

Fluctuations and Correlations of Baryonic Chiral Partners

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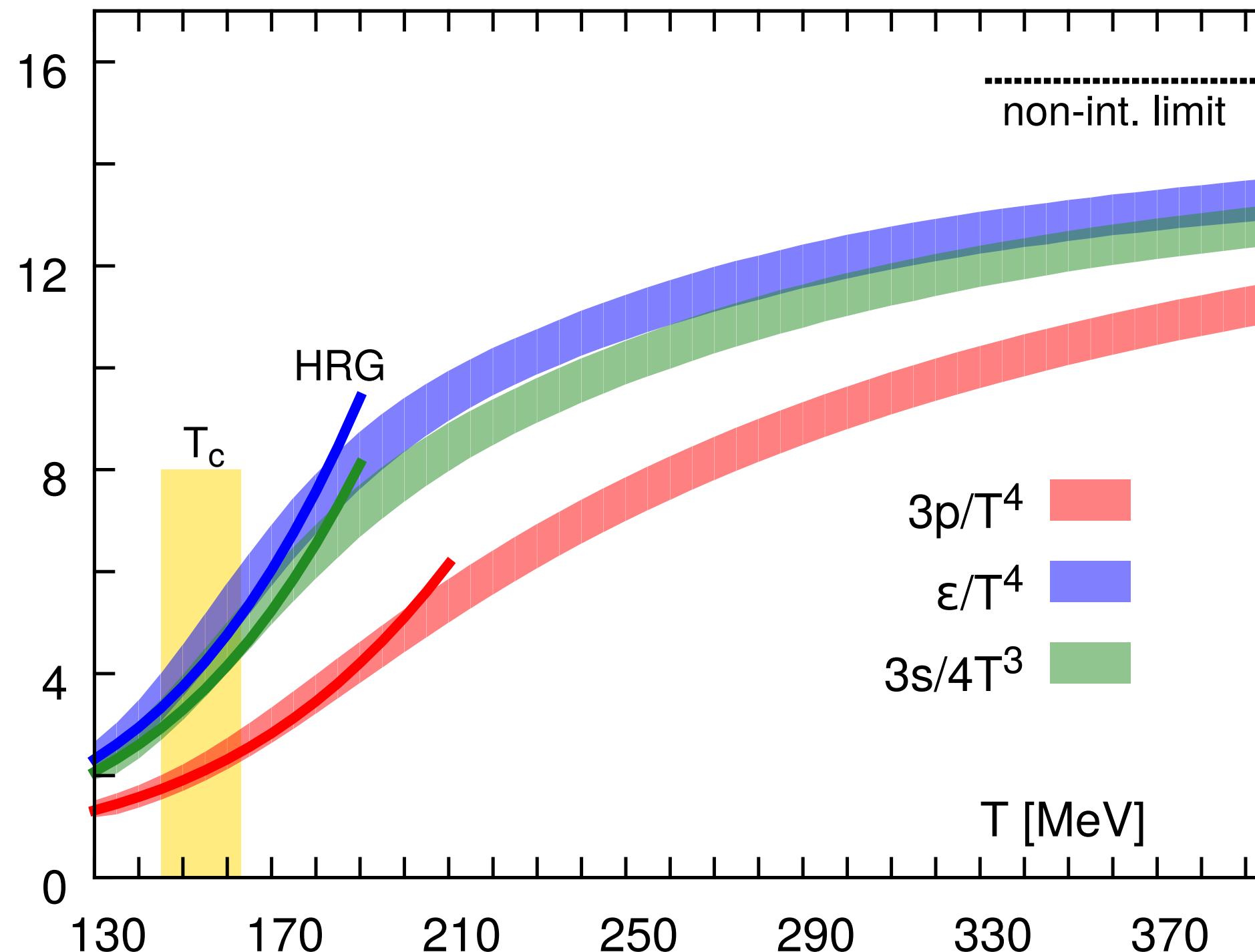
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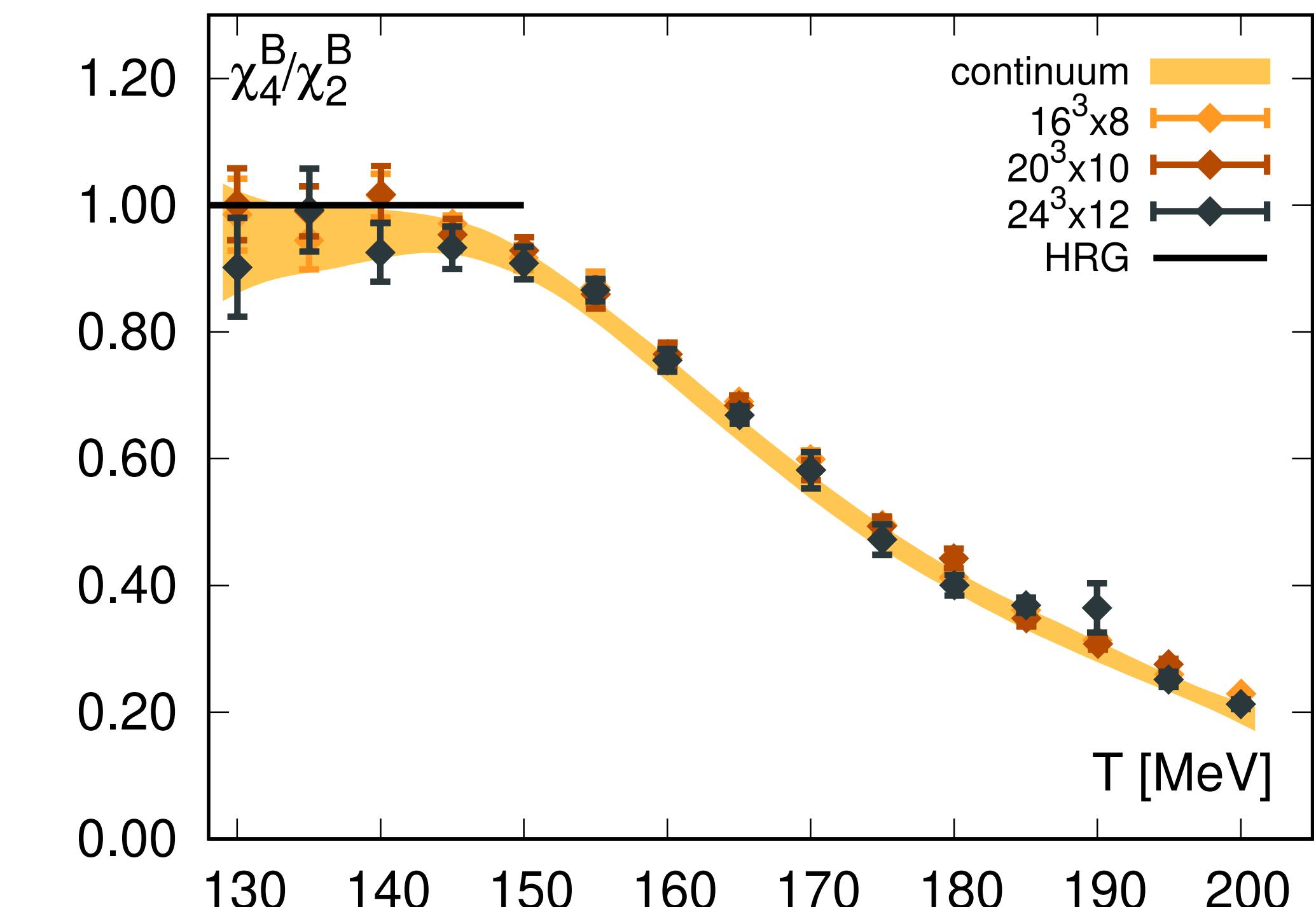
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Lattice QCD vs Hadron Resonance Gas

Bazavov et al, 2014



Borsányi et al, 2023



Pressure in the HRG model

$$P^{\text{HRG}} = \sum_{i \in \text{had}} P^{\text{id}}(T, \mu_i; m_i)$$

Agreement with LQCD EoS up to $\simeq T_c$

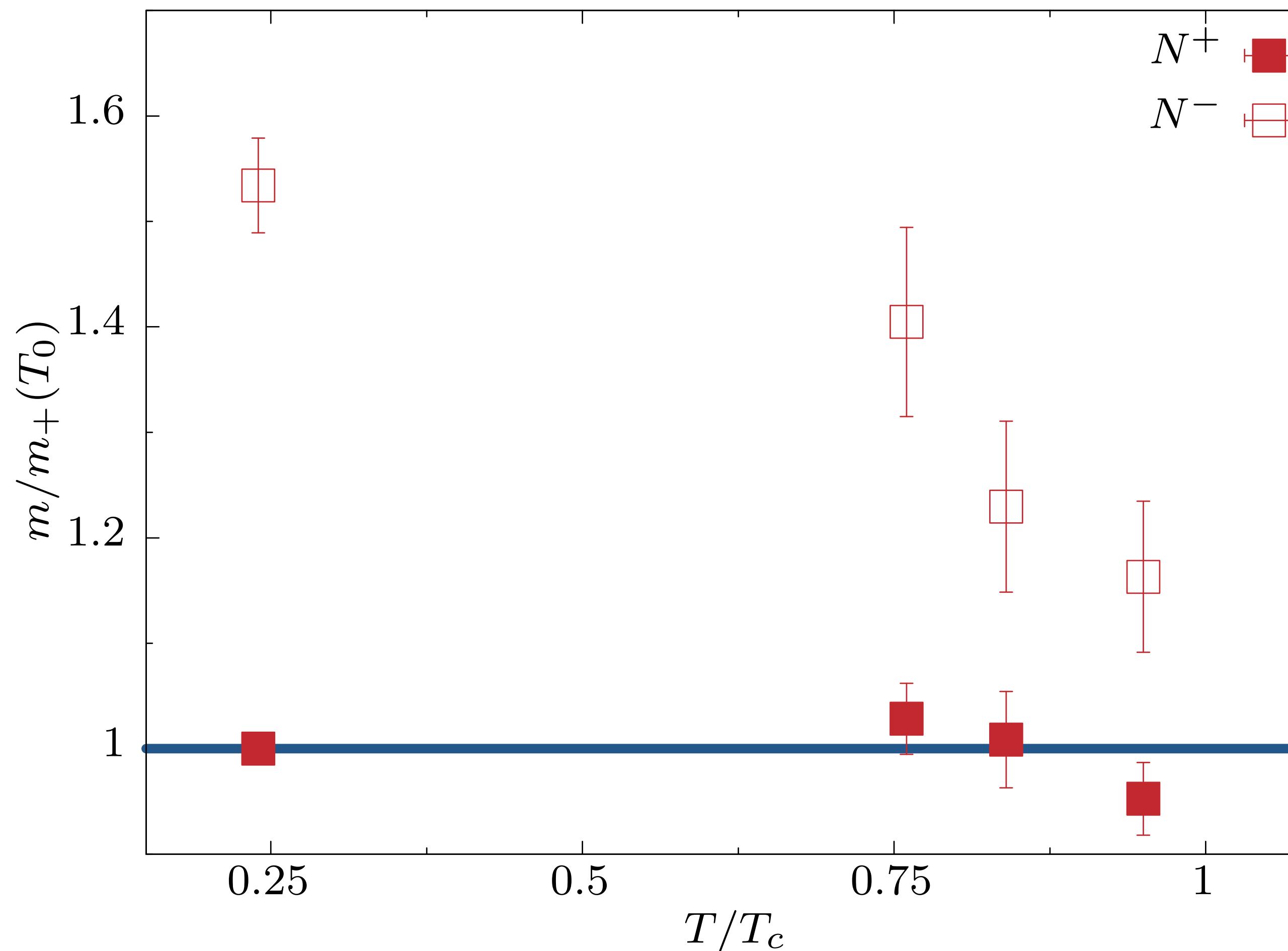
Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T} \right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial (\mu_B/T)^k}$$

Kurtosis: $\frac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Parity Doubling in Lattice QCD

