Correcting for volume effects in fluctuations studies of low-energy heavy-ion collisions

MARVIN NABROTH IN COLLABORATION WITH ANAR RUSTAMOV, JOACHIM STROTH, ROMAIN HOLZMANN

Outline

- ❑ Motivation
- ❑ Centrality estimation in heavy-ion experiments
- ❑ Data driven procedure for volume correction
  - ❑ Test in toy model
  - ❑ Test in HADES simulation
- ❑ Summary and Outlook

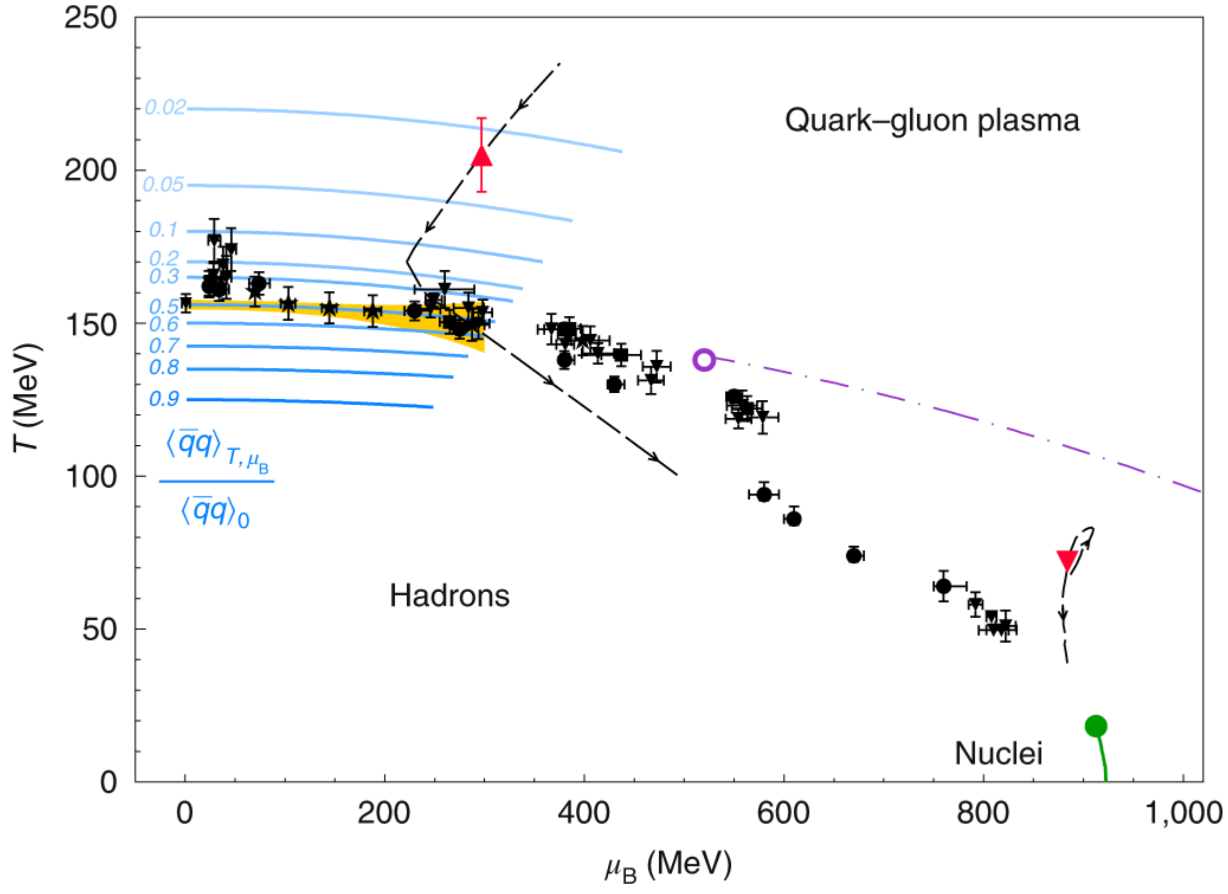
A model-free procedure to correct for volume fluctuations in E-by-E analyses of particle multiplicities
Anar Rustamov, Joachim Stroth, Romain Holzmann
Nucl.Phys.A 1034 (2023), 122641

Controlling volume fluctuations for studies of critical phenomena in nuclear collisions
Romain Holzmann, Volker Koch, Anar Rustamov, Joachim Stroth
e-Print: 2403.03598 [nucl-th]

Motivation – QCD at high μ_B

- Expected first order phase transition ending in a conjectured critical end point towards low μ_b
- High μ_b-region is not accessible by Lattice QCD calculations. (Sign-Problem)
- Only effective approaches

- **Experimental Access: Heavy-Ion collisions at energies at a few GeV**
- **Experimental observables:**
  - **Particle yields, Emisson spectra, Anisotropic flow, Femtoscopy and** E-by-E fluctuations of net-baryon number



The HADES Collaboration. Probing dense baryon-rich matter with virtual photons. *Nat. Phys.* **15**, 1040–1045 (2019)

Event-by-Event Fluctuations

- Study structure of freeze-out in detail by looking at **higher order moments of particle yields**
- Basic idea: Count number of particles emitted per event, extract moments or other distribution measures

Moments $< N^m > = \sum P(N) N^m$

Factorial Moments $C_m = \sum P(N) N(N-1) \dots (N-m+1)$

Moments ⇄ **Cumulants**

⇅ ⇅

Factorial Moments ⇄ **Factorial Cumulants**

Skewness $= \frac{\kappa_3}{\sqrt{\kappa_2^3}}$

Kurtosis $= \frac{\kappa_4}{\kappa_2^2}$

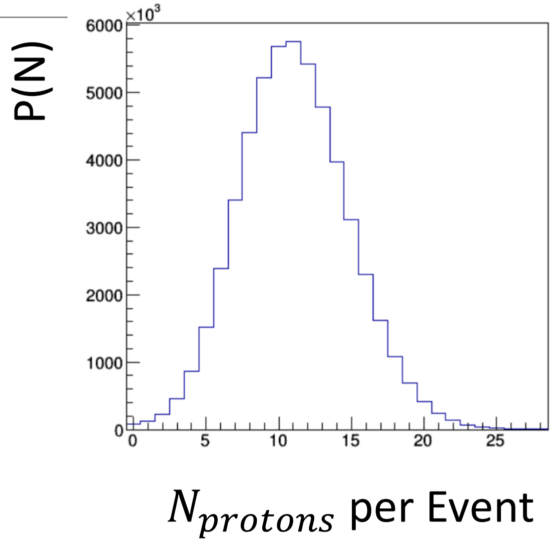
Cumulants

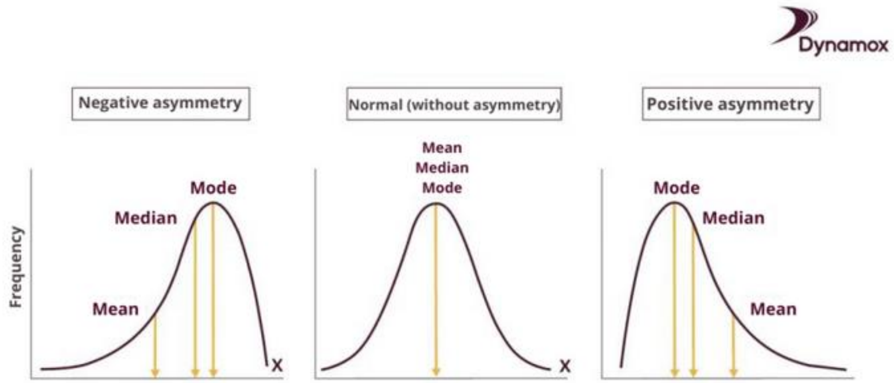
$\kappa_1 = Mean$

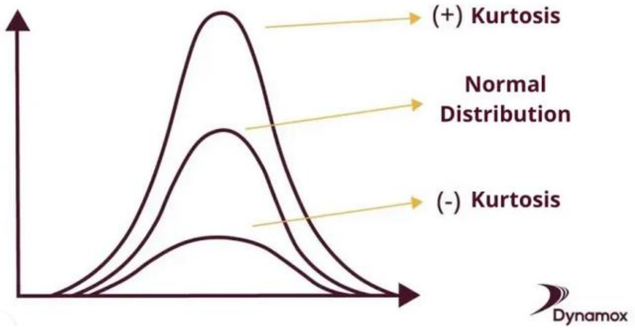
$\kappa_2 = Variance$

$\kappa_3 = < N^3 > - 3 < N^2 >< N > + 2 < N >^3$

$\kappa_4 = < N^4 > - 4 < N^3 >< N > + 3 < N^2 >^2 - 12 < N^2 >< N >^2 - 6 < N >^4$







Event-by-Event Fluctuations

Constant Volume

Equation of State

In the context of statistical model:

$$Z_{GCE}(V,T,\mu) = \sum \exp(-\frac{E_j - \mu N}{T})$$

- How to compare measurement with theory predictions?

Definition of Moments

$$< N^m > = \sum P(N) N^m$$

$$<N> = \frac{\partial \ \ln(Z_{GCE})}{\partial\left(\frac{\mu}{T}\right)}, \quad < N^2 > - < N >^2 = \frac{\partial^2 \ln(Z_{GCE})}{\partial\left(\frac{\mu}{T}\right)^2}, \ldots$$

Other distribution measures:

Central Moments

Cumulants

Factorial Cumulants

$$\kappa_n = \frac{\partial^n}{\partial t^n} g_X(t)$$

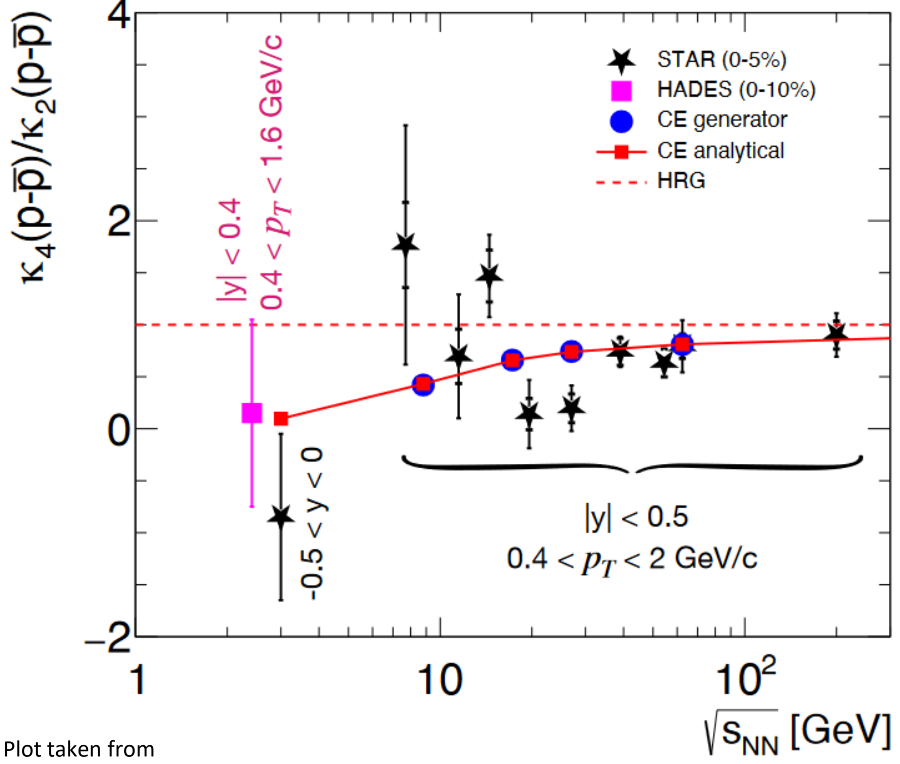
$$\frac{\partial^n \ln(Z_{GCE})}{\partial\left(\frac{\mu}{T}\right)^n}$$

"Generating Function"

→ **Cumulants of baryon number can be connected to n-order derivatives of the partition function**

Experimental goal:
Extract higher order cumulant ratios

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

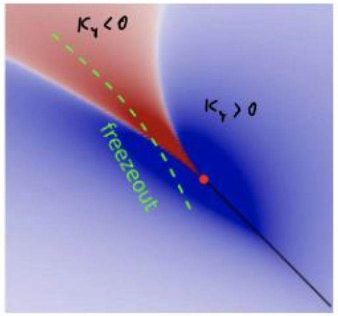


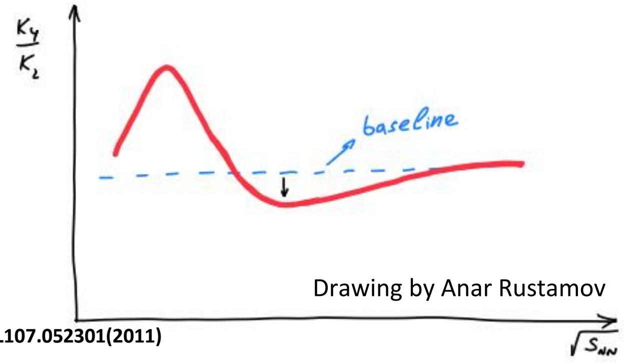
Plot taken from
Anar Rustamov DOI: 10.1051/epjconf/202327601007 , EPJ Web Conf. 276 (2023) 01007)

CPOD talk by Anar Rustamov, Mon 05/20

- ❑Measure energy excitation function of higher order cumulant ratios of net-baryons (protons as proxy in experiment)

  → Expected non-monotonic trend for critical behaviour





Drawing by Anar Rustamov

M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

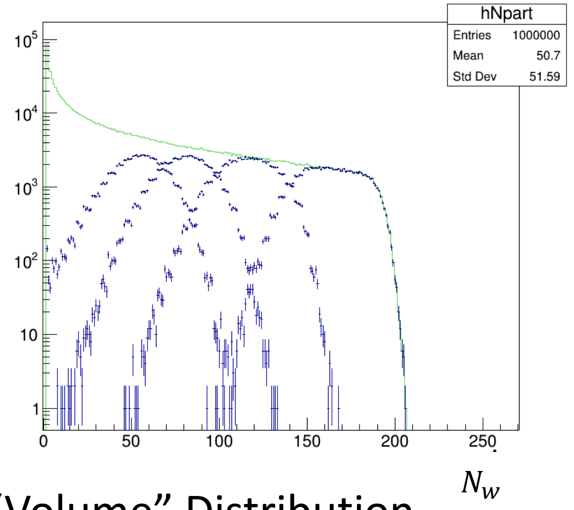
- ❑Probing low energy regime by STAR and HADES
- ❑HADES published proton cumulants data for **Au+Au at 1.23 AGeV**
- ❑Analysis of higher order cumulants for **Ag+Ag (1.58 AGeV) and Ag+Ag (1.23 AGeV)** is ongoing.
- ❑In March 2024, measurement of **Au+Au 0.8 AGeV**
- ❑Ratios do not cancel out volume effects!

