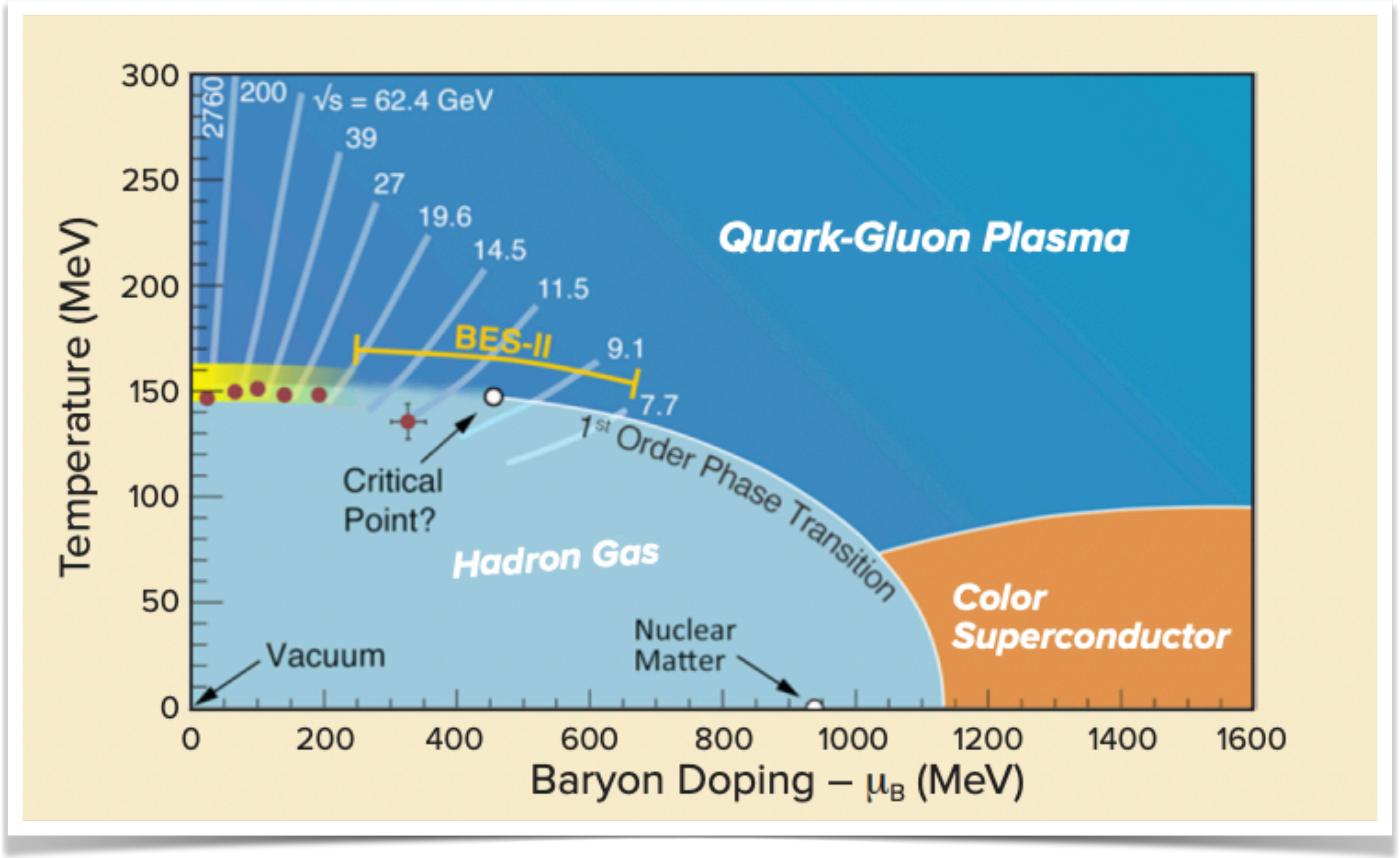


Theory Overview: Critical Point And First-Order Boundary

Critical Point and the Onset of Deconfinement, 2024

Maneesha Pradeep
University of Maryland at College Park

How can the theory community support the experimental efforts in discovering the critical point?

- Solve the sign problem for QCD —NP-Hard problem(?) Troyer and Wiese, PRL, 2005, Alexandru, Basar, Bedaque,Warrington, Rev. Mod. Phys. **94**, 015006,…
- Provide guidances for the location of the critical point
- Integrated framework : Observables ⟶ Equation of State

Current constraints on the location of the critical point from Lattice QCD

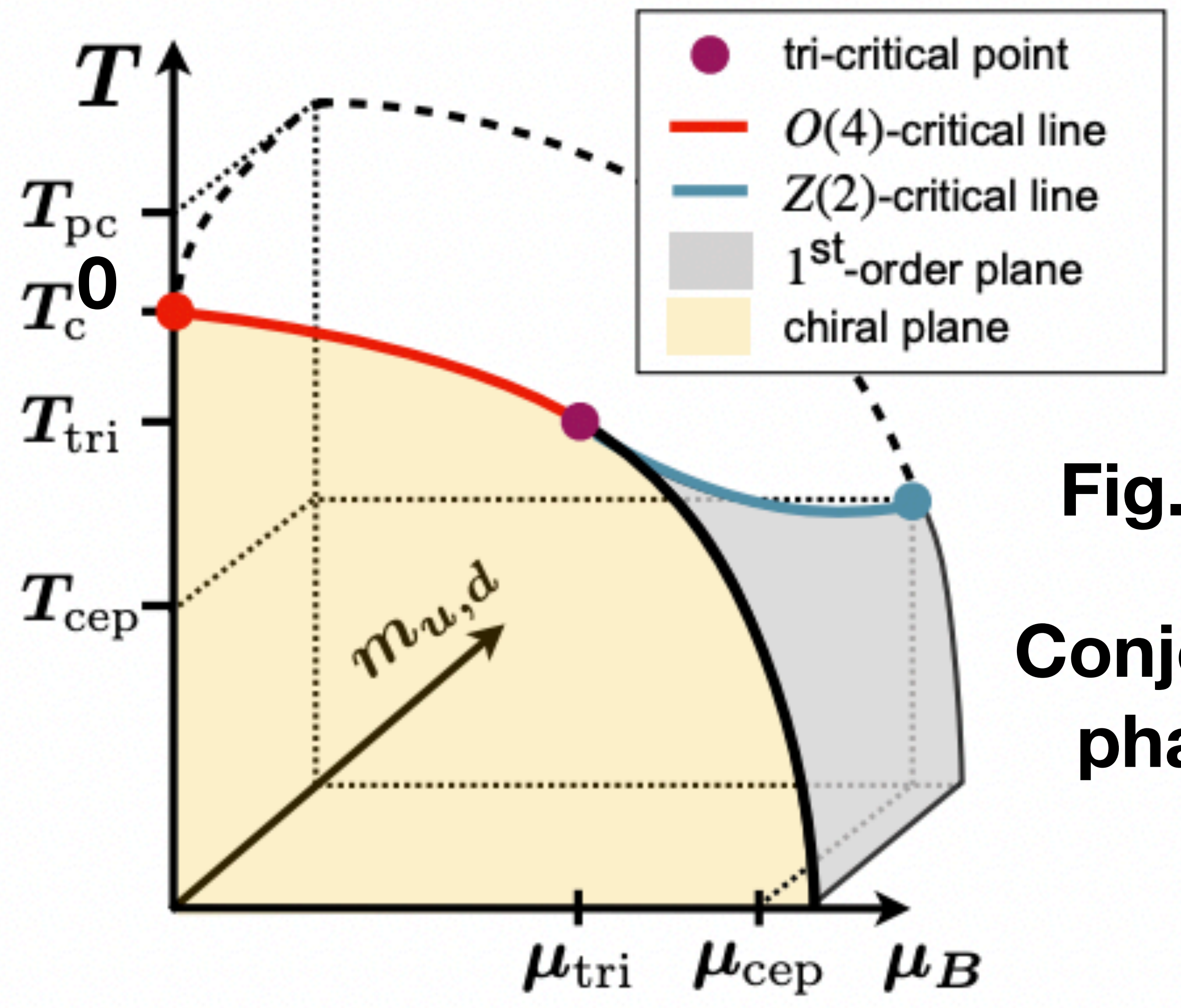


Fig. : Ding et al , 24

Conjectured chiral phase diagram

$T_c^0 > T_{\rm tri} > T_c$

Hatta, Ikeda, 03 $T_c^0 = 132^{+3}_{-6}\,\mathrm{MeV}$ **HotQCD, Ding et al, 19**

Pseudo-critical curve at physical quark mass

$$T_{\rm pc}(\mu_B) = T_{\rm pc}(0)\left[1+\kappa_2\left(\frac{\mu_B}{T_{\rm pc}}\right)^2\right]$$

Bazavov et al, 19, Ding et al,24

$$T_{\rm pc}(0) = 156 \pm 1.5\,\mathrm{MeV}\,,\ \kappa_2 = 0.012(4)$$

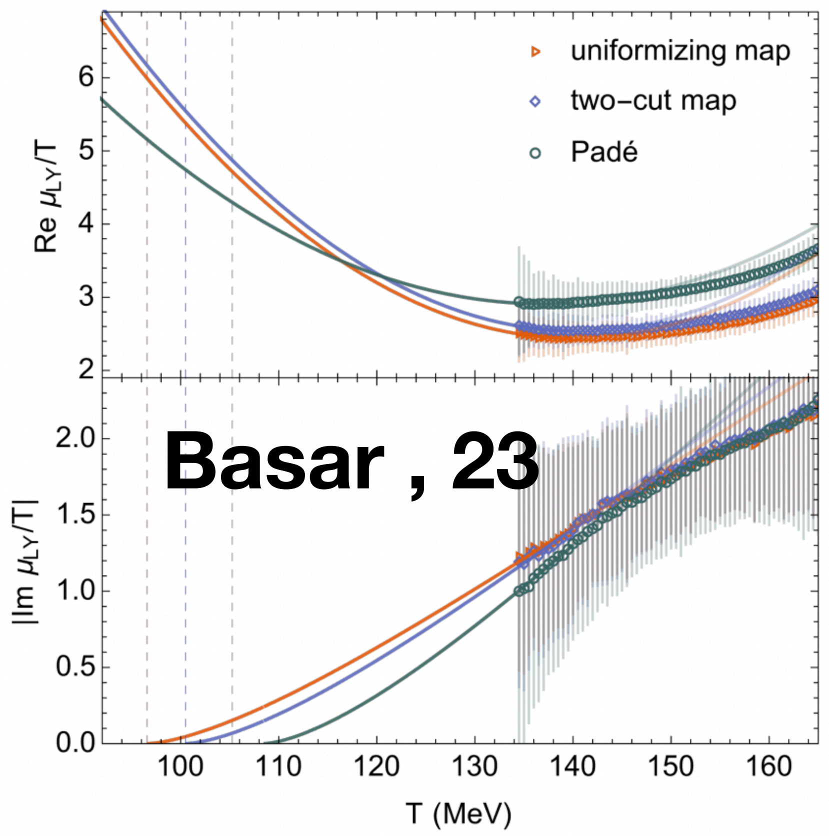
Borsanyi et al,20

$$T_{\rm pc}(0) = 158 \pm 0.6\,\mathrm{MeV}\,,\ \kappa_2 = 0.0153(18)$$

Critical point disfavored for $\mu_B/T < 3$

Discussions on Columbia Plot - S. Gupta, Thurs, 2:20 pm

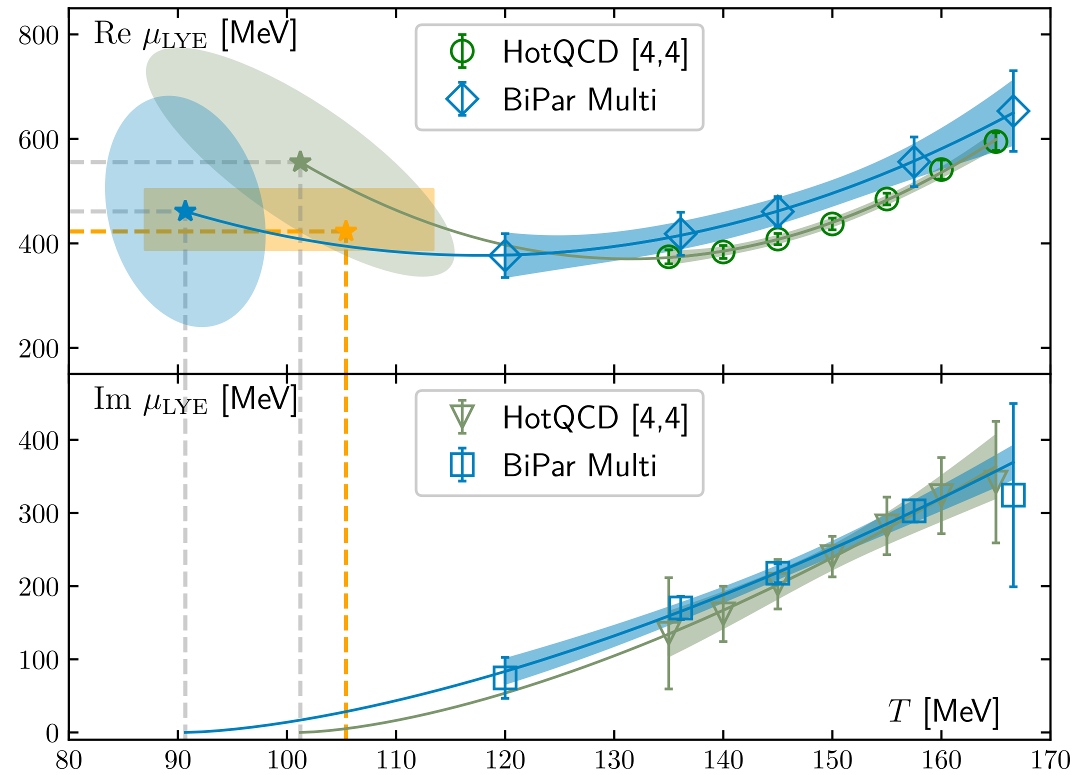
Guidances for location of the critical point



G. Basar, Fri, 11:40 am

Extrapolations of Lee-Yang edge singularities to real axis

$$(\mu_{BC}, T_c) \approx (580, 100)\ \mathrm{MeV}$$



C. Schmidt, Tue, 11:00 am

Clarke et al., 24

$$(\mu_{BC}, T_c) = (422^{+80}_{-35}, 105^{+8}_{-18})\ \mathrm{MeV}$$

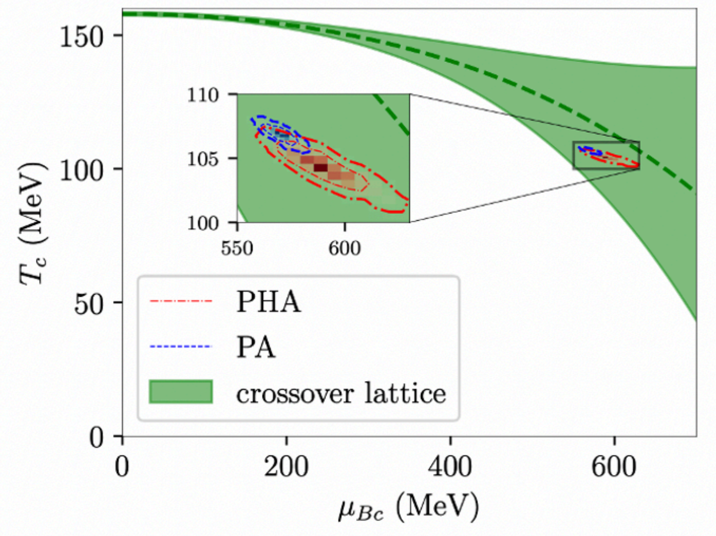
Bayesian Holography + Lattice input at $\mu = 0$

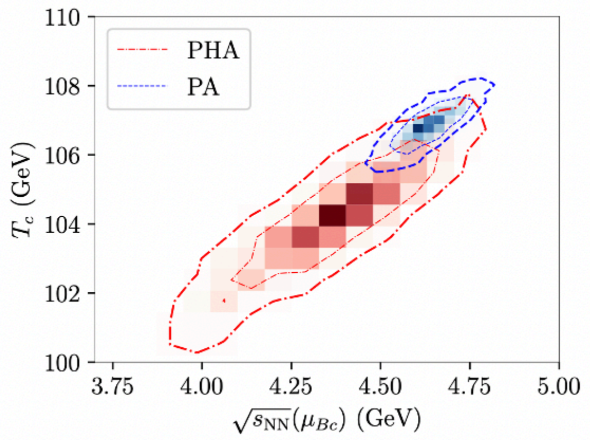
Hippert et al, e-Print: 2309.00579 [nucl-th]

Predict CEP (95% confidence level):

