High order fluctuations of conserved charges in the continuum limit

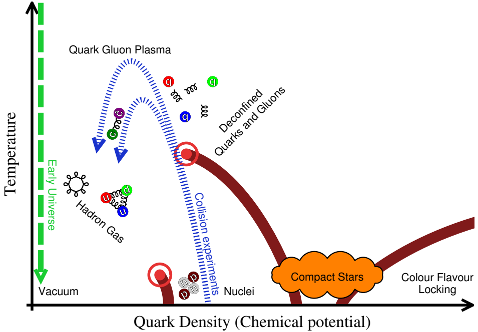
[2312.07528]

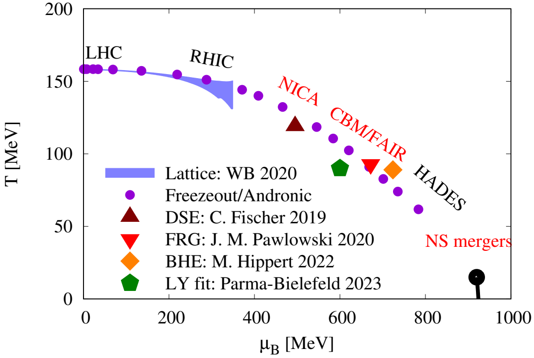
Szabolcs Borsányi

Wuppertal-Budapest collaboration.
Z. Fodor, J. N. Günther, S. D. Katz, P. Parotto, A. Pásztor, D. Pesznyák, K. K. Szabó and C. H. Wong

Bergische Universität Wuppertal

CPOD 2024, Berkeley





Zero μ_B: $T_c(\mu_B)$ crossover [Lattice QCD, Wuppertal-Budapest + BNL-Bielefeld 2006. . . 2020]
Low μ_B: transition line [Lattice QCD, Pisa group + Wuppertal-Budapest + BNL-Bielefeld 2015. . .]
Low μ_B: $T_{\mathrm{freeze-out}} \approx T_c(\mu_B)$ [Braun-Munzinger, Stachel, Wetterich, nucl-th/0311005]
High μ_B: $T_{\mathrm{freeze-out}} < T_c(\mu_B)$ [Floerchinger, Wetterich 1202.1671]

What can Lattice QCD still do about the phase diagram?

Baryon fluctuations in two scenarios

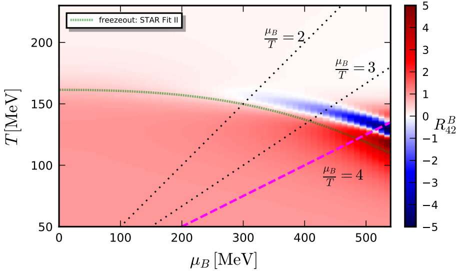
Stephanov [0809.3450,1104.1627] : Non-monotonic high order fluctuations near a critical end point.

Here : fourth cumulant $\langle B^4 \rangle_c$, normalized to the second cumulant $\langle B^2 \rangle_c$

FRG-Lattice assisted LEFT

Pawlowski et al. [2101.06035]

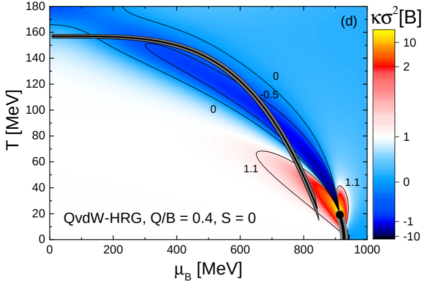
CEP $=$ (93 MeV, 672 MeV)



QVdW gas

Vovchenko et al [1906.01954]

Liquid-Gas CEP $=$ (19.7 MeV, 922 MeV)



Taylor coefficients

- Taylor coefficients of the pressure

$$\frac{p(\mu_B)}{T^4} = \frac{p(0)}{T^4} + \frac{1}{2!}\frac{\mu_B^2}{T^2}\chi_2^B(T) + \frac{1}{4!}\frac{\mu_B^4}{T^4}\chi_4^B(T) + \frac{1}{6!}\frac{\mu_B^6}{T^6}\chi_6^B(T) + \dots$$

