Lattice and Magnetic Field

Heng-Tong Ding
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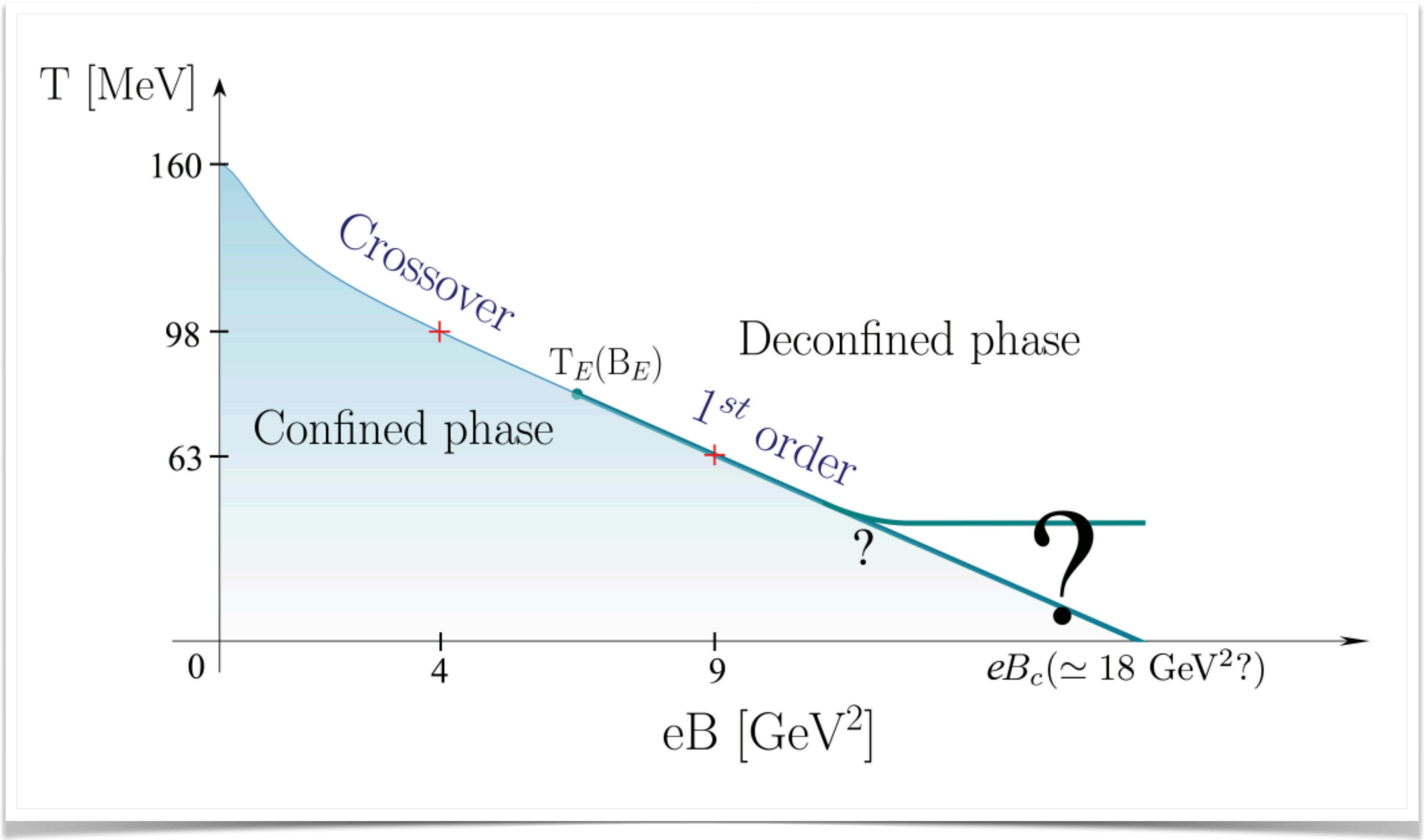
Baryon electric charge correlation as a Magnetometer of QCD

HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu,
Phys. Rev. Lett. 132 (2024) 201903, and work in progress

The 15th workshop on the Critical Point and Onset of Deconfinement

May 20-24, 2024 @ Berkeley, CA

A QCD CEP in very strong magnetic fields



M. D'Elia et al., Phys.Rev.D 105 (2022) 3, 034511

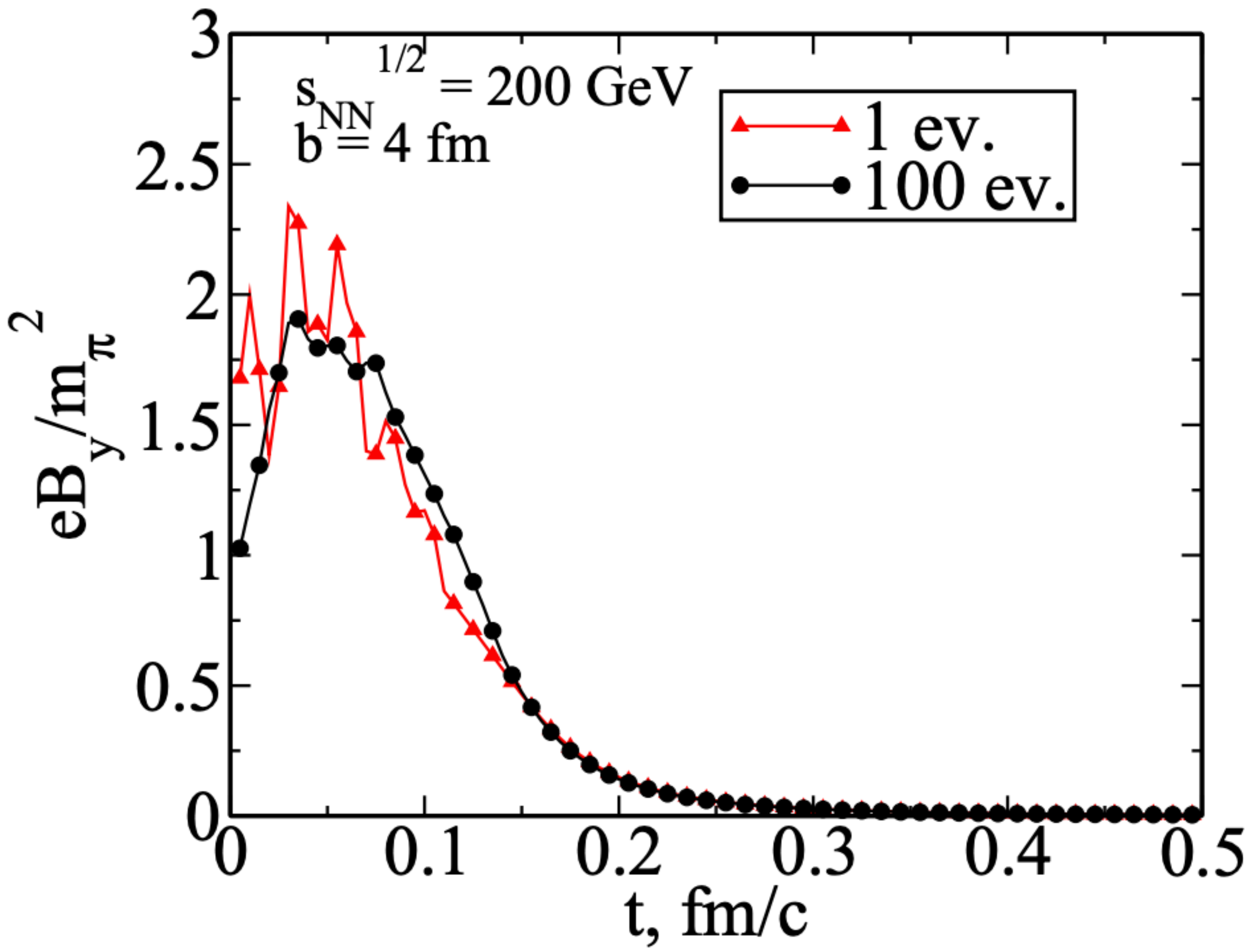
☑ No sign problem in LQCD simulations with eB