Aarts et al, 2017, 2019



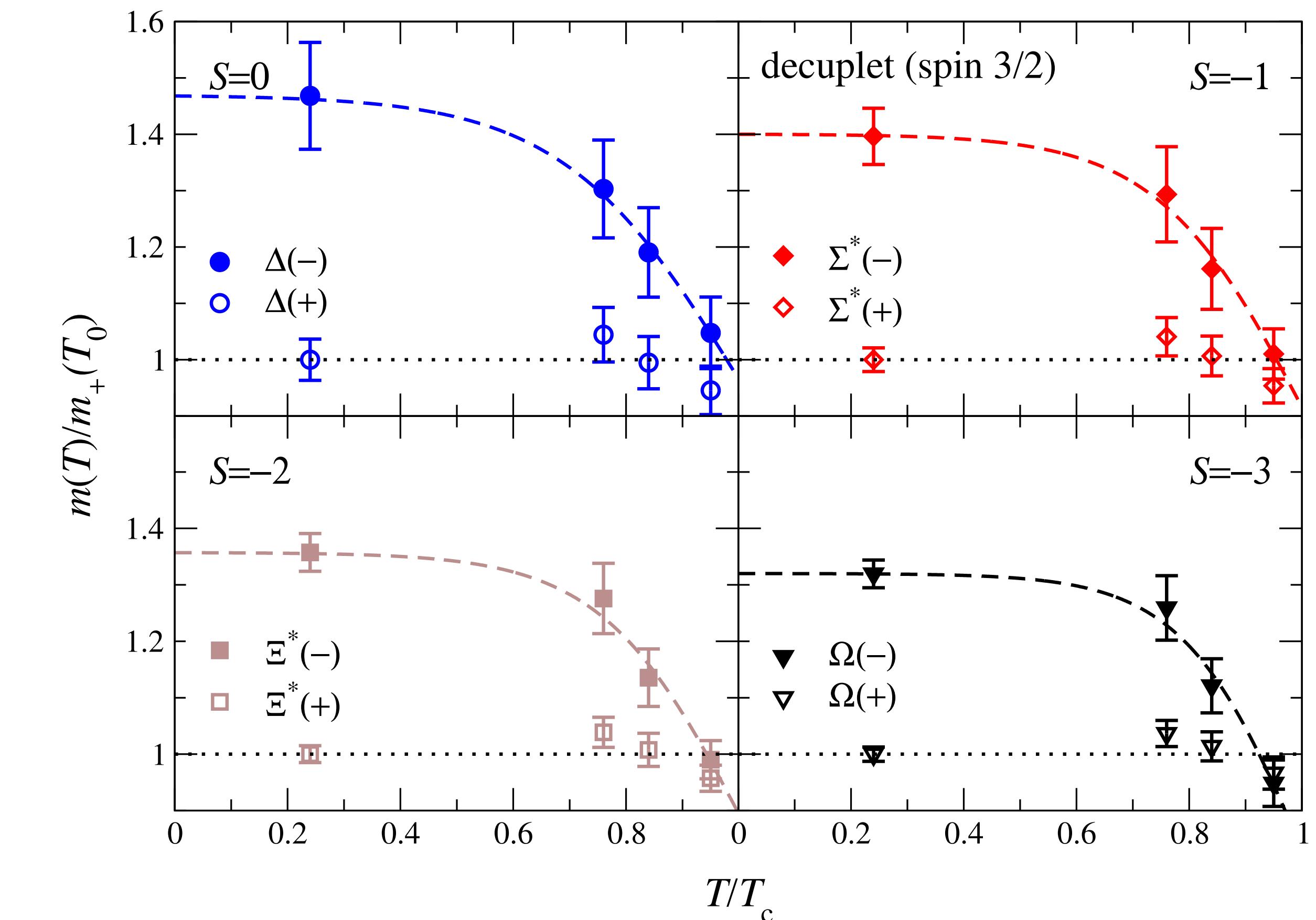
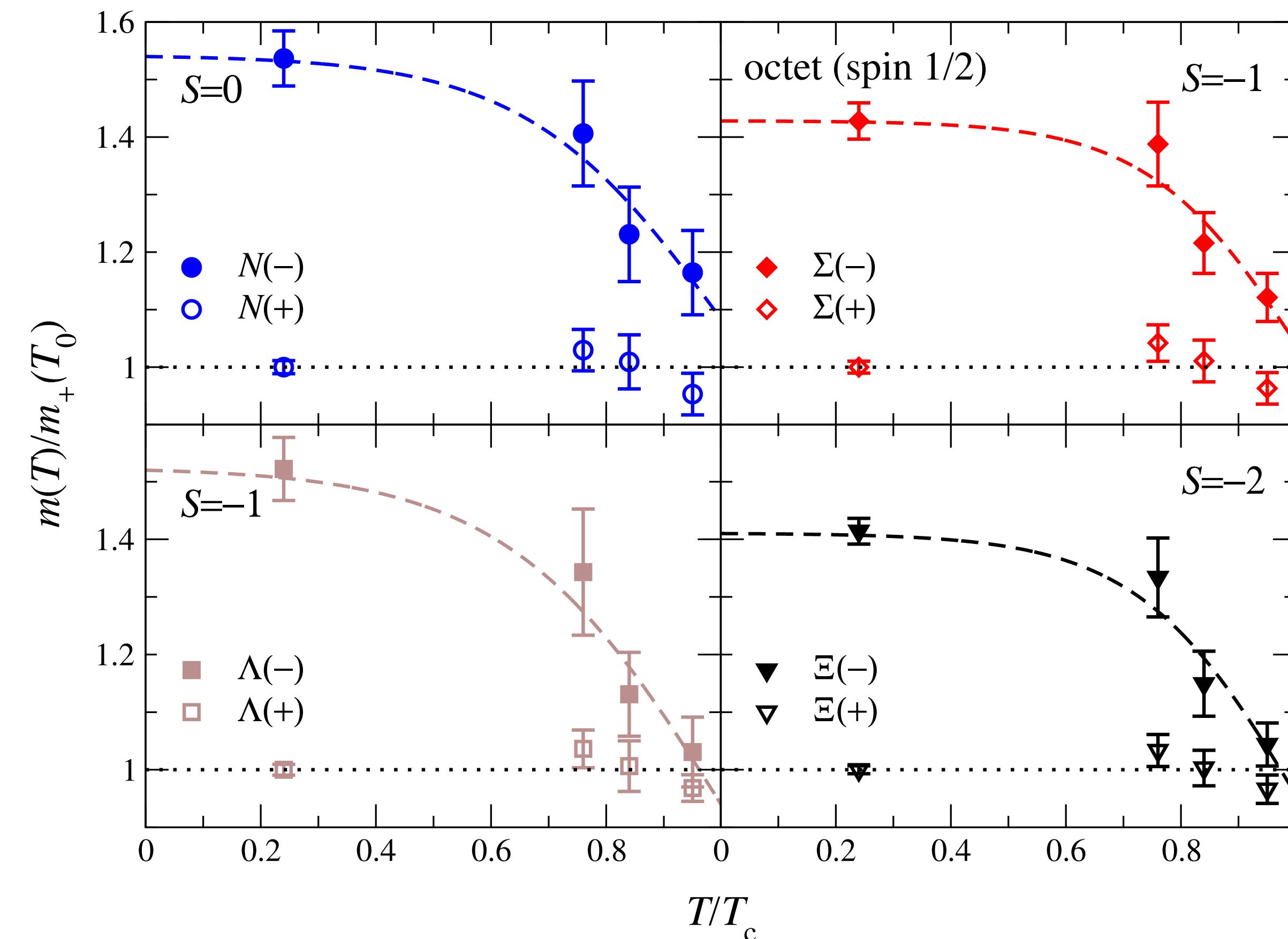
- N^+ nucleon stays nearly unchanged
- N^- chiral partner drops mass towards T_c
- Chiral partners N^\pm degenerate at T_c
- Chiral parents stay massive

Imprint of chiral symmetry restoration in the baryonic sector

LQCD results still obtained with heavy m_π far from continuum limit

Imprint of chiral symmetry restoration in the baryonic sector

Aarts et al, 2019



Clear evidence for partial restoration of chiral symmetry in the strange baryon sector

In-Medium Hadron Resonance Gas

Susceptibilities are sensitive probes of chiral dynamics

$$\chi_2^B = \frac{\partial^2 P/T^4}{\partial(\mu_B/T)^2}$$

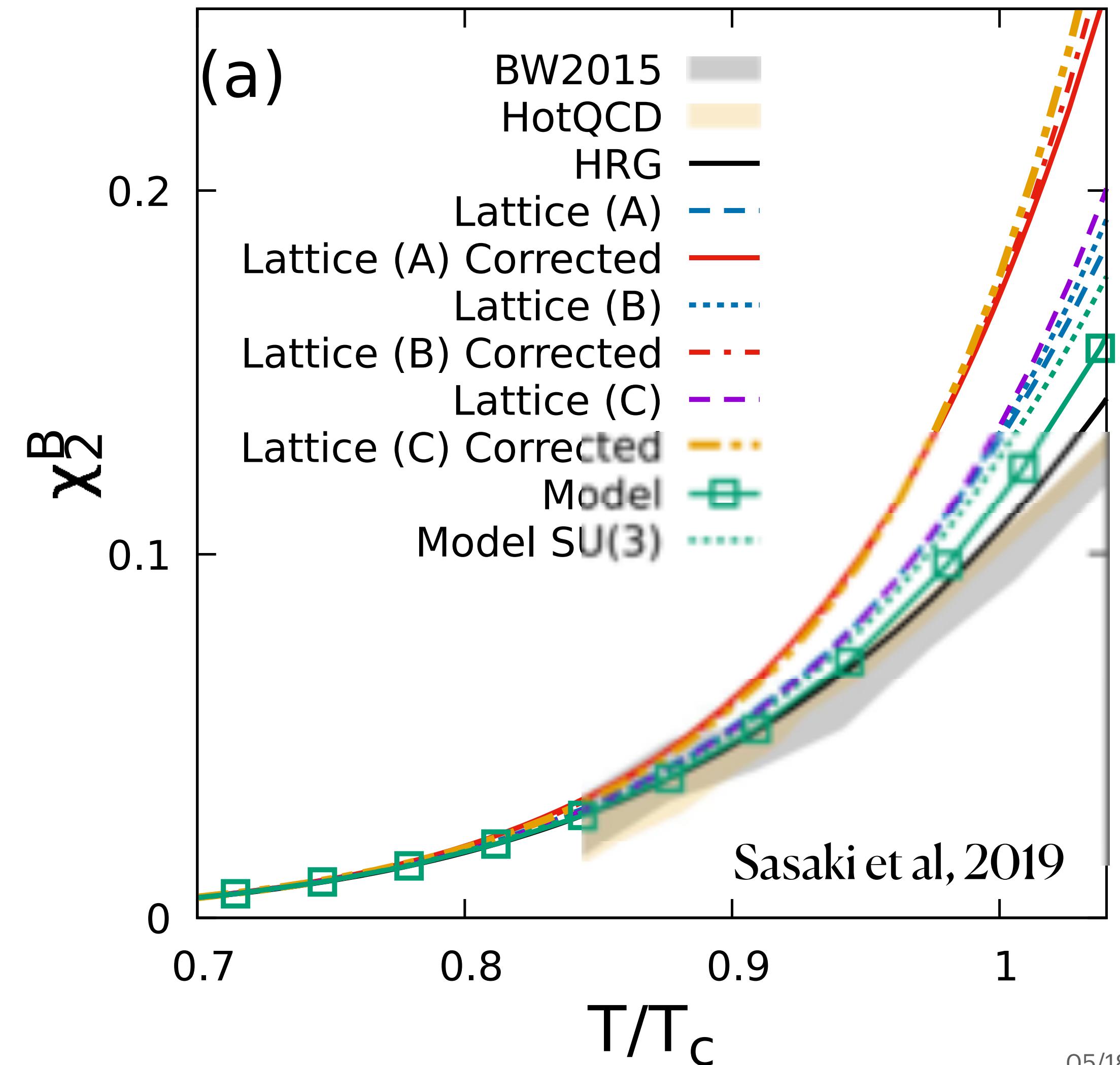
Fluctuations with chiral in-medium baryon masses