- These Taylor coefficients are equal to the Grand Canonical fluctuations

$$\langle B^2\rangle - \langle B\rangle^2 = \frac{1}{VT}\frac{\partial^2 \log Z(V,T,\mu_B,\mu_Q,\mu_S)}{\partial {\mu_B}^2} = \chi_2^B$$

- Higher fluctuations are the Taylor coefficients of lower fluctuations

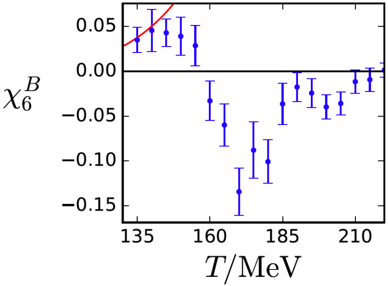
$$\chi_2^B(\mu_B) = \chi_2^B(\mu_B=0) + \frac{1}{2!}\frac{\mu_B^2}{T^2}\chi_4^B(\mu_B=0) + \frac{1}{4!}\frac{\mu_B^4}{T^4}\chi_6^B(\mu_B=0) + \dots$$

- Taylor coefficients can be used to reveal analytic structure of the thermodynamic potential
  - Repulsive interactions beyond ideal HRG
  - Searching the critical end point
- Hints for chiral O(4) universality

Two results without continuum extrapolation

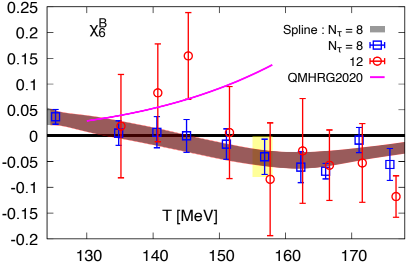
Wuppertal-Budapest, 1805.04445

4stout, $N_t = 12$



HotQCD, 2202.09184

HISQ, $N_t = 8, 12$



Is χ_6^B positive or negative at $T \approx 150$ MeV?
Does QCD agree with HRG at $T \approx 140$ MeV?

$$\chi_{200}^{uds} = +\langle B_u\rangle + \langle A_u A_u\rangle$$

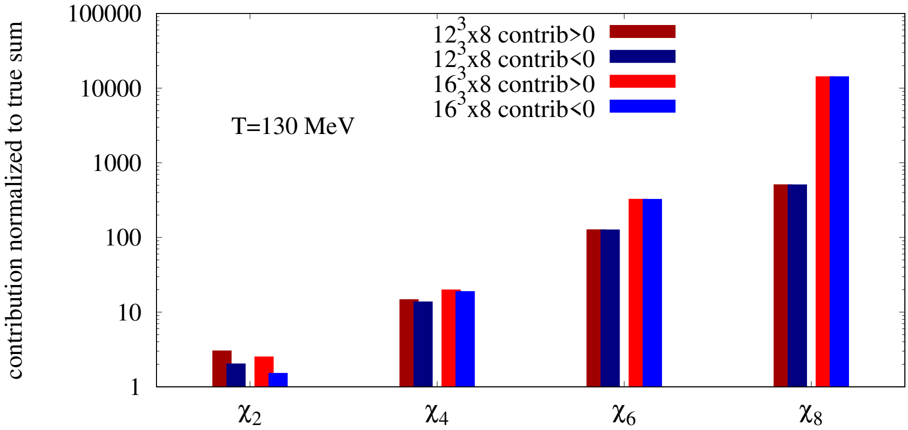
$$\begin{aligned}\chi_{400}^{uds} = & +\langle D_u\rangle + 3\langle B_u B_u\rangle - 3\langle B_u\rangle\langle B_u\rangle + 4\langle A_u C_u\rangle \\ & +\langle A_u A_u A_u A_u\rangle - 3\langle A_u A_u\rangle\langle A_u A_u\rangle + 6\langle A_u A_u B_u\rangle - 6\langle B_u\rangle\langle A_u A_u\rangle\end{aligned}$$