- A crossover transition exists still at eB=4 GeV^2
- A first order phase transition observed at eB=9 GeV^2
- There should exist a CEP at

$$4\ \mathrm{GeV}^2 < eB < 9\ \mathrm{GeV}^2$$

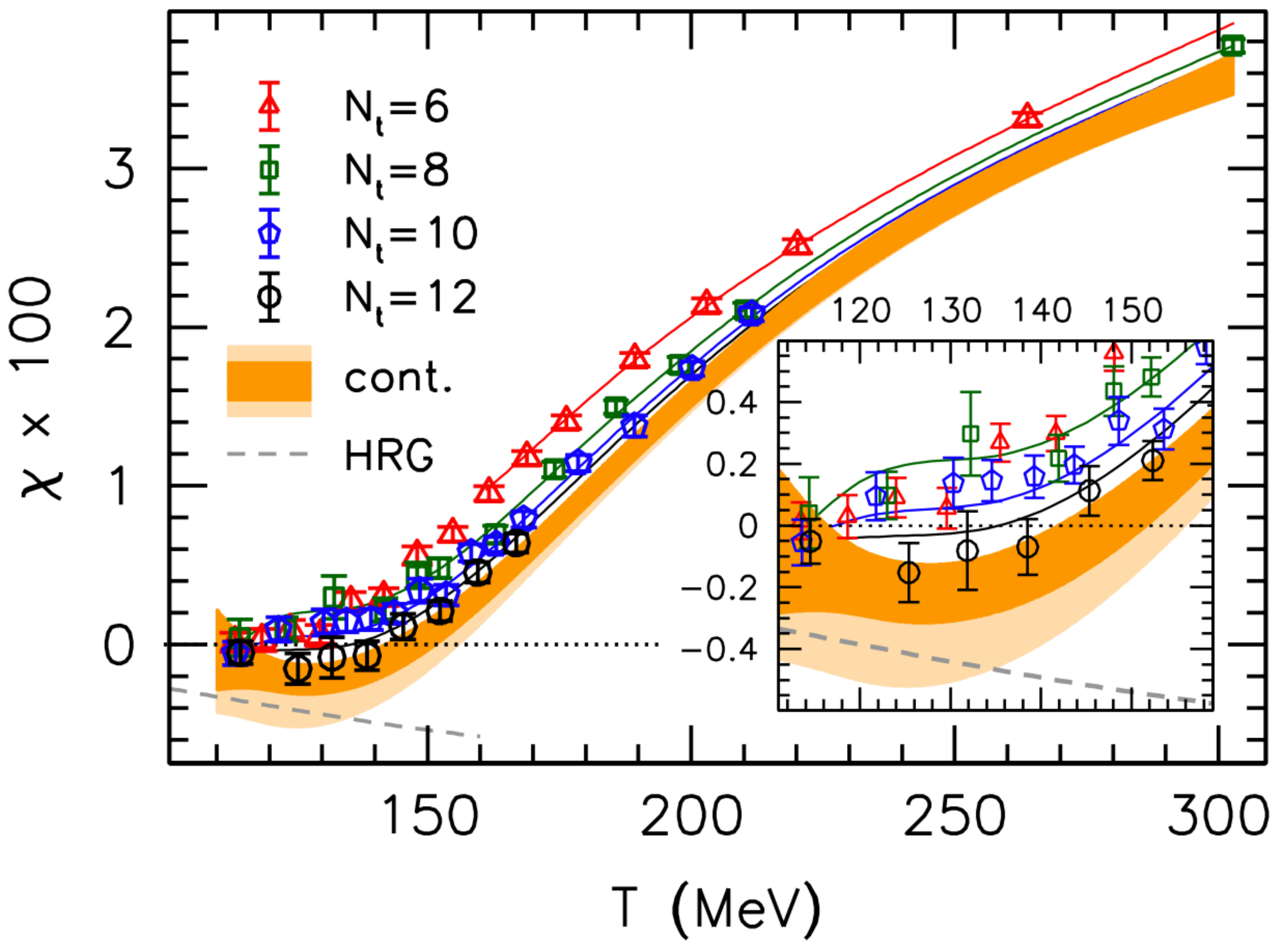
$$4\ \mathrm{GeV}^2 \sim 2 \times 10^{20}\ \mathrm{Gauss} \sim 200\ \mathrm{M}_\pi^2$$