Too large fluctuations



Correlations between parity partners



For multiplicity $N_B = N_+ + N_-$

Net-baryon number: $\langle N_B \rangle = \langle N_+ \rangle + \langle N_- \rangle$

Second-order fluctuations of the net-baryon number:

$$\langle \delta N_B \delta N_B \rangle = \langle (\delta N_+)^2 \rangle + \langle (\delta N_-)^2 \rangle + 2 \langle \delta N_+ \delta N_- \rangle$$

$$\langle \delta N_\alpha \delta N_\beta \rangle = VT^3 \chi_n^{\alpha\beta} \quad \longleftrightarrow \quad \chi_2^{\alpha\beta} = \frac{d^2 P / T^4}{d(\mu_\alpha / T) d(\mu_\beta / T)}$$

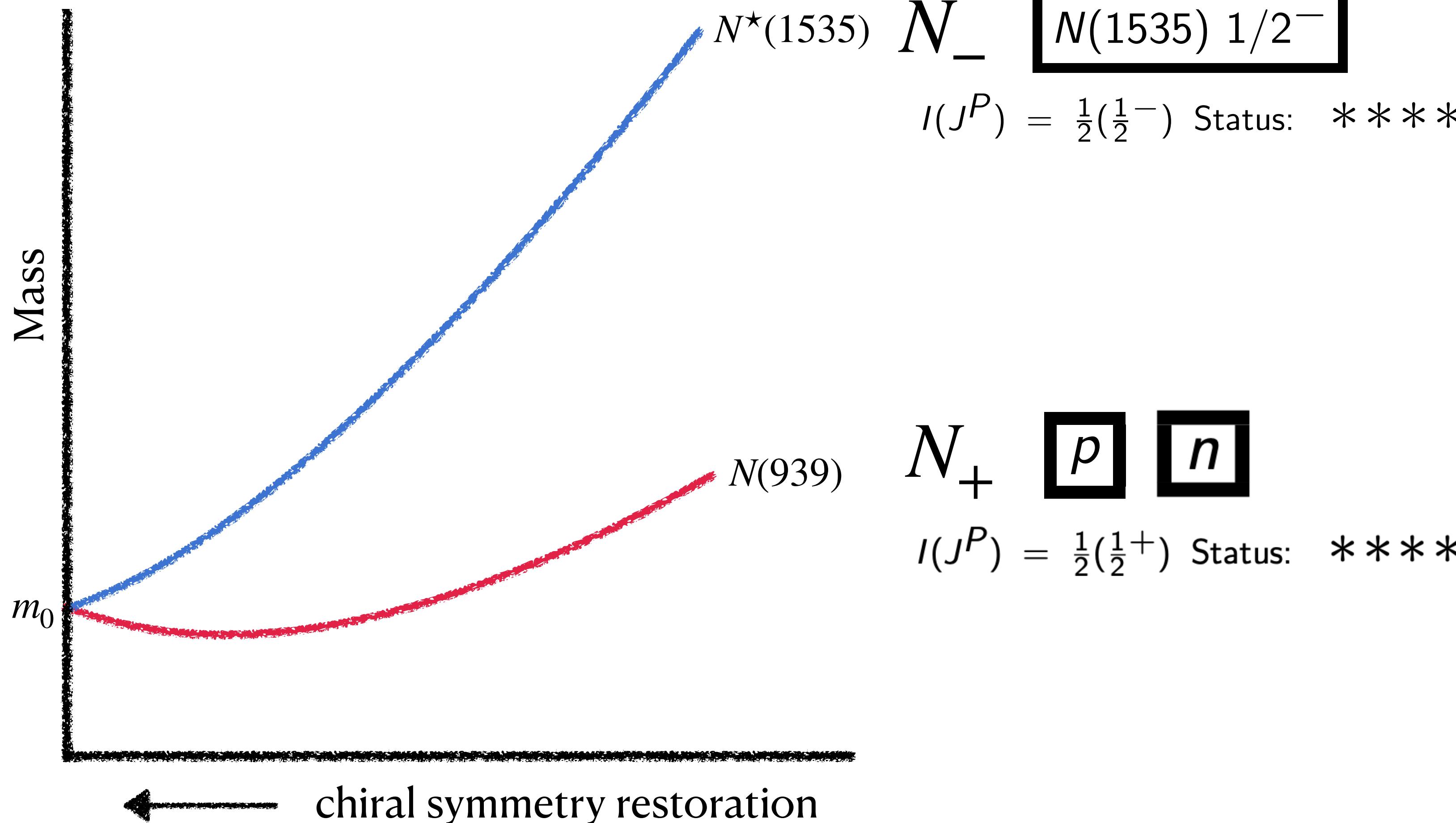
$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

- What are the individual contributions of parity partners N_+ and N_- ?
- What is the strength and sign of the correlation χ_2^{+-} ?
- Is net-proton a good proxy for net-baryon fluctuations? $\chi_2^B = \cancel{\chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}}$

Parity Doublet Model a'la DeTar, Kunihiro 1989

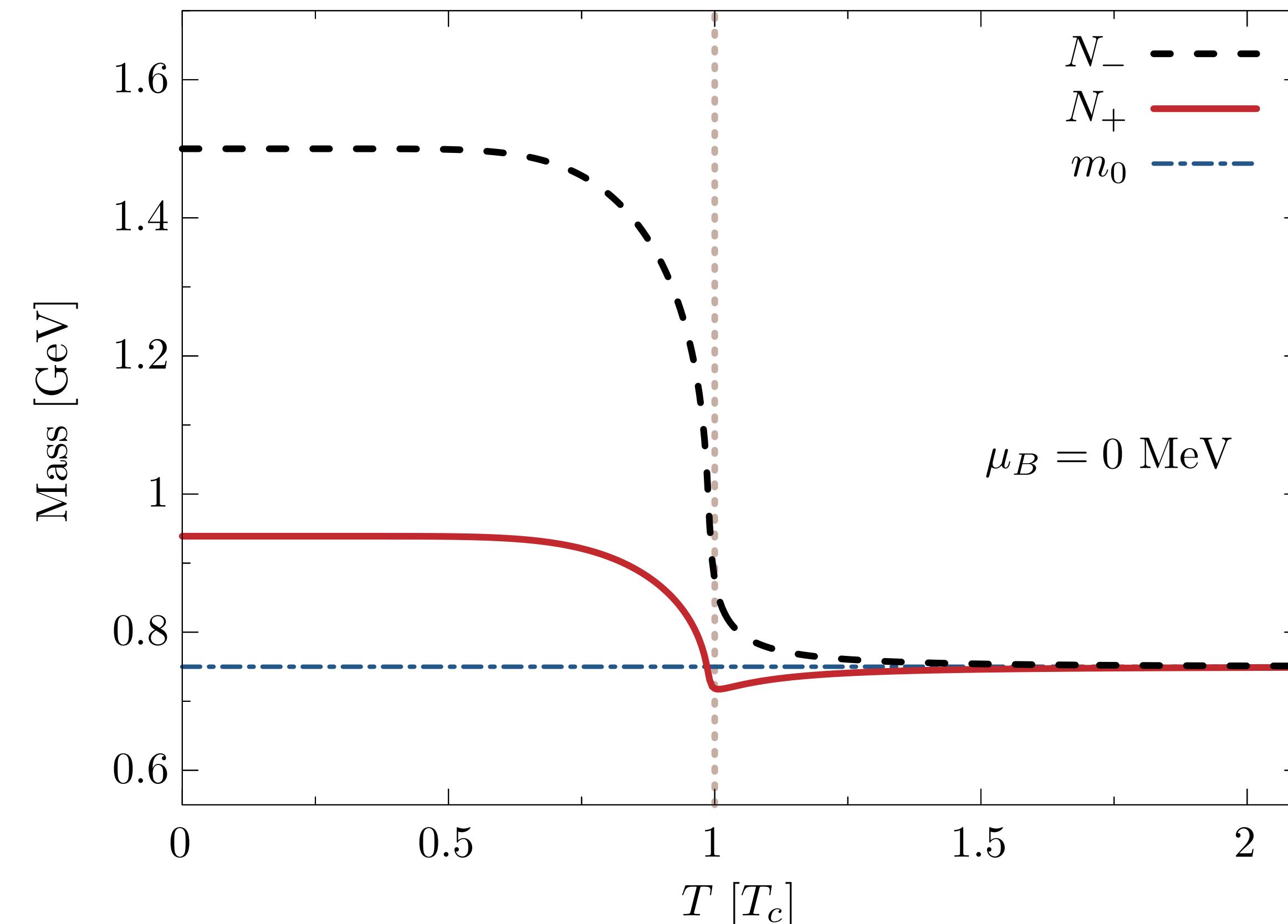
$$\mathcal{L}_{\text{mass}} \sim m_0 (\bar{\psi}_1 \gamma_5 \psi_2 + \bar{\psi}_2 \gamma_5 \psi_1)$$

$$M_{\pm} = \frac{1}{2} \left(\sqrt{4m_0^2 + a^2 \sigma^2} \mp b\sigma \right) \xrightarrow{\sigma \rightarrow 0} m_0$$