$$\begin{aligned}\chi_{600}^{uds} = & +\langle F_u\rangle + 10\langle C_u C_u\rangle + 15\langle B_u D_u\rangle + 15\langle B_u B_u B_u\rangle + 6\langle A_u E_u\rangle + 60\langle A_u B_u C_u\rangle + 15\langle A_u A_u D_u\rangle \\ & +45\langle A_u A_u B_u B_u\rangle + 20\langle A_u A_u A_u C_u\rangle + 15\langle A_u A_u A_u A_u B_u\rangle + \langle A_u A_u A_u A_u A_u A_u\rangle - 15\langle D_u\rangle\langle B_u\rangle \\ & -15\langle D_u\rangle\langle A_u A_u\rangle - 45\langle B_u\rangle\langle B_u B_u\rangle - 60\langle B_u\rangle\langle A_u C_u\rangle - 90\langle B_u\rangle\langle A_u A_u B_u\rangle \\ & -15\langle B_u\rangle\langle A_u A_u A_u A_u\rangle - 45\langle B_u B_u\rangle\langle A_u A_u\rangle - 60\langle A_u C_u\rangle\langle A_u A_u\rangle - 90\langle A_u A_u\rangle\langle A_u A_u B_u\rangle \\ & -15\langle A_u A_u\rangle\langle A_u A_u A_u A_u\rangle + 30\langle B_u\rangle\langle B_u\rangle\langle B_u\rangle + 90\langle B_u\rangle\langle B_u\rangle\langle A_u A_u\rangle \\ & +90\langle B_u\rangle\langle A_u A_u\rangle\langle A_u A_u\rangle + 30\langle A_u A_u\rangle\langle A_u A_u\rangle\langle A_u A_u\rangle\end{aligned}$$

$$\chi_{800}^{uds} = 79 \text{ terms}\ldots$$

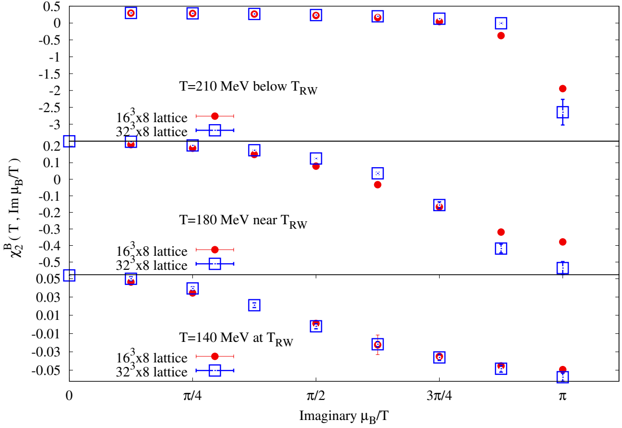
$A, B, C, \ldots$ are defined as $[\det M(\mu_u)]^{1/4} = [\det M(0)]^{1/4}\left(1 + A_u\mu_u + \frac{B_u}{2!}\mu_u^2 + \frac{C_u}{3!}\mu_u^3 + \ldots\right)$

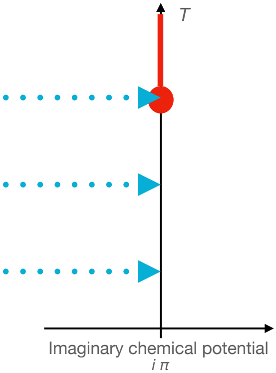
Data analysis uses computer generated code.


contribution normalized to true sum
100000
10000
1000
100
10
1
12^3x8 contrib>0
12^3x8 contrib<0
16^3x8 contrib>0
16^3x8 contrib<0
T=130 MeV
χ_2
χ_4
χ_6
χ_8