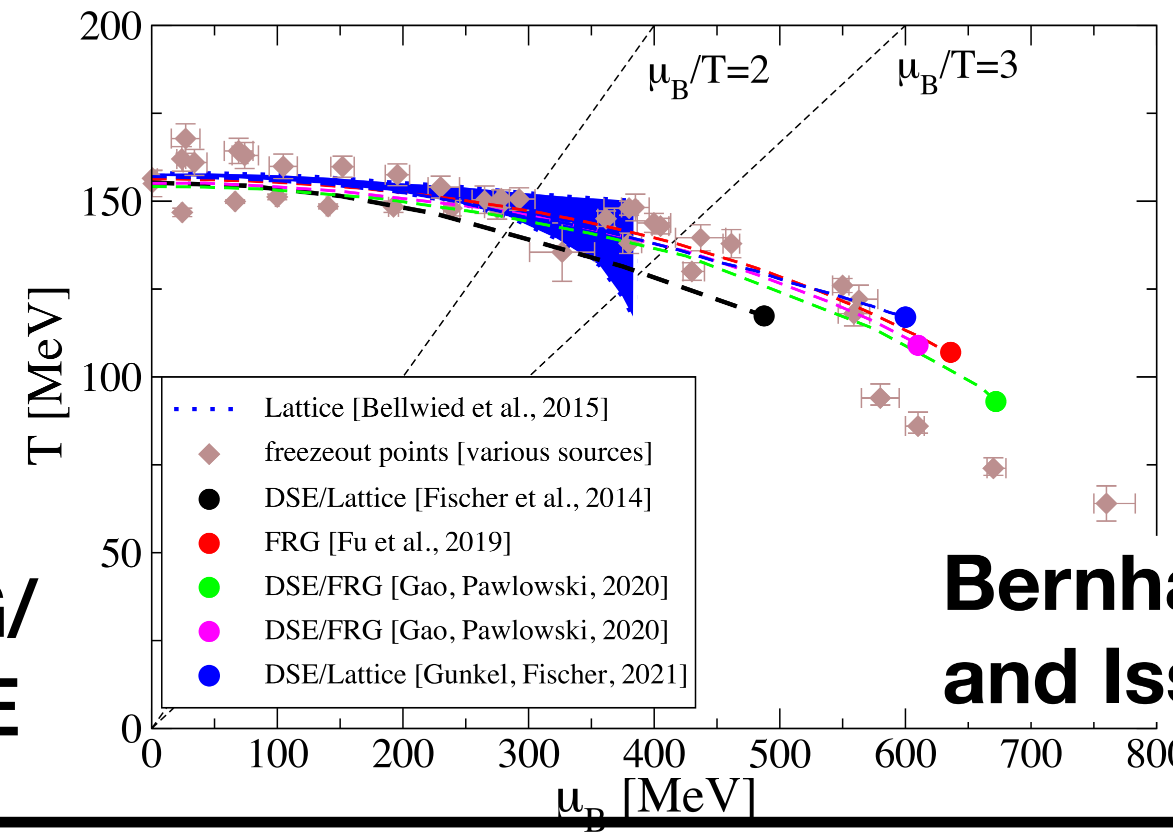
$$T_c = 101 - 108\ \mathrm{MeV} \quad \mu_c = 560 - 625\ \mathrm{MeV}$$

$$\sqrt{s} = 4.0\ -\ 4.8\ \mathrm{GeV}$$





J. Noronha, Tue, 11:40 am



FRG/ DSE

C. Fischer, Tue, 11:20 am

Bernhardt, Fischer and Isserstedt, 23

$$(\mu_{BC}, T_c) = (495 - 654, 108 - 119)\ \mathrm{MeV}$$

Equation of States with a QCD critical point

- Must agree with the Taylor Expanded EoS from lattice

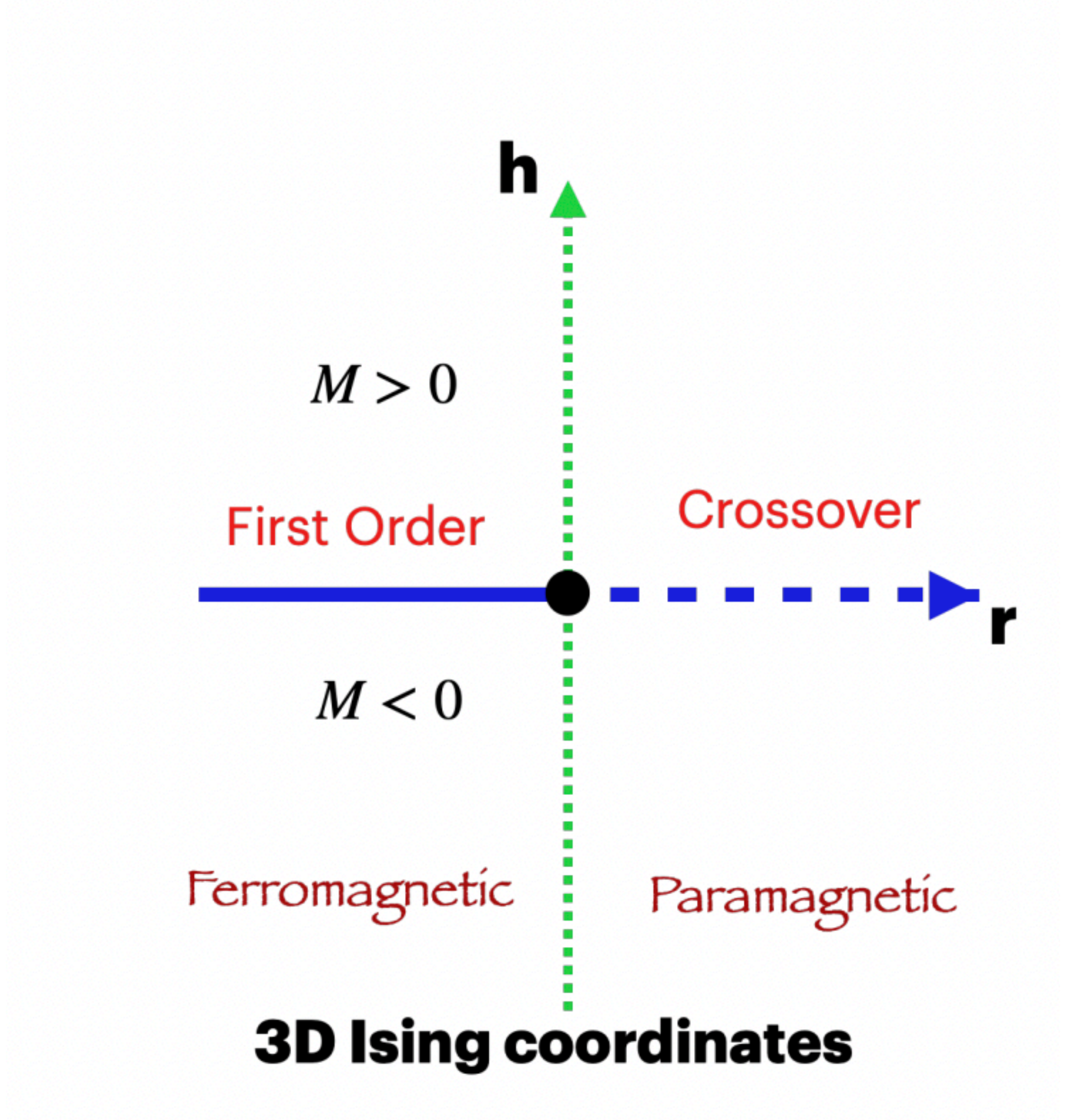
M. Kahangirwe, Wed, 12:10 pm

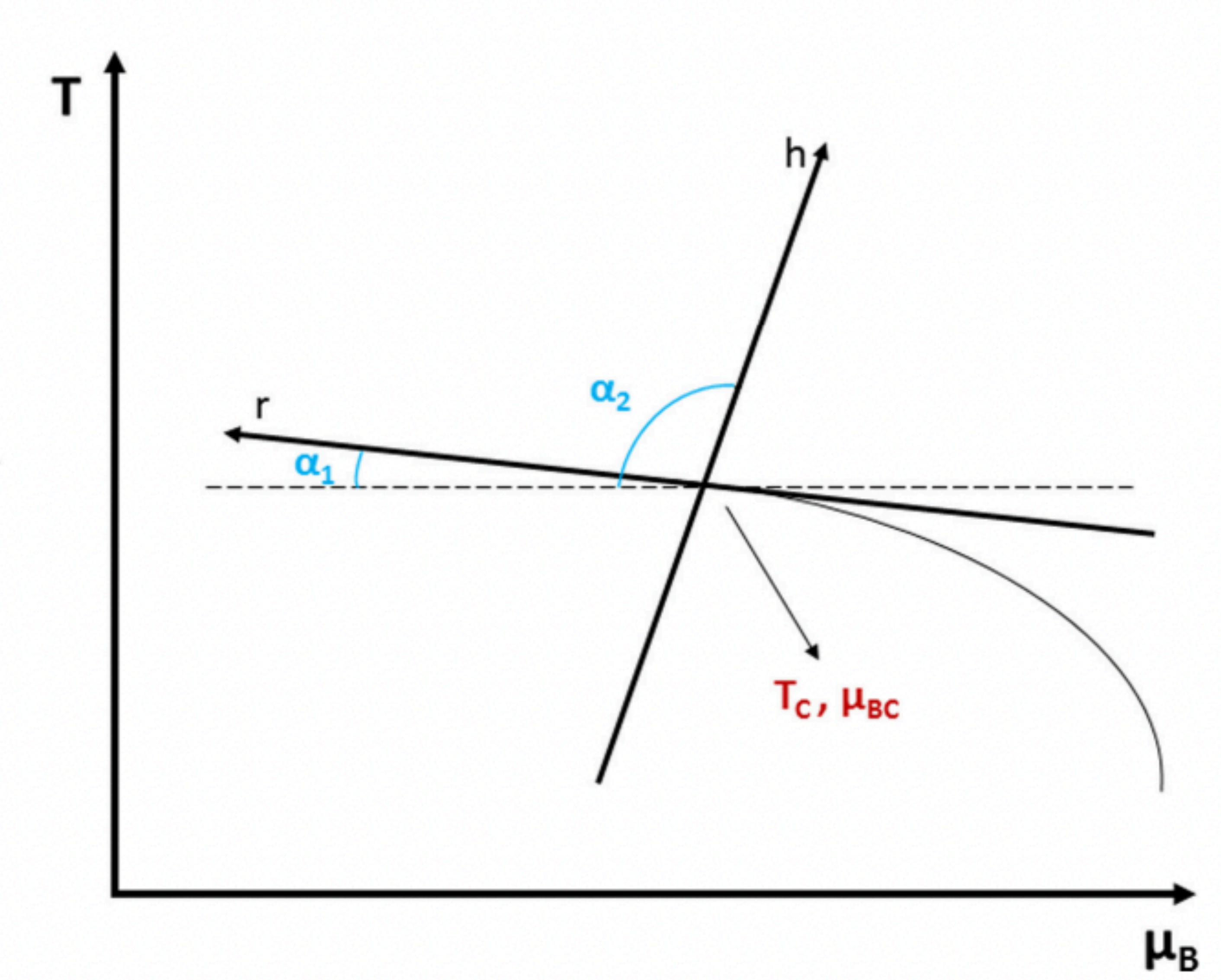
- Compatible with other limits : PQCD, HRG
- Critical point in the 3D Ising universality class

J. Noronha, Tue, 11:40 am

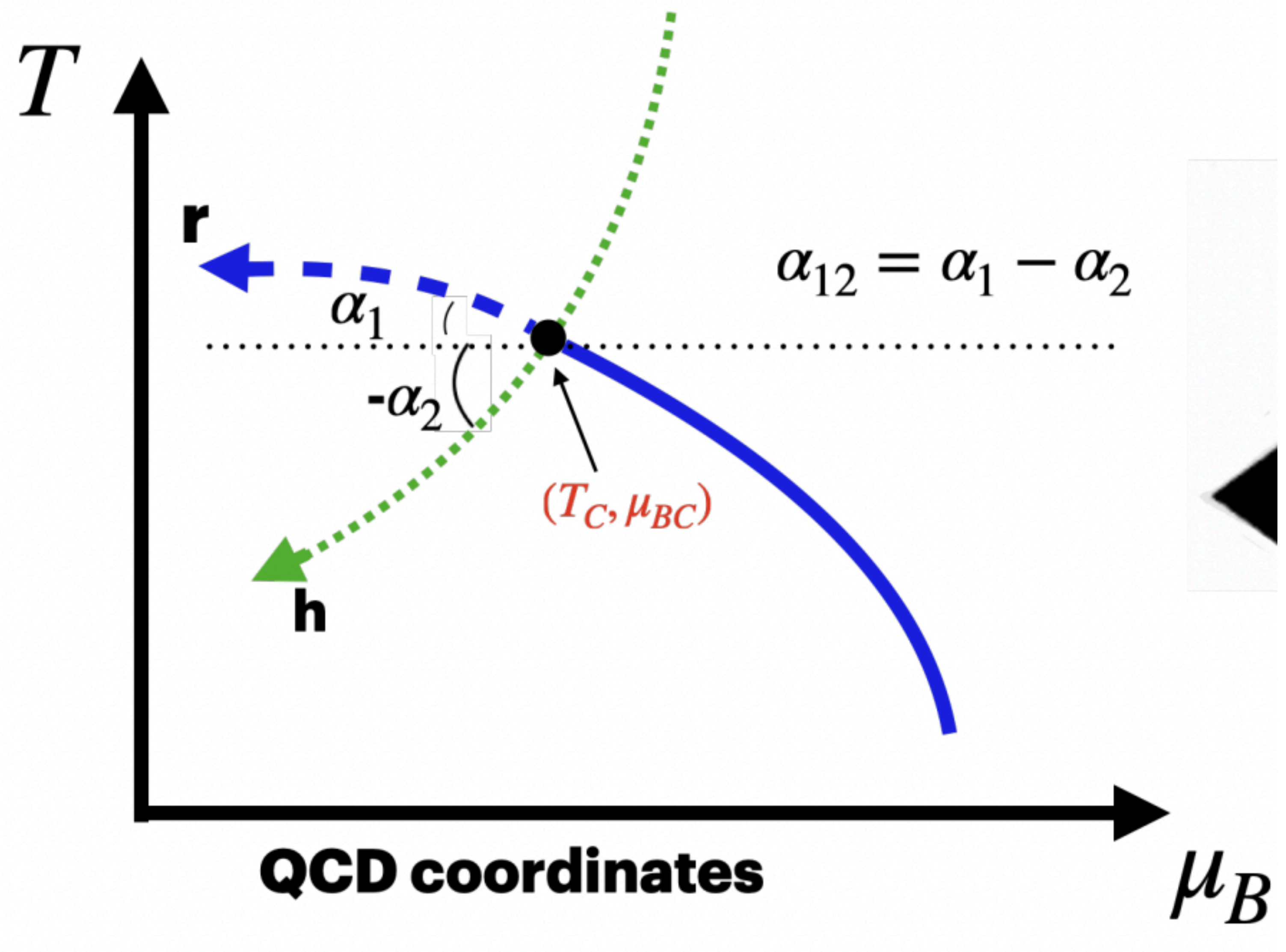
Examples of recently developed EoSs that have a CP in the Ising universality class but differ in their implementation: Parotto et al, 19, Karthein et al.,21, Grefa et al., 21,Kapusta&Welle,22, Kahangirwe et al.,24

QCD EoS near the Critical Point





Parotto et al., 18, Karthein et al., 21



M. Kahangirwe, Wed, 12:10 pm

Kahangirwe et al., 24

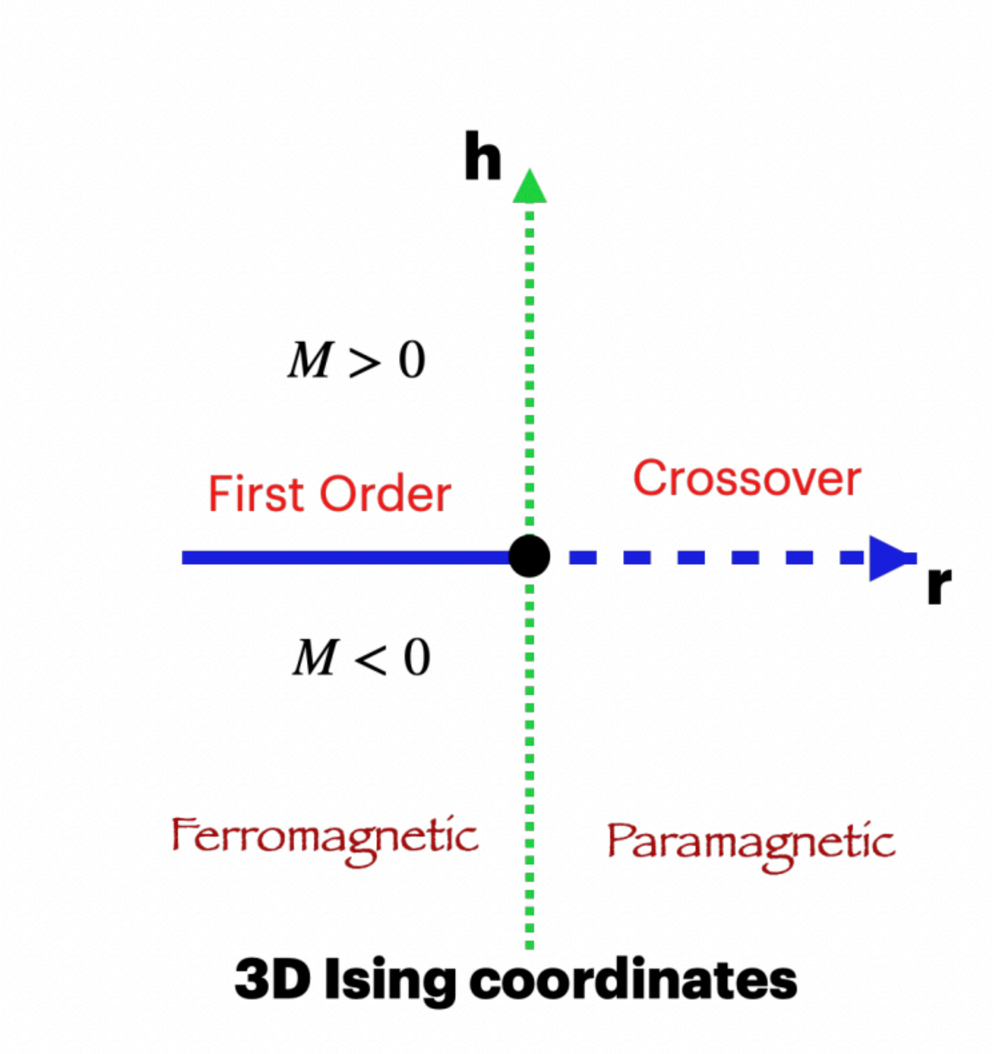
Summation scheme by WB collaboration
Borsanyi et al,21

