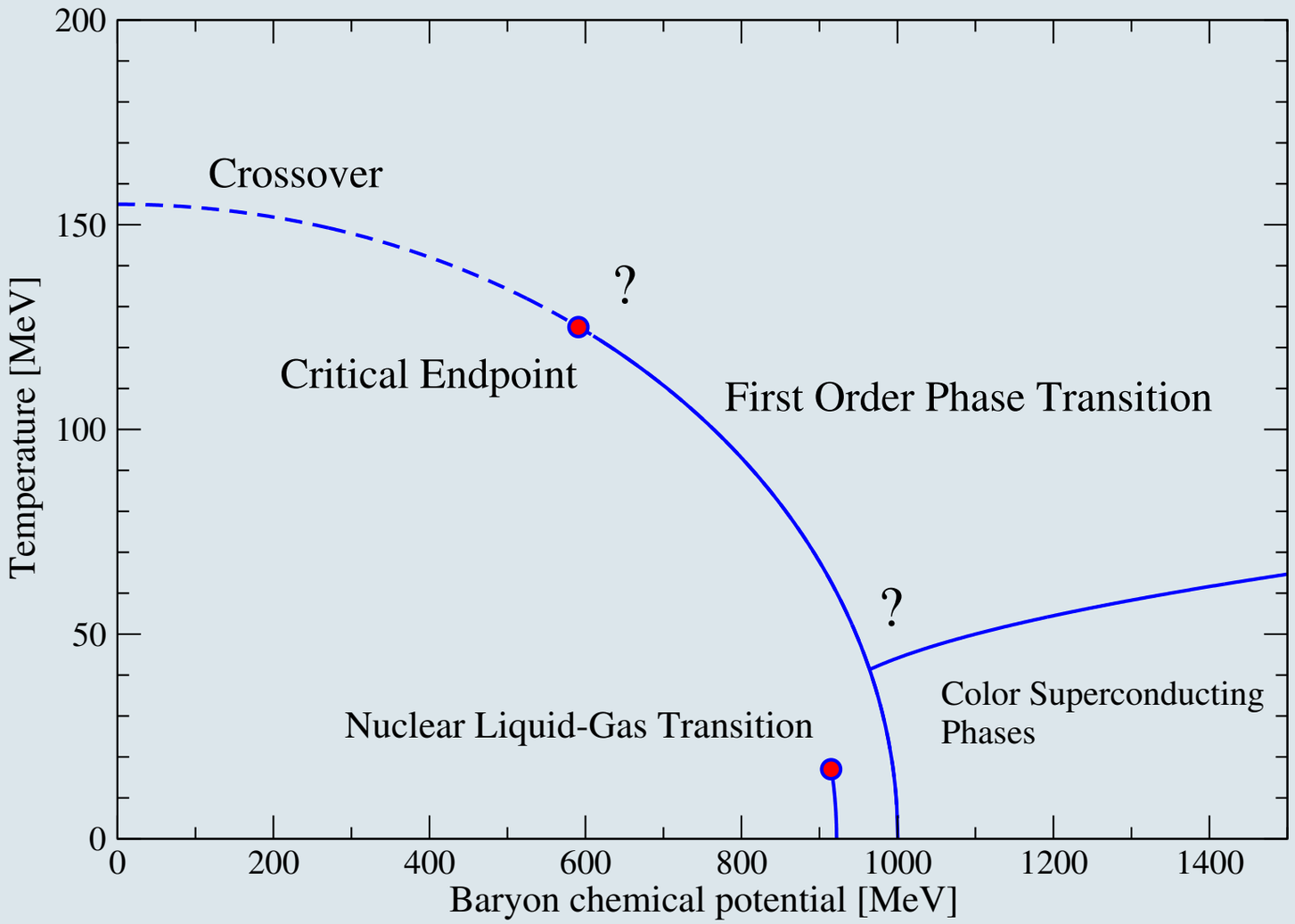


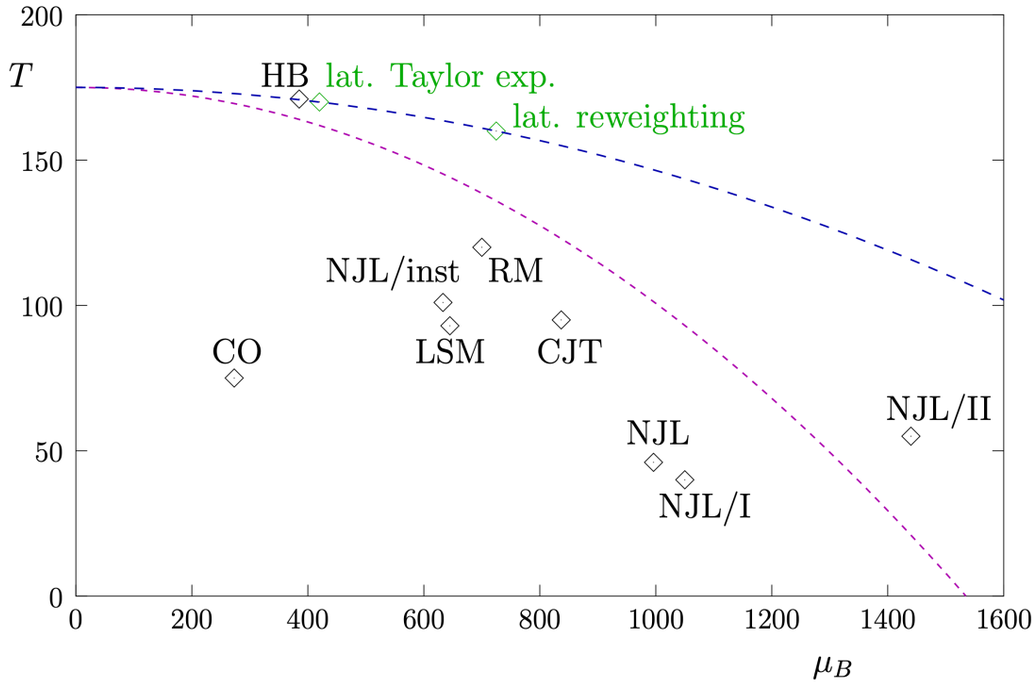
QCD CEP with functional methods

DSE: Bernhardt, CF, Gao, Gunkel, Isserstedt, Pawlowski, Welzbacher,...

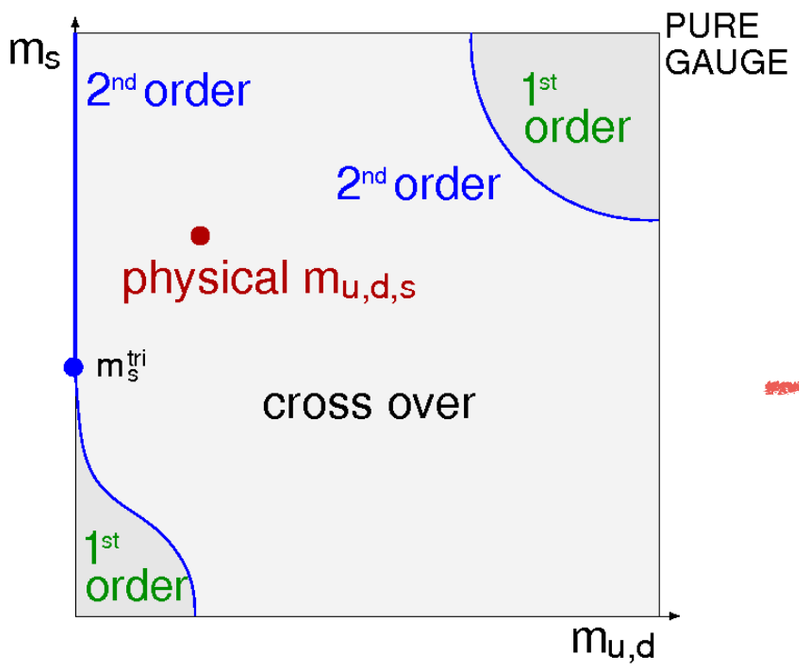
FRG: Braun, Fu, Rennecke, Pawlowski, Schaefer, Smekal, Wink, Yin,...

Take home message

2004

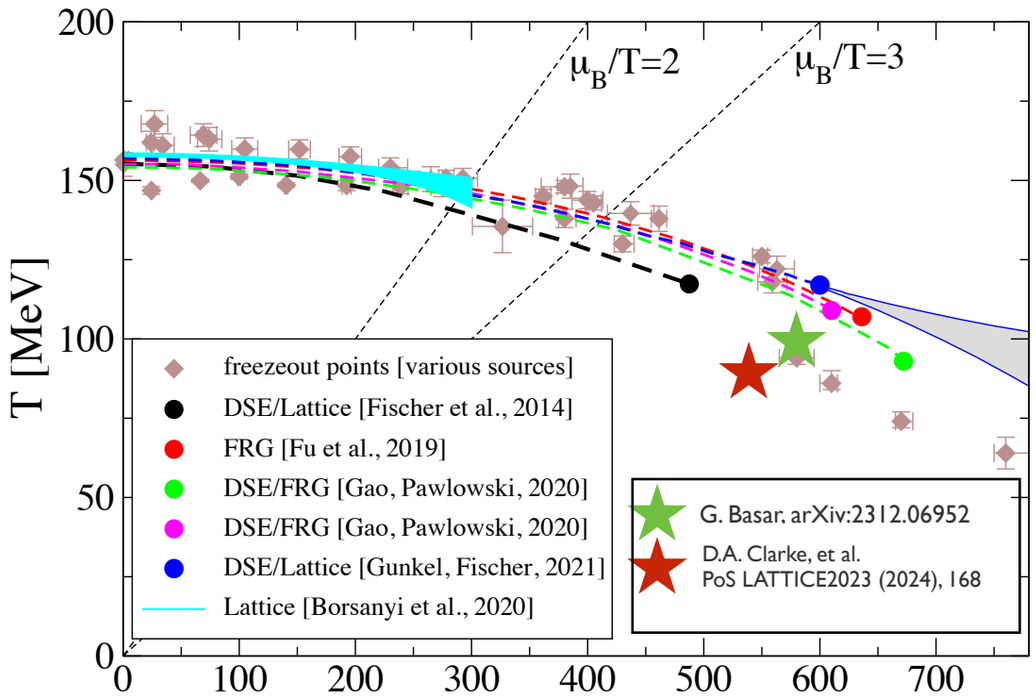


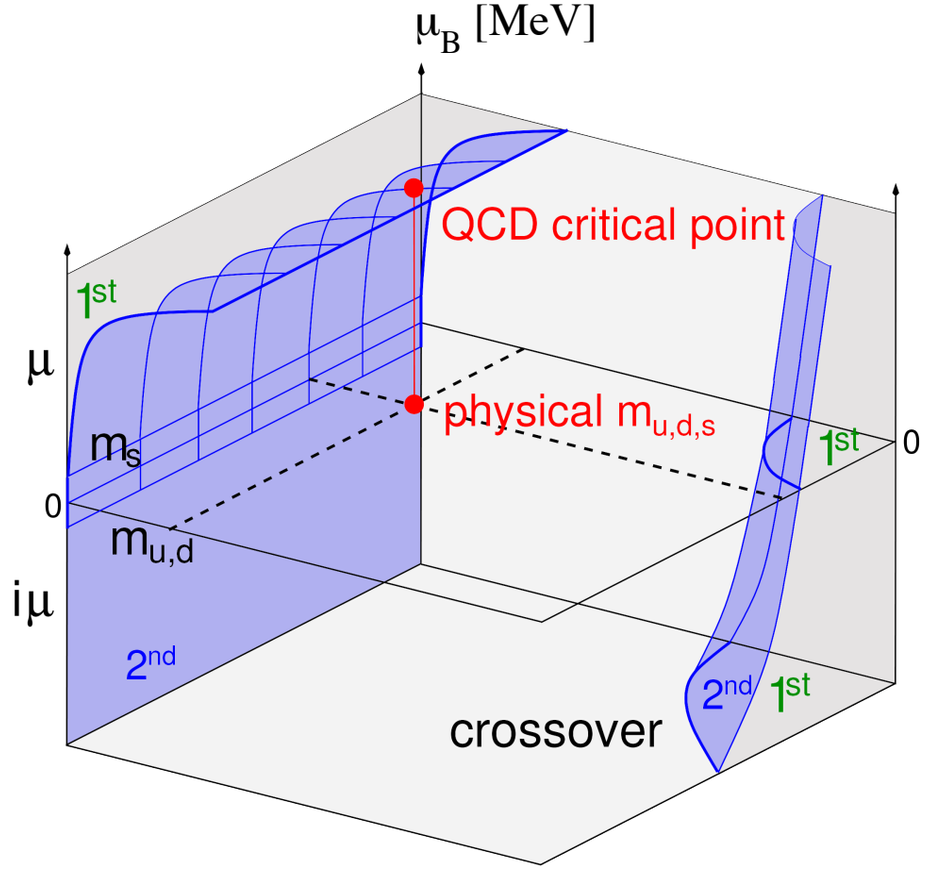
Stephanov, Prog. Theor. Phys. Suppl. 153 (2004)



Pisarski and Wilczek, PRD 29 (1984), 338-341

2024





Functional methods: DSE & FRG

DSE	FRG
nPI effective action	1PI effective action: flow equation
↓ field derivatives	↓ field derivatives
inf. tower of integral equations for m-point functions	inf. tower of int.-diff. equations for m-point functions

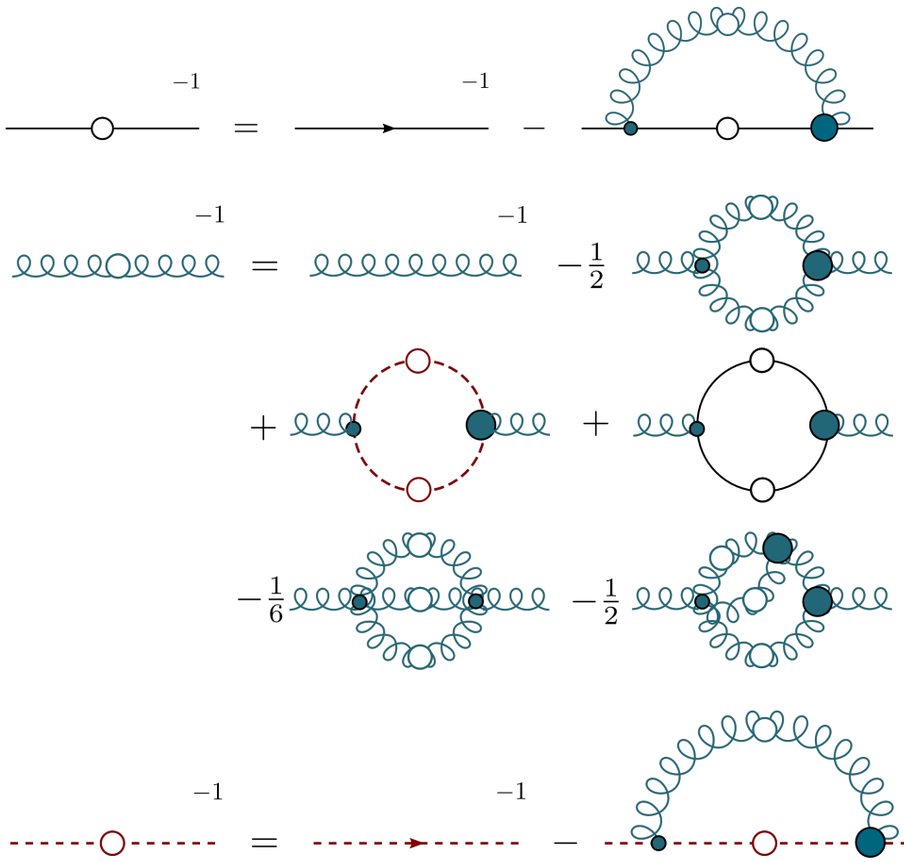
(slightly) different structure but same physics

complementary !

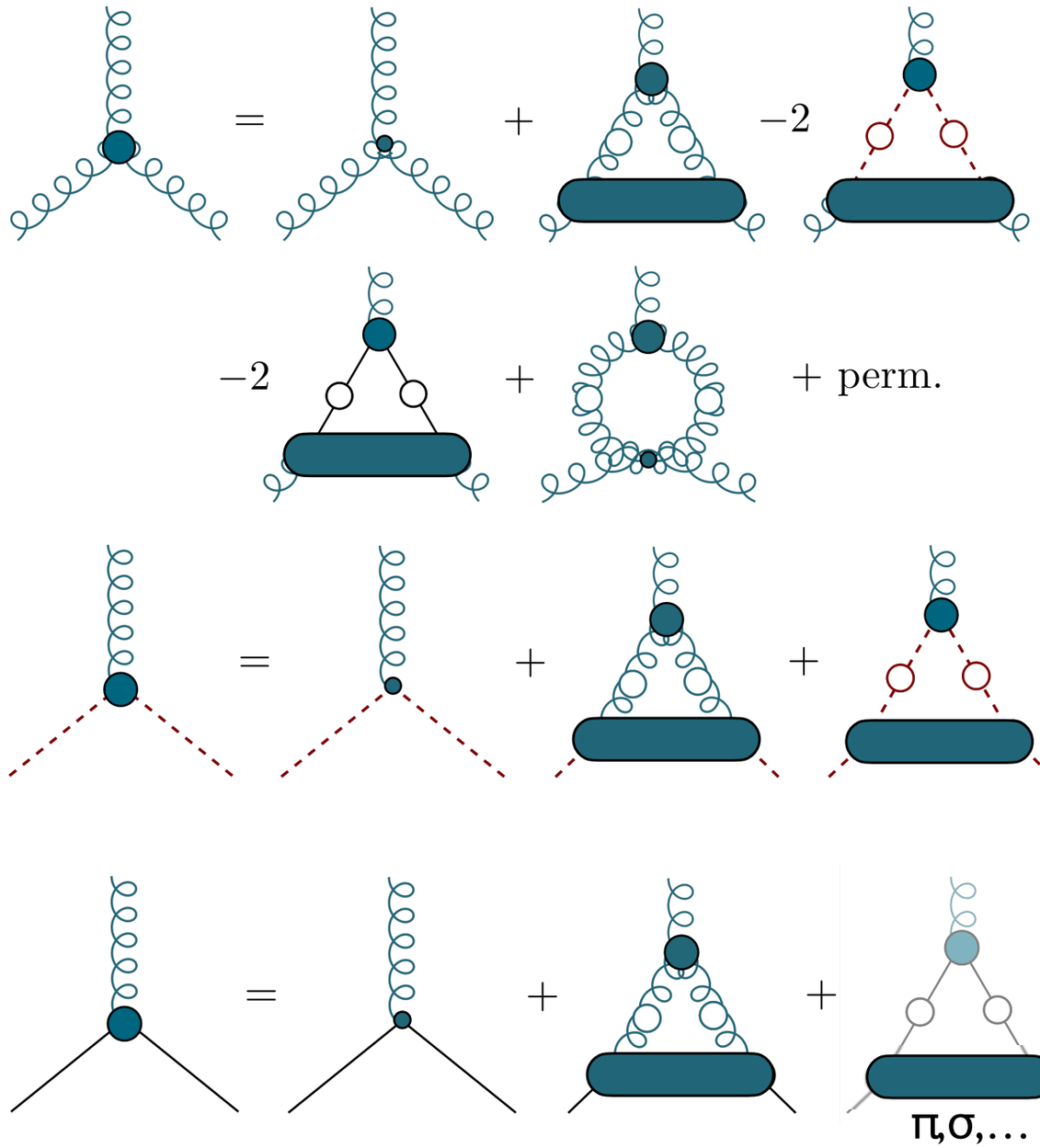
see e.g. CF, Pawlowski, PRD 75 (2007), PRD 80 (2009)

QCD with DSE

propagators



vertices

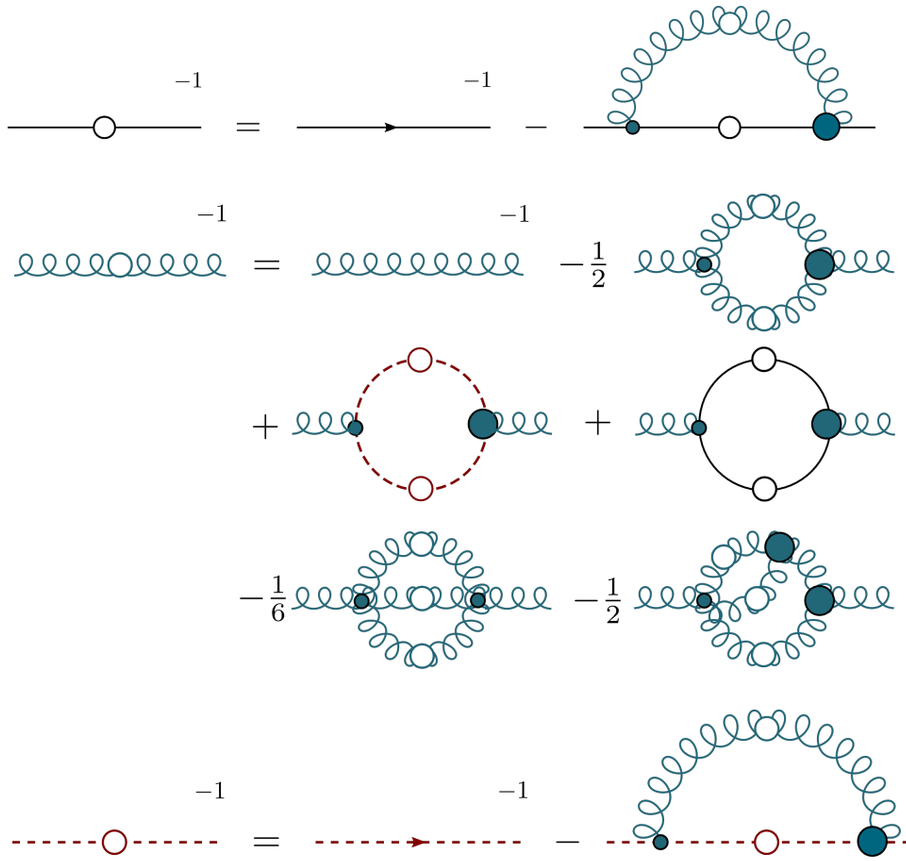


Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

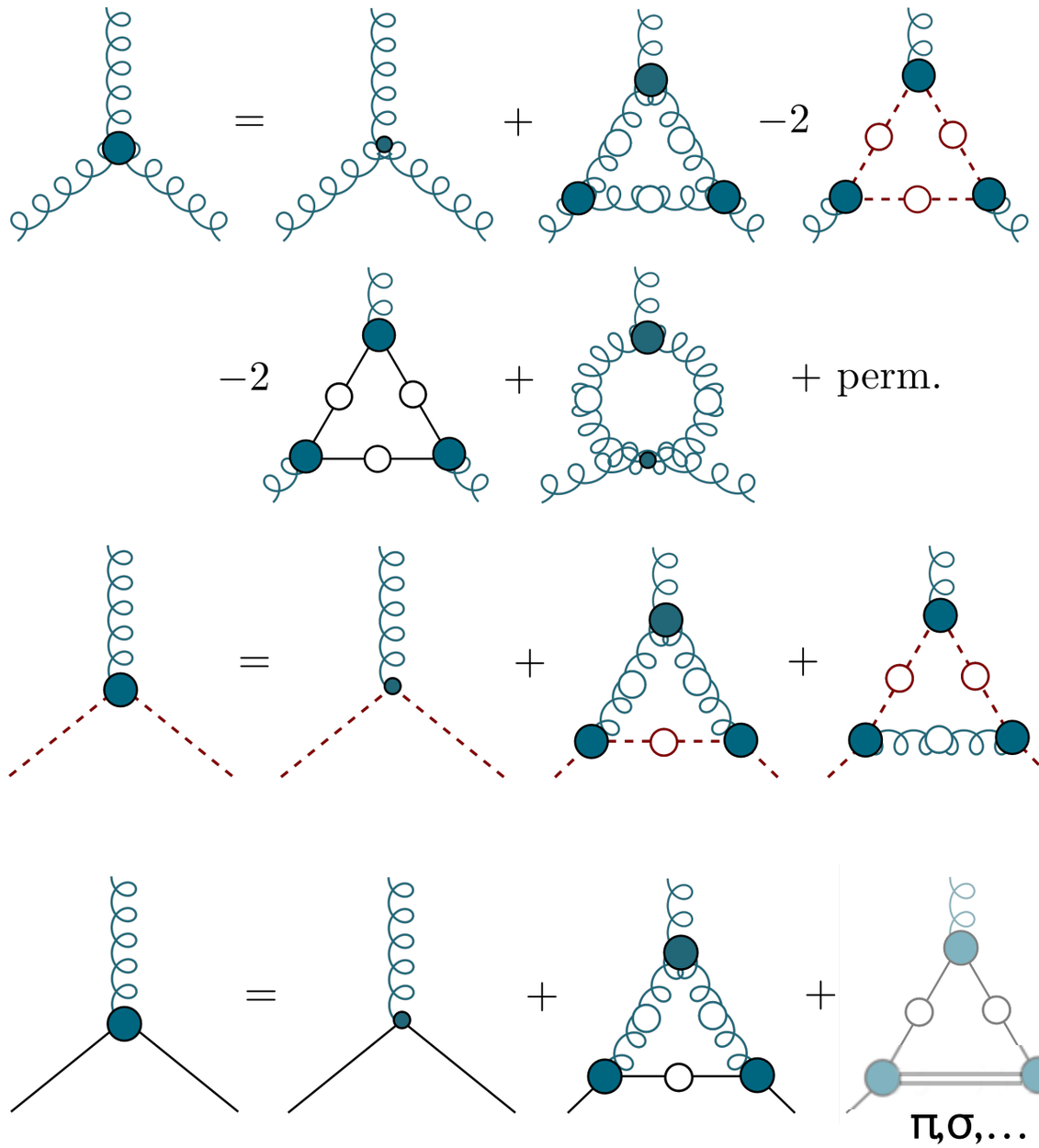
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

QCD with DSE

propagators



vertices



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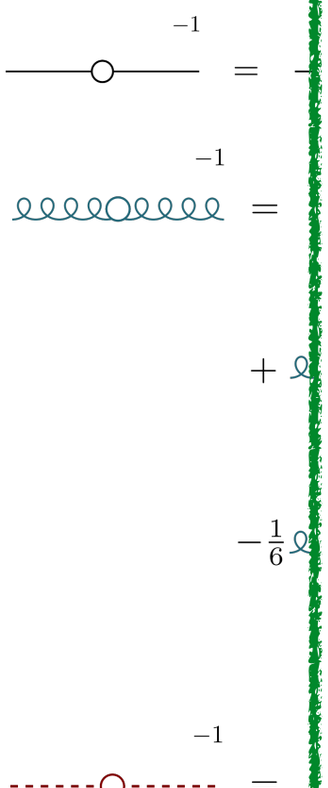
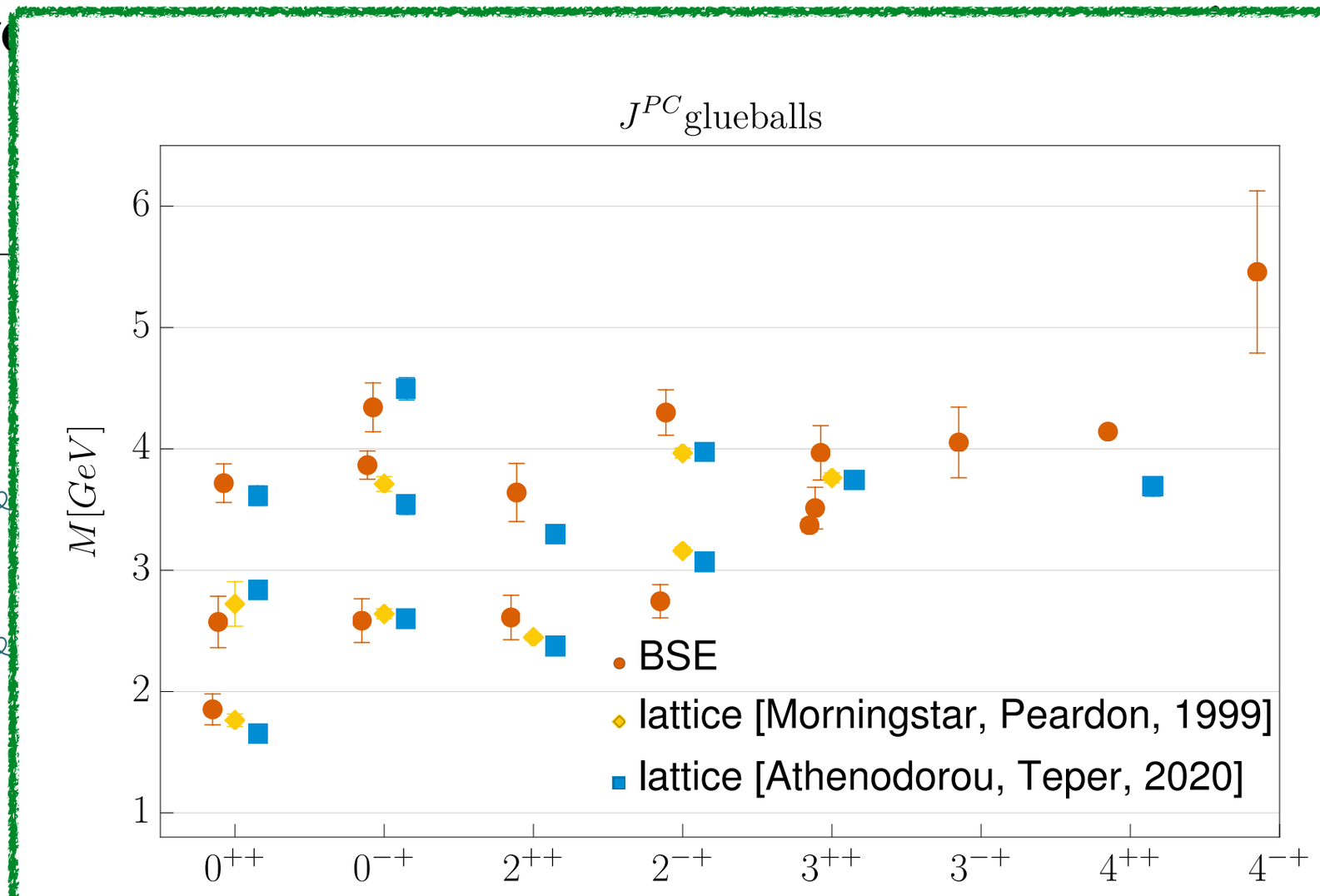
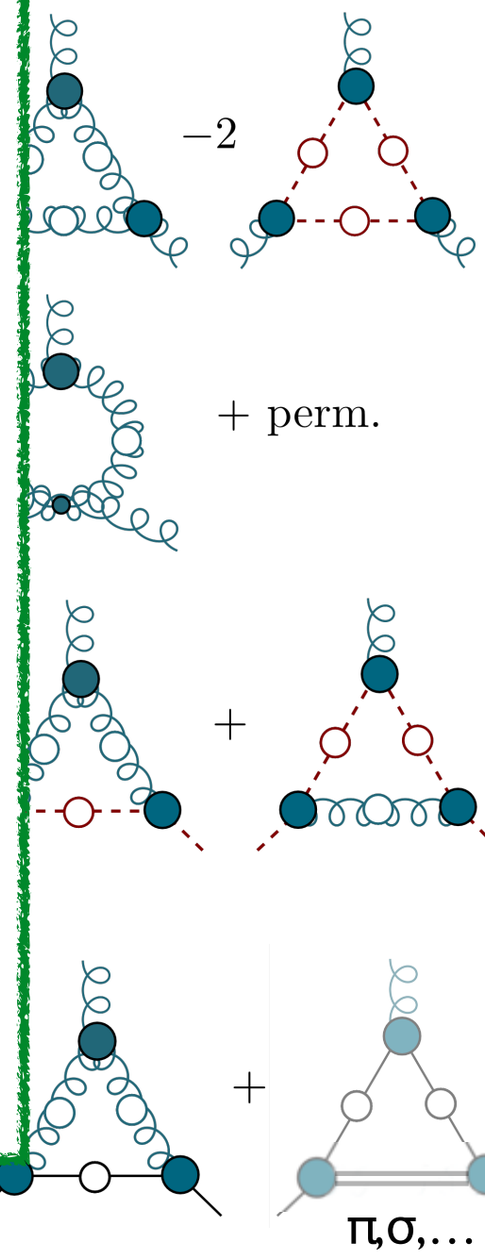
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

QCD with DSE

pr







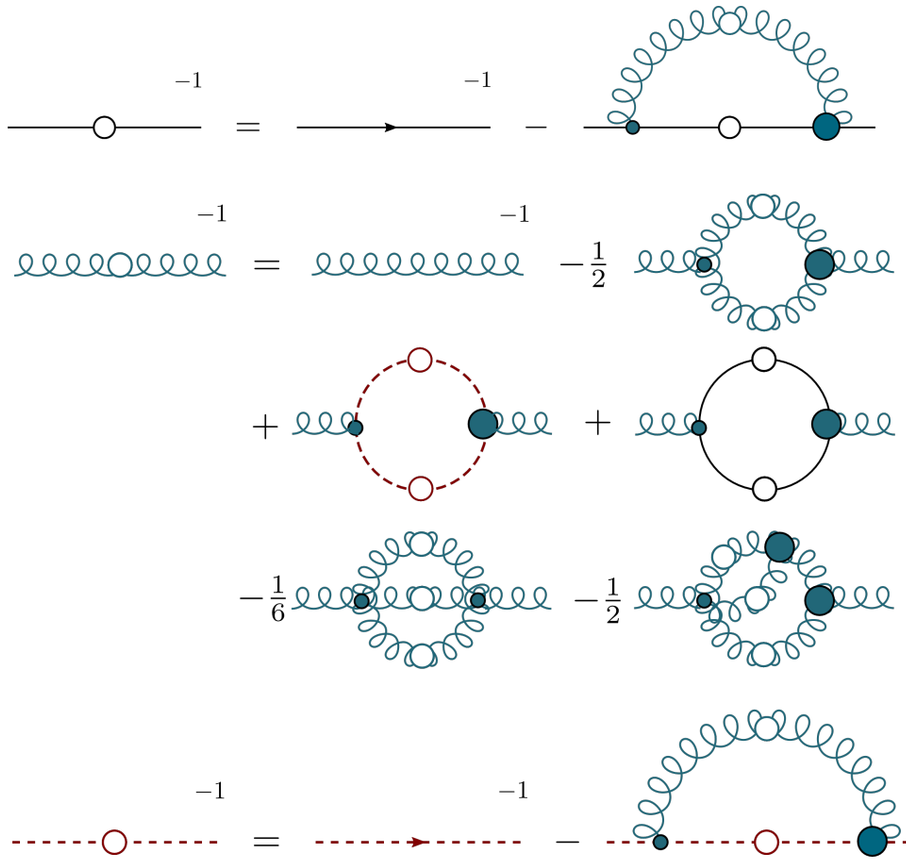
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