Baryon fluctuations near the Roberge Weiss point

at imaginary $\mu_B/T = \pi$





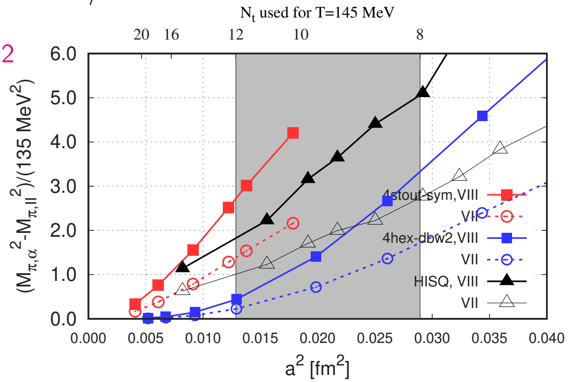
- 4 steps of HEX smearing + DBW2 gauge action
- Physical point defined by $m_\pi/f_\pi = 1.0337$, $m_s/m_{\text{light}} = 27.63$
- m_{light} tuned in the a range: $0.22 \ldots 0.072$ fm
- Thermodynamics runs: cca 50–100 k configurations / ensemble
  10 Temperatures $130 \ldots 205$ MeV
  3 lattice spacings $16^3 \times 8$, $20^3 \times 10$, $24^3 \times 12$

Taste breaking

(smaller is better)

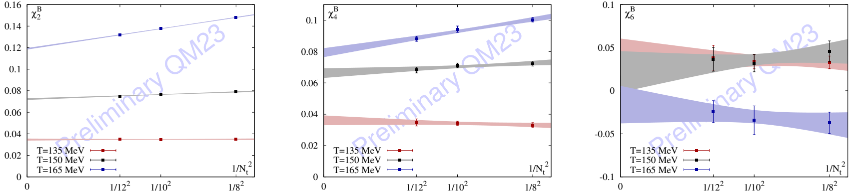
show relative quadratic discretization error on M_π^2

(HISQ: thanks to Peter Petreckzy)

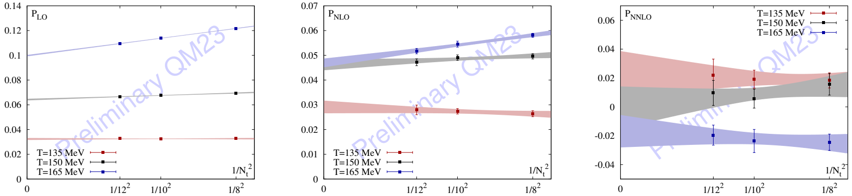


4HEX continuum extrapolations

Baryon Taylor coefficients



Strangeness neutral Taylor coefficients



We extrapolate : $\quad B(\mu_B, \mu_S, T) \quad S(\mu_B, \mu_S, T)$

Strangeness neutrality means that we solve for μ_S at any μ_B and T

$$S(\mu_B, \mu_S, T) \equiv 0$$

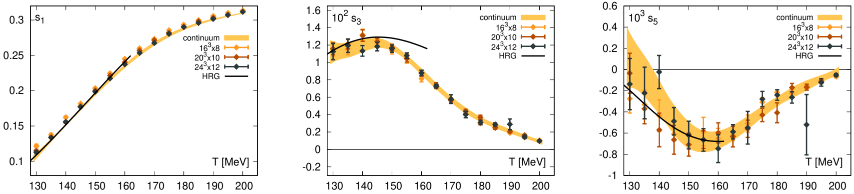
That requires a tuning of $\mu_S = \mu_S^*(\mu_B, T)$. *[For simplicity $\mu_Q \equiv 0$.]*