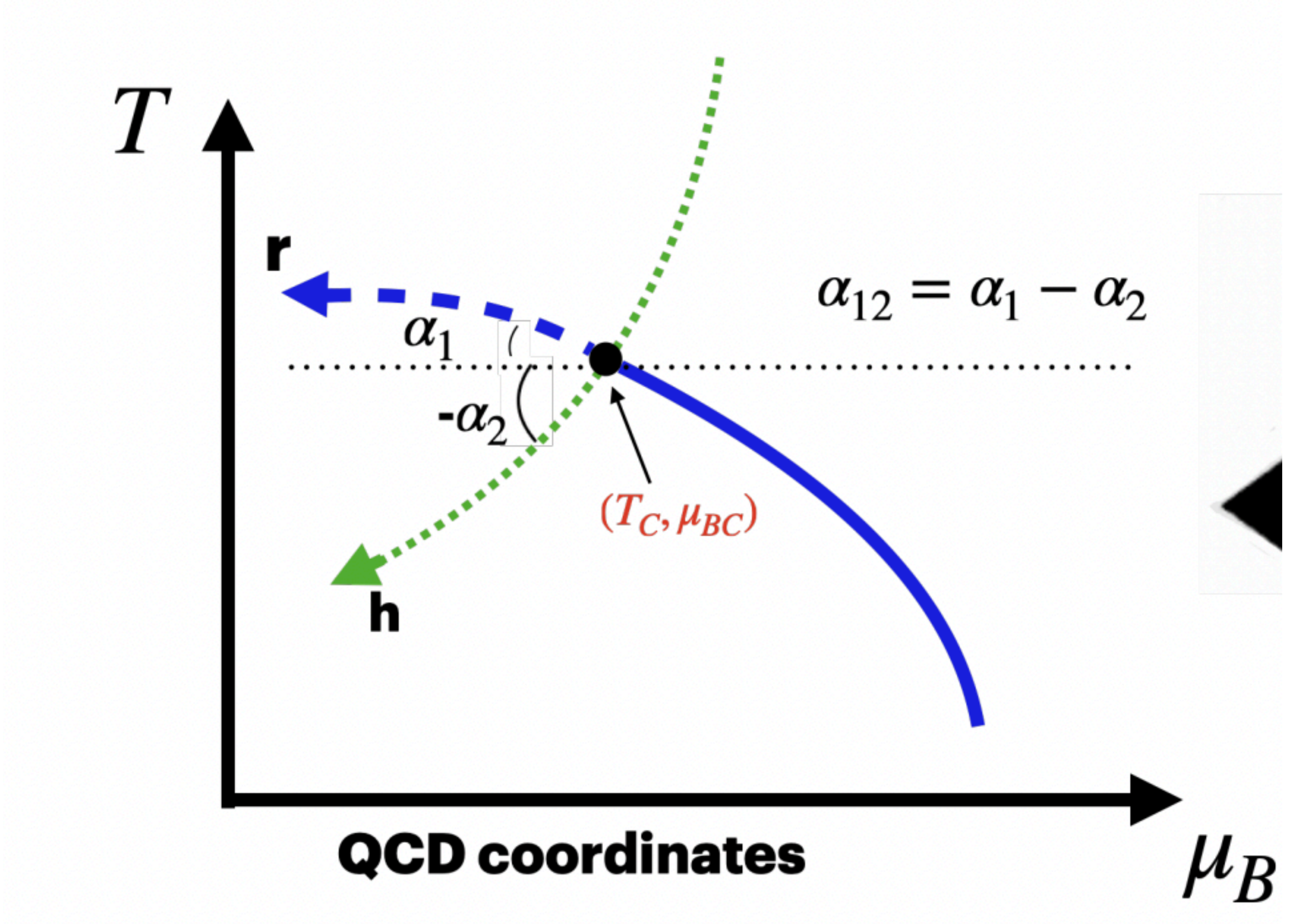
Non-universal map from QCD to Ising variables

$$P_{\rm QCD}(\mu, T) = P_{\rm BG}(\mu, T) + A\,G(r(\mu, T), h(\mu, T))$$

A general class of candidate EoSs

$$P_{\rm QCD}(\mu, T) \;=\; P_{\rm BG}(\mu, T) \,+\, A\, G(r(\mu, T), h(\mu, T))$$





7

Kahangirwe et al., 24

Independent & non-universal parameters

$$\mu_c \,,\; \alpha_{12} \,,\; \rho \,,\; w$$

Weakly constrained in the chiral limit

MP, Stephanov, 19

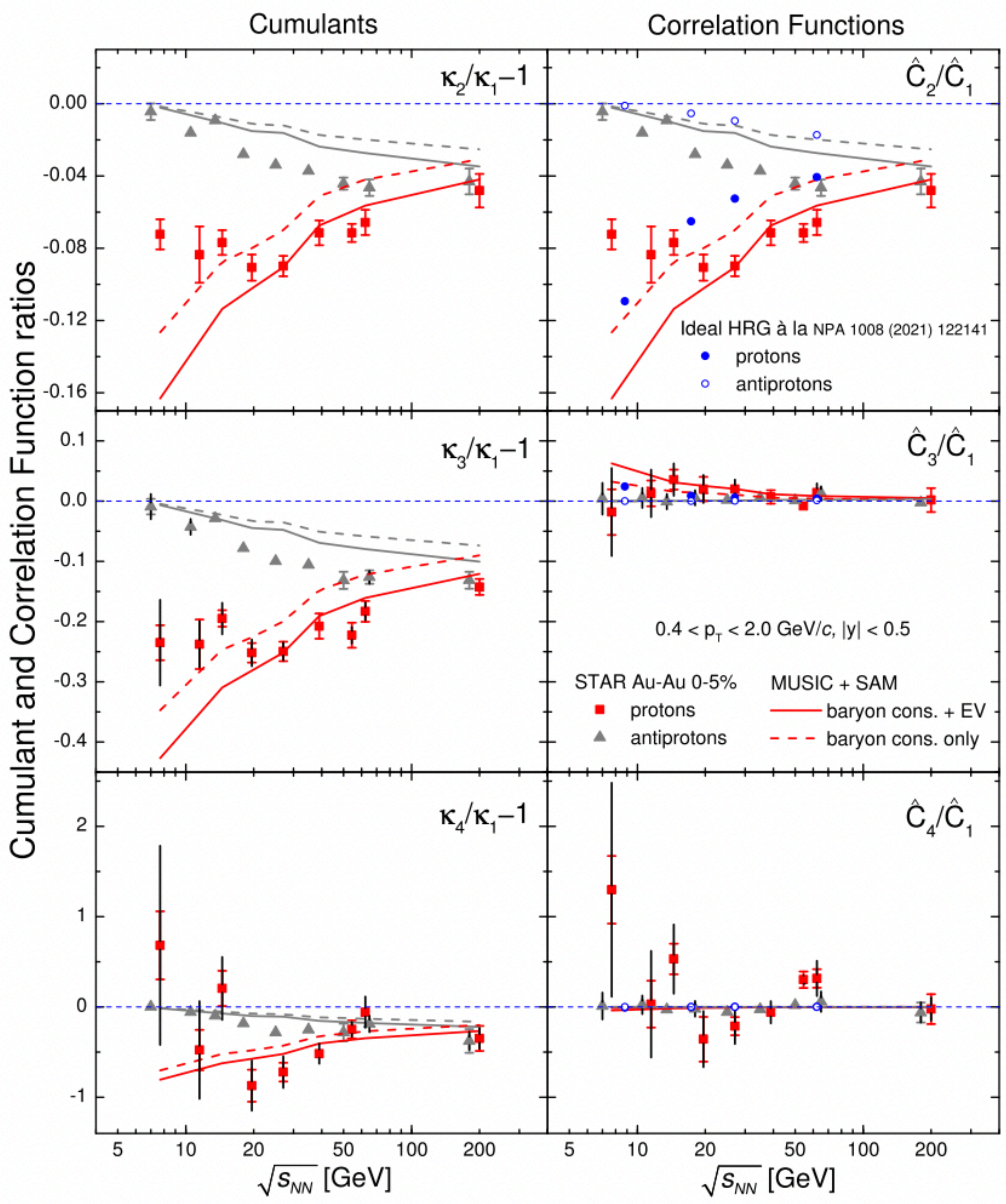
Range of Validity improved

$$0 \le \mu_B \le 700\,\text{MeV}\,,\; 25\,\text{MeV} \le T \le 800\,\text{MeV}$$

The new construction is causal and stable for a larger range of ρ and w

M. Kahangirwe, Wed, 12:10 pm

Deviation of cumulants of proton multiplicities relative to hydrodynamic (non-critical) baseline



Cumulants of proton multiplicities are expected to be highly sensitive to the critical point.

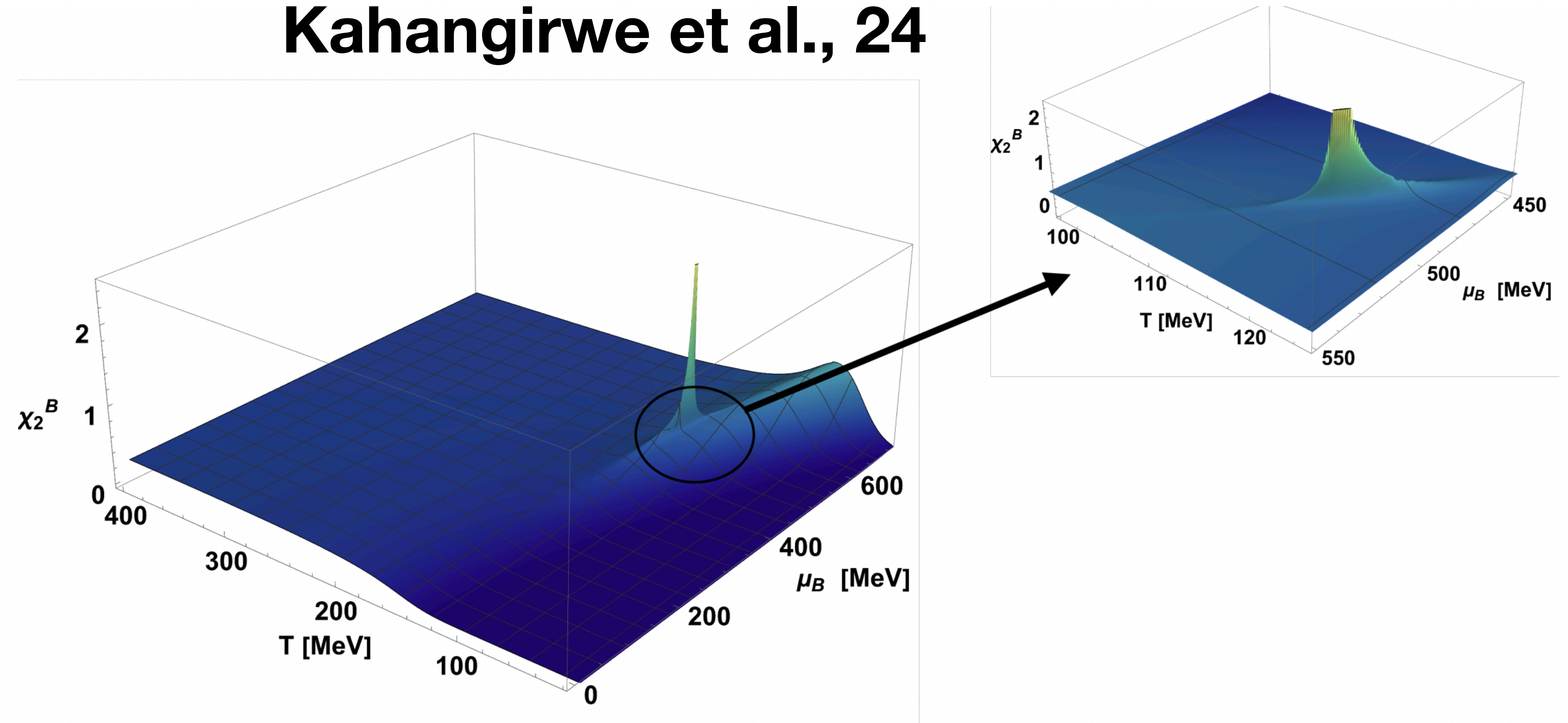
Stephanov, 09

A *clear excess* of scaled proton-number variance from non-critical baseline reported for $\sqrt{s_{NN}} \leq 10\,\mathrm{GeV}$

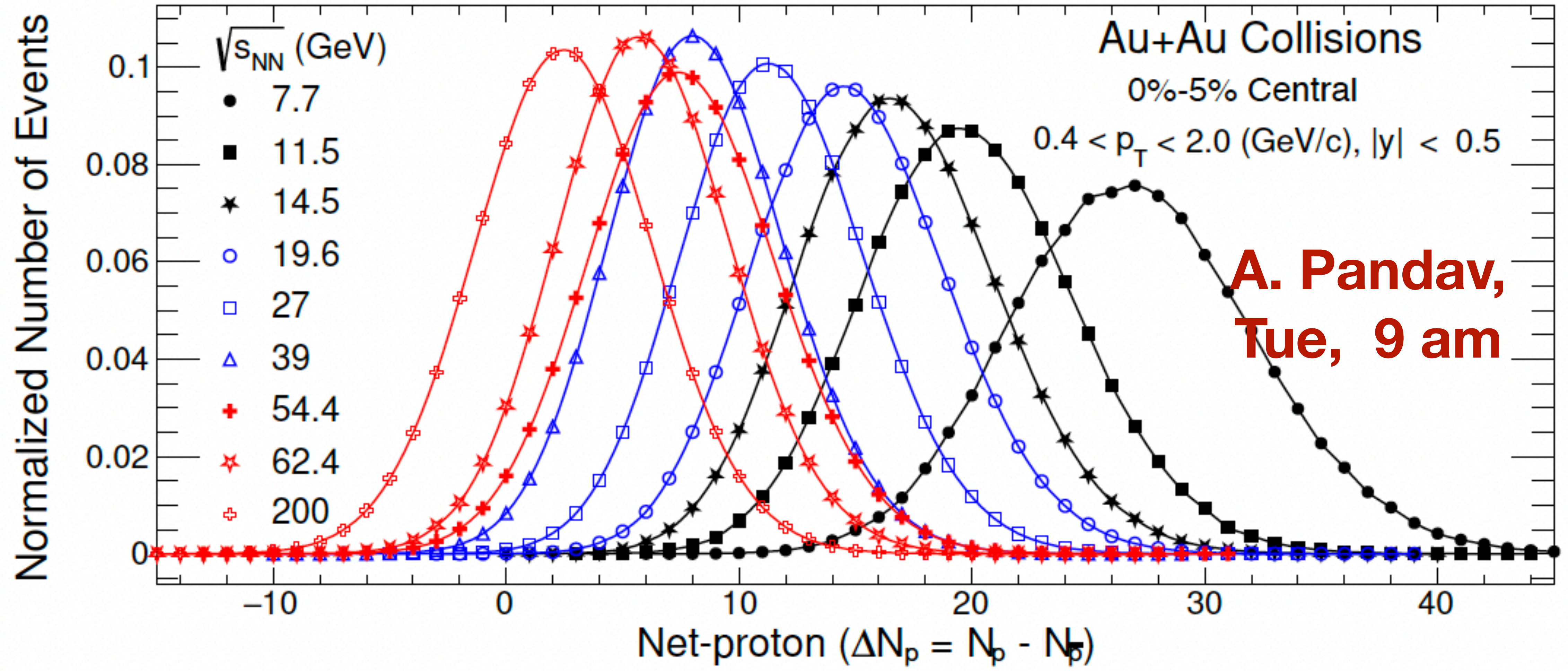
Vovchenko, Koch, Shen, 22

EoS ⟶ Cumulants of particle multiplicities

Kahangirwe et al., 24



Susceptibilities that diverge at CP



Imprints on Particle Distribution Functions?