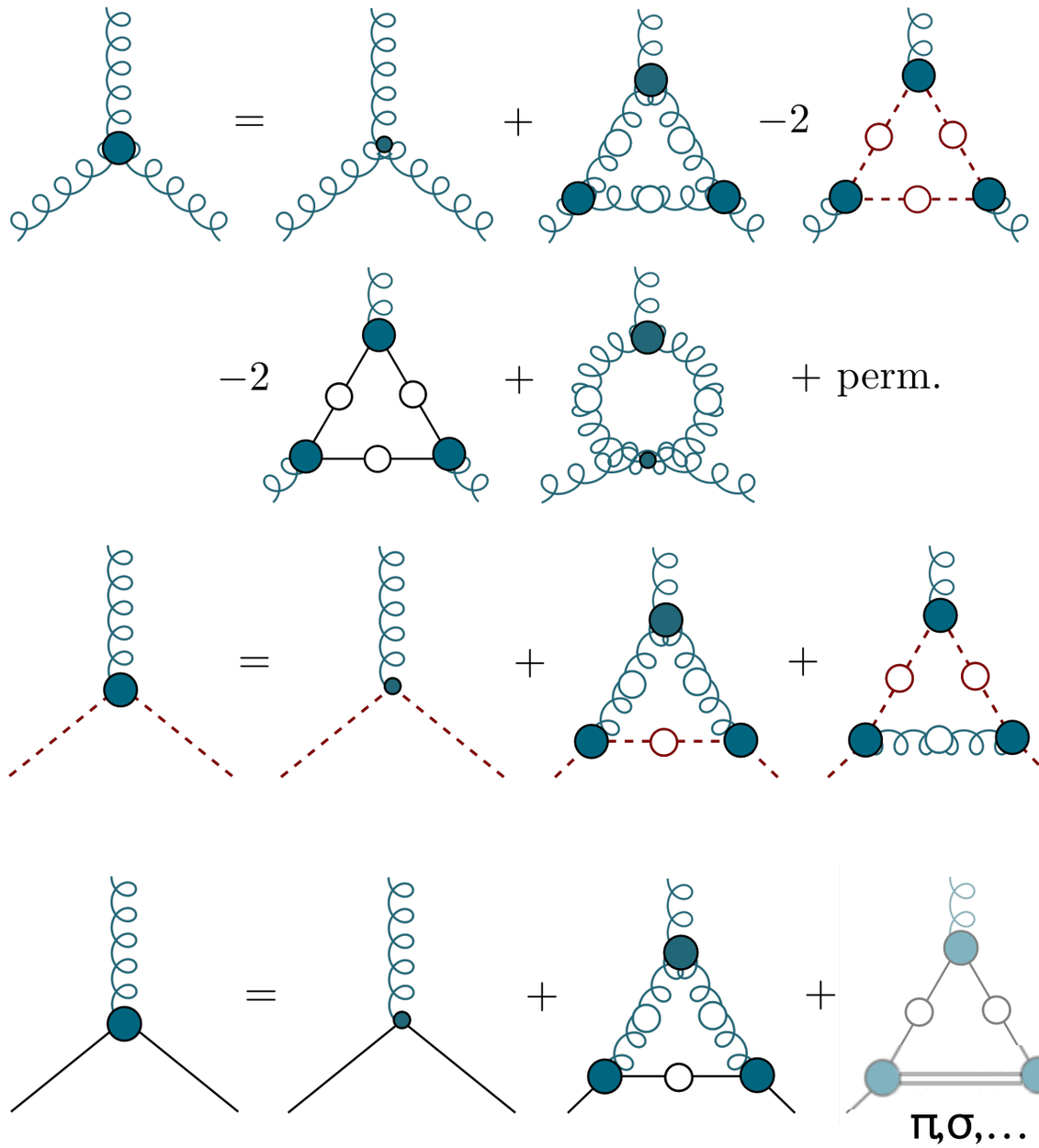
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

QCD with DSE

propagators



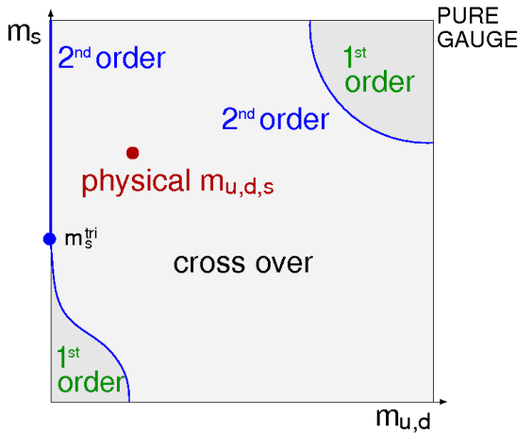
vertices



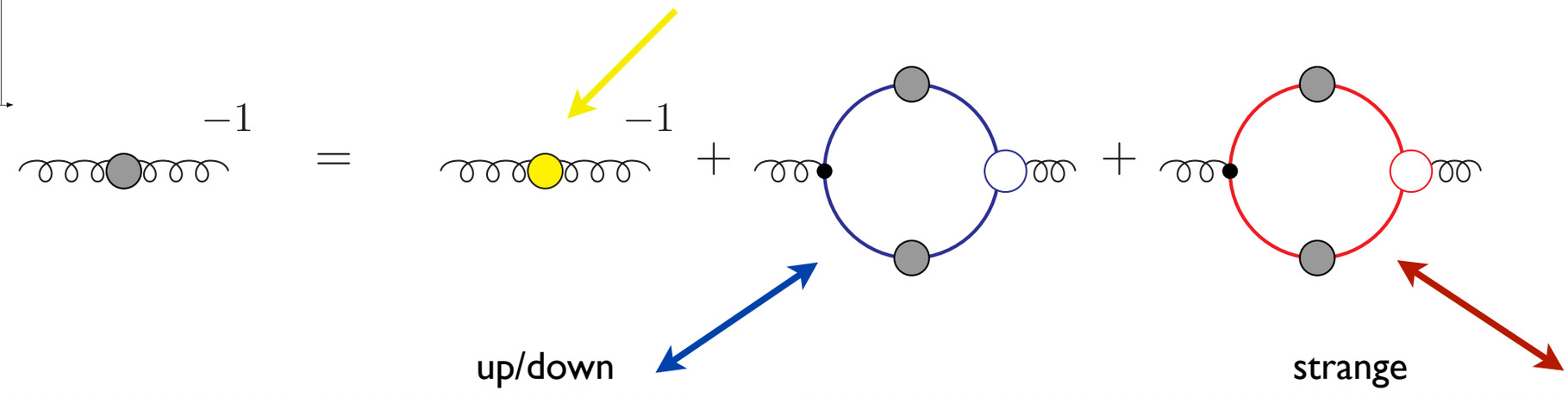
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

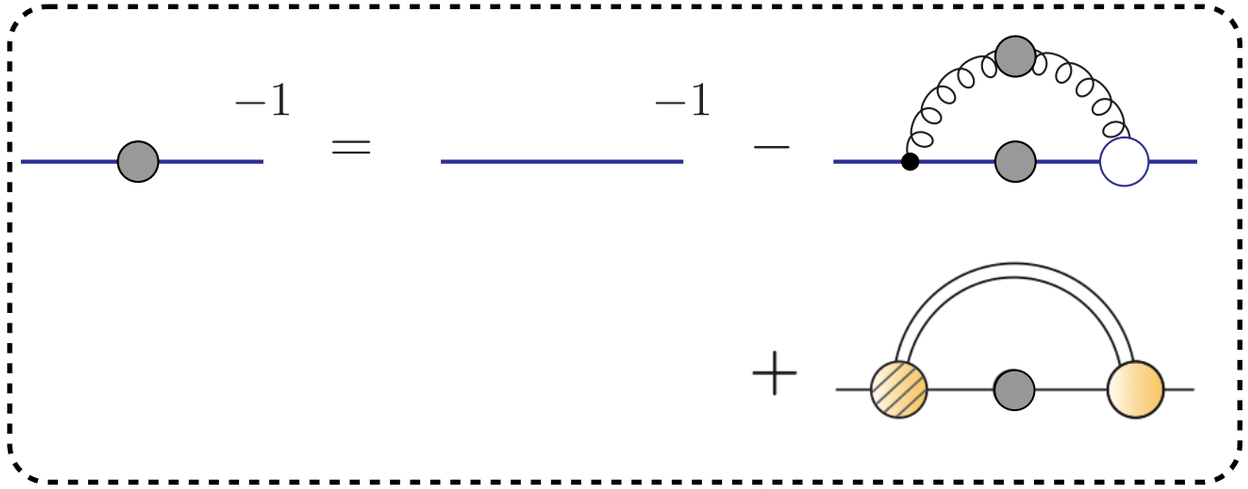
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

N_f=2+1-QCD with DSEs and meson backcoupling

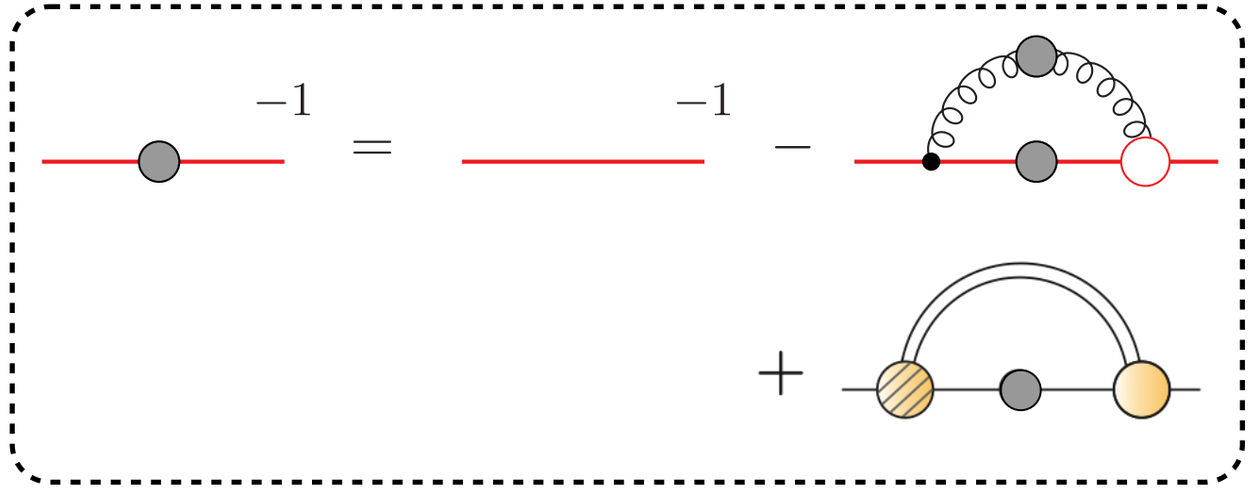


T-dep. propagator from lattice (pure glue; N_f=0)





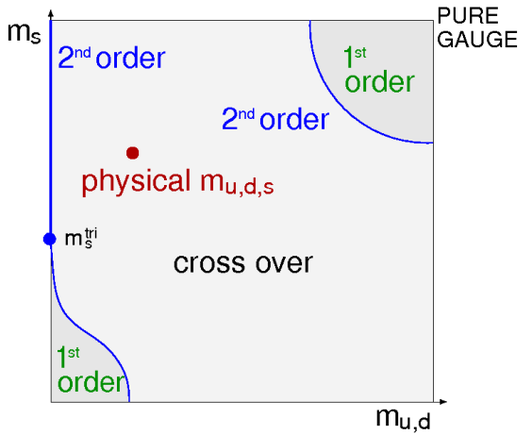
light mesons



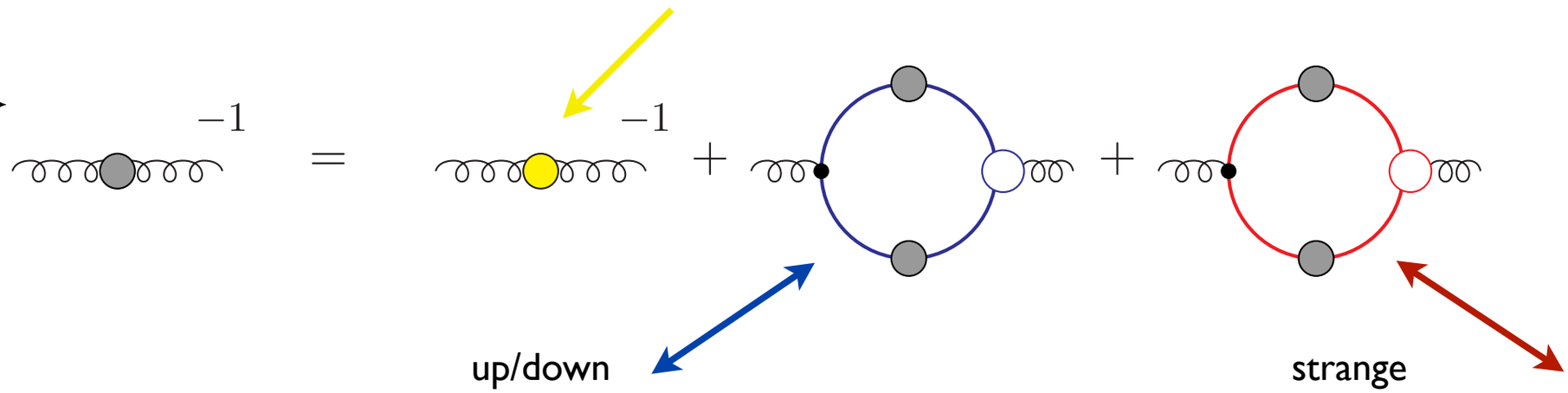
strange mesons

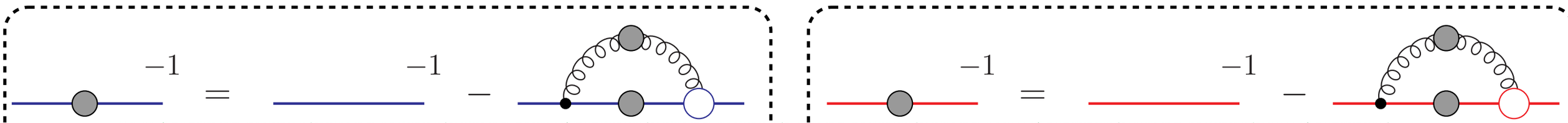
CF, Luecker, Welzbacher, PRD 90 (2014) 034022
Gunkel, CF, PRD 104 (2021) [2106.08356]

N_f=2+1-QCD with DSEs and meson backcoupling



T-dep. propagator from lattice (pure glue; N_f=0)

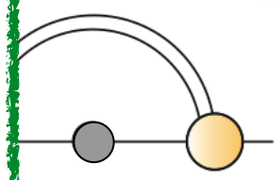




FRG: Fu, Rennecke, Pawlowski, PRD 101 (2020)

FRG/DSE: Gao, Pawlowski, PLB 820 (2021)

DSE: Gunkel, CF, PRD 104 (2021)



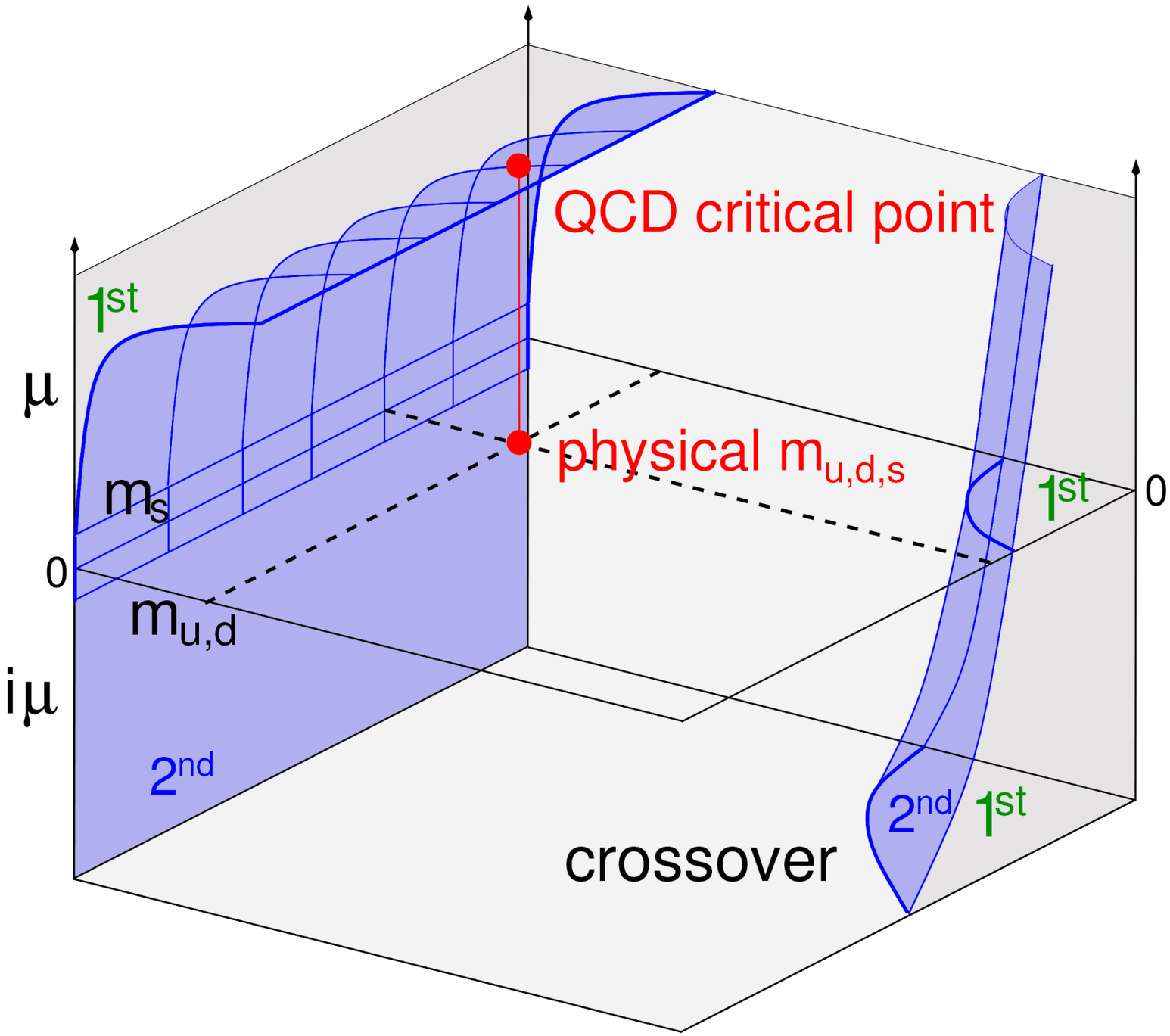
mesons

quality of truncations on same level

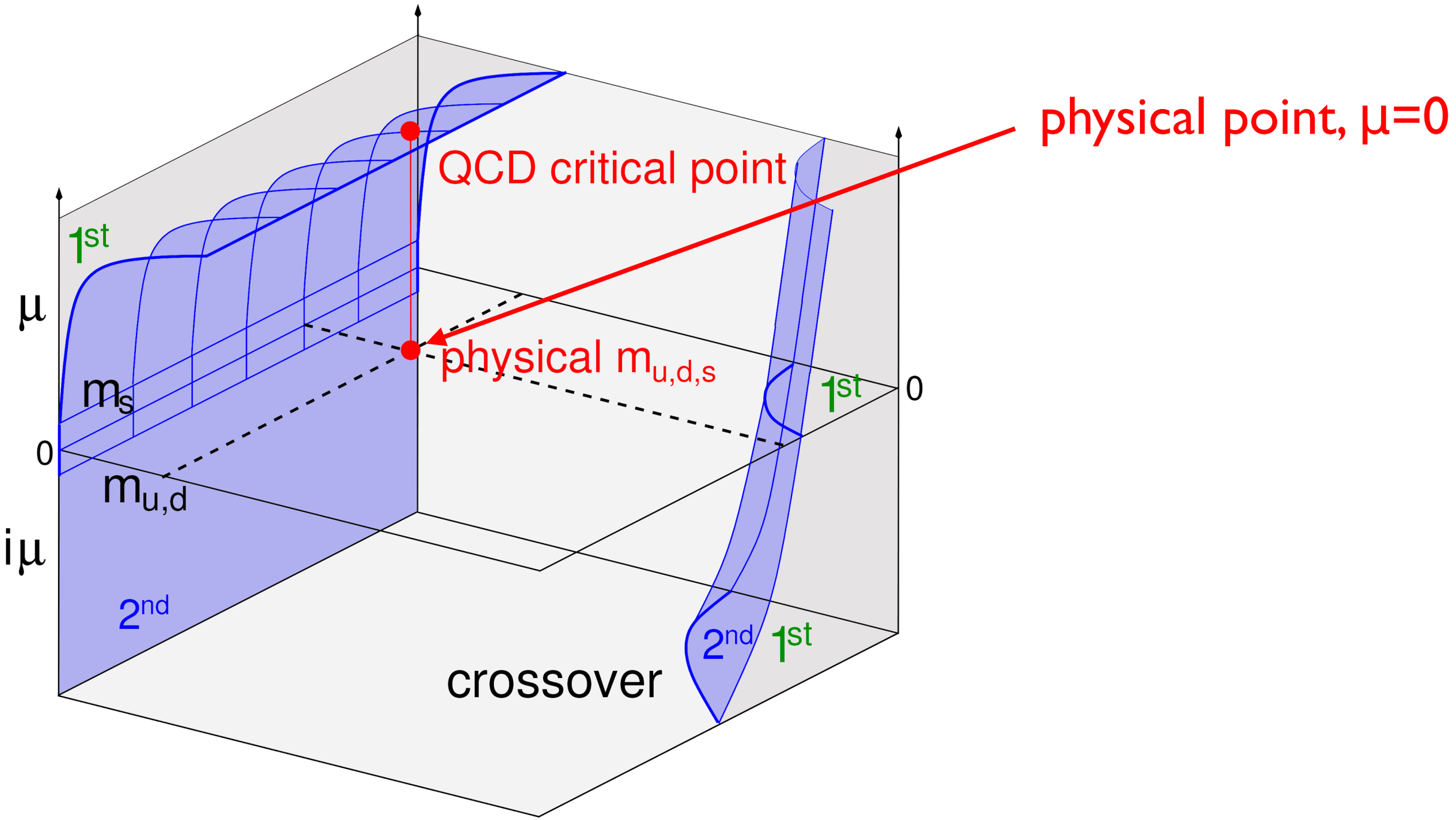
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Gunkel, CF, PRD 104 (2021) [2106.08356]

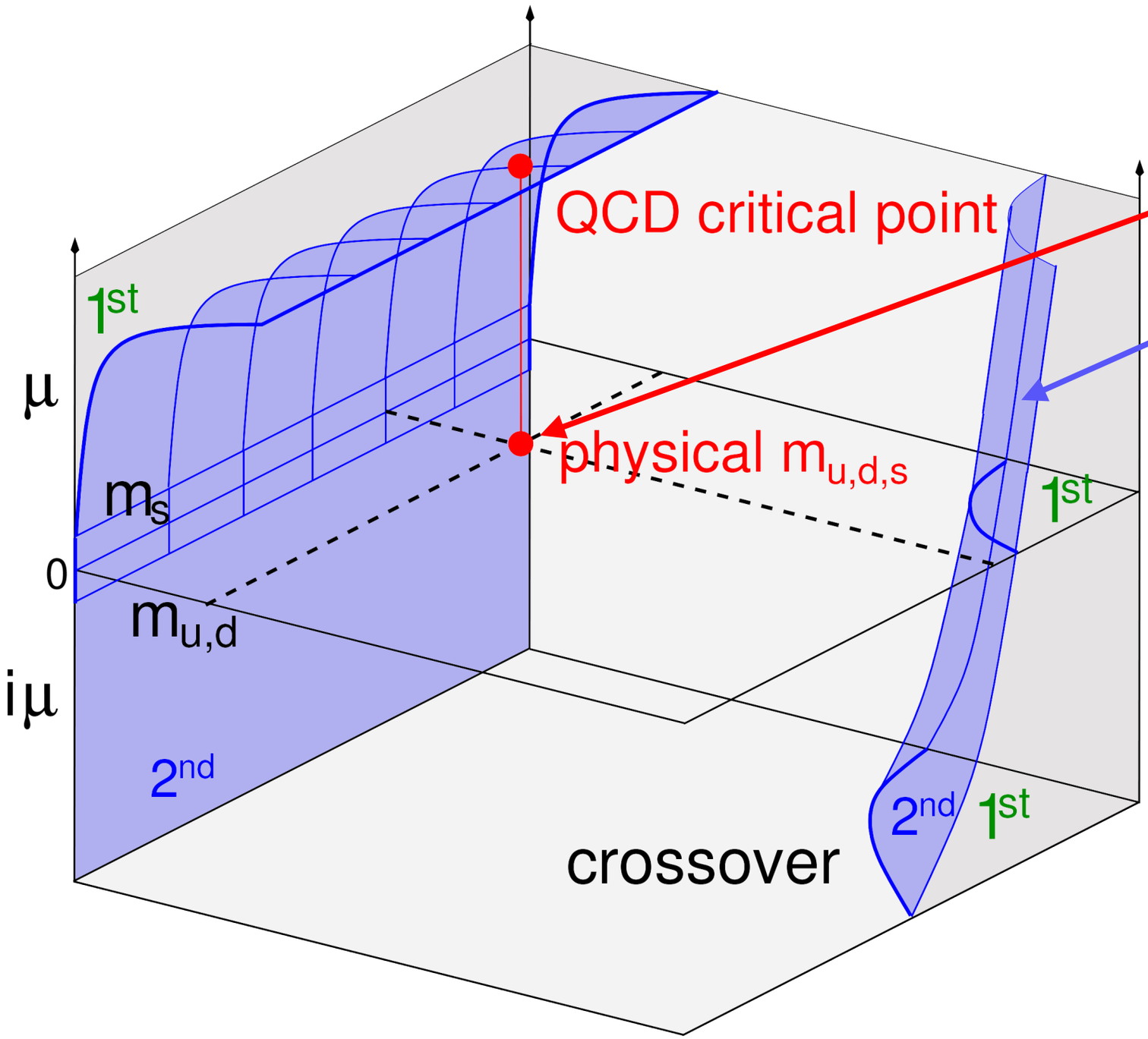
Roadmap (of longer talk….)



Roadmap (of longer talk….)



Roadmap (of longer talk….)

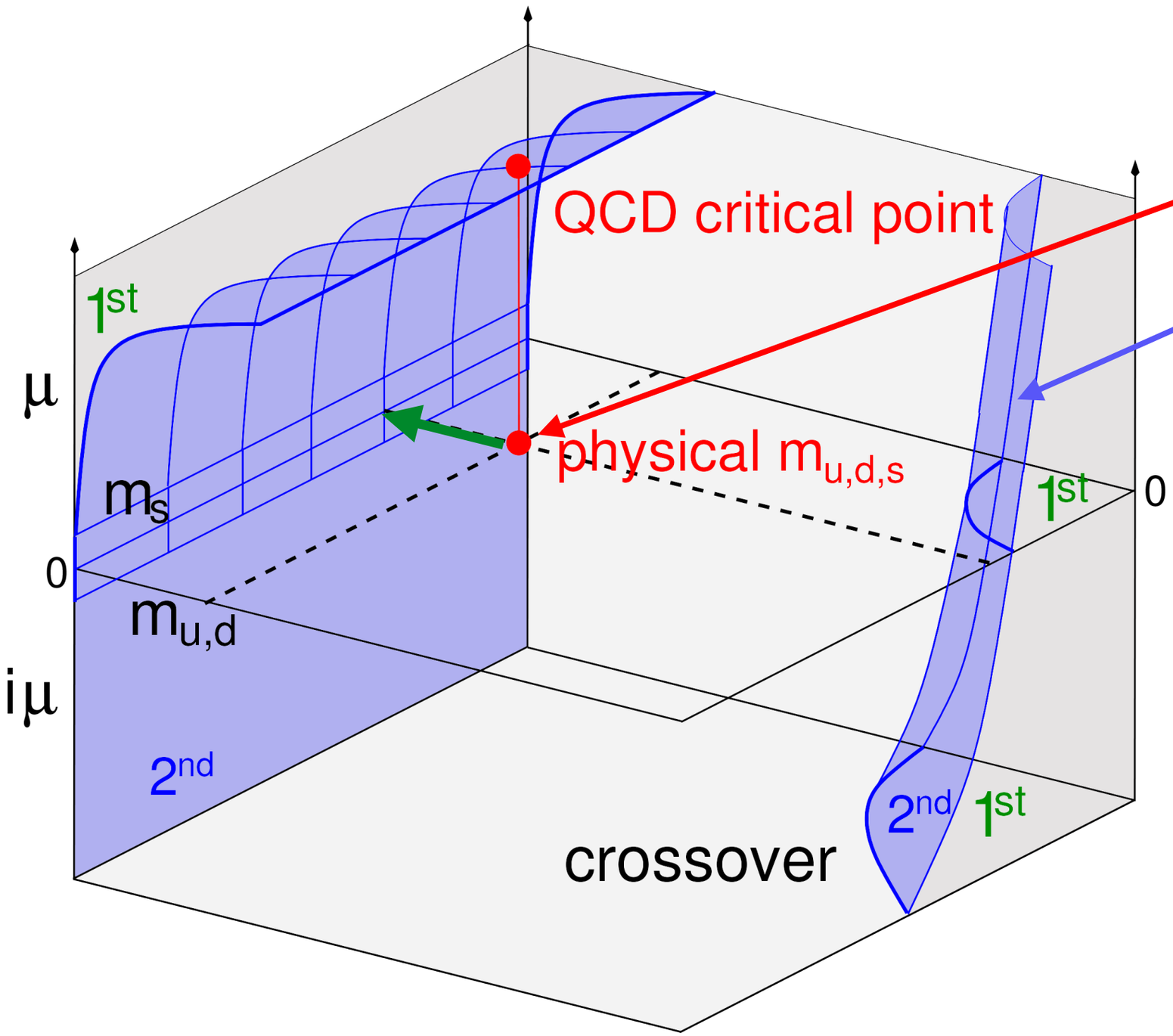


physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

Roadmap (of longer talk….)



physical point, μ=0

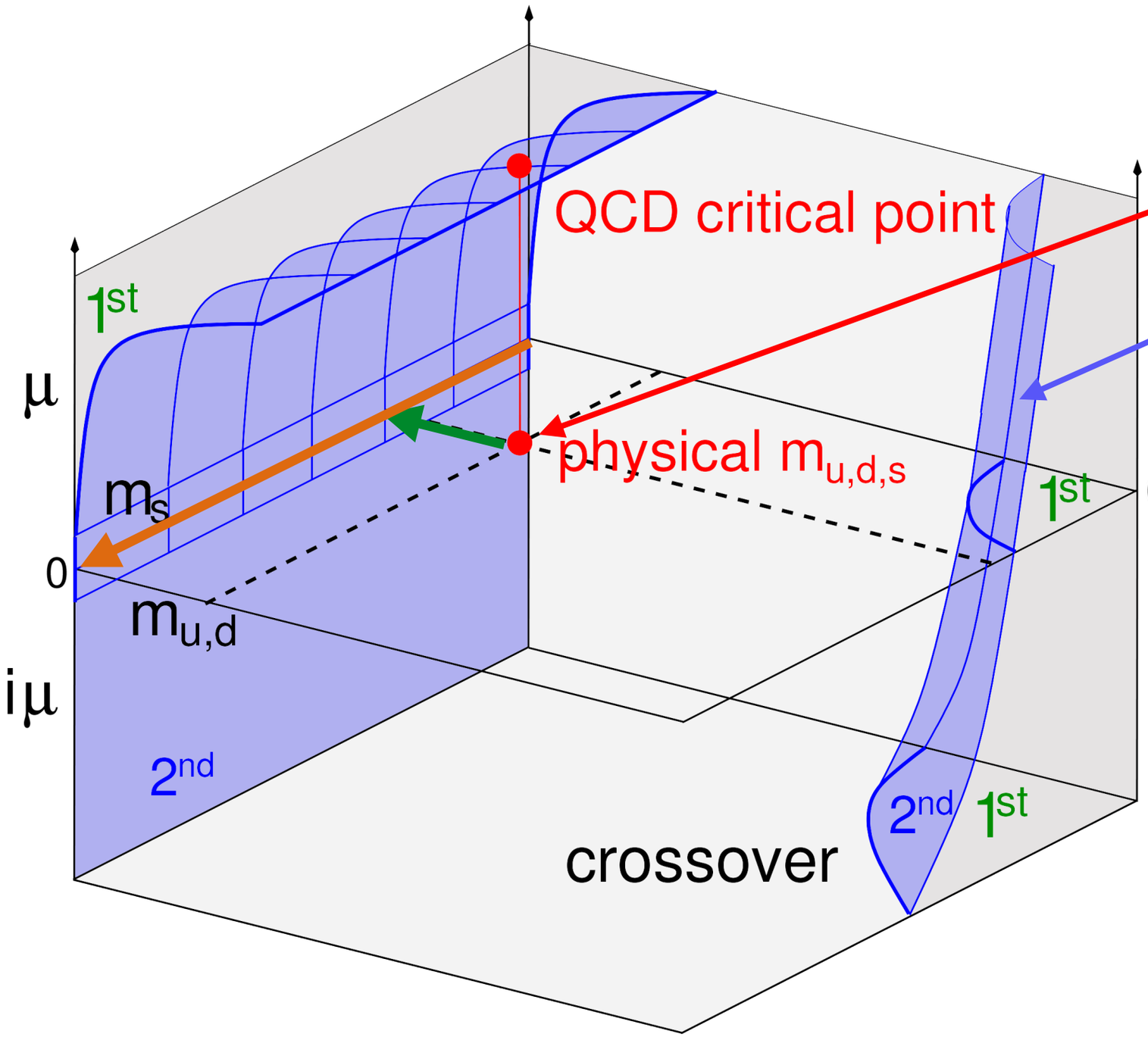
deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
FRG/DSE: Gao and Pawlowski PRD 105(2022) 094020
DSE: Bernhardt and CF, PRD 108 (2023) 114018

Roadmap (of longer talk….)



physical point, μ=0

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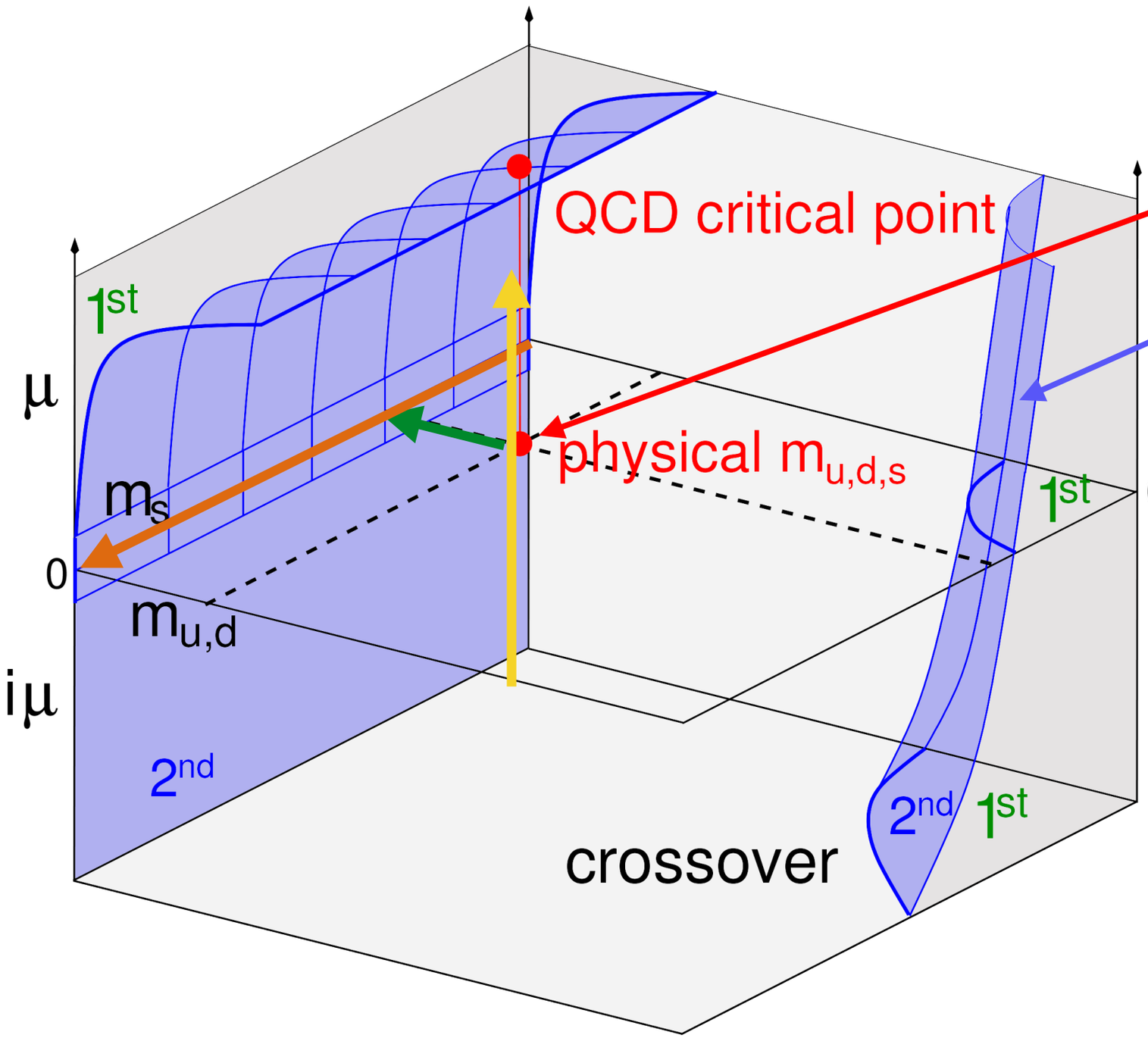
chiral limit