$$\mu_S^*(\mu_B, T) = s_1(T)\mu_B + s_3(T)\mu_B^3 + s_5(T)\mu_B^5 + \dots$$

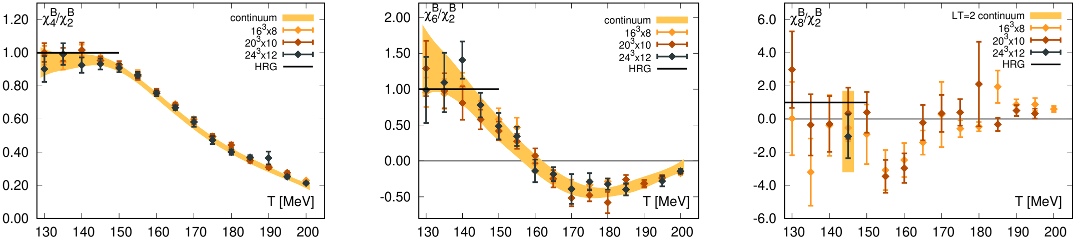
One obtains $s_1(T)$, $s_3(T)$ and $s_5(T)$ from the standard Taylor coefficients

[HotQCD 1208.1220; 1701.04325]

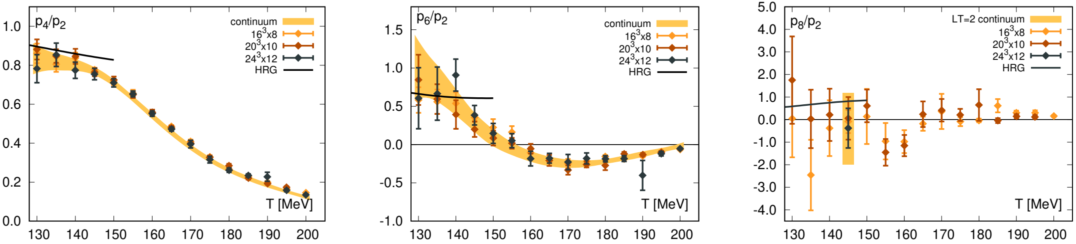
4HEX results:



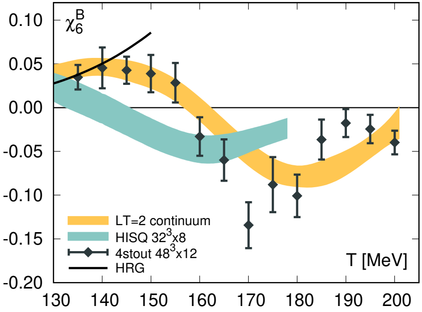
Baryon Taylor coefficients

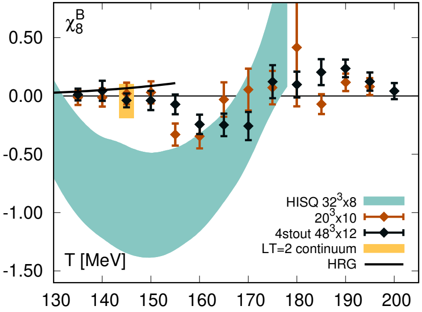


Strangeness neutral Taylor coefficients



Comparison with literature

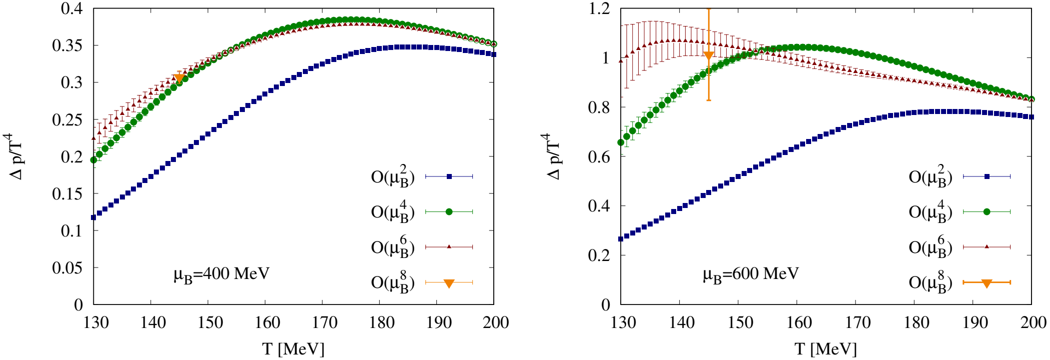




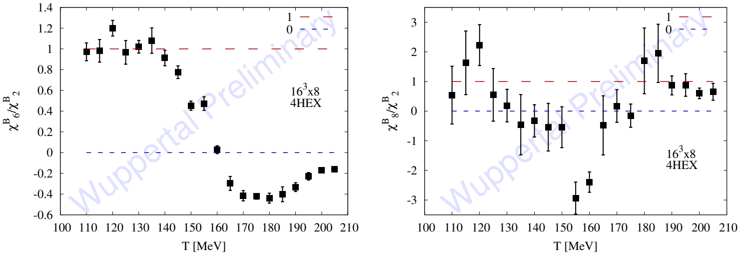
4stout data (imaginary μ_B, $48^3 \times 12$ lattice) : [Wuppertal-Budapest, 1805.04445]
HISQ data: ($\mu_B = 0$, $32^3 \times 8$ lattice, inexact charge conservation) [BNL-Bielefeld, 2202.09184,2212.09043]
4HEX result ($\mu_B = 0$, $16^3 \times 8$, $20^3 \times 10$, $24^3 \times 12$ lattices) [Wuppertal-Budapest, 2312.07528]

We calculate using continuum extrapolated coefficients *(in a modest volume $LT = 2$)*

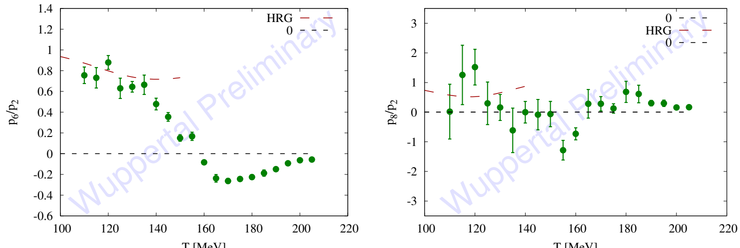
$$\frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} \approx \frac{p_0}{2!}\frac{\mu_B^2}{T^2} + \frac{p_0}{4!}\frac{\mu_B^4}{T^4} + \frac{p_0}{6!}\frac{\mu_B^6}{T^6} + \frac{p_0}{8!}\frac{\mu_B^8}{T^8}$$



Baryon cumulant ratios



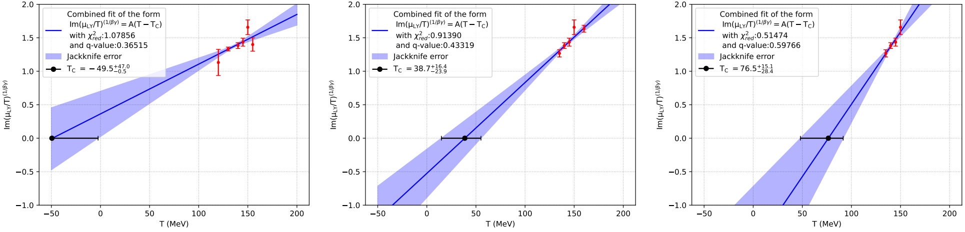
Strangeness neutral EoS cumulant ratios



The poles of the rational function

$$\chi_2(T,\hat{\mu}_B) \quad \approx \quad \frac{A_0 + A_1\hat{\mu}_B^2}{1 + B_1\hat{\mu}_B^2 + B_2\hat{\mu}_B^2} \overset{!}{=} \chi_2(T,0) + \frac{\hat{\mu}_B^2}{2!}\chi_4(T,0) + \frac{\hat{\mu}_B^4}{4!}\chi_6(T,0) + \frac{\hat{\mu}_B^4}{6!}\chi_8(T,0)$$