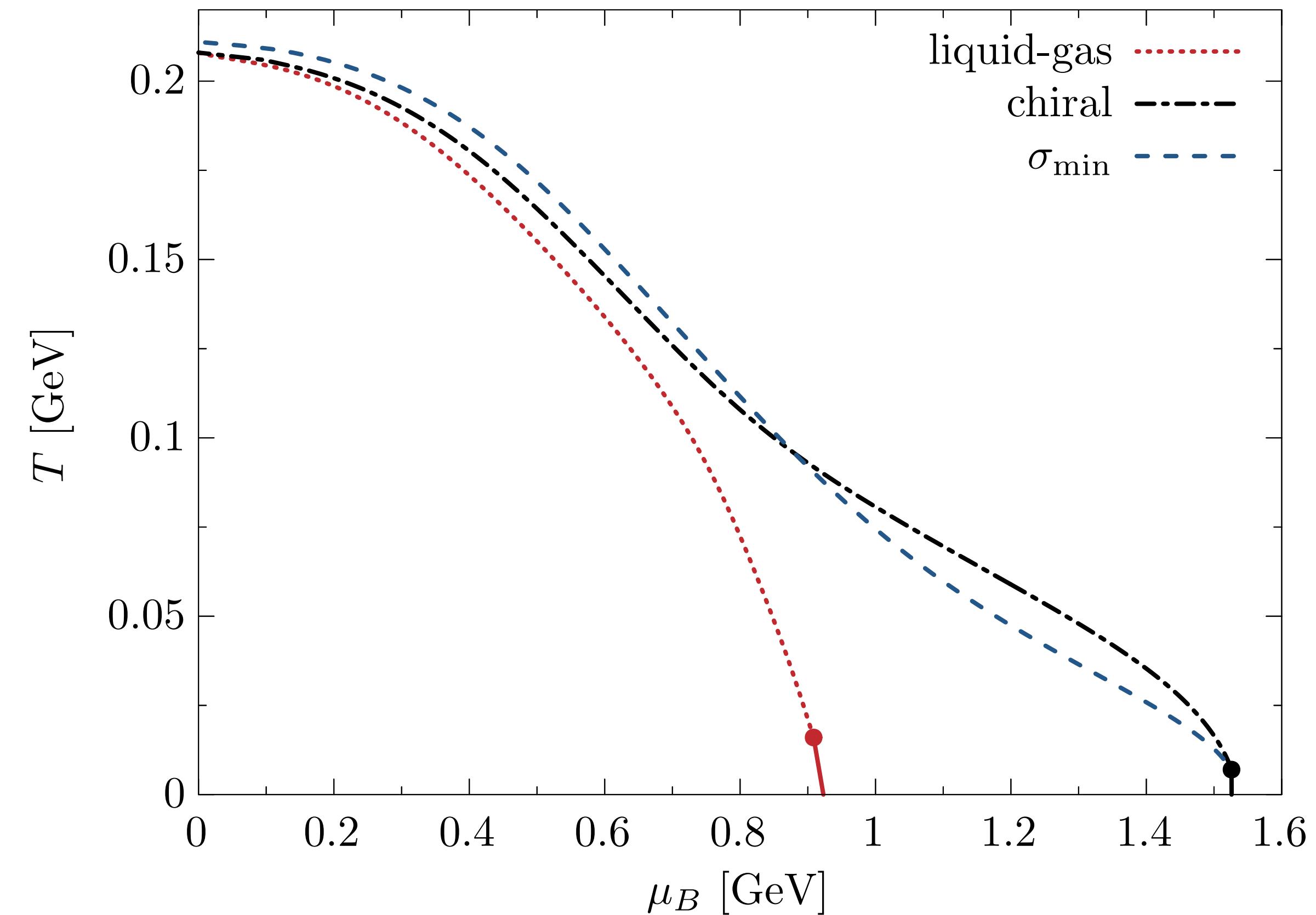


Chiral Criticality in Parity Doubling Model

In-medium masses



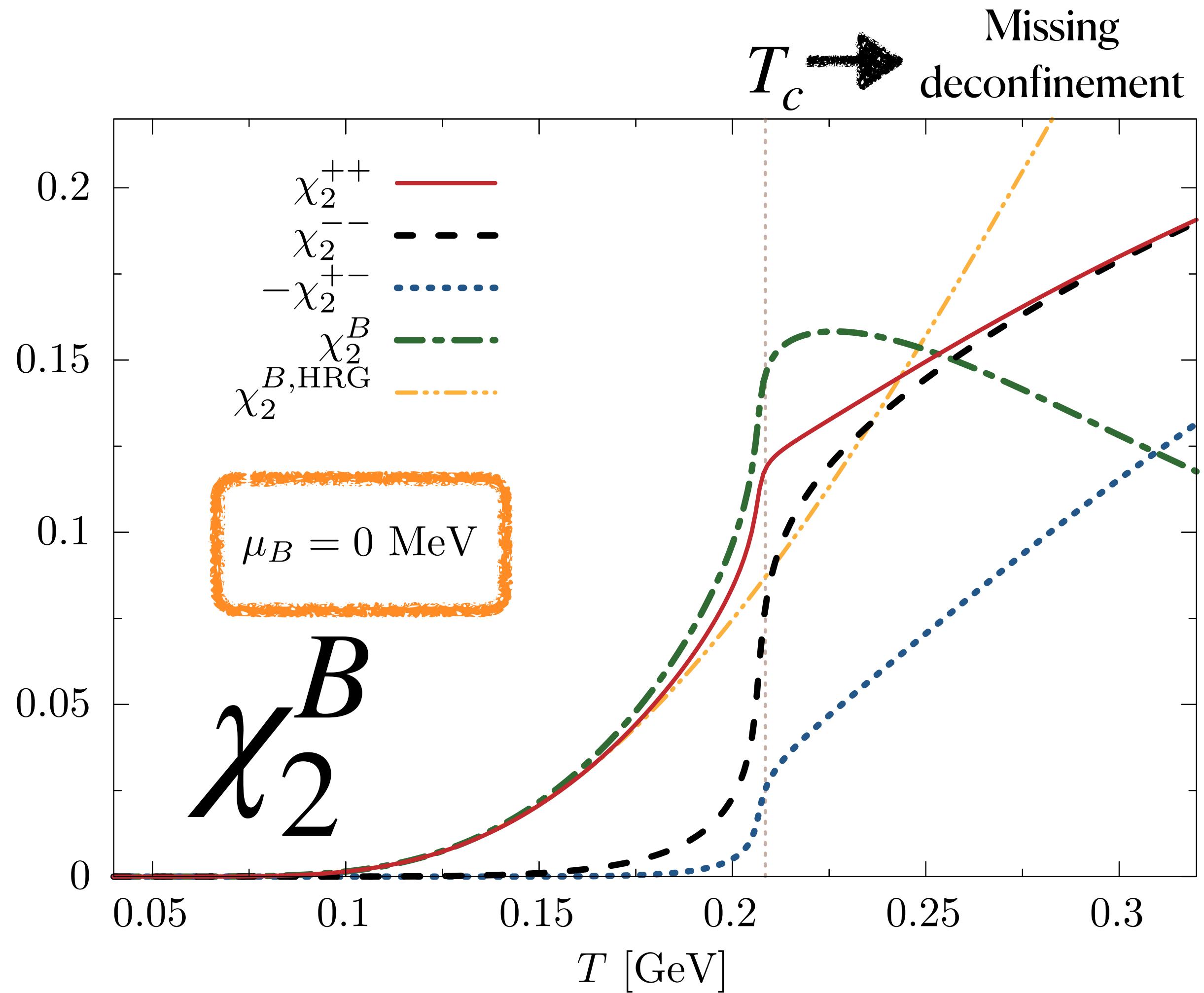
Phase diagram with liquid-gas and chiral PTs



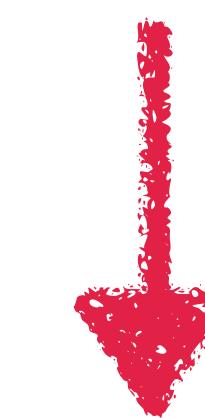
- M_- decreases monotonically
- M_+ has a minimum at $\sigma_{\min} = 2 \frac{b}{a} \frac{m_0}{\sqrt{a^2 - b^2}}$

- Position of σ_{\min} closely related to the chiral phase transition

Fluctuations of chiral partners near crossover at $\mu_B = 0$



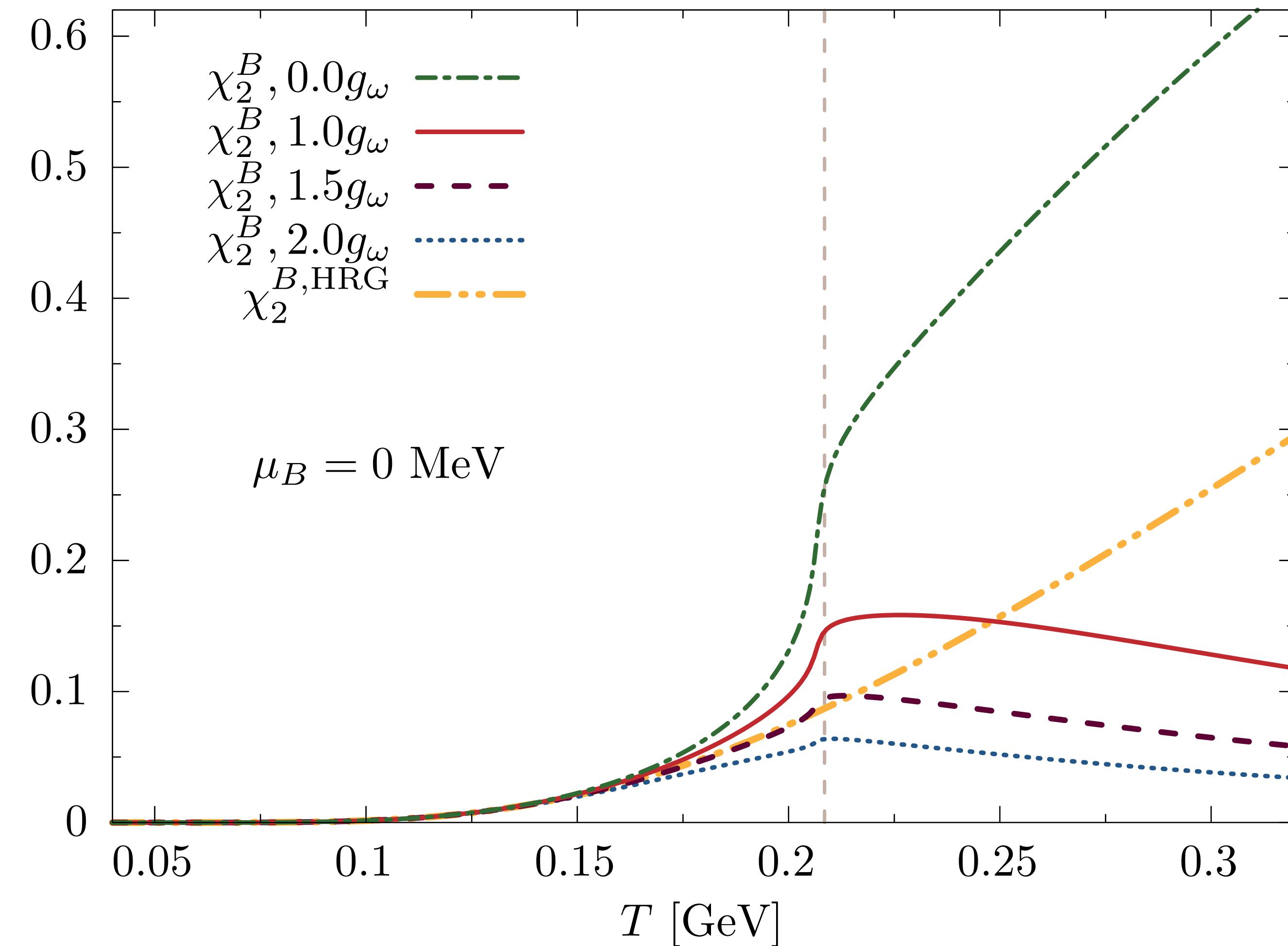
- χ_2^B dominated by nucleon (positive parity)
- N^- relevant only around T_c
- χ_2^{+-} relevant only around T_c and negative
- χ_2^{+-} more negative with repulsive interactions



Net-baryon number fluctuations sensitive to an interplay between repulsive interactions and chiral in-medium baryon masses