DSE: Bernhardt and CF, PRD 108 (2023) 114018

Roadmap (of longer talk….)



physical point, μ=0

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DSE: Bernhardt and CF, PRD 108 (2023) 114018

chiral limit

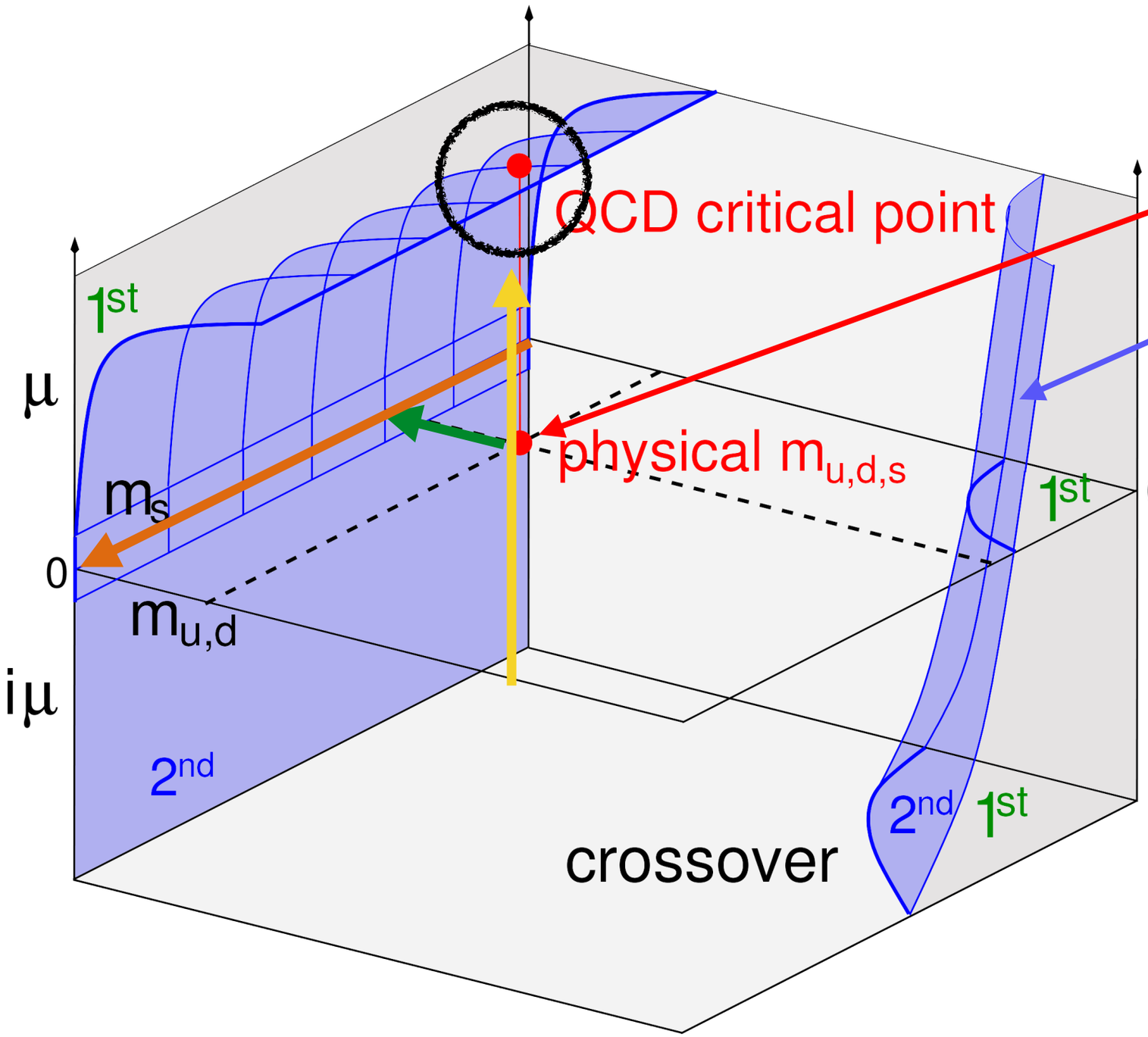
DSE: Bernhardt and CF, PRD 108 (2023) 114018

imaginary μ

FRG (Nf=2): Braun et. al. PRL 106 (2011)

DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

Roadmap (of longer talk….)



physical point, μ=0

deconfinement

DSE: CF, Luecker, Pawlowski, PRD 91 (2015) 1

towards chiral limit

FRG: Braun et al, PRD 102 (2020) 5, 056010
FRG/DSE: Gao and Pawlowski PRD 105(2022) 094020
DSE: Bernhardt and CF, PRD 108 (2023) 114018

chiral limit

DSE: Bernhardt and CF, PRD 108 (2023) 114018

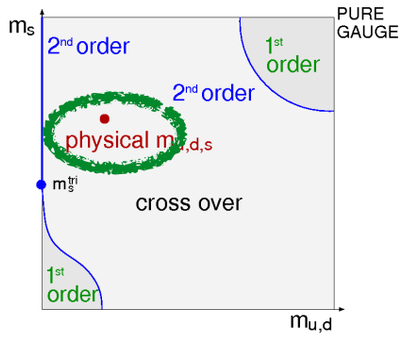
imaginary μ

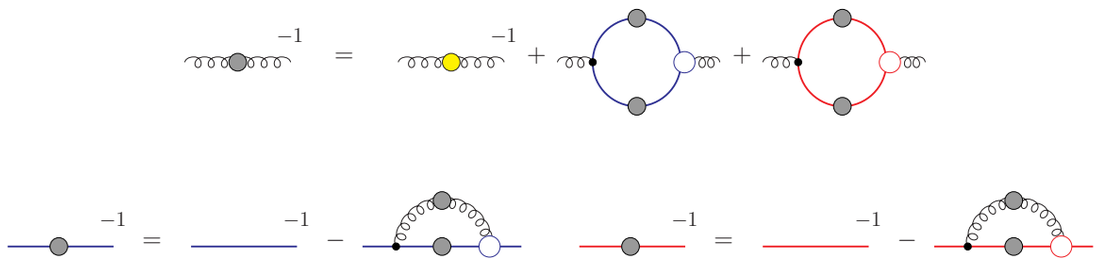
FRG (Nf=2): Braun et. al. PRL 106 (2011)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

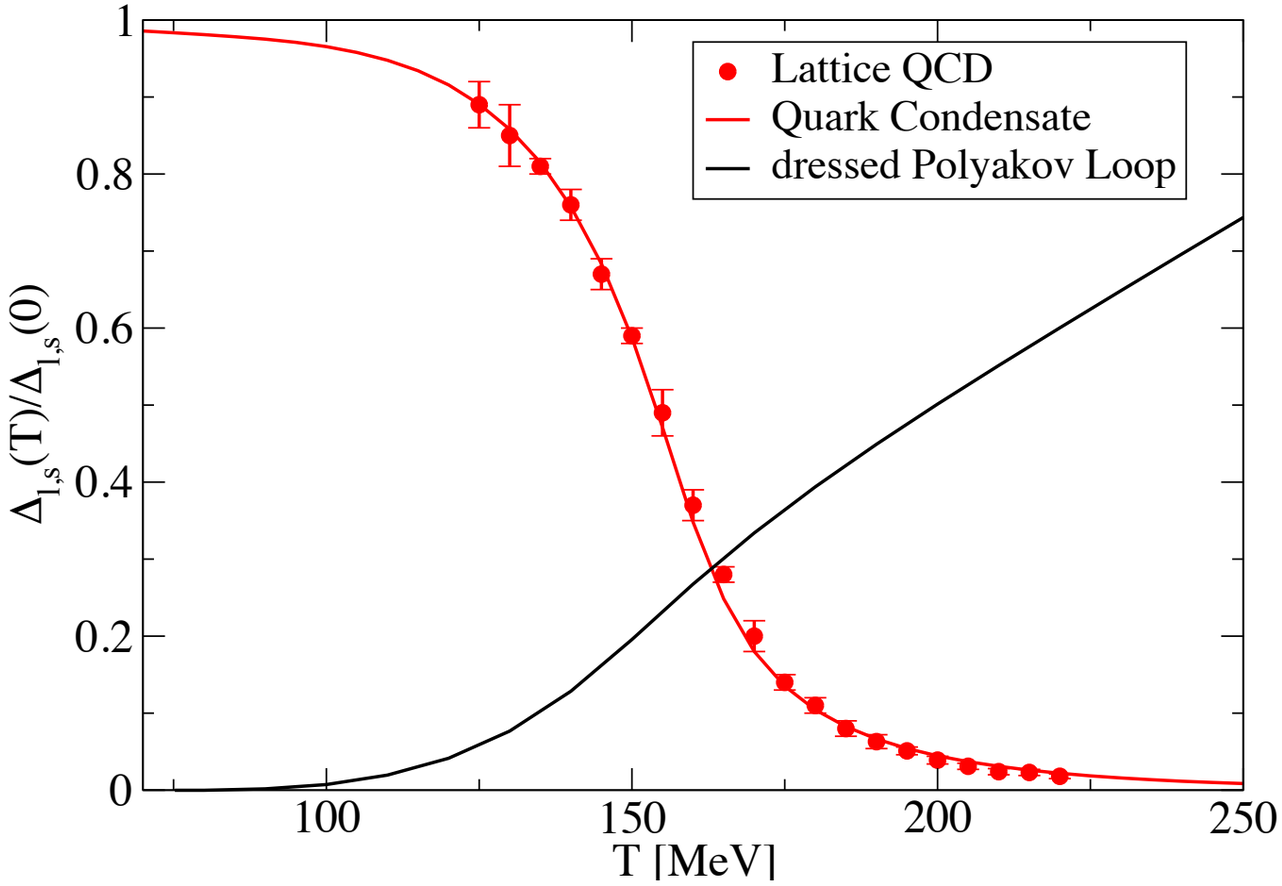
location of CEP

FRG: Fu, Rennecke, Pawlowski, PRD 101 (2020)
FRG/DSE: Gao, Pawlowski, PLB 820 (2021)
DSE: Gunkel, CF, PRD 104 (2021)

N_f=2+1, μ=0, physical point





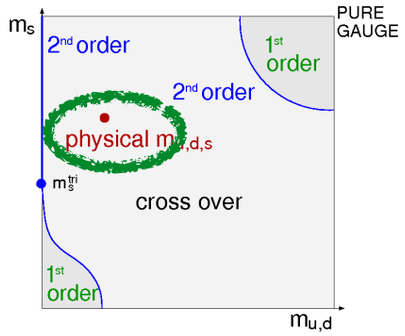


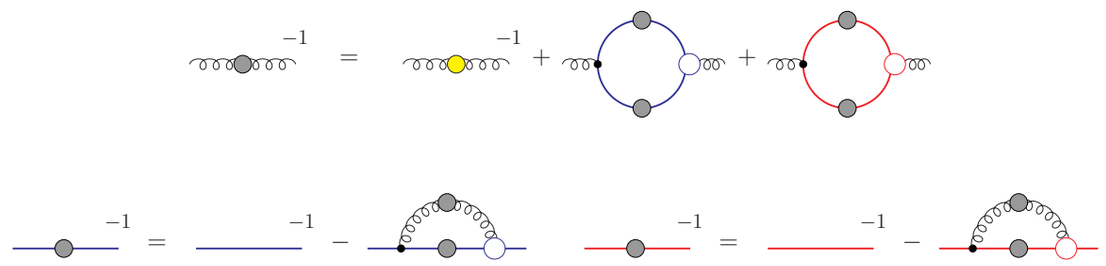
Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

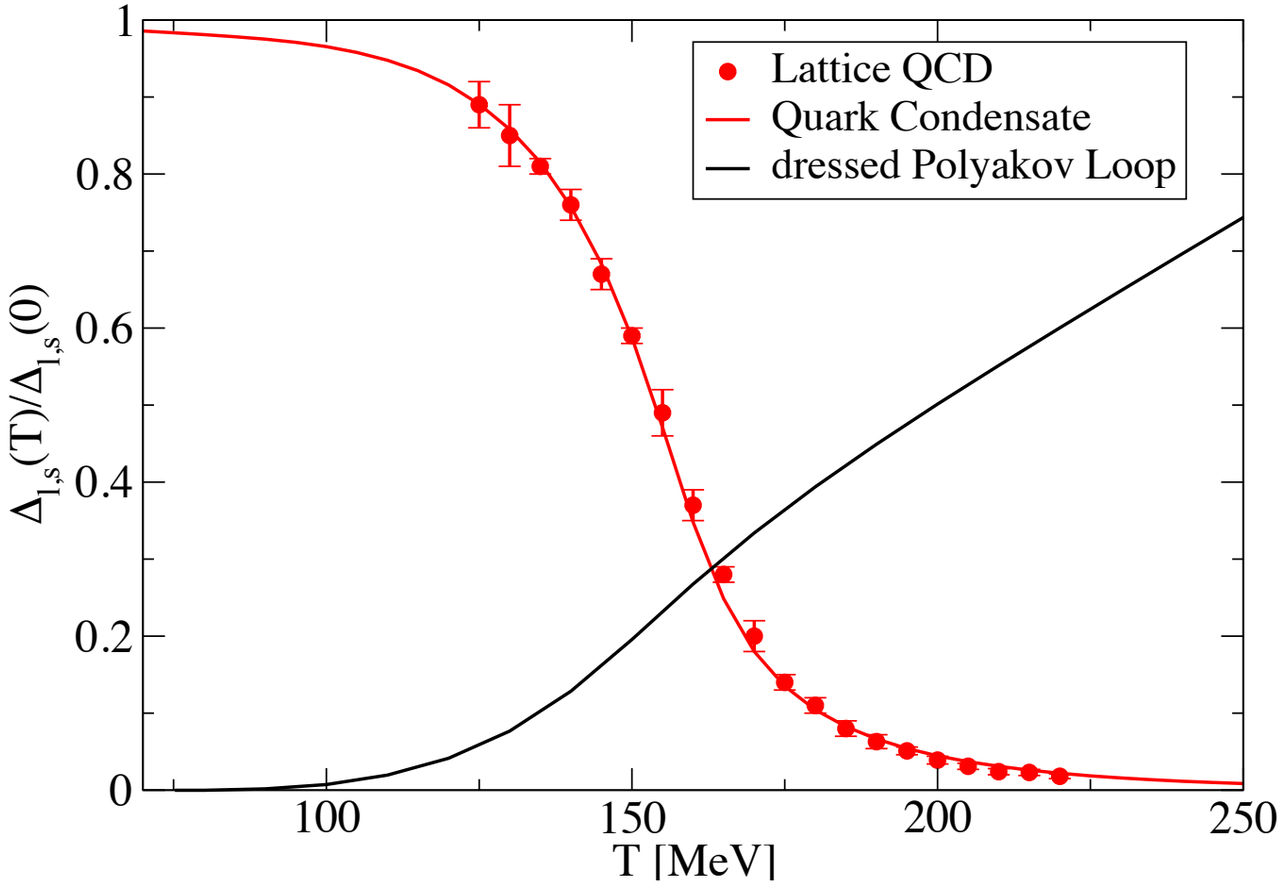
FRG: similar results

Fu, Rennecke, Pawlowski, PRD 101 (2020)

N_f=2+1, μ=0, physical point







Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
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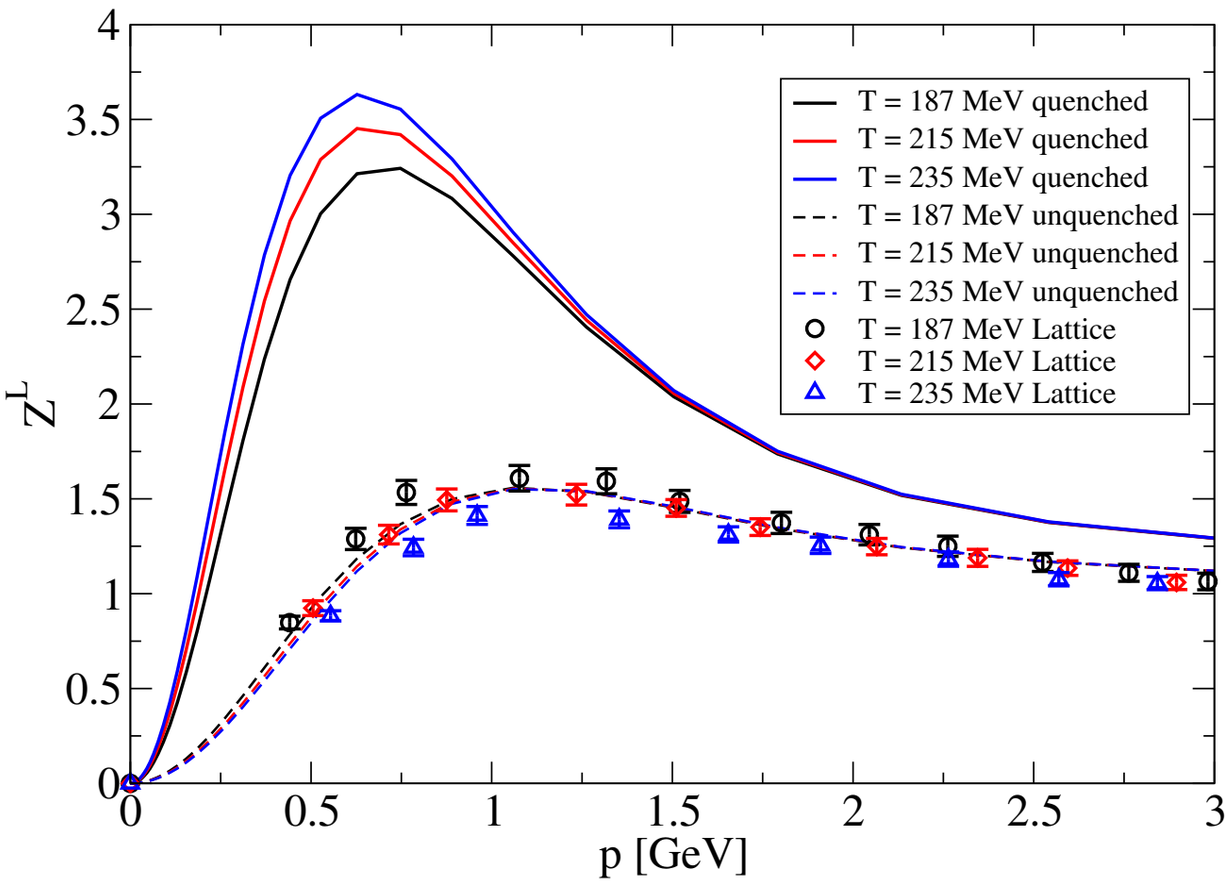
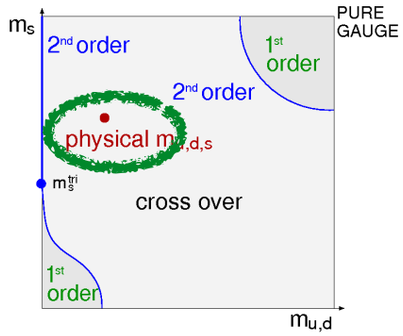
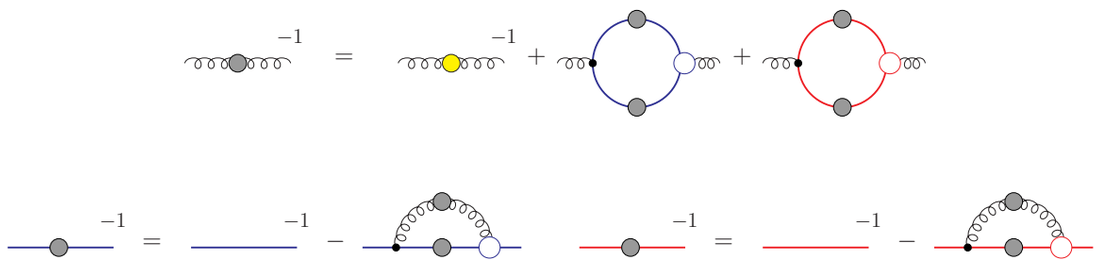


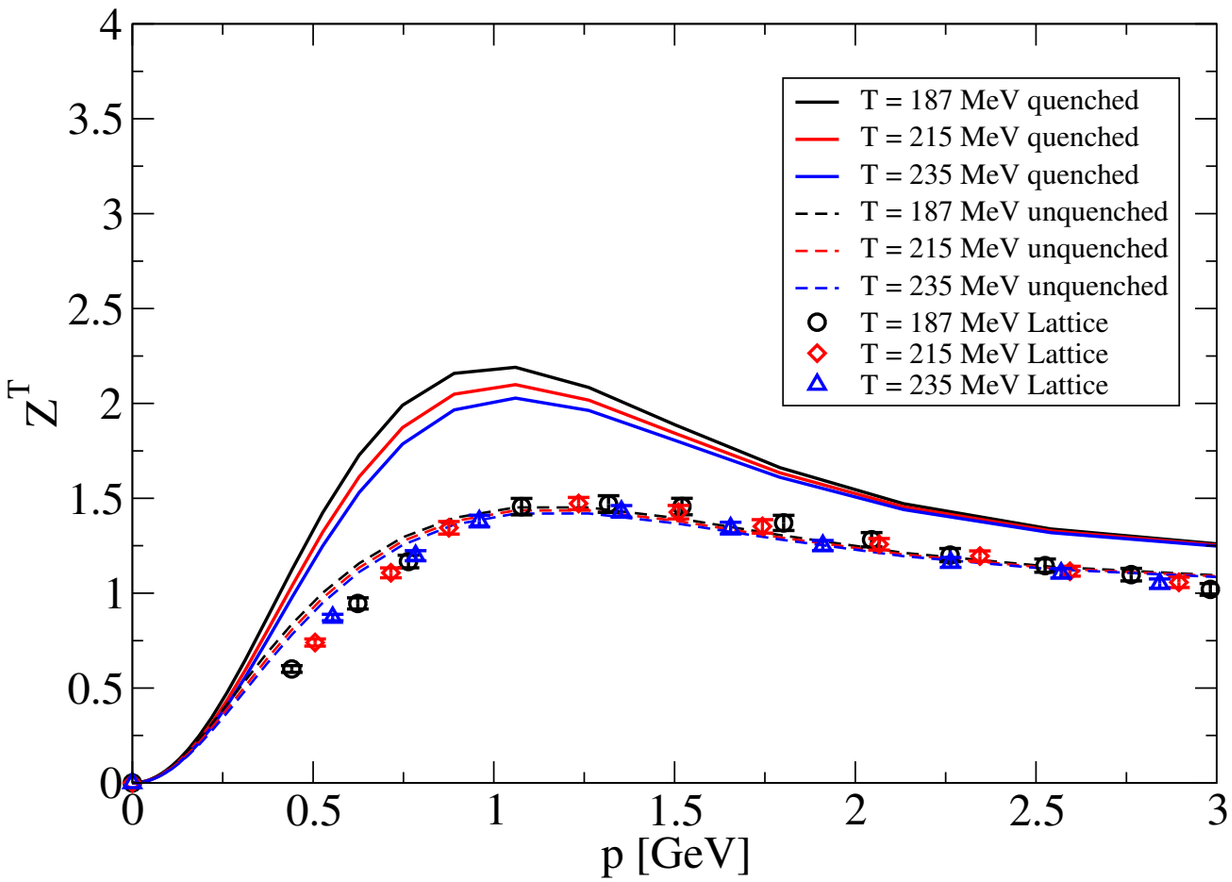
Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

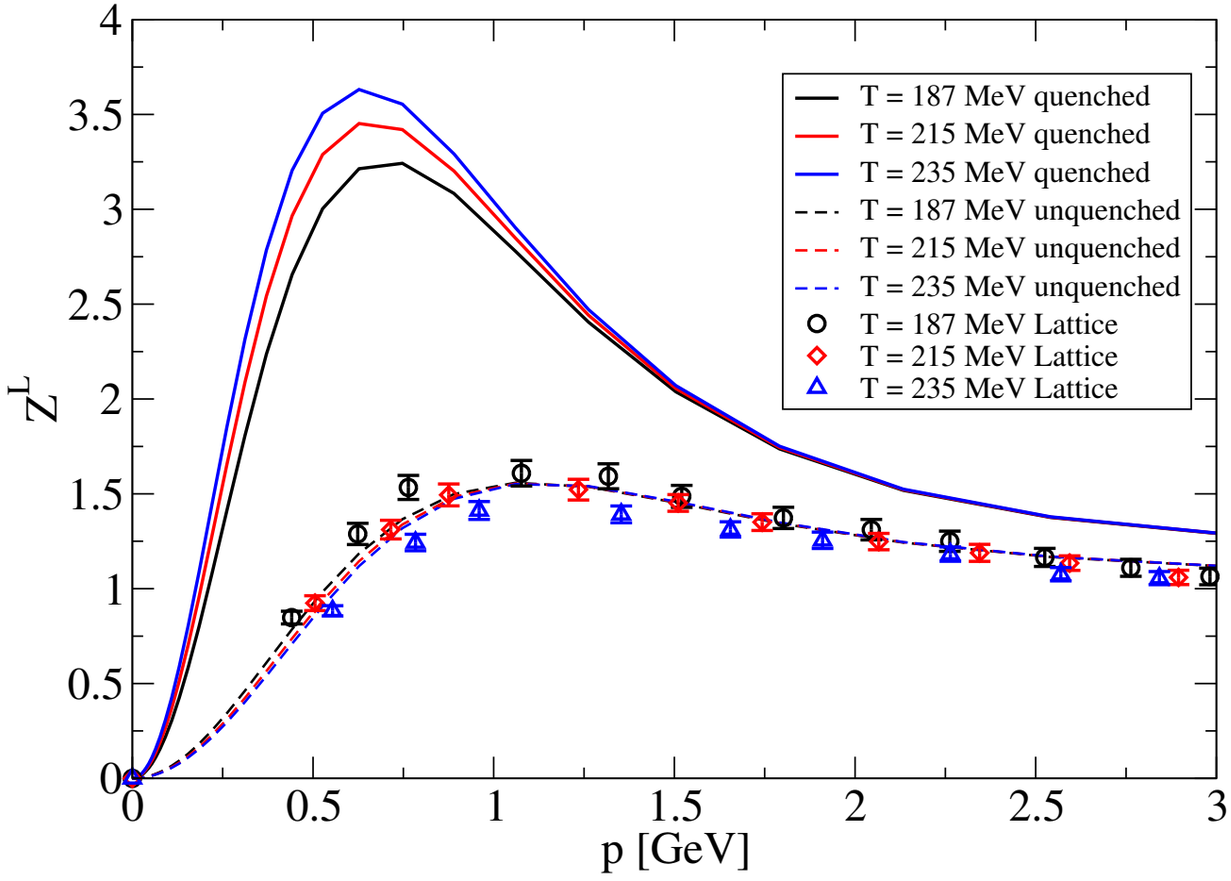
- quantitative agreement: DSE prediction verified by lattice
  FRG: similar results Fu, Rennecke, Pawlowski, PRD 101 (2020)

N_f=2+1, μ=0, physical point









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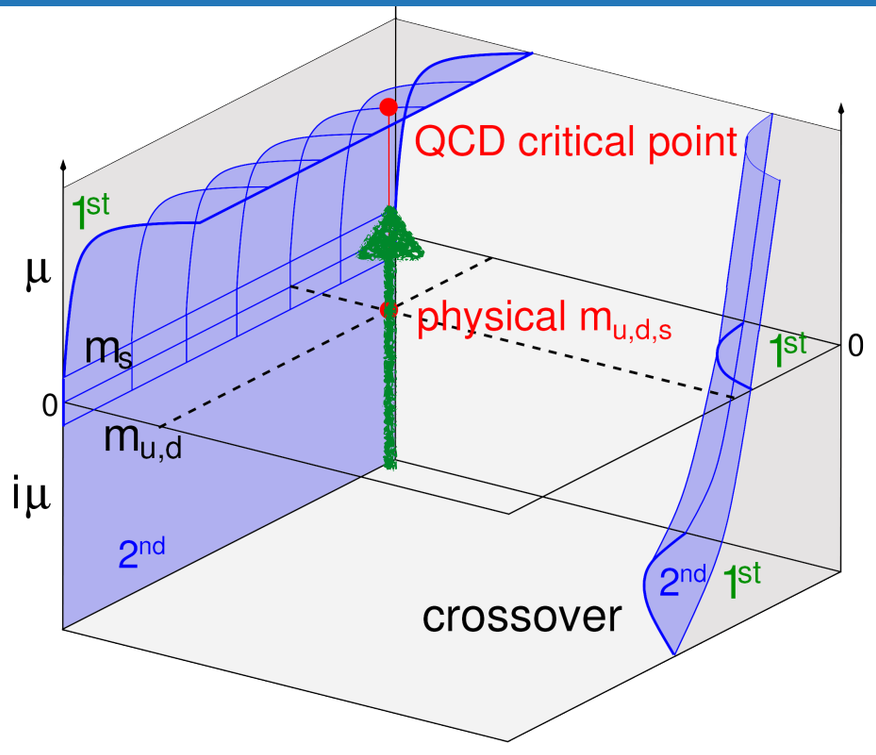
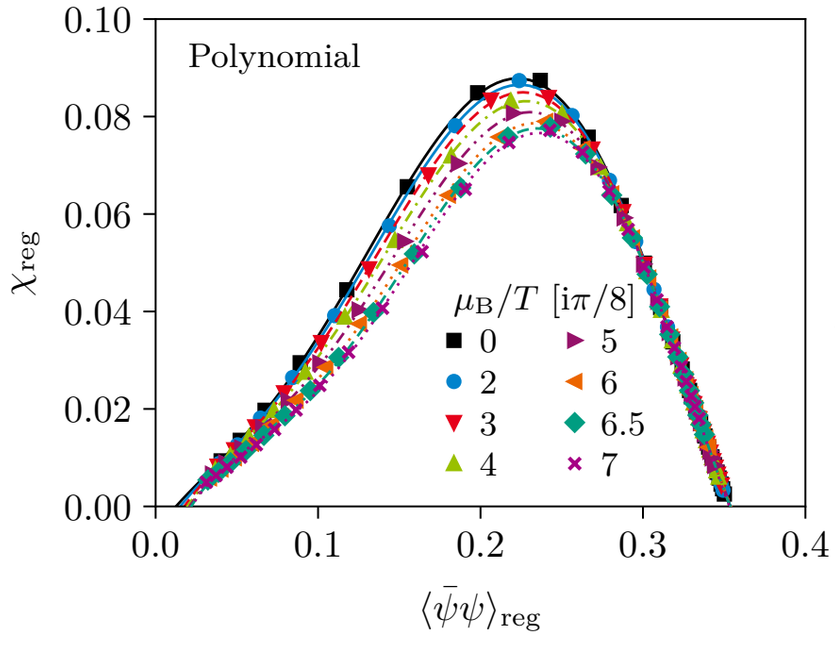
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

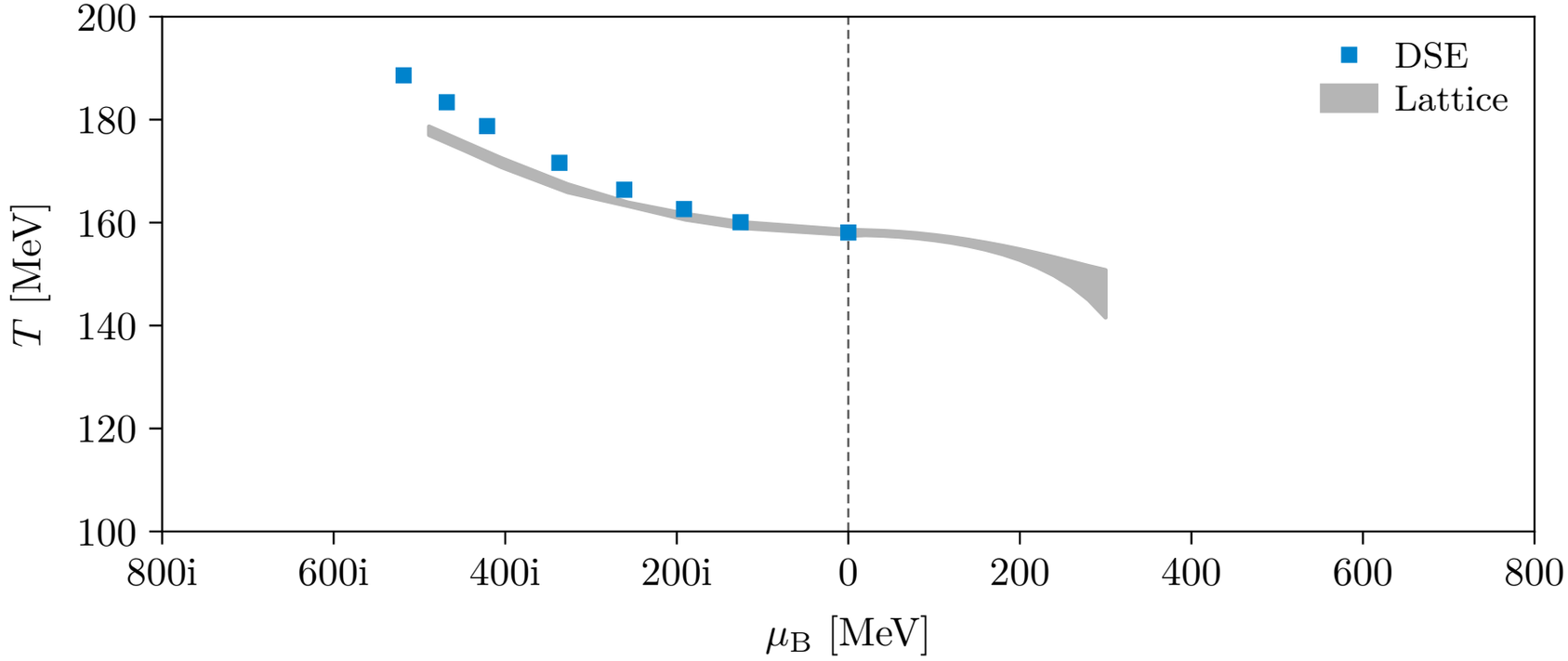
- quantitative agreement: DSE prediction verified by lattice
  FRG: similar results Fu, Rennecke, Pawlowski, PRD 101 (2020)

Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\rm u}}$$



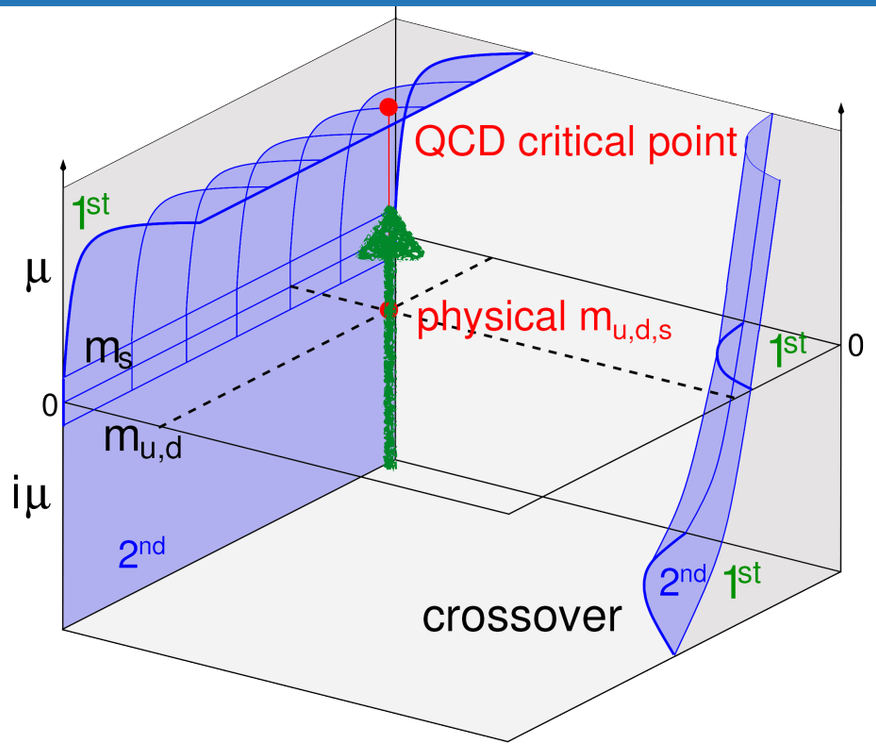
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

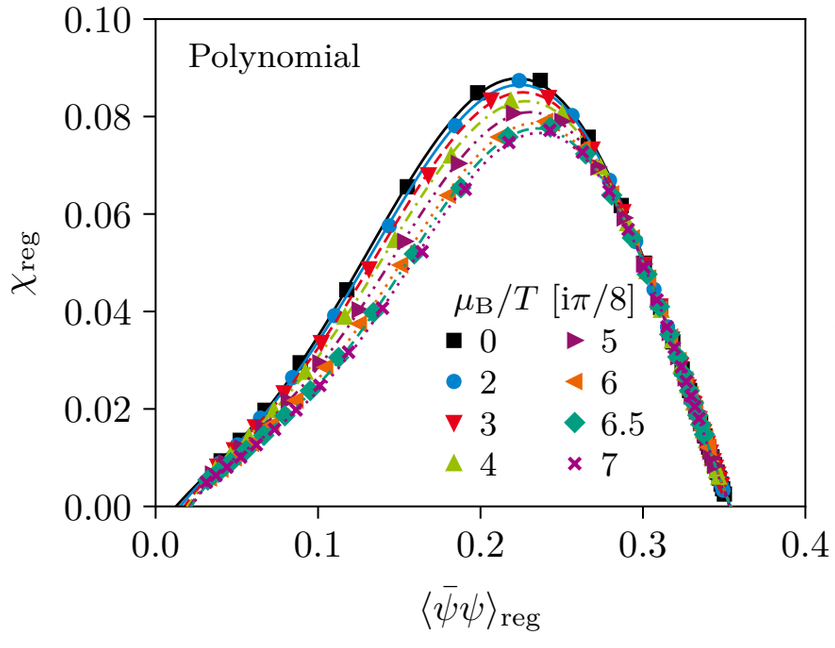
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

$$\kappa_2^{\rm poly} = 0.0196\,, \quad \kappa_4^{\rm poly} = 0.00015\,,$$

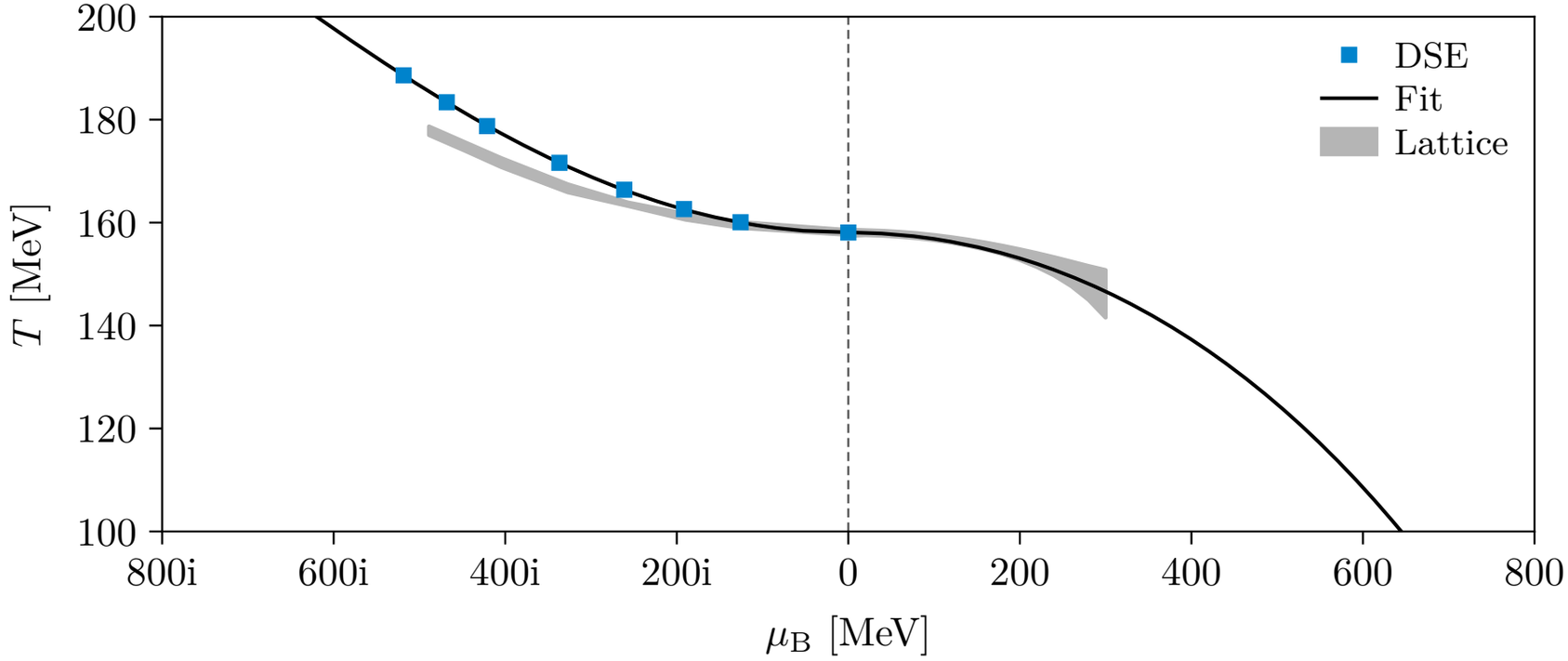
see also FRG (Nf=2): Braun et. al. PRL 106 (2011)

Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\rm u}}$$



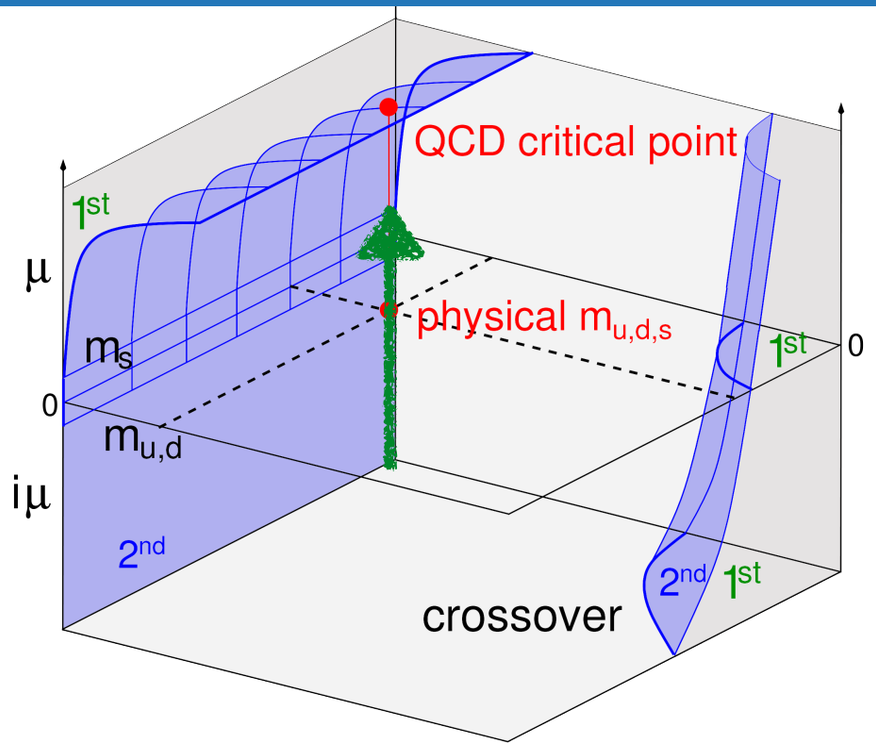
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

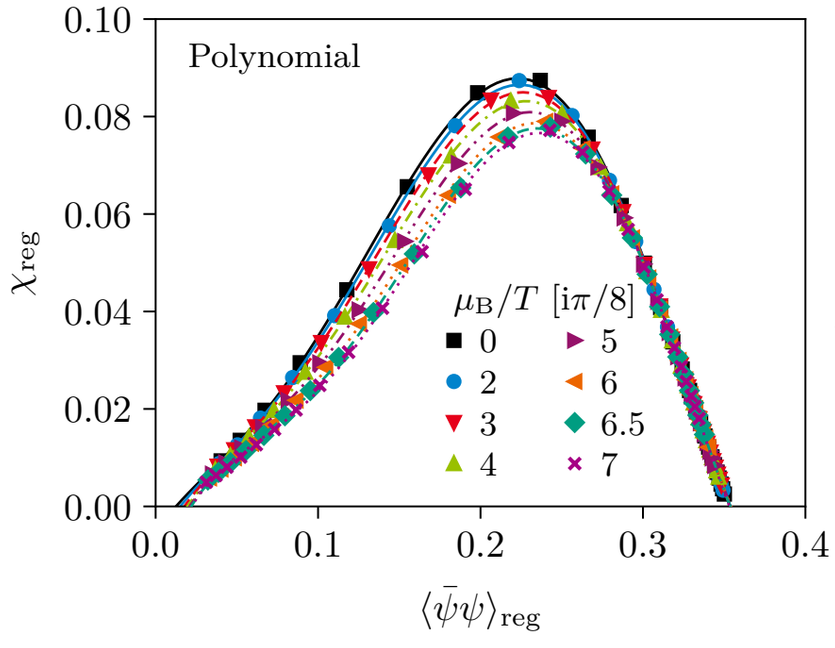
see also FRG (Nf=2): Braun et. al. PRL 106 (2011)

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

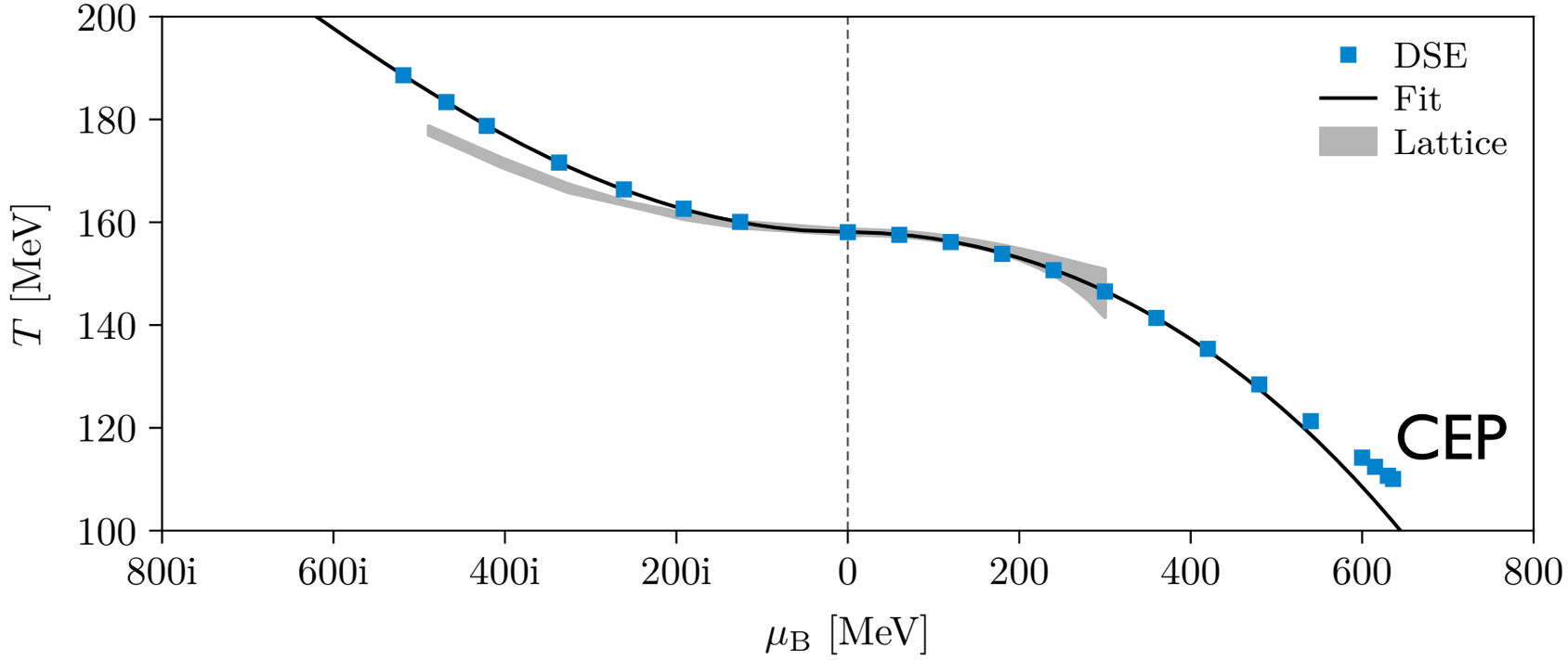
$$\kappa_2^{\rm poly} = 0.0196\,, \quad \kappa_4^{\rm poly} = 0.00015\,,$$

Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\mathrm{u}}}$$



- Extrapolation works very well!

see also FRG (Nf=2): Braun et. al. PRL 106 (2011)

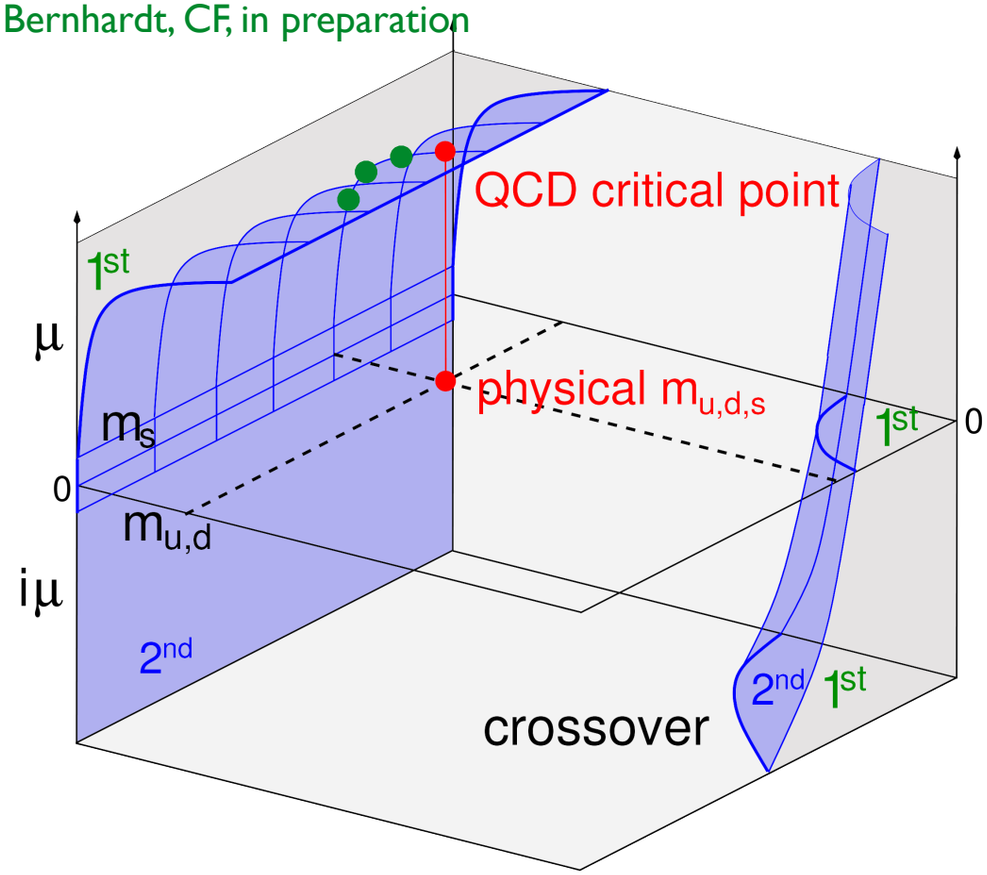
Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

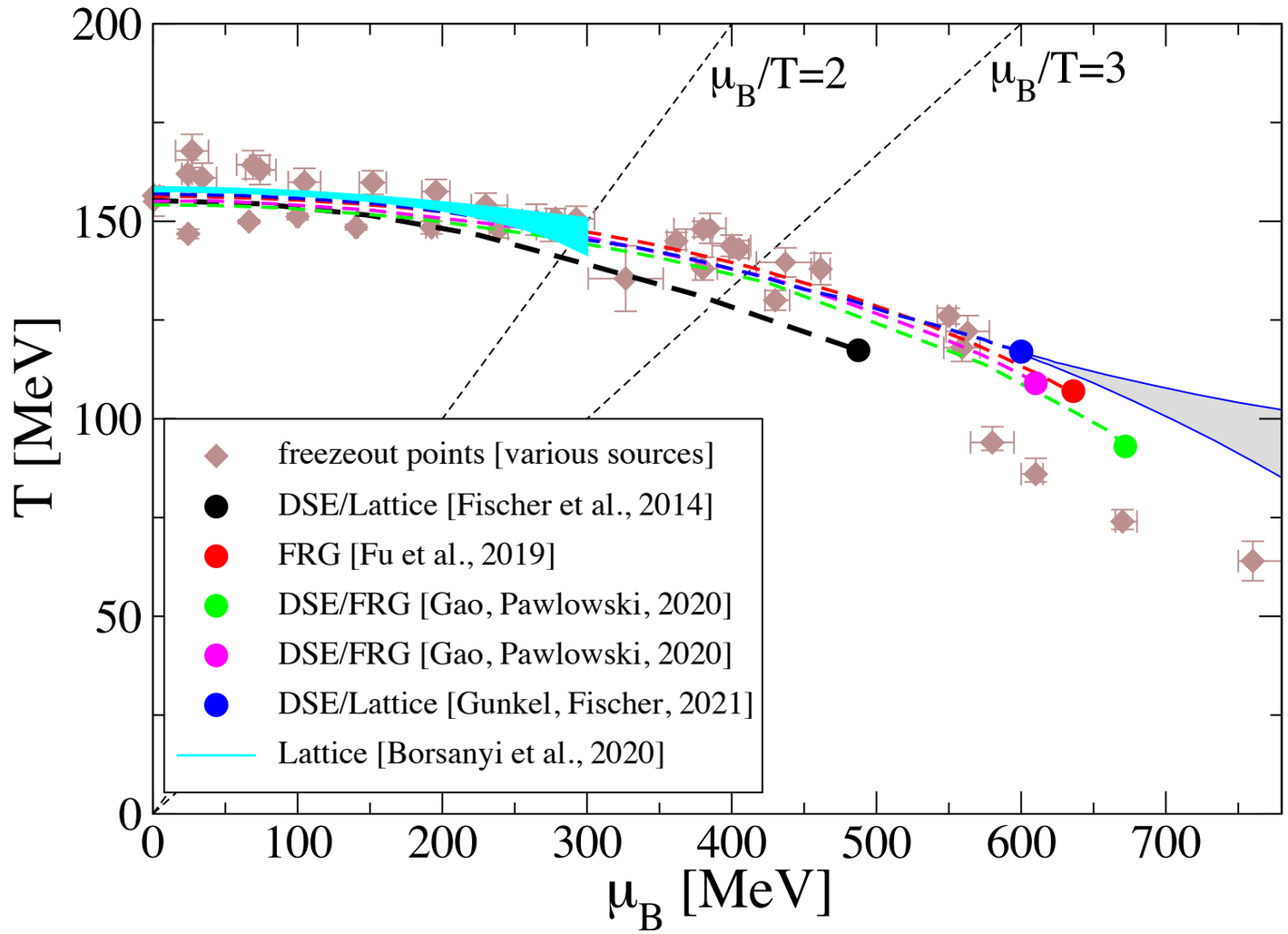
$$\frac{T_{\mathrm{c}}(\mu_{\mathrm{B}})}{T_{\mathrm{c}}} = 1 - \kappa_2\left(\frac{\mu_{\mathrm{B}}}{T_{\mathrm{c}}}\right)^2 - \kappa_4\left(\frac{\mu_{\mathrm{B}}}{T_{\mathrm{c}}}\right)^4$$

$$\kappa_2^{\mathrm{poly}} = 0.0196\,, \quad \kappa_4^{\mathrm{poly}} = 0.00015\,,$$

Location of CEP

Bernhardt, CF, in preparation

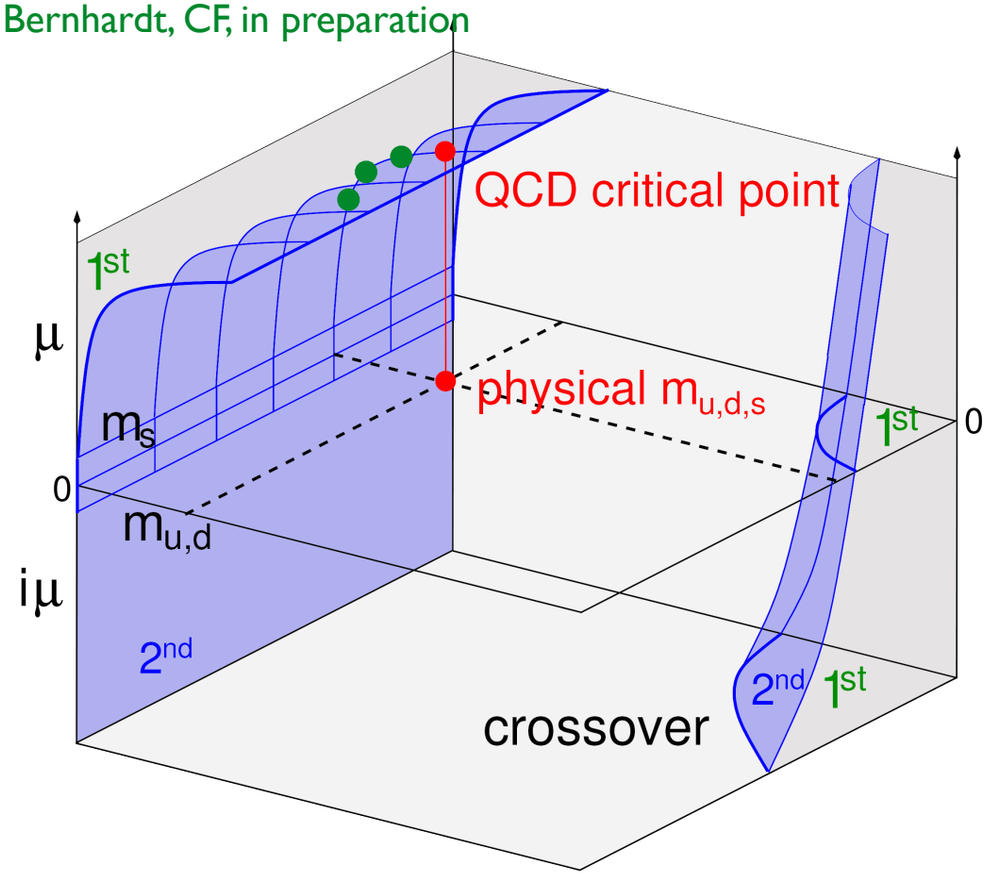


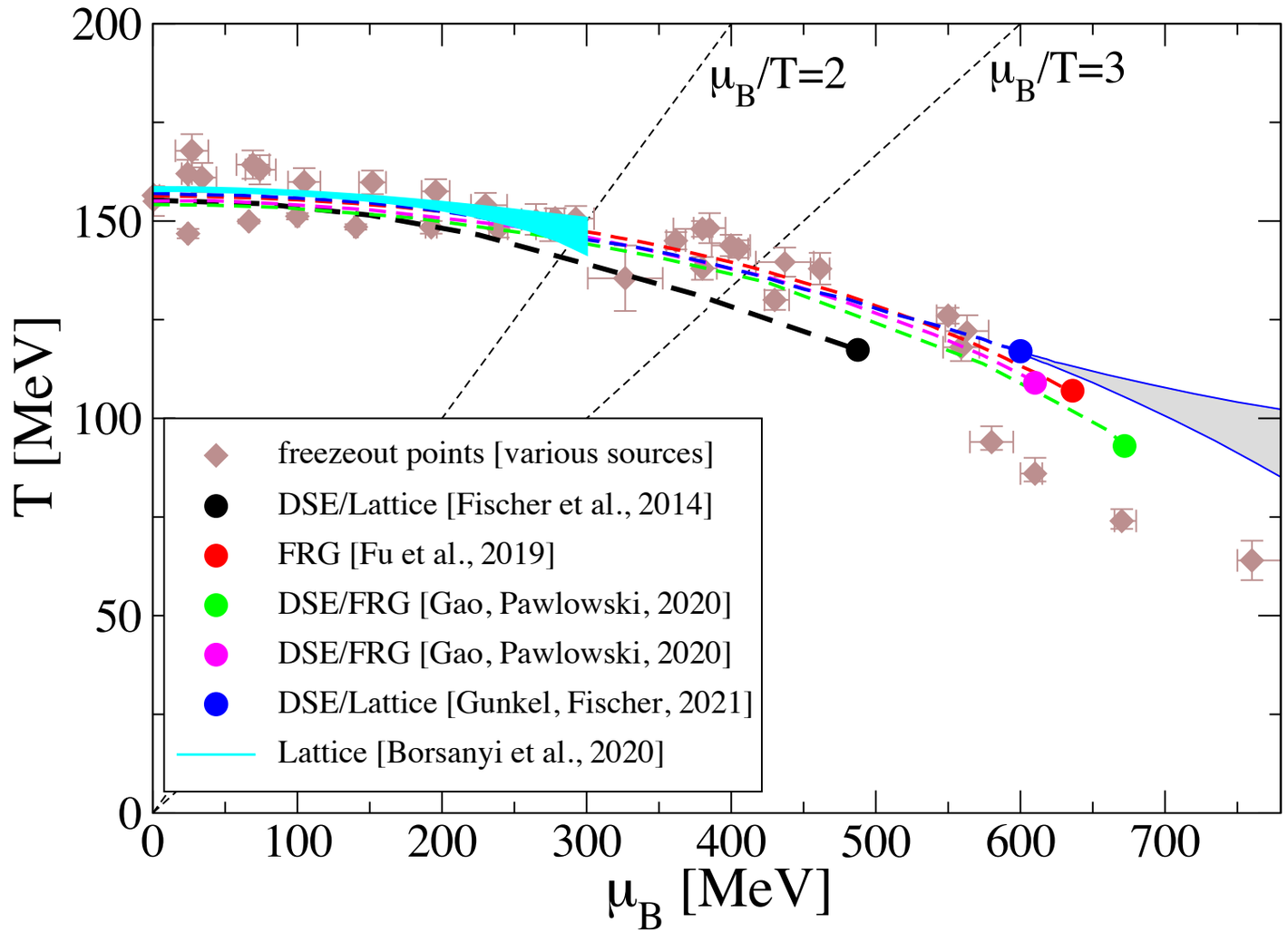


- how stable is this result ??
  - crosscheck DSE-FRG ✔

Location of CEP

Bernhardt, CF, in preparation



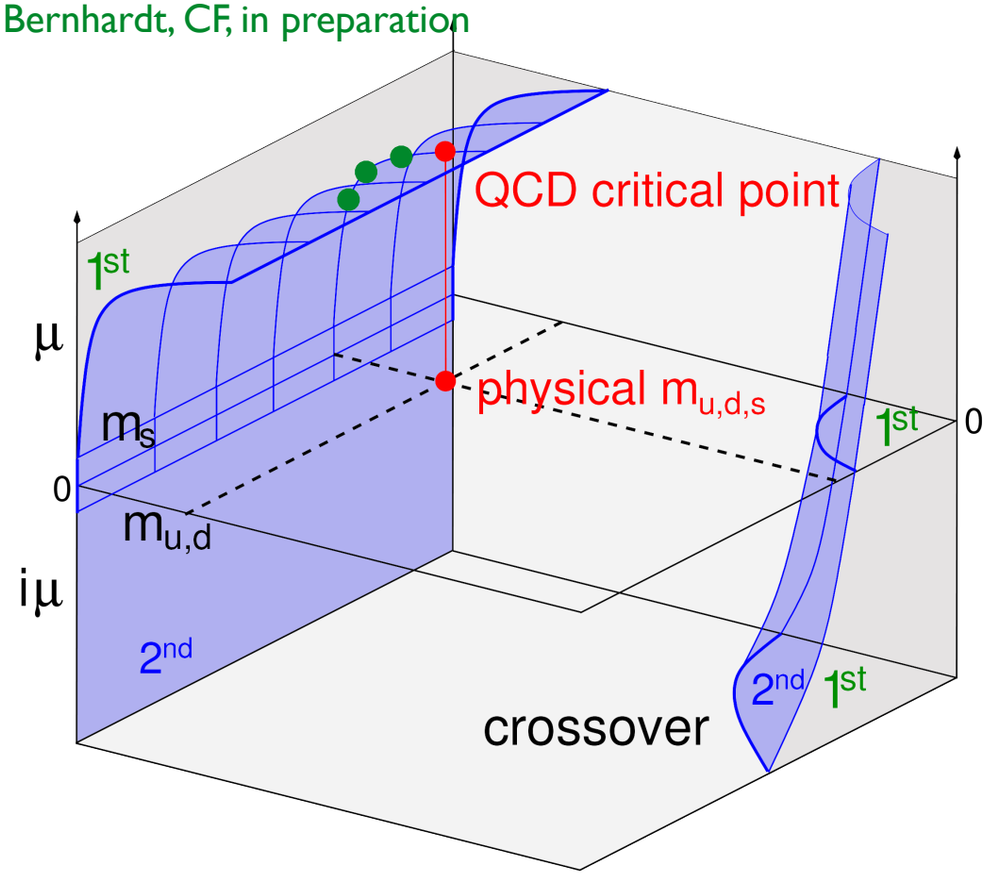


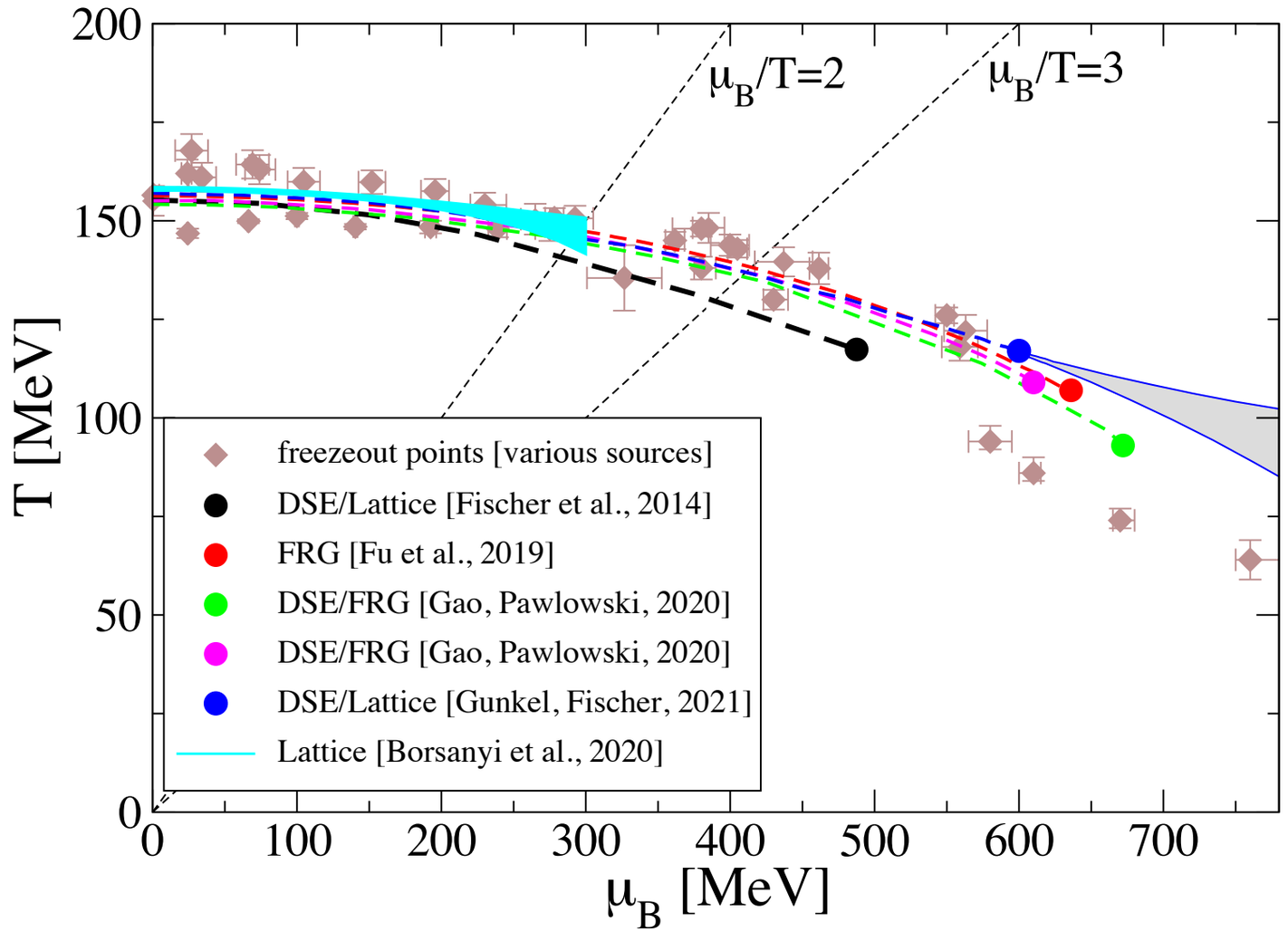
- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Location of CEP