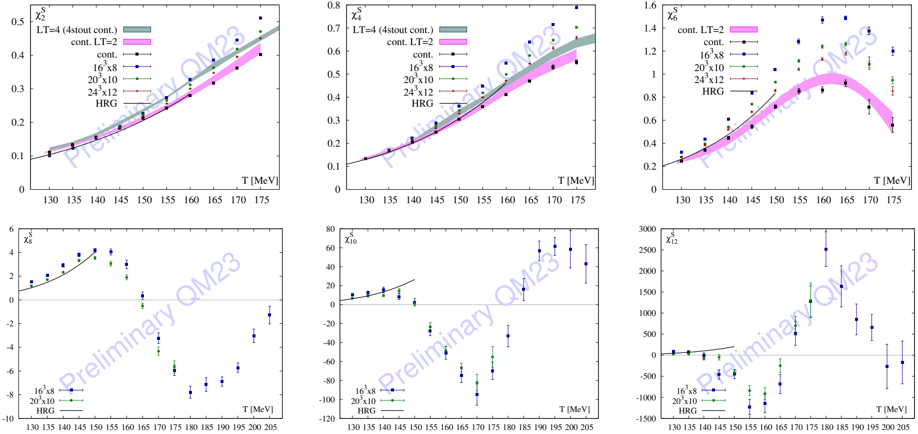
might reveal Lee-Yang zeros. Real Lee-Yang zeros indicate real tansitions.



i) Small temperatures could be more conclusive but require higher orders.

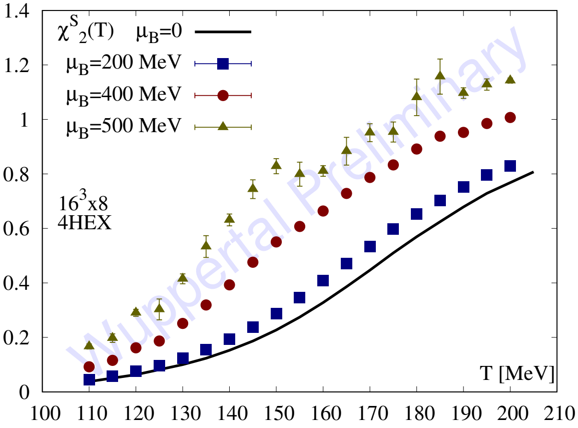
ii) Results come with large systematics (fit range dependence)

$$\chi_n^S = \frac{\partial^n (p/T^4)}{\partial \mu_S / T^n}$$



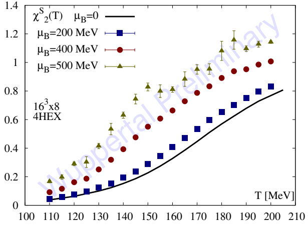
NNLO expansion on a $16^3 \times 8$ lattice

$$\chi_2^S(T,\mu_B) \approx \chi_2^S(T,0) + \frac{\mu_B^2}{2T^2}\chi_{22}^{BS}(T,0) + \frac{\mu_B^4}{24T^4}\chi_{42}^{BS}(T,0) + \frac{\mu_B^6}{720T^6}\chi_{62}^{BS}(T,0)$$



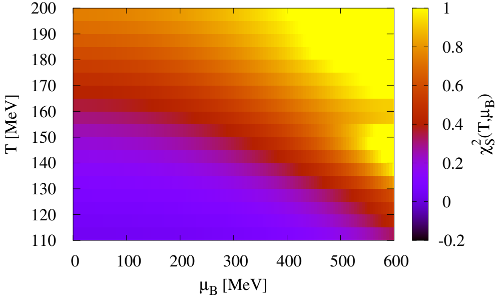
NNLO expansion on a $16^3 \times 8$ lattice

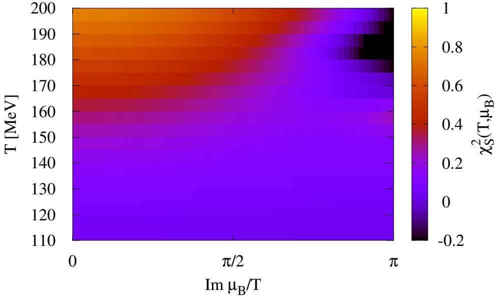
$$\chi_2^S(T,\mu_B) \approx \chi_2^S(T,0) + \frac{\mu_B^2}{2T^2}\chi_{22}^{BS}(T,0) + \frac{\mu_B^4}{24T^4}\chi_{42}^{BS}(T,0) + \frac{\mu_B^6}{720T^6}\chi_{62}^{BS}(T,0)$$



