NSF/MUSES, grant no OAC-2103680.

Illinois Center for Advanced Studies of the Universe

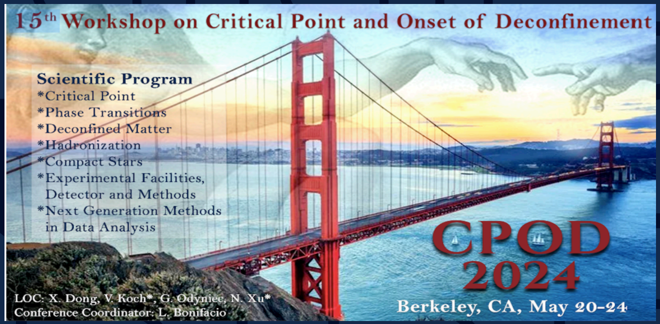
Location of the Critical Point from Holography

e-Print: 2309.00579 [nucl-th]

muses

Jorge Noronha

M. Hippert, J. Grefa, I. Portillo, J. Noronha-Hostler, C. Ratti, R. Rougemont and M. Trujillo



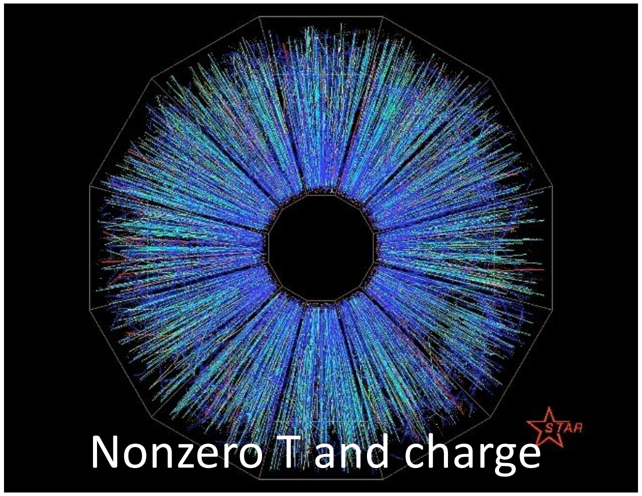
What is the holographic duality?

Maldacena, 1997; Witten, 1998, Gubser, Polyakov, Klebanov, 1998

“Calculations in strongly coupled plasmas using black holes”

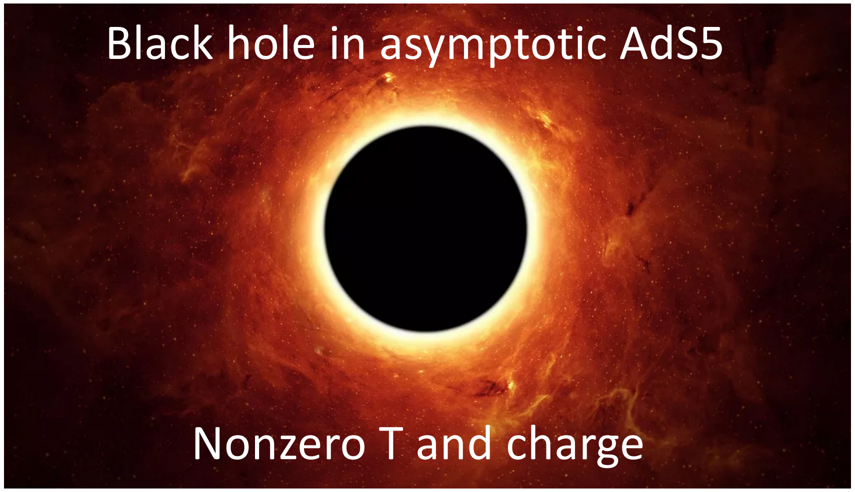
Strongly coupled non-Abelian plasma

Classical gravity in d>4



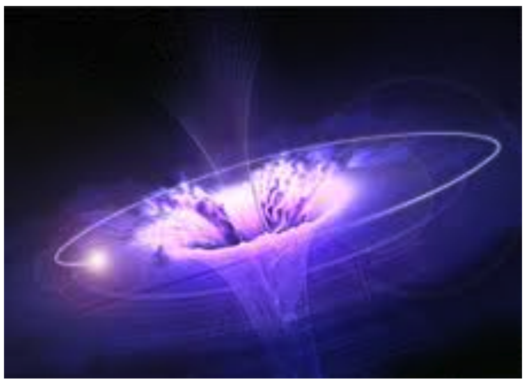
DUALITY

$\approx$



How is this used to find the QCD CP?

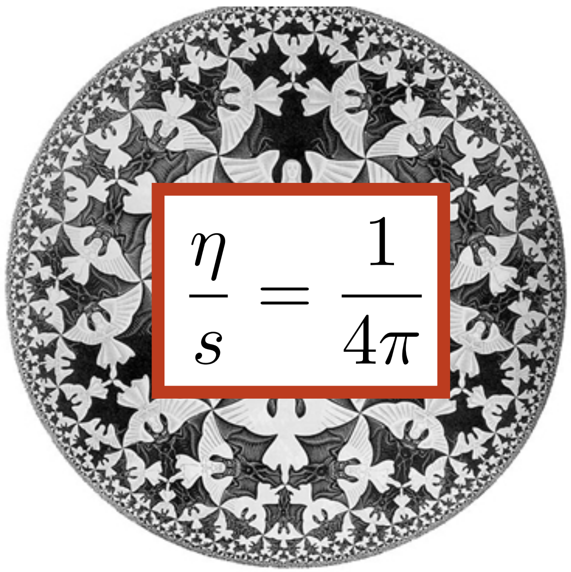
Black hole (brane)



Nonzero Hawking Temperature and charge

+

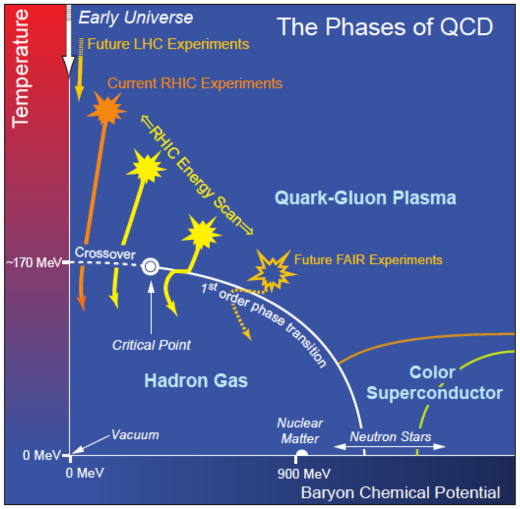
Anti-de Sitter (AdS) space



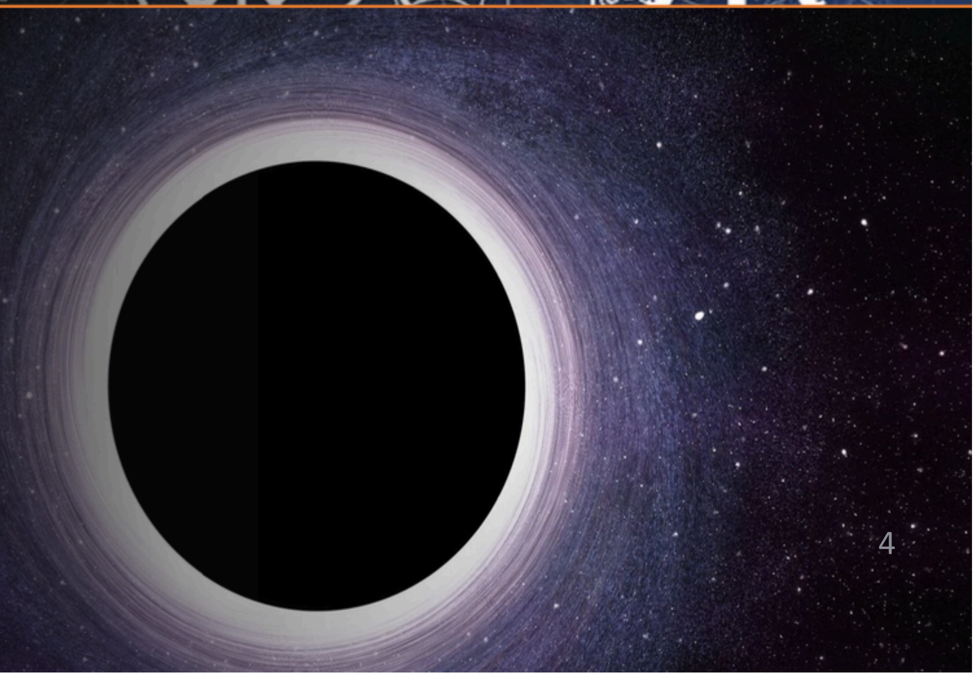
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Nearly perfect fluidity

=



Black Hole “Engineering”



DeWolfe, Gubser, Rosen, PRD (2011)
Finazzo et al, JHEP (2015)
Rougemont et al. PRL (2015), JHEP (2016),PRD (2017)
Critelli et al, PRD (2017)
Grefa et al, PRD (2021), PRD (2022)
Hippert et al., 2309.00579 [nucl-th]

+ MANY OTHERS!!!