Influence of the strength of the repulsive interactions

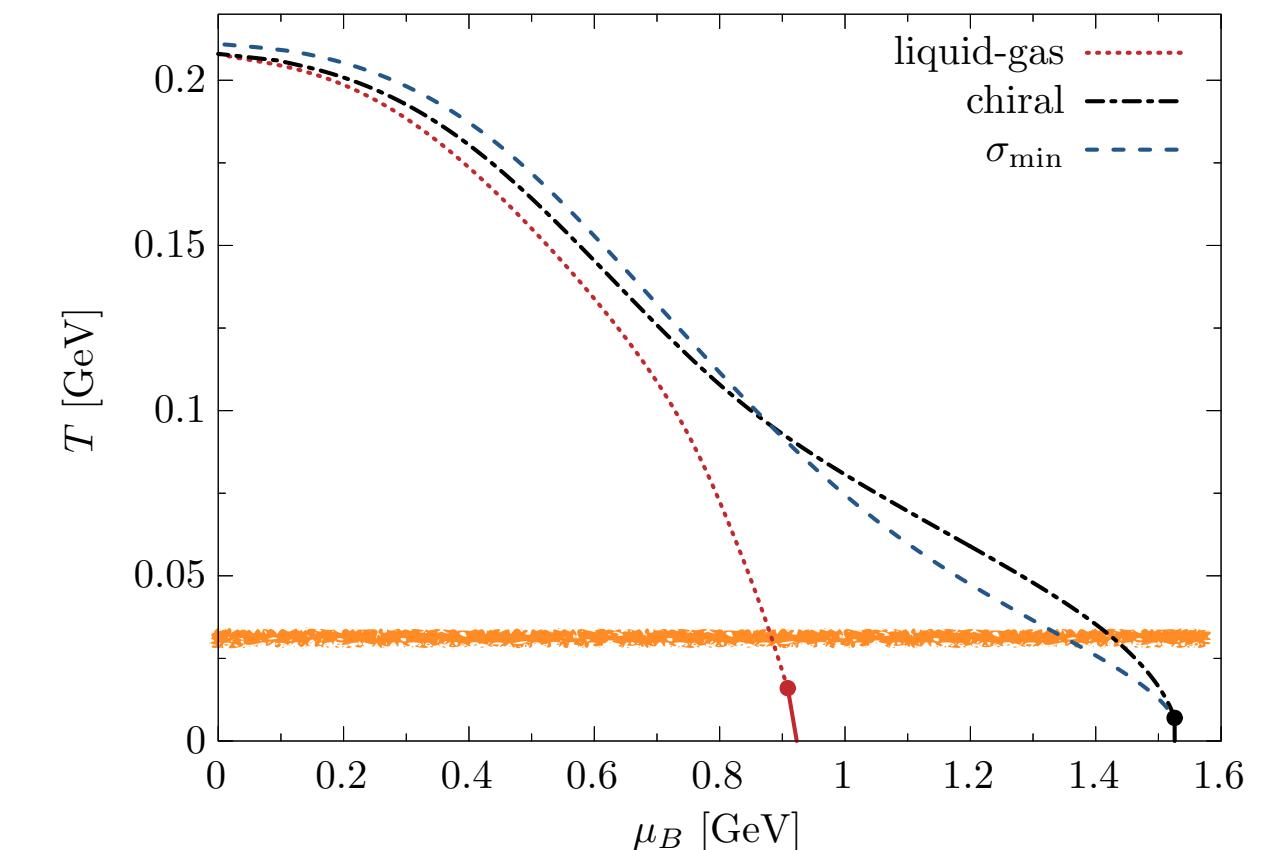
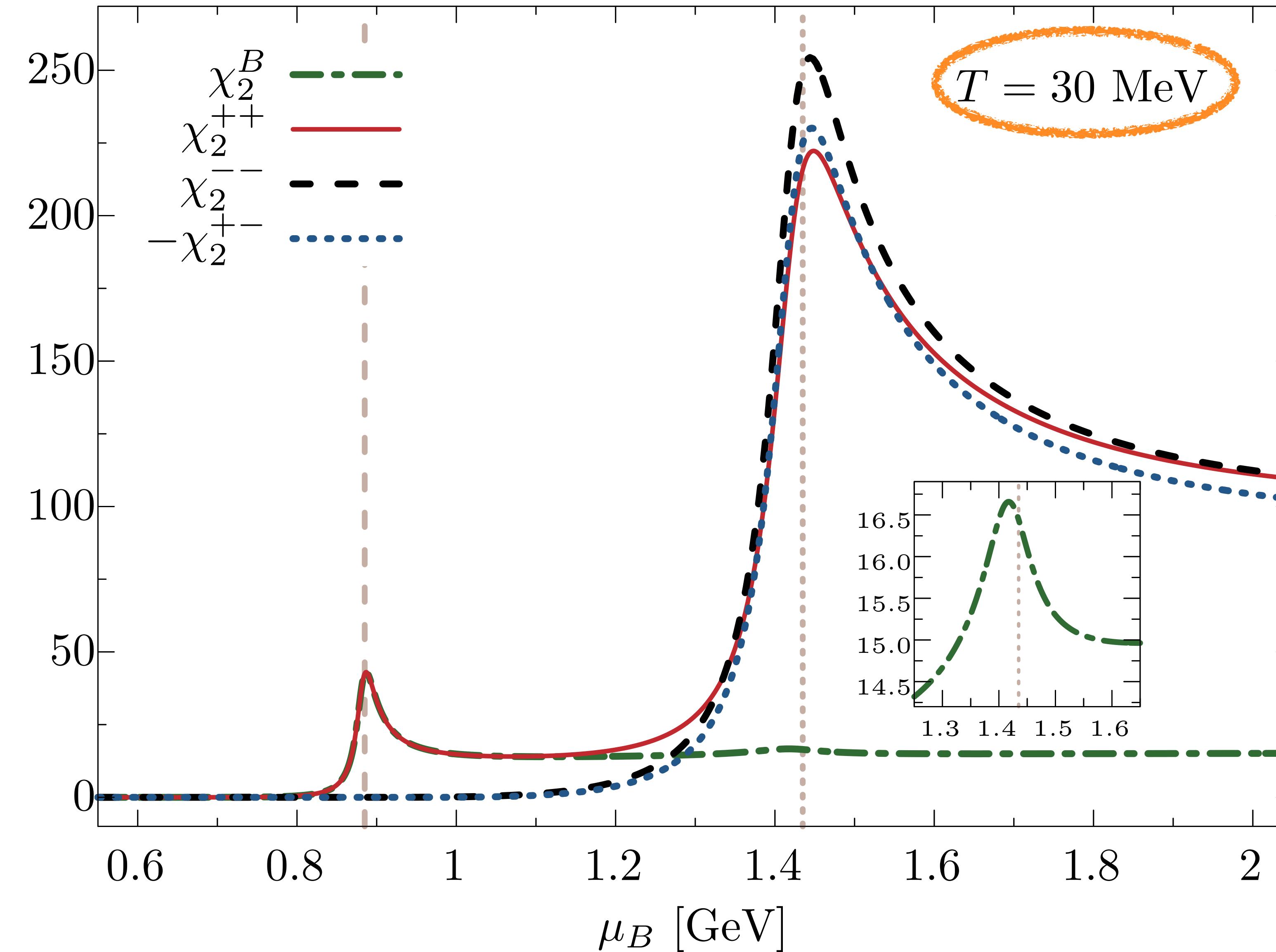
- Clear suppression of fluctuations with increasing repulsive vector interactions
- Increase of fluctuations due to in-medium chiral masses is reduced via negative correlations
- With particular repulsion strength, fluctuations are pushed down to HRG results with vacuum masses



Fluctuations at liquid-gas and chiral transitions

Liquid-Gas

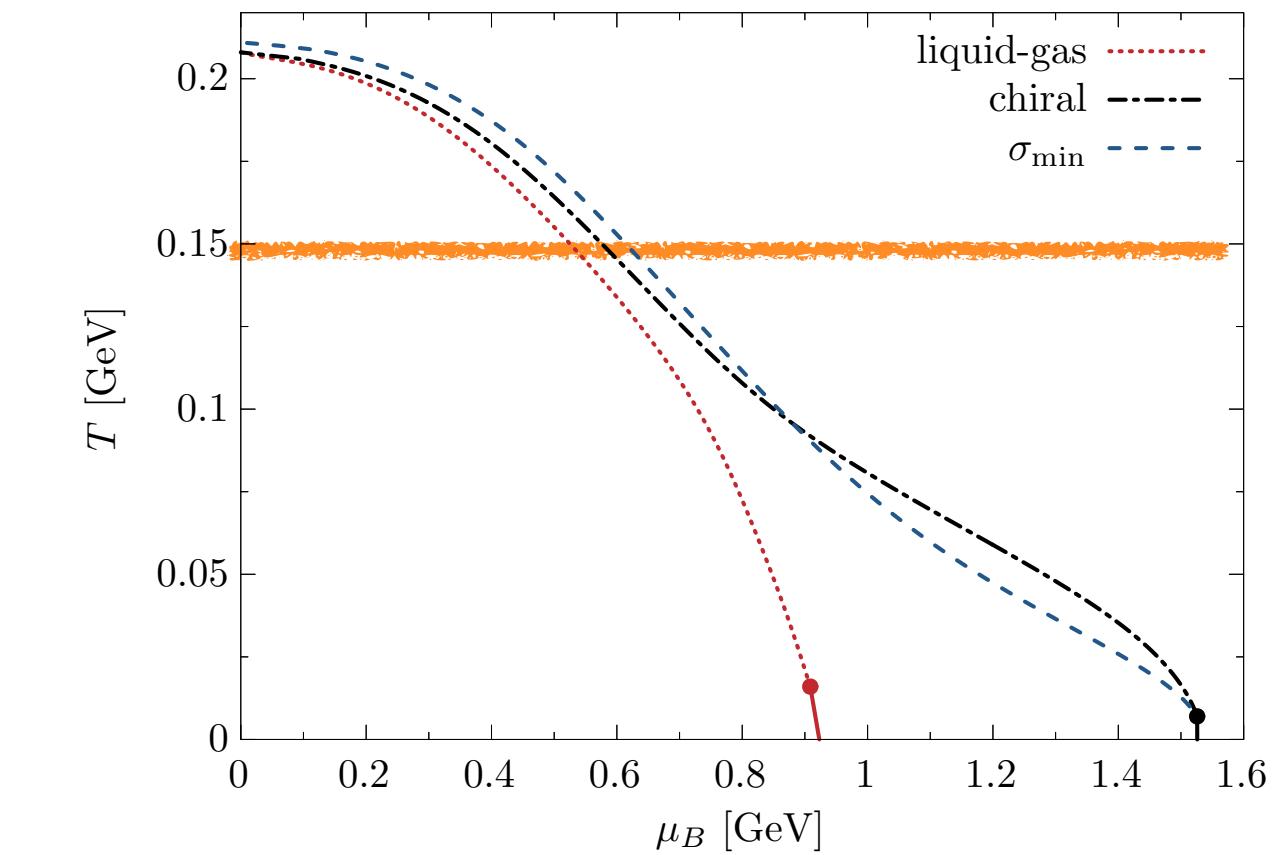
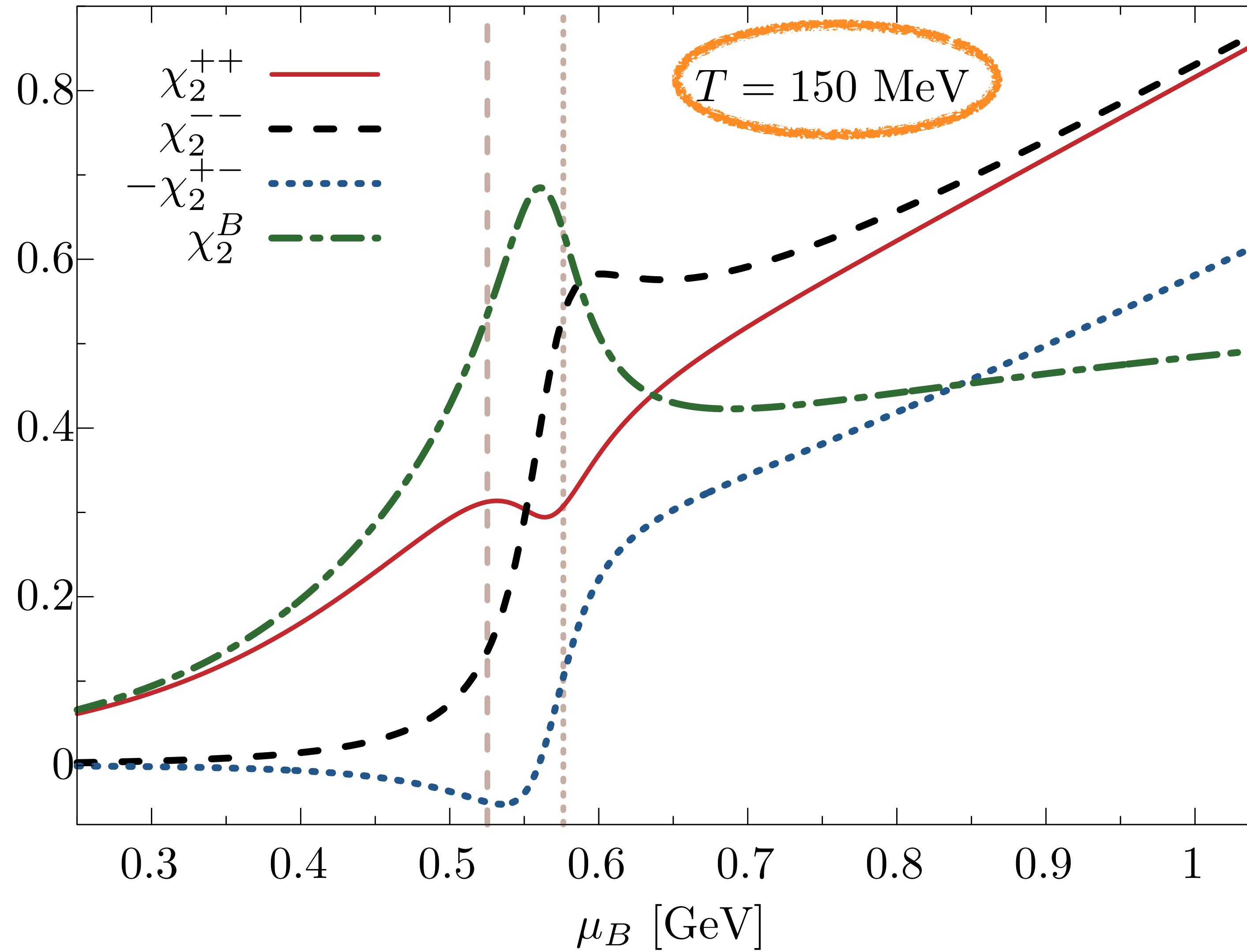
Chiral