Experimental challenges

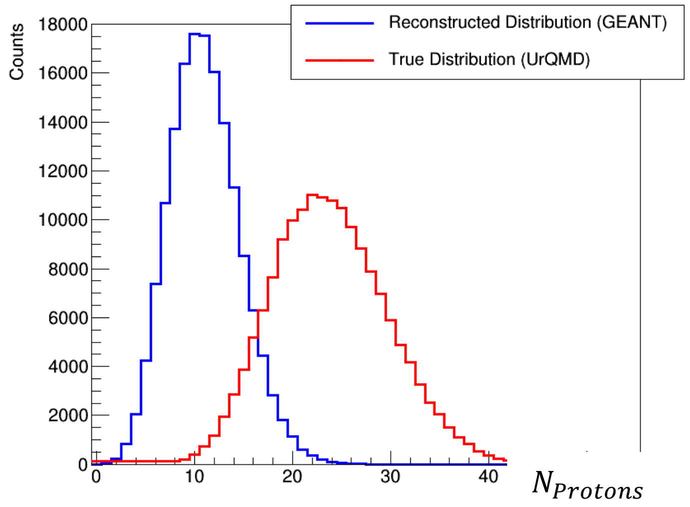
Dynamic/Signal fluctuations → Centrality selection: Signal fluctuation are **averaged over volume** → Imperfect detector efficiencies **distorts** signal → Measured distribution

- ❑ Beam quality variation
- ❑ "Bad" events (Pile-Up…)
- ❑ Contaminated event (Reaction with target frame...)
- ❑ Centrality selection window
- ❑ Centrality estimator can cause auto-correlations

→ We do not measure at constant volume!



"Volume" Distribution



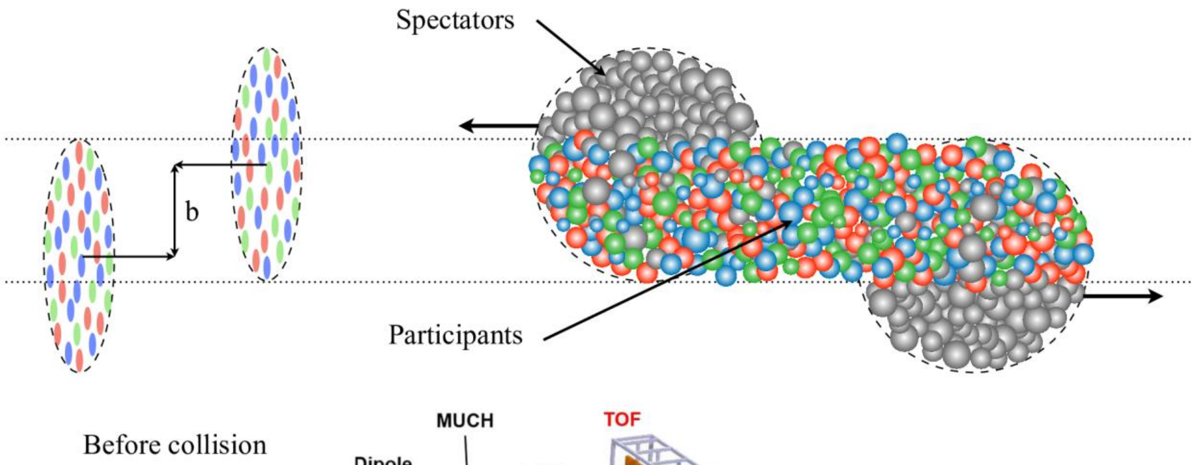
- ❑ Limited acceptance
- ❑ Detector tracking inefficiencies
- ❑ Particle mis-identification

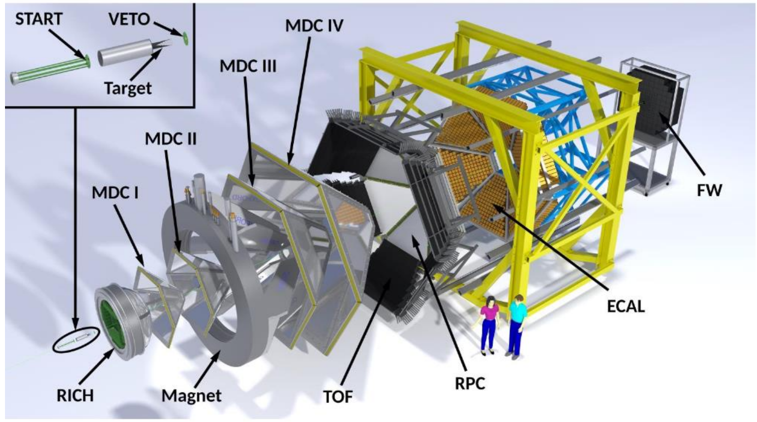
Centrality estimation in experimental data

- ❑Requirement for centrality selector: **Scales monotonically with number of participants or spectators**

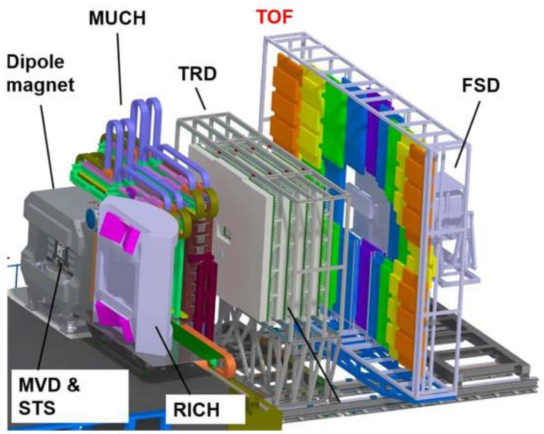
For fluctuation analysis:

- ❑Correlation of centrality estimator to protons should be as small as possible to **avoid auto correlations**





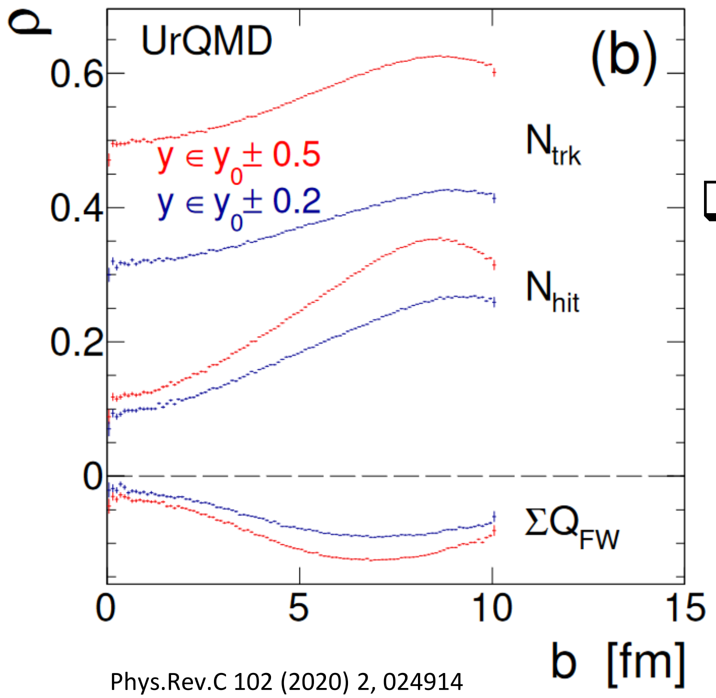
❑HADES
- ❑TOF + RPC Hits
- ❑Charged Tracks
- ❑Forward Wall



❑CBM:
- ❑STS + MVD Hits
- ❑Charged tracks
- ❑Charged Meson tracks
- ❑FSD Measurement

Centrality estimation in experimental data

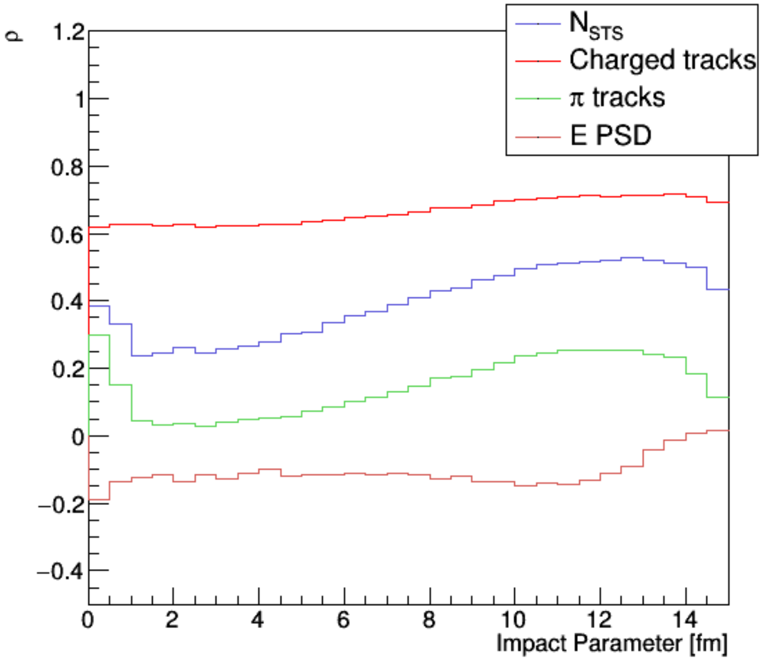
Transport + HADES GEANT Simulation Au+Au 1.23 AGeV



Phys.Rev.C 102 (2020) 2, 024914

- HADES
  - TOF + RPC Hits
  - Charged tracks
  - Forward Wall

PHQMD + CBM Simulation Au+Au 12 AGeV



- CBM:
  - STS + MVD Hits
  - Charged tracks
  - Charged meson tracks
  - FSD Measurement

$$\rho = \frac{cov(Centrality\ Estimator, N_{prot})}{\sigma_X \sigma_Y}$$

How to connect centrality estimator with volume quantities? – Glauber Model

TGlauberMC[1]

Sample impact parameter b

Randomly distribute nucleons according to Fermi Distribution

Perform collision based on straight line approximation
→ N_w, N_{col}

$$\boldsymbol{N_{soures} = f N_{part} + (1 - f) N_{coll}}$$

Sample Number of Hits $\boldsymbol{N_{tracks}}$
(Draw from e.g. **Negative Binomial, Gaussian, Poisson**)

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} * \frac{\left(\frac{\mu}{k}\right)^n}{\left(\frac{\mu}{k}+1\right)^{n+k}}$$

$\boldsymbol{N_{soures}\ times}$

Output:

N_{tracks}–Distribution, N_{part} – Distribution.

