D}^0 \to K^- (n\pi)^+$:
The GGSZ method

Giri, Grossman, Soffer, Zupan, PRD 68, 054018

• Exploit multibody $D^0$ decays
• Help resolve trig ambiguities in $\gamma$
• CP content depends on phase-space point
• Complex $D^0$ decay amplitude can be modeled (isobar+K-matrix, etc.) with a flavor-tagged $D^0$ sample:
Or:

Complex $D^0$ decay amplitude can be obtained model-independently with a CP-tagged $D$ sample:

$$\psi(3770) \rightarrow D^0_+D^0_0$$

$K^+K^- K_S \pi^+\pi^-$

CLEO-c (0903.1681)

- Belle (1509.01098) & LHCb (1408.2748) measured $\gamma$ in this model-independent method
- Stat error > model-dependent case, but no assumptions re: strong phases
# LHCb $\gamma$ combination

<table>
<thead>
<tr>
<th>$B$ decay</th>
<th>$D$ decay</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>GLW</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>ADS</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$\underline{D \rightarrow h^+h^+\pi^-\pi^+\pi^-}$</td>
<td>GLW/ADS</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$\underline{D \rightarrow h^+h^-\pi^0}$</td>
<td>GLW/ADS</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$D \rightarrow K^0_s h^+h^-$</td>
<td>GGSZ</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^+$</td>
<td>$D \rightarrow K_s^0 K^+\pi^-$</td>
<td>GLS</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^*K^+$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>GLW</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^{**}$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>GLW/ADS</td>
</tr>
<tr>
<td>$B^+ \rightarrow DK^{+\pi^+\pi^-}$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>GLW/ADS</td>
</tr>
<tr>
<td>$B^0 \rightarrow DK^{*0}$</td>
<td>$D \rightarrow K^+\pi^-$</td>
<td>ADS</td>
</tr>
<tr>
<td>$B^0 \rightarrow DK^{+\pi^-}$</td>
<td>$D \rightarrow h^+h^-$</td>
<td>GLW-Dalitz</td>
</tr>
<tr>
<td>$B^0 \rightarrow DK^{*0}$</td>
<td>$D \rightarrow K_s^0 \pi^+\pi^-$</td>
<td>GGSZ</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D^{+}_s K^{\pm}$</td>
<td>$D^{+}_s \rightarrow h^+h^-\pi^+$</td>
<td>TD</td>
</tr>
</tbody>
</table>

Underlined: modifications of above methods

LHCb-CONF-2017-004.pdf
LHCb $\gamma$ combination

$\gamma = (76.8^{+5.1}_{-5.7})^\circ$ (Up to a 180° ambiguity)
BABAR+Belle+LHCb combination

(Somewhat older LHCb results)
Summary & comments

• $|V_{ub}/V_{cb}|$ and $\gamma(\phi_3)$ are important SM parameters
• Crucial inputs to NP probing via unitarity triangle
• No significant inconsistency with the SM so far
• LHCb now competitive with BABAR+Belle in $|V_{ub}/V_{cb}|$
  – But relies on only one lattice calculation
• LHCb dominates measurement of $\gamma$
• Belle II will have $\sim$35 times more data than BABAR+Belle
• See M. Lubej’s talk for expected Belle II impact on $|V_{ub}/V_{cb}|$
  – Theory calculations will need to improve as well
• Belle II + LHCb should reach $\sigma_\gamma \sim 1^\circ$ through combination of different methods
Backup slides
• CLN expansion of form factor:

\[ F(w) = F(1) - \rho^2 (w - 1) + c(w - 1)^2 + \cdots \]

• BGN expansion:

\[ F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{n=0}^{\infty} a_n z^n \quad (94.6) \]

where the sum \( \sum |a_n|^2 \) is bounded. Furthermore, the function \( P(z) \) takes into account the resonances in the \( (\bar{c}b) \) system below the \( \bar{D}B \) threshold, and the weighting functions \( \phi_F(z) \) are derived from the unitarity constraint on the corresponding form factor. The values of \( z \) relevant to the decay are \( 0 \leq z \leq 0.06 \), hence the series in \( z \) converges rapidly and only very few terms are needed. Eq. (94.6) will be referred to as the “BGL” expansion.
$V_{ub}$ and $V_{cb}$ summary (PDG 2017)

Comparable experiment & theory uncertainties
1% common systematic uncertainty.
Good agreement between the two results

$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$ (inclusive)
$|V_{cb}| = (41.9 \pm 2.0) \times 10^{-3}$ (exclusive)

$|V_{ub}| = (4.49 \pm 0.15 \pm 0.16 \pm 0.17) \times 10^{-3}$ (inclusive)
$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$ (exclusive)

Experiment    Theory    Extrapolation of model to full kinematic range