Magnetic field created in the early stage of HIC



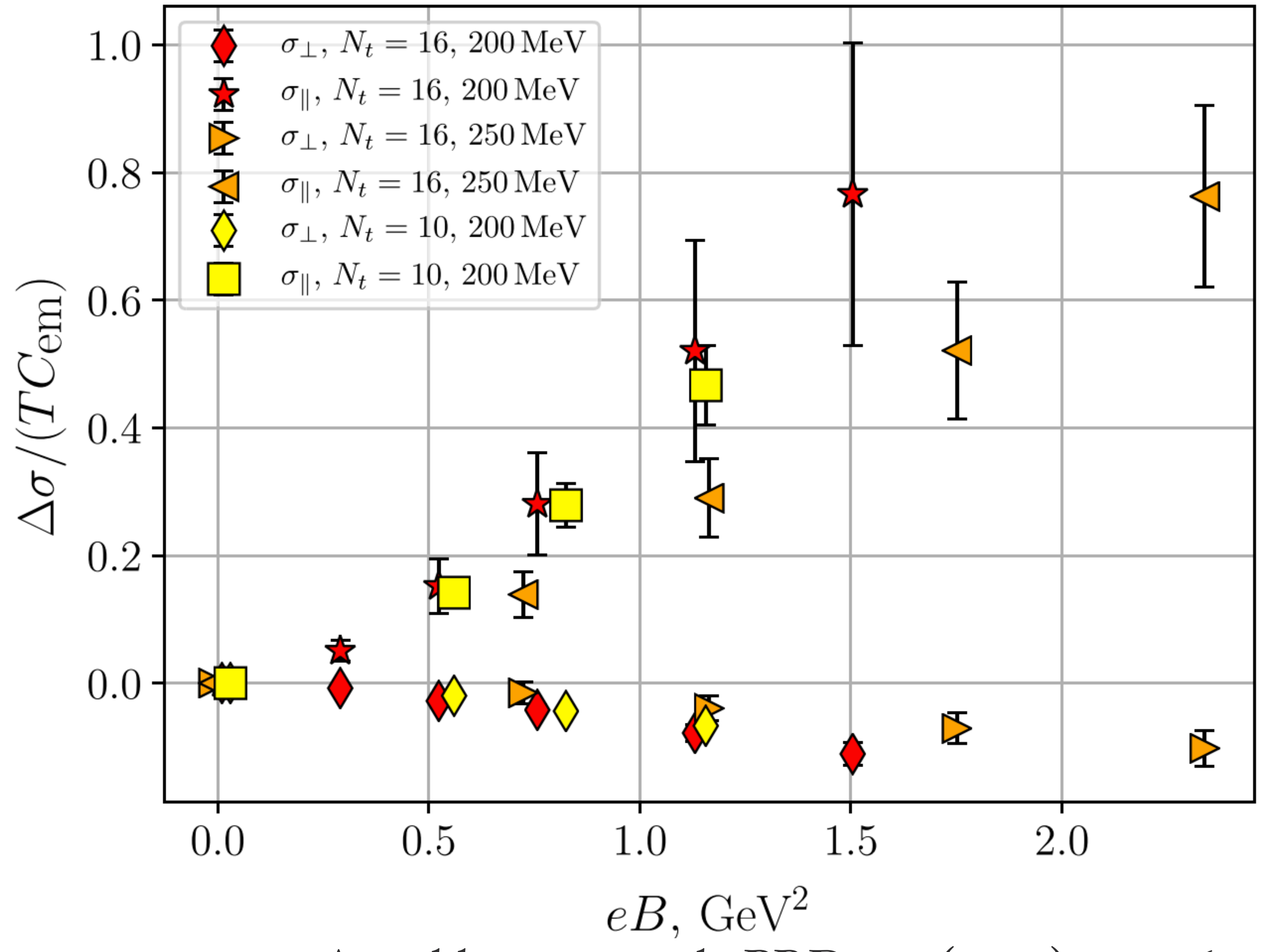
Skokov, Illarionov and Toneev, IJMPA 24 (2009) 5925

LQCD results on
Magnetic susceptibility



Bali, Endrodi, Piemonte, JHEP 07 (2020) 183

LQCD results on
difference to electromagnetic
conductivity at *eB*=0



Astrakhantsev et al., PRD 102 (2020) 054516

t=0: RHIC: $eB \sim 5M_\pi^2$

LHC: $eB \sim 70M_\pi^2$

T> 155 MeV: Paramagnetic

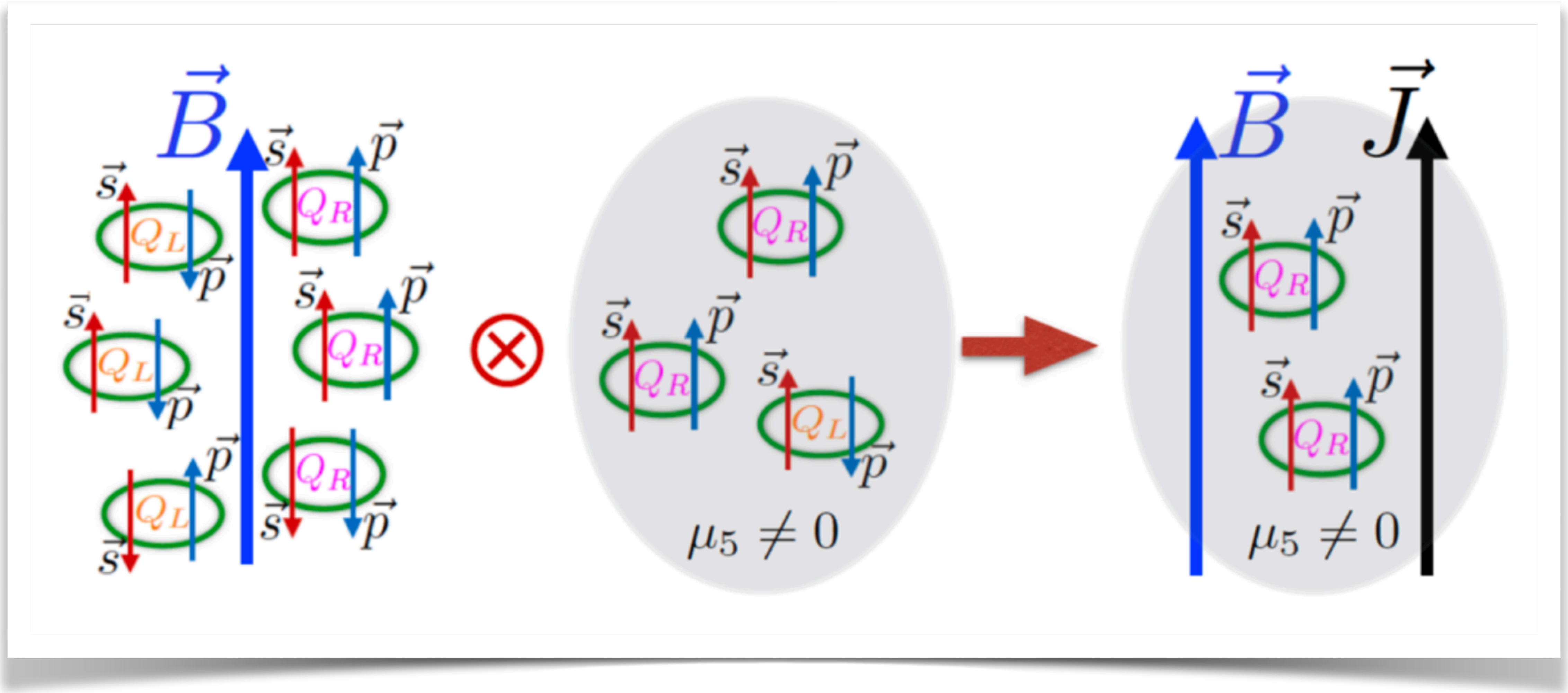
T<155 MeV: Diamagnetic

$eB\uparrow$ Parallel: $\uparrow$

Transverse: $\downarrow$

Whether imprints of a strong magnetic field exist in the final stage of heavy-ion collisions?