Correlation between N_{tracks} $and\ b, N_{part}$

NN Cross Section and Nuclei properties

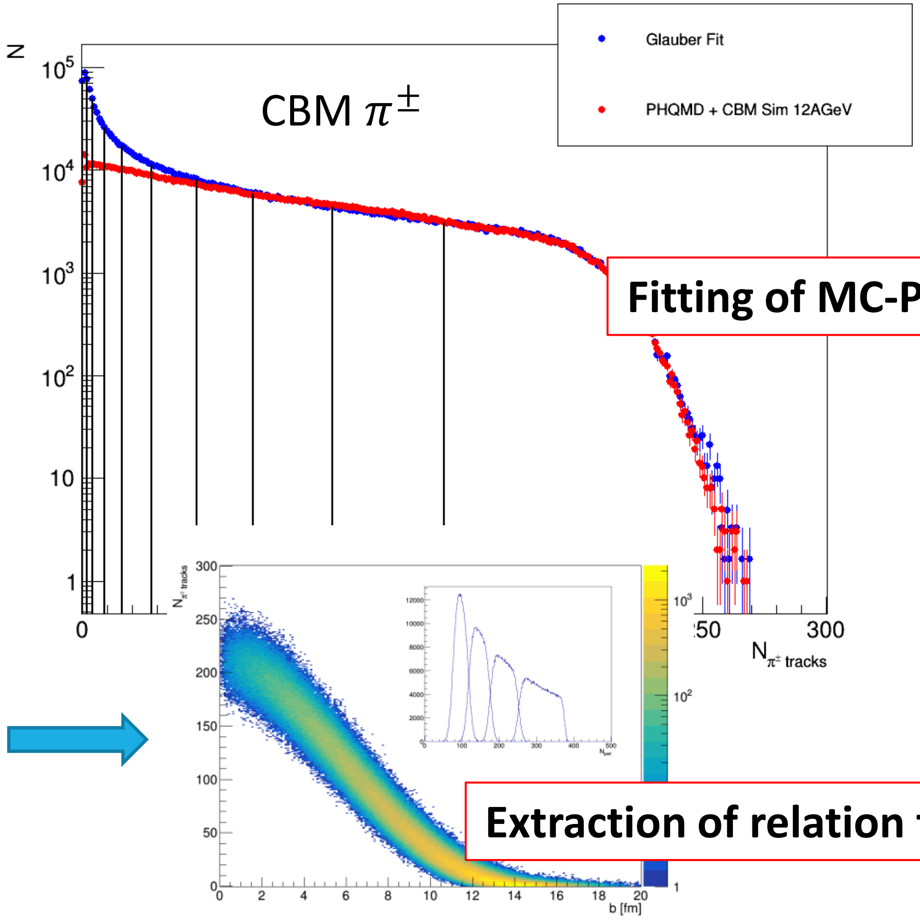
- Fit MC-distribution to the measured one → X^2 Minimization

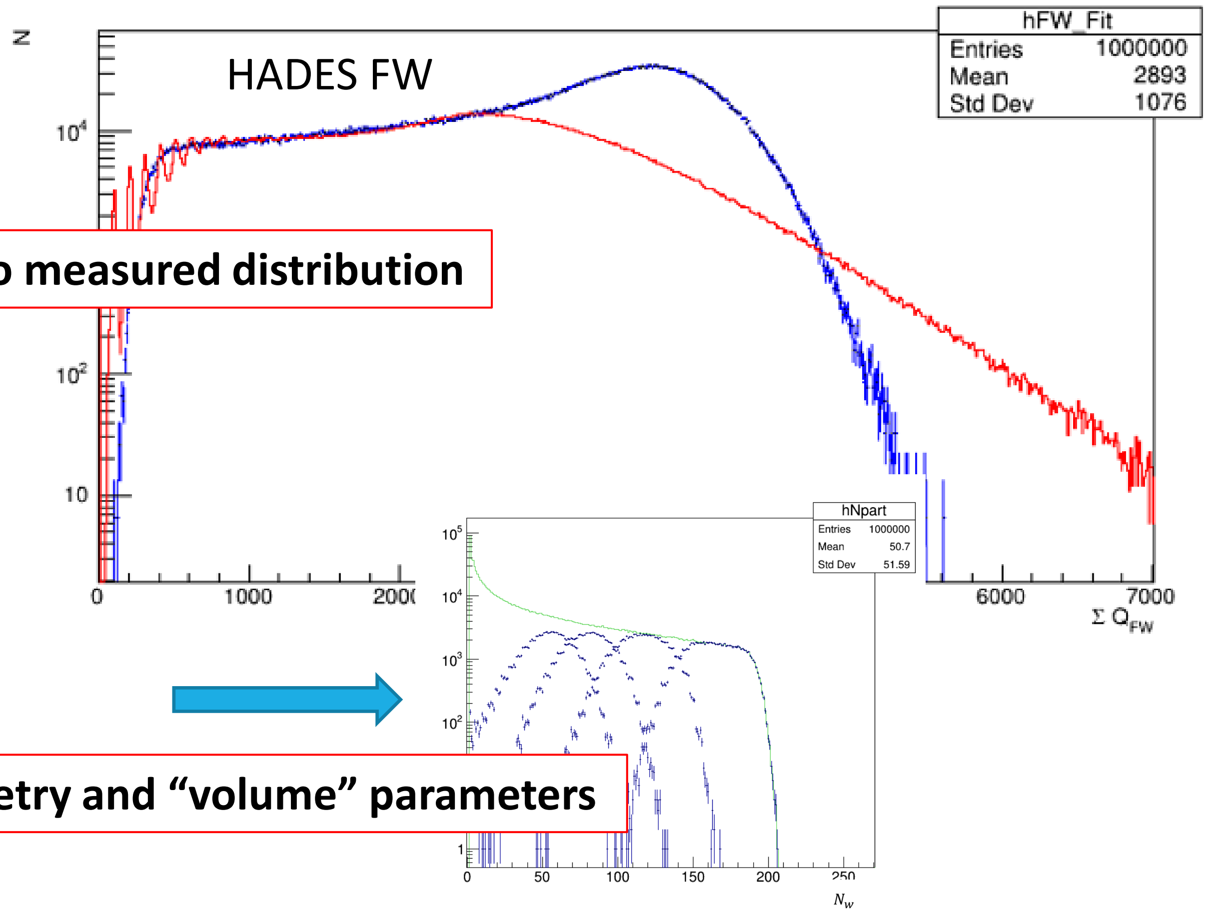
MC Process defined by

$\boldsymbol{(\mu, k, f)}$

[1] Improved version of the PHOBOS Glauber Monte Carlo
C. Loizides (LBL, Berkeley), J. Nagle (Colorado U.), P. Steinberg (Brookhaven)
SoftwareX 1-2 (2015) 13-18 , SoftwareX 1-2 (2015), 13-18

How to connect centrality estimator with volume quantities? – Glauber Model





Fitting of MC-Process to measured distribution

Extraction of relation to geometry and "volume" parameters

Definition of volume at low energies? Sources for particle emission?

N_{coll}

N_w

N_{part}

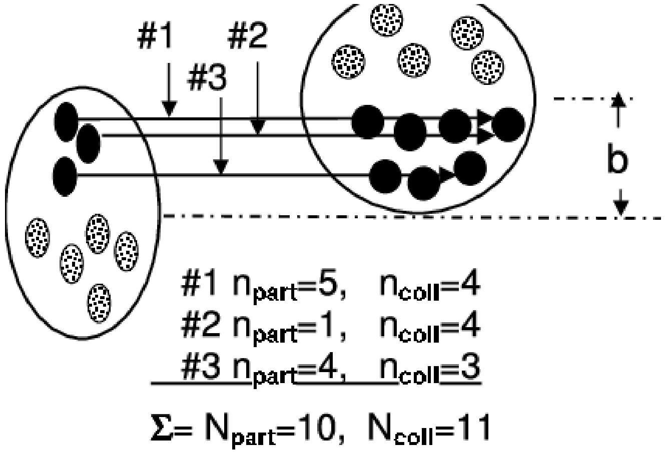
In literature usually / for Glauber: $\boldsymbol{N_{part} = N_w}$

Only accessible in transport models, not in Glauber

- *Number of binary collisions*

- *Nucleons that under went at least one inelastic collision*

- *Wounded nucleons + Nucleons that interacted with produced particles.*



Picture taken from

Carroll et. all (2003). PHOBOS at RHIC: Some global observations. Pramana. 61. 865-876. 10.1007/BF02704455

N_{eff}

- $N_{eff} = f\, N_{coll} + (1-f)N_w$

Furthermore, for low collision energies:

- Interaction of particles with spectators
- At low energies most of the protons are not produced, but rescattered

Volume effects on cumulants - Independent Source Model

- How to describe **volume contributions** in **observed particle cumulants**?
- Model with simplest assumption:
  - **Sources of particle production** are **statistical independent** [*][**]
  - Moment generating function factorizes

$$< N^m > (N_w) = \frac{d^m}{dt^m}[M_N(t)]$$

$$M_N(t) = [M_N(t)]^{N_{src}}$$

$$k_1(N) = < N_{Src} >< n >$$

$$k_2(N) = < N_{Src} > k_2(n) + < n >^2 k_2(N_{src})$$

$$k_3(N) = < N_{Src} > k_3(n) + 3k_1(n)k_2(n)k_2(N_{src}) + < \mathrm{n} >^3 \mathrm{k}_3(\mathrm{N}_{\mathrm{src}})$$

$$k_4(N) = < N_{Src} > k_4(n) + 4 < n > k_3(n)k_2(N_{src}) + 3k_2^2(n)k_2(N_{src}) + 6 < n >^2 k_2(n)k_3(N_{src}) + < n >^4 k_4(N_{src})$$

- Independent source model can also be applied to covariances

$k_m(N) \coloneqq$ *Oberserved particle cumulants*

$k_m(n) \coloneqq$ *Single source particle cumualnts*

$k_m(N_{src}) \coloneqq$ *Cumulants of sources\participants*

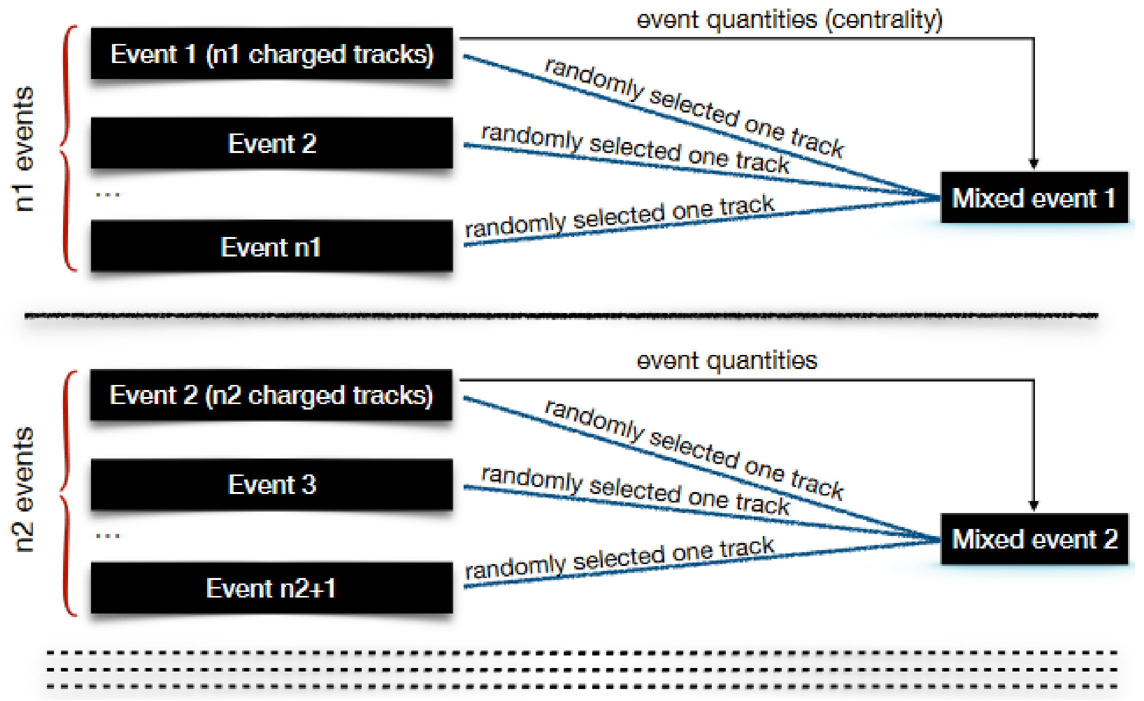
[*] V. Skokov, B. Friman, K. Redlich, Volume Fluctuations and Higher Order Cumulants of the Net Baryon Number, Phys. Rev. C 88 (2013) 034911. arXiv:1205.4756, doi:10.1103/PhysRevC.88.034911

[**] Bridging the gap between event-by-event fluctuation measurements and theory predictions in relativistic nuclear collisions
P. Braun-Munzinger (Darmstadt, EMMI and Heidelberg U.), A. Rustamov (Heidelberg U. and NNRC, Baku), J. Stachel (Heidelberg U.) Nucl.Phys.A 960 (2017), 114-130

Volume Correction – Novel Procedure Cumulants of sources

- To obtain single source particle cumulants, need to know cumulants of participants
  - Use cumulants from MC simulation (Transport, Glauber,…) → Relay on further assumptions / model dependence
  - **Better: Data driven approach**
- Novel procedure based on event mixing:

A model-free procedure to correct for volume fluctuations in E-by-E analyses of particle multiplicities
Anar Rustamov, Joachim Stroth, Romain Holzmann
Nucl.Phys.A 1034 (2023), 122641



Volume Correction
Cumulants of sources

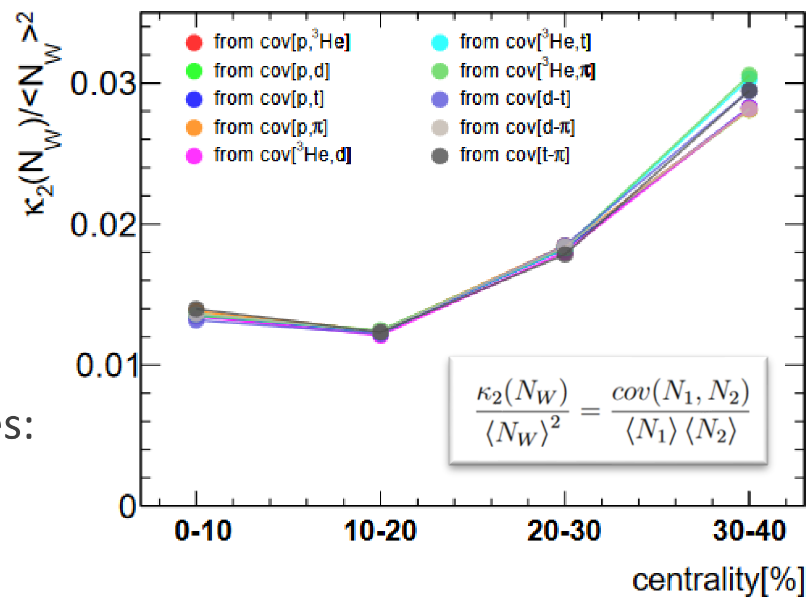
- Event mixing removes correlations between particles
  - In case of Poisson like behaviour one expects for the emission per sources:
    - $k_m(n) = \langle n \rangle, \; cov(n_1, n_2) = 0$

$$\kappa_2(N) = \langle N_W \rangle \langle n \rangle + \langle n \rangle^2 \kappa_2(N_W) = \langle N \rangle + \langle N \rangle^2 \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$

$$cov(N_1, N_2) = \langle n_1 \rangle \langle n_2 \rangle \kappa_2(N_W) = \langle N_1 \rangle \langle N_2 \rangle \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$

$$\Rightarrow \quad \frac{\kappa_2(N_W)}{\langle N_W \rangle^2} = \frac{cov(N_1, N_2)}{\langle N_1 \rangle \langle N_2 \rangle} \qquad \frac{\kappa_2(N_W)}{\langle N_W \rangle^2} = \frac{\kappa_2(N)}{\langle N \rangle^2} - \frac{1}{\langle N \rangle}$$

- Procedure can be extended with the same argument to higher orders.