Bernhardt, CF, in preparation

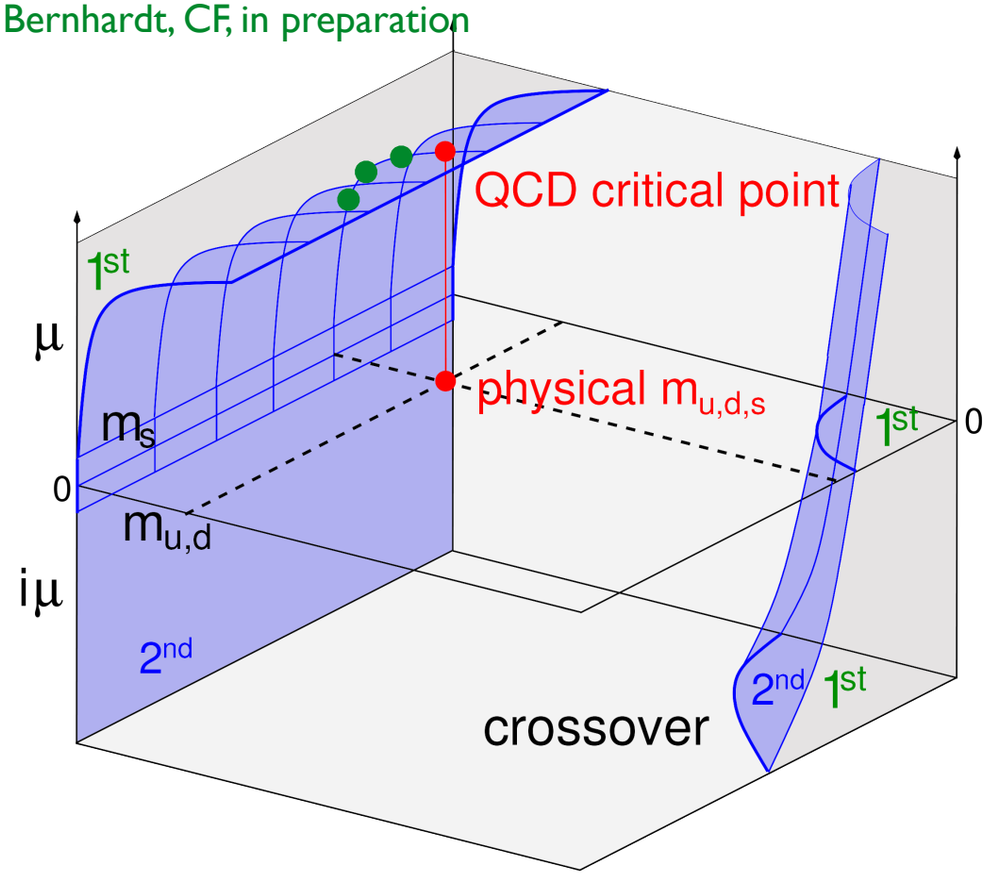


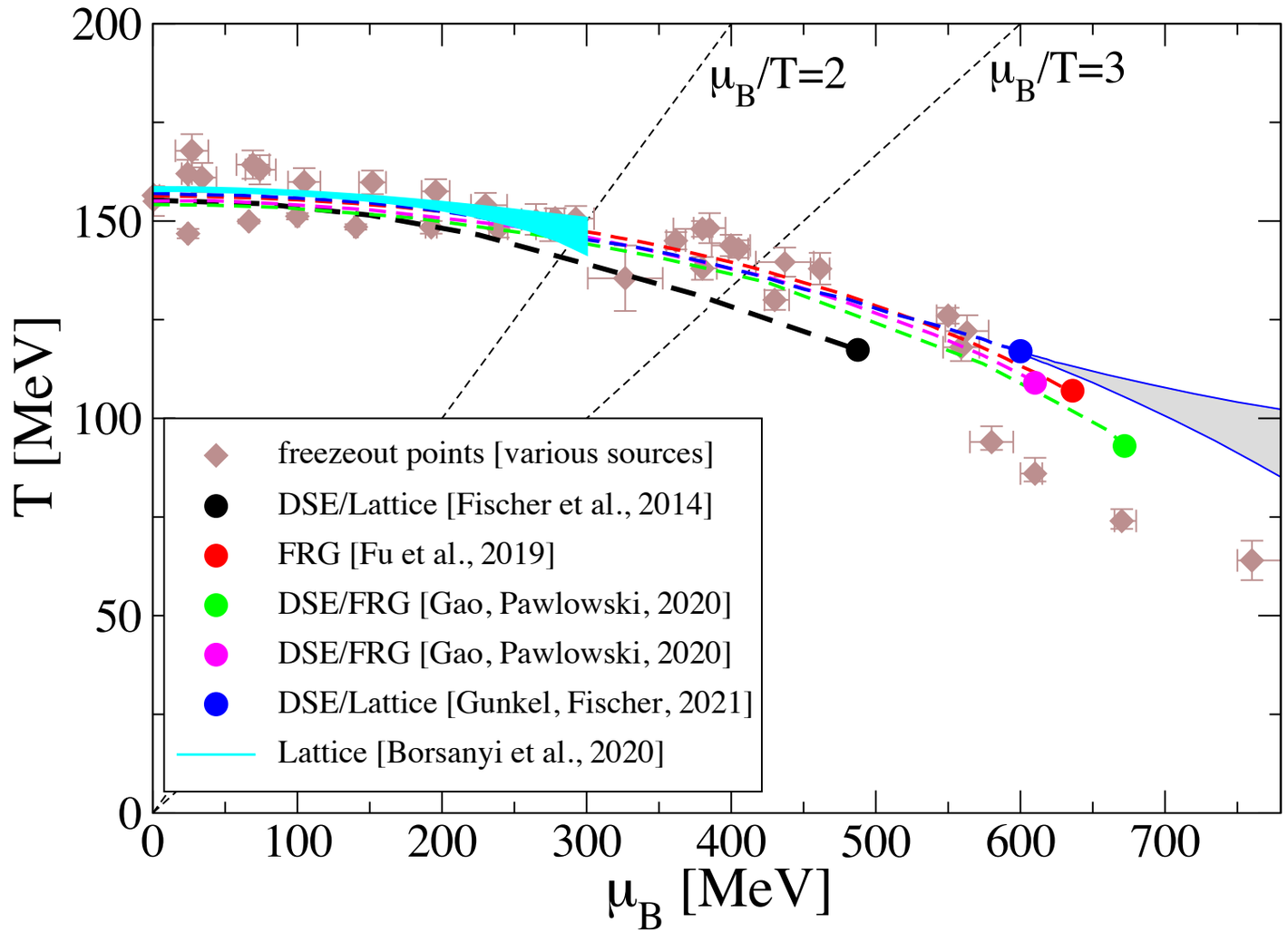


- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔ CF, Luecker, Welzbacher, PRD 90 (2014) 034022
  - baryon effects ✔ Eichmann, CF, Welzbacher, PRD93 (2016)

Location of CEP

Bernhardt, CF, in preparation

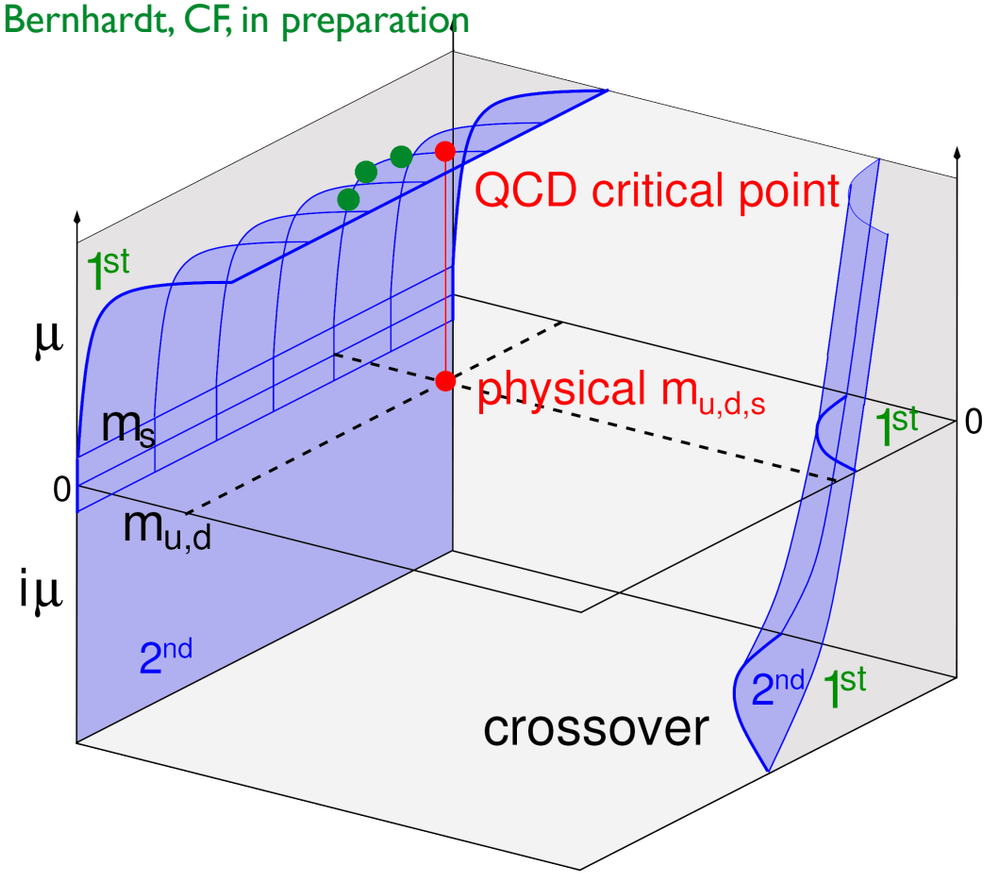


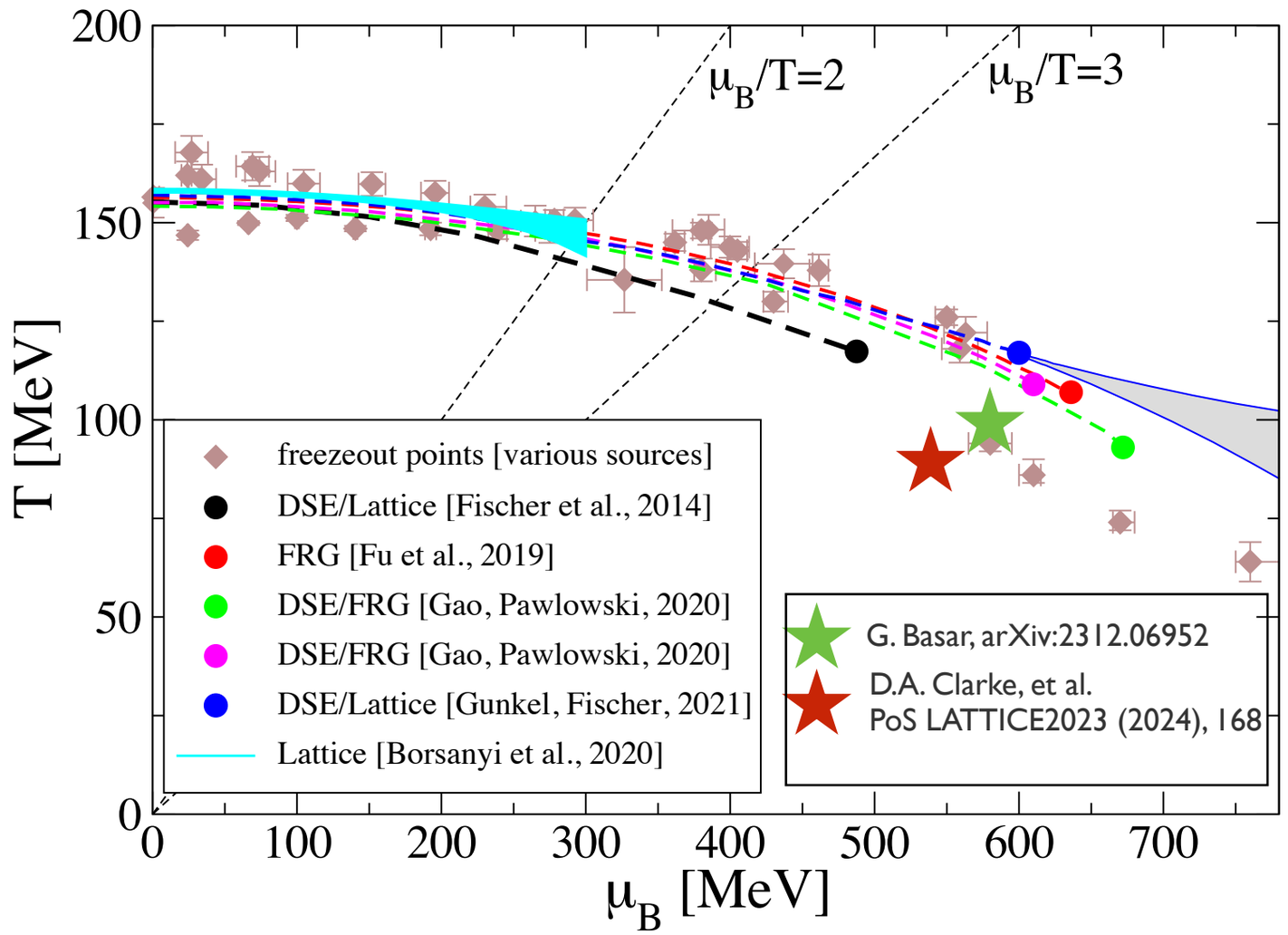


- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔ CF, Luecker, Welzbacher, PRD 90 (2014) 034022
  - baryon effects ✔ Eichmann, CF, Welzbacher, PRD93 (2016)
  - inhomogenuous phases T. F. Motta, J. Bernhardt, M. Buballa and CF, PRD 108 (2023)

Location of CEP

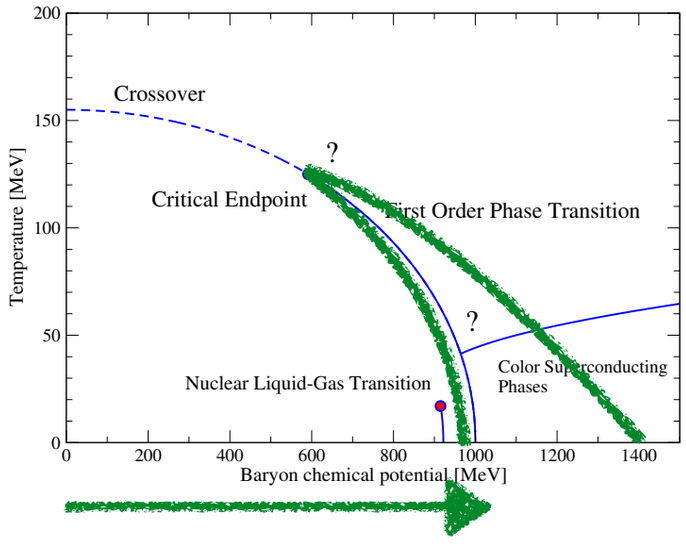
Bernhardt, CF, in preparation

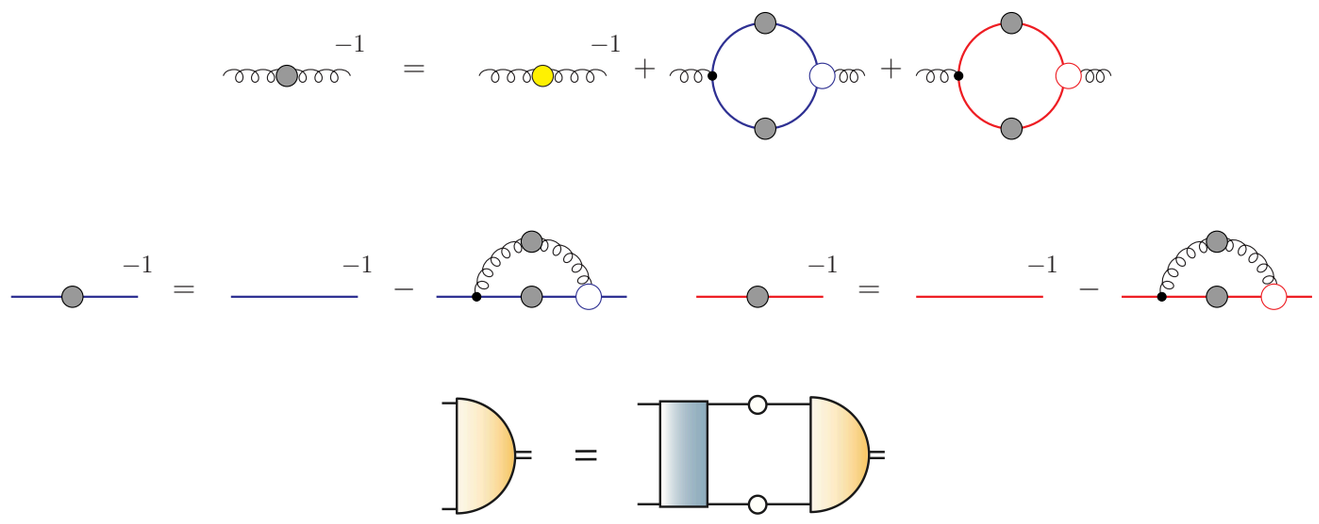




- how stable is this result ??
  - crosscheck DSE-FRG ✔
  - N_f=2+1+1 ✔ CF, Luecker, Welzbacher, PRD 90 (2014) 034022
  - baryon effects ✔ Eichmann, CF, Welzbacher, PRD93 (2016)
  - inhomogenuous phases T. F. Motta, J. Bernhardt, M. Buballa and CF, PRD 108 (2023)
  - cross-check with lattice ✔

Meson properties at finite chemical potential

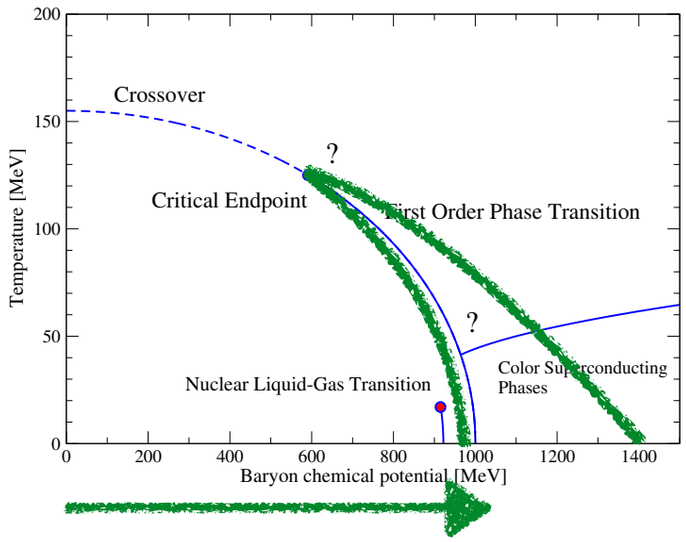


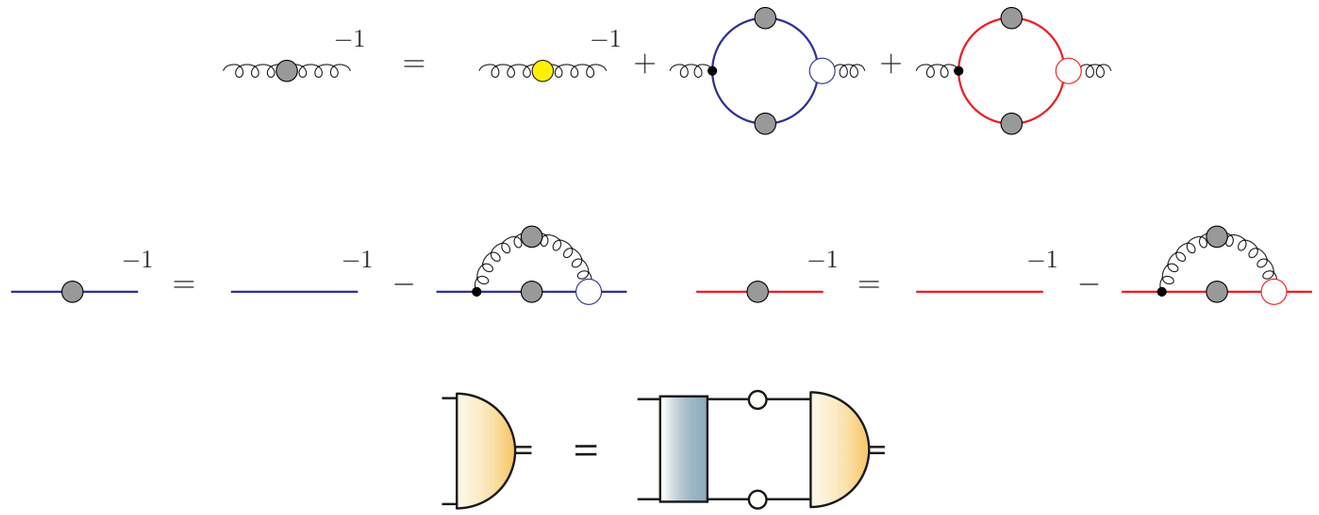


- Quarks/meson wave functions do change !

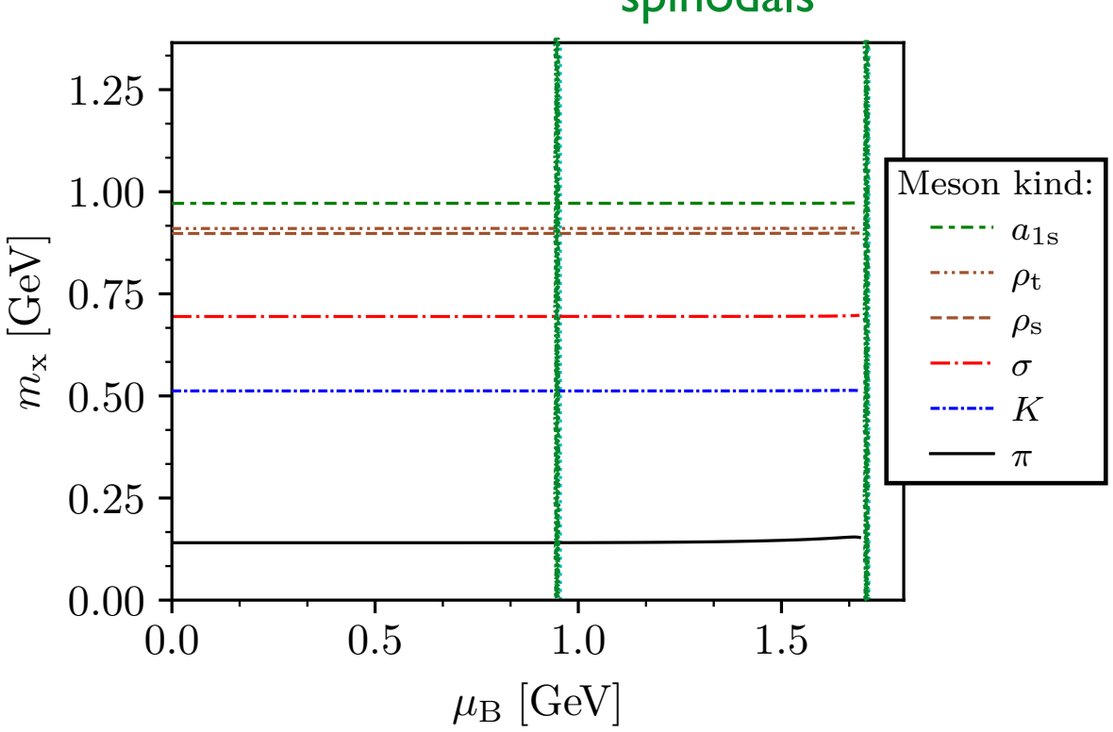
Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

Meson properties at finite chemical potential

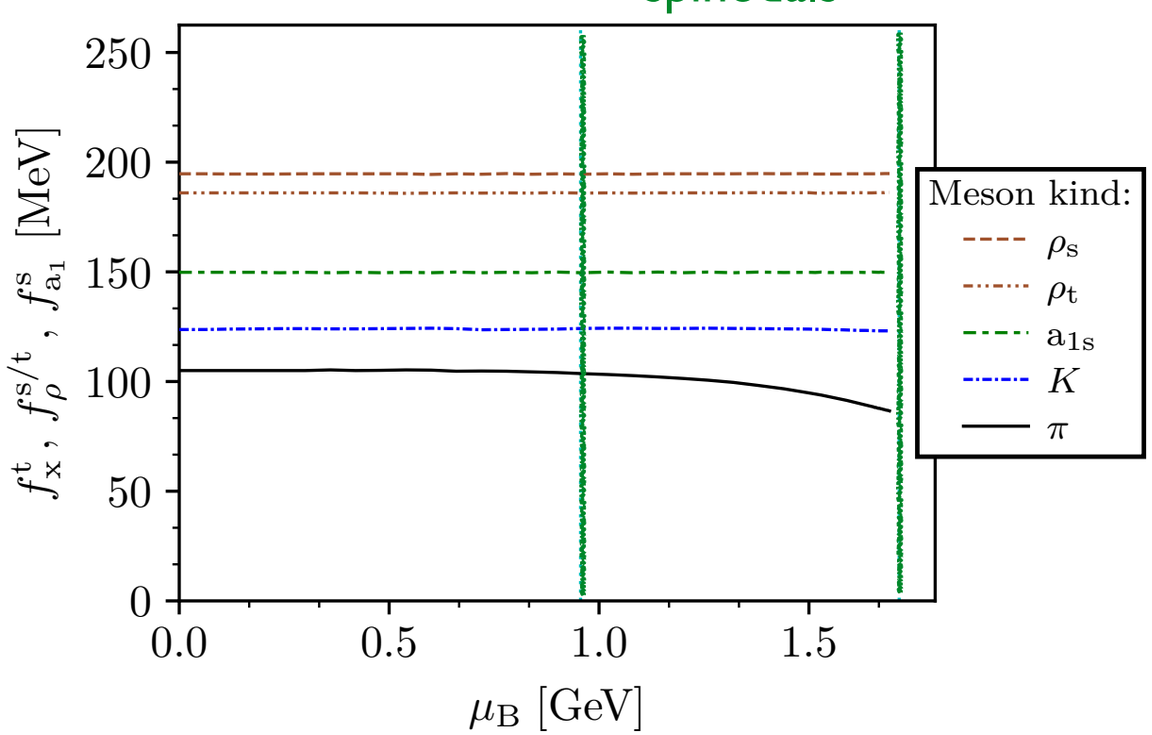




spinodals



spinodals



- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

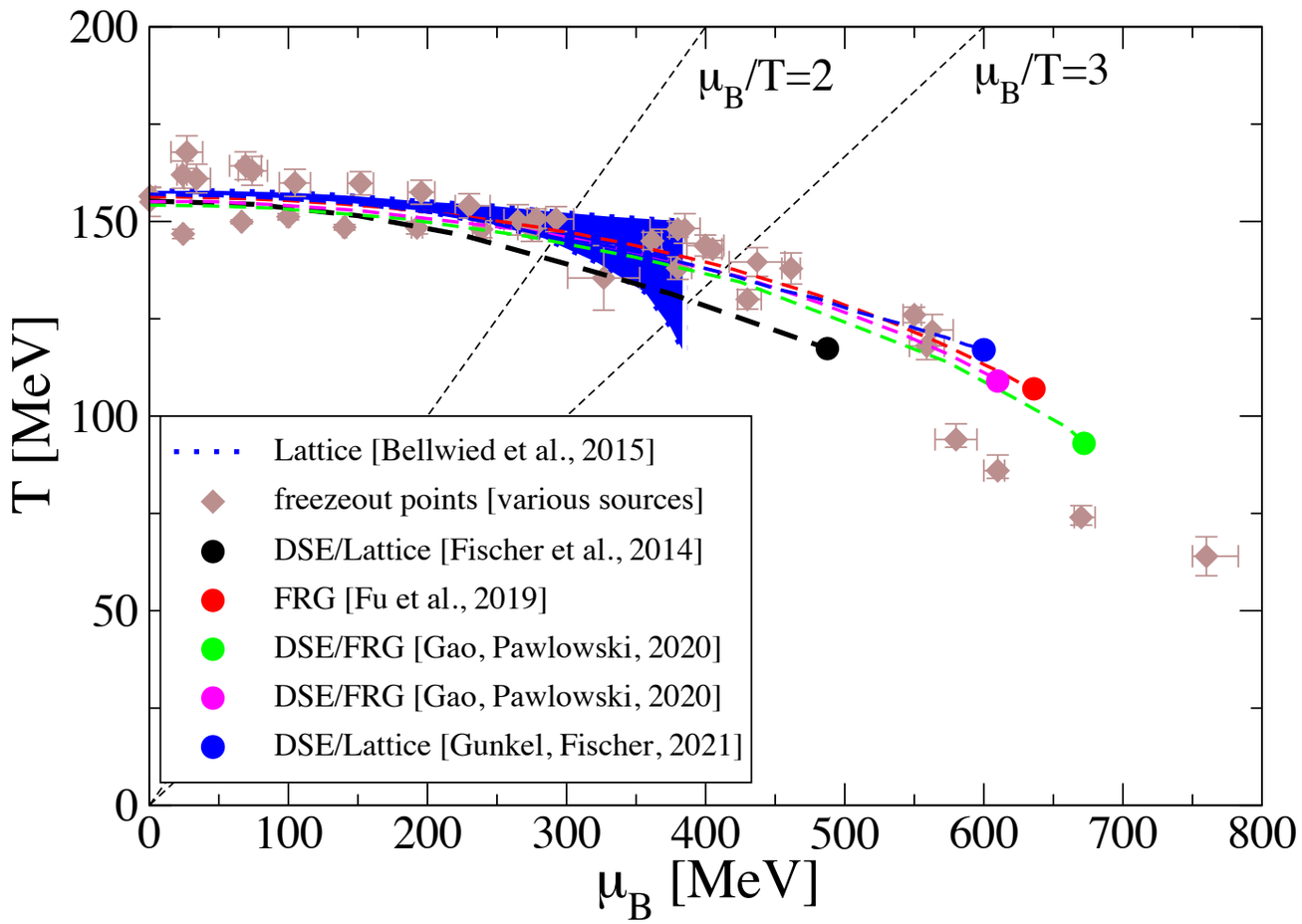
Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147
T. D. Cohen, PRL 91 , 222001 (2003)

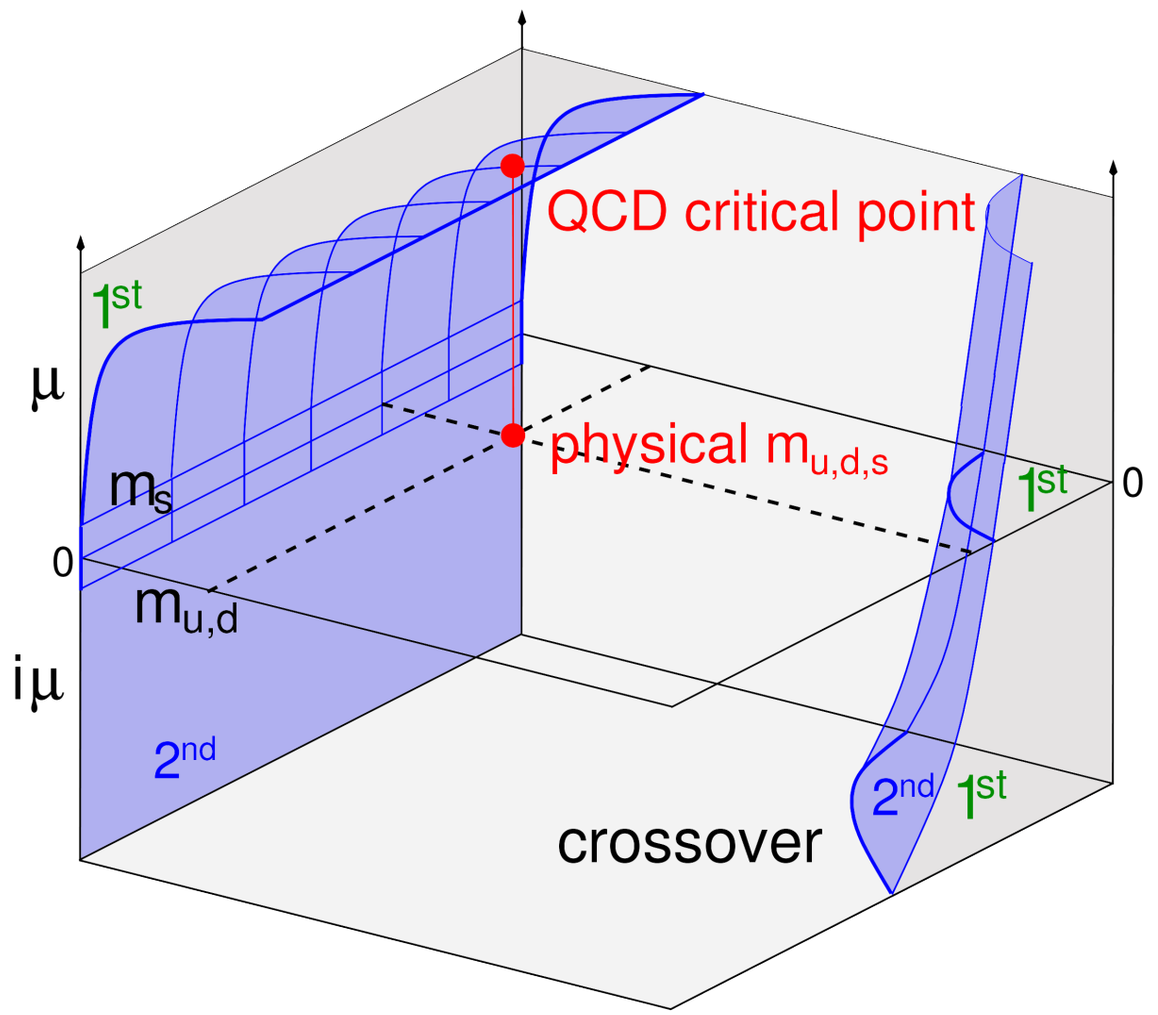
Summary: QCD with functional methods

Main goals:

- **one** framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

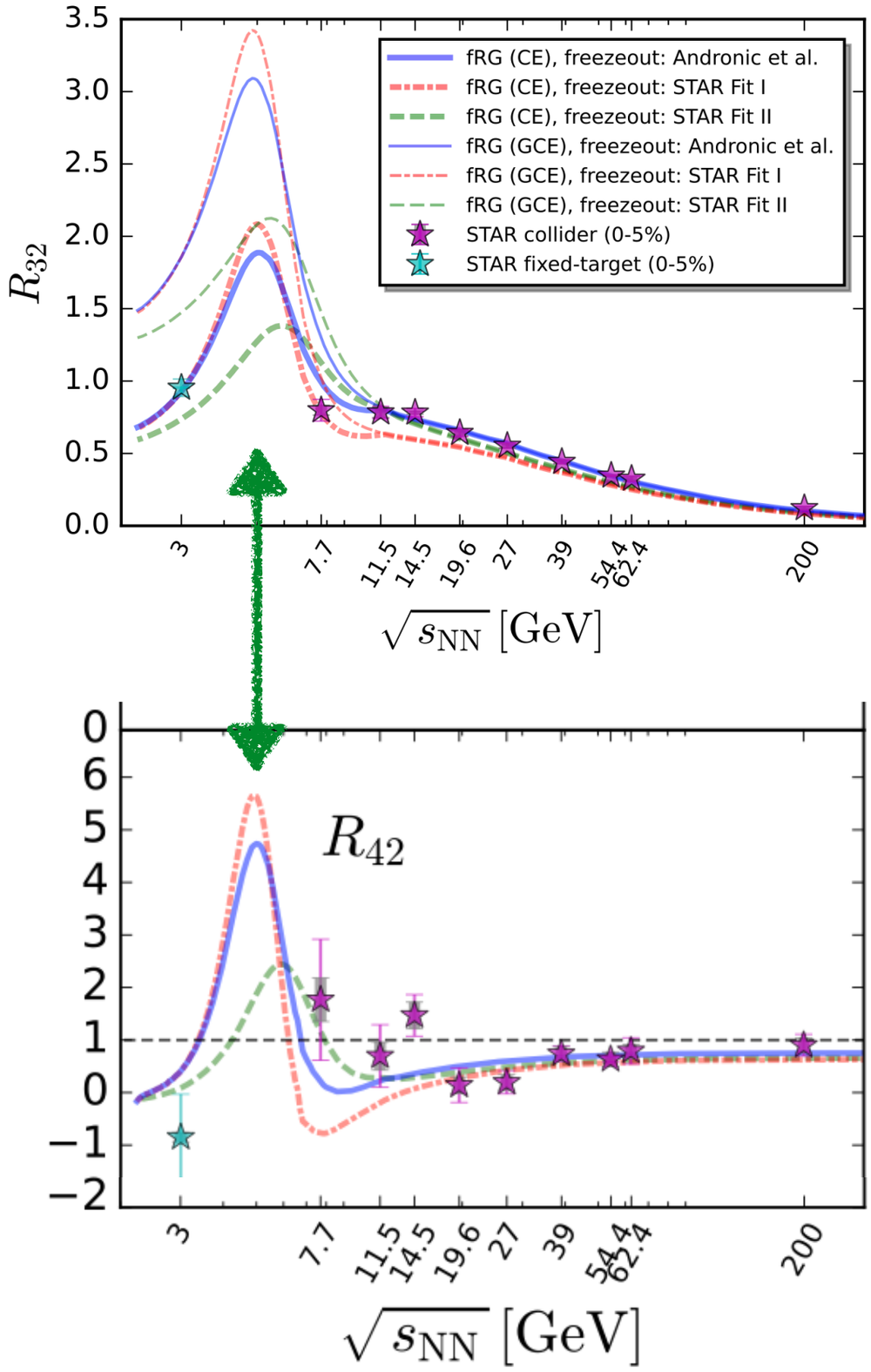
Main results:

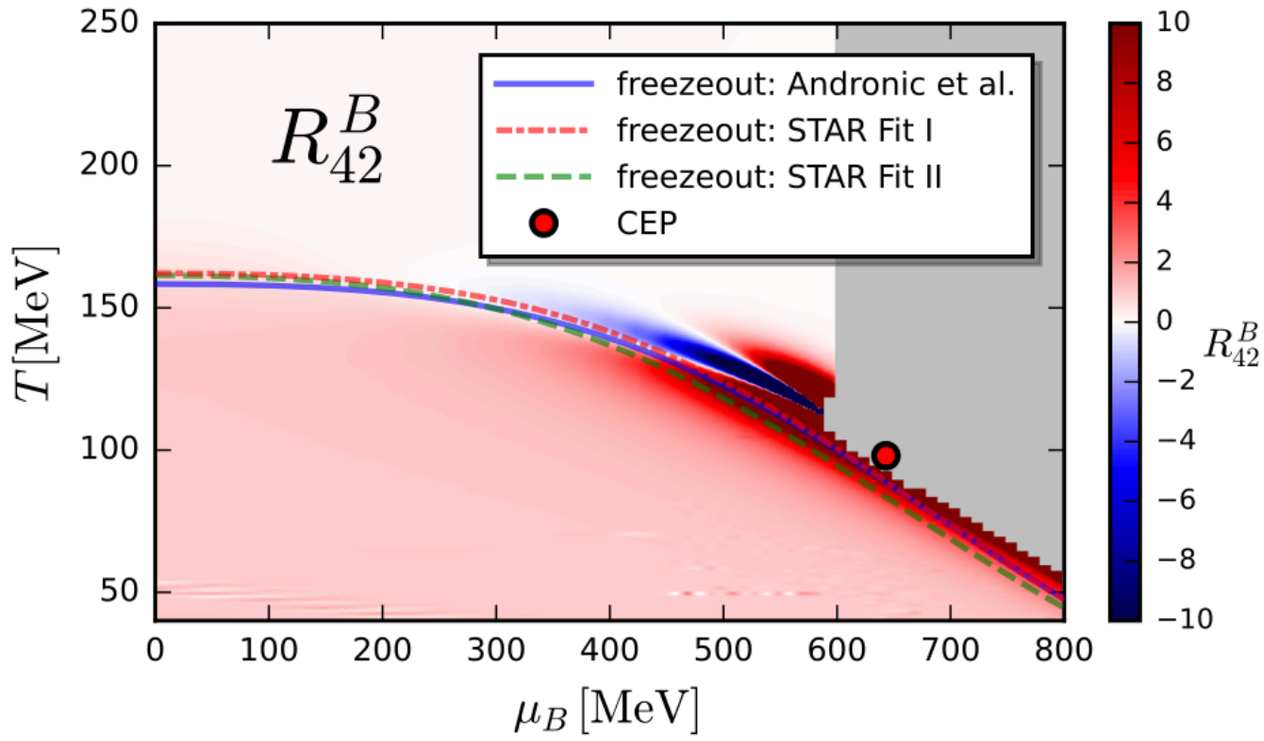




Backup

Results for fluctuations: FRG of effective theory





- location of peak: sensitive to details of freeze-out
- height of peak: sensitive to location of CEP!

