Light cluster formation in the PHQMD transport approach

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PHQMD

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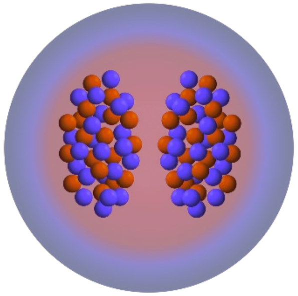
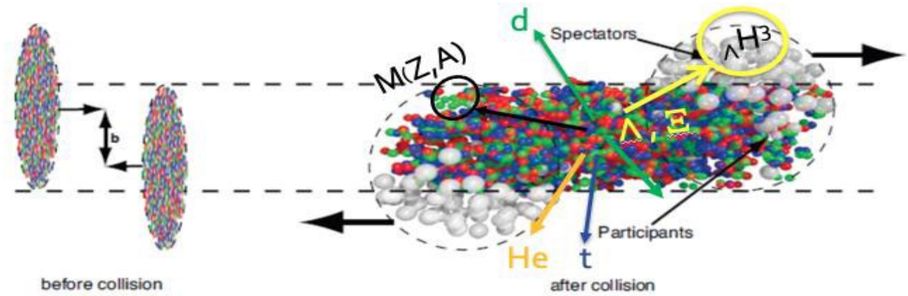
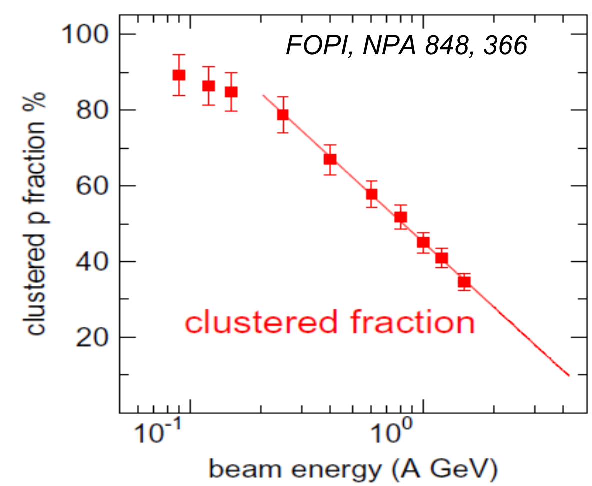


Cluster production in heavy-ion collisions



Clusters and (anti-) hypernuclei
are observed experimentally at all energies

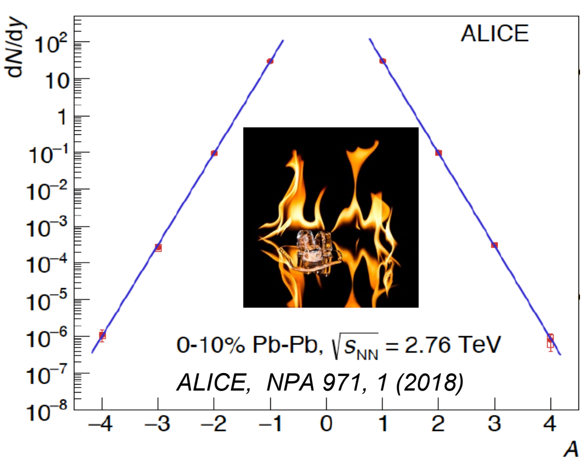
Au+Au



- **Clusters are abundant at low energy**

- **High energy HIC:**
  ‚Ice in a fire‘ puzzle:
  how the weakly bound objects can be formed and survive in a hot enviroment?!

Pb+Pb, central. midrapidity



➔ **Mechanisms of cluster formation in strongly interacting matter are not well understood**

Cluster production in heavy-ion collisions is a continous process from $\sqrt{s}$ =2 GeV to $\sqrt{s}$=10 TeV

Cluster formation at midrapidity happens from

E_{kin} =1 GeV to $\sqrt{s}$ = 200 GeV in a very continuous way

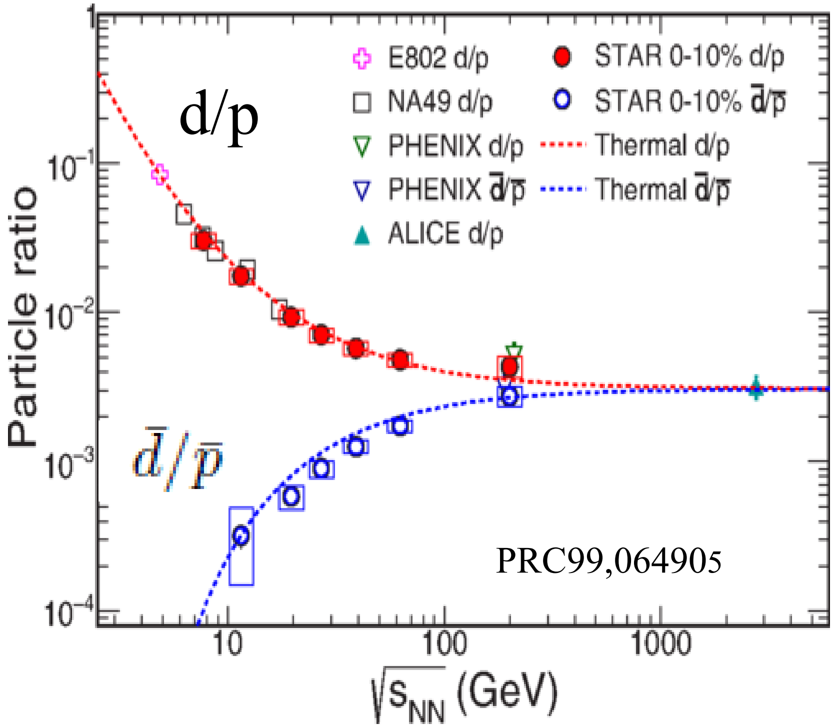
although environment changes drastically:

E_{kin} = 1GeV 90% nucleons 10% pions

$\sqrt{s}$ = 200 GeV 5% <(anti)baryons

95% mesons

hadronic environment → QGP



The slope of the transverse energy spectra is rather similar

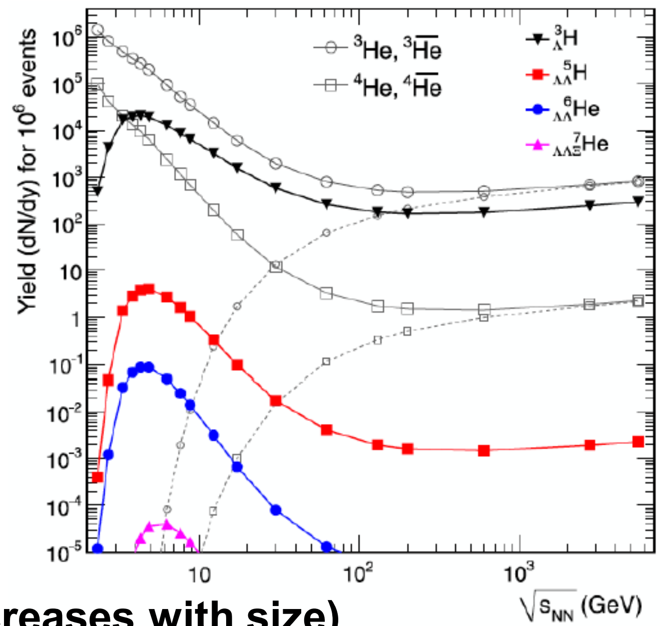
T ≈ 100 MeV

→ To study cluster production we should explore all data (which cover often a larger rapidity interval than at RHIC/LHC and where models have to make less assumptions than at RHIC/LHC)

Models for cluster and hypernuclei formation

- ❑ <u>Existing models for cluster formation:</u>

A. Andronic et al., PLB 697, 203 (2011)



- ❑ **statistical model:**
  - **- assumption of thermal equilibrium**
  - **no hadronic interactions → spectra wrong**

Dynamical Models:

- ❑ **coalescence model:**
  - **determination of clusters at a freeze-out time by coalescence radii in coordinate and momentum space**
  - **ad hoc model with free parameters (number increases with size)**
  - **third body for d-production?**

- ❑ **collisions NNN → dN ; NNπ → dπ (kinetic deuterons)**
  - **corrections in the dense medium (d needs space)**
  - **complicated 3 body process (detailed balance)**
  - **only for deuterons**

- ❑ **formation by potential interactions (potential deuterons)**
  - **(the same as applied during the whole HI collision)**

PHQMD

PHQMD

PHQMD: a unified n-body microscopic transport approach for the description of heavy-ion collisions and dynamical cluster formation from low to ultra-relativistic energies

Realization: combined model PHQMD = (PHSD & QMD) + (MST/SACA)