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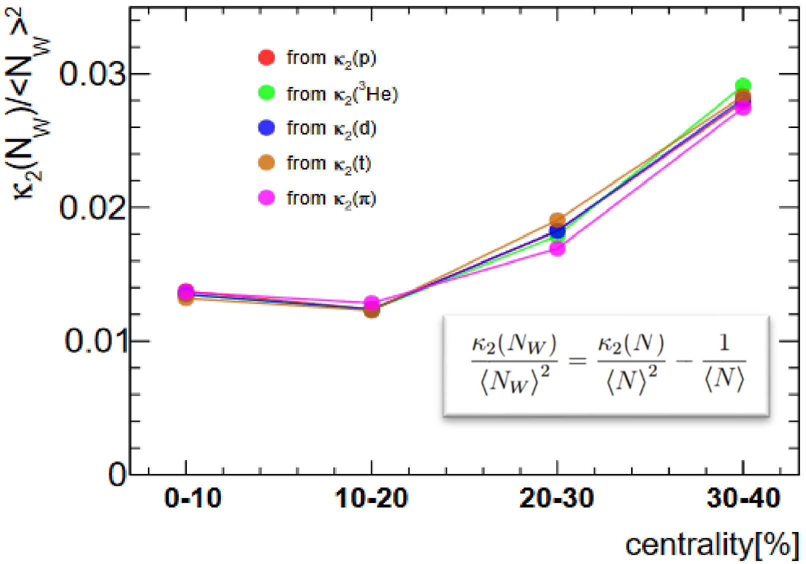


Volume Correction
Generalized correction scheme

- New paper by **V. Koch, A. Rustamov, R. Holzmann & J. Stroth**
  - Mathematical generalization using factorial cumulants
  - They found that event mixing is equivalent to charged tracks
  - Additional Bias terms that account for difference to Poisson distribution

Controlling volume fluctuations for studies of critical phenomena in nuclear collisions
Romain Holzmann, Volker Koch, Anar Rustamov, Joachim Stroth
e-Print: 2403.03598 [nucl-th]
https://arxiv.org/pdf/2403.03598.pdf

- Independent w.r.t to average Binomial efficiency

$$\frac{\kappa_2[N_w]}{\langle N_w\rangle^2} = \frac{C_2[M]-\bar{C}_2[M]}{\langle M\rangle^2}$$

$$\frac{\kappa_3[N_w]}{\langle N_w\rangle^3} = -3\frac{\bar{C}_2[M]}{\langle M\rangle^2}\frac{\kappa_2[N_w]}{\langle N_w\rangle^2} + \frac{C_3[M]-\bar{C}_3[M]}{\langle M\rangle^3}$$

$$\frac{\kappa_4[N_w]}{\langle N_w\rangle^4} = -6\frac{\bar{C}_2[M]}{\langle M\rangle^2}\frac{\kappa_3[N_w]}{\langle N_w\rangle^3} - \frac{4\bar{C}_3[M]\langle M\rangle + 3\bar{C}_2[M]^2}{\langle M\rangle^4}\frac{\kappa_2[N_w]}{\langle N_w\rangle^2} + \frac{C_4[M]-\bar{C}_4[M]}{\langle M\rangle^4}$$

$\overline{C_n}(M) \coloneqq Factorial\ Cumulants\ of\ charged\ tracks, for\ \boldsymbol{fixed}\ \boldsymbol{N_w}$

$C_n(M) \coloneqq Factorial\ Cumulants\ of\ charged\ tracks, \boldsymbol{averaged\ over}\ \boldsymbol{N_w}$ **:= what we can measure in experiment**

Volume Correction - Test
Toy model - Reco. of participant cumulants

Participant generation
Glauber Model

→

Particle Generation
Poisson, Negative Binomial

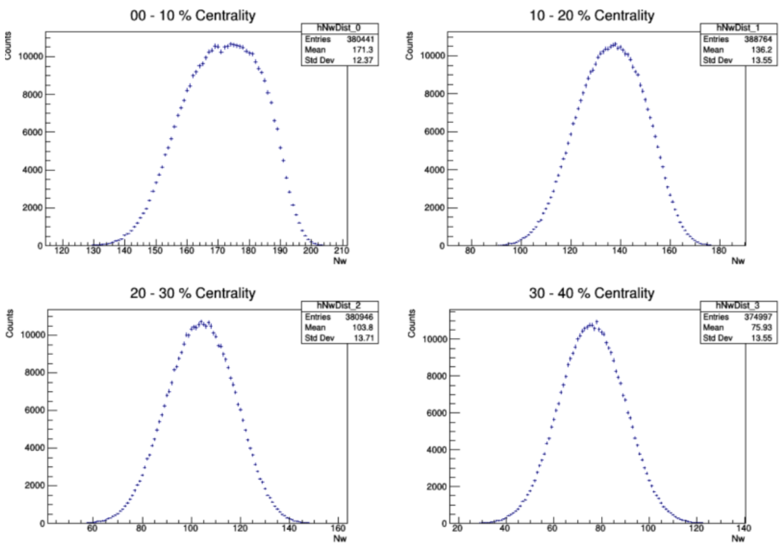
→

Detector Effects

→

Particle Number Distribution folded with Volume Distribution

Negative Binomial



- HADES, Ag+Ag 1.58 AGeV (Real Data):

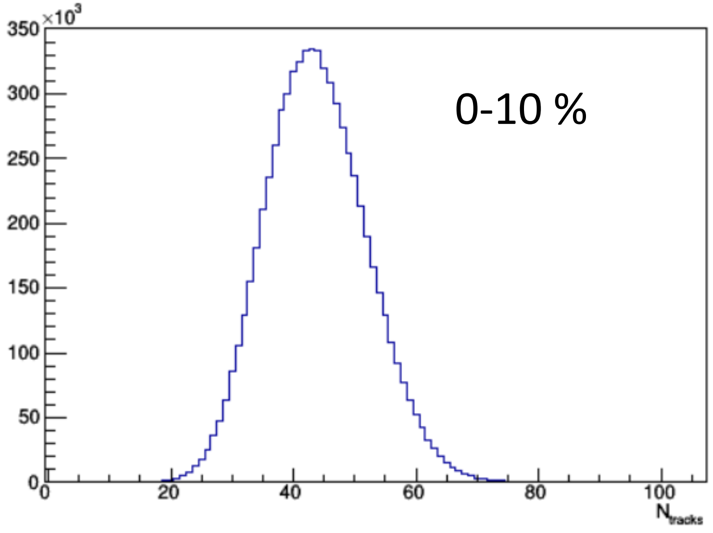
$$f = 1, \mu = 0.35, k = 24$$

Cumulants of Negative Binomial

$$C_n(n) = (n-1)!\frac{p^n r}{(1-p)^n}$$

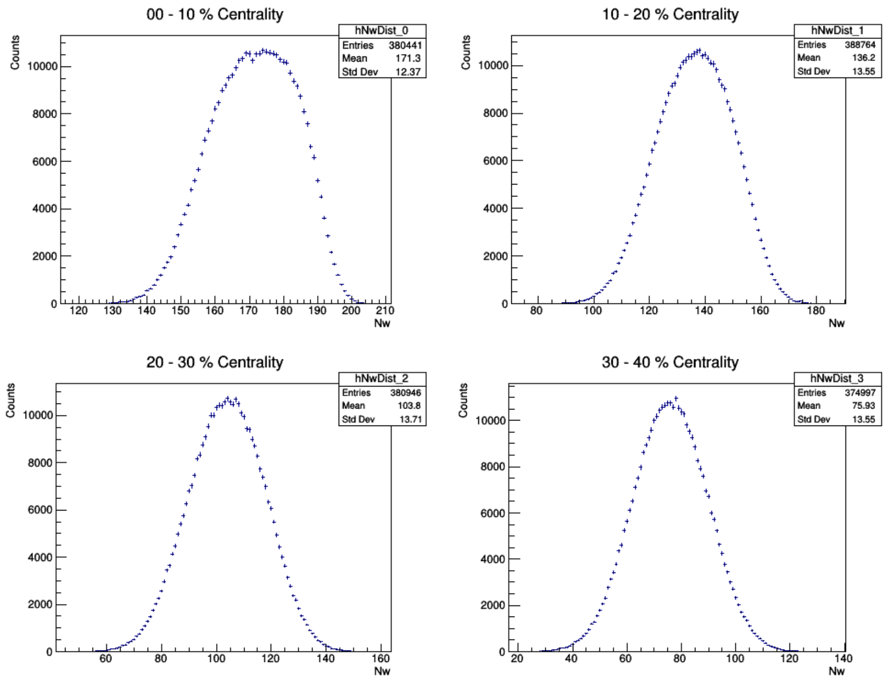
Volume Correction
Test in HADES Simulation

Charged Track Distribution



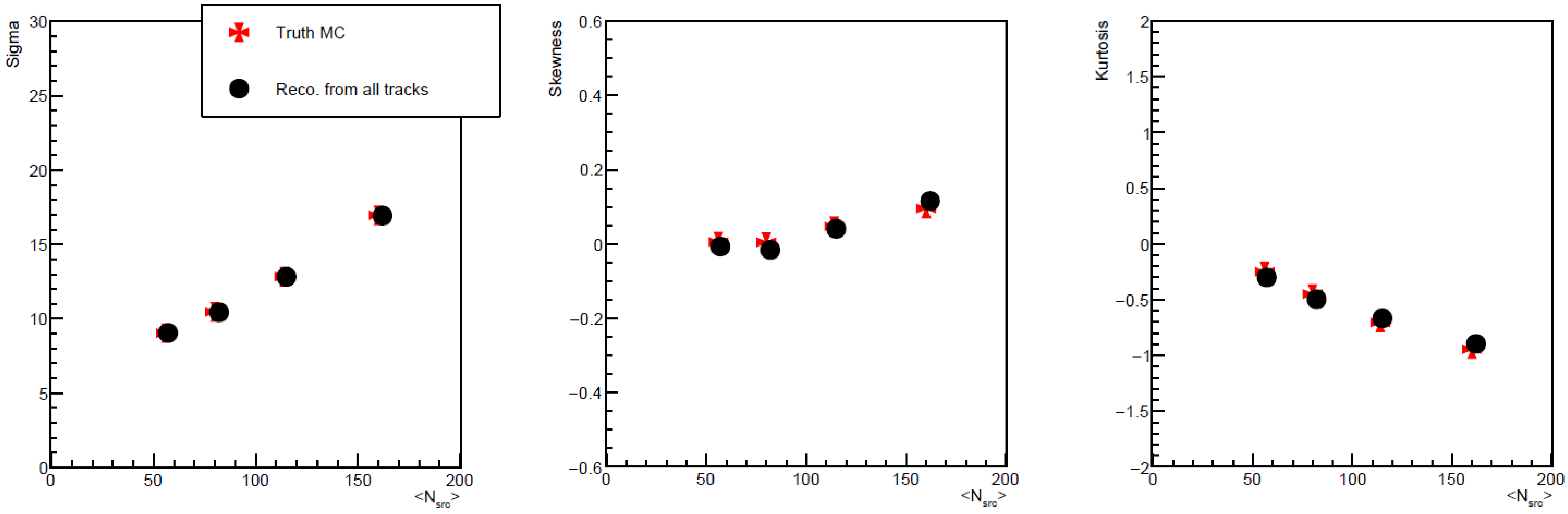
Aim:
Reconstruct cumulants of participants from cumulants of all charged tracks or mixed event single particle sample

