Emergent low-energy symmetry and phases of 3 flavour QCD

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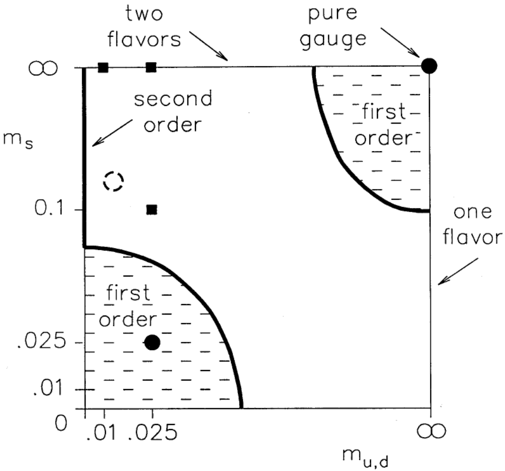
CPOD 2024
Berkeley USA (May 23, 2024)

The original Columbia plot

Lattice QCD has given detailed knowledge of hot QCD and enough information to begin to construct its detailed phase diagram.



Phases of QCD

Already in 1990 the Columbia plot captured an outline of what was known
[Brown etal doi:10.1103/PhysRevLett.65.2491]

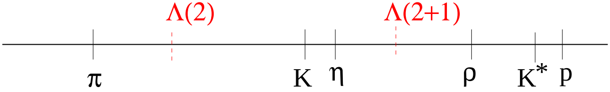
- Search for critical mass in QCD with $N_f = 2 + 1$: critical quark masses less than 12% of physical
  [Endrodi et al doi:10.22323/1.042.0182]
- Search for critical point in QCD with $N_f = 3$: only crossover even when π(K) mass is 80 MeV
  [Dini et al doi:10.1103/PhysRevD.105.034510]

A mystery of low scales. Why? I will give an explanation using EFTs. Big clue: the crucial role of chiral symmetry.

Related questions: How to extrapolate to chiral limit? What determines the cross over line? How to extrapolate to real time? All these are related to the properties of pions at $T > 0$.

Using an EFT to extend lattice studies

Start with a EFT for $T > 0$ which incorporated the chiral symmetry of quarks. Since it is an EFT, there is a cutoff Λ: different for $N_f = 2$ and $N_f = 2 + 1$.



SG, Sharma doi:10.1103/PhysRevD.97.036025

$$D = 3 \qquad L_3 = d_3 \Lambda \overline{\psi}\psi \qquad \text{mass term}$$

$$D = 4 \qquad L_4 = \frac{1}{2}\overline{\psi}\partial\!\!\!/_0\psi + \frac{1}{2} d_4 \overline{\psi}\nabla\!\!\!\!/\,\psi \qquad \text{kinetic terms}$$

$$D = 6 \qquad L_6 = \frac{1}{\Lambda^2}\sum_{i=1}^{10} d_{6,i}\left(\overline{\psi}\Gamma_i\psi\right)\left(\overline{\psi}\Gamma_j\psi\right) + \frac{1}{\Lambda^2} d_{6,11}\overline{\psi}\nabla^2\nabla\!\!\!\!/\,\psi,$$

where Γ_i are flavour×Dirac matrices; $d_{6,i}$ constrained by chiral symmetry and CPT. Not a model: EFT uses all allowed terms

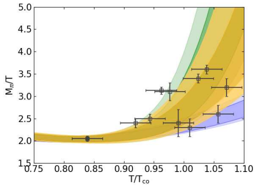
The $N_f = 2$ EFT

Construct mean field theory: only a combination d_6 enters. Introduce flucuations ϕ to linearize L_6 (integrate over ψ to prevent double counting). Gives a theory of (pseudo-)Goldstone bosons:

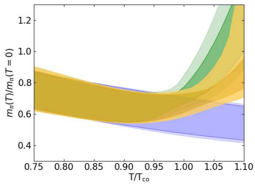
$$L = m_\pi^2\phi^2 + \frac{1}{2}(\partial_0\phi)^2 + \frac{1}{2}u_\pi^2(\nabla\phi)^2 + \lambda_\pi\phi^4 + \cdots$$

m_π/Λ, u_π, λ_π functions of d_3, d_4 and d_6. Super-daisy resummation of pion propagator and fit to lattice predicts pole mass m_π given screening mass m_π/u_π. All parameters fixed by 3 inputs, everything else prediction.

pion screening mass:



pion pole mass:



SG, Sharma

doi:10.1142/S0217751X20300215

The $N_f = 3$ EFT: an emergent symmetry

Lagrangian up to $D = 6$ has U(3)×U(3) symmetry: hence nonet of Goldstone bosons after χSB. Same emergent symmetry with $D = 8$ terms. Symmetry is broken by two $D = 9$ terms: suppressed by three powers of UV cutoff Λ.

Emergent symmetry: a bug or a feature? Usually not a bug;

- baryon number conservation of standard model
- graphene and the effective Lorentz symmetry of electron gas near its Fermi surface
- emergent gauge symmetries in Hubbard models
- near degeneracies of masses and weak decays of heavy quarks in HQET

In the meson spectrum the $(m_{\eta'} - m_\eta)/(m_{\eta'} + m_\eta) \simeq 0.27$, which is large. No effect of emergent symmetry?