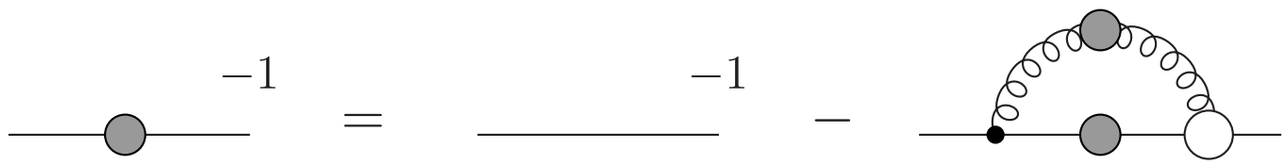
Fu, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Order parameters from propagators (DSE+FRG)

Chiral order parameter:

$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_\omega \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$

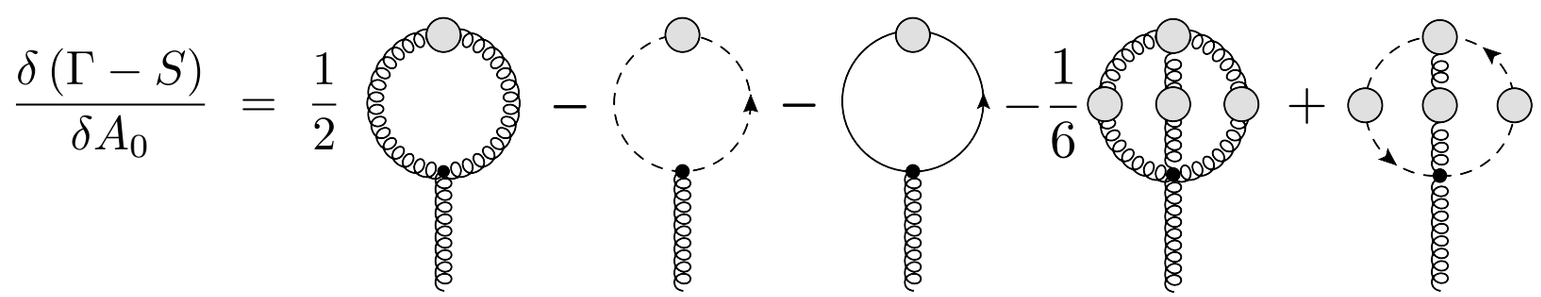
spatially homogenuous



$$S^{-1}(\omega_p,\vec{p}) = i\vec{\not{p}}\, A(\omega_p,\vec{p}) + i\gamma_4\omega_p C(\omega_p,\vec{p}) + B(\omega_p,\vec{p})$$

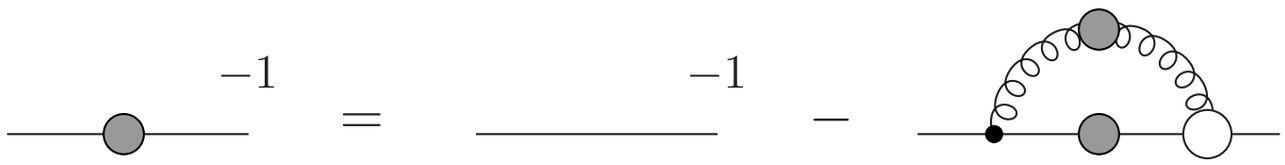
Deconfinement:

- Polyakov loop potential $\quad L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)

Modeling and eff. theories: not adequate…

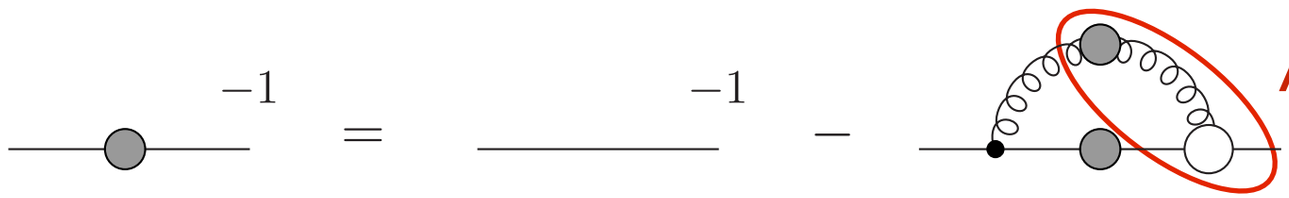


Models/approximations:

I) NJL model (DSE,FRG):

II) PQM model (FRG):

III) Rainbow-ladder (DSE):



→ valuable for exploratory studies

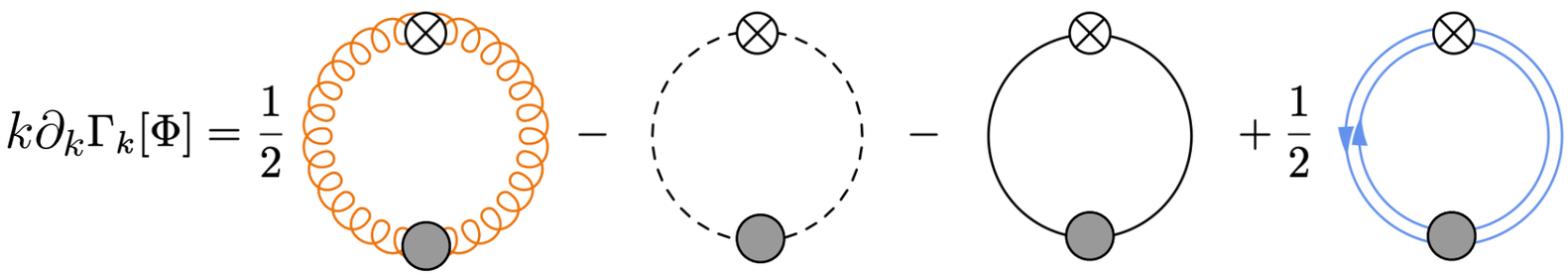
→ not good enough for quantitative and/or systematic studies at finite T, μ

necessary: solve tower of FRGs/DSEs

CF, PPNP 105 (2019) [1810.12938]

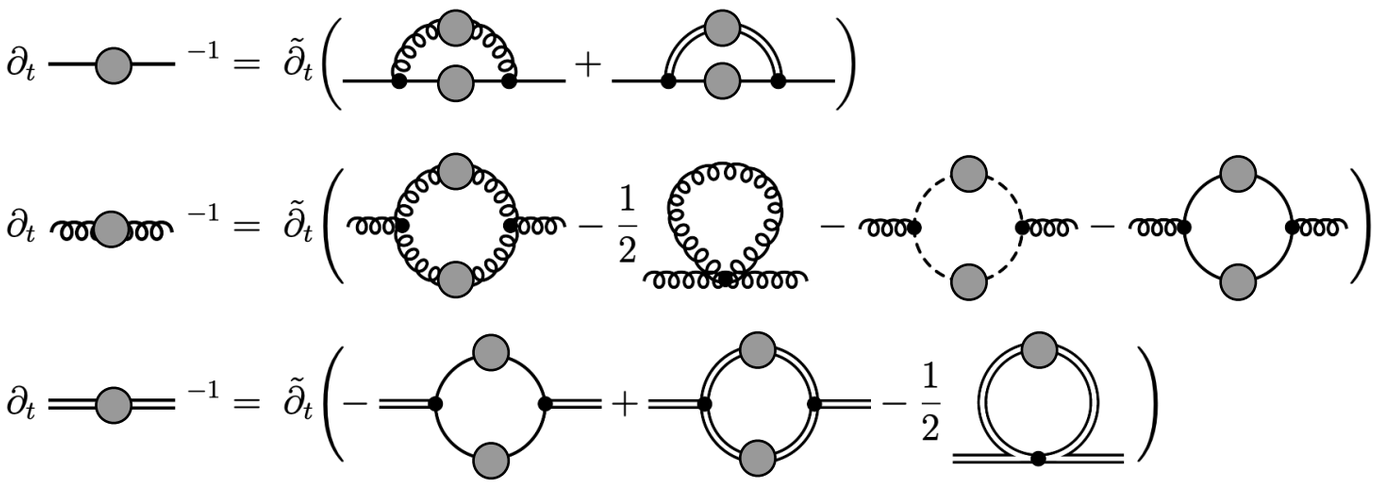
QCD with FRG

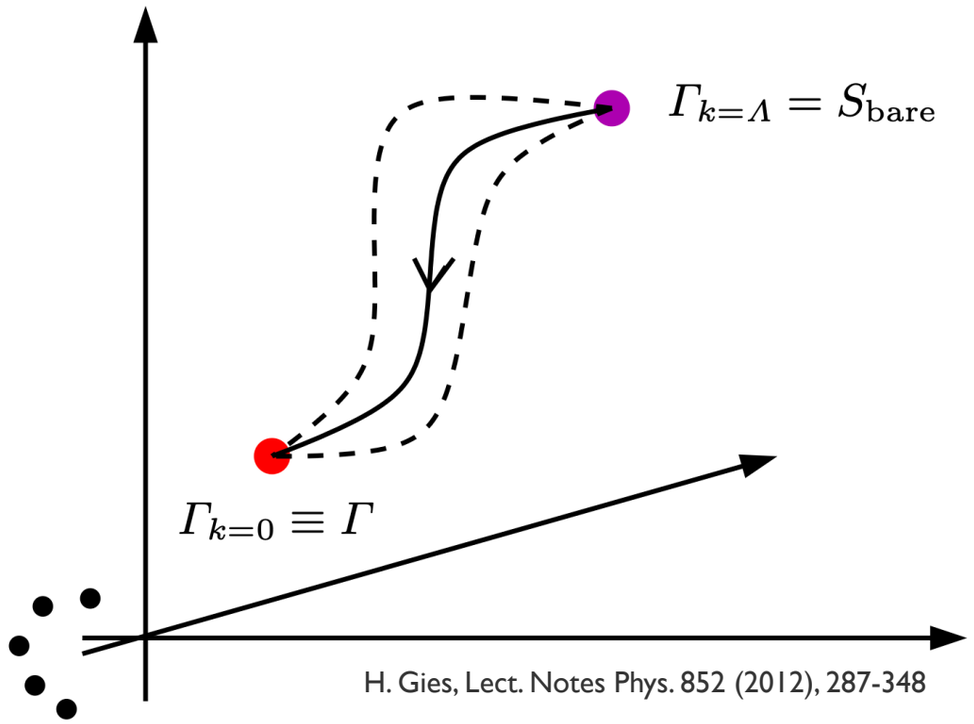
1PI effective action Γ



k: regulator scale

field derivatives

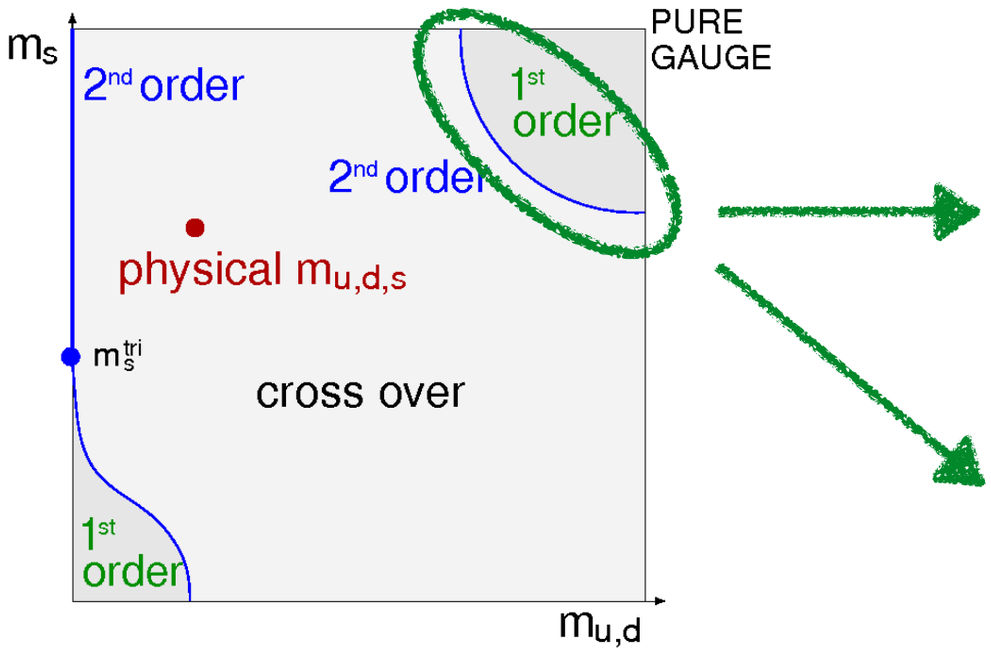




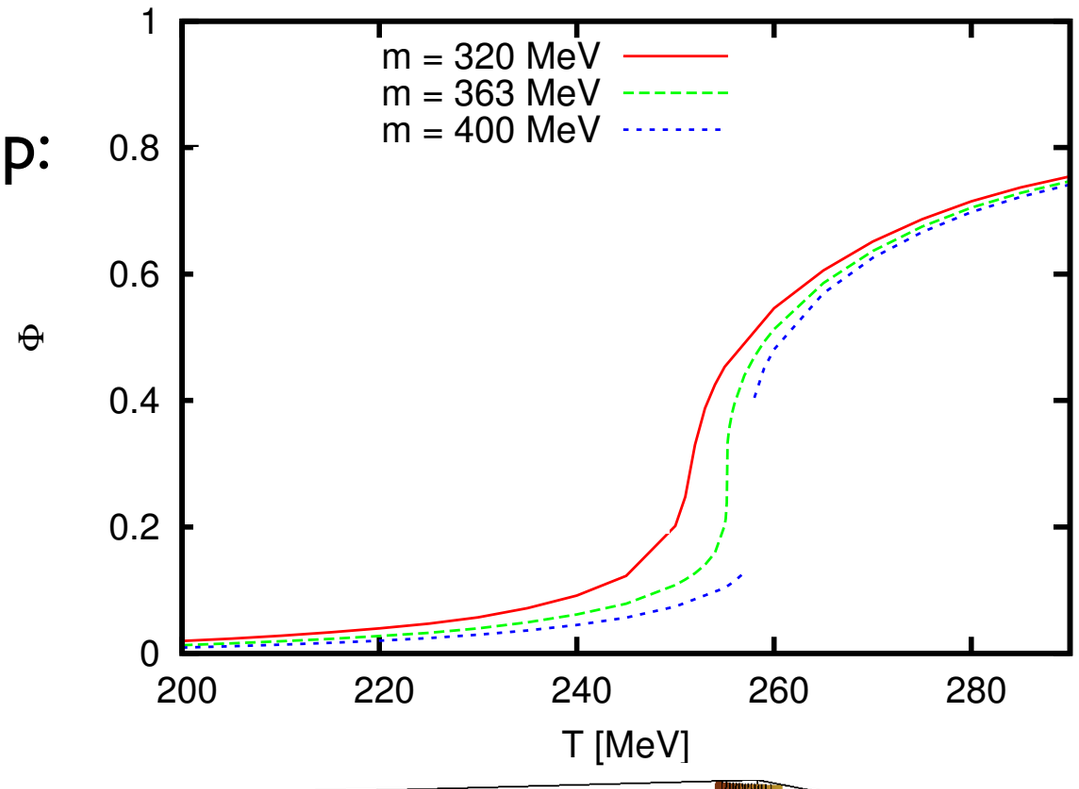
H. Gies, Lect. Notes Phys. 852 (2012), 287-348

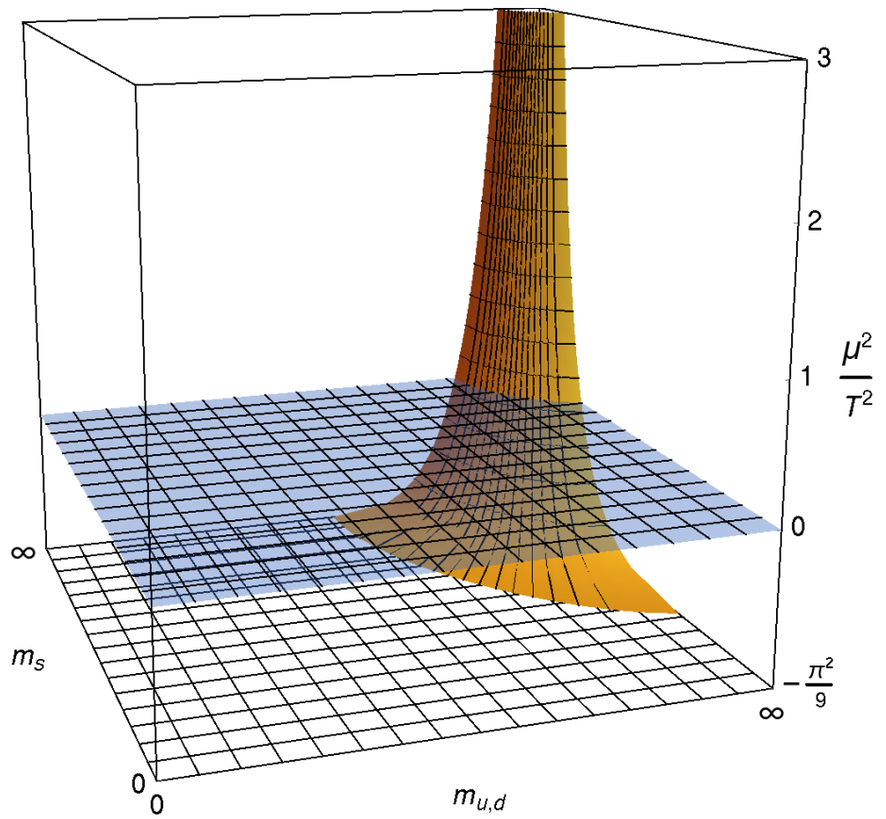
Fu, Rennecke, Pawlowski, PRD 101 (2020)

Critical line/surface for heavy quarks



Polyakov Loop:



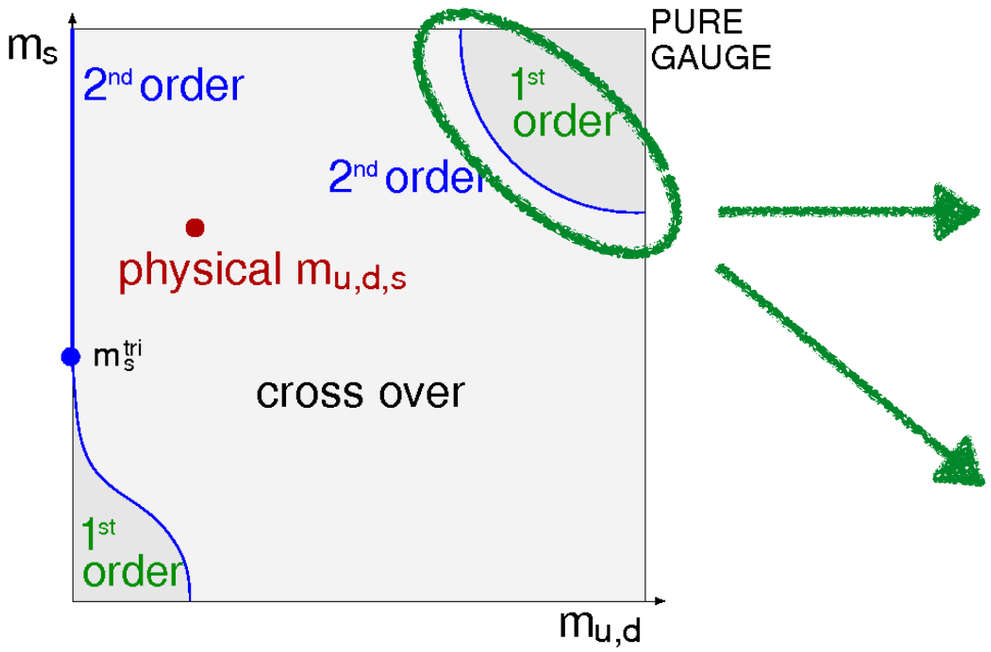


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

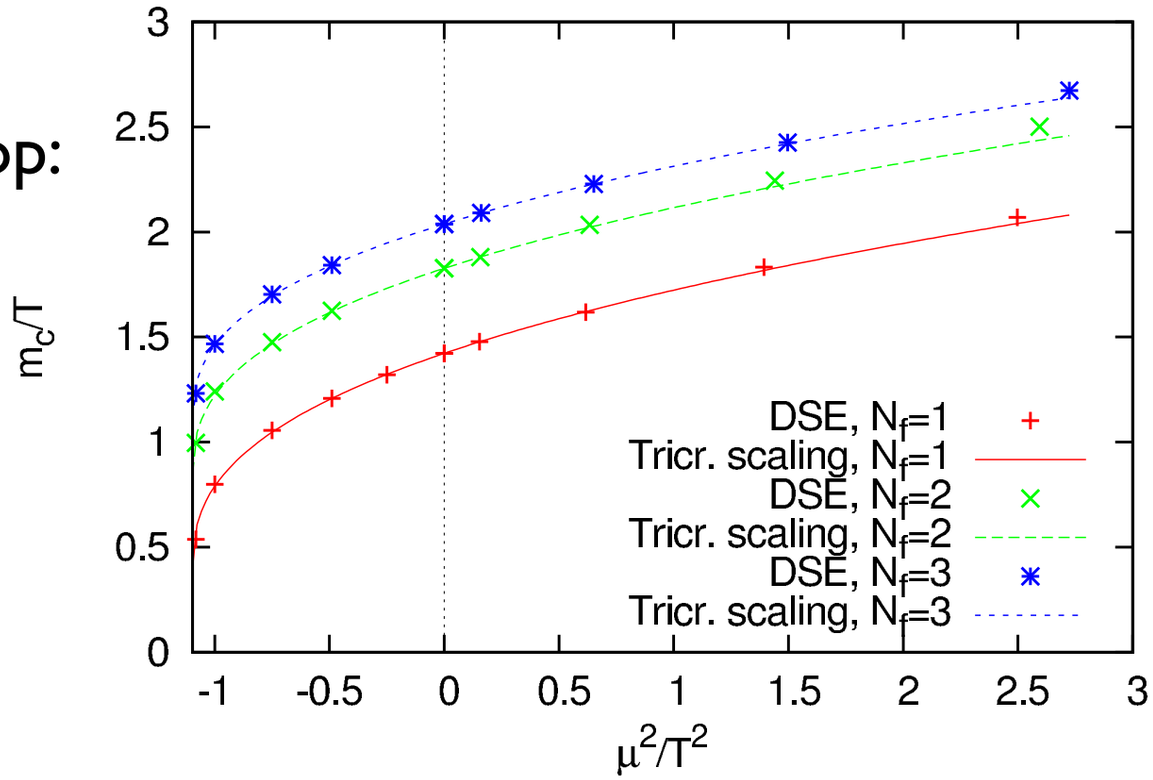
Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

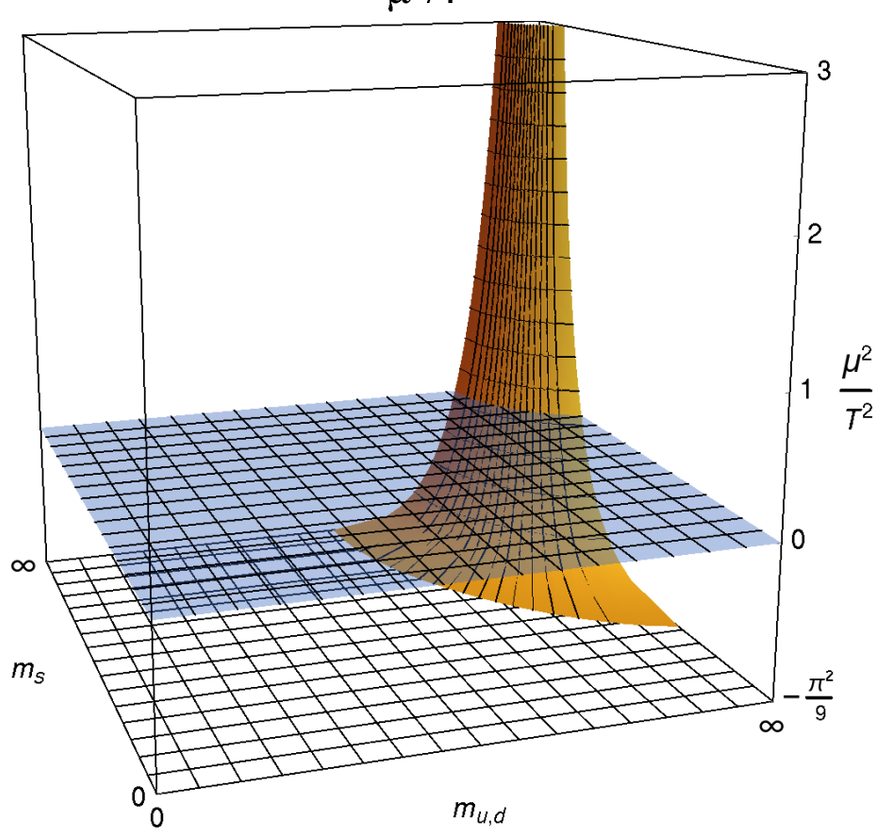
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Critical line/surface for heavy quarks



Polyakov Loop:





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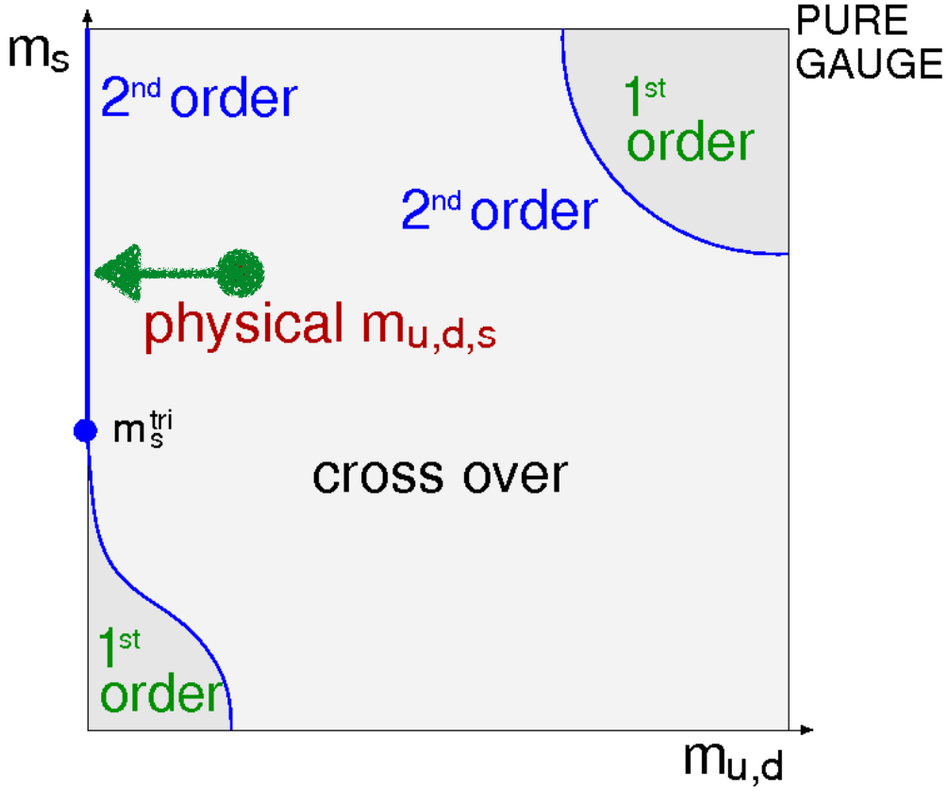
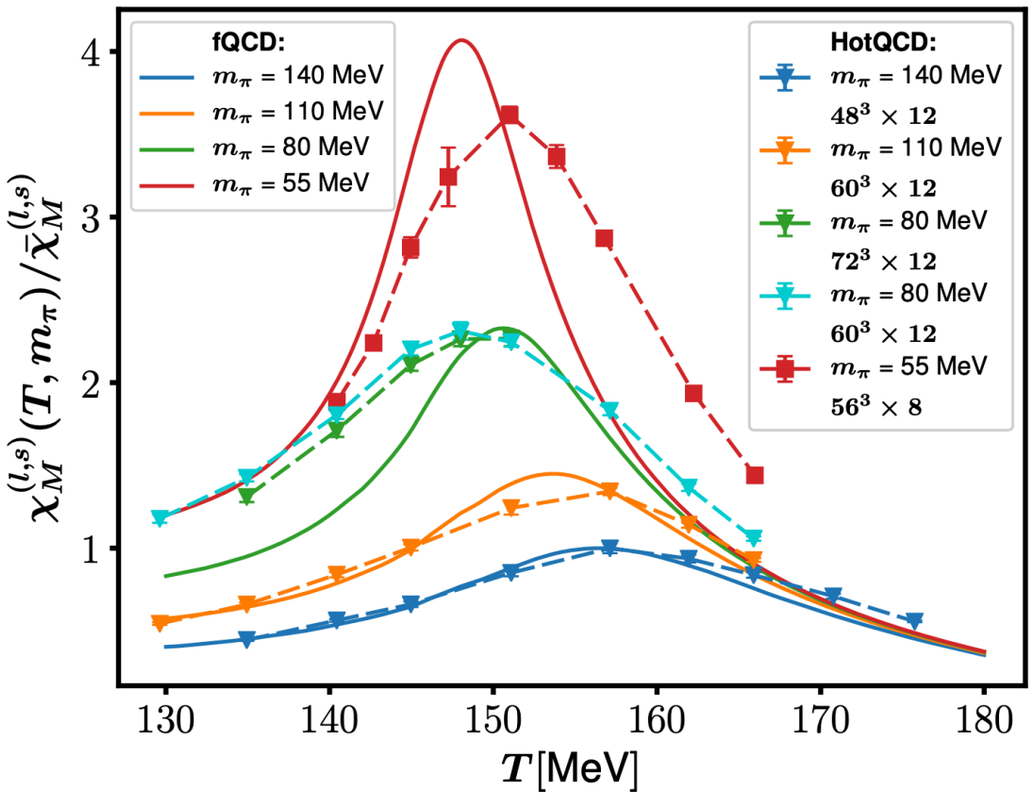
CF, Luecker, Pawlowski, PRD 91 (2015) 1

Towards the chiral limit… (FRG+DSE)