Chiral magnetic effect



See recent reviews e.g.
D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

☑ Axial U(1) anomaly

HTD, Li, Mukherjee, Tomiya, Wang & Zhang, PRL 126 (2021) 082001

HTD, Huang, Mukherjee & Petreczky, PRL 131 (2023) 161903

Kaczmarek, Shanker & Sharma, PRD108 (2023) 094501

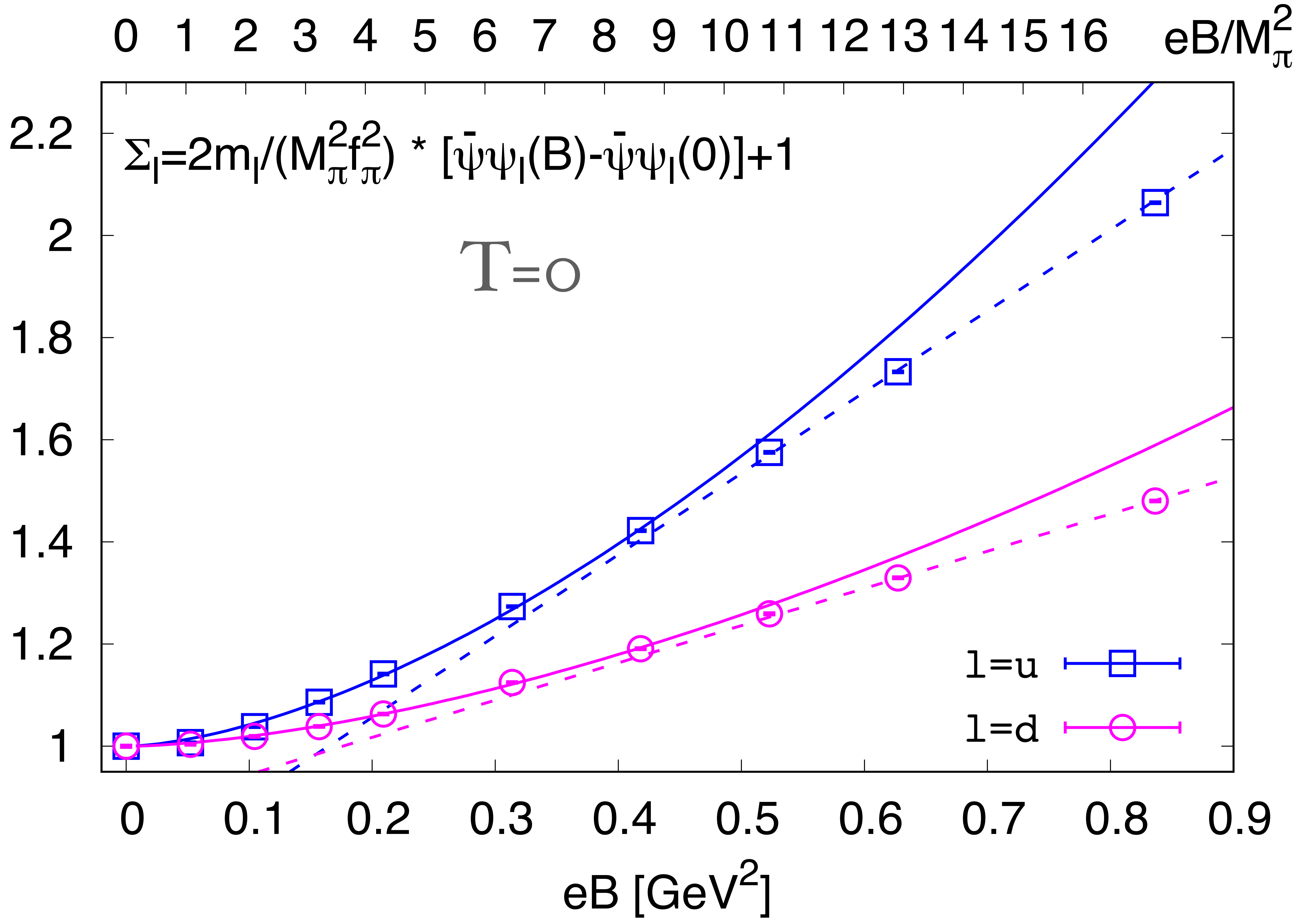
Alexandru, Bonano, D'Elia & Horvath, arXiv:2404.12298