Parton-Hadron-Quantum-Molecular Dynamics

PHSD mode

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

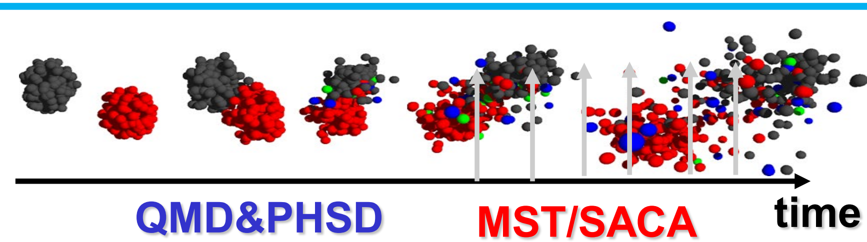
J.Aichelin, Phys. Rept. 202 (1991)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

Cluster recognition:
MST (Minimum Spanning Tree)
or SACA (Simulated Annealing Clusterization Algorithm)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266



PHQMD:
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
G. Coci et al., PRC 108 (2023) 1, 014902

QMD time evolution

Dirac-Frenkel-McLachlan approach
A. Raab, Chem. Phys. Lett. 319, 674
J. Broeckhove et al., Chem. Phys. Lett. 149, 547

- **Generalized Ritz variational principle**: $$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$$

Many-body wave function:

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ **for N particles (neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

[Aichelin, Phys. Rept. 202 (1991)]

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) in coordinate and momentum space:**

$$\boxed{\dot{r}_{i0} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}}$$

Many-body Hamiltonian: $$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j\neq i} V_{i,j})$$

2-body potential: $$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t)$$

Antisymmetrization is neglected since it would be impossible to formulate collision term

Local momentum dependent potential in PHQMD

New!

- Nucleon-nucleon **local** two-body **momentum dependent potential**:

$$V_{ij} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t)$$
$$= V_{\mathrm{Skyrme\ loc}} + V_{\mathrm{mom}} + V_{\mathrm{Coul}}$$
$$= \boxed{\frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma+1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j)\, \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t)}$$

Skyrme

$$+ \boxed{V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_{i0}, \mathbf{p}_{j0})} + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|},$$

momentum dependent

Coulomb

- The single-particle potential <V> resulting from the convolution of the distribution functions f_i and f_j with the interactions V_{Skyrme} + V_{mom} (local interactions including their momentum dependence) for symmetric nuclear matter:

1) Skyrme potential ('static') :

$$\langle V_{Skyrme}(\mathbf{r}_{i0}, t)\rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0}\right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0}\right)^{\gamma}$$

with modified interaction density (with relativistic extension):

$$\rho_{int}(\mathbf{r}_{i0}, t) \rightarrow C \sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r}^{\mathrm{T}}_{i0}(t) - \mathbf{r}^{\mathrm{T}}_{j0}(t))^2}$$
$$\times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}^{\mathrm{L}}_{i0}(t) - \mathbf{r}^{\mathrm{L}}_{j0}(t))^2},$$

Momentum dependent potential → EoS in PHQMD

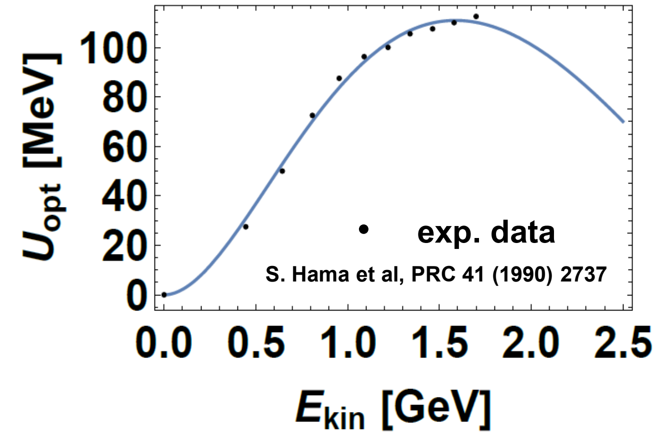
2) Momentum dependent potential :

$$\boxed{V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1 - \mathbf{r}_2)}$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters a, b, c are fitted to the "optical" potential (Schrödinger equivalent potential U_{SEP}) extracted from elastic scattering data in pA:

$$U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p} - \mathbf{p}_1) dp_1^3}{\frac{4}{3}\pi p_F^3}$$



- **In infinite matter a potential corresponds to the EoS:**

$$\boxed{E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)}$$

$$V_{mom} = (a\Delta p + b\Delta p^2)\ exp(-c\sqrt{\Delta p})\ \frac{\rho}{\rho_0}$$

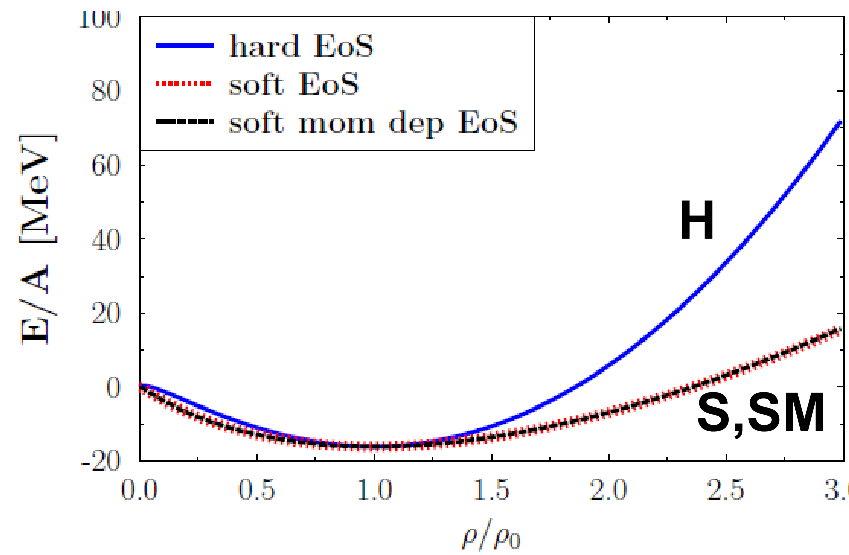
$$V_{Skyrme} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200
	a $[MeV^{-1}]$	b$[MeV^{-2}]$	c$[MeV^{-1}]$	
	236.326	-20.73	0.901	

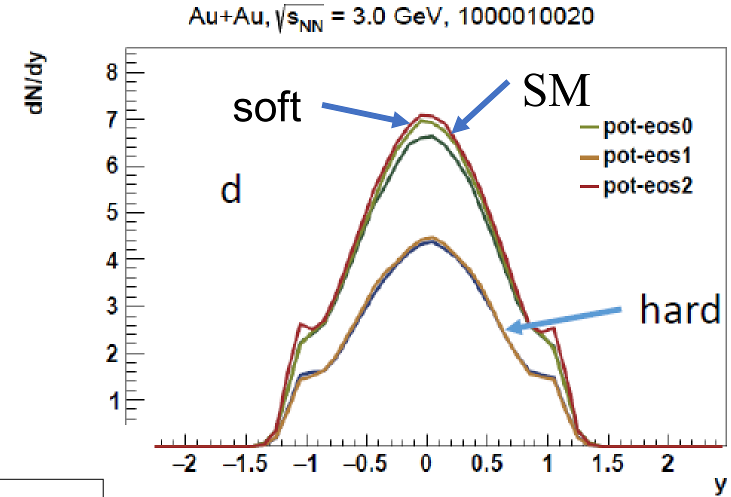
EoS for infinite cold nuclear matter at rest



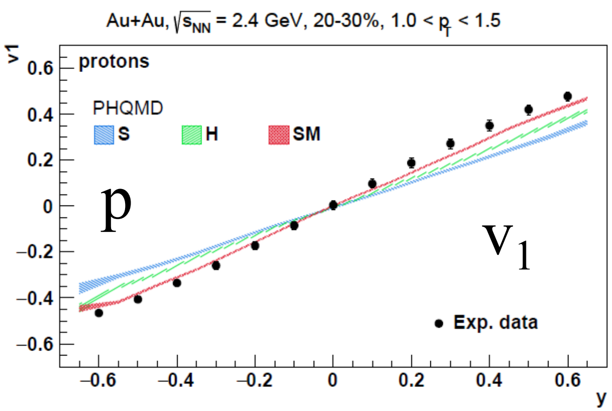
EoS dependence of flow observables

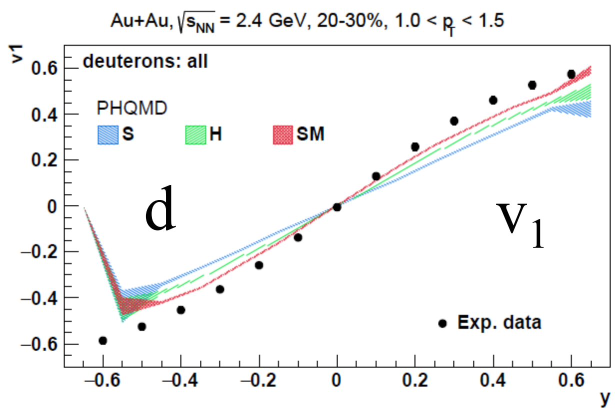
SM potential acts differently on different observables:

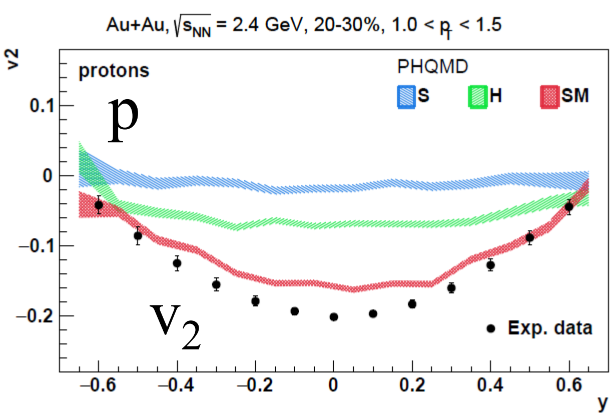
- **yield (dN/dy) like a soft EoS**
- **flow harder than a hard EoS**

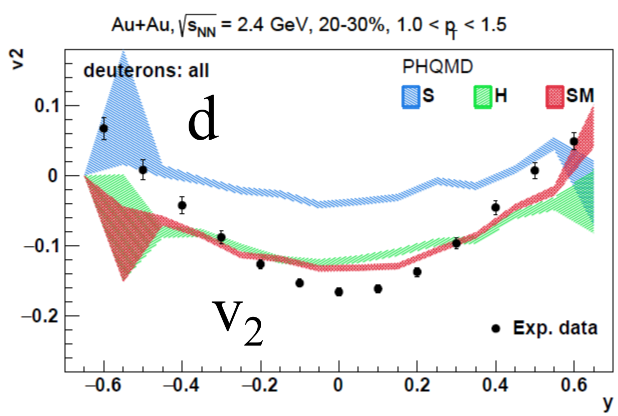


Au+Au, $\sqrt{s_{NN}}$ = 3.0 GeV, 1000020030









Mechanisms for cluster production in PHQMD:

I. potential interactions (recongnized by MST)

&

II. kinetic reactions

III. Coalescence (discussed later)

PHQMD

I. Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

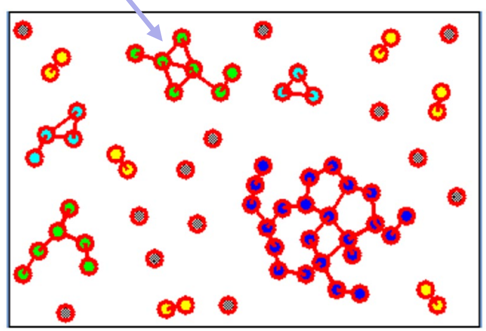
The **Minimum Spanning Tree** (MST) is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

The MST algorithm searches for **accumulations of particles in coordinate space:**

1. Two particles are **'bound'** if their **distance in the cluster rest frame** fulfills

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm} \quad \text{(range of NN potential)}$$

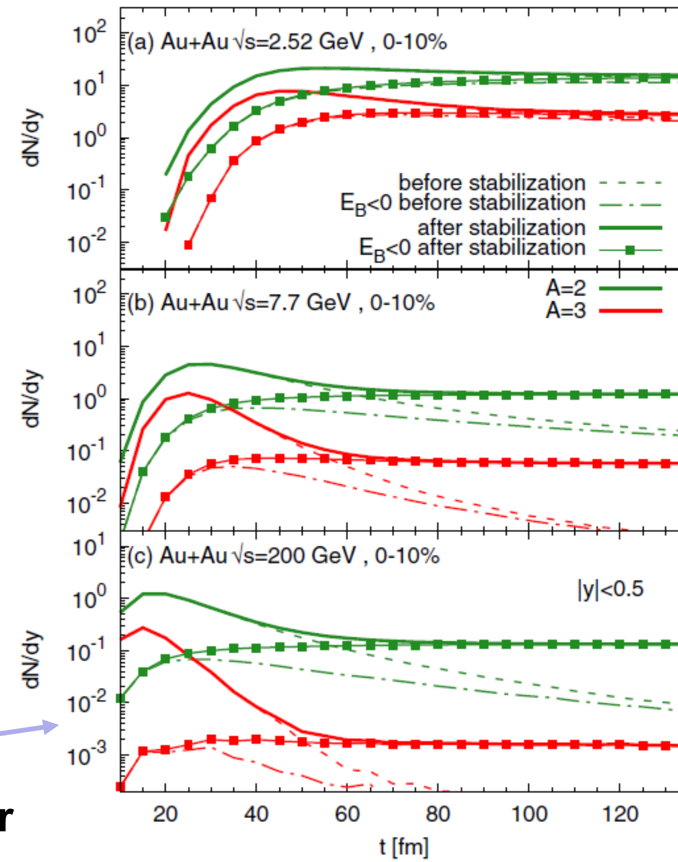
2. Particle is **bound to a cluster** if it **binds with at least one particle** of the cluster



* Remark: inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are almost never at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)

Advanced MST (aMST)

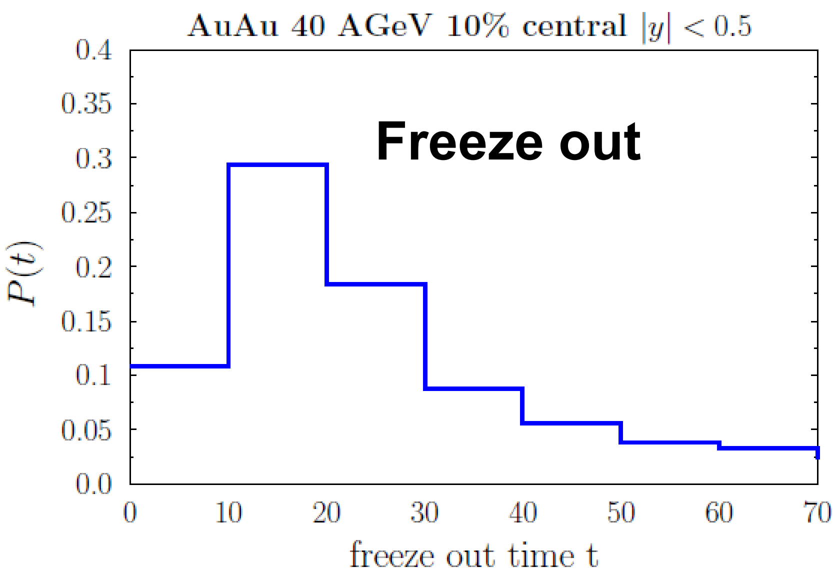
- MST **+ extra condition: E_B<0** **negative binding energy** for identified clusters
- **Stabilization procedure** – to correct artifacts of the semi-classical QMD: recombine the final "lost" nucleons back into cluster if they left the cluster without rescattering



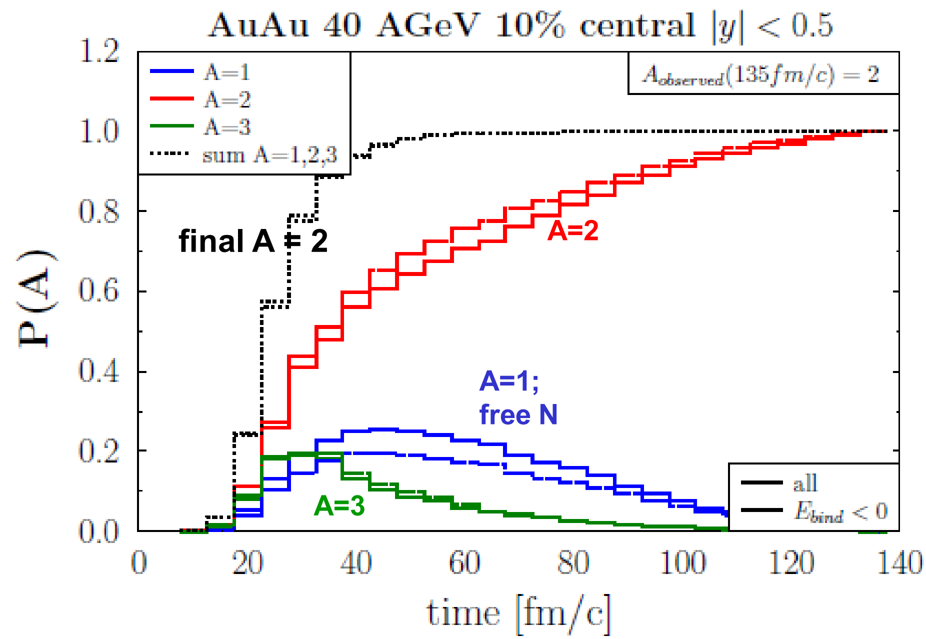
G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

When are the A=2 clusters formed?