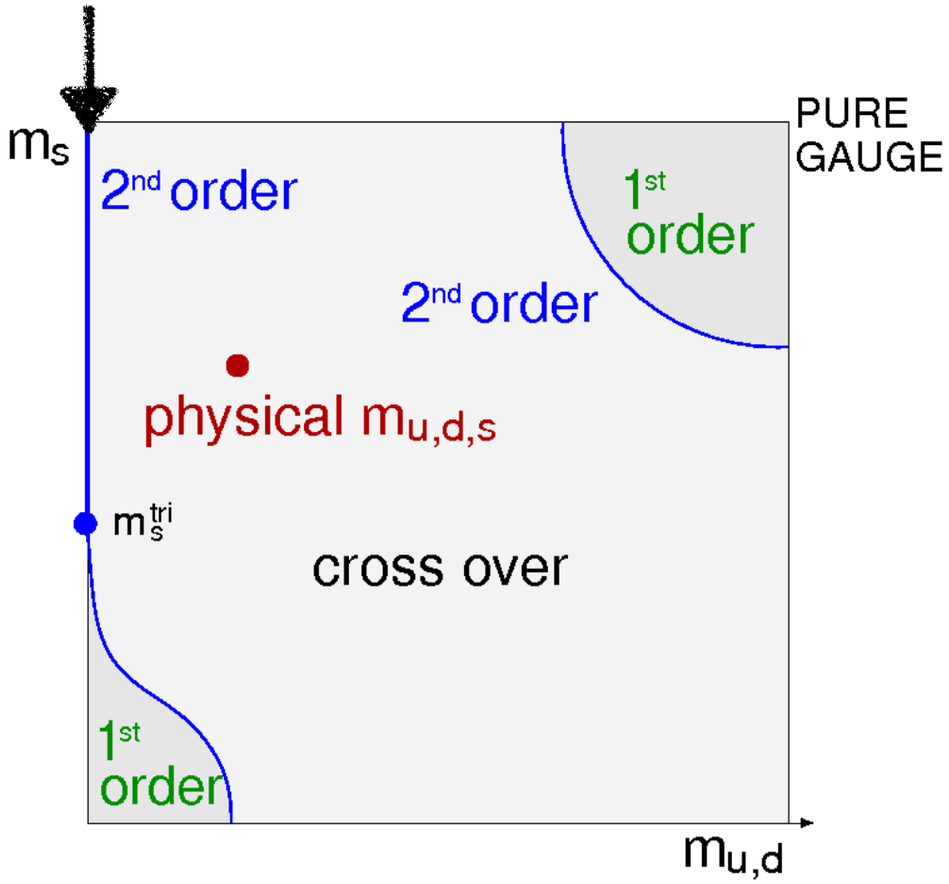


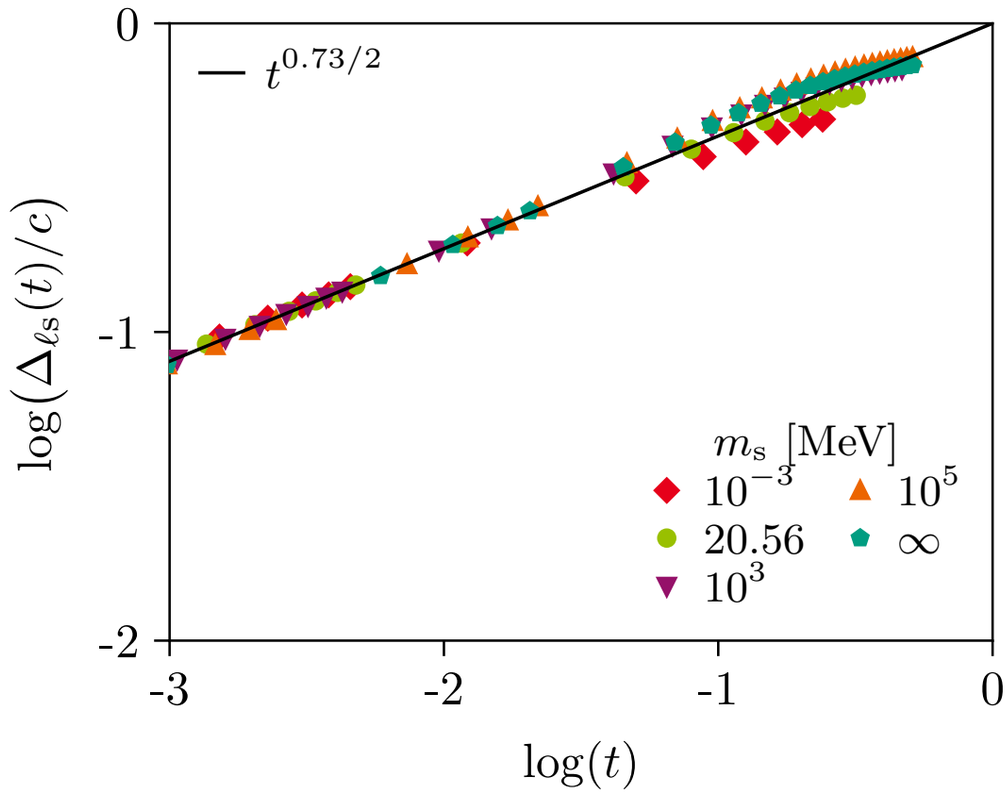


HotQCD: Ding et al. PRL 123, 062002 (2019)
FRG: Braun et al, PRD 102 (2020) 5, 056010
FRG/DSE: Gao and Pawlowski PRD 105(2022) 094020
DSE: Bernhardt and CF, PRD 108 (2023) 114018

m_π [MeV]		0	55	80	110	140
T_c [MeV]	DSE	146.7	149.9	151.6	154.0	156.7
	FRG [63]	142	148.0	150.5	153.6	156.3
	FRG–DSE [96]	141.3	146.5	149.1	152.1	155.4
	HotQCD ($N_\tau = 12$) [95]	-	-	$149.7^{+0.3}_{-0.3}$	$155.6^{+0.6}_{-0.6}$	$158.2^{+0.5}_{-0.5}$
	HotQCD ($N_\tau = 8$) [95]	-	$150.9^{+0.4}_{-0.4}$	$153.9^{+0.3}_{-0.3}$	$157.9^{+0.3}_{-0.3}$	$161.0^{+0.1}_{-0.1}$

At the chiral limit…





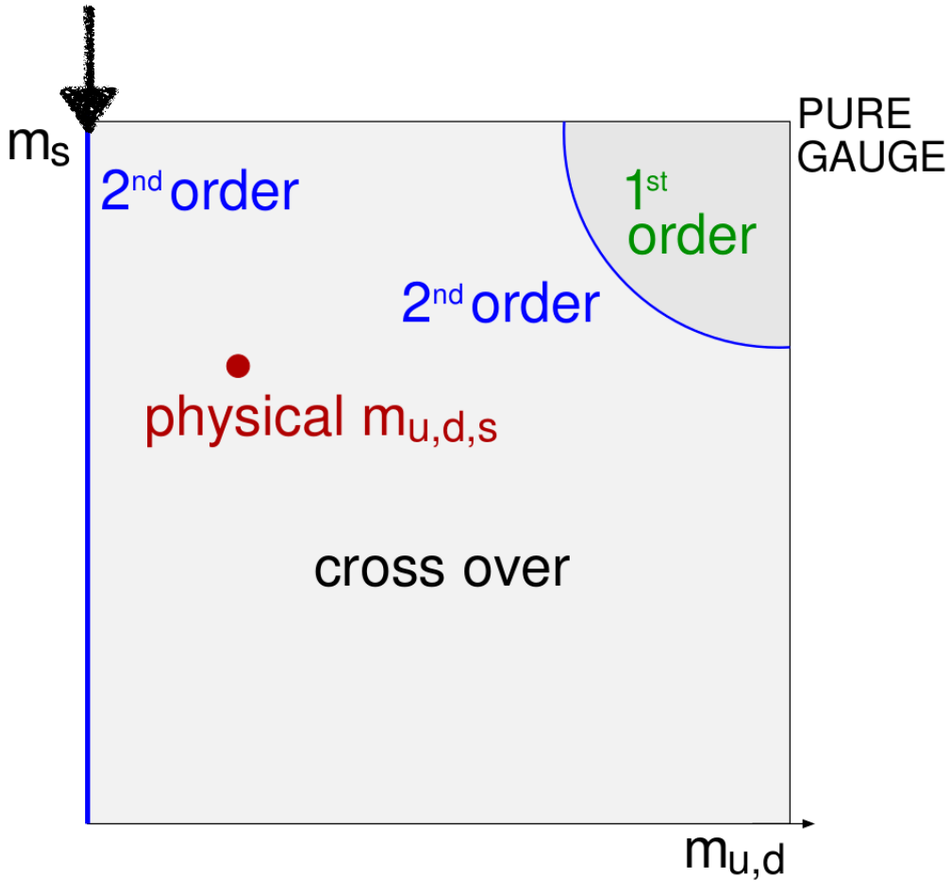
Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141

Bernhardt and CF, PRD 108 (2023) no.11, 114018

DSE: Bernhardt and CF, PRD 108 (2023) 114018

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

At the chiral limit…

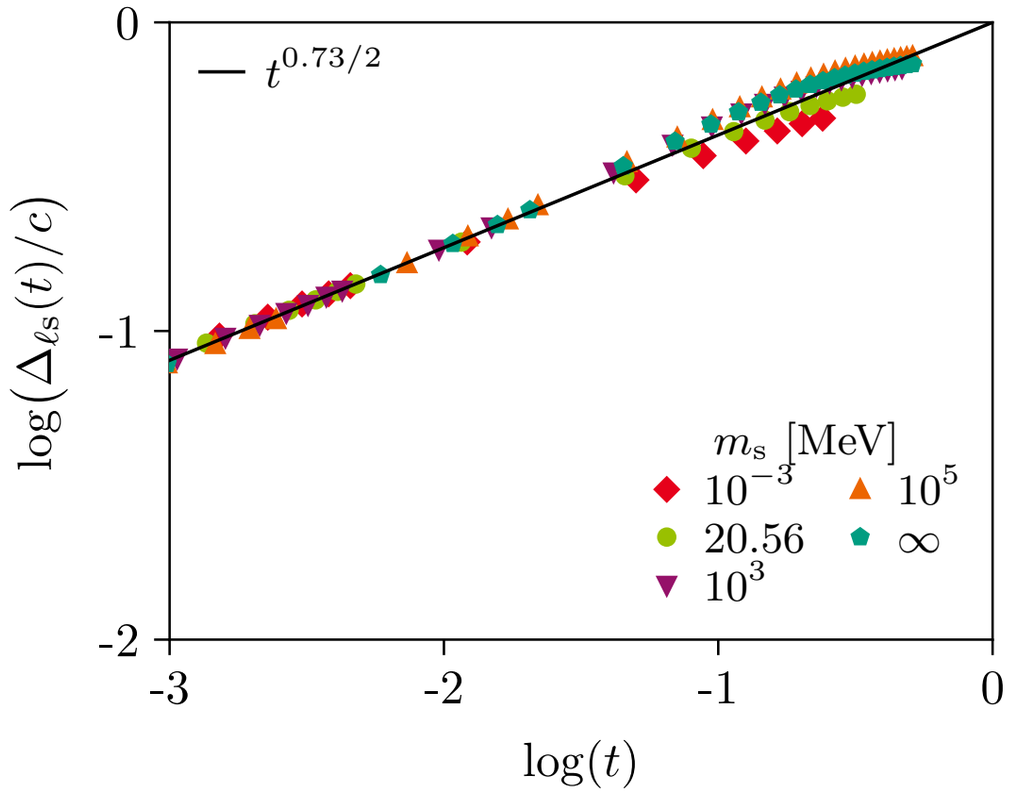


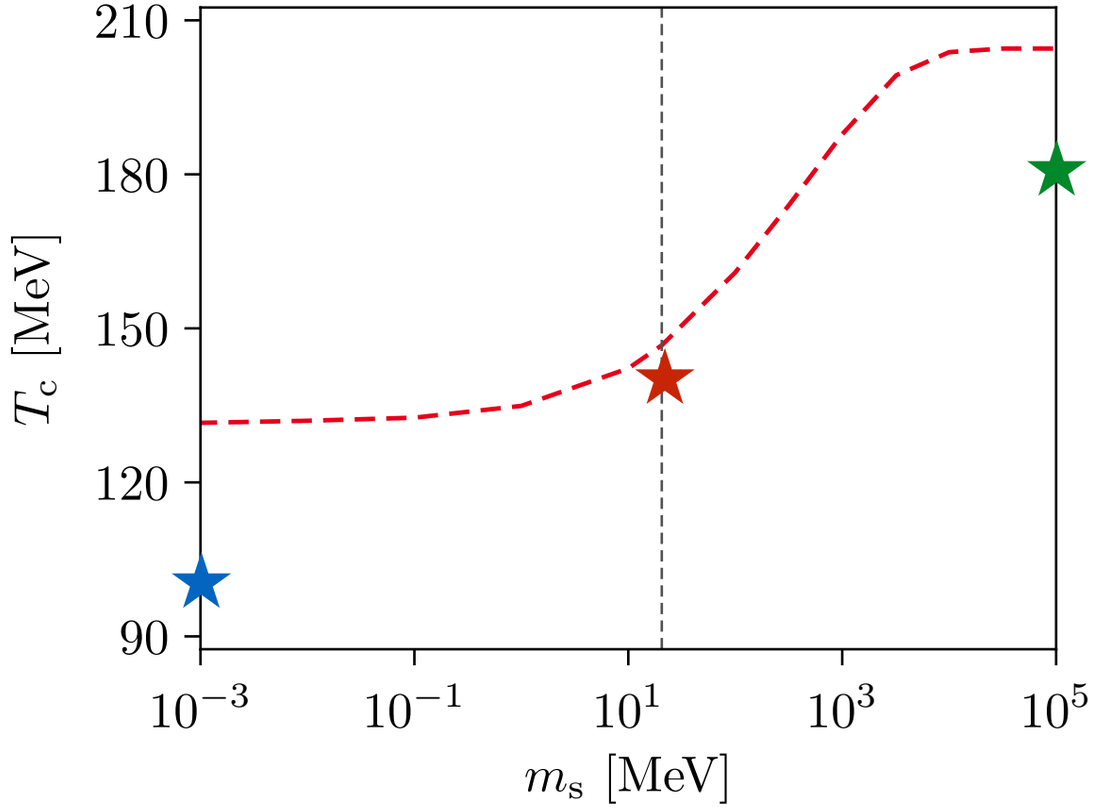
Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141

Bernhardt and CF, PRD 108 (2023) no.11, 114018

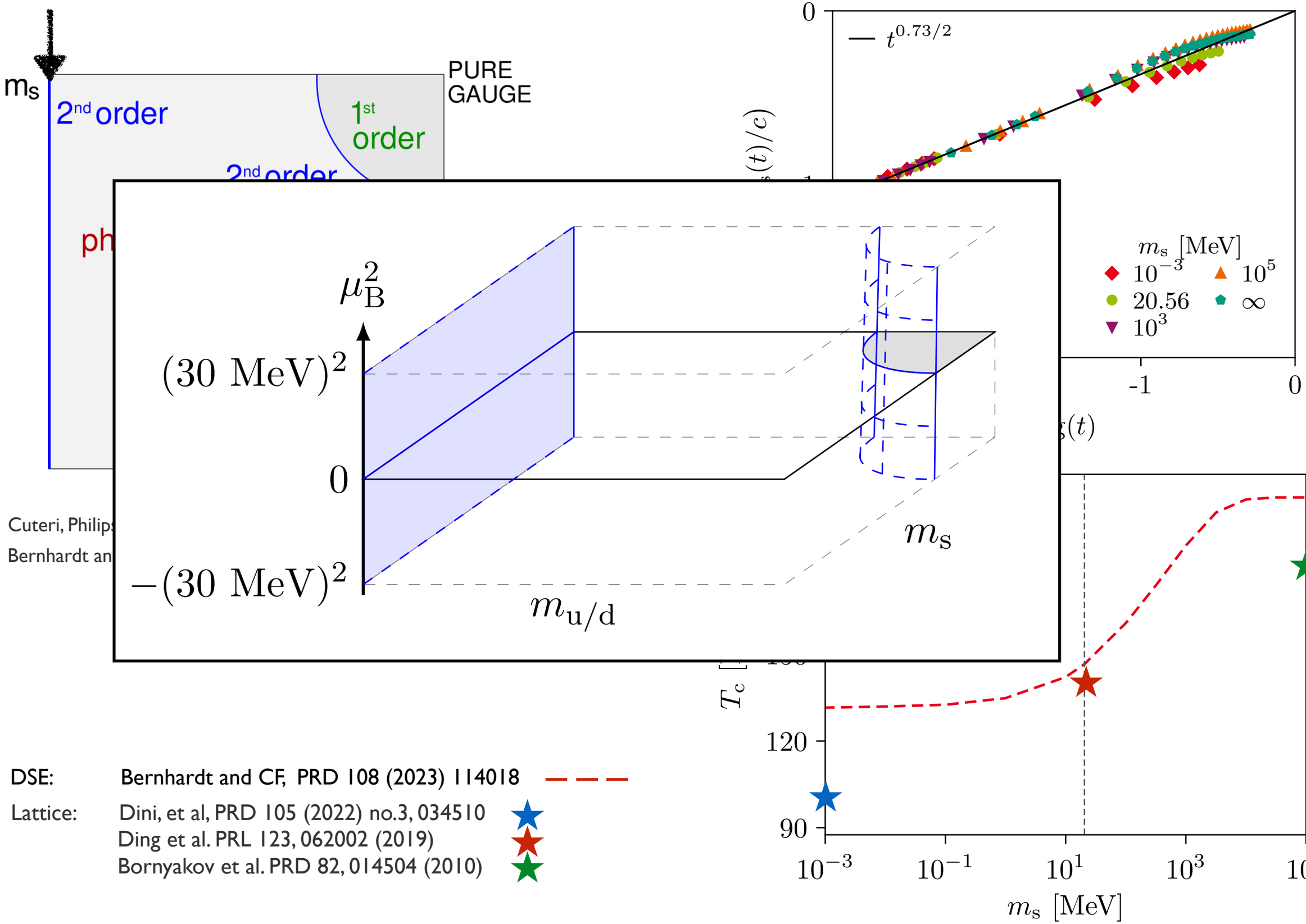
DSE: Bernhardt and CF, PRD 108 (2023) 114018

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)





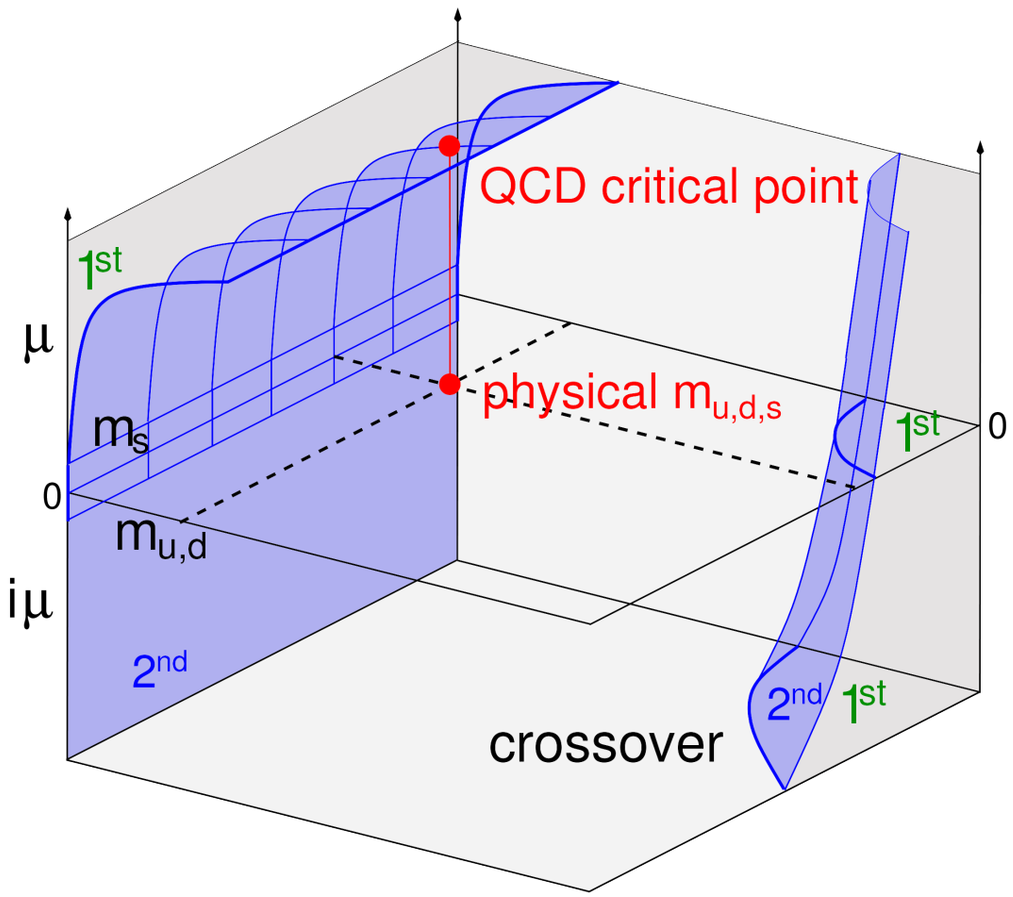
At the chiral limit…

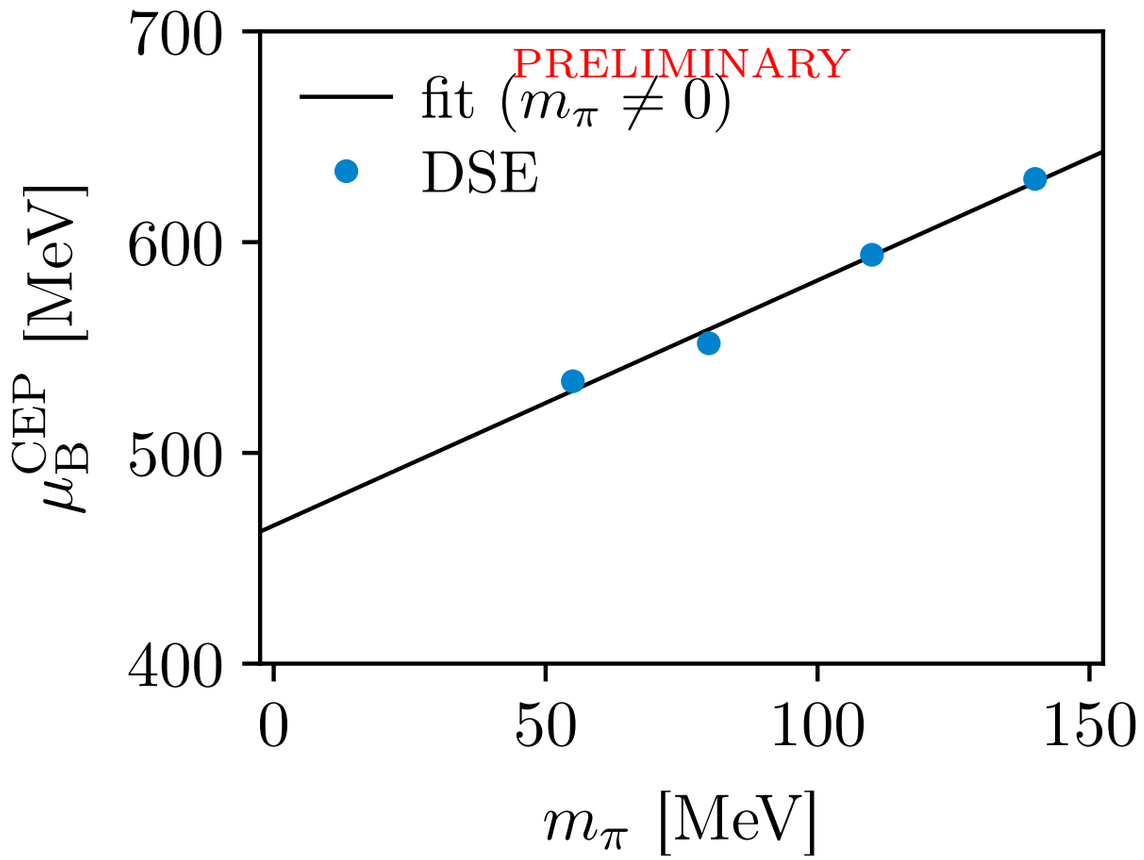


DSE: Bernhardt and CF, PRD 108 (2023) 114018

Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

Evolution of CEP with $m_{u/d}$



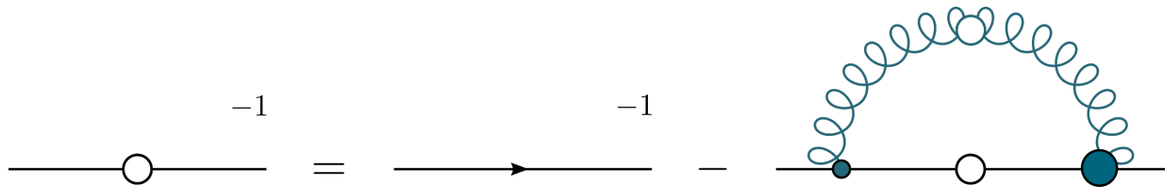


- 'large' pion masses: linear
- close to chiral limit: non-linear…

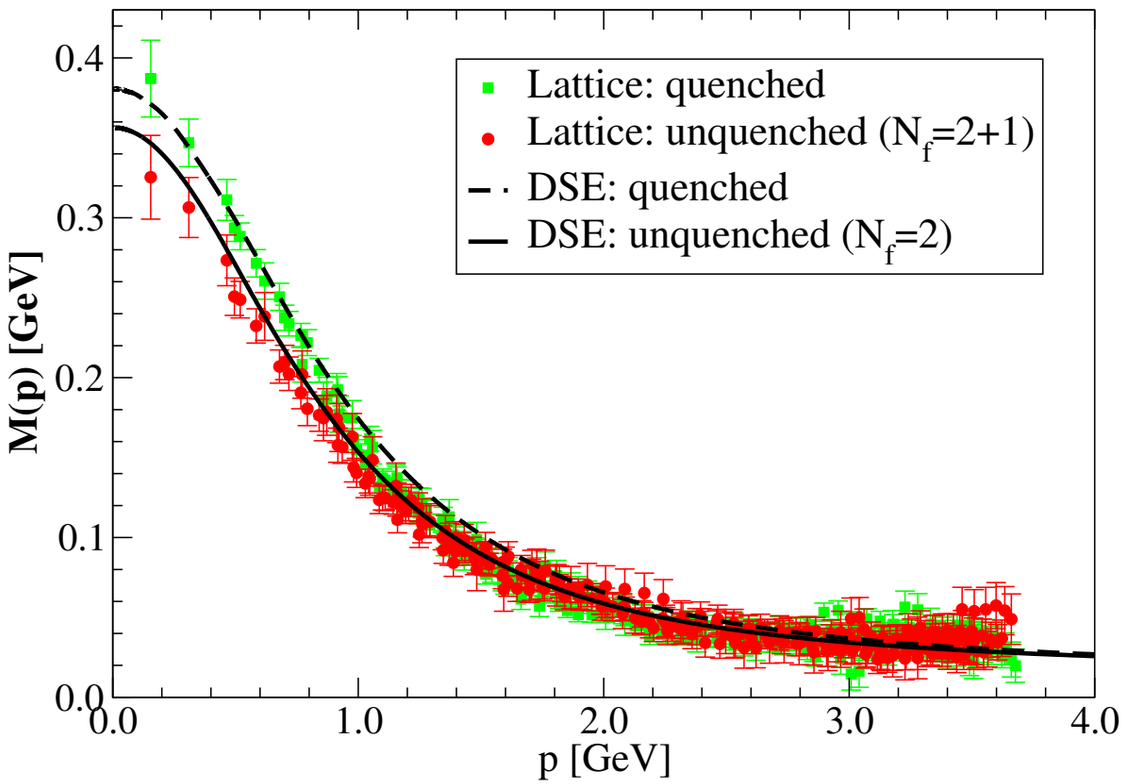
Bernhardt, CF, in preparation

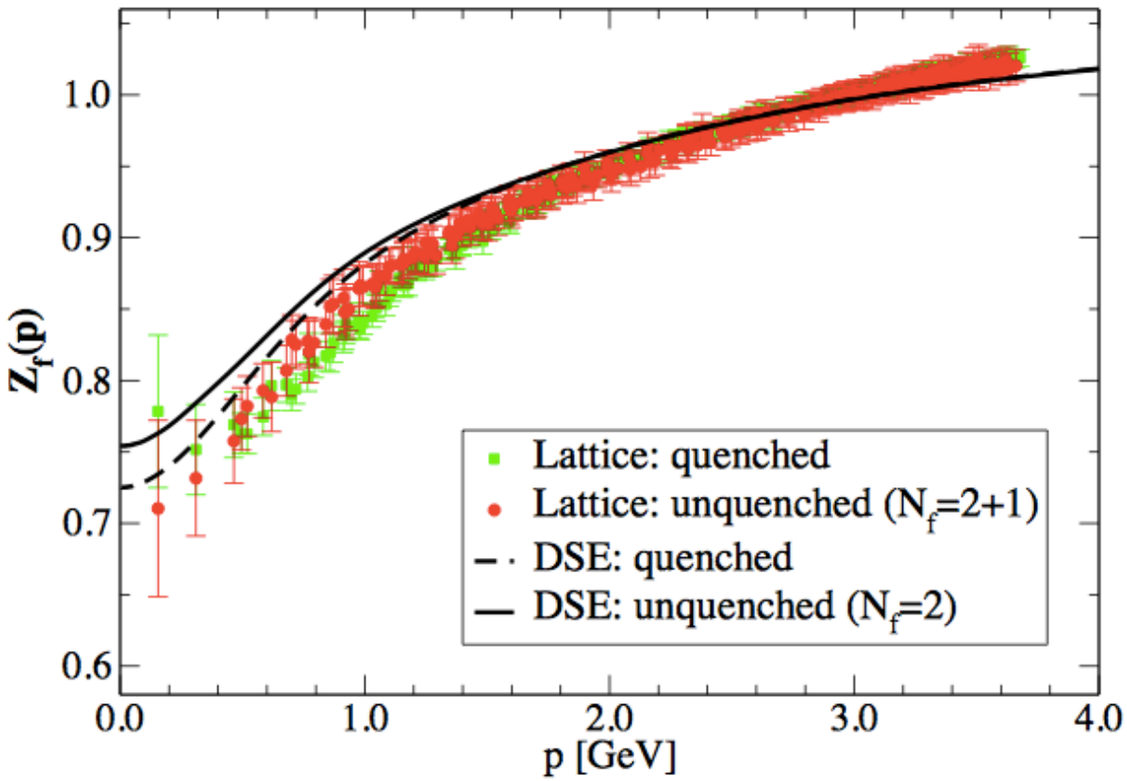
Dynamical mass generation (T=0)

$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





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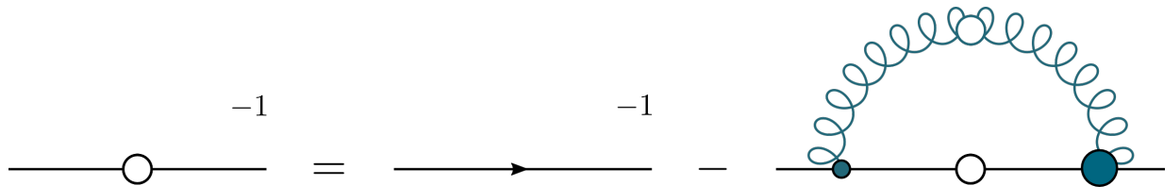

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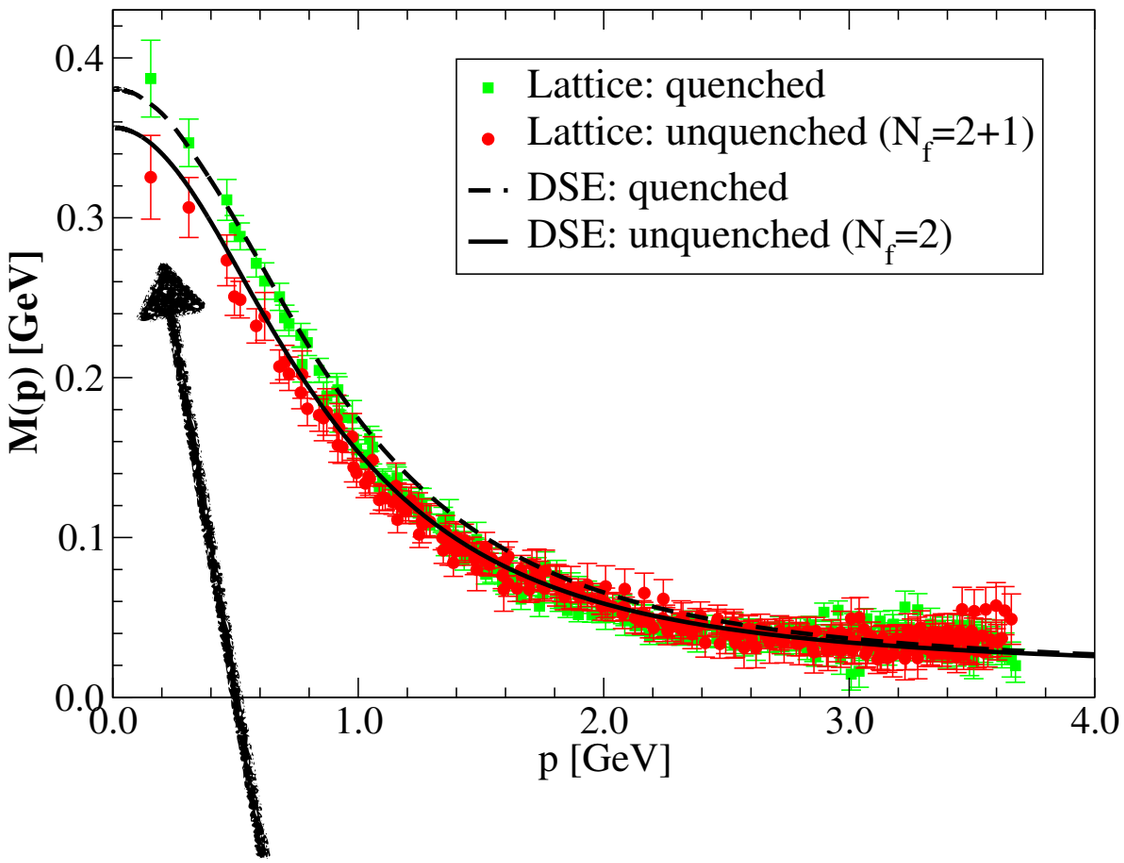
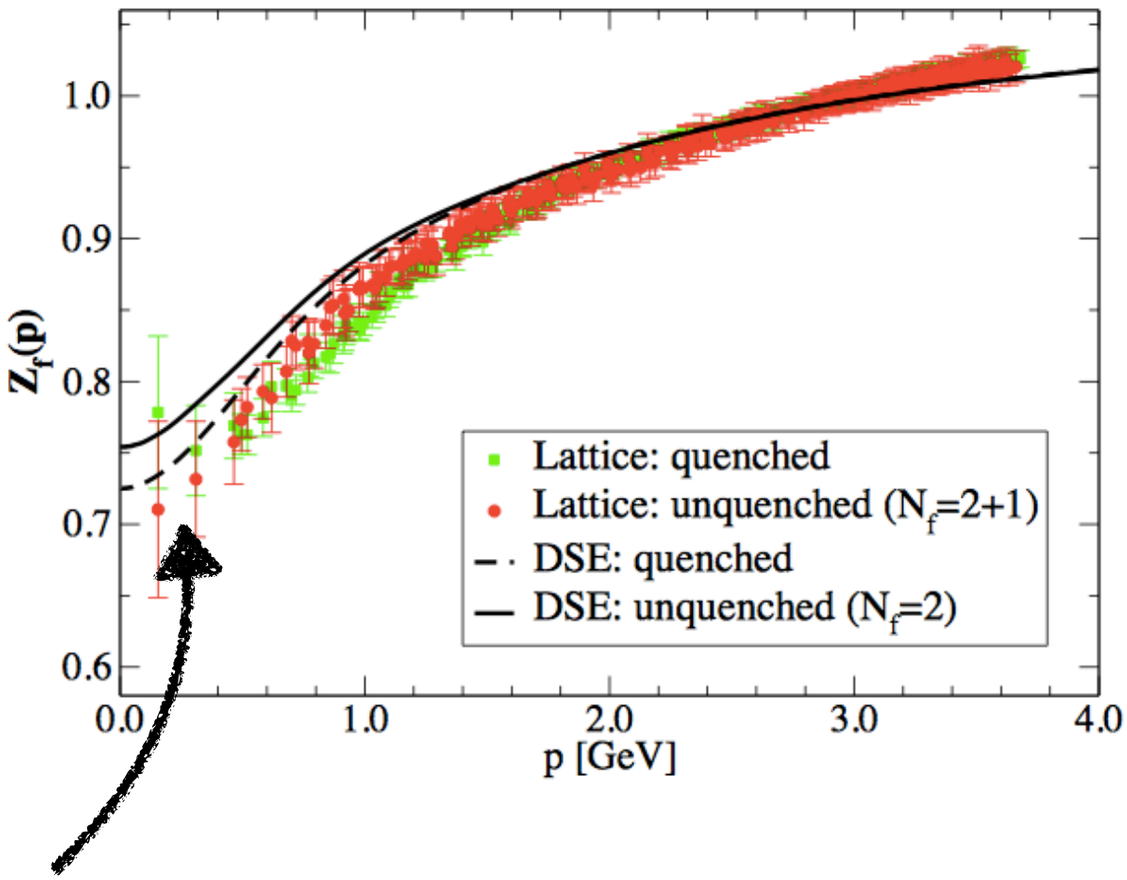
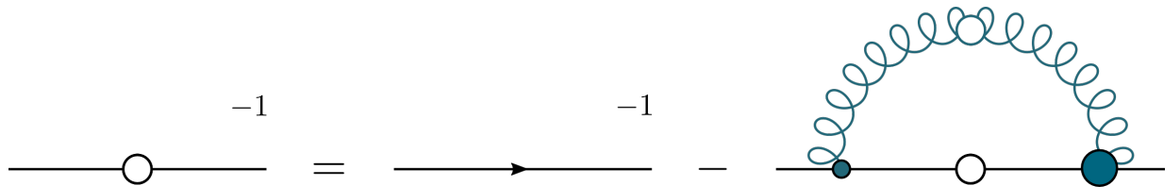