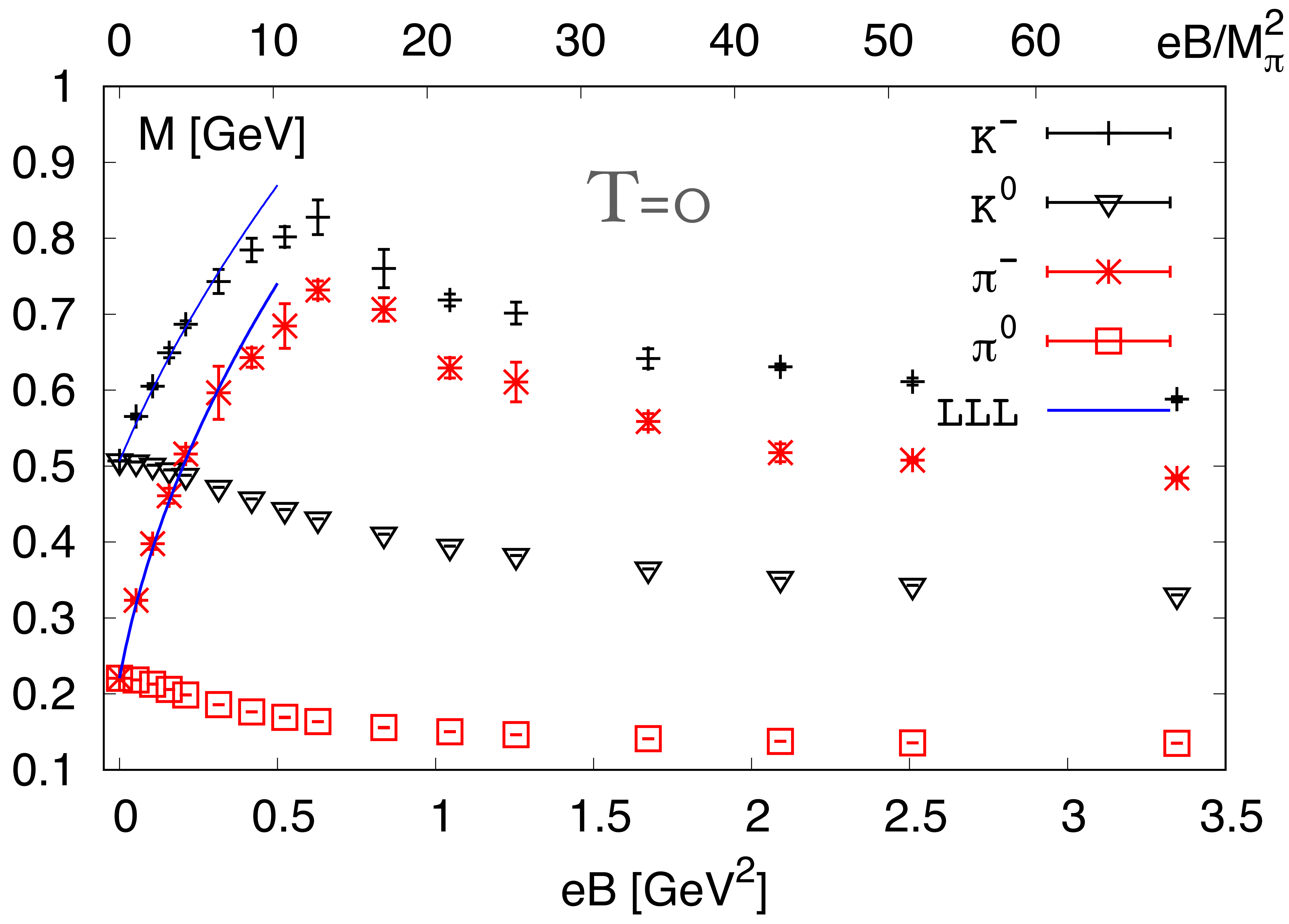
… …

▷ Strong magnetic field

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates and meson masses

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, $32^3 \times 96$ lattice swith $a^{-1} \approx 1.7$ GeV and HISQ action





HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

See also in e.g. Bali et al., Phys.Rev.D86(2012)071502

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505, Luschevskaya et al., NPB 898 (2015) 627

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates and meson masses

N_f=2+1 QCD, $M_\pi(eB=0) \approx 220$ MeV, $32^3 \times 96$ lattice swith $a^{-1} \approx 1.7$ GeV and HISQ action

Not accessible in HIC experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

See also in e.g. Bali et al., Phys.Rev.D86(2012)071502

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505, Luschevskaya et al., NPB 898 (2015) 627

Fluctuations of net baryon number (B), electric charge (Q) and strangeness (S)

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

See talks of
S. Borsanyi and C. Schmidt
on Tue

$$\hat{\chi}_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_{\mathrm{B}}/T\right)^i \partial\left(\mu_{\mathrm{Q}}/T\right)^j \partial\left(\mu_{\mathrm{S}}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}}\,, \quad \mu_d = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}}\,, \quad \mu_s = \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}}\,.$$

- At $eB \neq 0$ a lot more could be explored

HRG: Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301
Bhattacharyya et al., EPL115(2016)62003

PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009

Lattice setup

- HISQ/tree action, line of constant physics adopted from the HotQCD collaboration

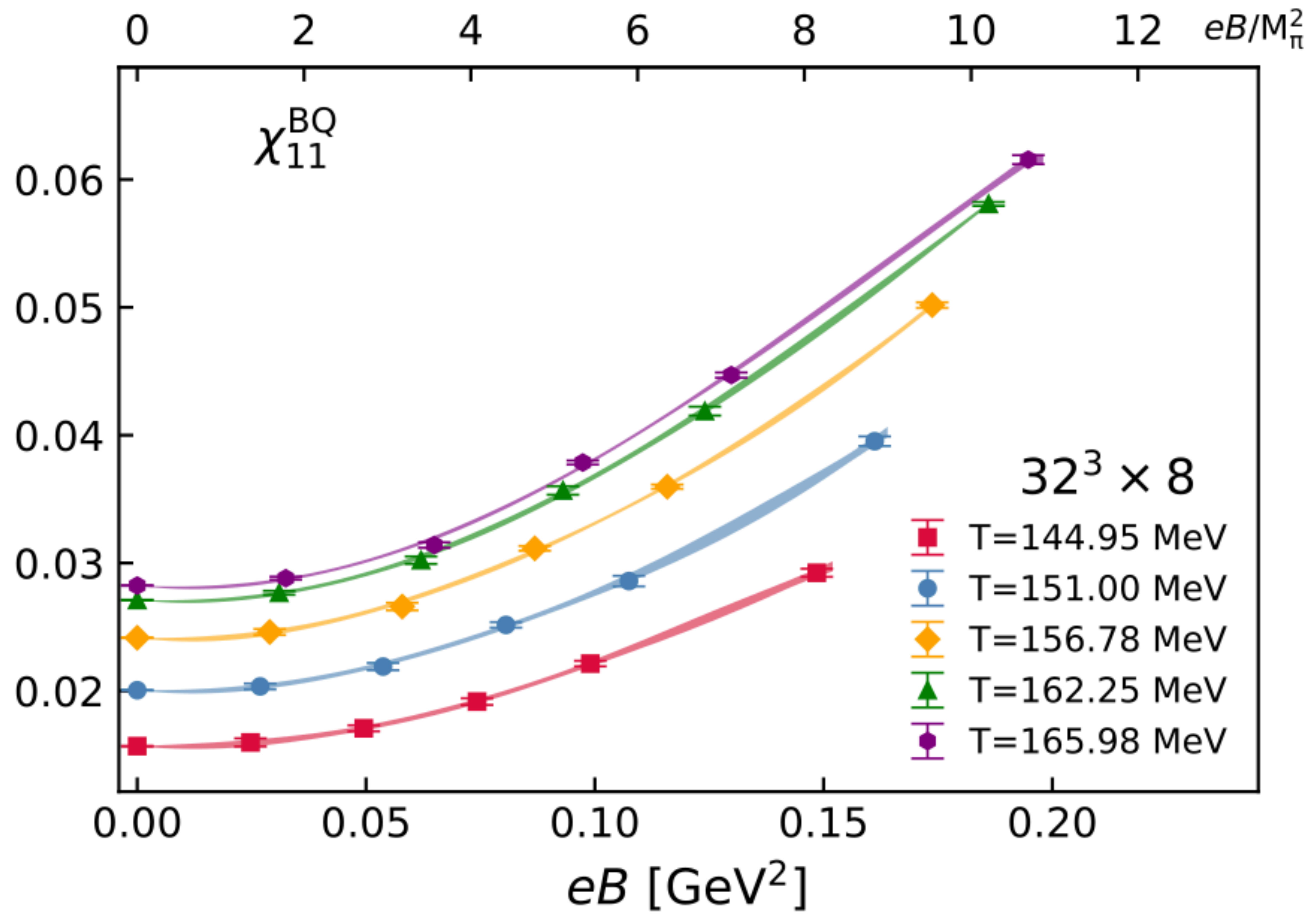
- Magnetic field strength: $eB = 6\pi/N_s^2 * N_b * a^{-1}$ with $N_b = 1,2,3,4,6,12,16,24$, i.e. $eB \lesssim 0.8\ \mathrm{GeV}^2 \sim 45\mathrm{m}_\pi^2$