- The normalized distribution of the **freeze-out time of baryons** (nucleons and hyperons) which are finally observed at mid-rapidity |y|<0.5



- The conditional probability **P(A)** that the nucleons, which are finally observed in A=2 clusters at time 135 fm/c, were at time *t* members of A=1 (free nucleons), A=2 or A=3 clusters



➔ **Stable clusters** (observed at 135 fm/c) **are formed shortly after the dynamical freeze-out**

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

II. Deuteron production by hadronic reactions

"Kinetic mechanism"
1) hadronic inelastic reactions NN ⟷ dπ , πNN ⟷ dπ , NNN ⟷ dN
2) hadronic elastic π+d, N+d reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
J. Staudenmaier et al., PRC 104 (2021) 034908
AMPT: R.Q. Wang et al. PRC 108 (2023) 3

- **Collision rate** for hadron "*i*" is the number of reactions in the **covariant volume** $d^4x = dt*dV$
- With **test particle ansatz the transition rate for 3→2 reactions**:

W. Cassing, NPA 700 (2002) 618

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

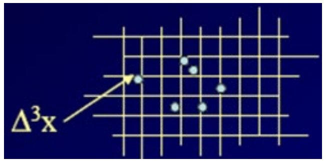
$$\boxed{P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

$$\boxed{P_{2,3}(\sqrt{s}) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}}$$

➔ solved by **stochastic method**



- Numerically tested in **"static" box:** PHQMD provides a good agreement with analytic solutions from rate equations and with SMASH for the same selection of reactions
- **New in PHQMD: π+N+N⟷ d+π** inclusion of **all possible isospin channels** allowed by total isospin T conservation ➔ enhancement of the d production

$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$
$\pi^- + p + p \leftrightarrow \pi^0 + d$
$\pi^+ + n + n \leftrightarrow \pi^0 + d$
$\pi^0 + p + p \leftrightarrow \pi^+ + d$
$\pi^0 + n + n \leftrightarrow \pi^- + d$

G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Modelling finite-size effects in kinetic mechanism

G. Coci et al., PRC 108 (2023) 014902

How to account for the **quantum nature of deuteron**, i.e. for

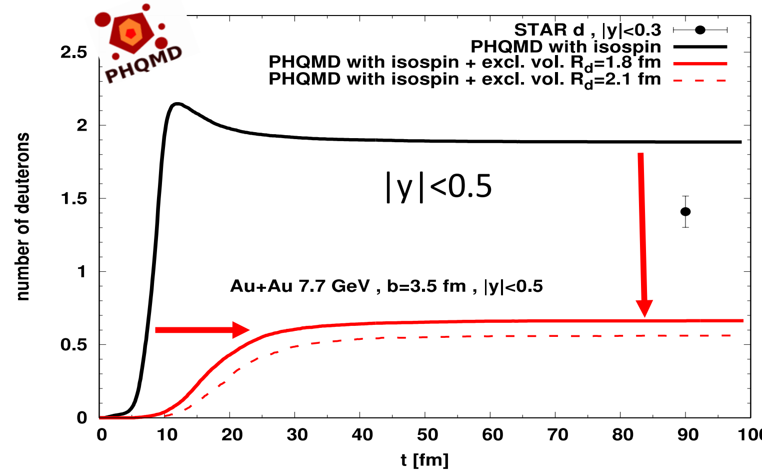
1) the **finite-size of *d* in coordinate space** (*d* is not a point-like particle) – for in-medium d production
2) the **momentum correlations of *p* and *n*** in the entrance channel

Realization:

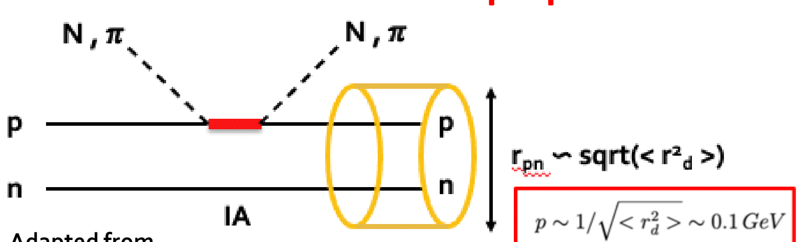
1) assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume':

Excluded-Volume Condition: $\boxed{|\vec{r}(i)^* - \vec{r}(d)^*| < R_d}$

- ❑ **Strong reduction of d production**
- ❑ p_T slope is not affected by excluded volume condition

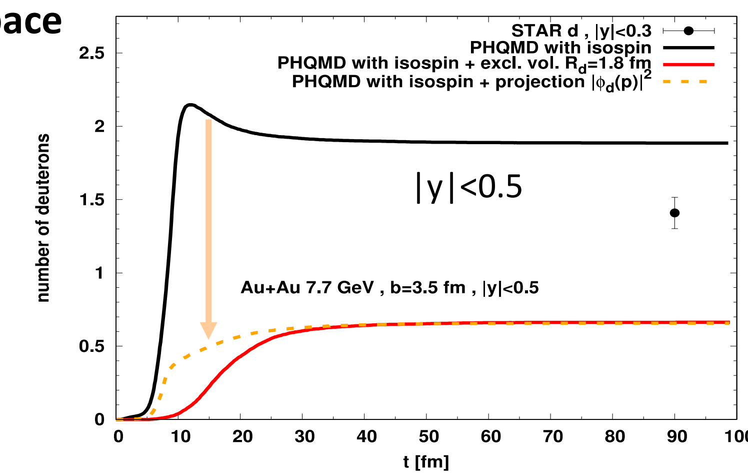


2) QM properties of deuteron must be also in momentum space
→ **momentum correlations of pn-pair**



$$p \sim 1/\sqrt{<r_d^2>} \sim 0.1\,GeV$$

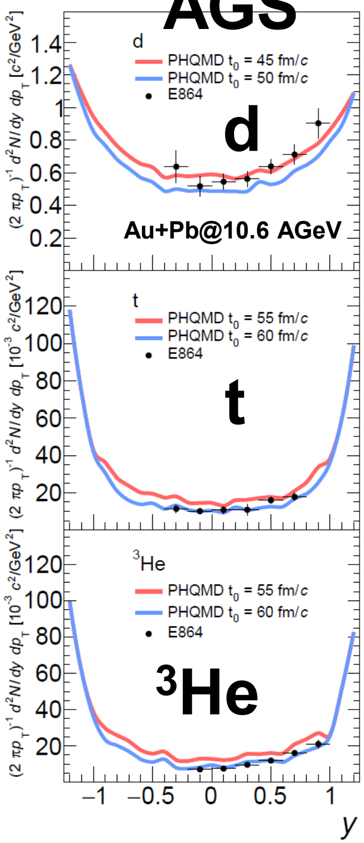
Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]
Same spirit as AMPT [K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742]

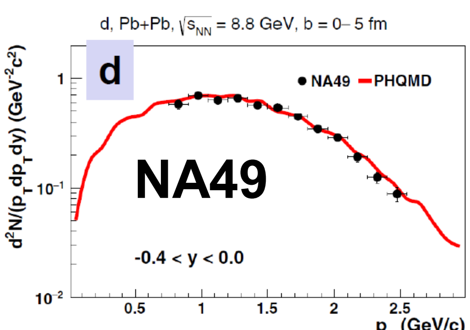


- ❑ **Strong reduction of d production at early times** by projection on DWF $|\phi_d(p)|^2$