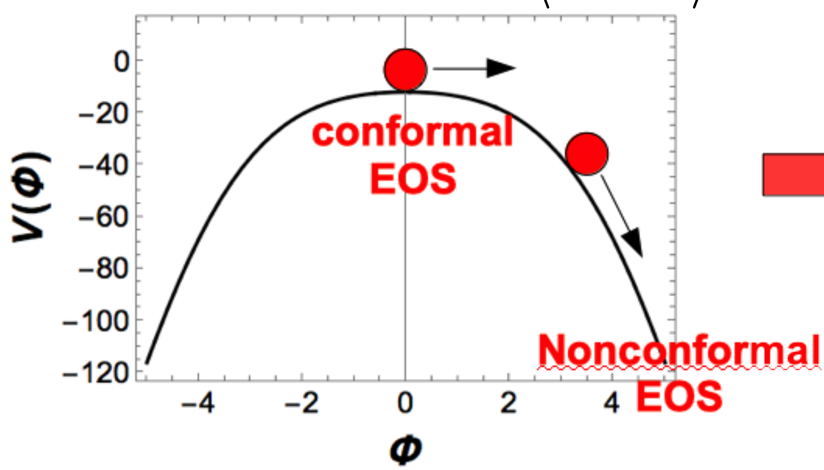
Black hole engineering in AdS

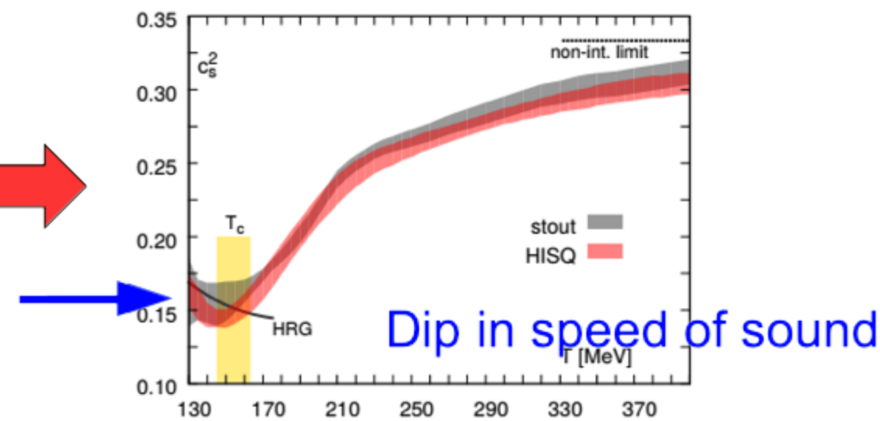
- Black hole solutions of Einstein-Maxwell-Dilaton equations

See the review by Rougemont et al, Progress in Particle and Nuclear Physics 135 (2024)

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - V(\phi) - \frac{f(\phi)F_{\mu\nu}^2}{4}\right]$$

Dilaton field dual to $\langle \mathrm{Tr}F^2 \rangle$





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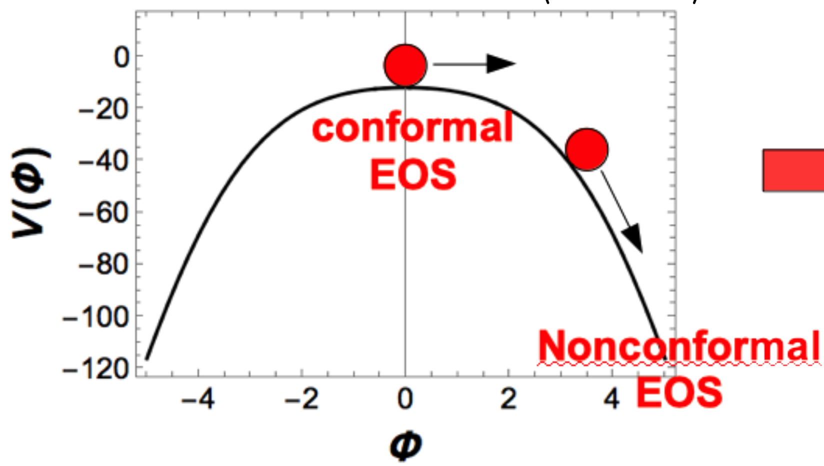

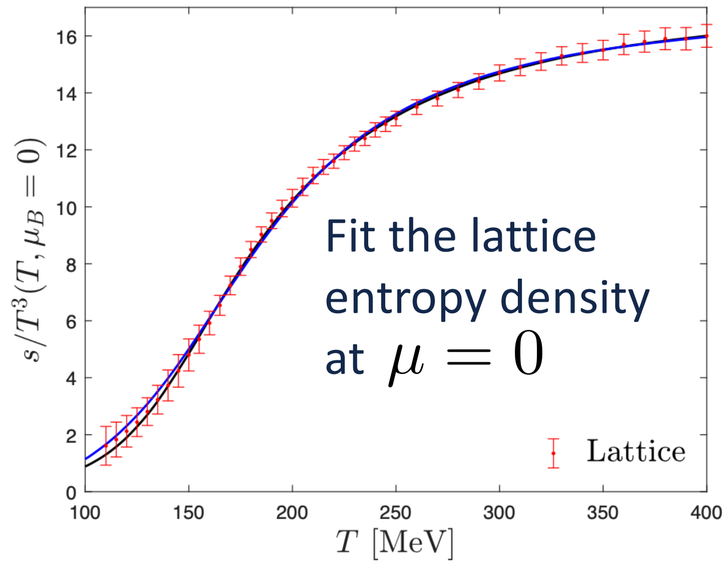

Fit the lattice entropy density at $\mu = 0$

Black hole engineering in AdS

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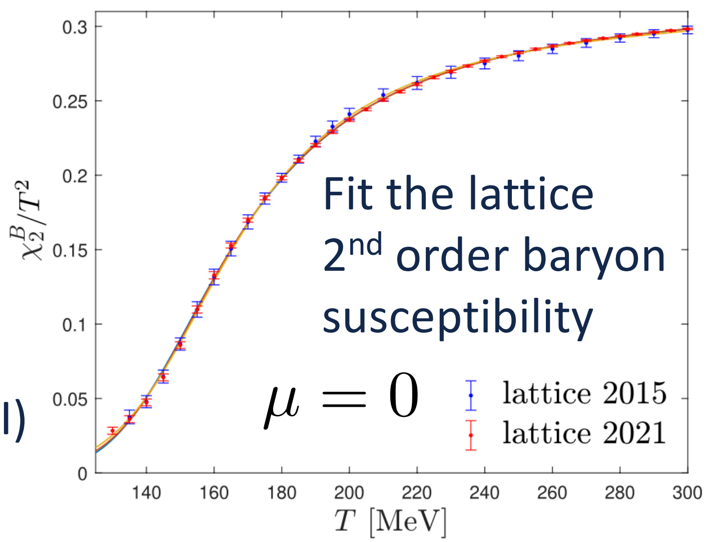
5d gauge field	Conserved baryon charge	Dilaton-Maxwell coupling
$A_\mu \Longrightarrow$	$U(1)_B$	$f(\phi)$

Backreaction from quark flavor branes (approx. of DBI)
(describes chirally symmetric phase)



Finite density properties

Grefa et al, Phys. Rev. D 104, 034002 (2021)
Grefa et al, Phys. Rev. D 106 (2022) 034024