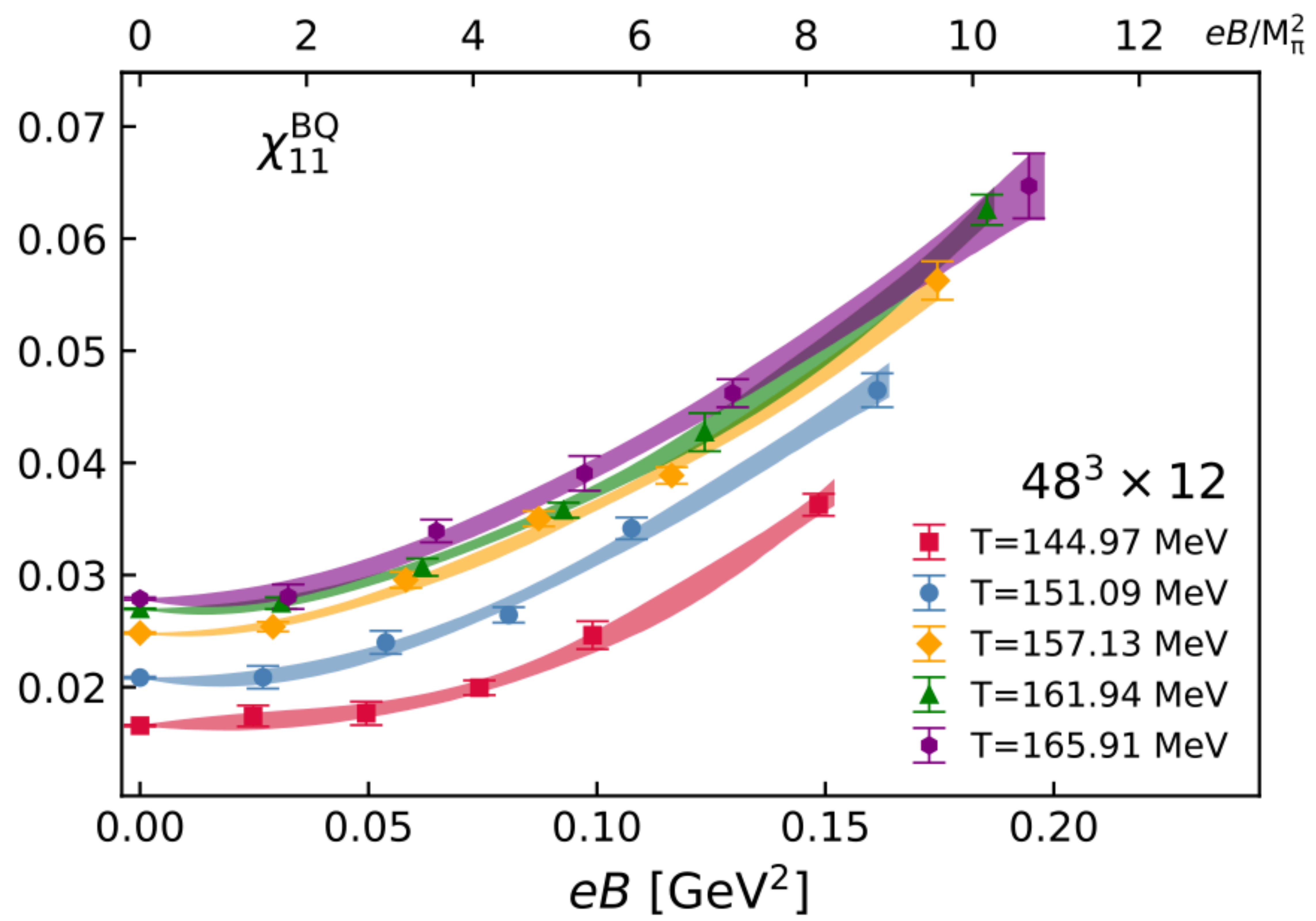
- Temperature ranges from ~145 MeV to ~166 MeV

- Lattice sizes: 32^3x8 and 48^3x12, with additional 64^3x16 at one single value of *eB*

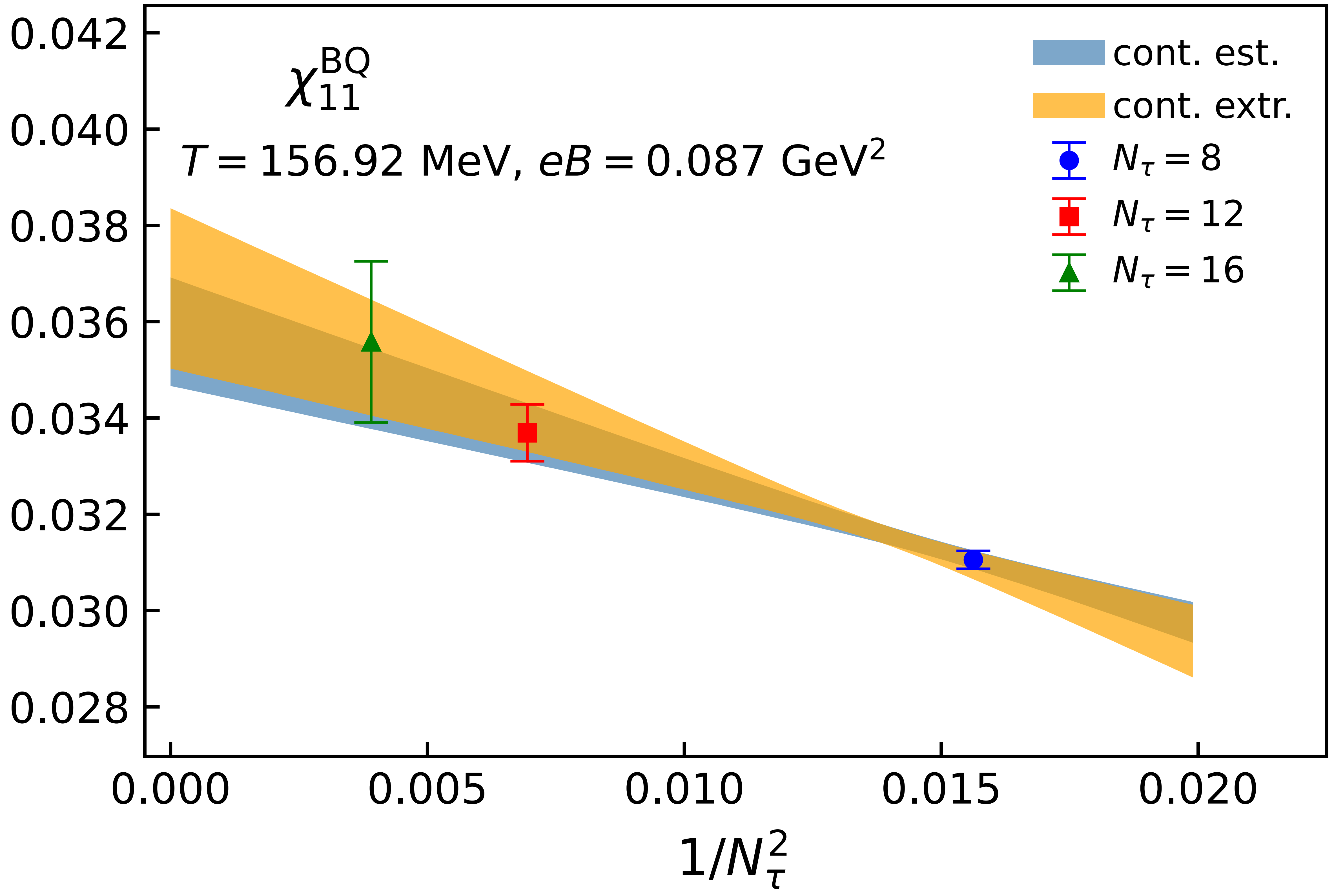
Nuclear Science Computing Center at CCNU