Increasing T

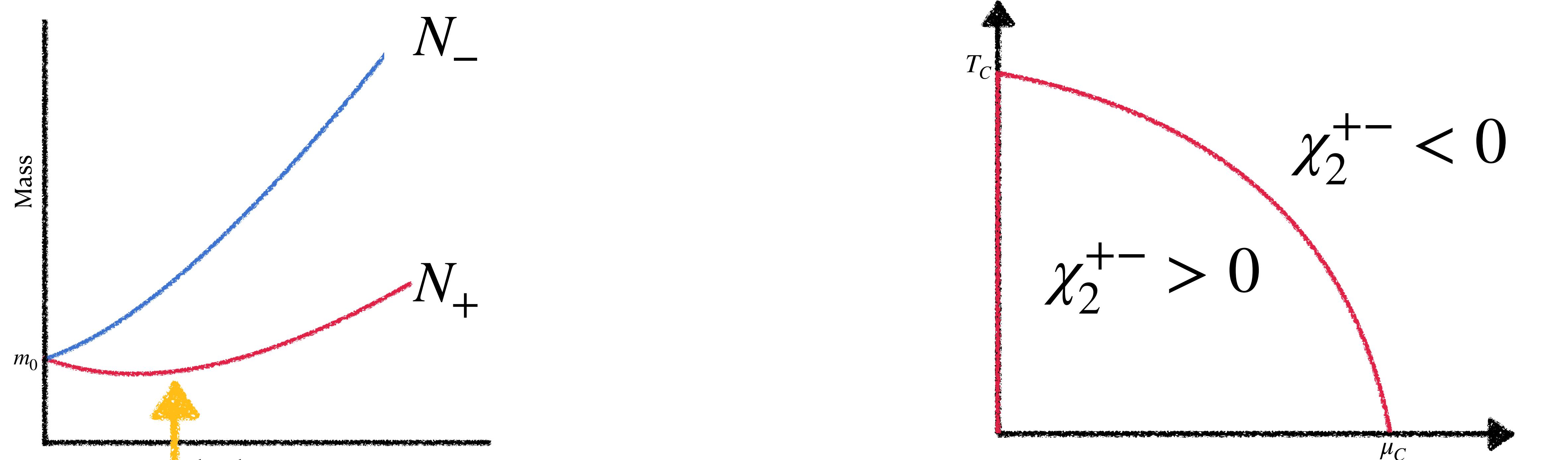
peaks get closer

Liquid-Gas Chiral



- Qualitative difference of χ_2^{++} and χ_2^{--}
- Stronger signal left in χ_2^B

Idealized behavior of the χ_2^{+-} -correlator \rightarrow no repulsive forces



$$\chi_2^{+-} \sim \frac{\partial m_+}{\partial \sigma} - \frac{\partial m_-}{\partial \sigma}$$

but also repulsion

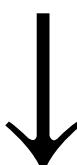
Correlations of between different baryon species e.g., $N^\pm \Delta^\mp$, behave similarly