The $N_f = 2+1$ EFT

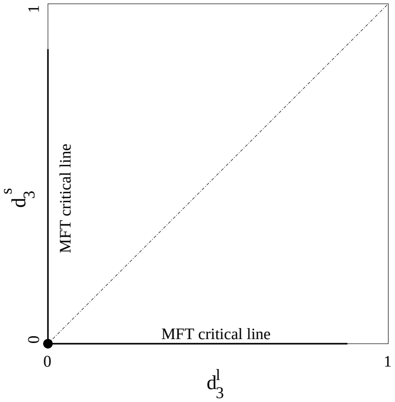
Break 3 flavour symmetry explicitly to 2+1 by changing the mass term. Projection operators $\Pi_\ell = (\sqrt{2}\, T^0 + T^8)/\sqrt{3}$ and $\Pi_s = (T^0/\sqrt{2} - T^8)/\sqrt{3}$. Use them to project the light and strange quark pieces out of the remaining terms in the Lagrangian.

$$
\begin{array}{lll}
D = 3 & L_3 = L_3^\ell + L_3^s = d_3^\ell \Lambda \overline{\psi}_\ell \psi_\ell + d_3^s \Lambda \overline{\psi}_s \psi_s & \text{mass terms} \\
D = 4 & L_4 = L_4^\ell + L_4^s \quad \text{kinetic terms} & \\
D = 6 & L_6 = L_6^{\ell\ell} + L_6^{ss} + L_6^{\ell s} &
\end{array}
$$

Emergent symmetry remains: U(2)×U(2) in the light sector.

1. Integrate over the momenta between $\Lambda(2+1)$ and $\Lambda(2)$ to obtain the $N_f = 2$ action. Emergent symmetry remains.
2. Use the $N_f = 2+1$ action to construct the thermal MFT and phase diagram. Group theory of Dirac×chiral-flavour removes $L_6^{\ell s}$ terms from the MFT: no couplings between Σ^ℓ and Σ^s!

The emergent Columbia plot



Columbia plot in MFT with $D = 6$ Lagrangian

U(2)×U(2) ≃ O(4)×O(2)
Transition may be first or second order in a lattice computation **[Kamikado, doi:10.22323/1.251.0207]**. Confused situation in other approaches. U(3)×U(3) transition second order in conformal bootstrap **[Kousvos, Stergiou, doi:10.21468/SciPostPhys.15.2.075]**

Breaking emergent symmetry at D=9

At $D = 9$ one adds the 't Hooft determinant term

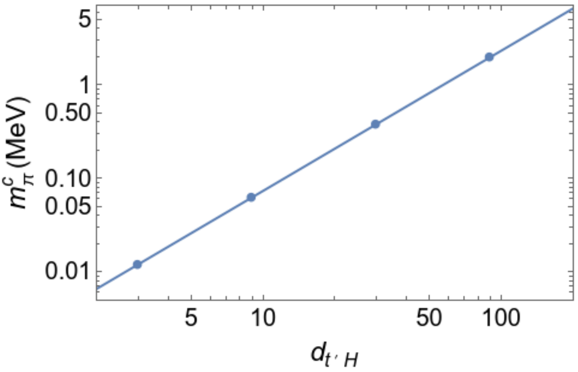
$$L_9 = \frac{d_{9,t'H}}{\Lambda^5}\,\epsilon_{abc}\epsilon_{a'b'c'}\overline{\psi}^a\psi^{a'}\overline{\psi}^b\psi^{b'}\overline{\psi}^c\psi^{c'}.$$

$d_{9,t'H}$ captures interaction with instantons. We discover a second operator: two of the currents have a tensor structure. Not considered before.

Integrating over strange quarks and all modes with $\Lambda(2+1) > k > \Lambda(2)$, gives U(1) symmetry breaking in the $N_f = 2$ theory with additional coefficients to the $D = 6$ term of the order of $(m_s/\Lambda)^3$.

For $N_f = 3$ the MFT has a Σ^3 term as expected, and therefore gives a first order transition in the chiral limit. Therefore there is a critical mass.

Critical mass on the $N_f = 3$ line



Approximate scaling exponent: $m_\pi^c \simeq d_{9,t'H}^{3/2}$. T_c depends on $d_{9,t'H}$ rather weakly.

Polynomial approximation

Free energy of the model is given by a quartic polynomial in Σ plus an integral which is analytic in Σ and goes to zero faster than Σ^4 as Σ vanishes.

We are able to bound the effect of the integral, so that in the range $m/\Lambda < 1$ it is enough to examine the quartic polynomial.

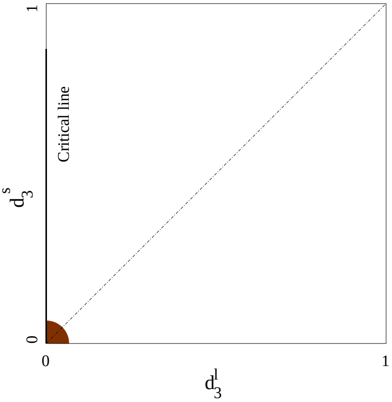
The zeroes of the (cubic) gap equation give information about the critical behaviour of the theory. Only at the critical mass, as T changes, a degenerate pair of solutions appears at T_c.

One can show

$$\frac{m_c}{\Lambda} \simeq \frac{\pi^4}{839808 N_f^2} \times \frac{d_4^6 d_{9,t'H}^3}{d_{6,NJL}^8}$$

This agrees remarkably well with the numerical results.

The emergent Columbia plot



The first order region shrinks as $d^3_{9,t'H}$.

Summary

1. Lattice computations indicate that the critical masses in the Columbia plot are too small for current generation of computations to reach.
2. EFT approach is known to have an emergent symmetry $U(N_f)\times U(N_f)$ which is broken only at order $1/\Lambda^{3N_f-4}$. In hadron physics this extra symmetry has no consequence.
3. For $T > 0$ this emergent symmetry explains the lattice results on the smallness of the critical quark masses.
4. Full description of lattice results will require using the $D = 8$ terms. Task for later.