STAR Collaboration, PRL 2014, PRL 2021

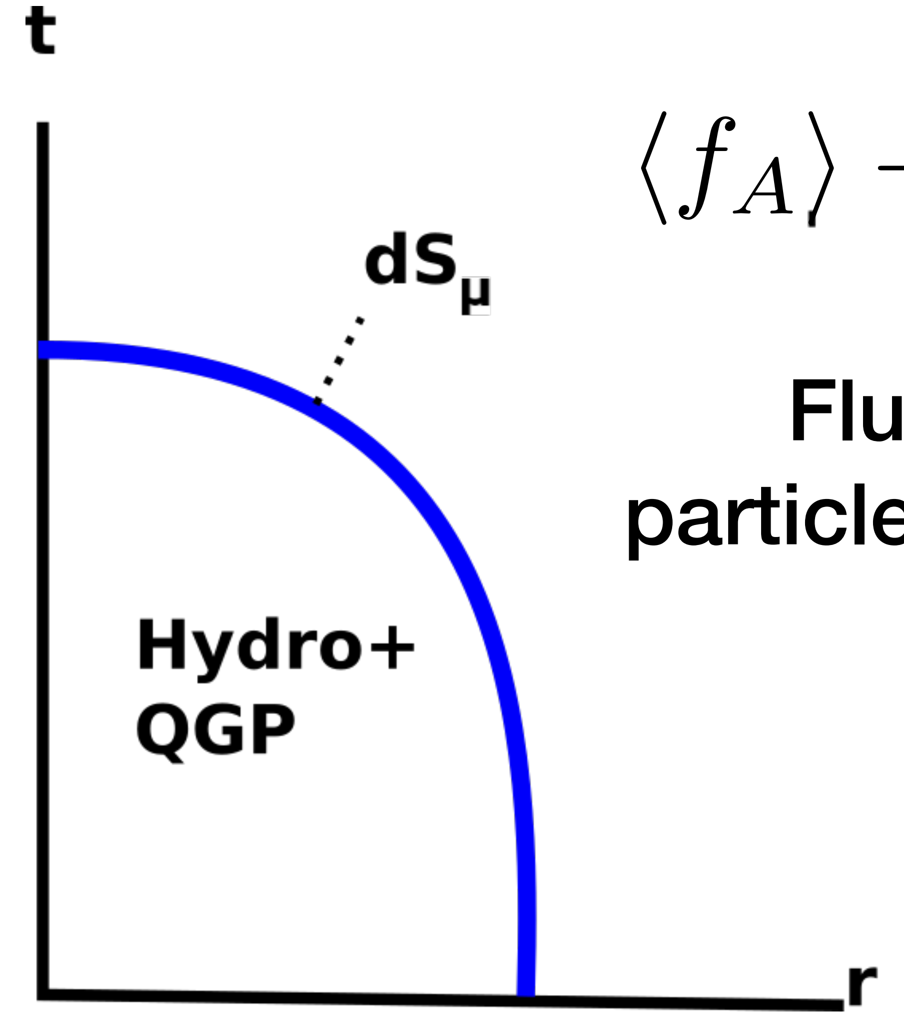
$$\chi_j(\mu, T; \mu_c, \alpha_{12}, w, \rho) \stackrel{?}{\to} C_A^k(\mu_F(T_F); \mu_c, \alpha_{12}, w, \rho, \Gamma)$$

Generalization to Cooper-Frye (74) freeze-out based on maximum entropy principle

MP, Stephanov, 23

Hydrodynamics + Correlation Functions $\rightarrow$ HRG Ensemble with non-trivial phase space correlations

- Hydrodynamics + Correlations contain information about critical EoS
- Infinitely many ensembles that would match with hydro+correlations
- Cooper-Frye (74) maximizes thermodynamic entropy of HRG
- We maximize the *nPI entropy* of the HRG ensemble with fluctuations subject to matching conditions



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution

ME gives least biased ensemble of HRG that matches with hydro+correlations

- Natural generalization of factorial cumulants (IRCs, or irreducible relative cumulants)

Depends only on reference distribution

$$\hat{\Delta} G_{ABC\ldots} = \mathcal{F}(\bar{H}, \bar{G})\, \hat{\Delta} H_{ABC\ldots}$$

Phase space correlation functions of the gas variables (IRCs)

Correlation functions of the hydrodynamic variables (IRCs)

Equilibrium estimates for the critical contribution to the factorial cumulants of proton multiplicity

$$\omega_p^k = \frac{C_k}{C_1}_{\text{crit}} \approx \frac{T^3}{n_p}\left(\frac{X^T \bar{H}^{-1} P}{w \sin(\alpha_1 - \alpha_2)}\right)^k \kappa_k(\mu, T)$$

Cumulants in Ising model mapped to QCD

$\bar{H}^{-1}P$ HRG

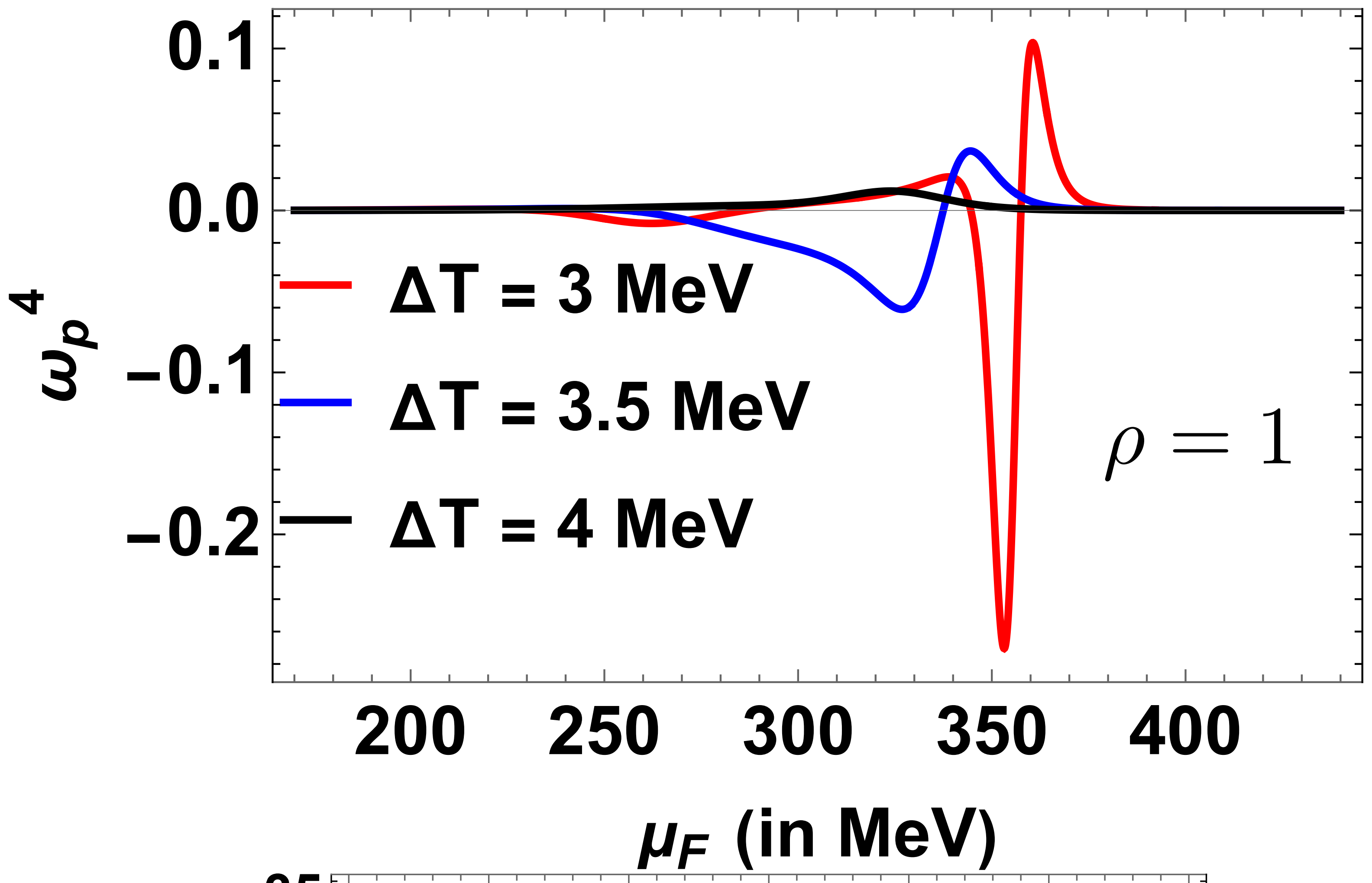
X depends on the mapping to Ising

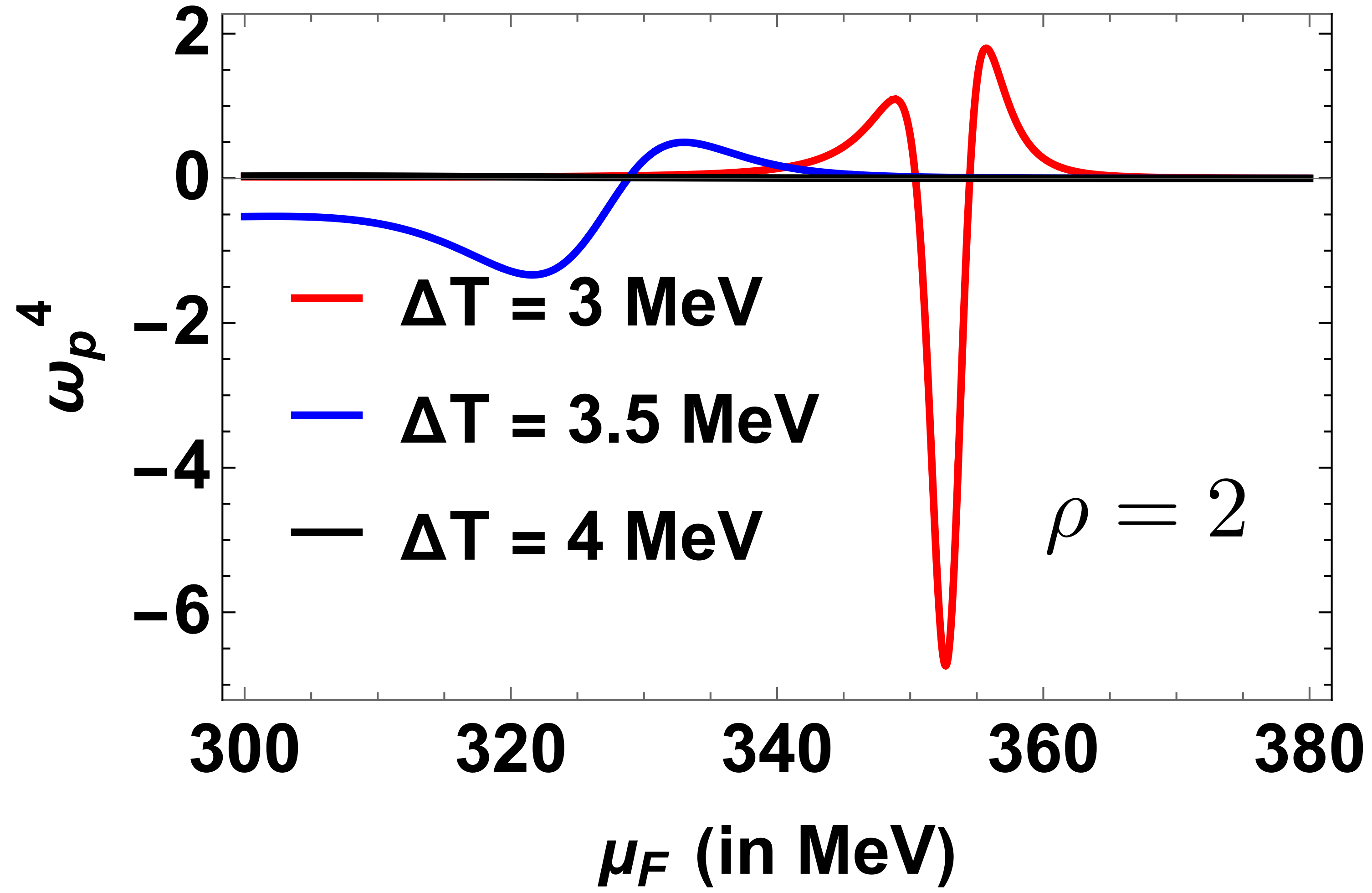
Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

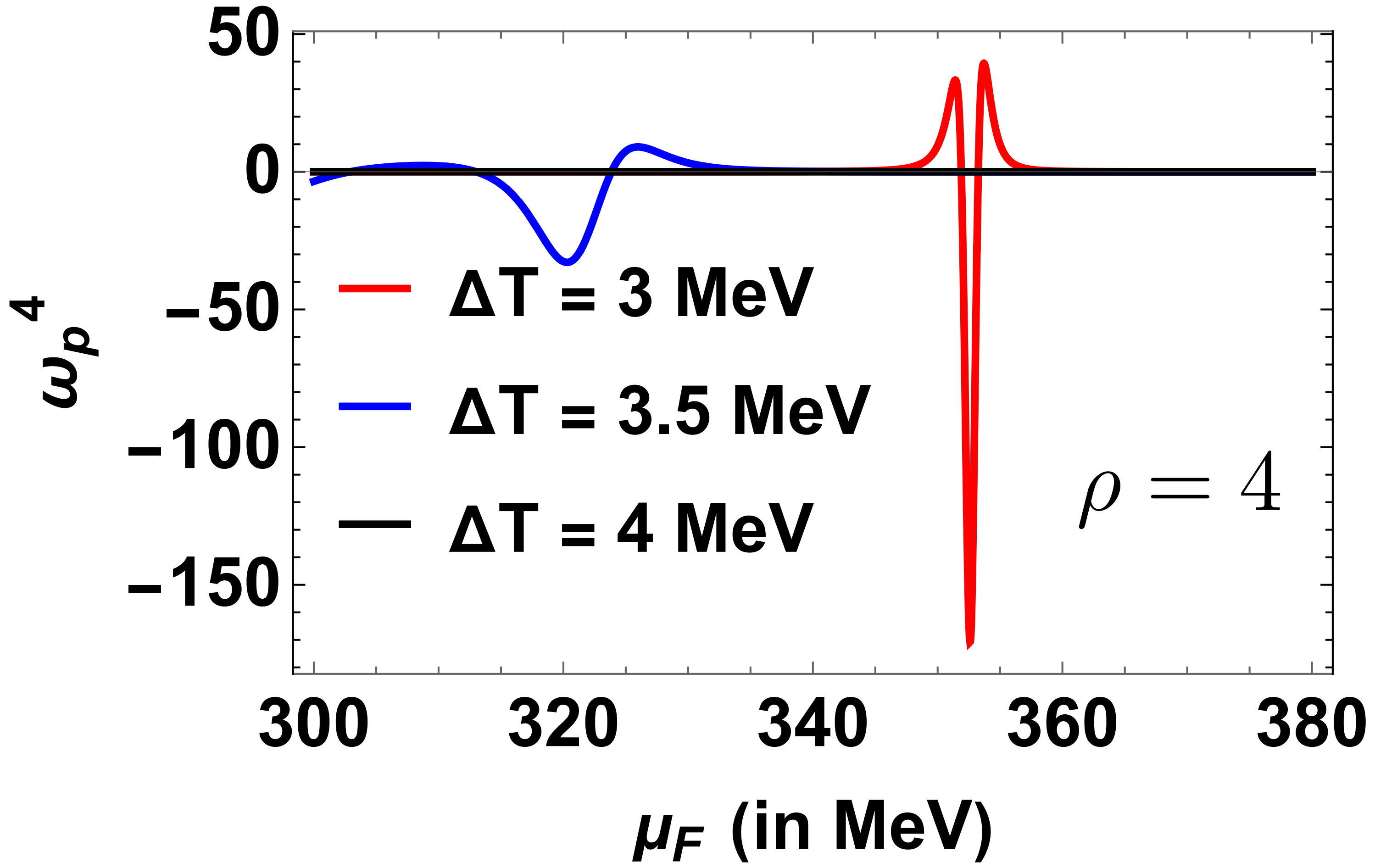
$$\omega_p^k = \left(\frac{C_k}{C_1}\right)_{\text{crit}}$$

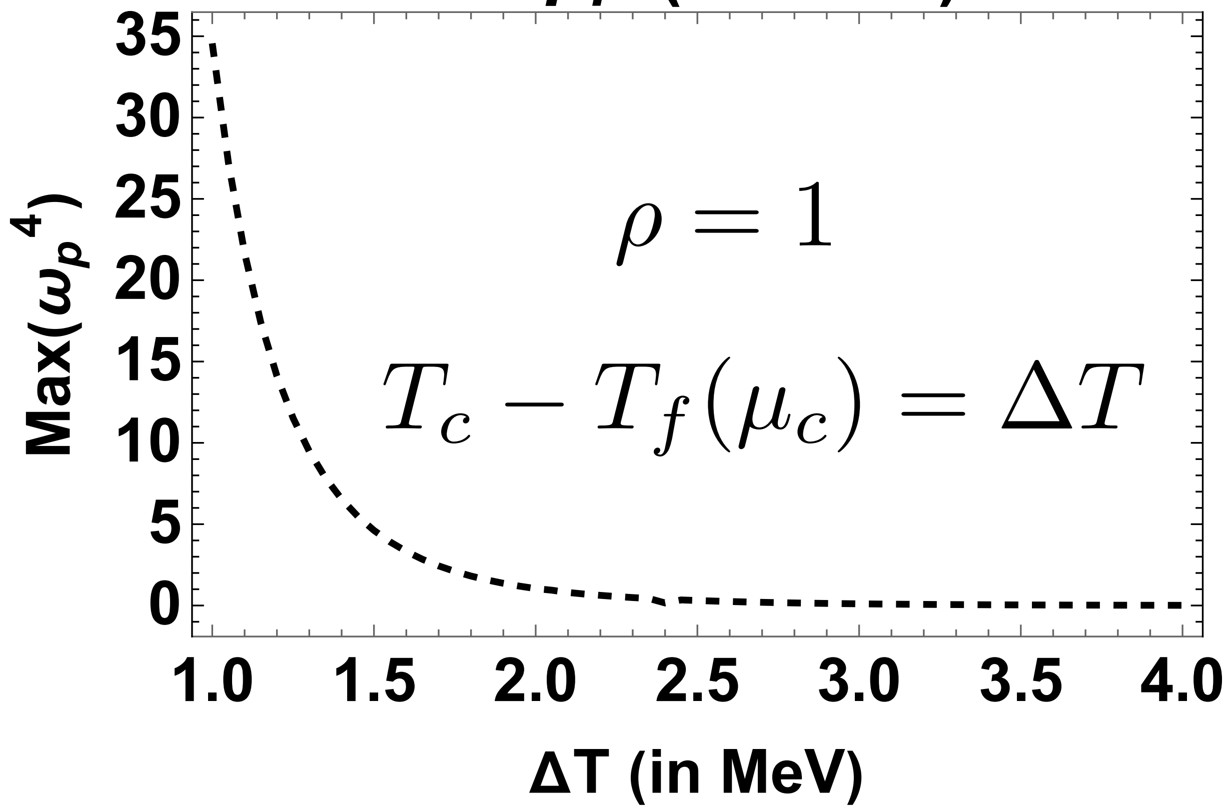
$w = 30\,,\ \alpha_{12} = 10^\circ\,,\ \mu_c = 420\,\text{MeV}$