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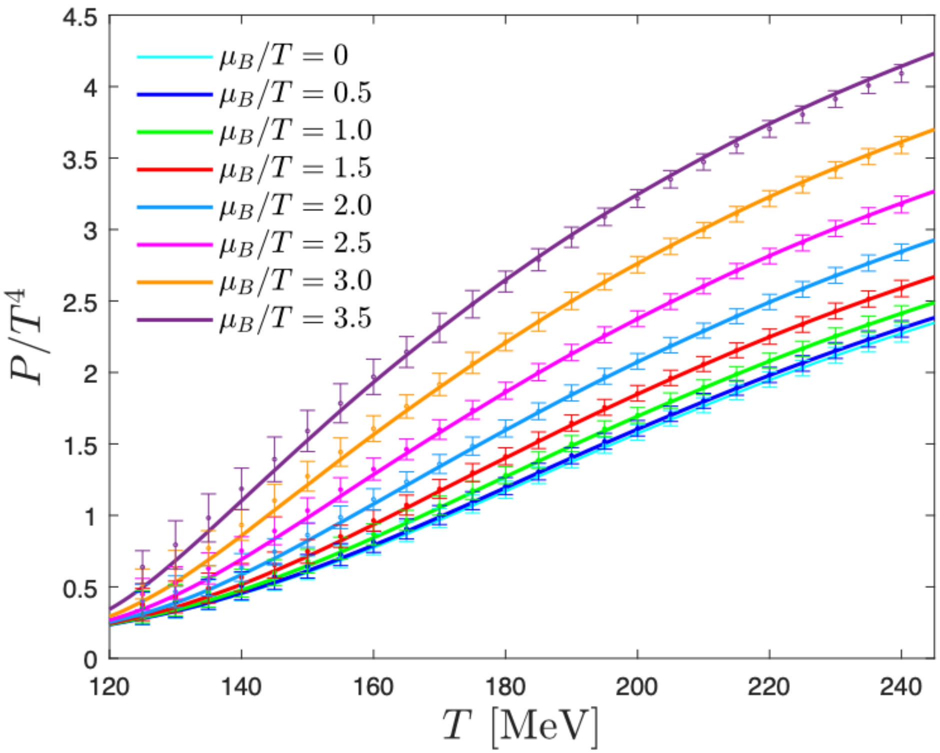
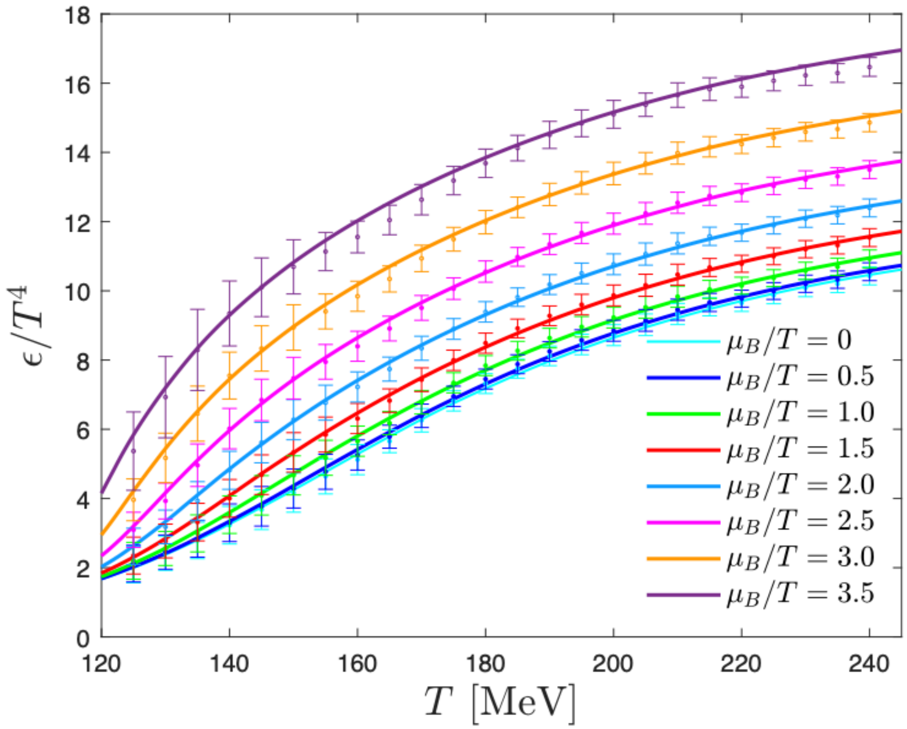


Lattice: Borsányi et al, PRL 126 (2021)



- Powerful, predictive model for baryon-rich QGP
- Real-time calculations (e.g. transport, hydro) possible

Precise determination of phase diagram

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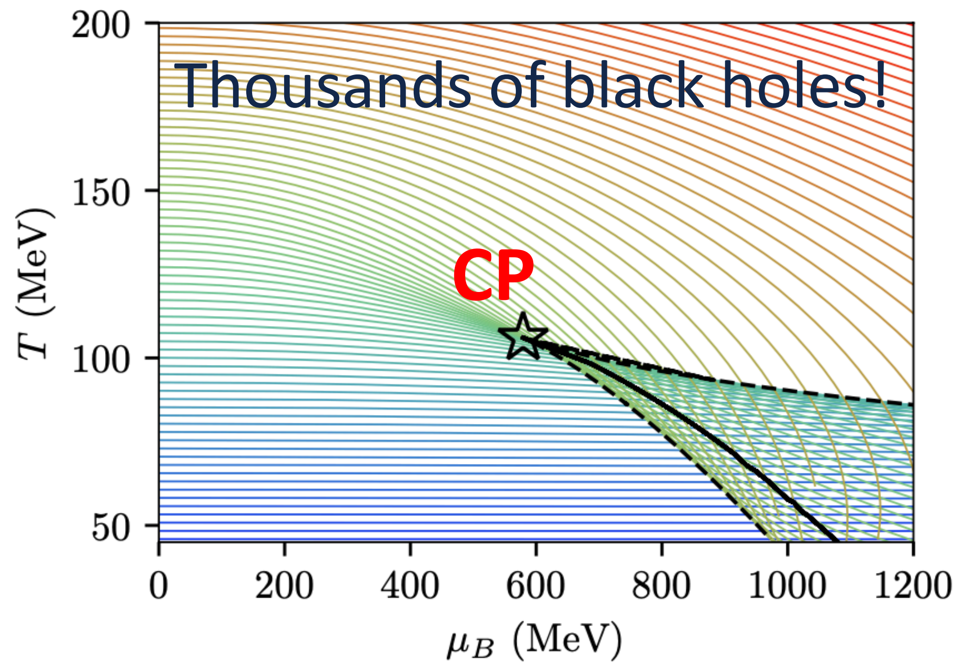
- Dilaton and electric fields at horizon: ϕ_0 and Φ_1 fully specify the physical state.
- Lines of constant ϕ_0 can cross.
- Metastable states, spinodal lines.
- Critical point: where crossings start.
  Fast algorithm to find CP!
- Maxwell construction: first-order line.



Fast and precise new algorithm = ripe for statistical analysis!

How do the many parameters affect CP?