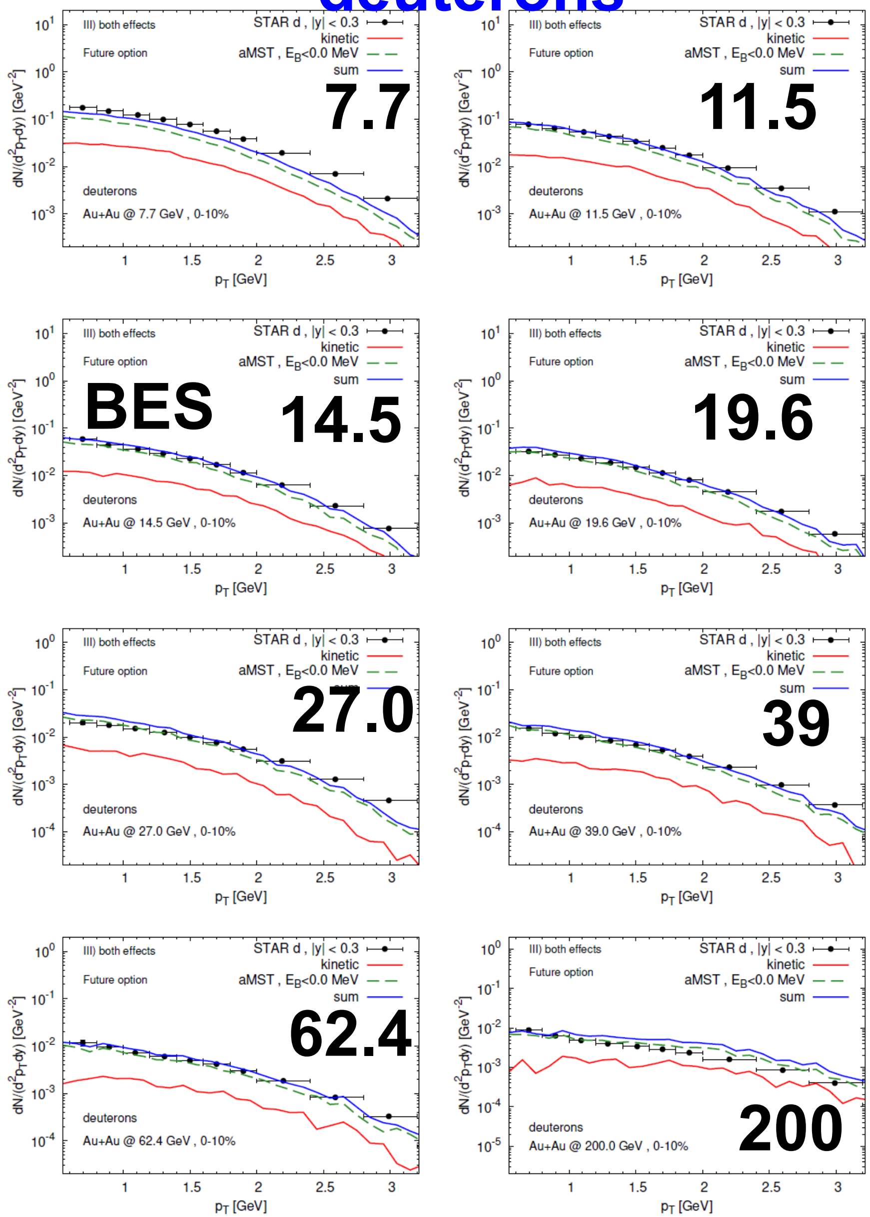
Highlights: PHQMD cluster and hypernuclei dynamics from SIS to RHIC

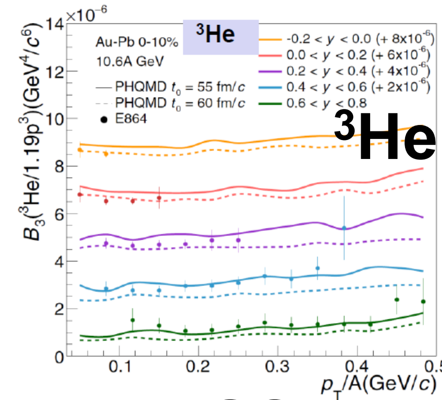
AGS





deuterons





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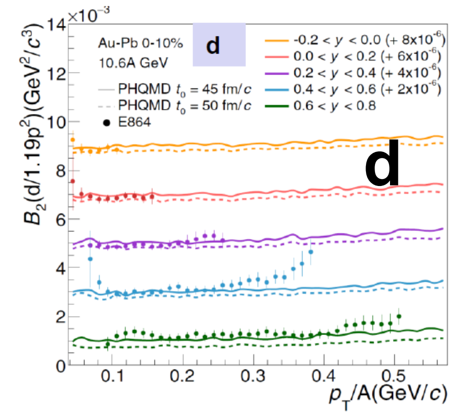

PHQMD:
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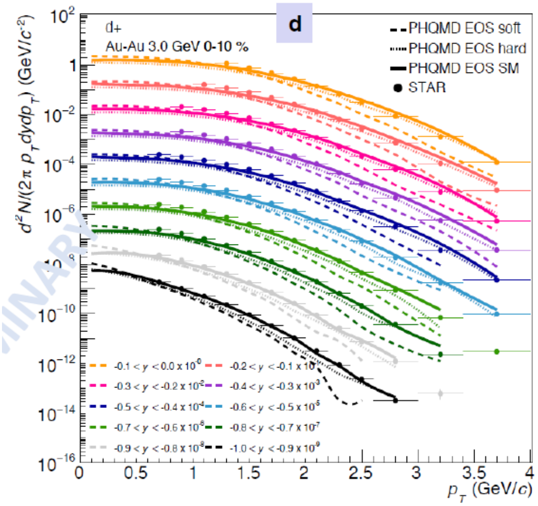
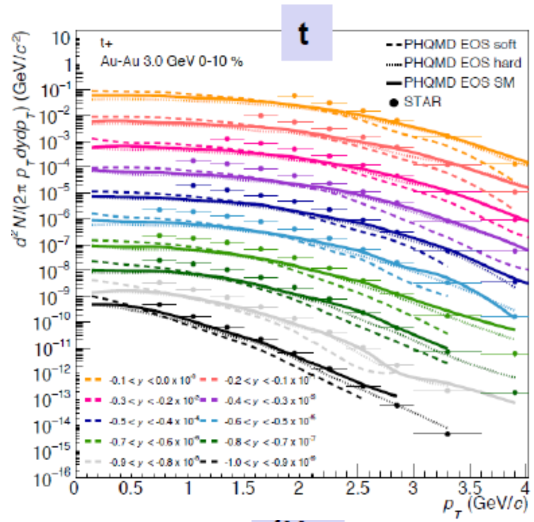
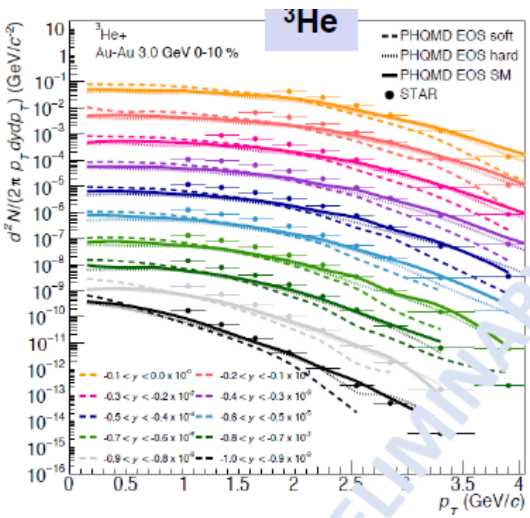
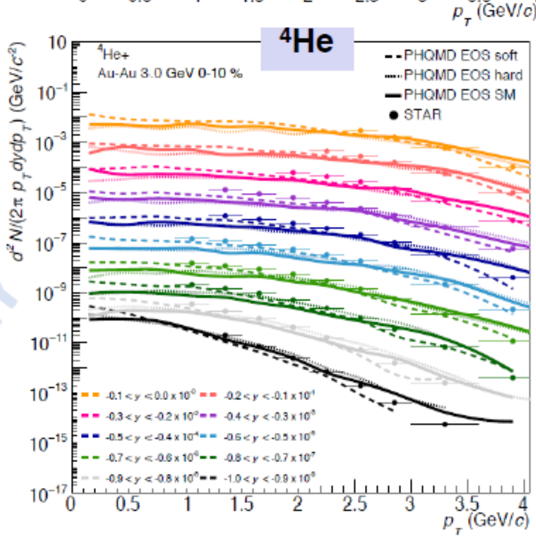
Light cluster production at $s^{1/2}$ = 3 GeV

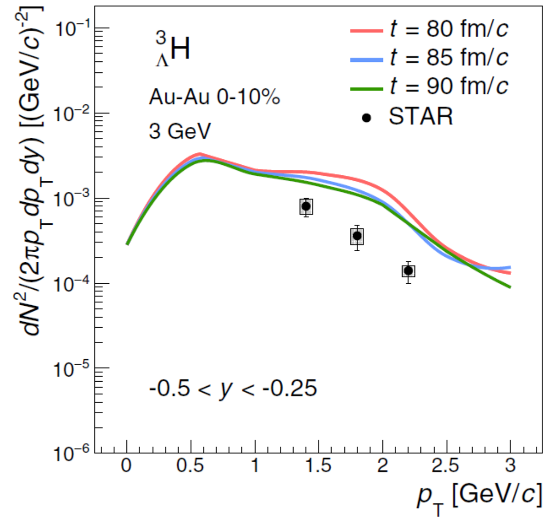
The PHQMD comparison with recent STAR fixed target p_T distribution of p, d, t, ^{3}He, ^{4}He from Au+Au central collisions at $\sqrt{s} = 3$ GeV