Example **choice of Mapping Parameters**







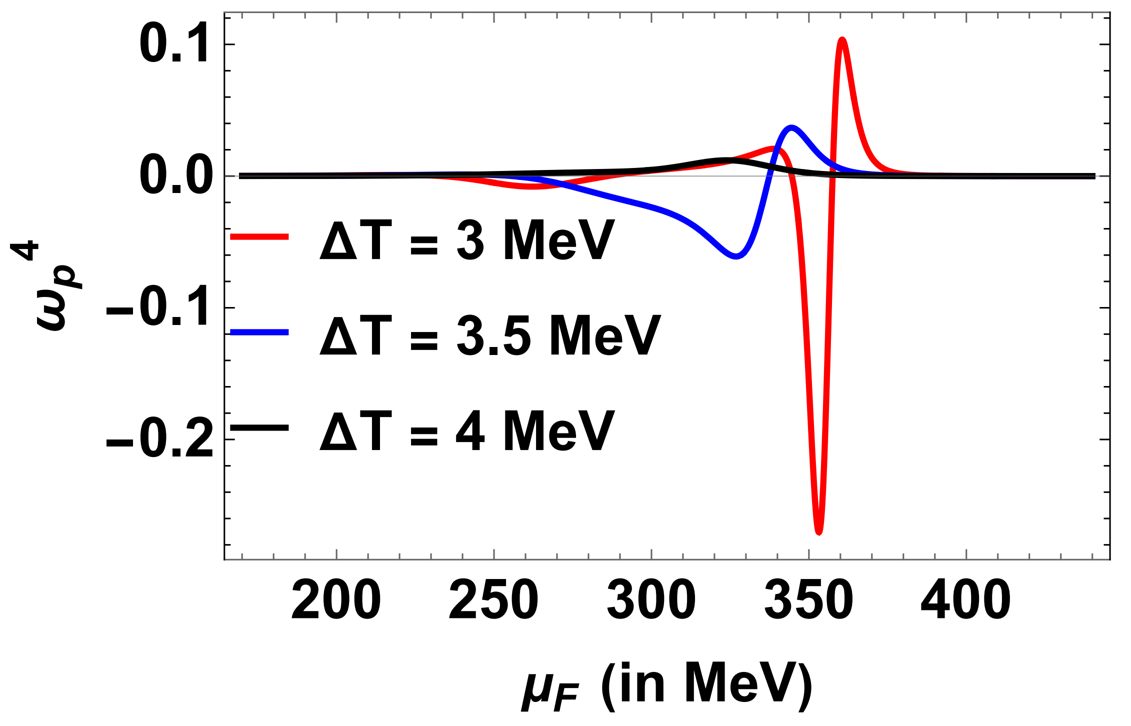


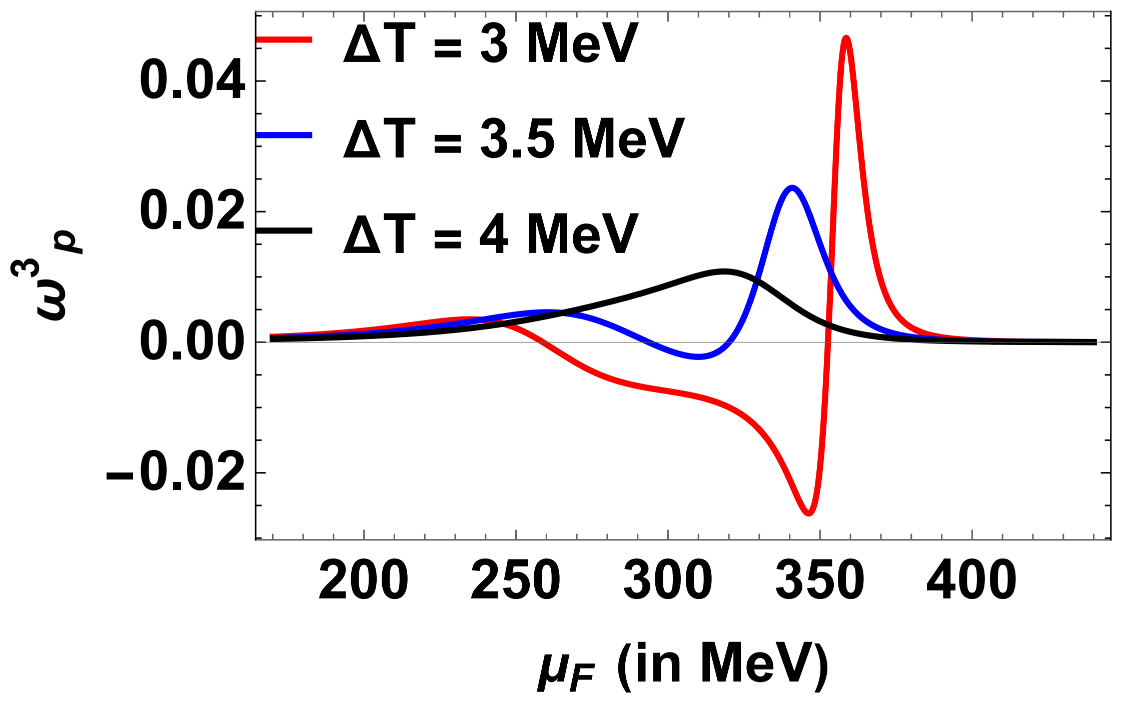
Freeze-out parametrization adapted from Andronic et al, 18

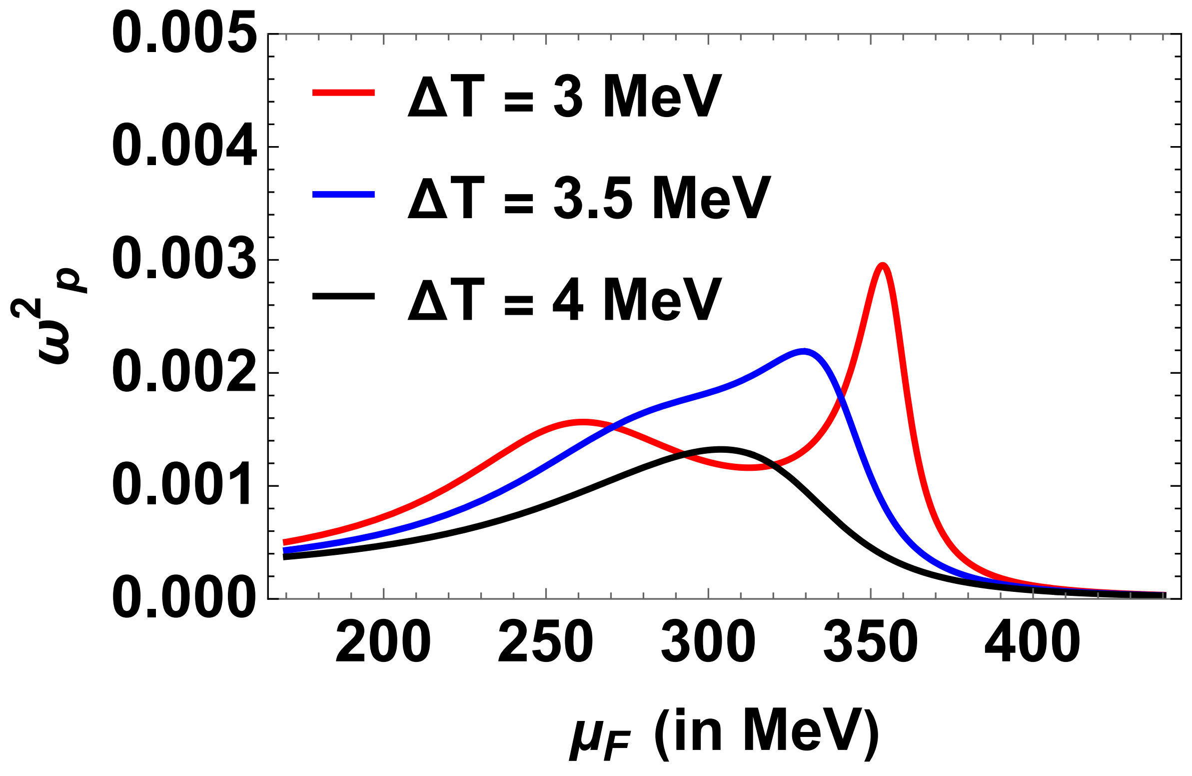
$$\chi_k(\mu, T; \mu_c, \alpha_{12}, w, \rho) \overset{\text{ME}}{\to} C_p^k(\mu, T; \mu_c, \alpha_{12}, w, \rho)$$

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

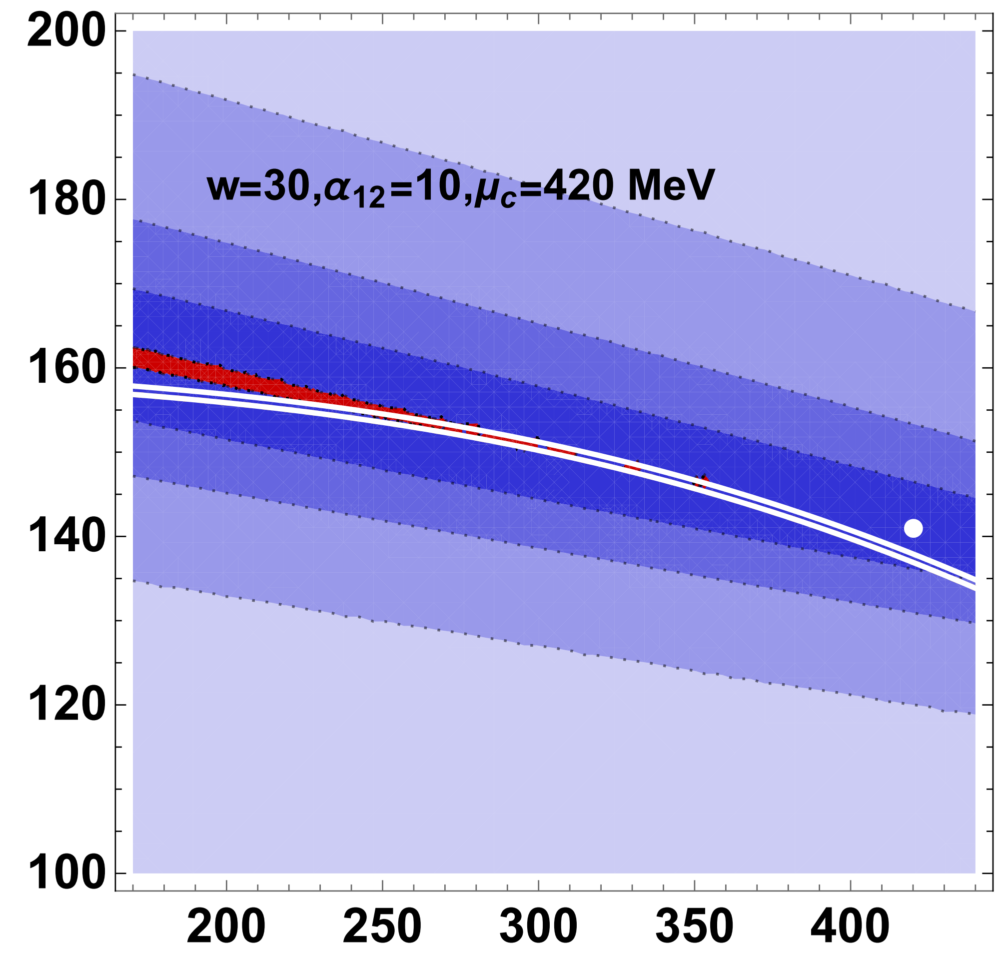


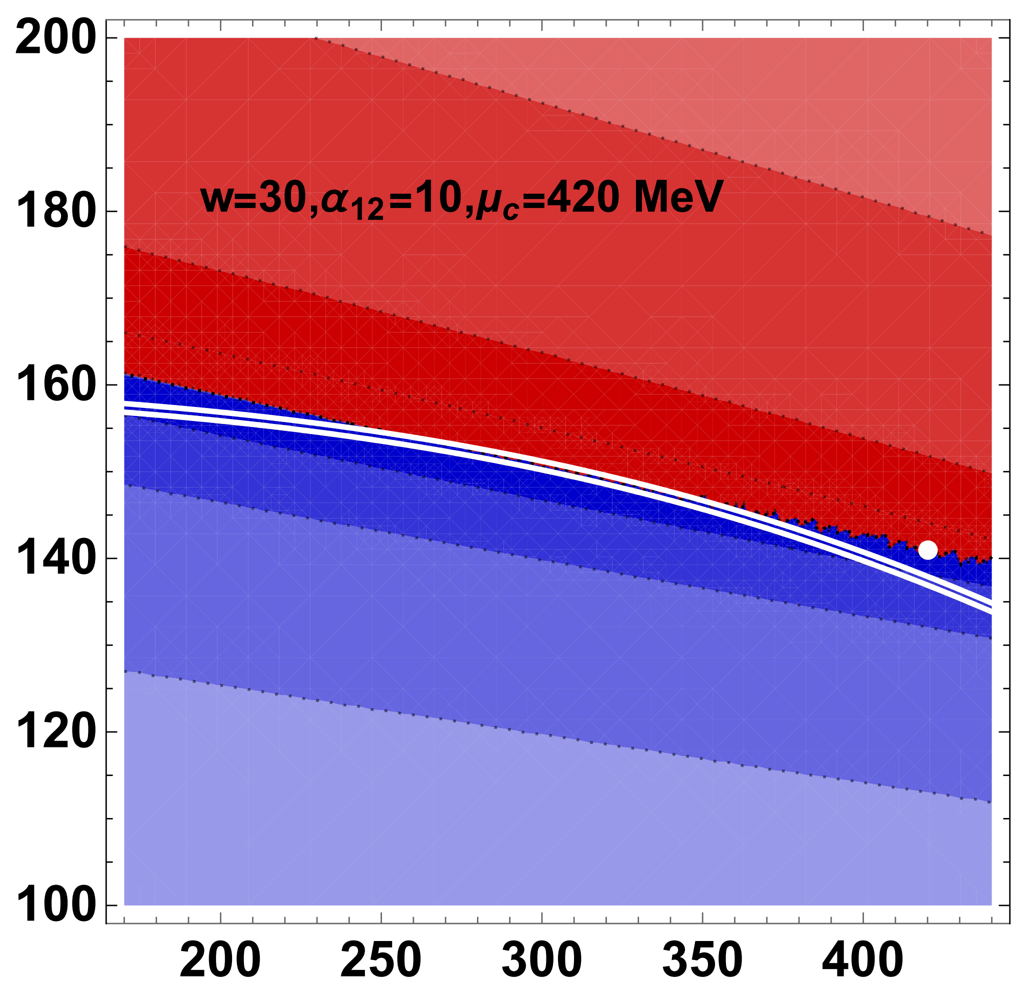


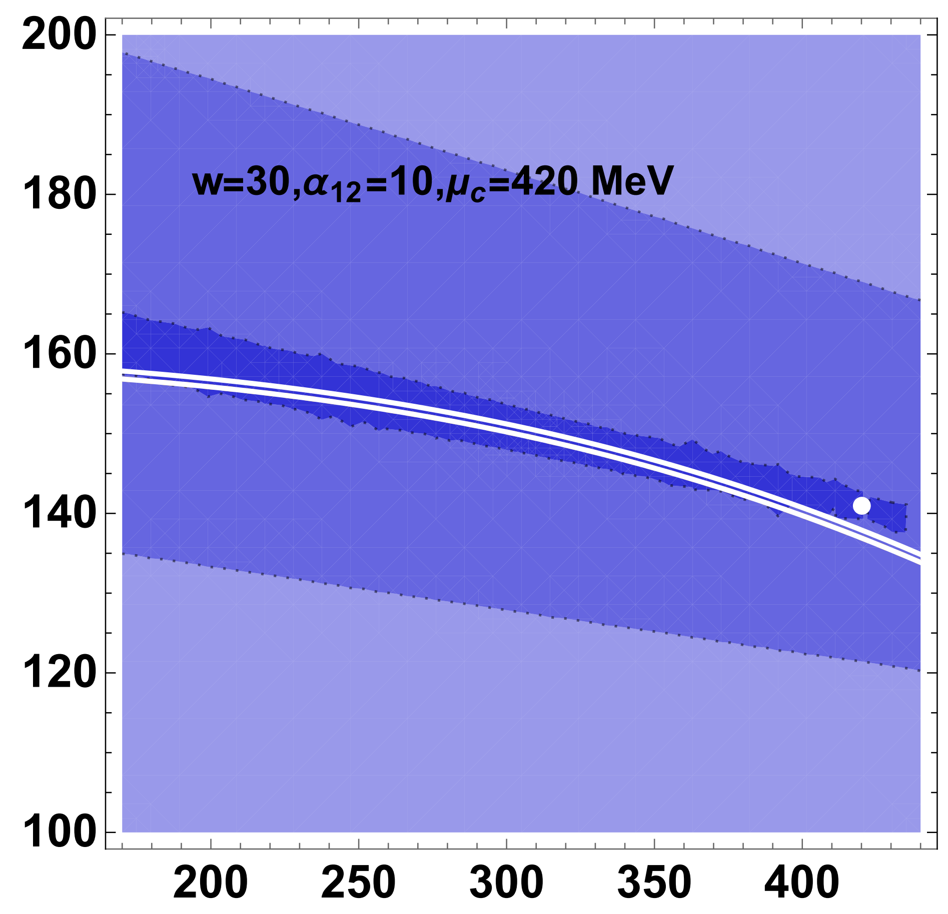


Specific features depend on the orientation (slope & curvature) of freeze-out curve relative to that of pseudo-critical curve