Polynomial-Hyperbolic Ansatz (PHA)

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- Interpolates between Phys. Rev. D 96 (2017) and Phys. Rev. D 104 (2021)

$$V(\phi) = -12\cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

$$f(\phi) = \frac{\mathrm{sech}(c_1\phi + c_2\phi^2 + c_3\phi^3)}{1+d_1} + \frac{d_1}{1+d_1}\,\mathrm{sech}(d_2\phi)$$

Parametric Ansatz (PA)

- Similar shapes, more interpretable parameters Phys. Rev. D 96 (2017)

$$V(\phi) = -12\cosh\left[\left(\frac{\gamma_1\,\Delta\phi_V^2 + \gamma_2\,\phi^2}{\Delta\phi_V^2 + \phi^2}\right)\phi\right]$$

$$f(\phi) = 1 - (1 - A_1)\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi - \phi_1}{\delta\phi_1}\right)\right] - A_1\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi - \phi_2}{\delta\phi_2}\right)\right]$$

Bayesian black hole engineering

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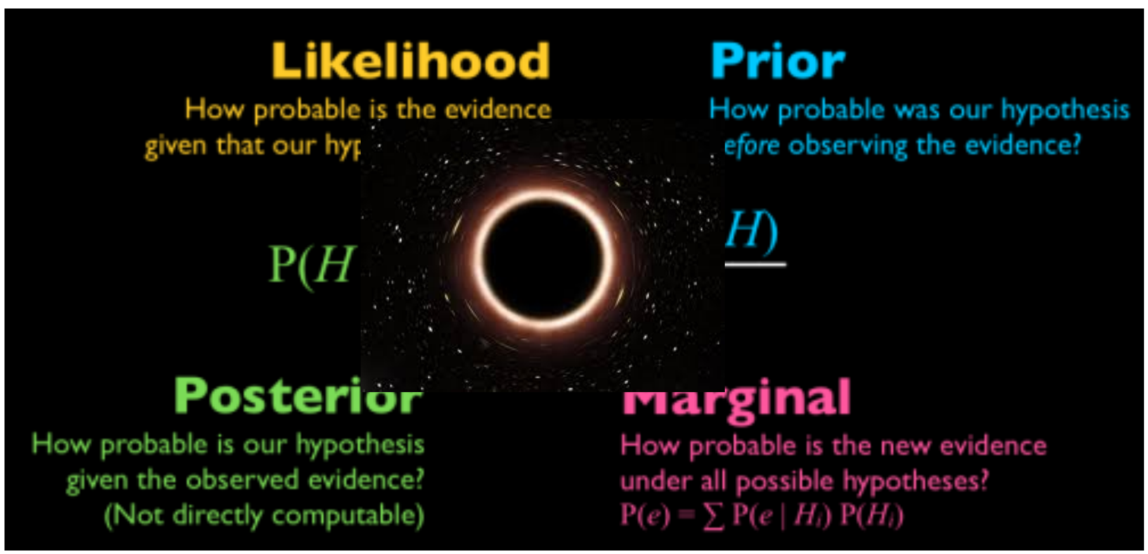
- How do we make sense of these parameters?
- Are CPs always present in these models?
- How do lattice results affect CP location/existence?
- Bayesian inference analysis is needed!



Bayesian black hole engineering

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Assigning probabilities

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Bayes' Theorem

$$\underbrace{P\left(\text{model} \,|\, \text{results}\right)}_{\text{posterior } \mathcal{P}} \times P\left(\text{results}\right) = \underbrace{P\left(\text{results} \,|\, \text{model}\right)}_{\text{likelihood } \mathcal{L}} \times \underbrace{P\left(\text{model}\right)}_{\text{prior knowledge}}$$

Gaussian Likelihood

$$\mathcal{L} = \exp\left\{-\frac{1}{2}\boldsymbol{\delta x}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\delta x} - \frac{1}{2}\log\det\boldsymbol{\Sigma} + \text{constant}\right\}$$

- $\boldsymbol{\delta x}$: deviation for $s(T)$ and $\chi_2^{(B)}(T)$ at $\mu = 0$.
- Correlation $\Gamma \equiv \exp\left(-\Delta T/\xi_T\right)$ between neighboring points $\rightarrow$ extra model parameter.

Bayesian black hole engineering

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Mauricio Hippert

Markov Chain Monte-Carlo (MCMC)

- Random evolution to sample from posterior.
- Transition probabilities such that $\mathcal{P}$ is stationary limit.
- Differential evolution MCMC: suited for correlations.

C.J.F. Ter Braak, Statistics and Computing **16** (2006)

Differential evolution

1. Use other chains j, k to update chain $i \neq j \neq k$: $\theta_i \to \theta_i + \frac{b}{\sqrt{2d}}(\theta_j - \theta_k) + \xi_i$.
2. Compute $\mathcal{P}$ from model EoS.
   - If $\mathcal{P}/\mathcal{P}_0 > 1$, transition to new parameters.
   - Otherwise, accept transition with probability $\mathcal{P}/\mathcal{P}_0$.
3. Repeat.

Inputs: Baryon susceptibility and entropy density from the lattice.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, PRL **730** (2014)
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

Bayesian black hole engineering

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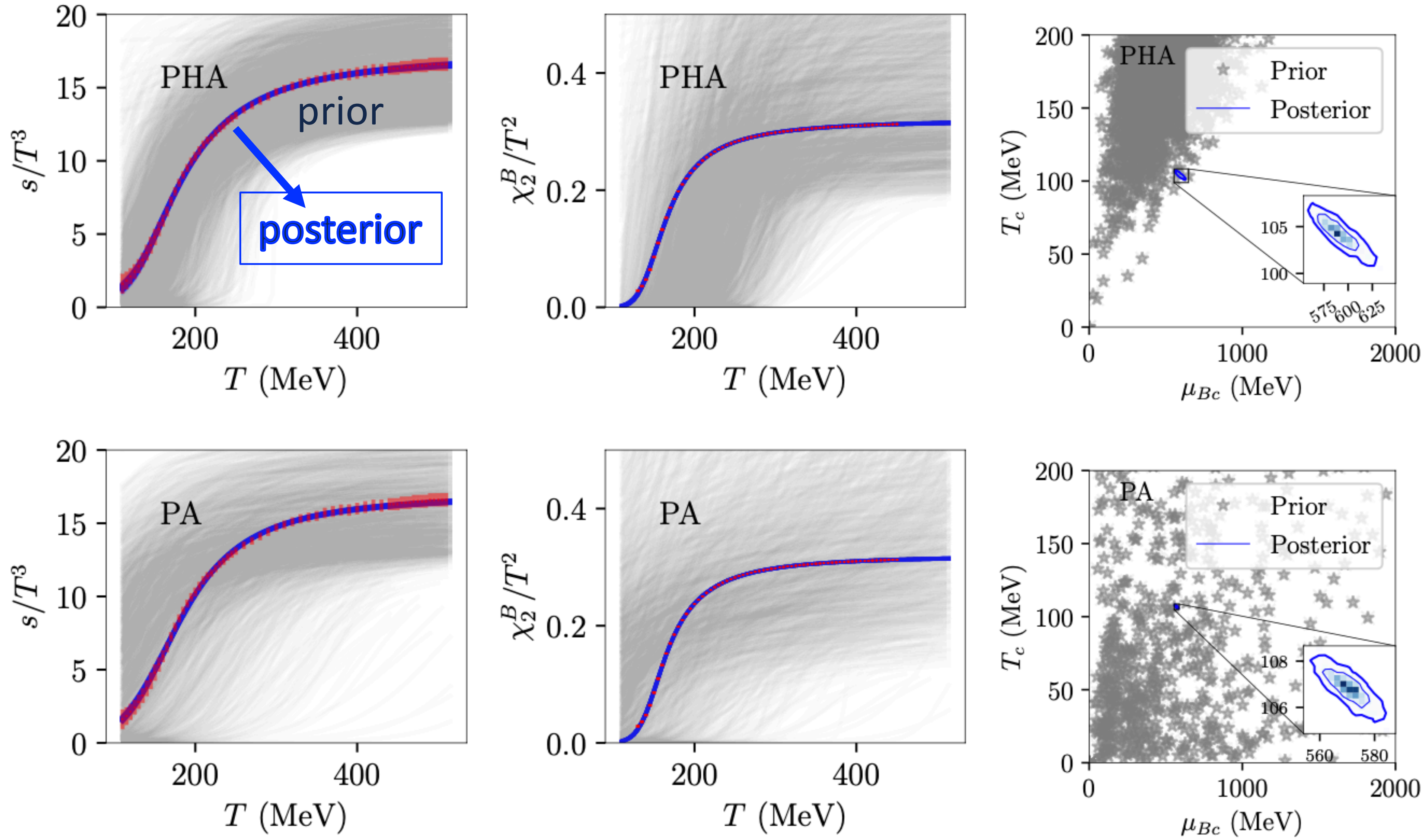
- Flat prior for parameters
- 20% of prior samples give no CP