Distribution **of Source / Participants**



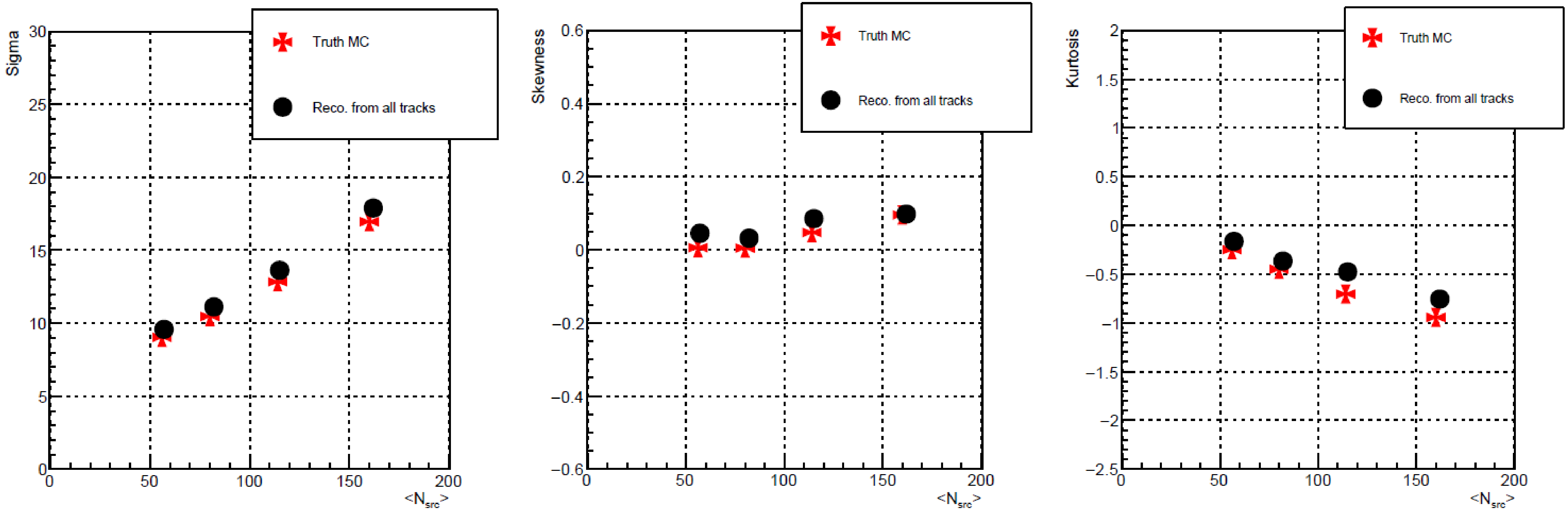
Volume Correction
Toy model – Reco. of participant cumulants

Poisson Toy Model



- For Poissonian particle emission bias terms vanish, good agreement with MC Truth
  - Formulars are implemented correctly

Volume Correction
Toy model - Rec. of participant cumulants

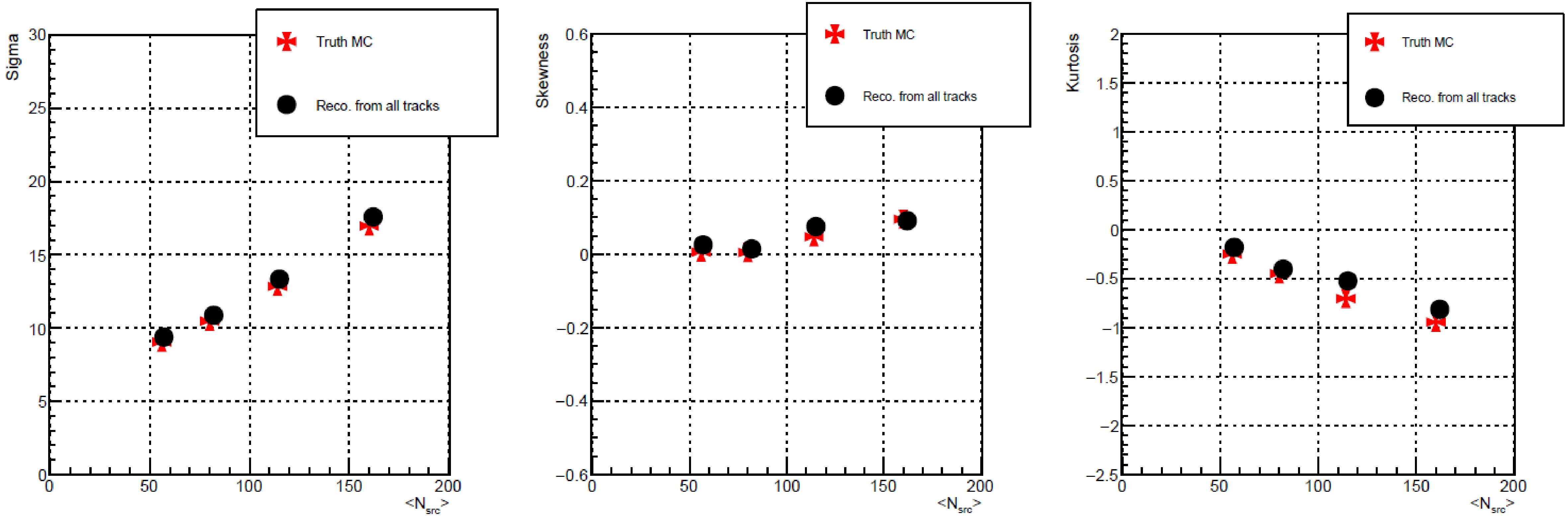


- ❑ HADES Negative Binomial Toy Model exhibit good agreement with MC Truth

Volume Correction
Toy model - Rec. of participant cumulants

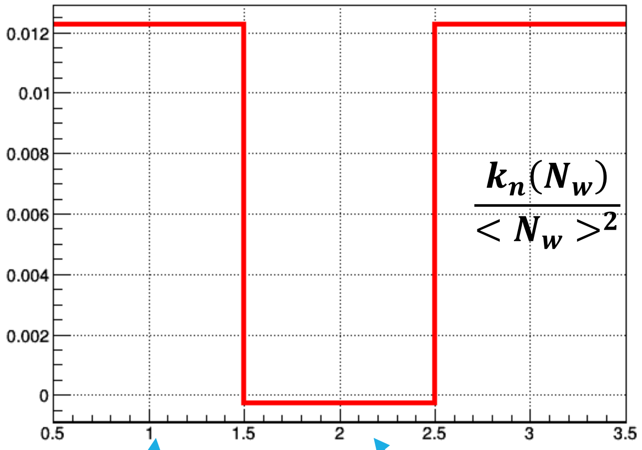
HADES Toy Model

With Bias Terms

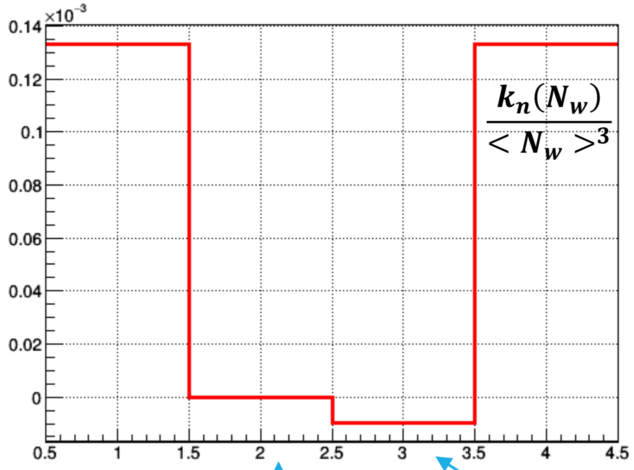


- HADES Negative Binomial Toy Model exhibit good agreement with MC Truth
- Contribution from Bias terms are small

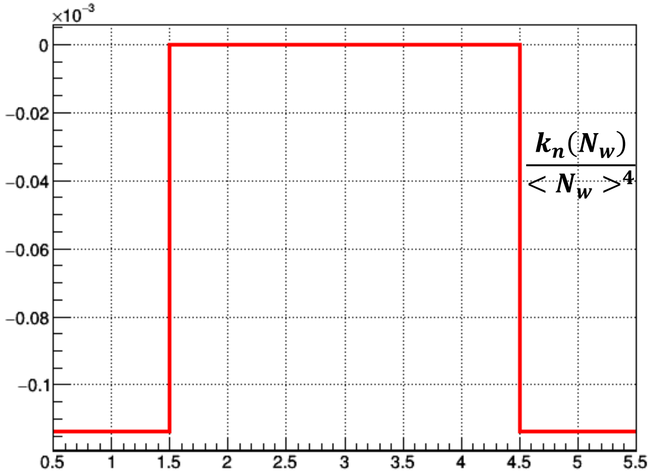
Volume Correction
Toy model - Influence of Bias terms



$$\frac{C_2[M]}{< M >^2} + \frac{-\overline{C_2}[M]}{< M >^2}$$



$$\frac{C_3[M]}{<M>^3} + \frac{-\overline{C_3}[M]}{<M>^3} + \frac{-3\,\overline{C_2}[M]}{<M>^2}\frac{\kappa_2[N_w]}{<N_w>^2}$$



$$\frac{C_4[M]}{<M>^4} + \frac{-\overline{C_4}[M]}{<M>^4} + \frac{-6\,\overline{C_2}[M]}{<M>^2}\frac{\kappa_3[N_w]}{<N_w>^3} + \left(-\frac{4\,\overline{C_3}[M]<M>+3\overline{C_2}[M]^2}{<M>^2}\frac{2[N_w]}{<N_w>^2}\right)$$

- Bias terms for HADES-Neg.-Bino-Toy model are small as cumulants are close to Poisson

Volume Correction Test in HADES Simulation

Fluctuation measure between particle A and B

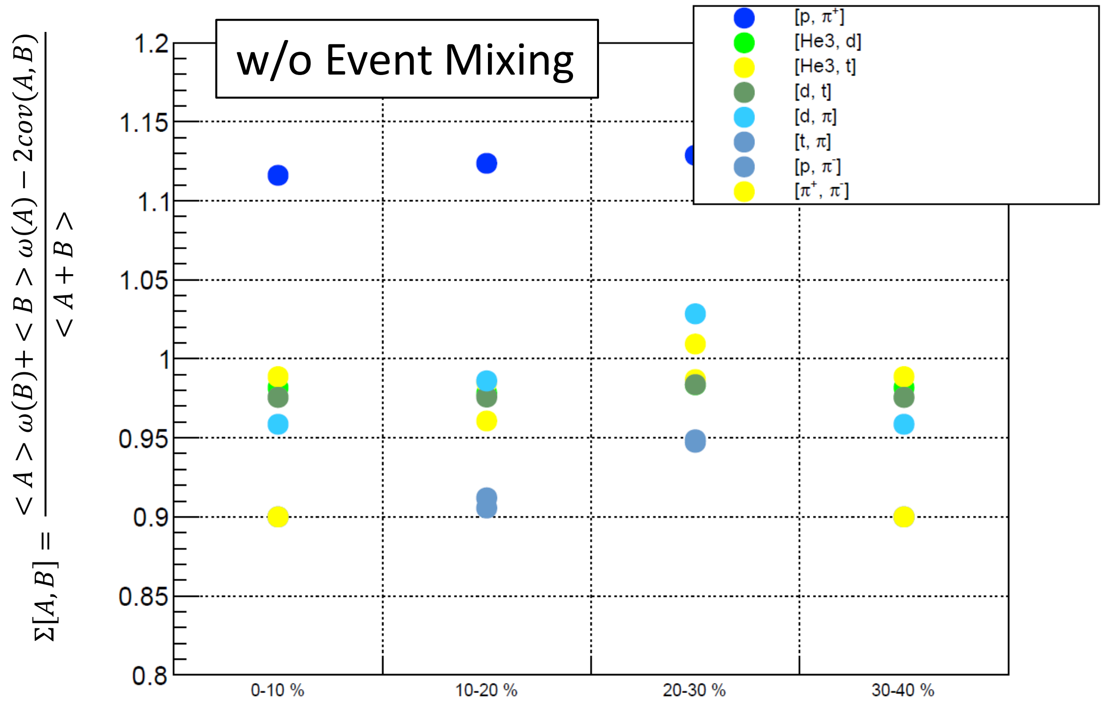
$$\Sigma[A,B] = \frac{<A> \omega(B) + <B> \omega(A) - 2cov(A,B)}{<A+B>}$$