T







Critical slowing down would affect the magnitude of cumulants at freeze-out - fluctuation dynamics

MeV $\mu \longrightarrow$

$$\rho = 1\,,\; w = 30\,,\; \alpha_{12} = 10^\circ\,,\; \mu_c = 420\ \text{MeV}$$

Example **choice of Mapping Parameters**

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Fluctuation dynamics

Hydrodynamic evolution of the fireball and collectivity **L. Du, Next talk**

Stochastic Approach

- Stochastic differential equations for hydro fields
- Implementation using Metropolis algorithm developed and applied to various models, including Model H

T. Schaefer, Wed, 9:30 am

Schaefer and Skokov, 22, Chattopadhyay et al, 23, Florio et al, 21, Basar et al, 24,Chattopadhyay et al, 24…

Deterministic Approach

X. An, Tue, 4pm

- Deterministic evolution for hydro correlation functions
- Semi-realistic estimates for two-point correlation functions computed and connected to observables

Teaney, Akamatsu, Mazeliauskas, 16 along with Yan,Yin , 19, Stephanov, Yin,17, An, Basar, Stephanov,Yee, 19,20,22,24…

An, Basar, Stephanov,Yee, 22,24

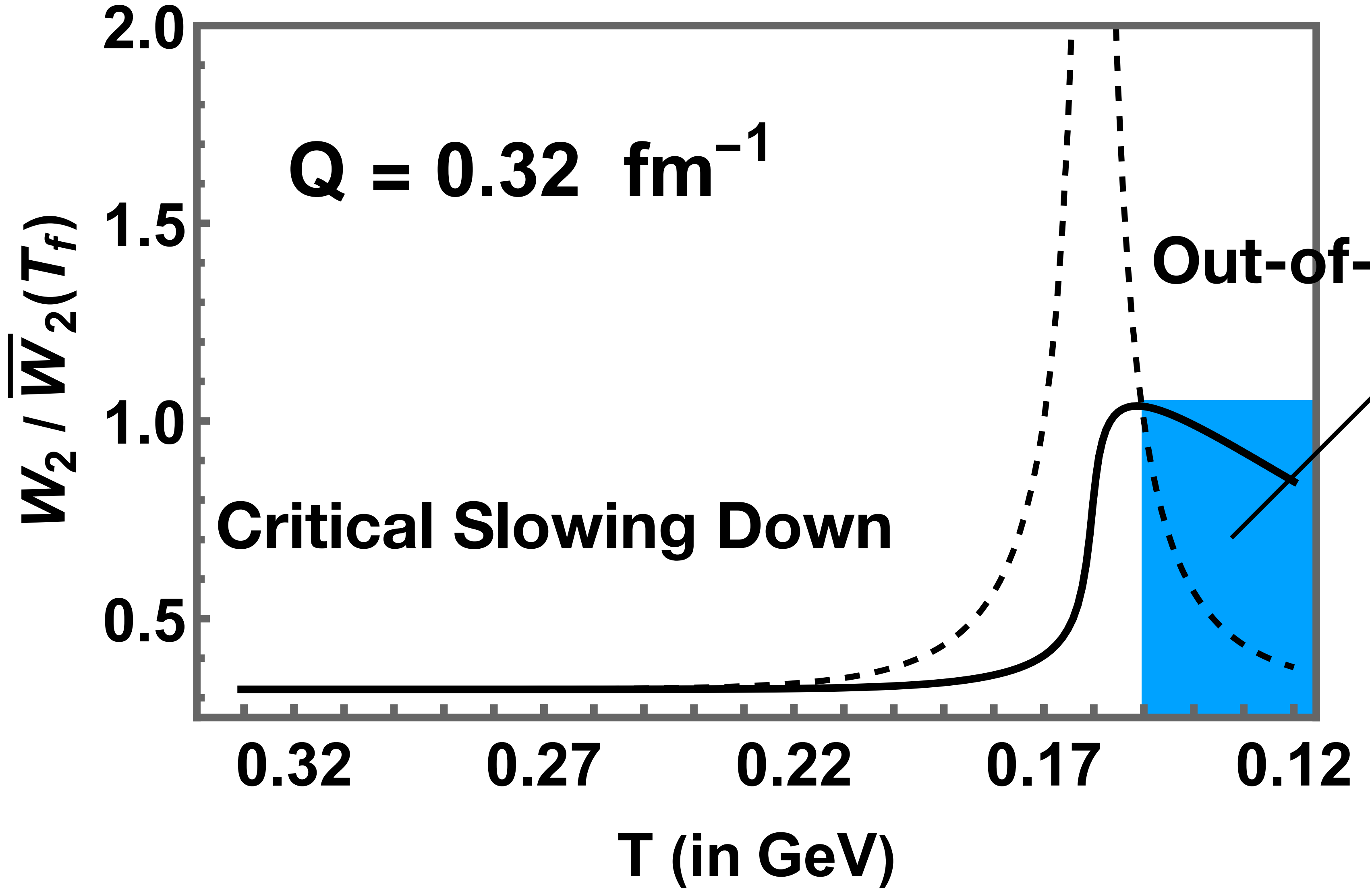
Rajagopal,Ridgway,Weller,Yin,19,Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22,Mukherjee, Venugpalan, Yin 15

Out-of-equilibrium effects of fluctuations near the CP

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\, W_2(\mathbf{Q})\,,\ \mathbf{\Delta x} = x_+ - x_-$$

Persistence of critical imprints in the fluctuation observables until freeze-out

D_0= 1 fm **Suppression of peak**



Out-of-equilibrium>Equilibrium

Prolonged memory of CP

Rajagopal,Ridgway,Weller,Yin,19
Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22
Mukherjee, Venugopalan, Yin 15

Quantitative estimation of non-Gaussian cumulants, work in progress.

Finite size scaling of proton cumulants : A. Sorensen, Fri, 9:50 am

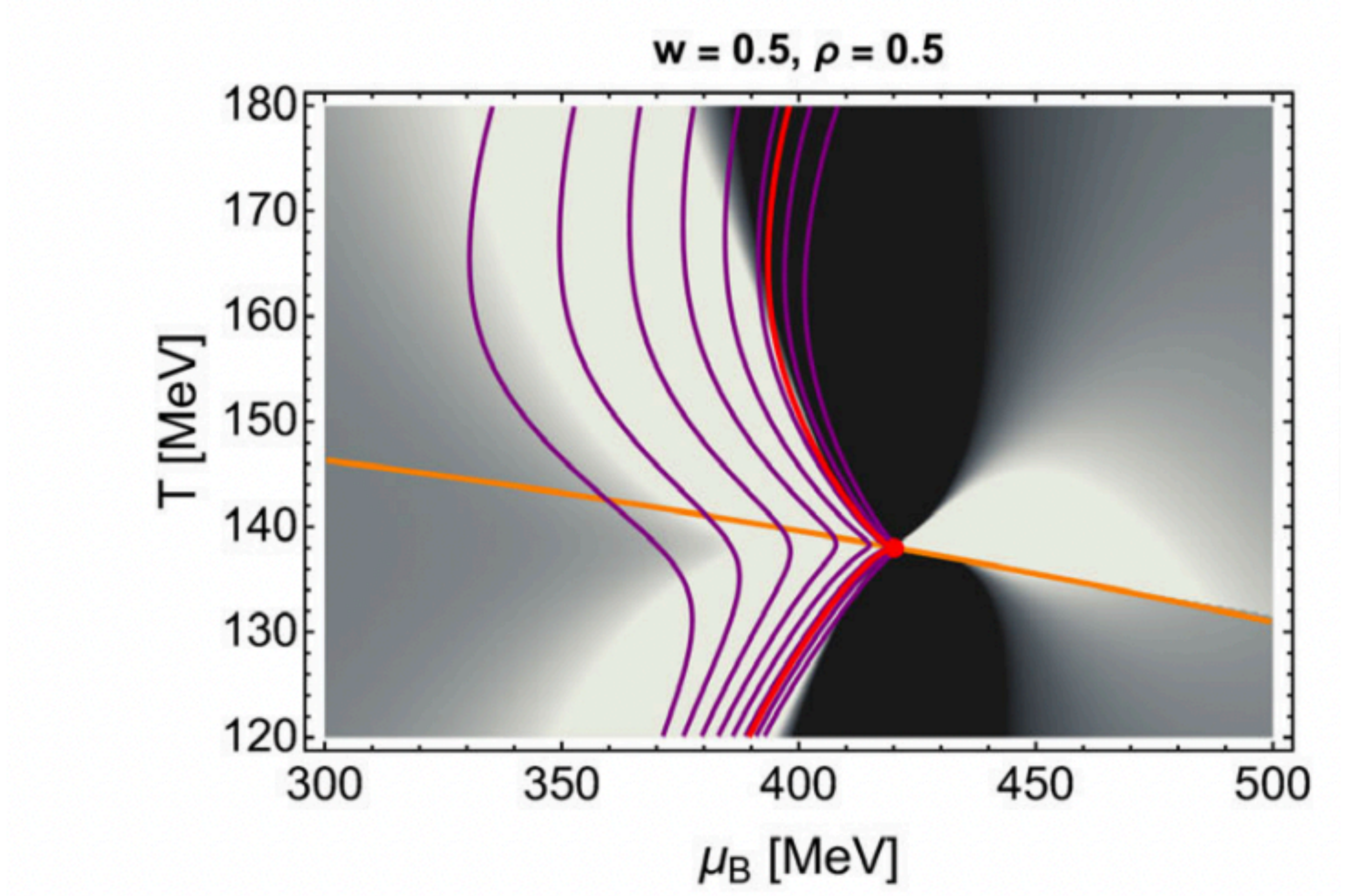
CP fluctuations in MD simulations : V. Kuznietsov, Thurs 2:40 pm

Deformation of hydrodynamic trajectories near CP

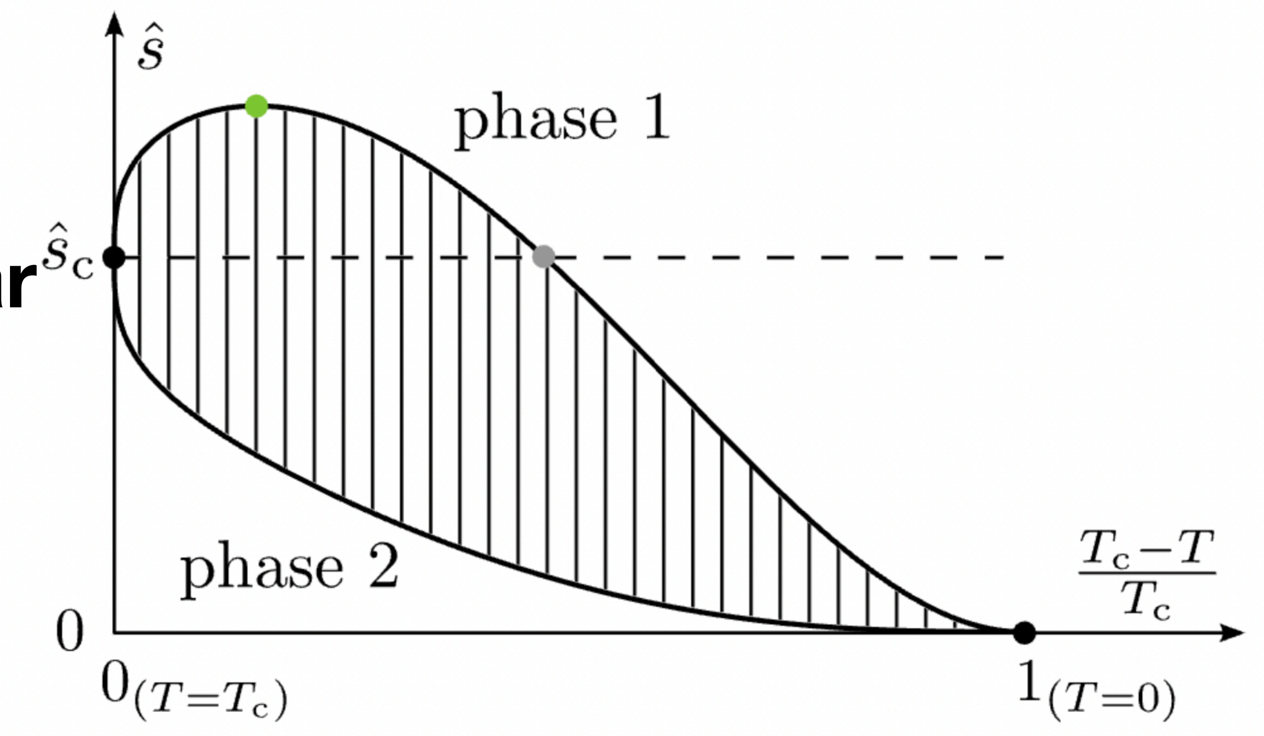
MP, Sogabe, Stephanov, Yee,24

Deformations can be broadly classified based on the value of the mapping parameter α_2