Change of the sign of χ_2^{+-} linked to the chiral phase boundary

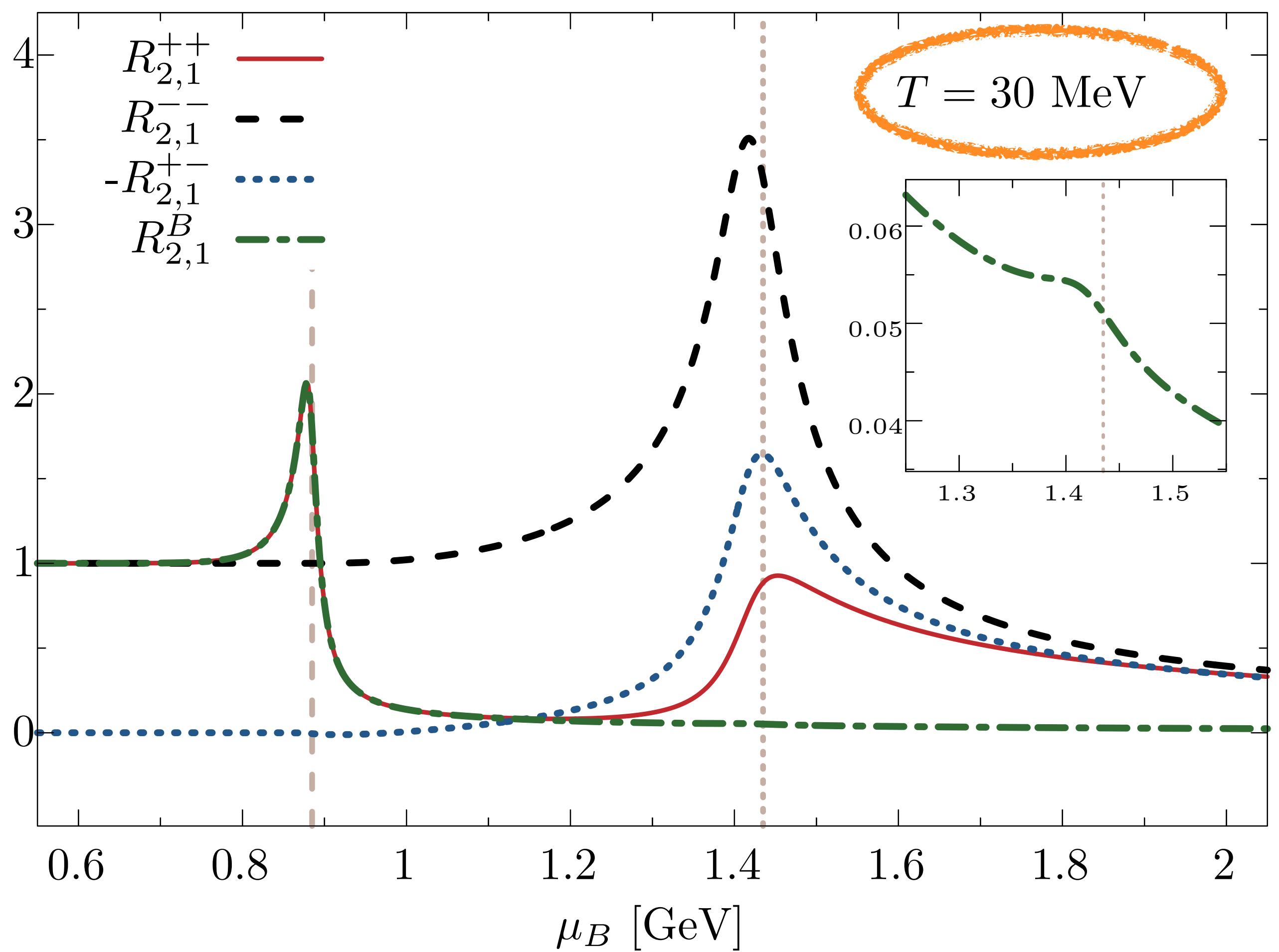


interesting quantity to calculate in LQCD

Cumulants $C_n \sim V$

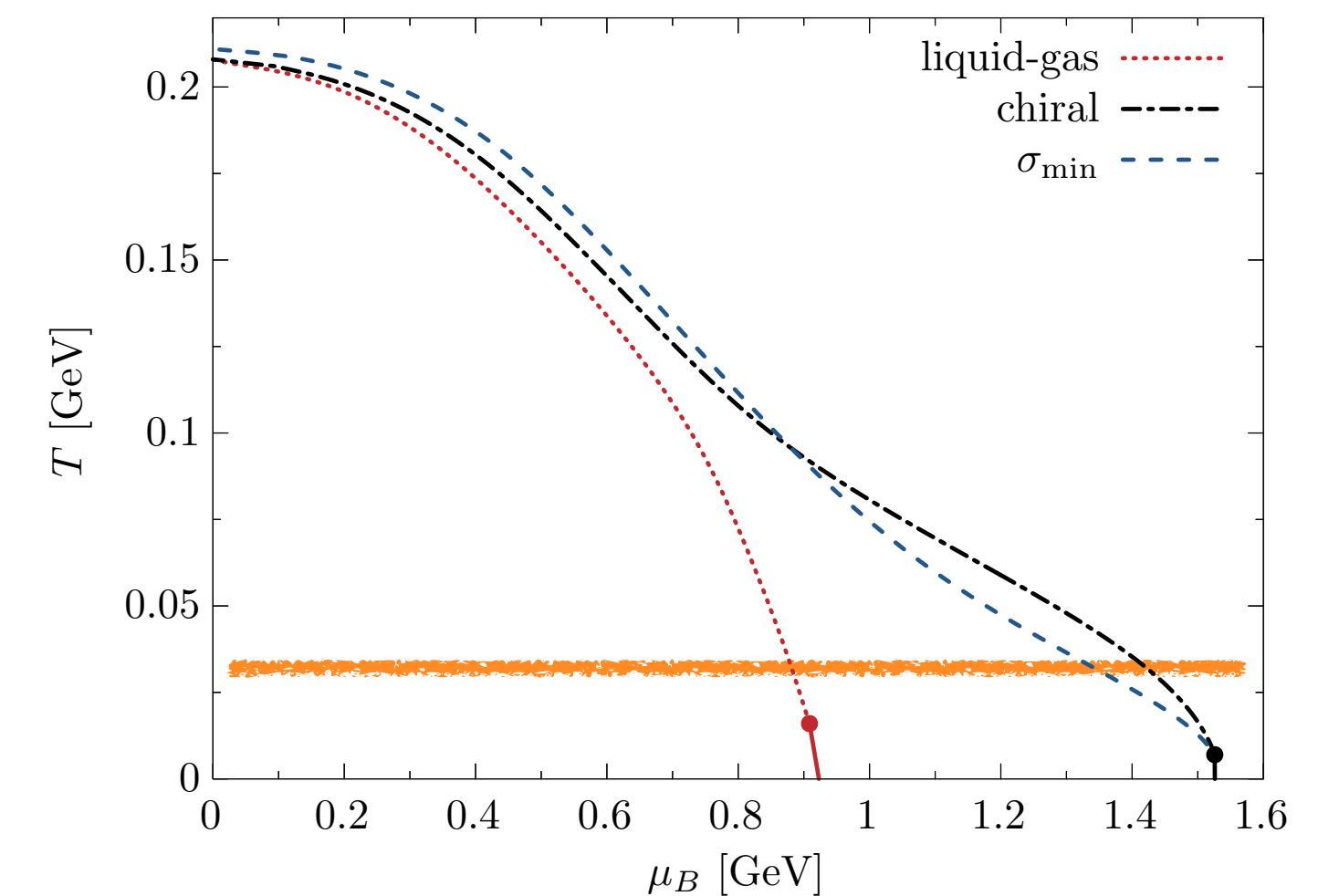


volume cancels in ratios

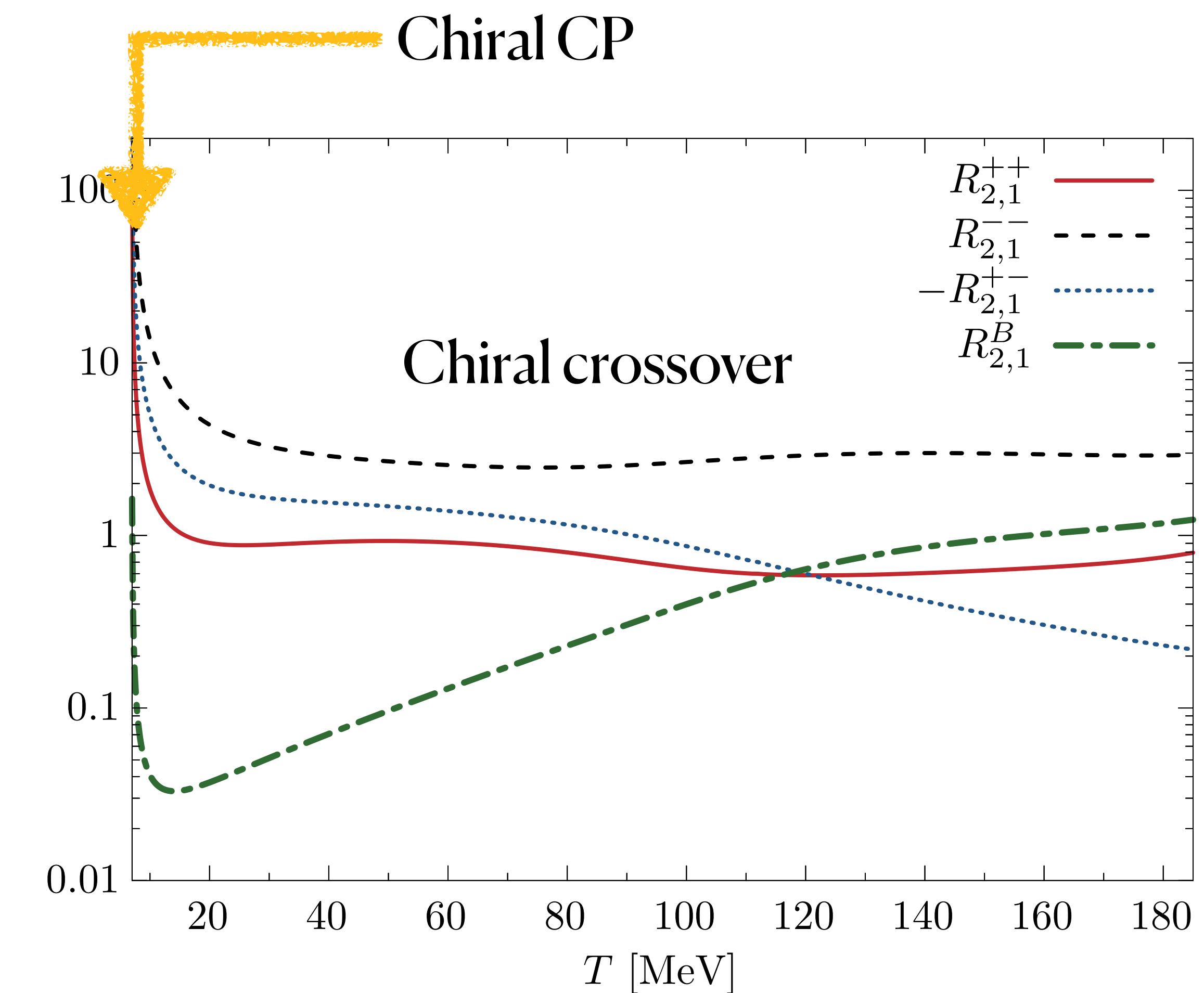
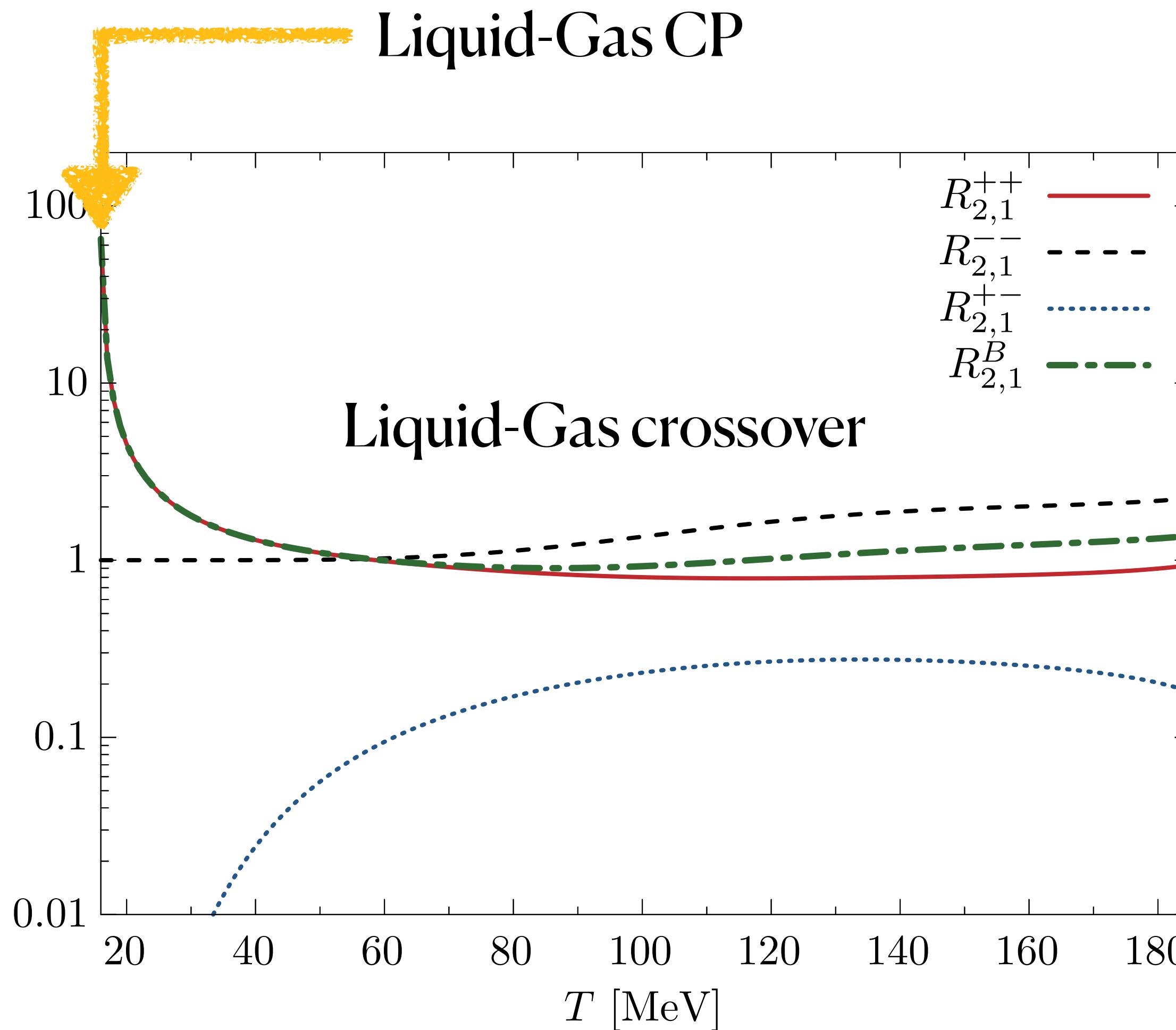


Scaled variance

$$R_{2,1}^{\alpha\beta} \equiv \frac{C_2^{\alpha\beta}}{\sqrt{C_1^\alpha C_1^\beta}} = \frac{\chi_2^{\alpha\beta}}{\sqrt{\chi_1^\alpha \chi_1^\beta}} = \frac{\sigma^2}{M}$$



Approaching CPs



Fluctuations dominated by positive parity



Net-baryon \sim Net-nucleon

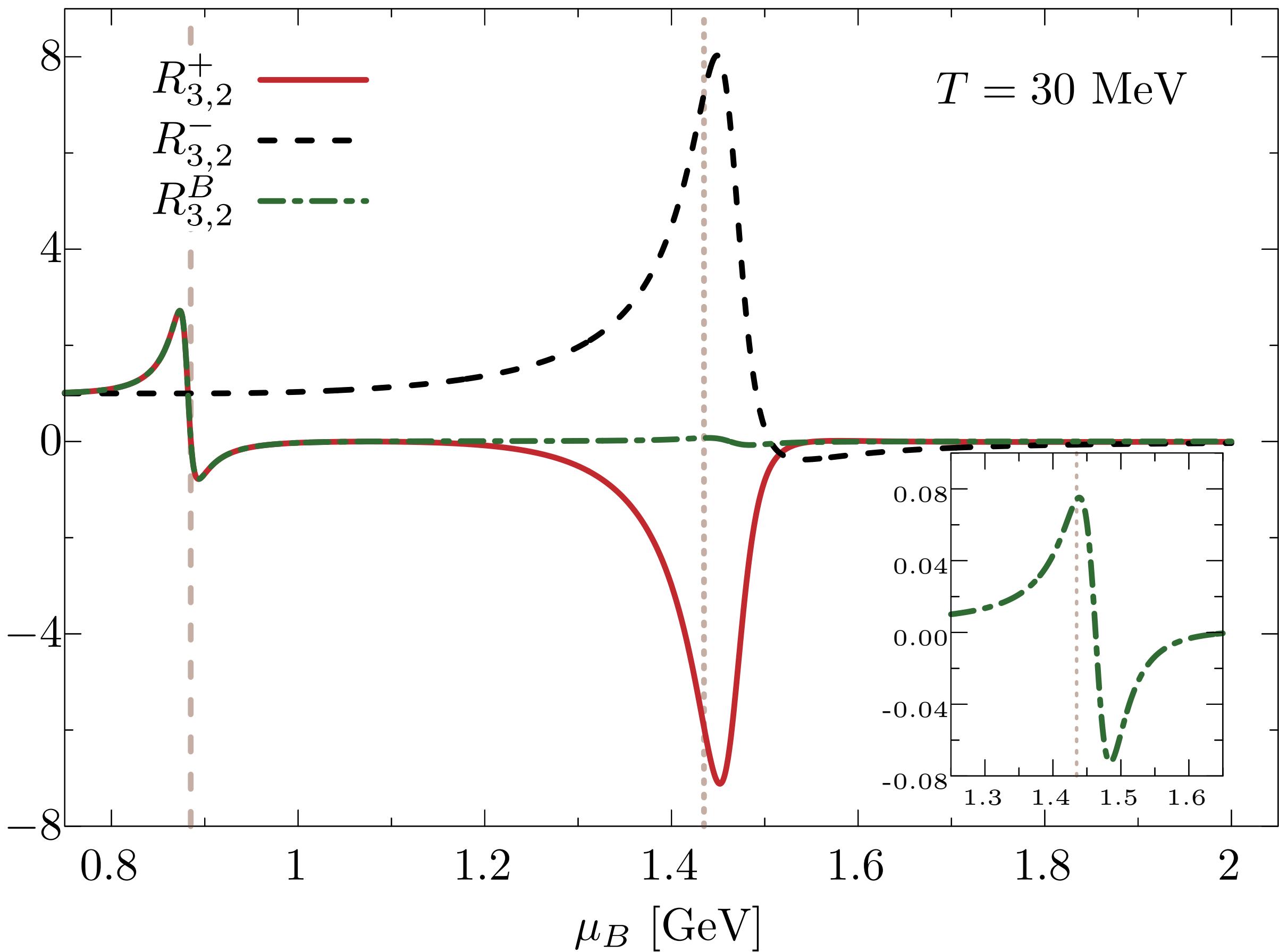
Presence of chiral partners + correlations