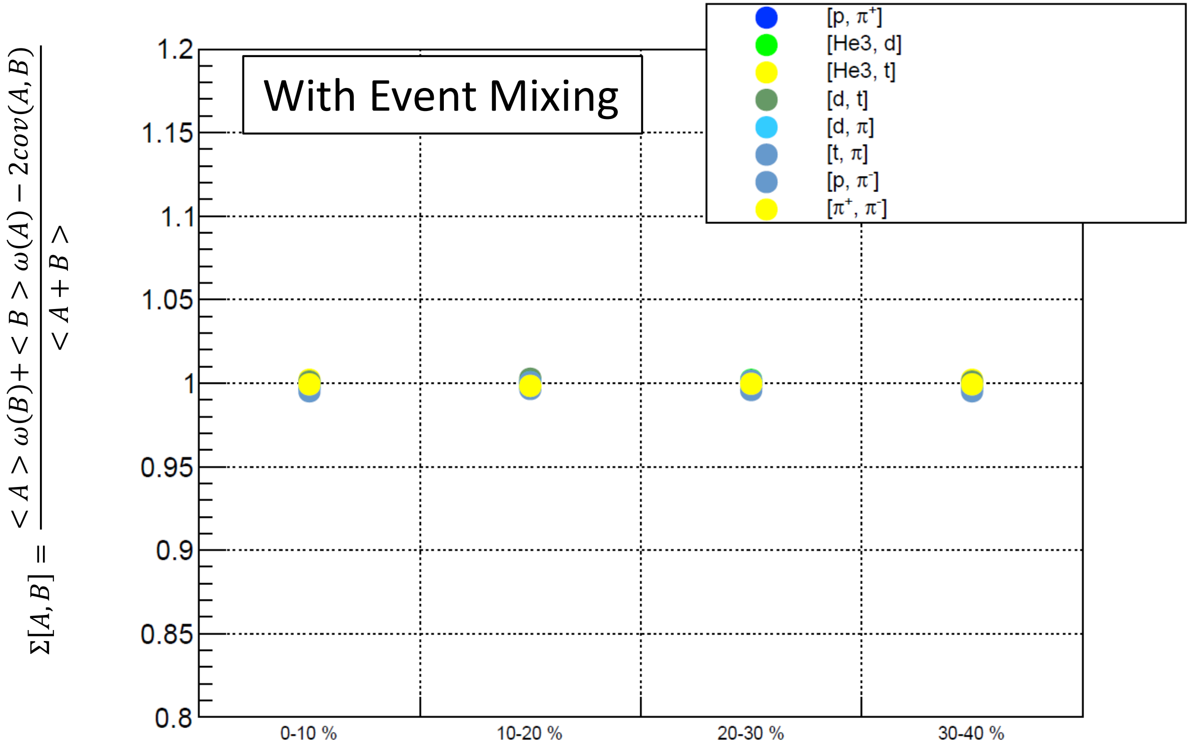
$\omega(X)$:== Scaled Variances

No correlations → $\Sigma[A,B]$ = 1

M. I. Gorenstein, M. Gazdzicki, Strongly Intensive Quantities, Phys. Rev. C 84 (2011) 014904

- Simulation: Smash+(Clustering) Ag+Ag 1.58 AGeV





✓ Event mixing scheme successfully removes correlations

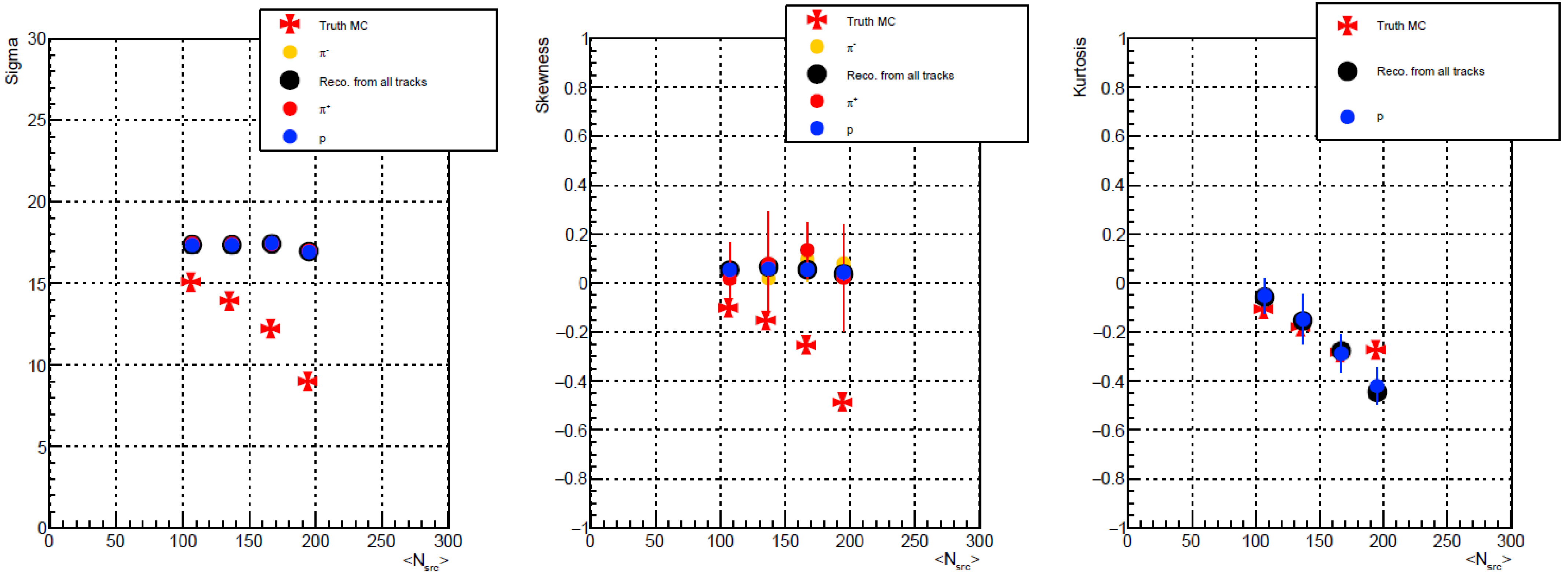
Volume Correction Test in HADES Simulation

No acceptance restriction

$N_{src} = N_{part}$

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}



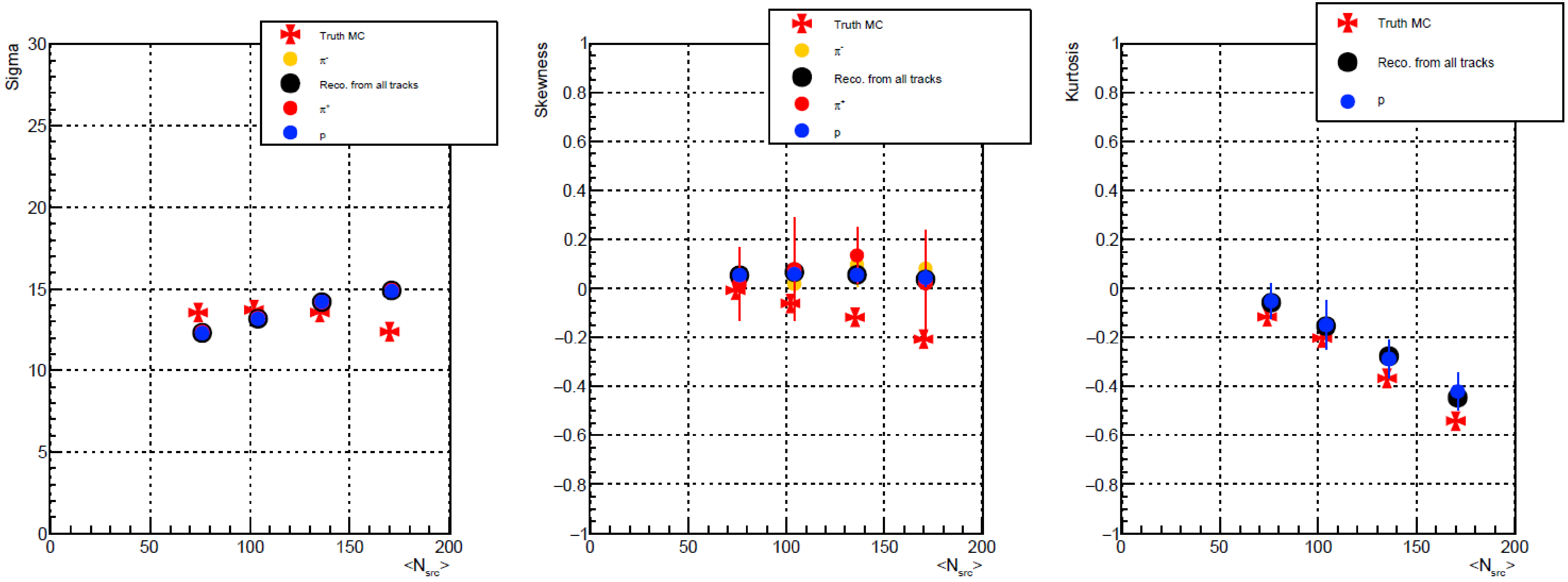
Volume Correction Test in HADES Simulation

No acceptance restriction

$N_{src} = \mathrm{N}_w$

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}



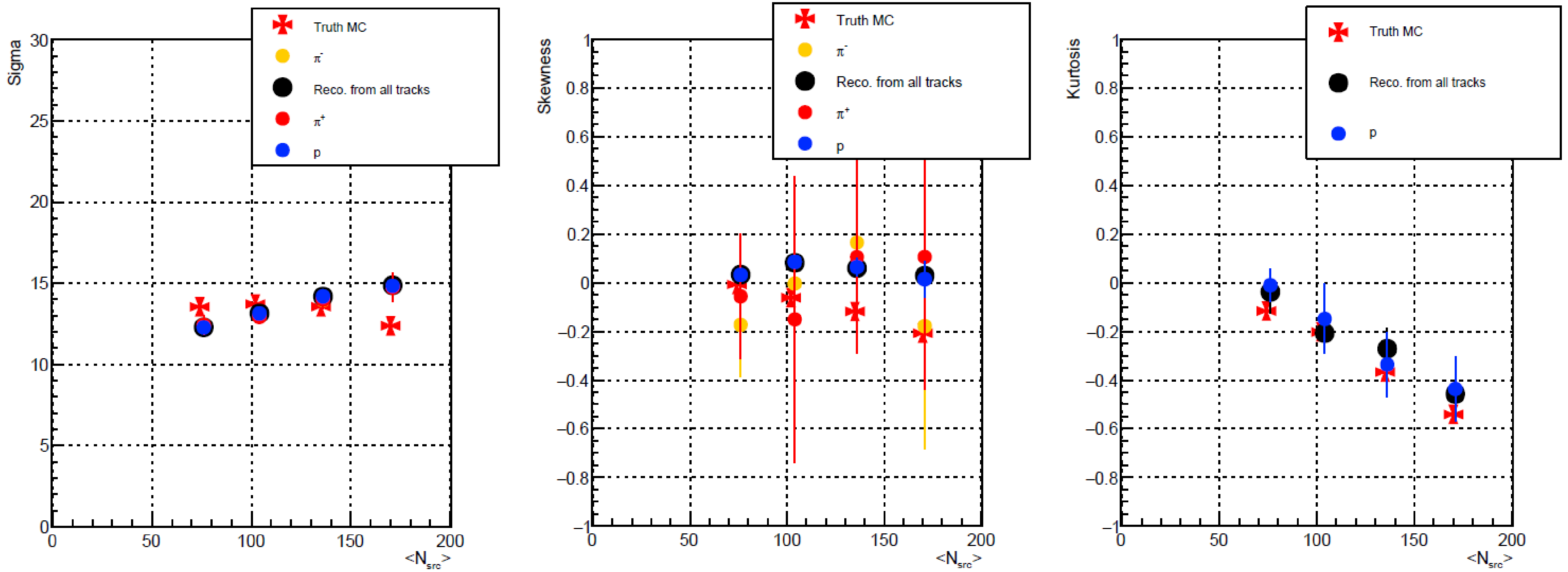
Volume Correction Test in HADES Simulation

Acceptance Restriction: |ycm| < 0.4

$N_{src} = \mathrm{N_w}$

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}



Volume Correction
Test in HADES Simulation

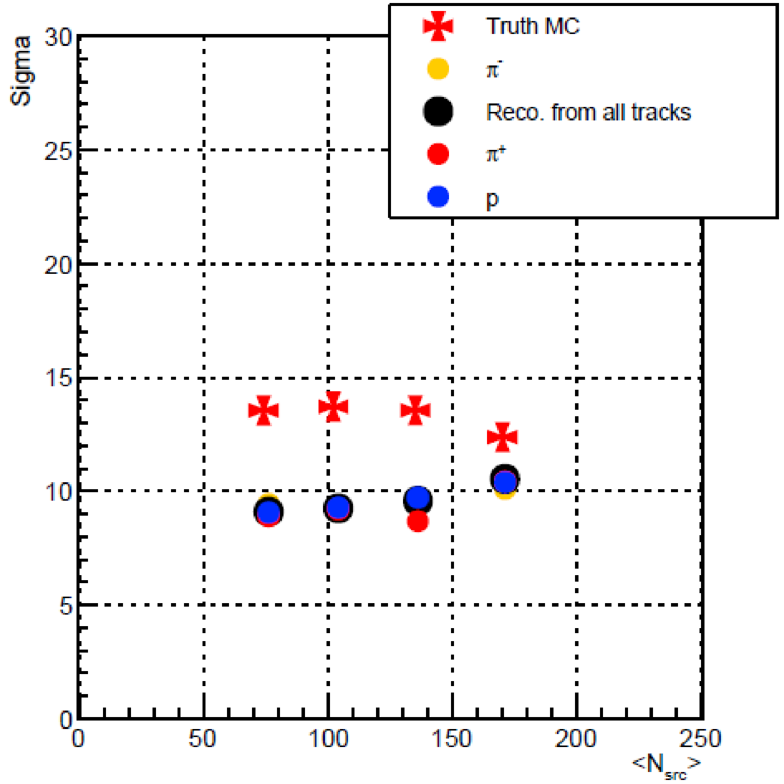
No acceptance restriction

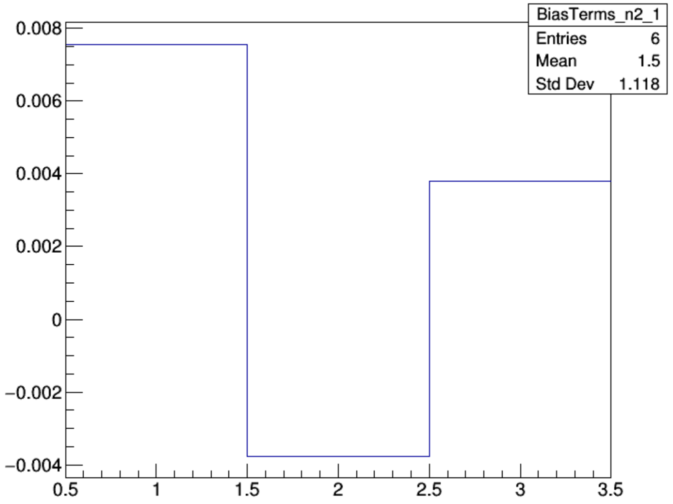
$N_{src} = N_w$

Bias Terms

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}





$$\frac{C_2[M]}{< M >^2} + \frac{-\overline{C_2}[M]}{< M >^2}$$

- Bias terms lead to an over correction
- For higher order statistic of used simulation not sufficient.