N. Sogabe, Thurs, 3pm



Universal scaling near CP



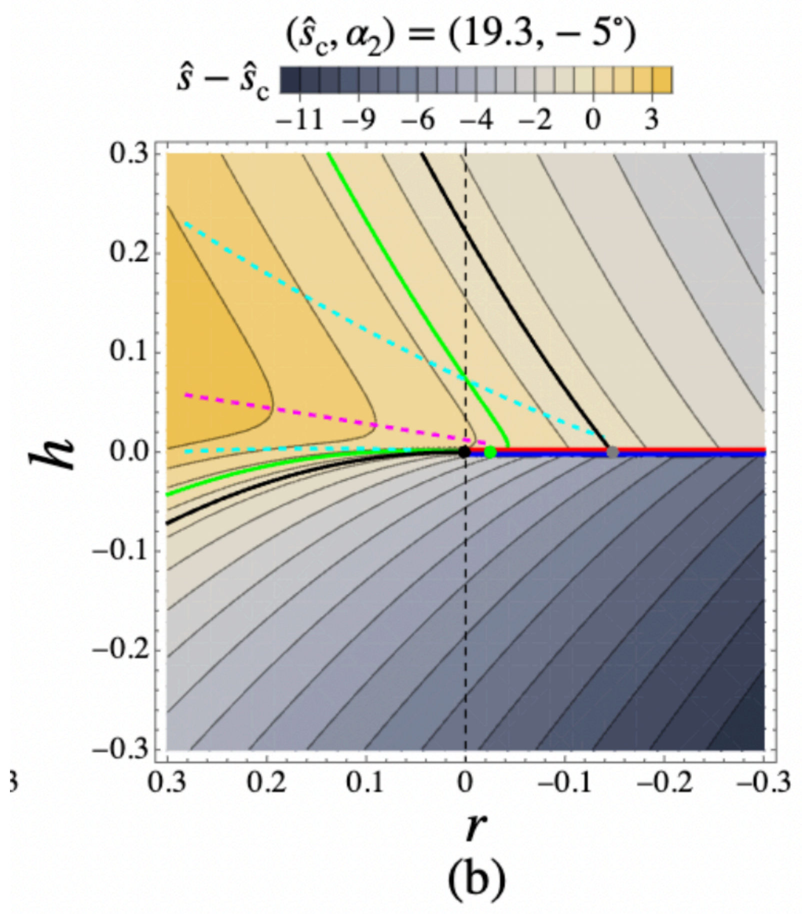
Third Law

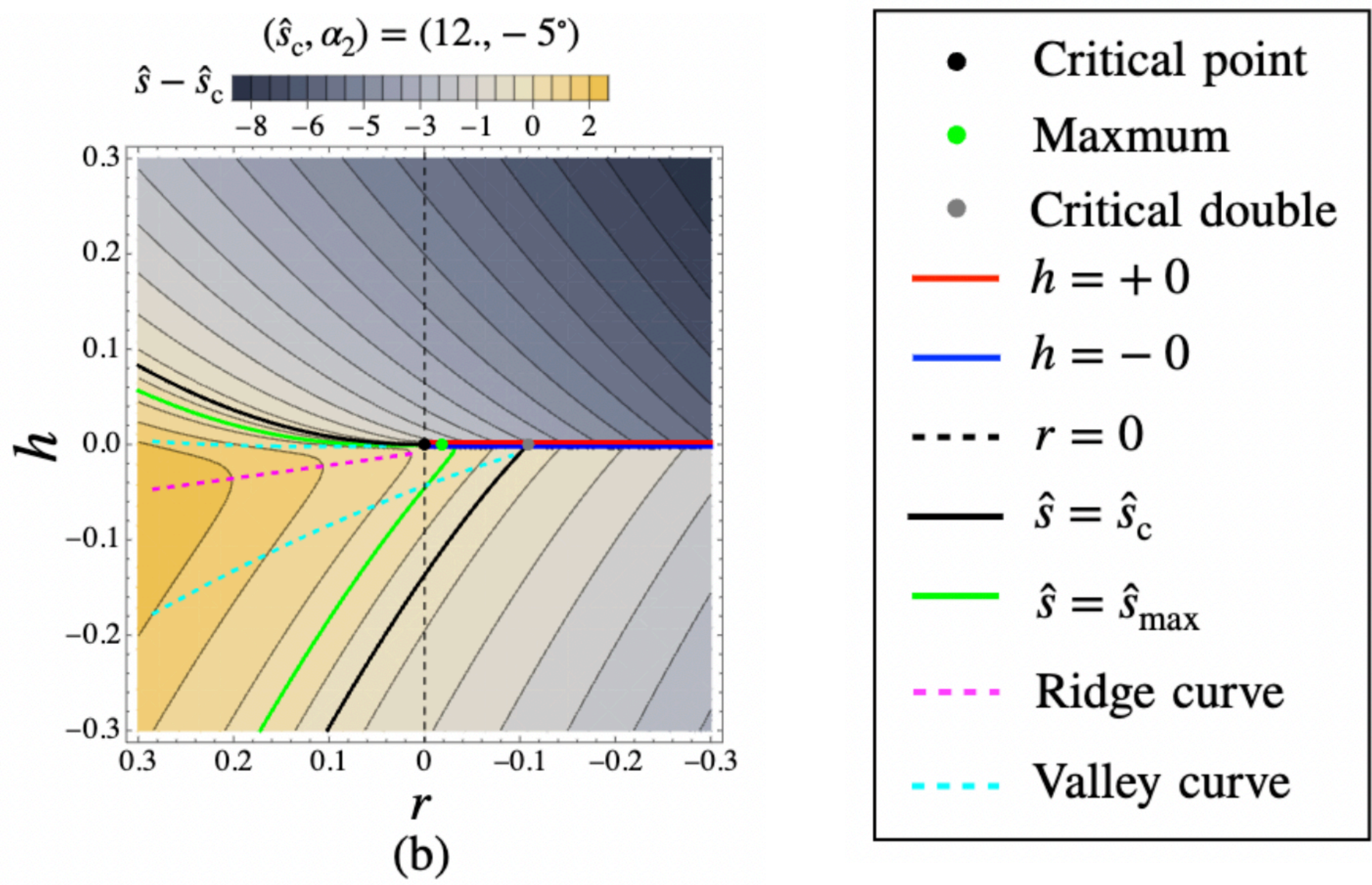
- Specific entropy is non-monotonic along one of the branches on the first-order curve

Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

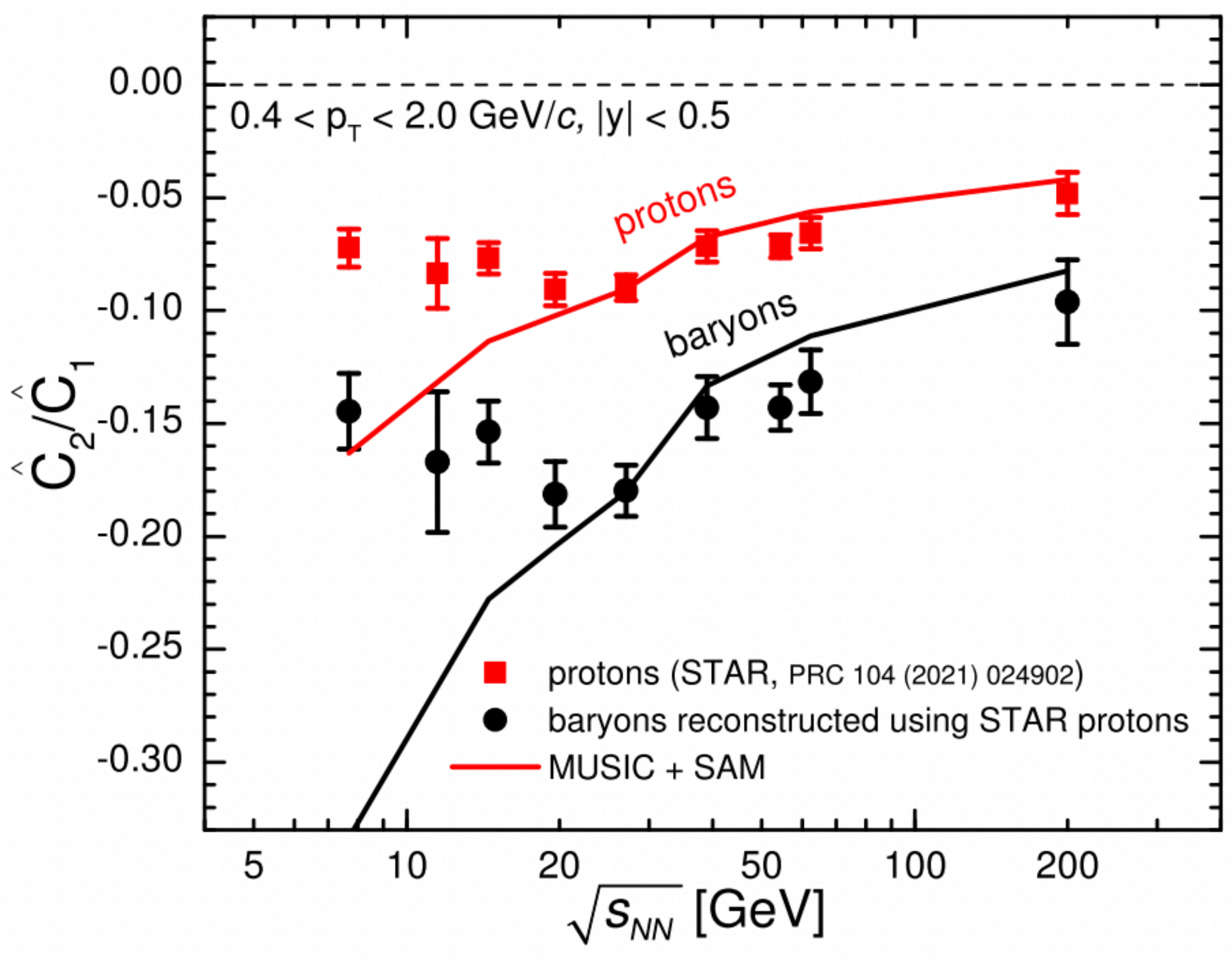
- Consequence of universal ridge-like structure of the isentropes near CP

- Phenomenological implications





Summarizing & Looking forward



Vovchenko,Koch,Shen,22

Non-critical ***baselines*** important!

A ***family of candidate EoSs with a CP*** that match with the lattice have been developed

$$\chi_j(\mu, T; \mu_c, \alpha_{12}, w, \rho) \overset{\text{ME}}{\rightarrow} C_A^k(\mu_F(T_F); \mu_c, \alpha_{12}, w, \rho, \Gamma)$$

Dynamics

Quantitative estimates for ***out-of equilibrium*** corrections to higher order cumulants - need of the hour

Bayesian Analysis of experimental data pertaining to ***multiple observables*** with the theoretical framework may possibly help us learn about QCD EoS near CP, if it exists in the regime scanned by HICs

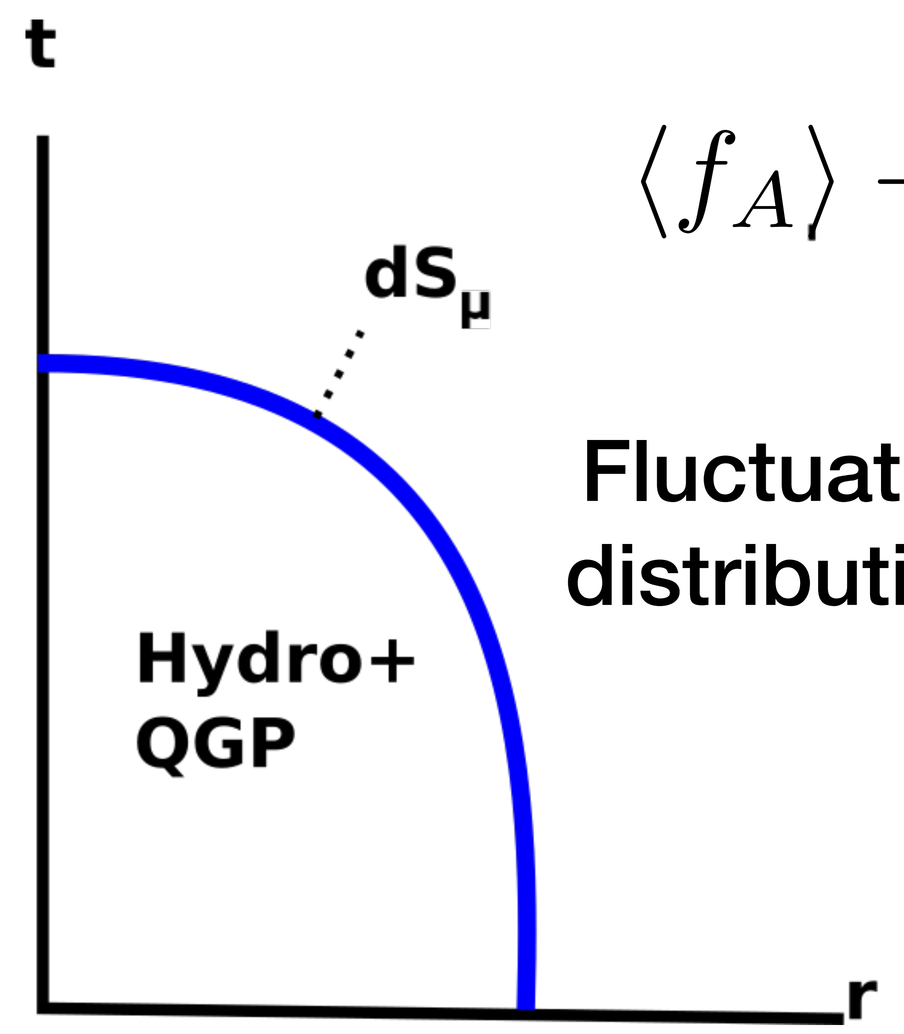
Thank you!

BACK UP SLIDES

Freeze-out : Transition from hydrodynamics to hadron gas

Hydrodynamic mean densities

$$\{\langle \epsilon u^{\mu} \rangle\,,\, \langle n \rangle\} \equiv \Psi^a$$



$$\langle f_A \rangle + \delta f_A$$

Fluctuating particle distribution function

Conserved energy, momentum and charge densities and their correlations

Hydrodynamic correlations

$$\Psi^a,\, \left\langle \delta\Psi^a \delta\Psi^b \right\rangle \equiv H^{ab}, \ldots\, H^{abc\ldots}$$

Particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A\,,\, \langle \delta f_A \delta f_B \rangle = G_{AB}\,,\, \langle \delta f_A \delta f_B \delta f_C \rangle = G_{ABC}\, \ldots$$

f_A Is the phase space distribution function for species A

Matching conditions at freeze-out

$$\langle \epsilon \, u^\mu \rangle = \sum_A \int_{p_A} \bar{f}_A \, p_A^\mu \, , \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

$$\Psi^a = \sum_A \int_{p_A} \bar{f}_A P_A^a$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

$$H^{abc\ldots} = \sum_{A,B,C,\ldots} \int_{p_A p_B p_C \ldots} G_{ABC\ldots} P_A^a P_B^b P_C^c \ldots$$

- Matching conditions for averages of conserved densities
- Infinitely many sets of distribution functions that satisfy these matching conditions
- Freeze-out prescription corresponds to choosing one of these sets - **How to choose**?

Equilibrium

$$S[\bar{f}]$$

Which entropy to maximize?

Generalized

$$S[P(f)]$$

$$S[P(f)] = S_0\left[\bar{f}\right] + \int_f P(f) \log \frac{P_{\text{eq}}(f)}{P(f)}$$

$$S[\bar{f}, G_2, G_3, \ldots]$$

G s are the correlation functions in the Hadron Gas description

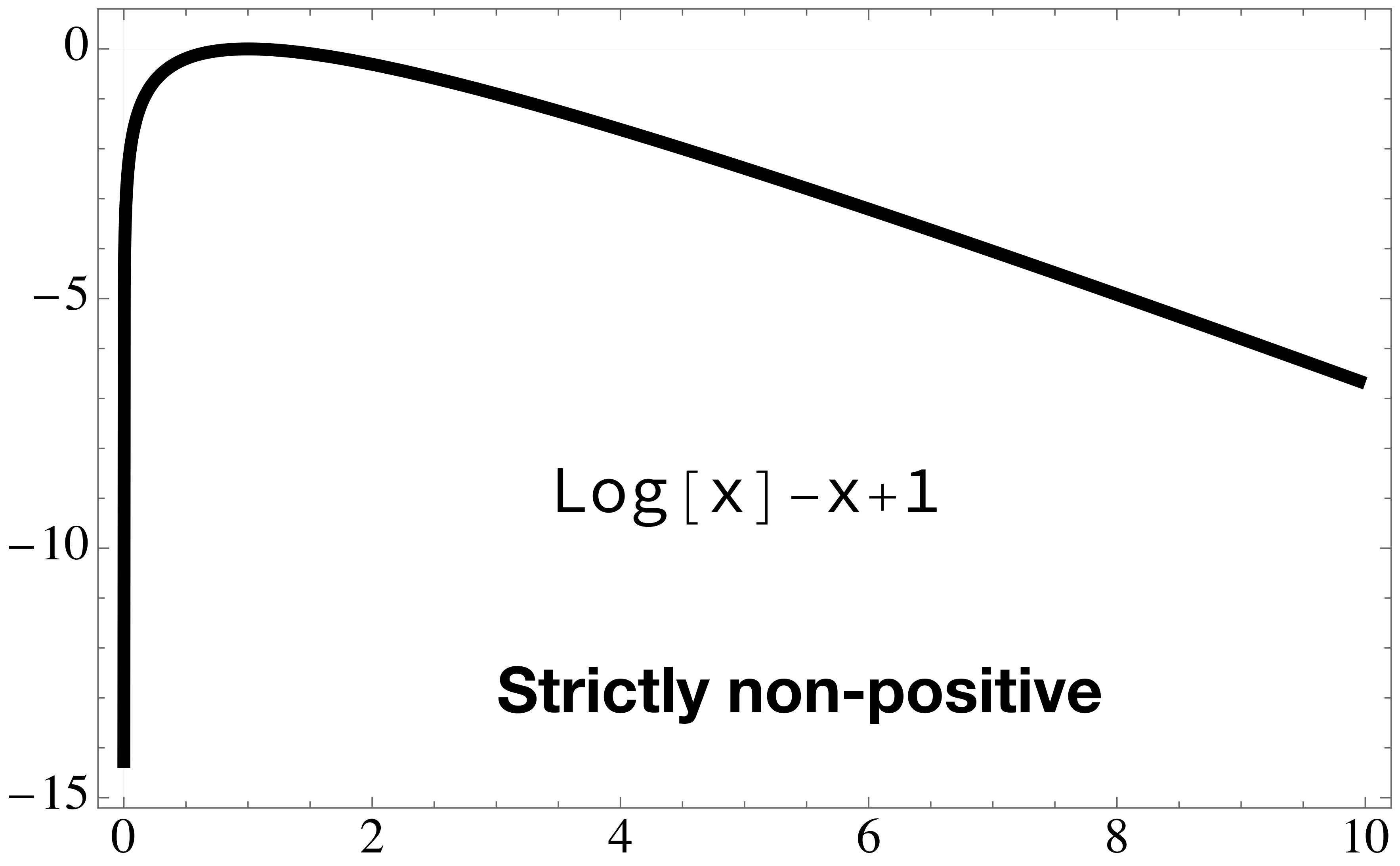
$$S_0[\bar{f}] = -\int_f P_{\text{eq}}(f) \log P_{\text{eq}}(f)$$

- Maximize the relative entropy when correlations are out of equilibrium
- Constraints from matching conditions

Entropy to describe out-of equilibrium two-point correlations in ideal HRG

$$S_2 = S_0 + \frac{1}{2}\mathrm{Tr}\left[\log G\bar{G}^{-1} - G\bar{G}^{-1} + 1\right] \,,$$

Equilibrium



Similar 2-PI action

Berges, 04, Stephanov, Yin, 17…

2-PI entropy

Upon maximizing the 2PI entropy, subject to constraints of conservation

$$G^{-1}_{AB} = \bar{G}^{-1}_{AB} + (H^{-1} - \bar{H}^{-1})^{ab}(\bar{H}^{-1}P\bar{G})^a_A(\bar{H}^{-1}P\bar{G})^b_B$$

When all but two-point correlations are in equilibrium, the solution given above is exact.