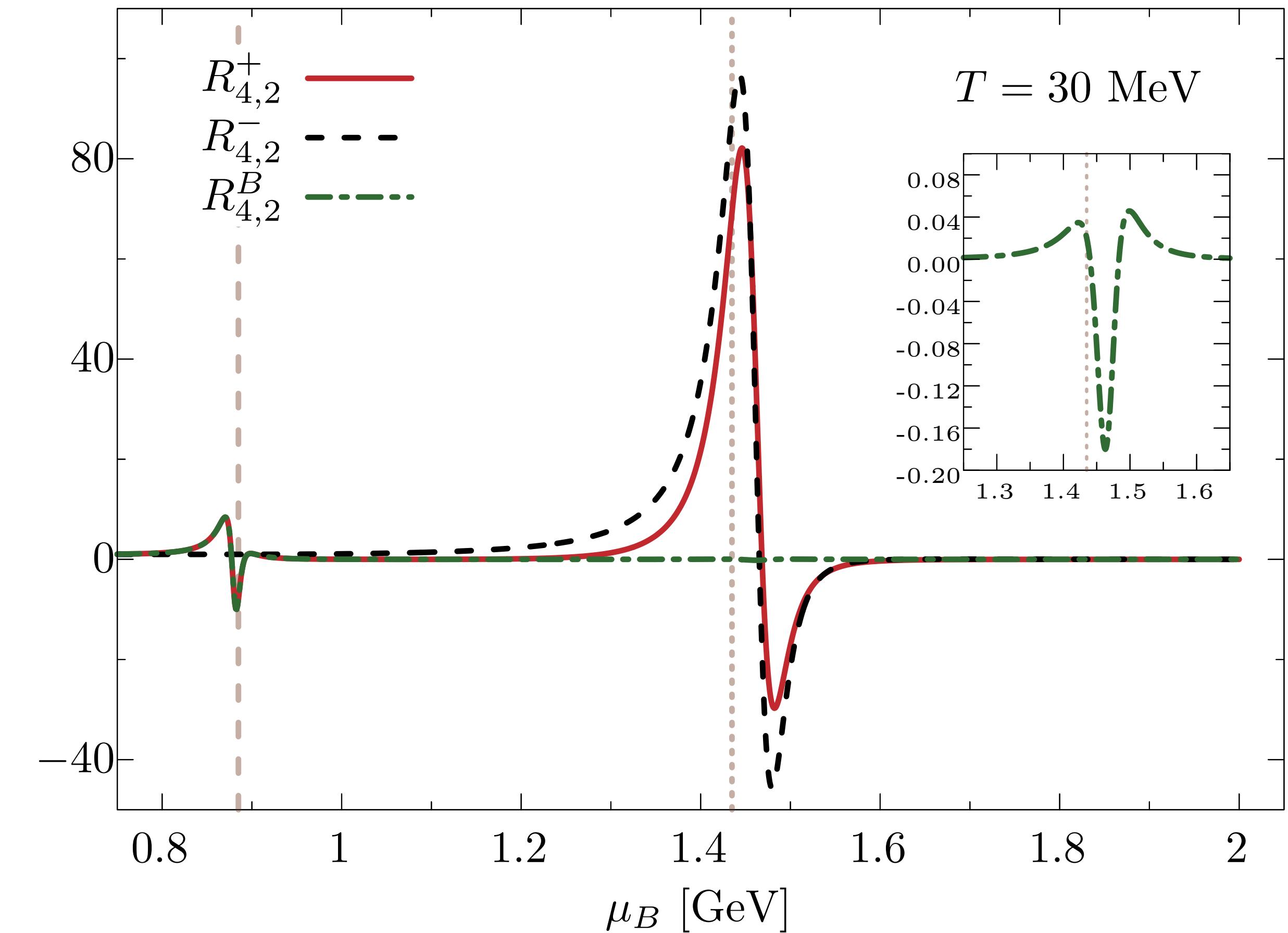
Net-baryon \ll Net-nucleon

Higher-Order Fluctuations of Parity Partners

$$R_{3,2} = \chi_3 / \chi_2$$



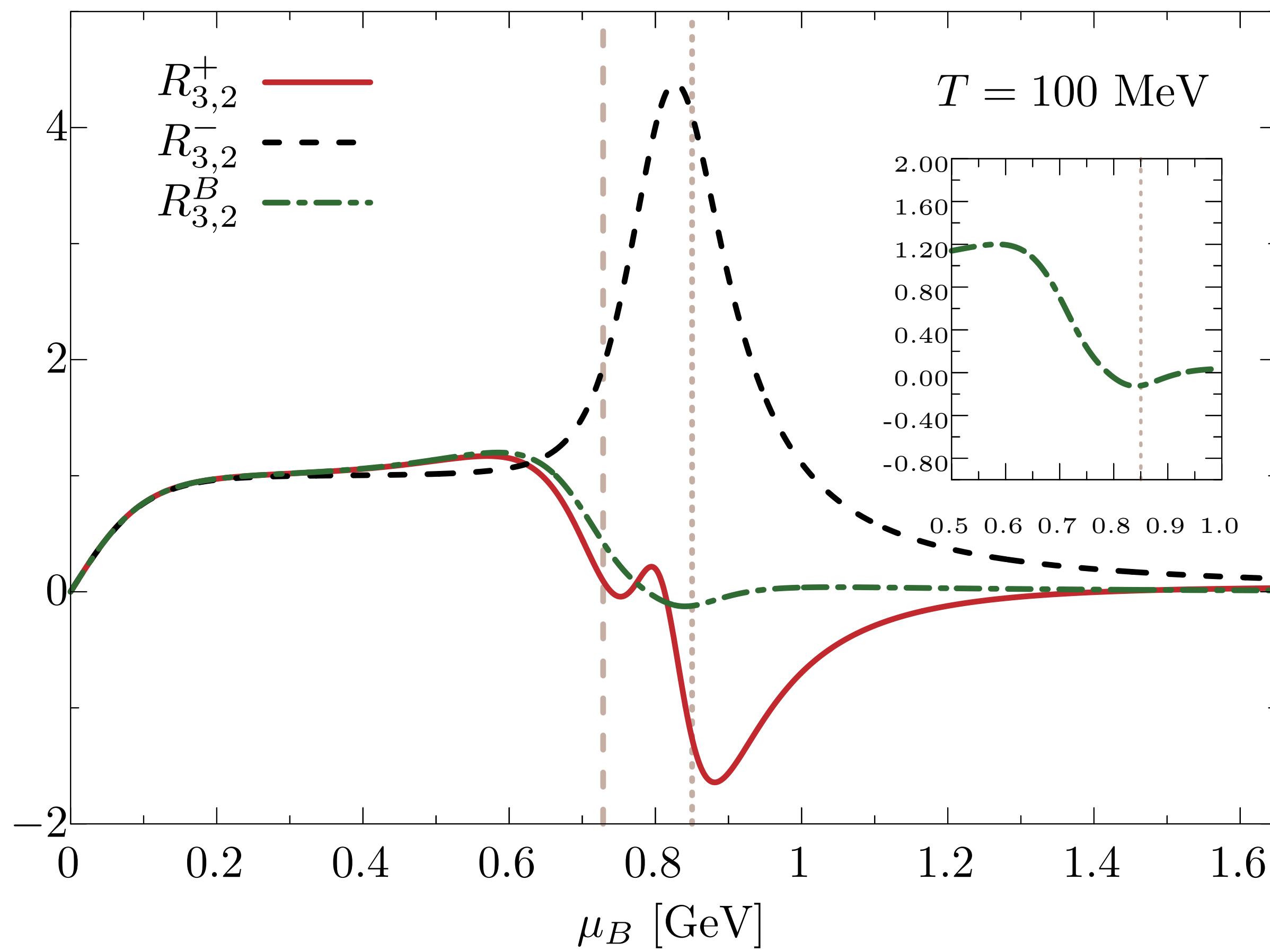
$$R_{4,2} = \chi_4 / \chi_2$$



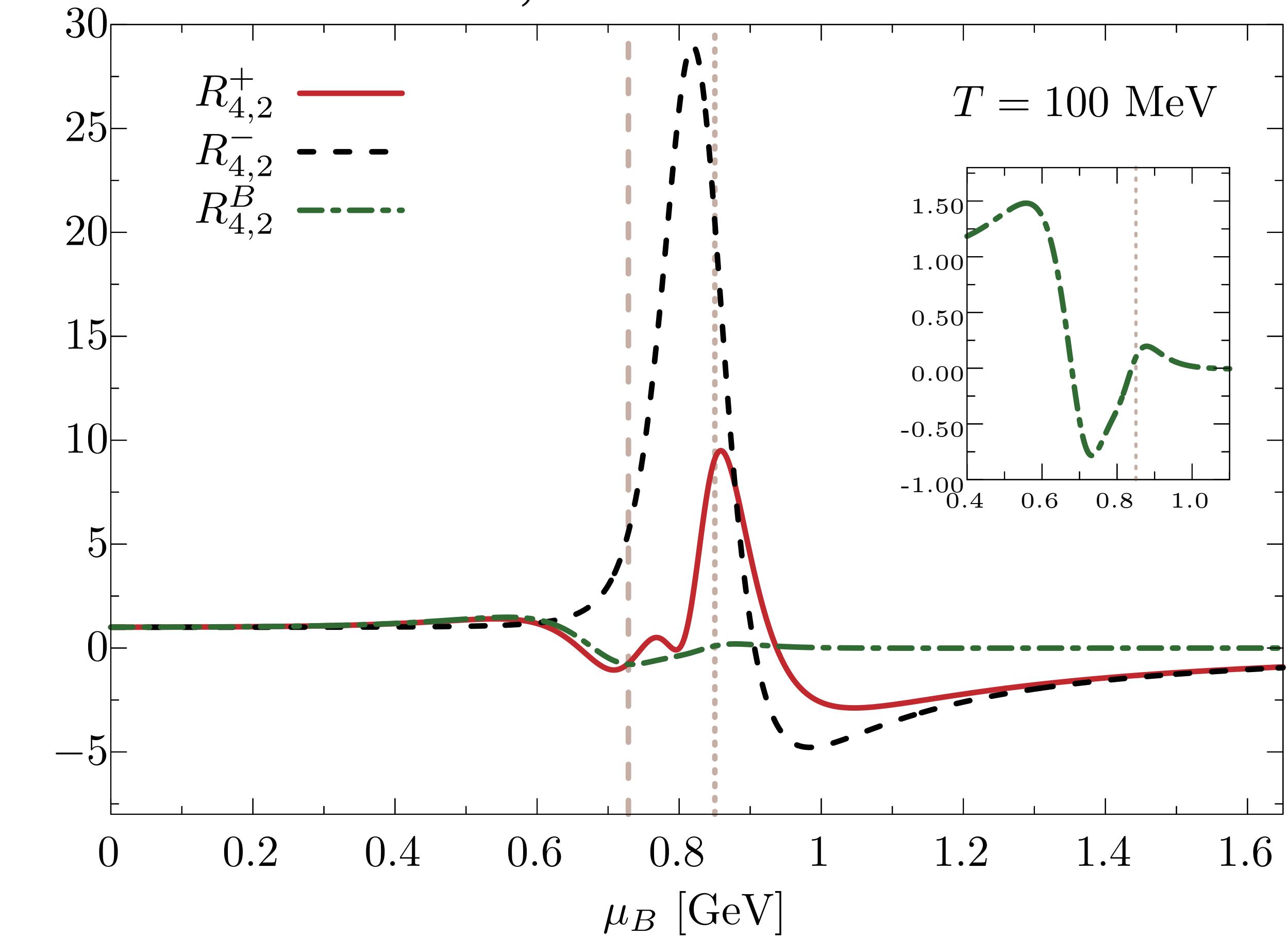
- Very different properties of N^\pm in the ratios
- Especially $R_{3,2}^\pm$ are quite different close to the chiral phase boundary

Higher-Order Fluctuations of Parity Partners

$$R_{3,2} = \chi_3 / \chi_2$$



$$R_{4,2} = \chi_4 / \chi_2$$



net-proton number \neq net-baryon number fluctuation ratios

Summary

Negative correlations between baryonic chiral partners

χ_2^{proton} may not reflect χ_2^B at the chiral phase boundary

Interesting to calculate $\chi_2^{++}, \chi_2^{--}, \chi_2^{+-}$ in other non-perturbative approaches

Thank You

Cumulants vs Susceptibilities

Mean: M	$\langle N_B \rangle$	C_1
Variance: σ^2	$\langle (\delta N_B)^2 \rangle$	C_2
Skewness: S	$\langle (\delta N_B)^3 \rangle / \sigma^3$	$C_3/C_2^{3/2}$
Kurtosis: K	$\langle (\delta N_B)^4 \rangle / \sigma^3 - 3$	C_4/C_2^2

$$C_n \equiv VT^3 \frac{d^n P/T^4}{d(\mu_B/T)^n} \Bigg|_T \quad \longleftrightarrow \quad \chi_n^B \equiv \frac{d^n P/T^4}{d(\mu_B/T)^n} \Bigg|_T$$

$$C_n = VT^3 \chi_n^B$$

STAR, 2023