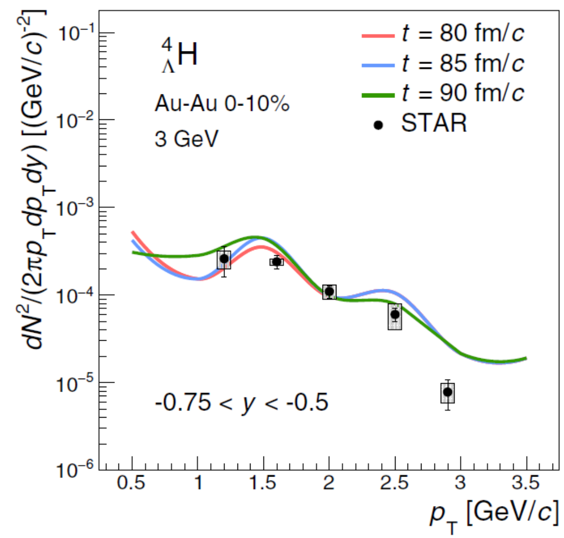




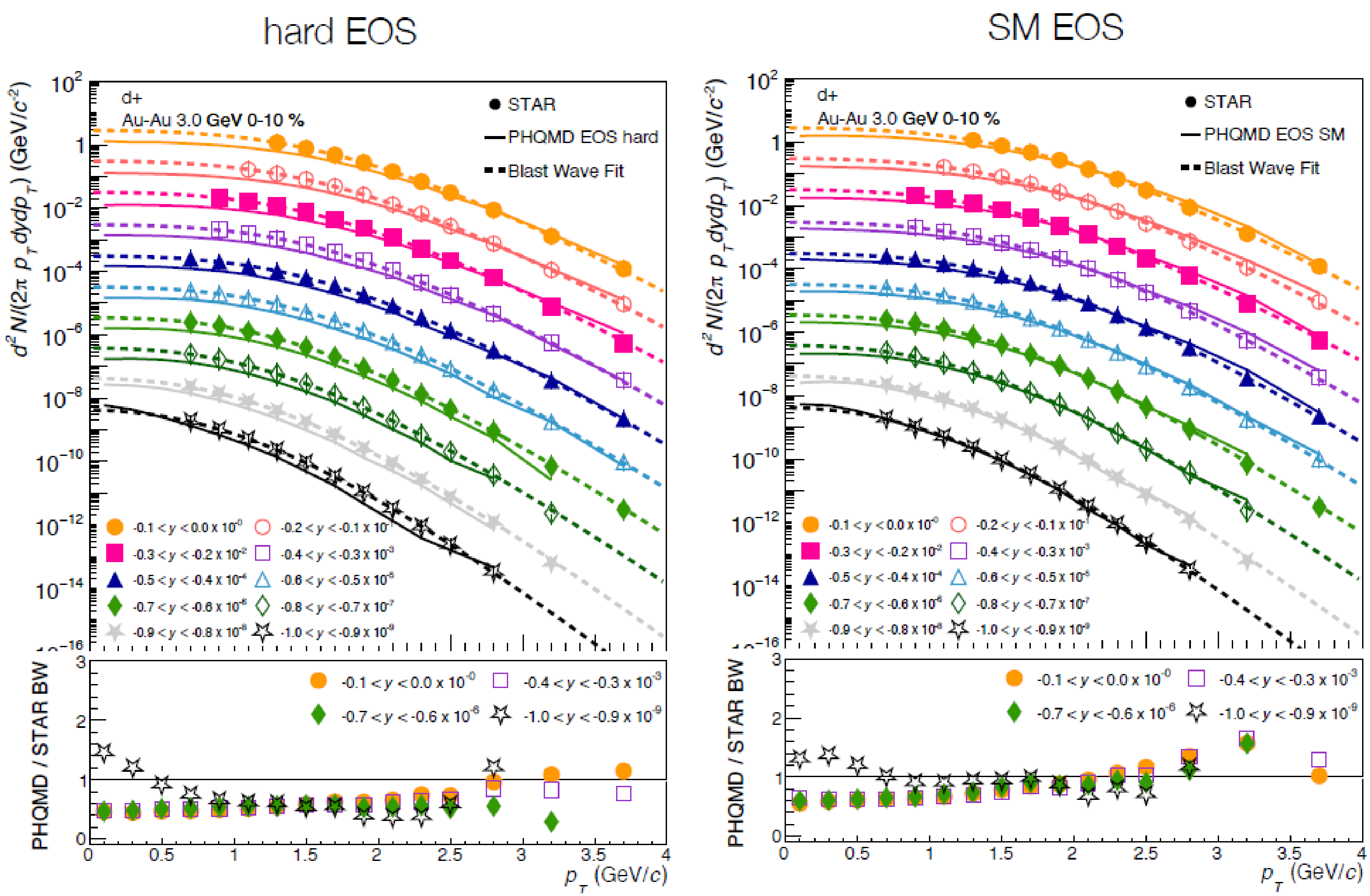








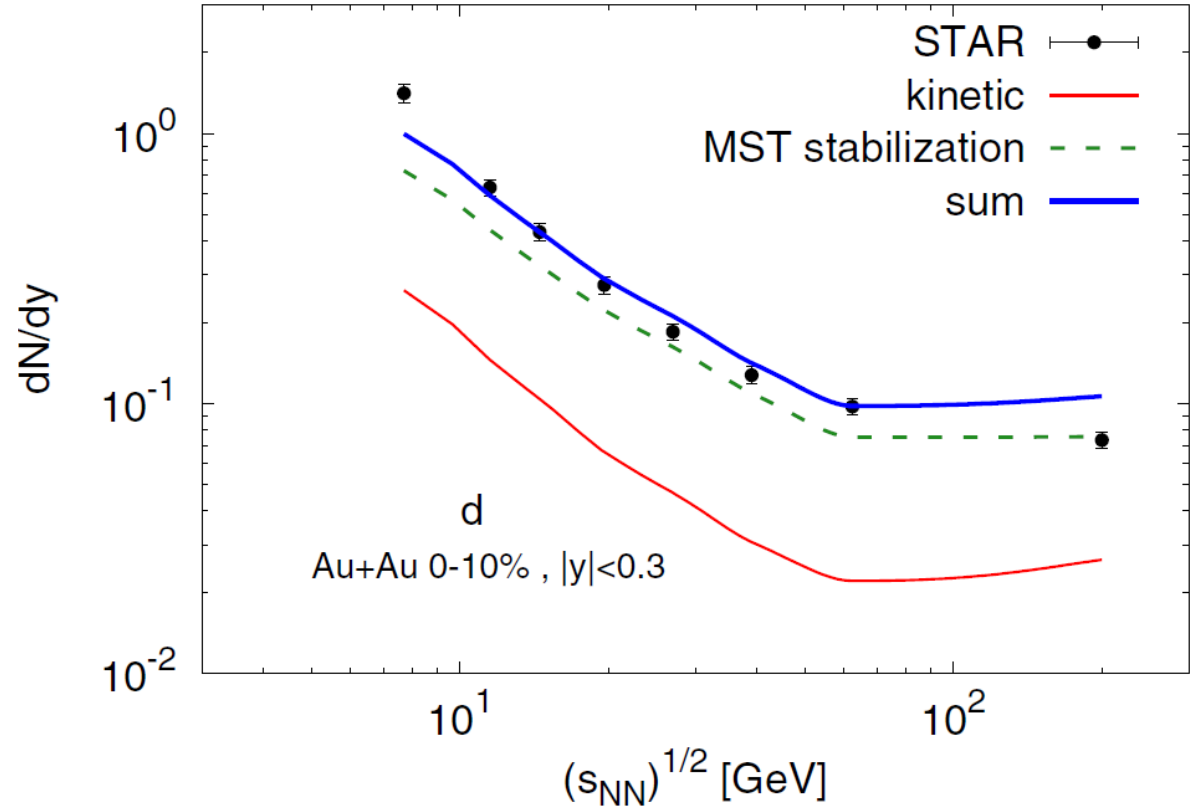
More in detail



SM describes data best
difference PHQMD-data at low p_T → blast wave fits ok?

Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



- **Very continuous as a function of $\sqrt{s}$**
- **Functional form similar for kinetic and potential deuterons**
- **PHQMD provides a good description of STAR data**
- **The potential mechanism is dominant for d production at all energies!**

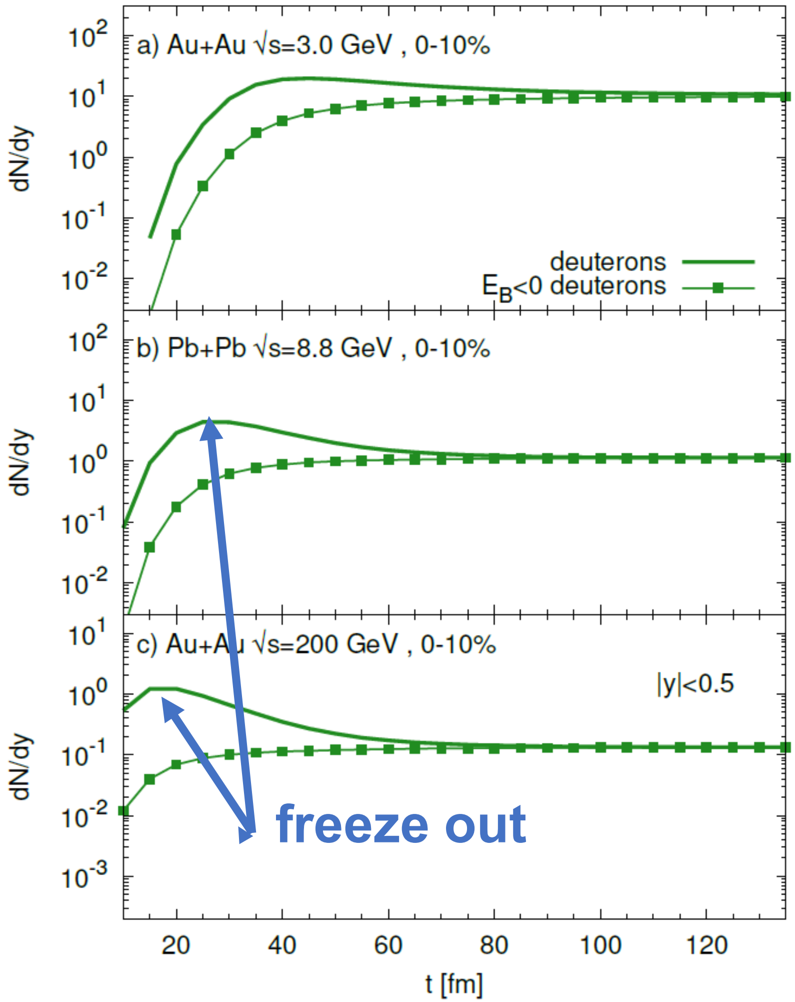
G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Can the production mechanisms be identified experimentally?

PHQMD

Coalescence in PHQMD and UrQMD

MST deuterons



Why may the observables be different in coalescence and in MST?

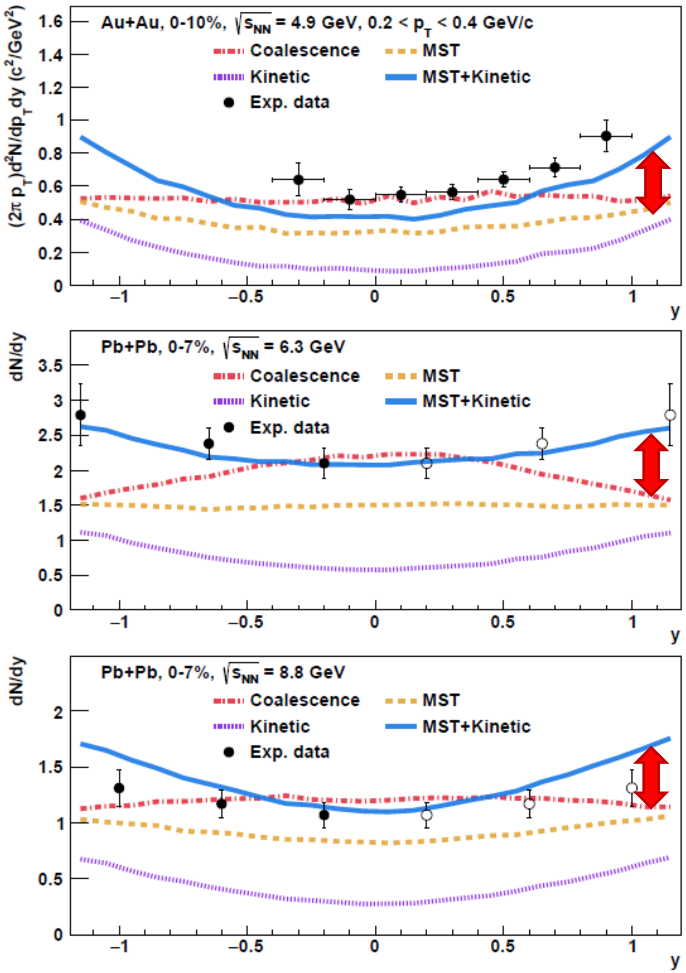
Same simulation

- **Coalescence deuterons produced earlier**
- **Most of the coalescence deuterons unbound**
- **Factor 3/8 brings them to the physical value**
- **Many surrounded by other hadrons when produced**

Coalescence parameters from UrQMD → in PHQMD

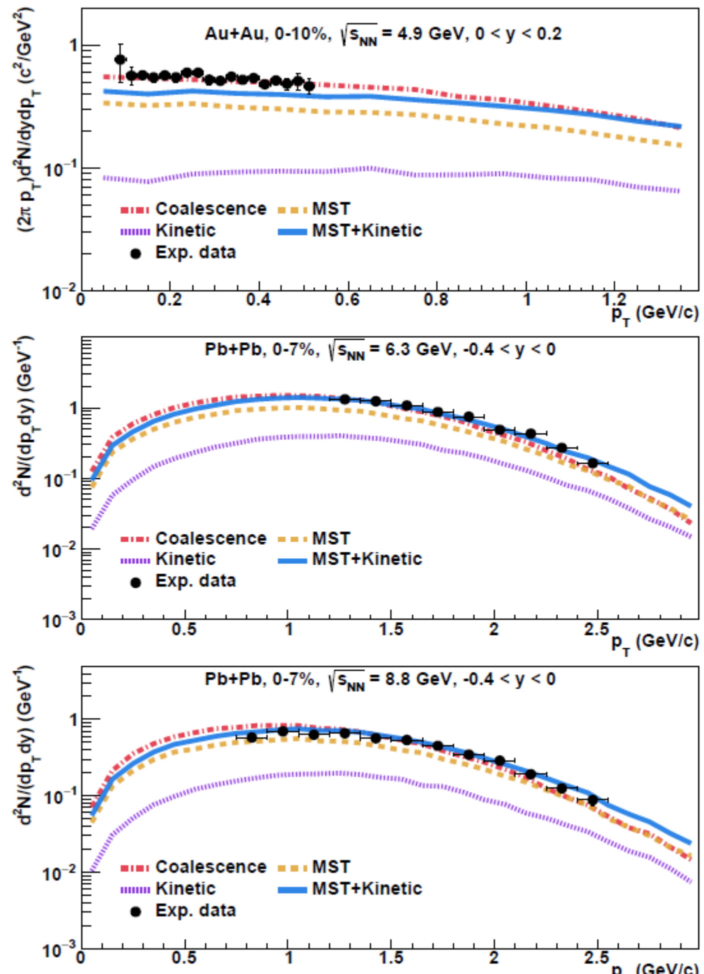
Coalescence and MST (potential) deuterons calculated in the same PHQMD run

Mechanism for cluster production
coalescence and MST ⟺ experimental data



Deuterons:

- p_T distributions similar for coalescence/ MST-kinetic
- y- distributions show differences

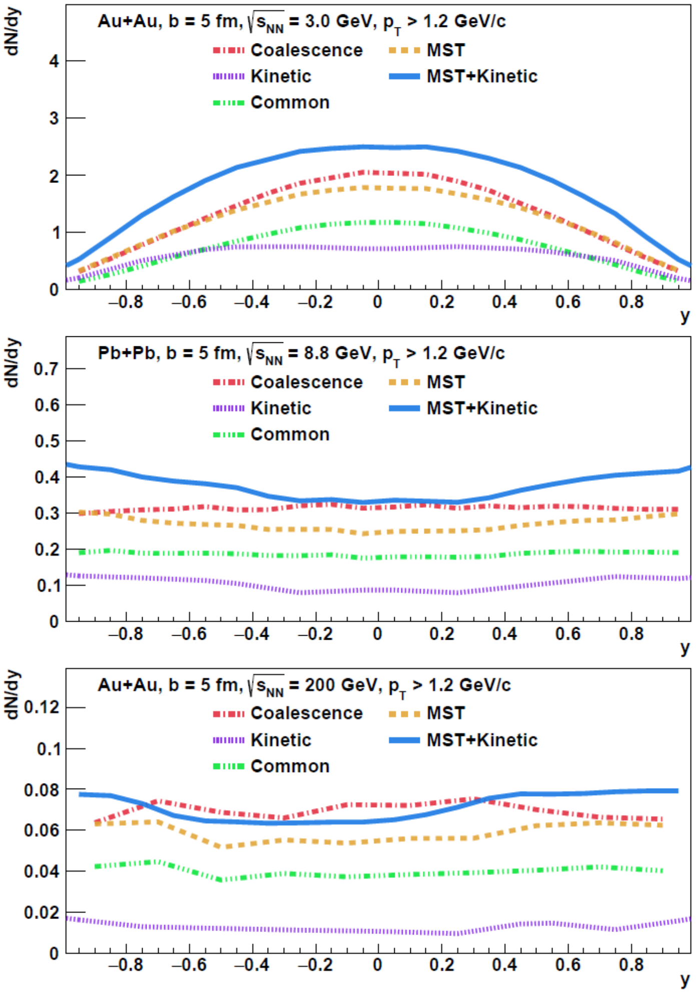


The analysis of the presently available data points tentatively to the MST + kinetic scenario but further experimental data are necessary to establish this mechanism.