Bayesian black hole engineering

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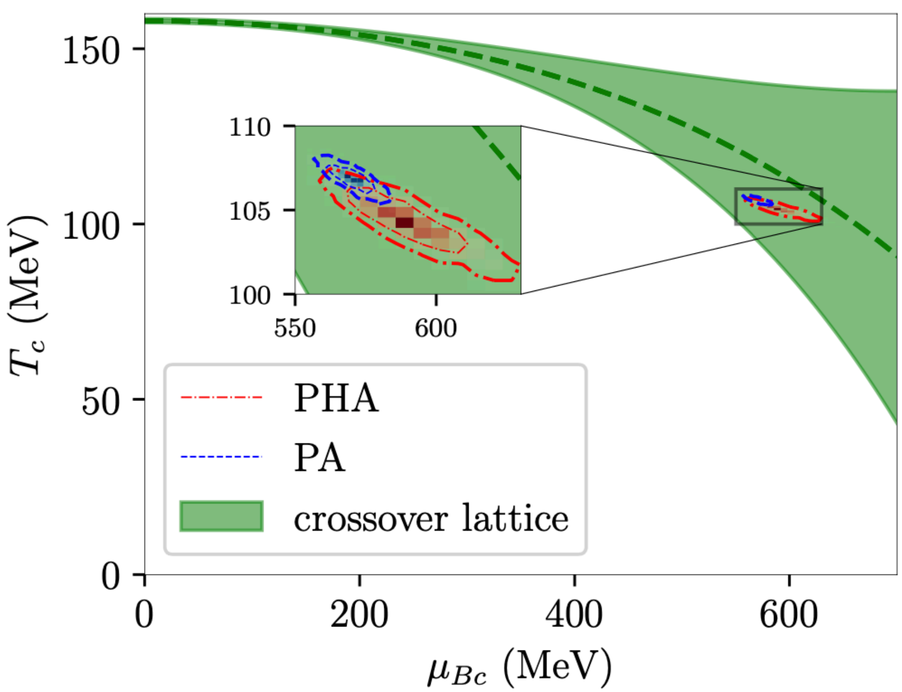
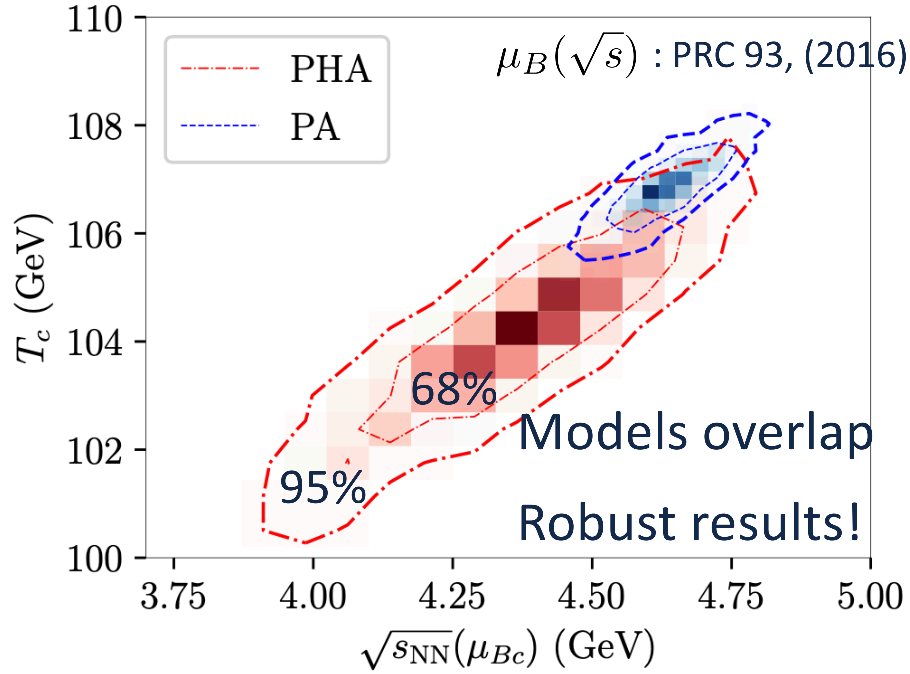
All posterior predictions for CP location collapse around these regions

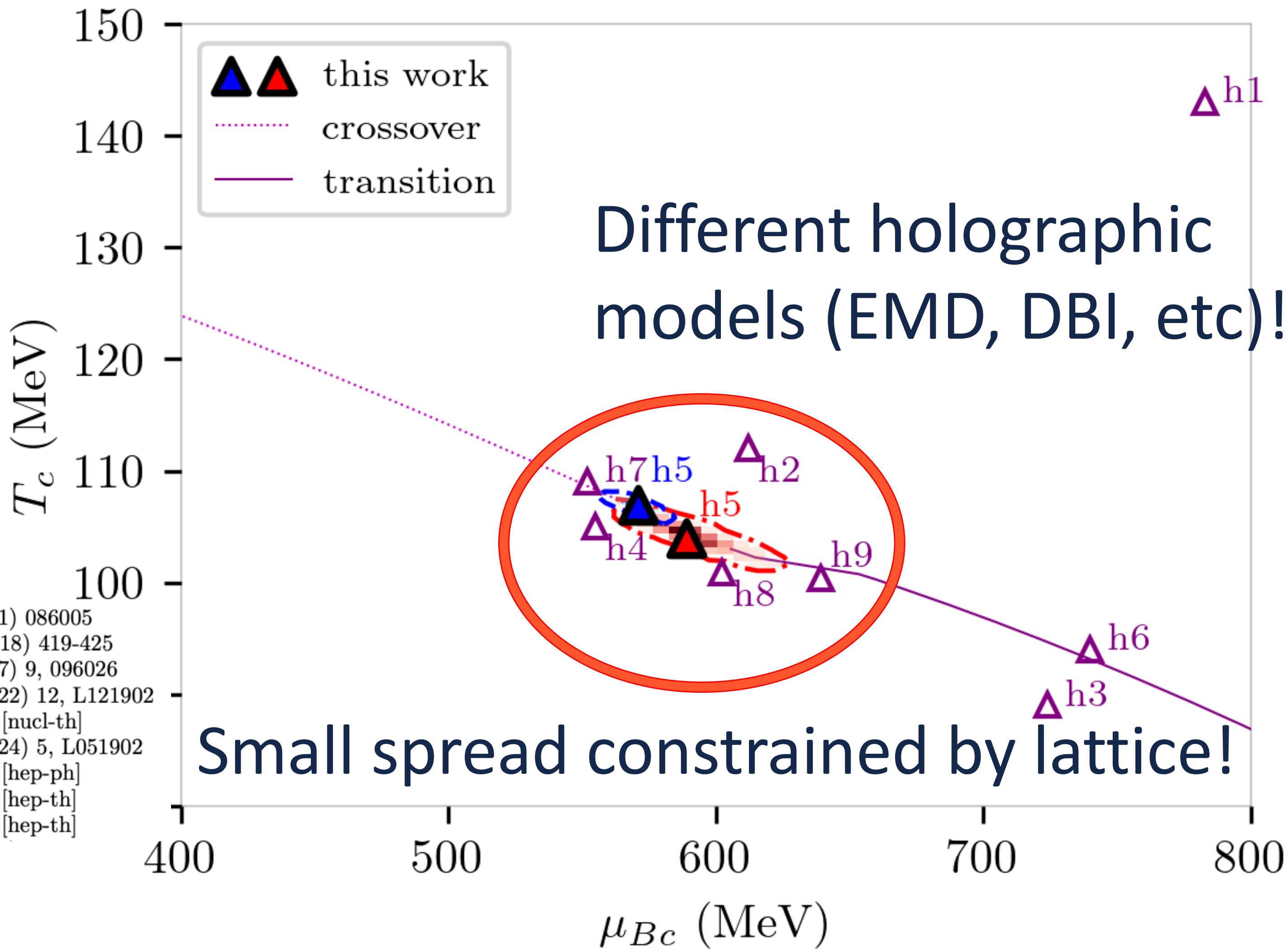
Posterior critical points

$(T_c,\, \mu_{Bc})_{PHA} = (104 \pm 3,\, 589^{+36}_{-26})$ MeV, $(T_c,\, \mu_{Bc})_{PA} = (107 \pm 1,\, 571 \pm 11)$ MeV.





Location of CP from holography



Different holographic models (EMD, DBI, etc)!