Real μ_B

Imaginary μ_B





High order fluctuations from lattice is an expensive endeavour

- Earlier (2018) we exploited analiticity in the (imaginary) chemical potential to substantially cut costs on $48^3 \times 12$ lattices *two years of INCITE access*

- Today I presented continuum extrapolated fluctuations in a smaller volume ($LT = 2$) thanks to the 4HEX discretization using a fraction of resources $\frac{1}{2}$ *years in Jülich*

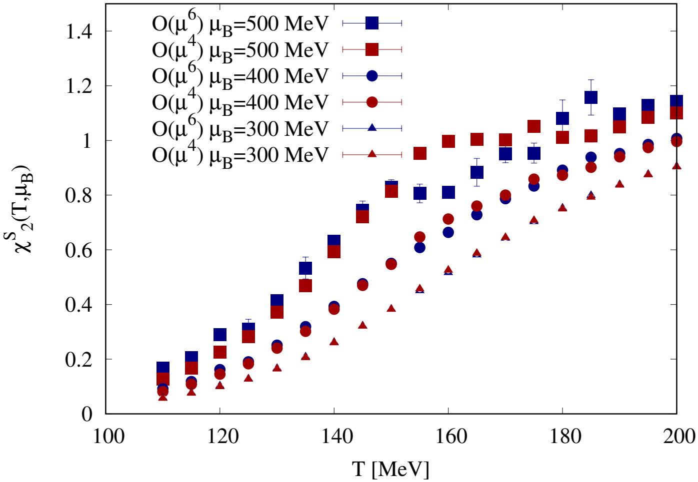
  - Cut-off effects are significant (staggered fermions)
    New action (4HEX) permits continuum extrapolation from $N_t = 8, 10$ and 12.

  - Large volume kills the signal *(as expected in the presence of a sign problem)*;
    small volume could already show hints for criticality

- From very high statistics ensembles *(an other $\frac{1}{2}$ years in Jülich + 2 weeks on LUMI)* one can attempt to extrapolate to so far unattainable parts of the phase diagram.
  (today only on coarse lattices)

backup

$$\chi_2^S(T,\mu_B) \approx \chi_2^S(T,0) + \frac{\mu_B^2}{2T^2}\chi_{22}^{BS}(T,0) + \frac{\mu_B^4}{24T^4}\chi_{42}^{BS}(T,0) + \frac{\mu_B^6}{720T^6}\chi_{62}^{BS}(T,0)$$

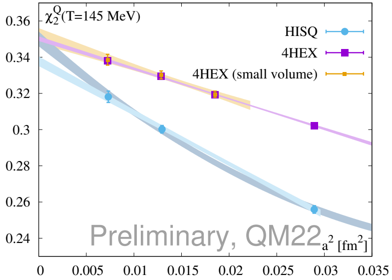


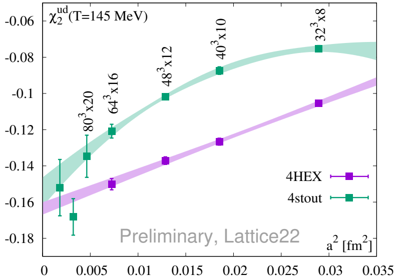
Testing continuum limits with the 4HEX action

4HEX staggered action with strongly reduced taste breaking.

Let's look at those fluctuations e.g. chanrge, that is sensitive to it.

Continuum extrapolation $T = 145$ MeV with large volume up to $N_t = 16$





4STOUT: Wuppertal-Budapest (2013–2023[Wuppertal-Budapest [1507.04627]]

HISQ: BNL-Bielefeld (2011–. . .)[HotQCD [2107.10011]]

4HEX: Wuppertal-Budapest (2022–. . .) [Wuppertal-Budapest **QM2022**]