V. Kireyeu et al., PRC109, 044906

Difference big enough for an experimental decision?

p_T >1.2 GeV (experimental acceptance)



Difference between COAL and MST mostly at low p_T

In the measured p_T range signal is gone for $\sqrt{s}$ = 3 GeV

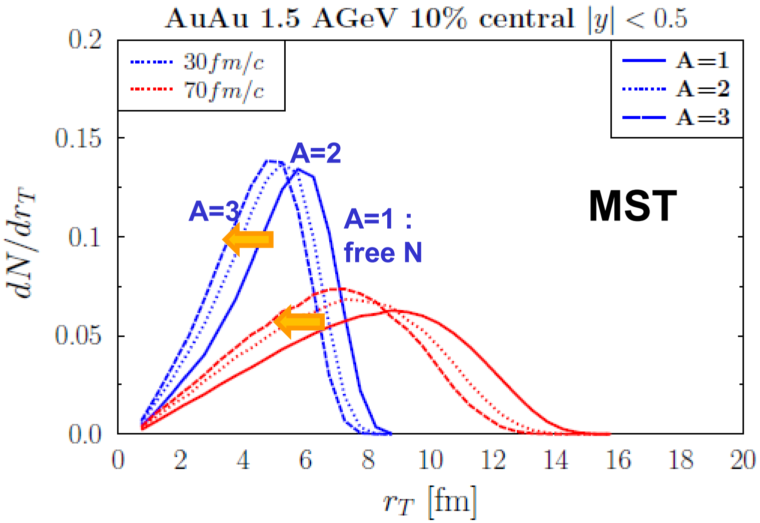
But: there seems to be a sweet spot around $\sqrt{s} = [6-8]$ GeV to identify the reaction mechanism

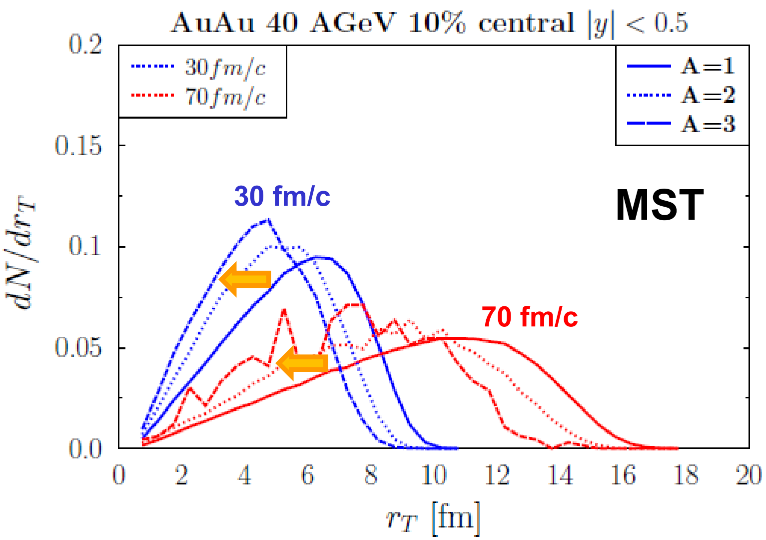
➔ We have to wait for more precise rapidity distributions

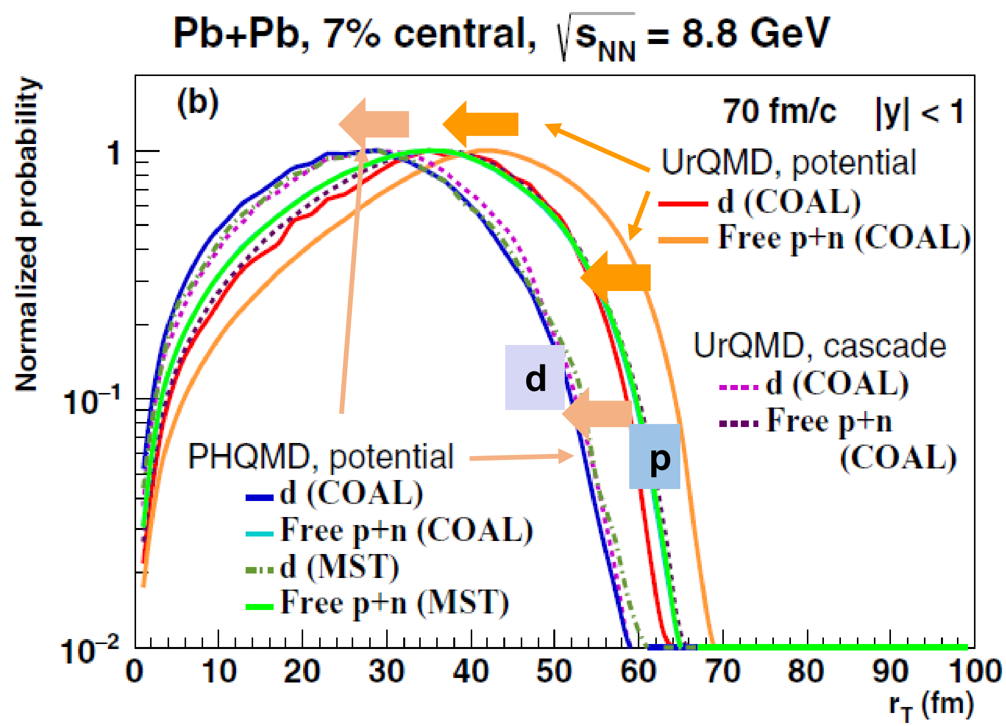
Where the clusters are formed?

PHQMD

PHQMD and UrQMD: Where clusters are formed?







➔ COAL(escence) as well as the MST show that the deuterons remain in transverse direction closer to the center of the heavy-ion collision than free nucleons

➔ deuterons are behind the fast nucleons (and the pion wind)

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

Clusters are formed dynamically

1) by potential interactions among nucleons and hyperons
   Novel development: momentum dependent potential with soft EoS
2) for d also by kinetic mechanism: hadronic inelastic reactions $NN \leftrightarrow d\pi$, $\pi NN \leftrightarrow d\pi$, $NNN \leftrightarrow dN$ with inclusion of all possible isospin channels which enhance d production
   + accounting for quantum properties of d, modelled by the finite-size excluded volume effect in coordinate space and projection of relative momentum of p+n pairs on d wave-function in momentum space leads to a strong reduction of d production

- PHQMD reproduces cluster and hypernuclei data of dN/dy and dN/dp_T as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from AGS to top RHIC energies.
- Measurement of dN/dy beyond mid-rapidity seems to distinguish the mechanisms for cluster production: coalescence versus dynamical cluster production recognized by MST + kinetic mechanism for deuterons

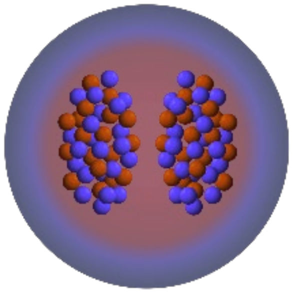
- Dependencee of y- and p_T-spectra (and v_1,v_2) on EoS - soft, hard, soft-mom. dependent - at SIS energies
- The influence of U(p) decreases with increasing collision energy since the modelled $U_{SEP}(p)$ has a maximum at energy 1.5 GeV and decreases for large p ← no exp. data for extrapolation of $U_{SEP}(p)$ to large p!
- HADES data data on v_1,v_2 STAR data at 3 GeV favour a soft momentum dependent potential (SM)

What did we learn
(besides that PHQMD describes the data)?

- Cluster production at midrapidity is a smooth process from $\sqrt{s}$ 2.4 GeV to 5 TeV
- Stable clusters are formed (shortly) after elastic and inelastic collisions have ceased
- They are formed behind the front of the expanding energetic hadrons
- They can survive the expansion because '' ice does not meet the 'fire'
- This result is the same for the PHQMD and UrQMD transport approaches
  (and very probably this is true for all other transport approaches)
- Coalescence as well as MST(+kinetic) can describe the data
  however: to describe A[2-4] (and at low energy larger A)
  MST does not need any (free) parameters for cluster production
  Coalescence needs two for deuterons, 4 for ^{3}He,t ….. + problem of hadrons close by

Major problem to be solved:

- complete relativistic kinematics
- how to project classical phase space distributions on quantum states



Thank you for your attention !

Thanks to the Organizers !