Small spread constrained by lattice!

h1: Phys.Rev.D 83 (2011) 086005
h2: Phys.Lett.B 778 (2018) 419-425
h3: Phys.Rev.D 96 (2017) 9, 096026
h4: Phys.Rev.D 106 (2022) 12, L121902
h5: e-Print: 2309.00579 [nucl-th]
h6: Phys.Rev.D 109 (2024) 5, L051902
h7: e-Print: 2404.12109 [hep-ph]
h8: e-Print: 2405.02394 [hep-th]
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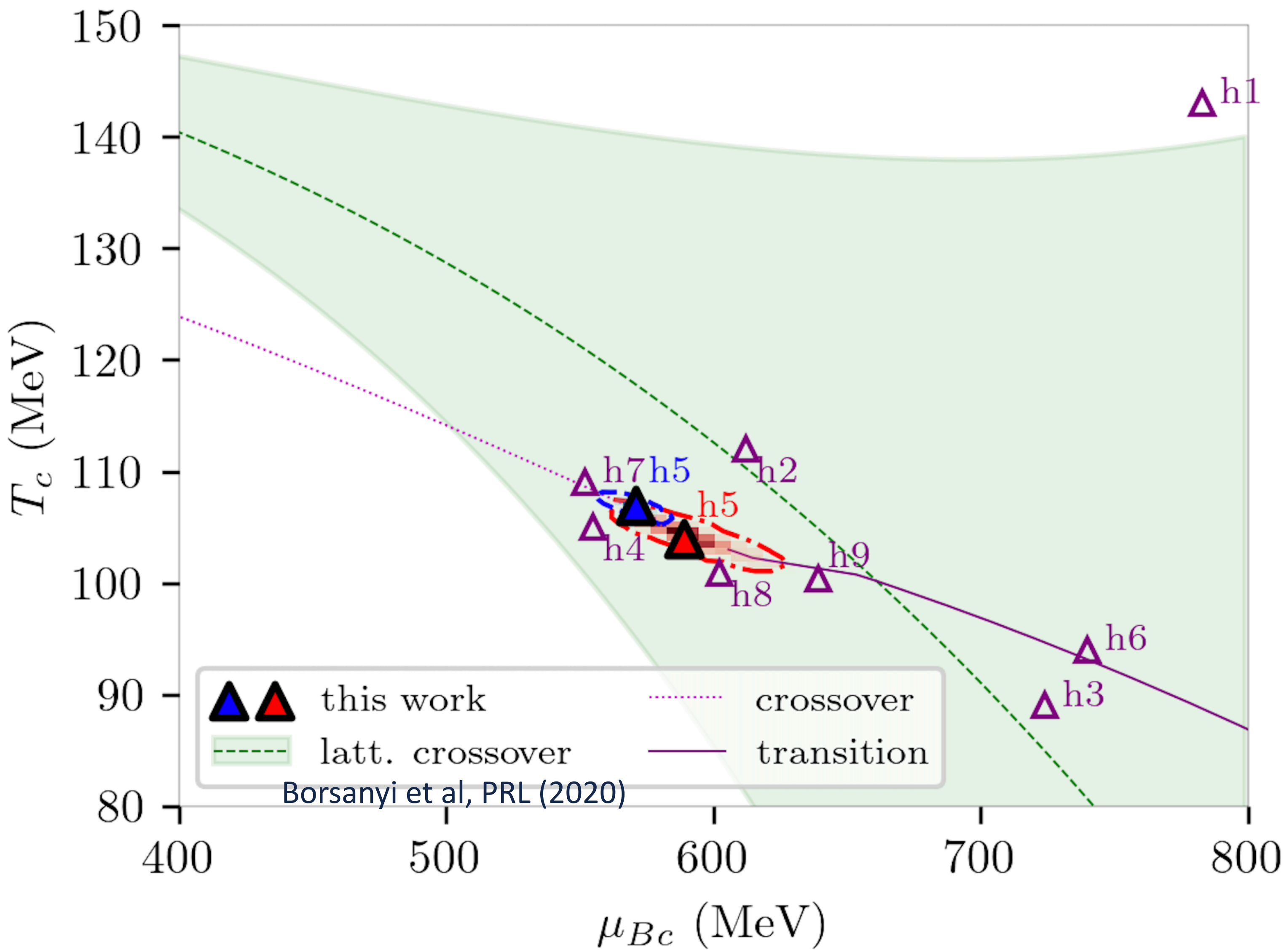
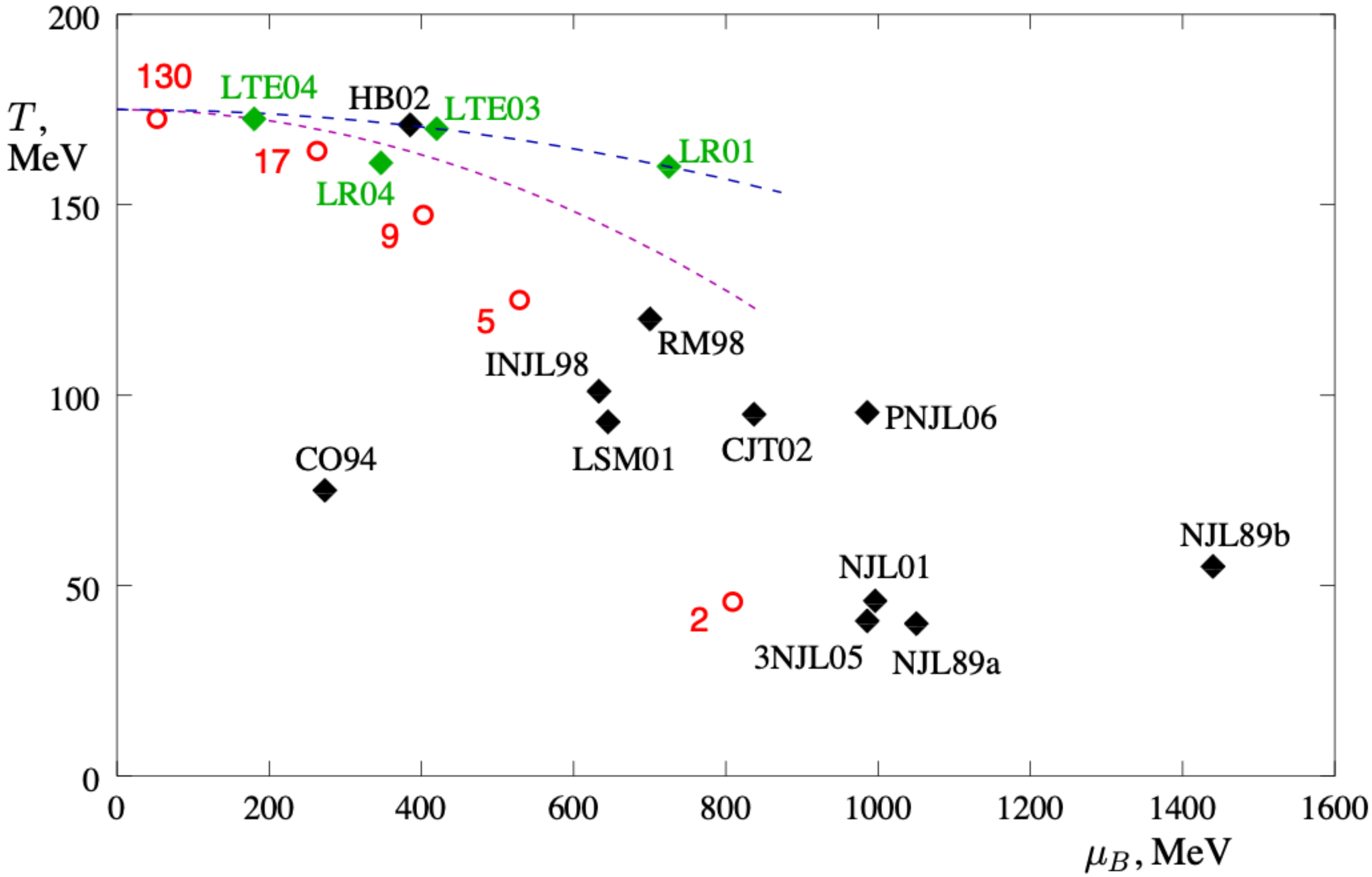


Borsanyi et al, PRL (2020)