Linearizing,

$$G_{AB\ldots} = \bar{G}_{AB} + \Delta H_{ab}(\bar{H}^{-1}P\bar{G})^a_A(\bar{H}^{-1}P\bar{G})^b_B + \ldots,$$

Self correlations

Contribution of self correlations to hydrodynamics is subtracted

$$\Delta H^{ab} = H^{ab} - \bar{H}^{ab}$$

Contribution of self correlations to hydrodynamics

$$\bar{H}_{ab} = \sum_{A,B}\int \bar{G}_{AB}P^a_A P^b_B$$

Generalization to Non-Gaussian Correlations

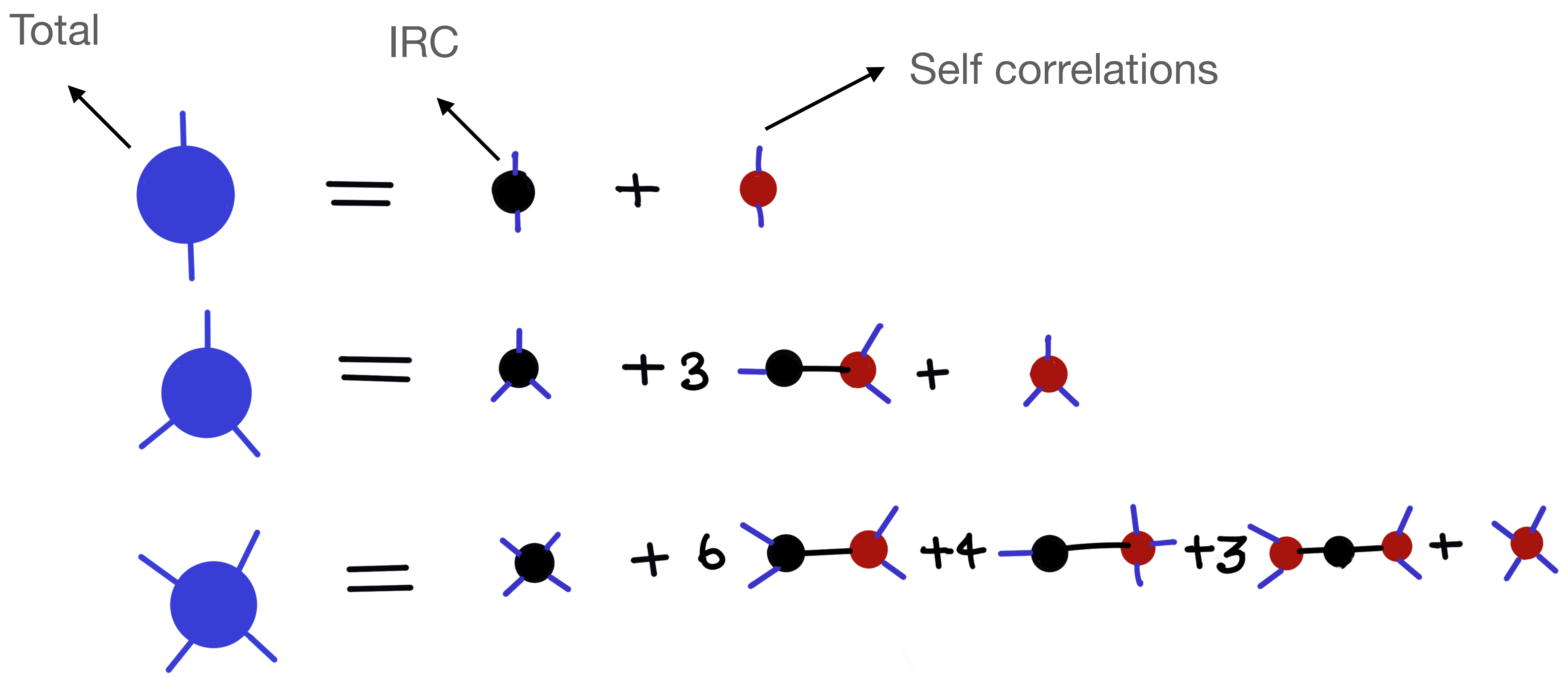
IRC

Equilibrium

IRC

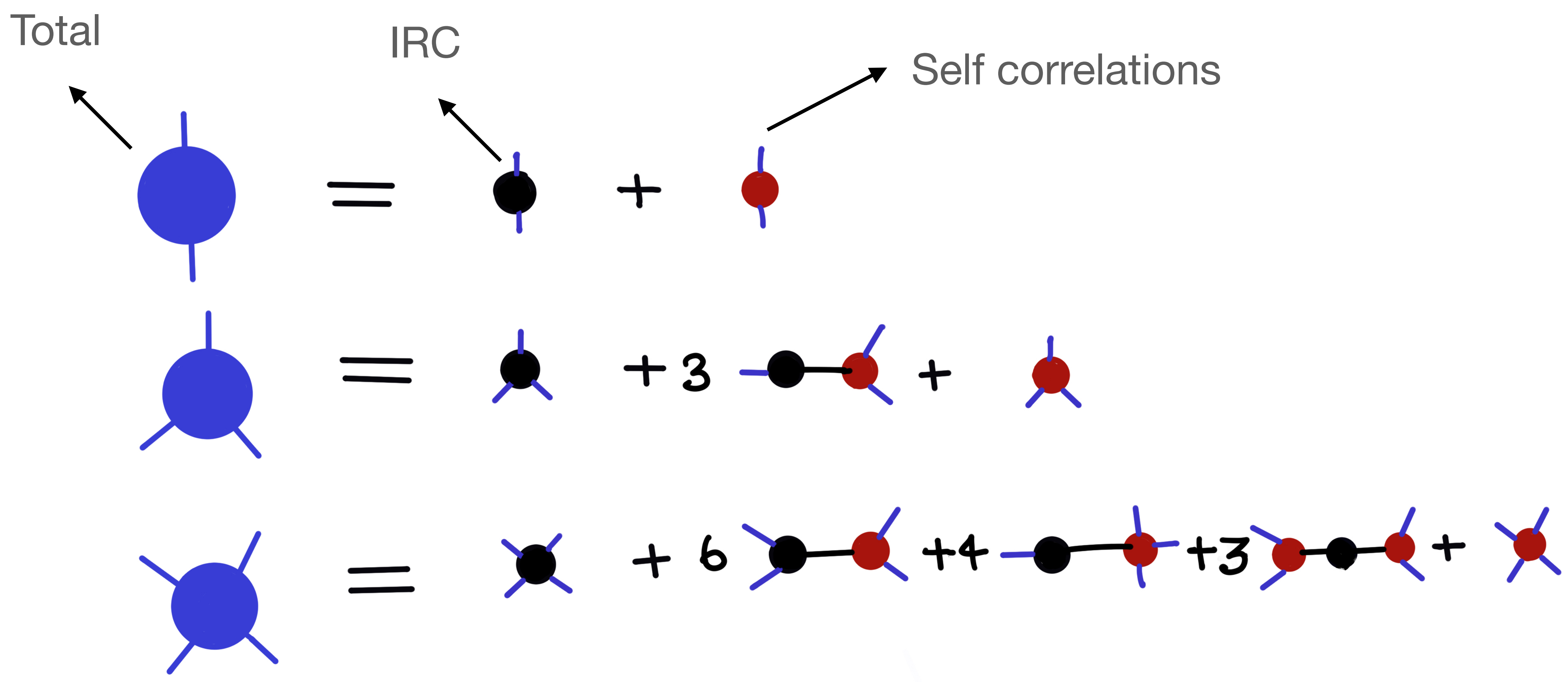
$$\hat{\Delta}G_{ABC\ldots} = \mathcal{F}(\bar{H}, \bar{G})\,\hat{\Delta}H_{ABC\ldots}$$

For classical gas, irreducible relative cumulants (IRCs) reduce to so called “factorial cumulants”.



Irreducible relative cumulants

For classical gas, irreducible relative cumulants (IRCs) reduce to so called “factorial cumulants”.



- For gases obeying different statistics, IRCs quantify the non-trivial correlations
- Non-trivial correlations relative to any specified baseline distribution

Equilibrium estimates for the critical contribution to the factorial cumulants of proton multiplicity

$$\omega_p^k = \frac{C_k}{C_{1\,\mathrm{crit}}} \approx \frac{T^3}{n_p}\left(\frac{X^T \bar{H}^{-1} P}{w \sin(\alpha_1 - \alpha_2)}\right)^k \kappa_k(\mu, T)$$

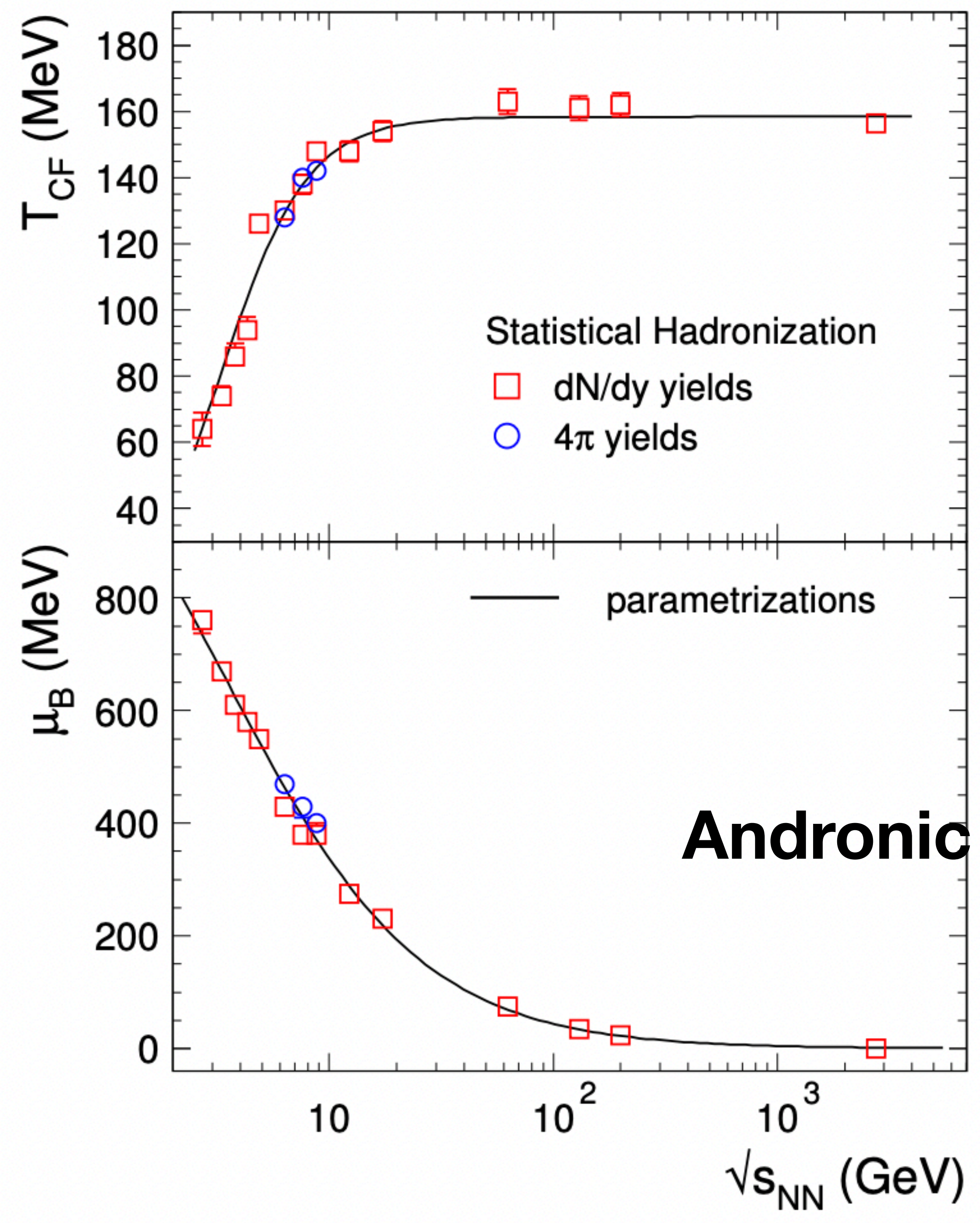
Cumulants in Ising model mapped to QCD

$\bar{H}^{-1}P$ HRG

X depends on the mapping to Ising

$$X = \begin{pmatrix} s_1 \\ c_1 + \mu_c s_1/T_c \end{pmatrix}, \bar{H} = \int_H f'_H \begin{pmatrix} (E_H/T_c)^2 & (E_H/T_c) q_H \\ (E_H/T_c) q_H & q_H^2 \end{pmatrix}, P_A = \int_A f'_A \begin{pmatrix} E_A/T_c \\ q_A \end{pmatrix}$$

Freeze-out parametrization



Andronic et al, 18

We use freeze-out parametrization from Andronic et al, 18 and add a variable additive constant such that

$$T_c - T_f(\mu_c) = \Delta T$$

We study sensitivity of observables to ΔT

Novel summation scheme from WB collaboration

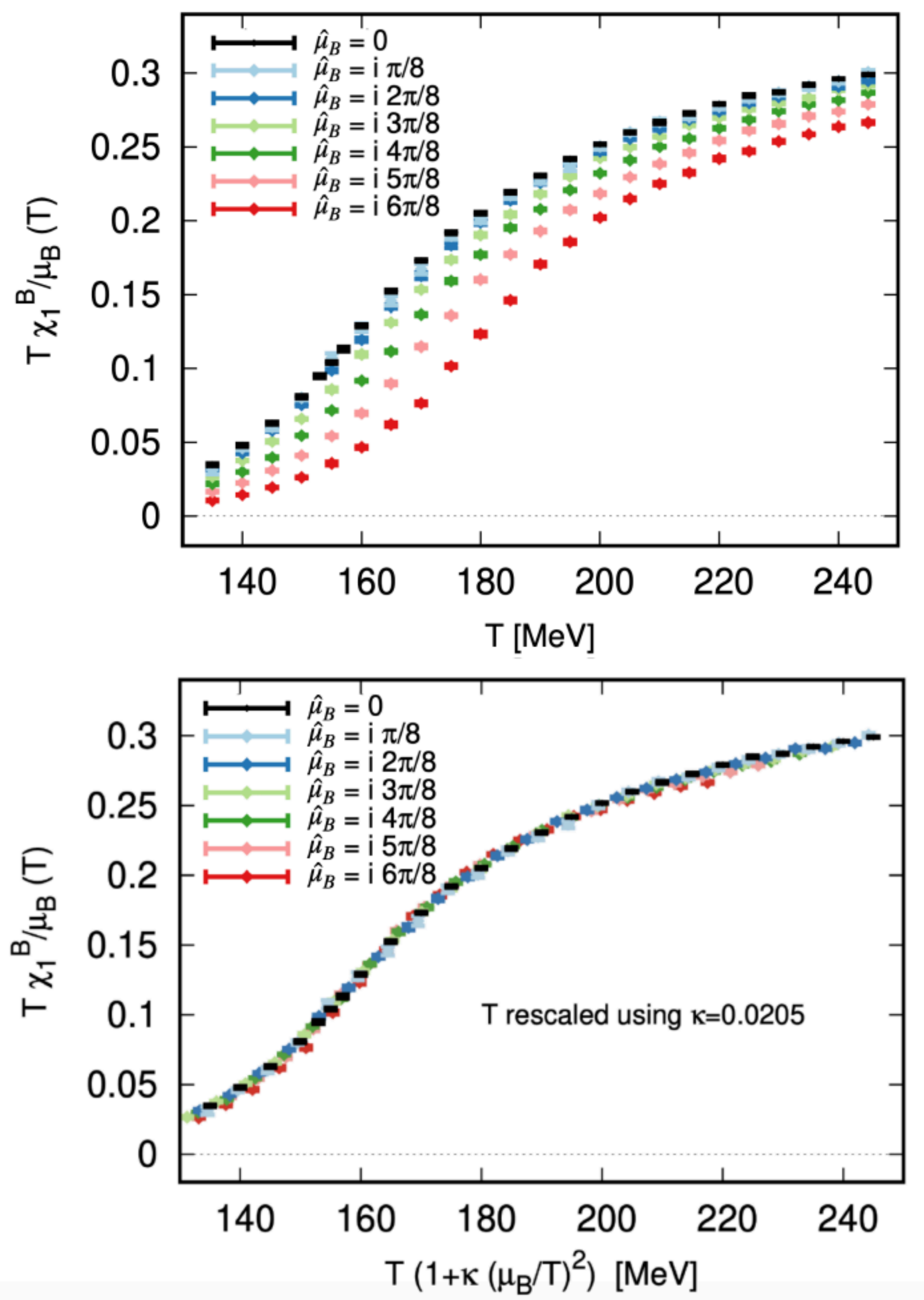


FIG. 1. Upper panel: scaled baryon density $\chi_1^B(T,\mu_B)/\hat{\mu}_B$, as a function of temperature for different values of scaled imaginary baryon chemical potential $\hat{\mu}_B \equiv \mu_B/T$ (labeled using different colors). Lower panel: the same quantity, but with the temperature rescaled by a factor $1+\kappa\hat{\mu}_B^2$, with $\kappa = 0.0205$. In terms of the rescaled temperature the curves representing different $\hat{\mu}_B$ collapse onto the same curve. The points labeled $\hat{\mu}_B = 0$ correspond to the limit $\mu_B \to 0$ which is the baryon number susceptibility $\chi_2^B(T,0)$ (The figure is taken from Ref. [13]).

$$\frac{T\chi_1(\mu_B,T)}{\mu_B} = \chi_2(T^{'},0)$$

Borsanyi et al,21,

Kahangirwe et al., 24