‘constituent quark’: large mass - very composite

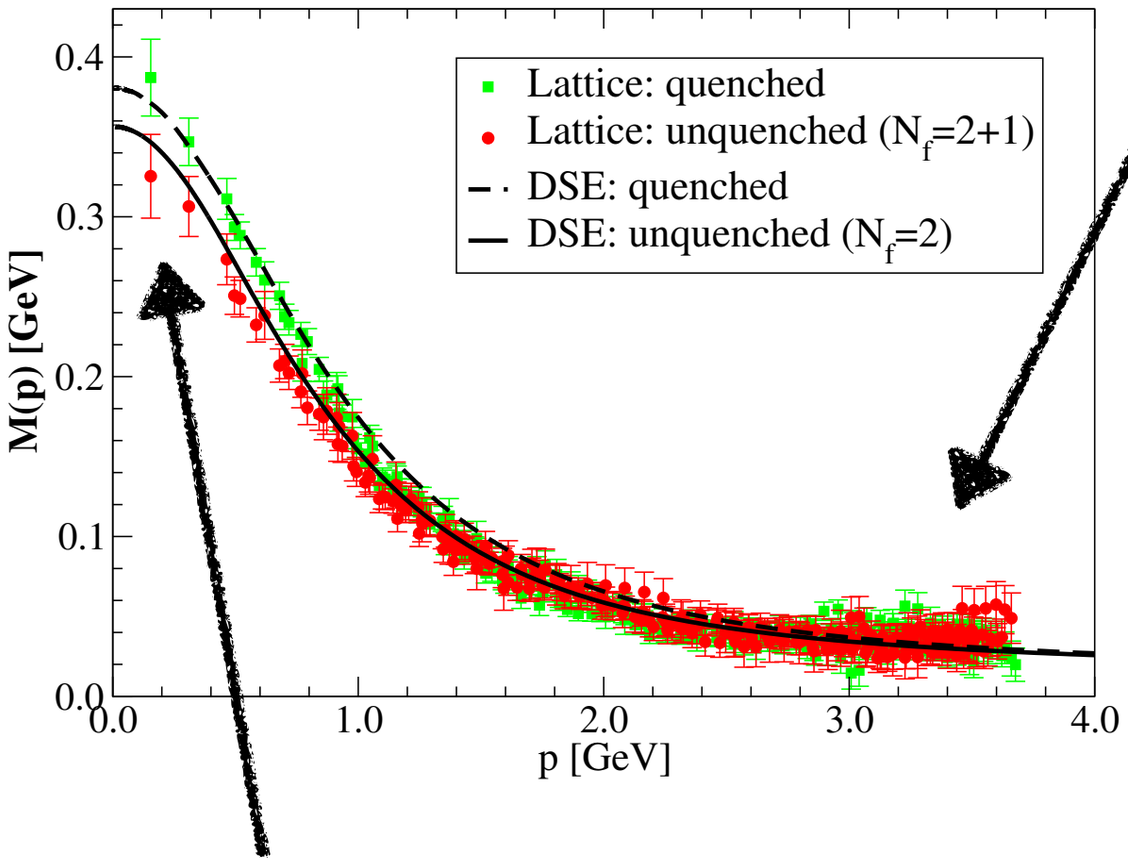
Dynamical mass generation (T=0)

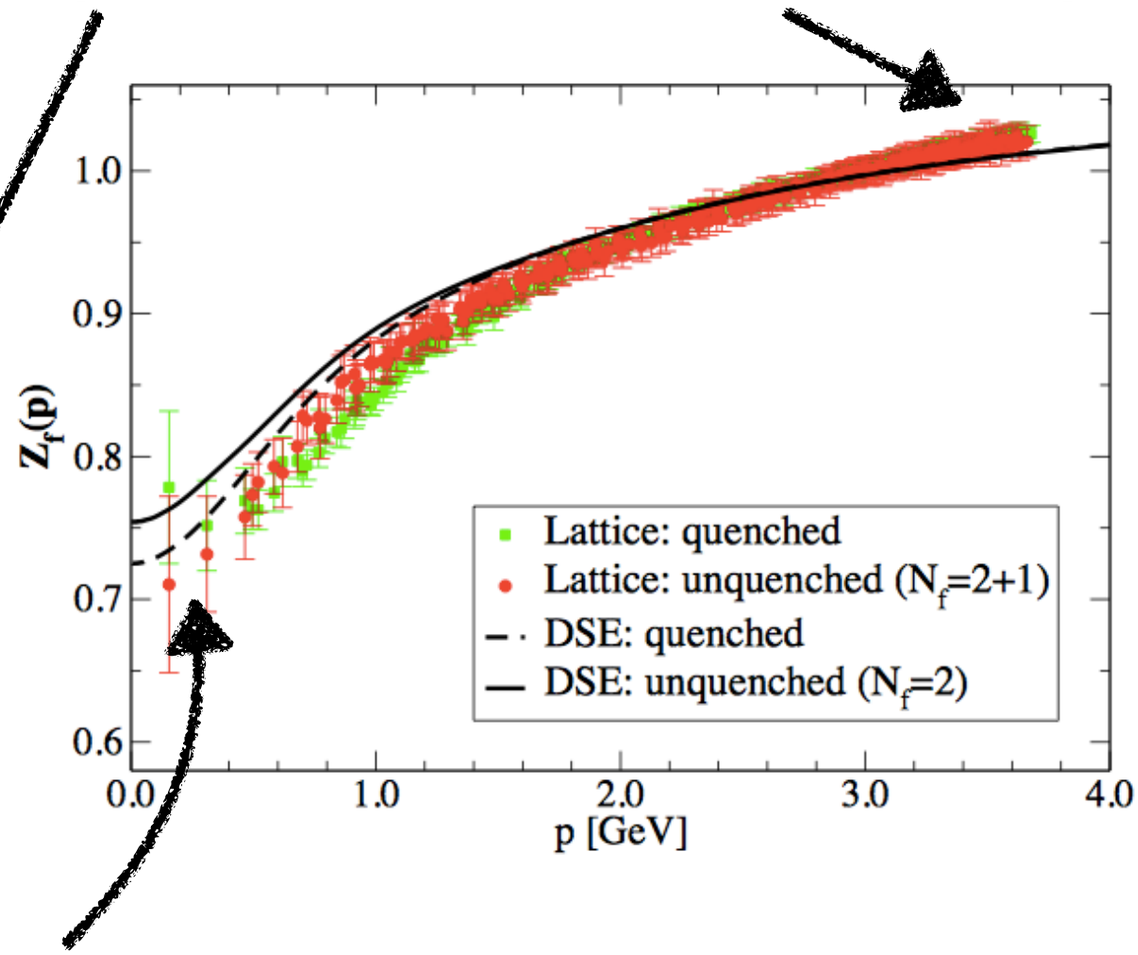
$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

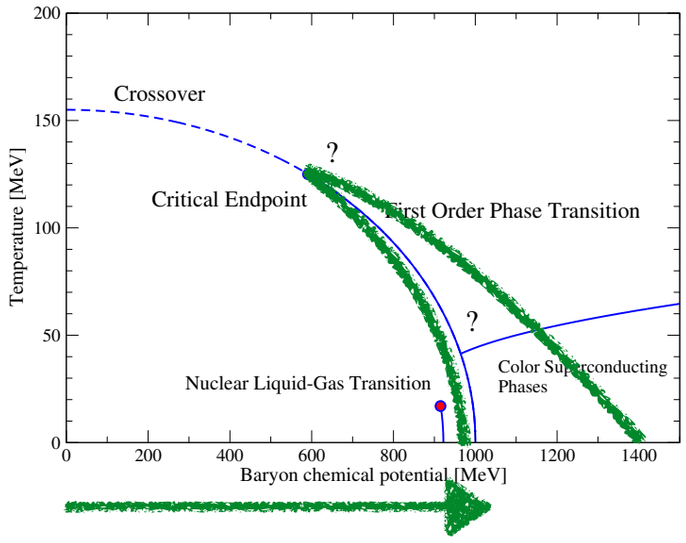
‘current quark’: small mass; non-composite

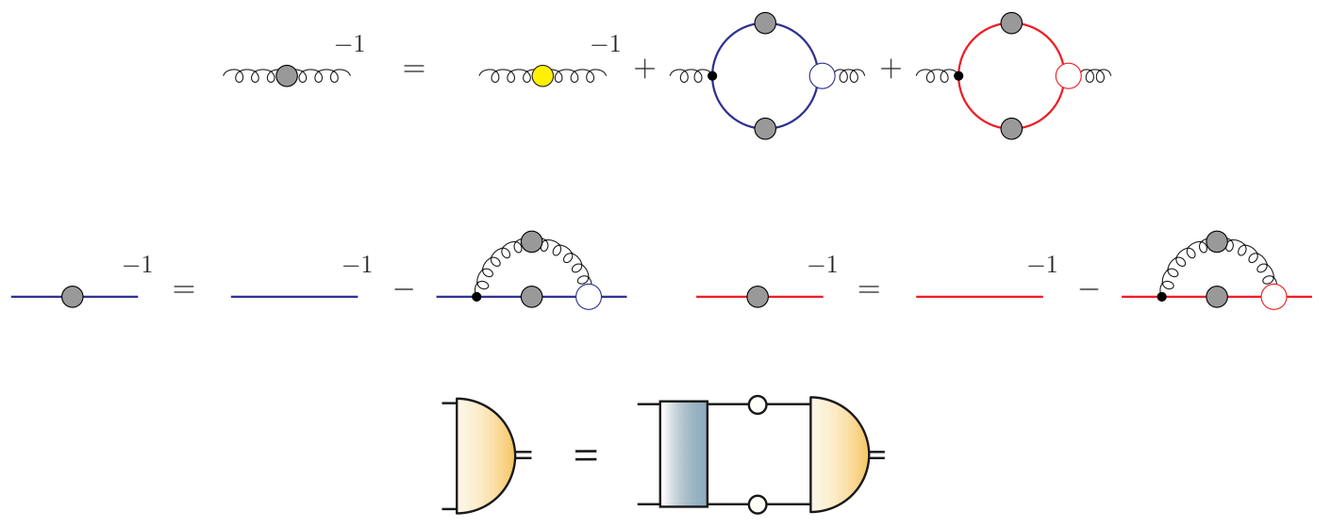




‘constituent quark’: large mass - very composite

Meson properties at finite chemical potential

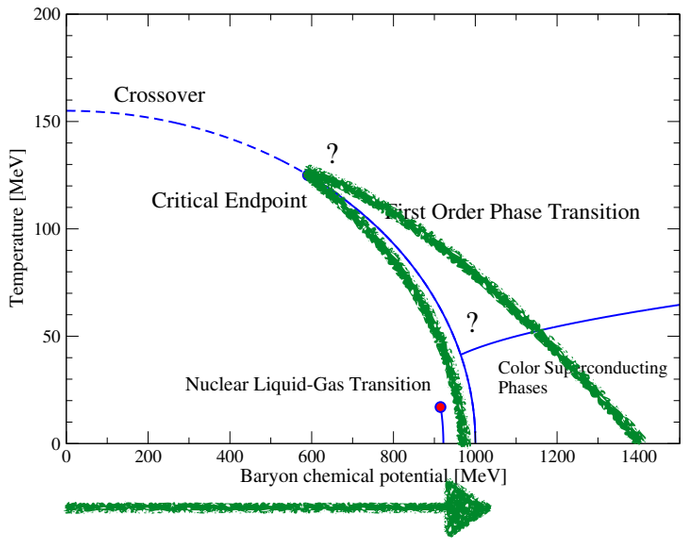


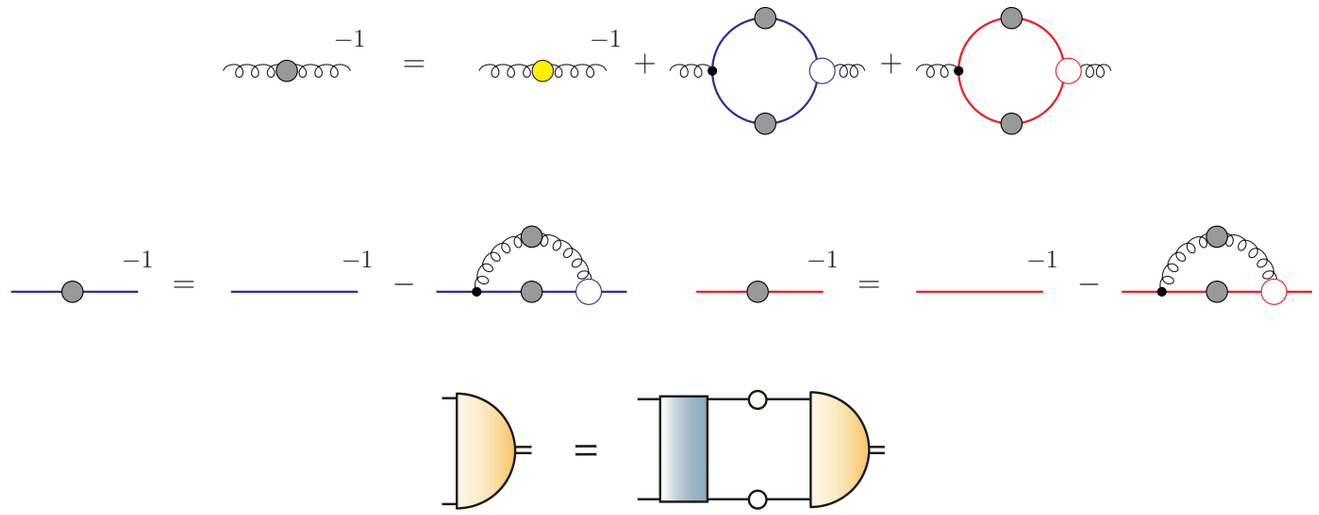


- Quarks/meson wave functions do change !

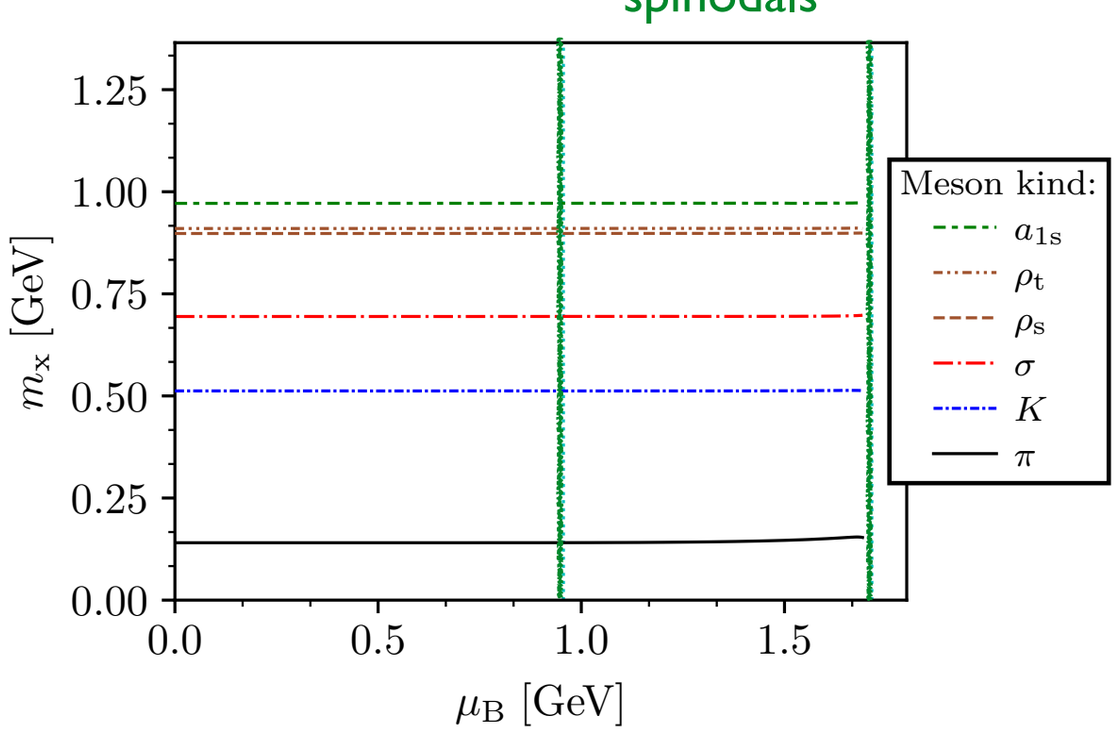
Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

Meson properties at finite chemical potential

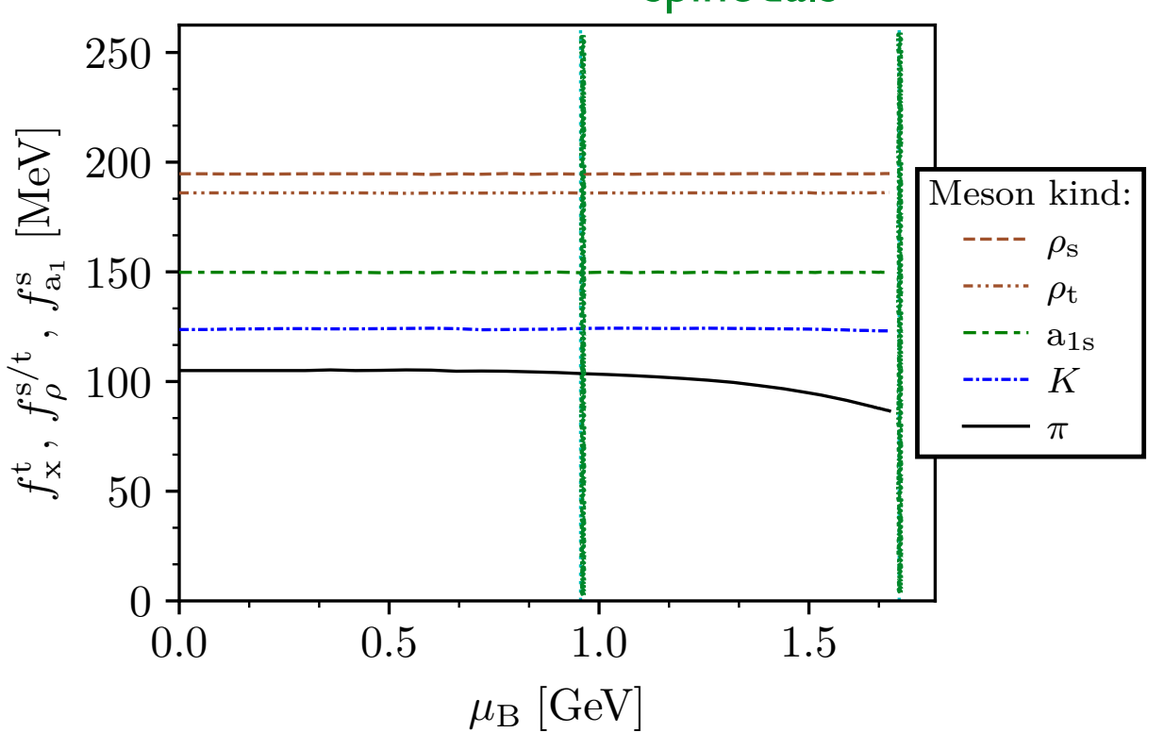




spinodals



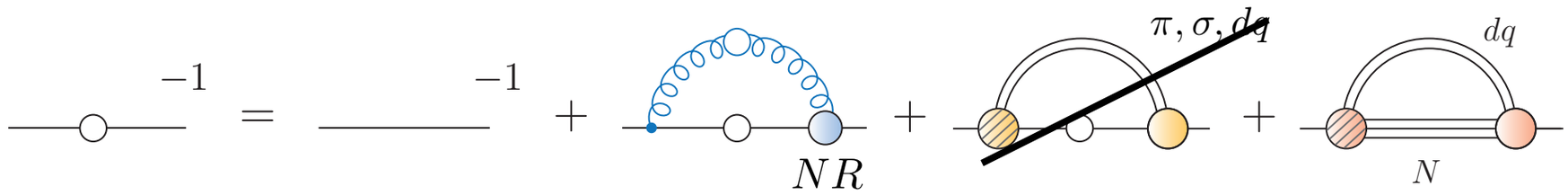
spinodals

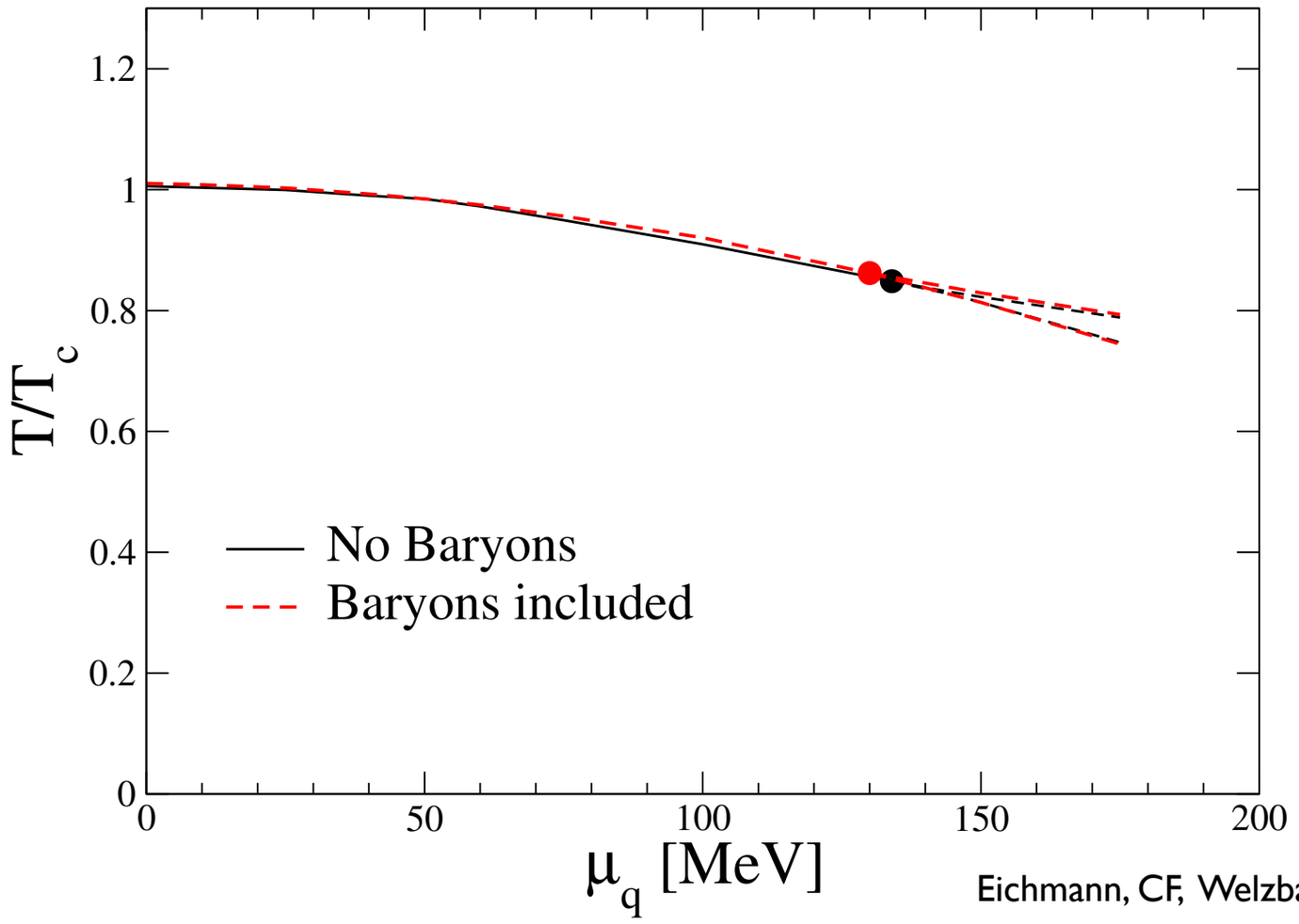


- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147
T. D. Cohen, PRL 91 , 222001 (2003)

Baryon effects on the CEP - results (N_f=2)

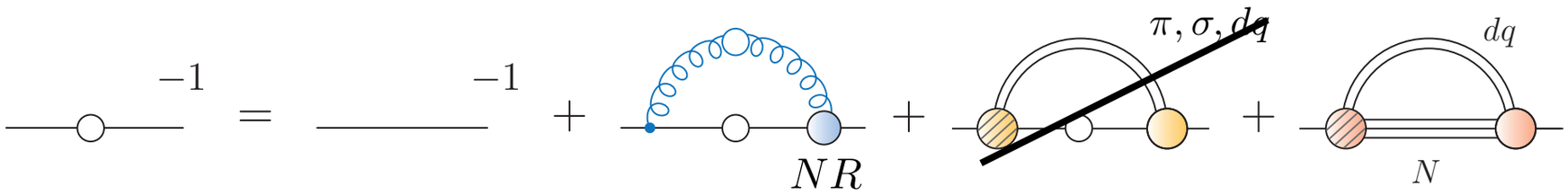


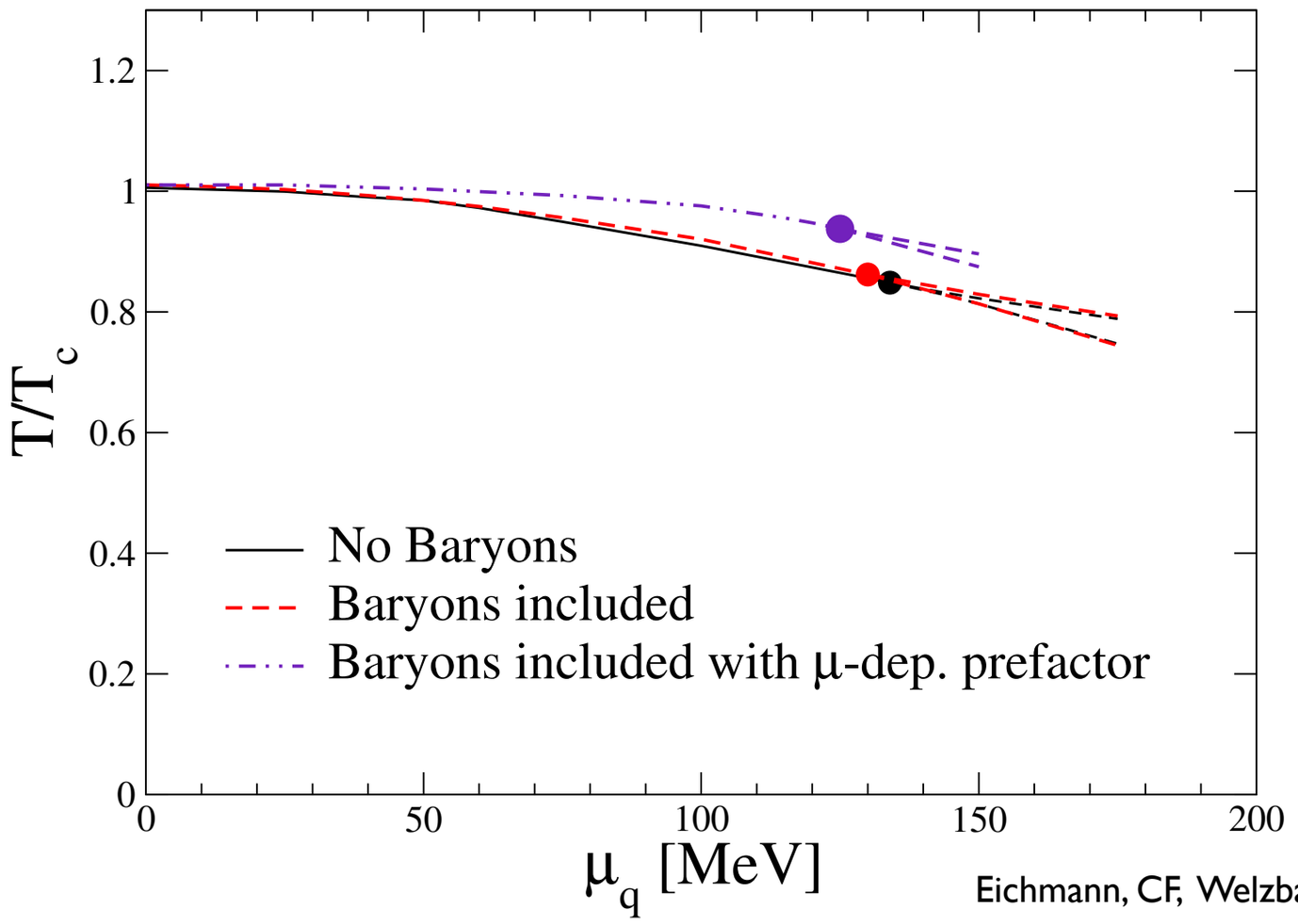


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

- Small chemical potential: no effect
- almost no effect on location of CEP

Baryon effects on the CEP - results (N_f=2)



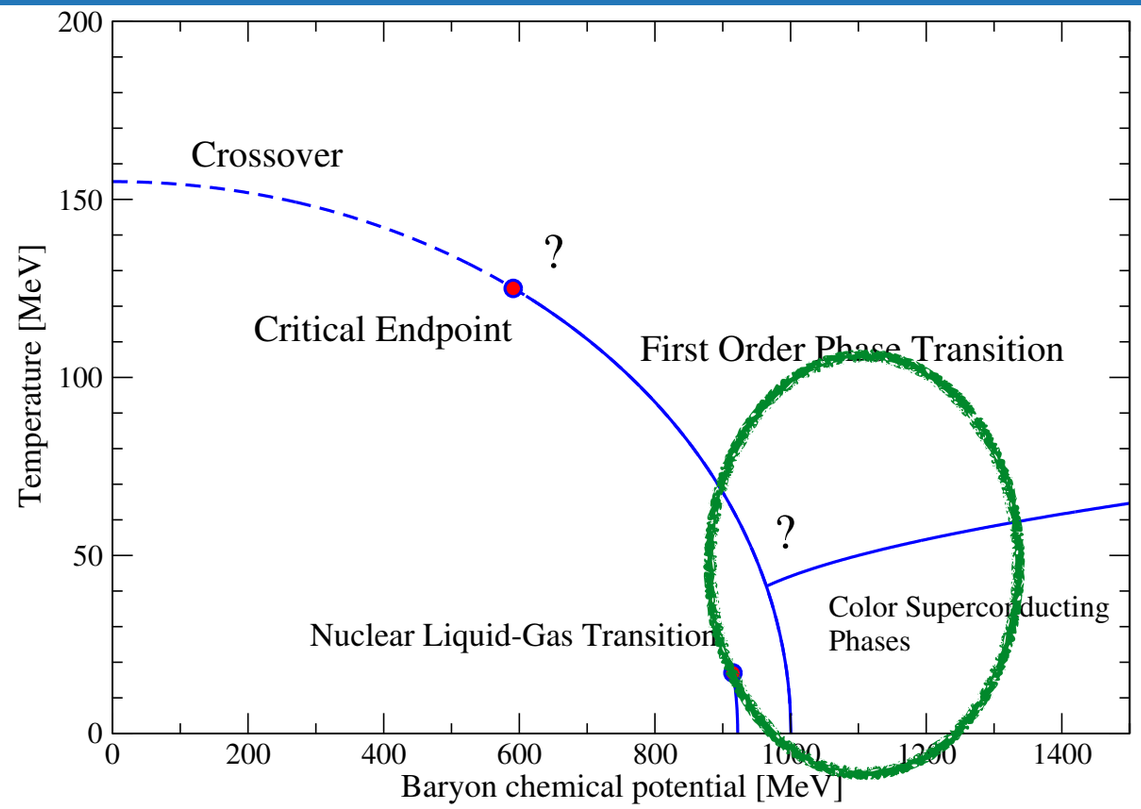


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

- Small chemical potential: no effect
- almost no effect on location of CEP
- But: strong μ-dependence of baryon wave function may change situation…

Equation of state from QCD

Gefördert durch
DFG Deutsche Forschungsgemeinschaft



EoS from microscopic QCD (functional approach):

- chirally broken phase
  - quarks, mesons
  - baryons

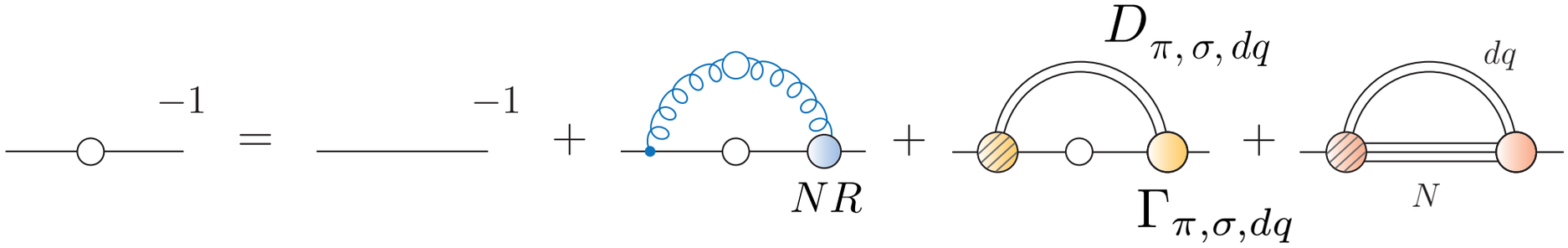
  ✔ our work

  work in progress (DFG-ind.)
- superconducting phase(s)

  ✔ Buballa et al.
  Müller, Buballa, Wambach, arXiv:1603.02865
- inhomogenuous broken (‘cristaline’) phase(s)

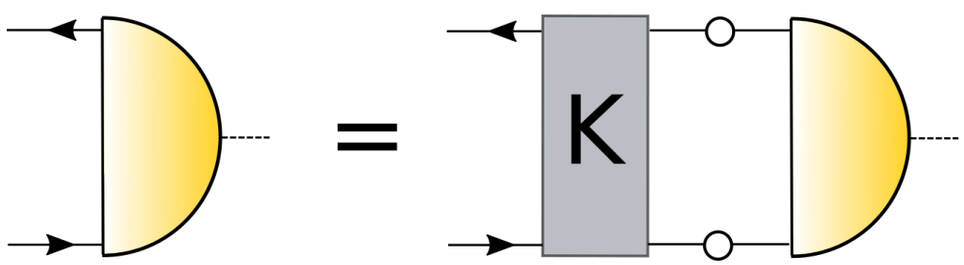
  work in progress (CRC, A03)
  Motta, Bernhardt, Buballa, CF, arXiv:2306.09749

Meson effects at finite T and μ



$$D_\pi(p) = \frac{1}{p_4^2 + u^2(\vec{p}^{\,2} + m_\pi(T,\mu)^2)} \qquad u = \frac{f_s}{f_t}$$

Son, Stephanov, PRD 66 (2002) 7



$$\Gamma_\pi(P,q) = \gamma_5\, E(P,q,T,\mu) + \dots$$

chiral limit: $\Gamma_\pi = \gamma_5 \dfrac{B}{f_t}$