Location of CP from models: 18 years ago

M. Stephanov, Lattice 2006 Plenary talk, arXiv:hep-lat/0701002



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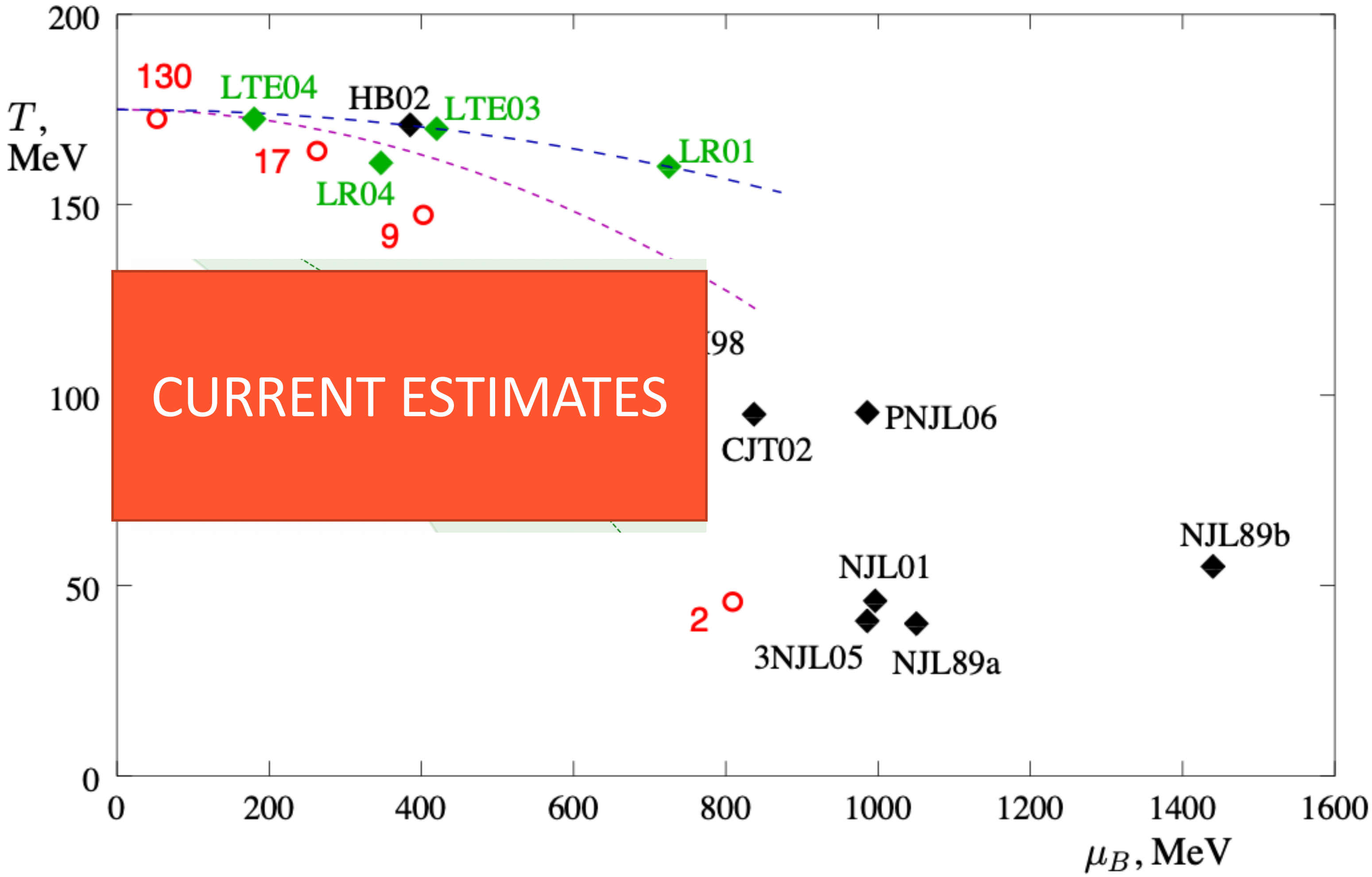

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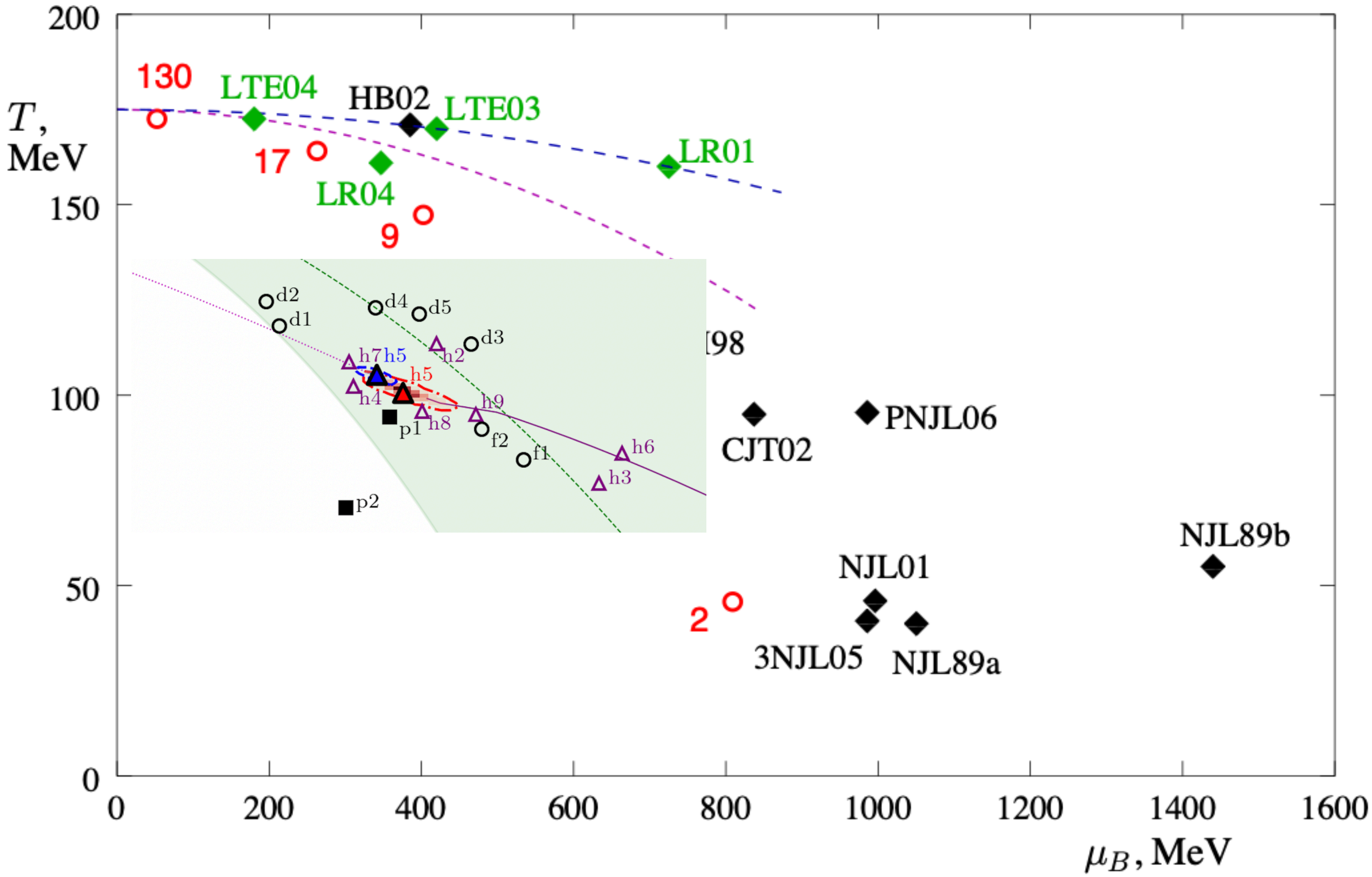
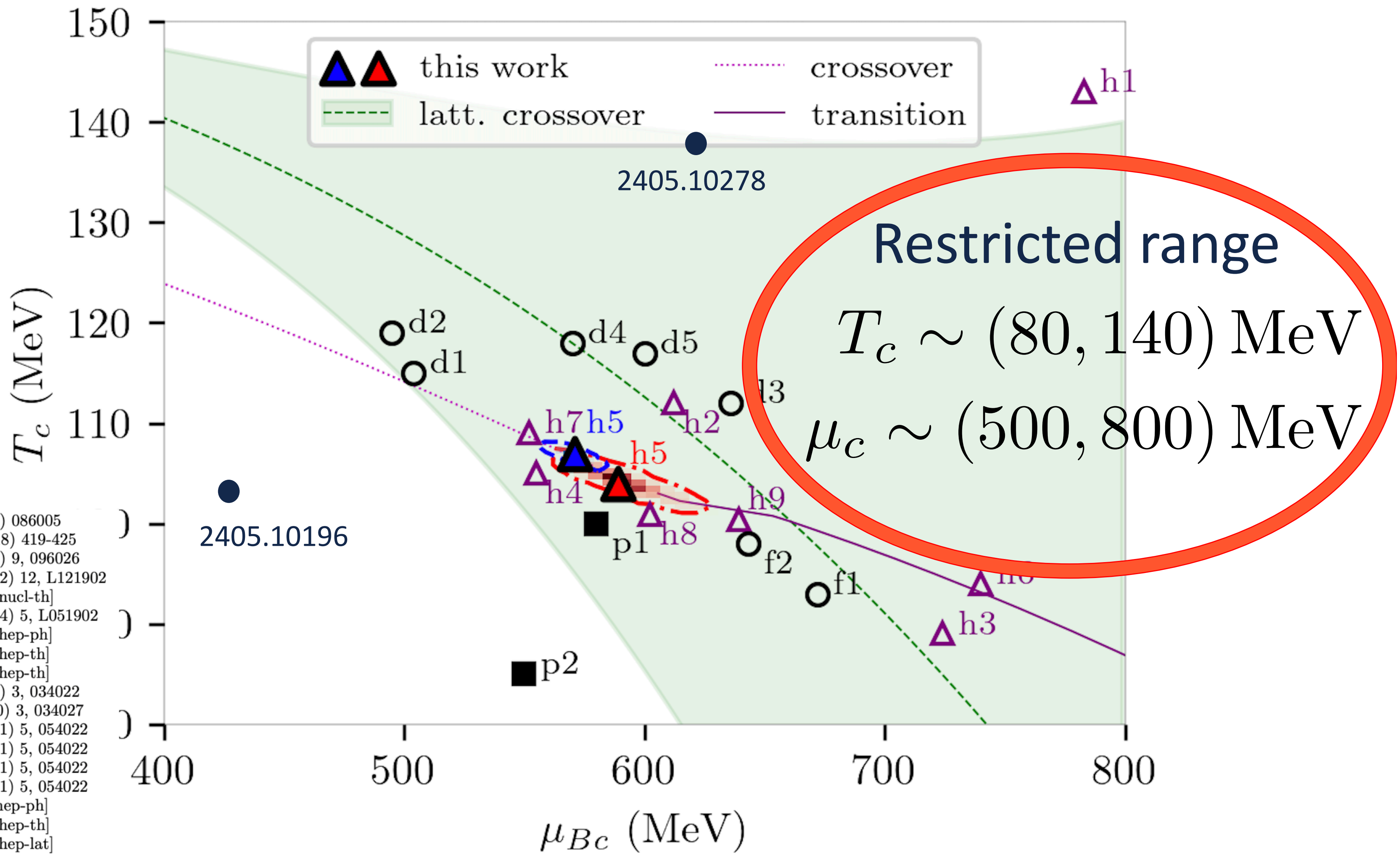