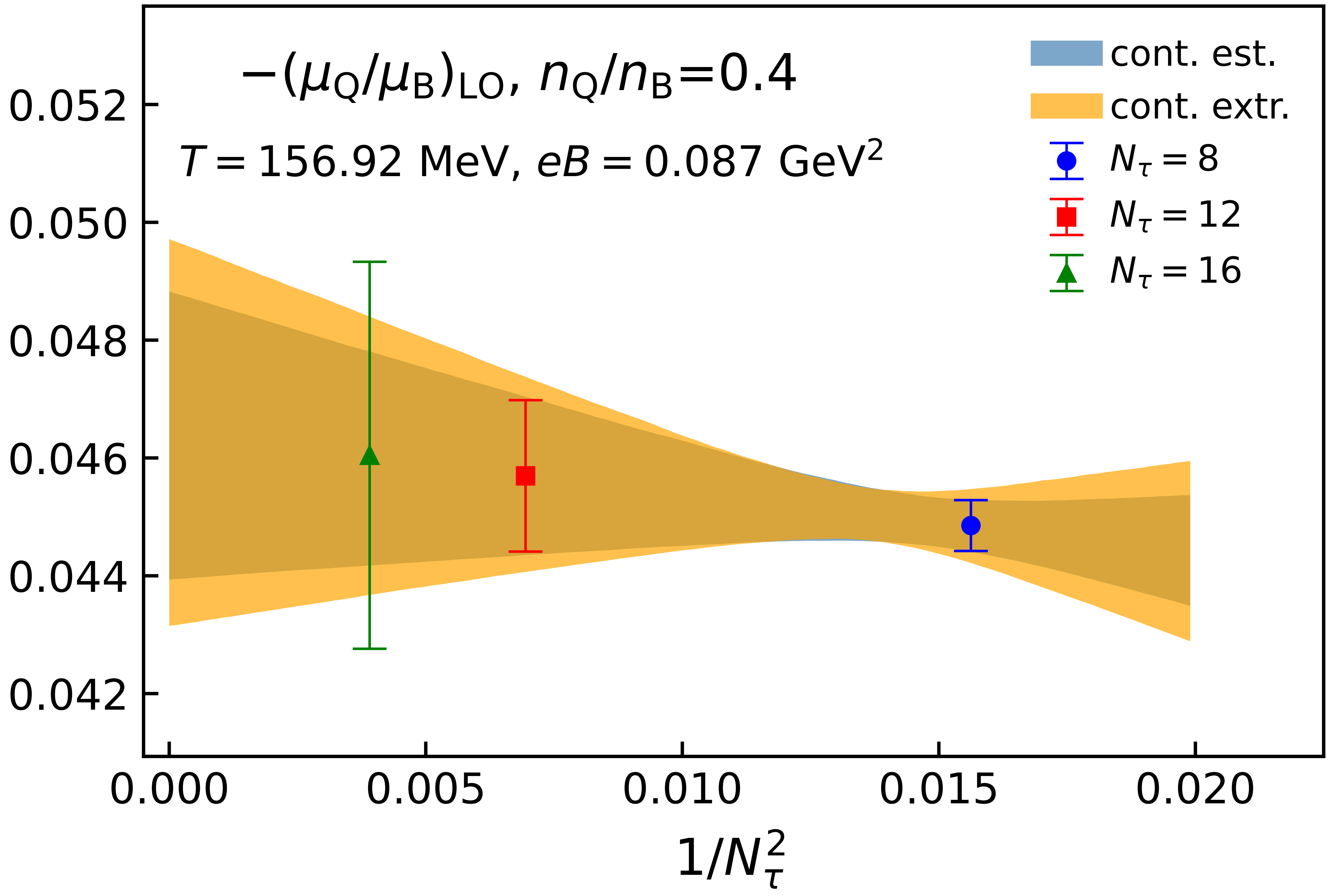
Lattice data on N$_\tau$ =8 and 12 lattices