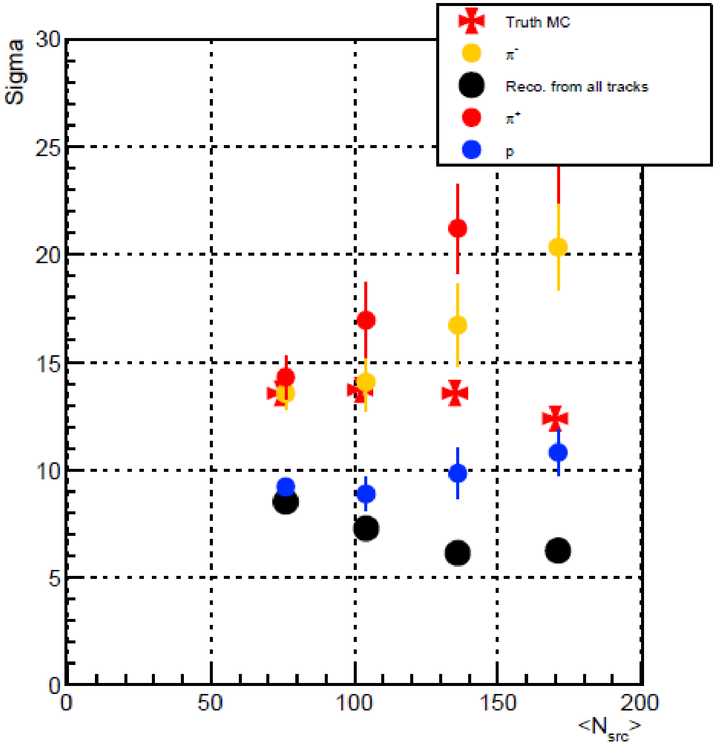
Volume Correction Test in HADES Simulation

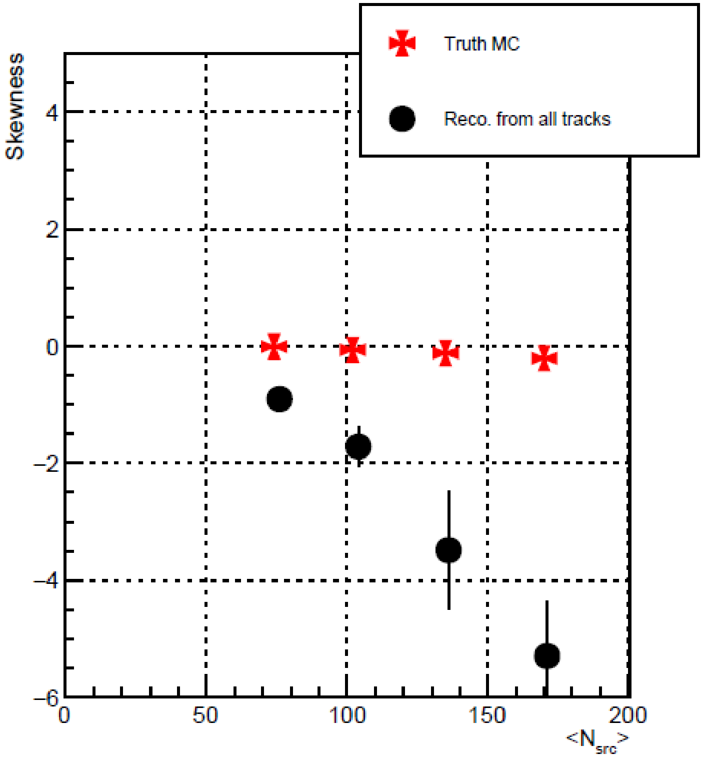
Acceptance Restriction: |ycm| < 0.4

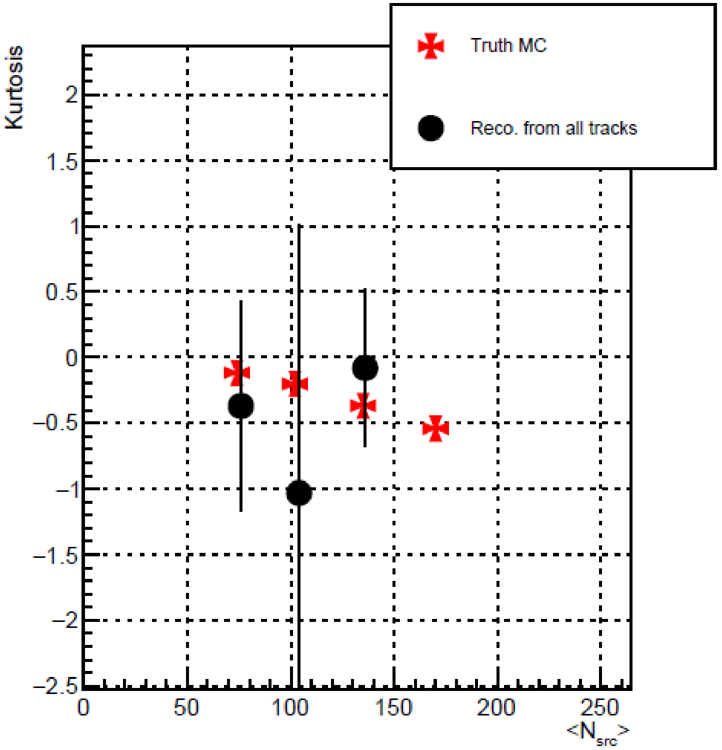
$N_{src} = N_w$

- ❑ Smash+(Clustering) Ag+Ag 1.58 AGeV
- ❑ Test after HADES track reconstruction (GEANT 3.4)

MC Truth: Cumulants of N_{src}







Summary

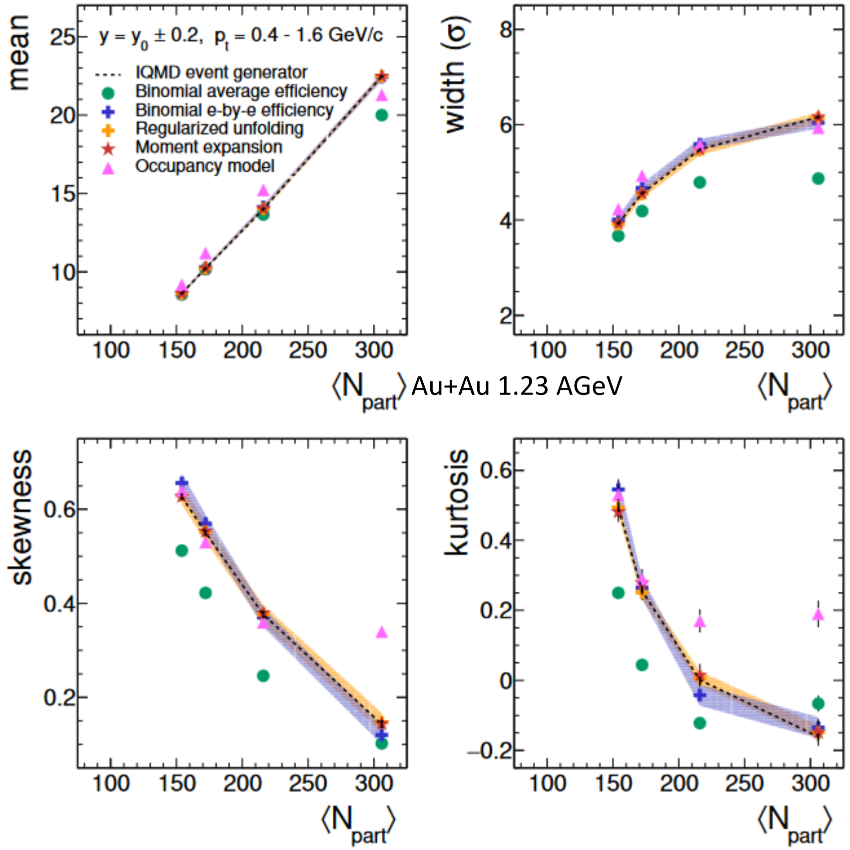
- In experiment, extracted cumulants are averaged over distribution of participants
  - Need to correct for this effect, before doing any comparisons with theory predictions

- Centrality selection and connection to volume parameters in experiment is done via implementation of the MC Glauber Model, Fits to measured centrality selectors, e.g. hits, number of tracks (For fluctuation studies → auto-correlations!)
  - Use Forward Wall (Spectator measurement), Meson tracks

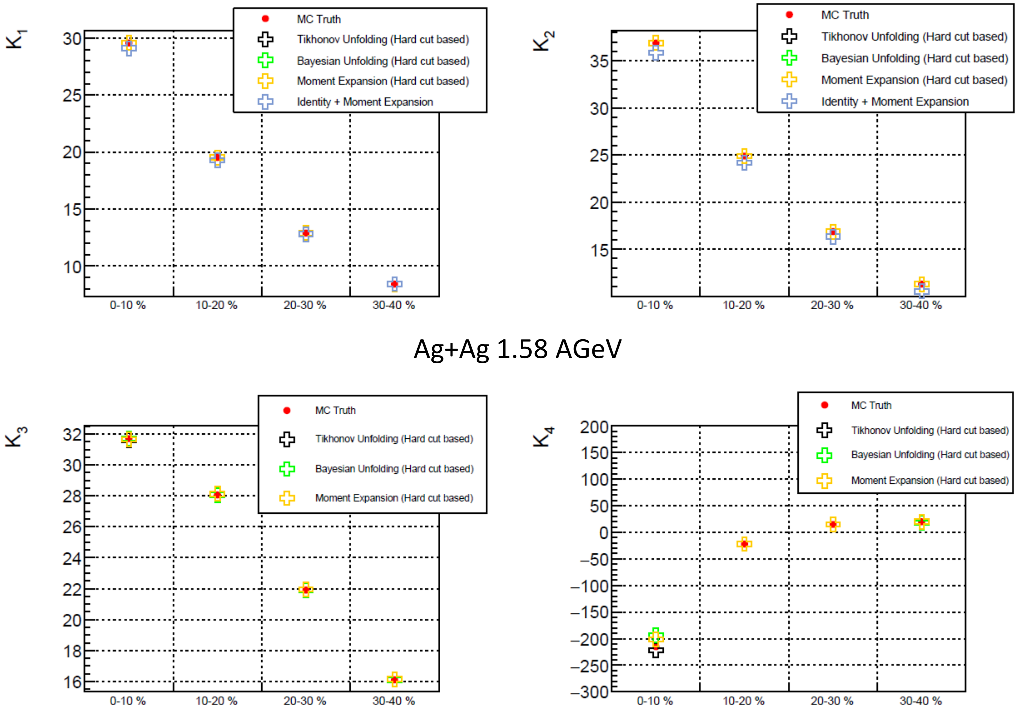
- Independent Source Model describes how single particle cumulants are entangled with cumulants of participants

- **For the extraction of the participant's cumulants a novel procedure based on event mixing and charged tracks has been presented and tested**
  - **Test in HADES-Neg.Bino.-Model show reasonable agreement with MC Truth → Particle emission close to Poisson**
  - **For Transport simulation, method seems to be sensitive to N_w, not N_{part}**
  - **Reasonable performance for Variance and Kurtosis, Deviations for Skewness!**
  - **Over correction in transport when considering bias terms → Assumption of independent source model violated?**
  - **Not invariant w.r.t to efficiency correction → No average binomial behaviour in HADES experiment**

Outlook
Handle efficiency issue



J. Adamczewski-Musch, et al., Proton-number fluctuations in √sN N =2.4 GeV Au + Au collisions studied with the High-Acceptance Di-Electron Spectrometer (HADES), Phys. Rev. C 102 (2) (2020) 024914