Current estimates for the CP location



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f1: Phys.Rev.D 102 (2020) 3, 034027
d2: Phys.Rev.D 104 (2021) 5, 054022
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d4: Phys.Rev.D 104 (2021) 5, 054022
d5: Phys.Rev.D 104 (2021) 5, 054022
f2: e-Print: 2308.15508 [hep-ph]
p1: e-Print: 2312.06952 [hep-th]
p2: e-Print: 2401.08820 [hep-lat]

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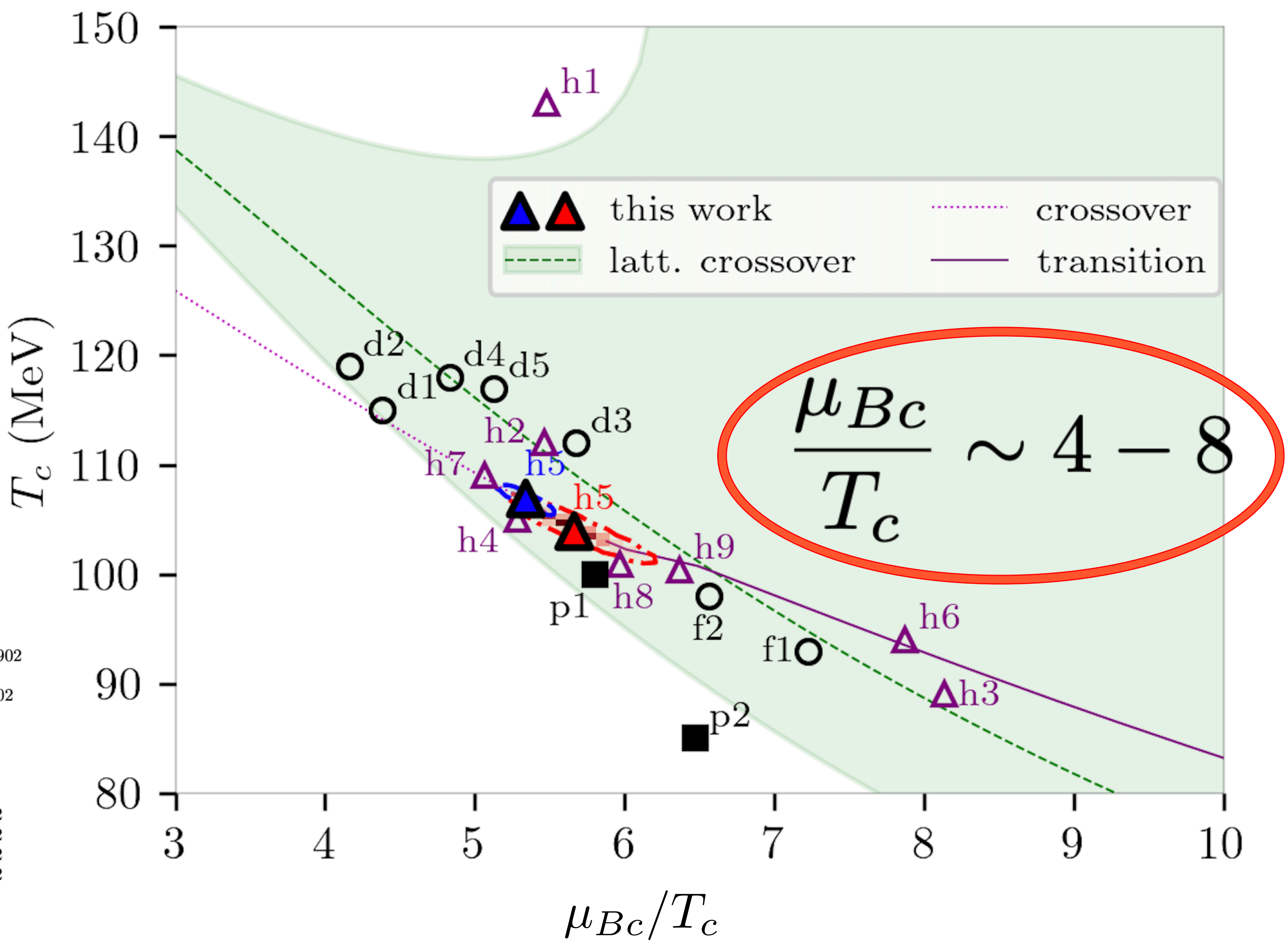

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Conclusions

- First Bayesian inference analysis of CP location constrained by lattice results at zero net baryon density:

Predict CEP (95% confidence level):

$$T_c = 101 - 108 \text{ MeV}$$
$$\mu_c = 560 - 625 \text{ MeV}$$

$$\frac{\mu_c}{T_c} \sim 5 - 6$$

- Other approaches (FRG,DSE, FSS, Pade, etc) predict CP in a similar narrow range.

- They didn't have to agree, but they do: *who ordered that?*

12:00

Discussions

Building 50 Auditorium