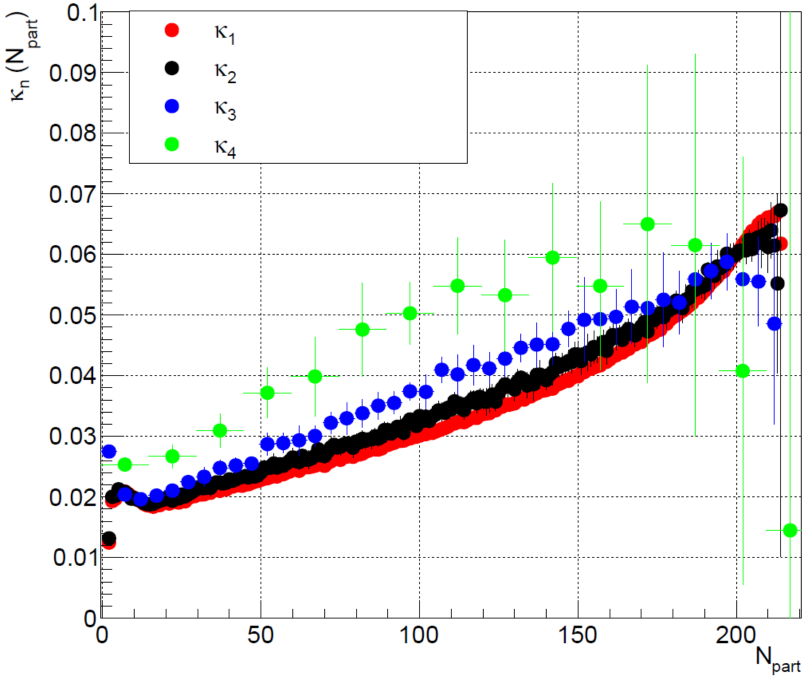


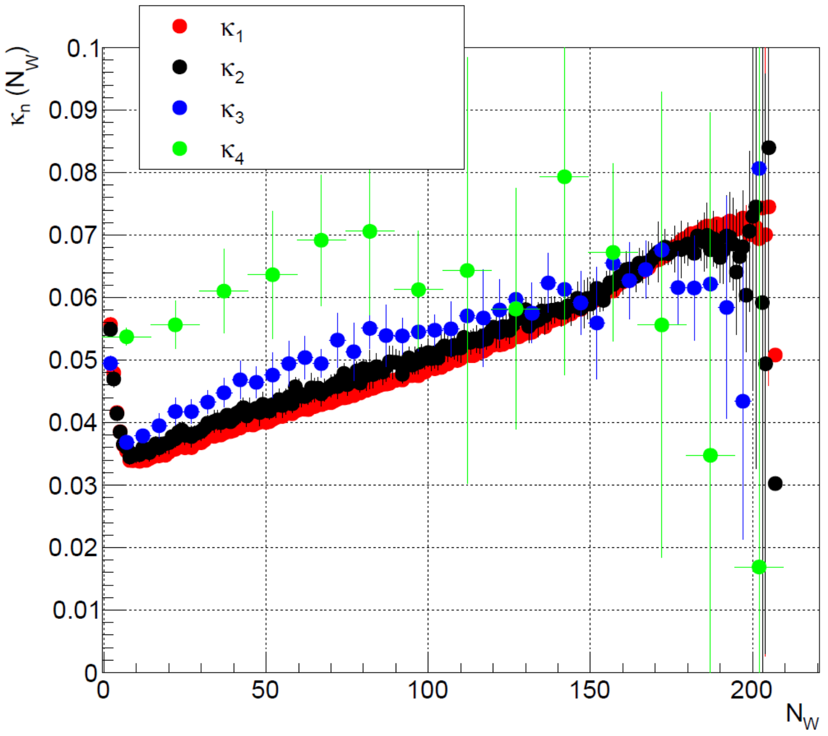
- Various methods available to correct for efficiency: E-by-E Binomial (Multiple Eff. Bins), Reg. Unfolding, Moment Expansion, Identity method

Statistical independent sources → $\kappa_n(N_w) = \frac{K_n(N_w)}{N_w}$

Outlook
Is the independent source model valid?

Smash with Clustering Ag+Ag 1.58 AGeV $-0.2 < y_{cm} < 0.2 \quad 400 < p_t < 1600\ MeV/c$





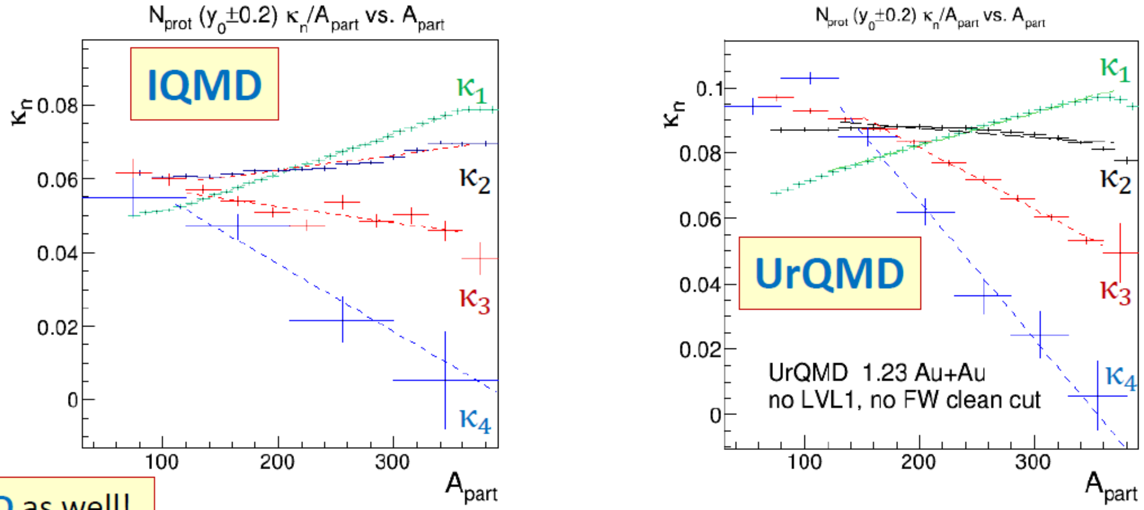
- ❑ Scaling of reduced proton cumulants is not constant → Assumption of independent sources not fulfilled in transport model. Experimental data we can not test, here we relay on Negative Binomial Toy model

Outlook
Is the independent source model valid?

Statistical independent sources ⟶ $\kappa_n(N_w) = \frac{K_n(N_w)}{N_w}$

Definition: $\kappa_n = K_n/V$ with $V = A_{part}$

See also Sugiura, Nonaka & Esumi PRC 100 (2019)



→HSD as well!

→ Skokov et al. assumption (κ_n = cst) is not fulfilled!

→ Extend formalism to NLO: $\kappa_n \rightarrow \kappa_n + \boldsymbol{\kappa'_n} \cdot (V - \bar{V})$
and N2LO: $\kappa_n \rightarrow \kappa_n + \boldsymbol{\kappa'_n} \cdot (V - \bar{V}) + \boldsymbol{\kappa''_n}\,(V - \bar{V})^2$

details given in Adamczewski-Musch et al. PRC 102 (2020)

Potential Solution:

- ❑ Find effective quantity for Number of Sources
  - ❑ N_{part} dependence, incorporate non-linearity
- ❑ Combine mathematical ansatz with Taylor expansion (as proposed by HADES) considering also slopes?

Thank you for your attention!

Back Up

Volume Correction Test in HADES Simulation

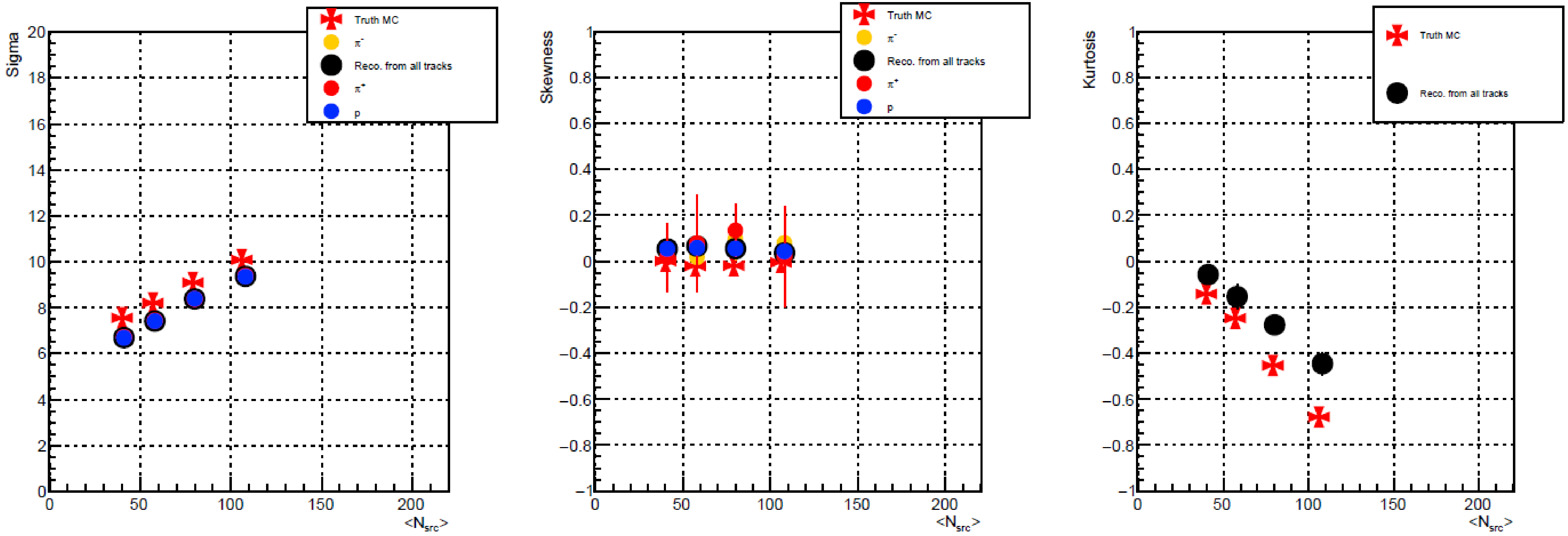
Acceptance Restriction: |ycm| < 0.4

$$N_{src} = fN_{coll} + (1-f)N_{part}$$

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}

$f = 0$



Volume Correction Test in HADES Simulation

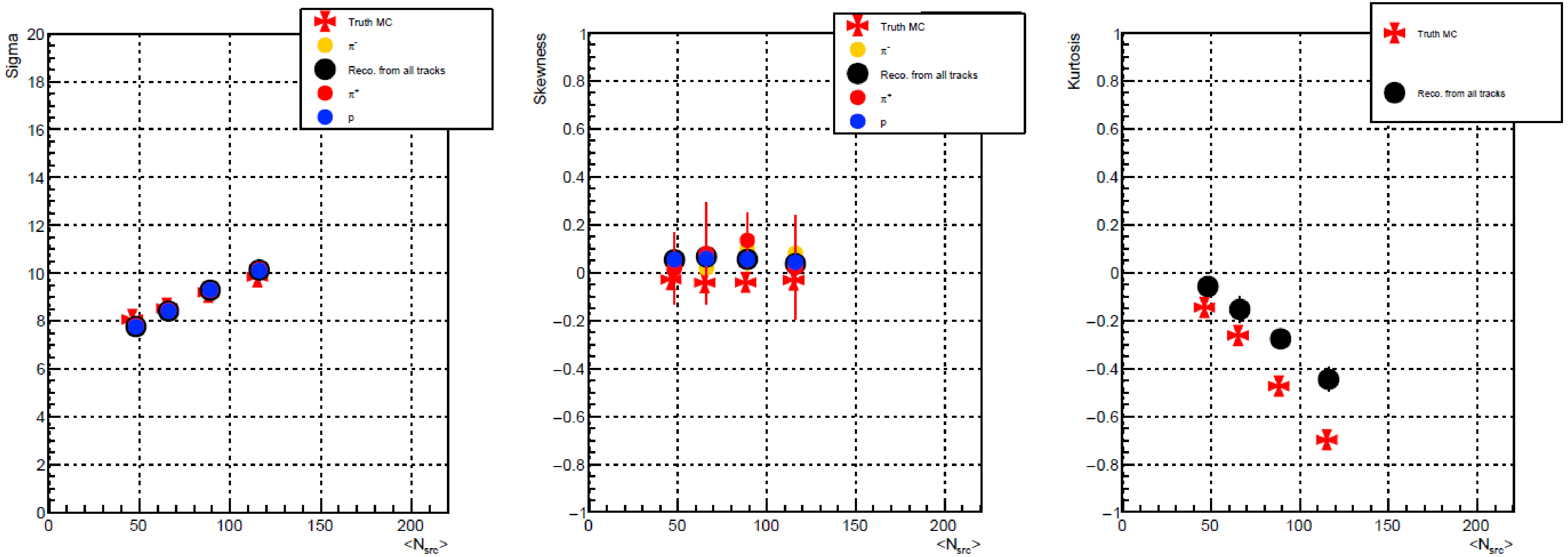
Acceptance Restriction: |ycm| < 0.4

$$N_{src} = f N_{coll} + (1-f) N_{part}$$

- Smash+(Clustering) Ag+Ag 1.58 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}

$f = 0.1$



Volume Correction Test in HADES Simulation

Acceptance Restriction: |ycm| < 0.4

$N_{src} = \mathrm{N_w}$

- UrQMD+(Clustering) Au+Au 1.23 AGeV
- Test on **Generator level**

MC Truth: Cumulants **of** N_{src}