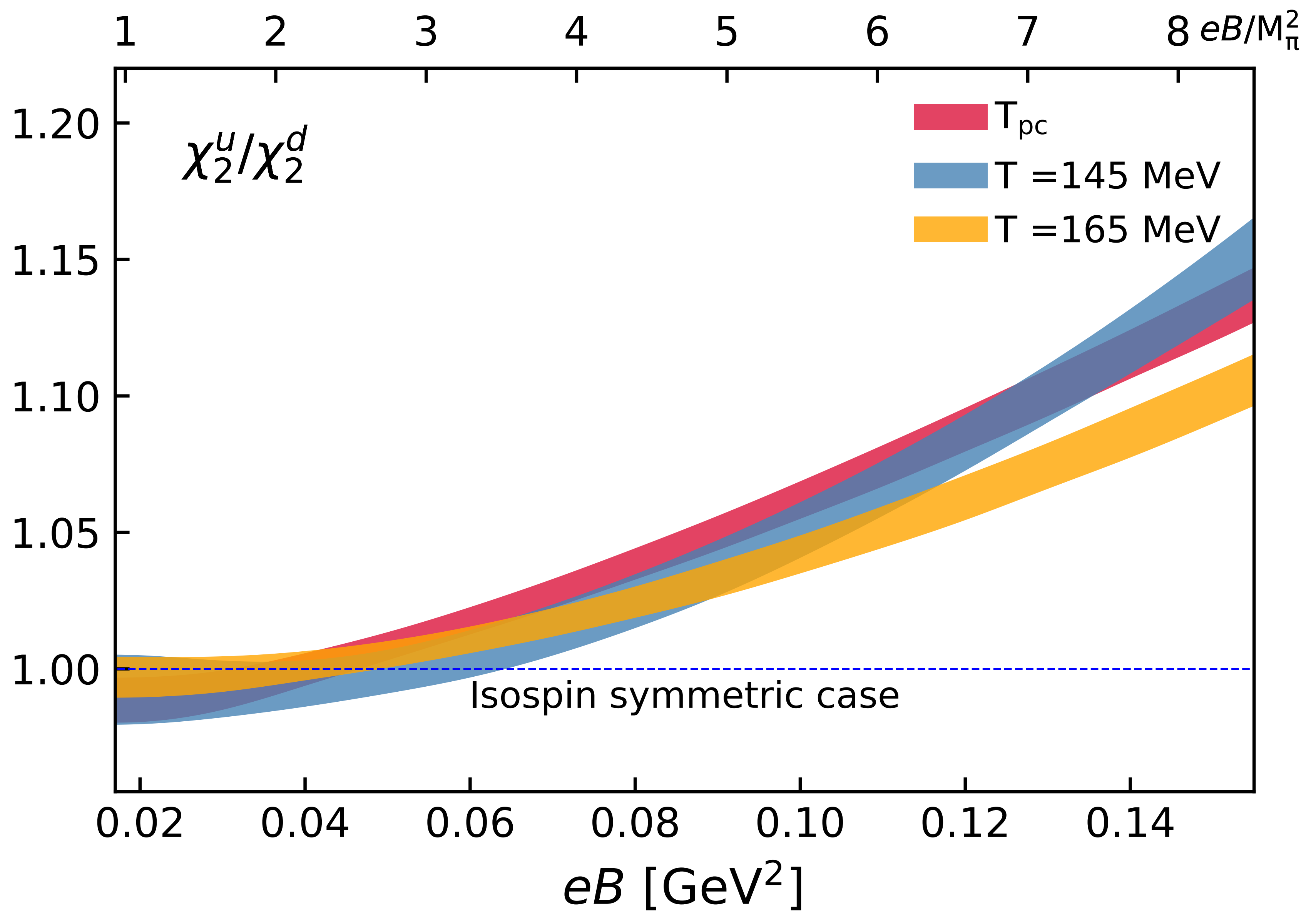


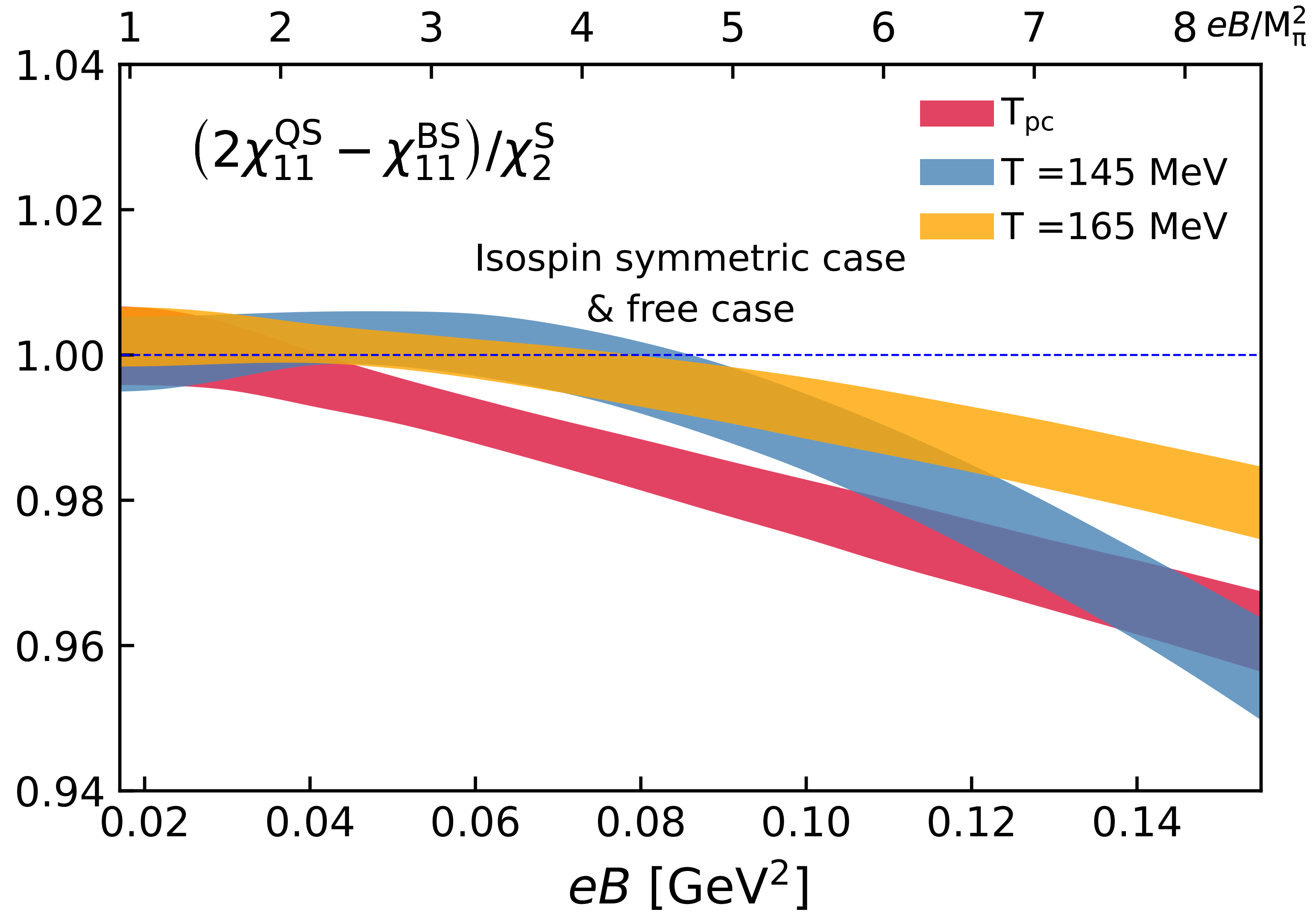
Continuum estimate v.s. continuum limit





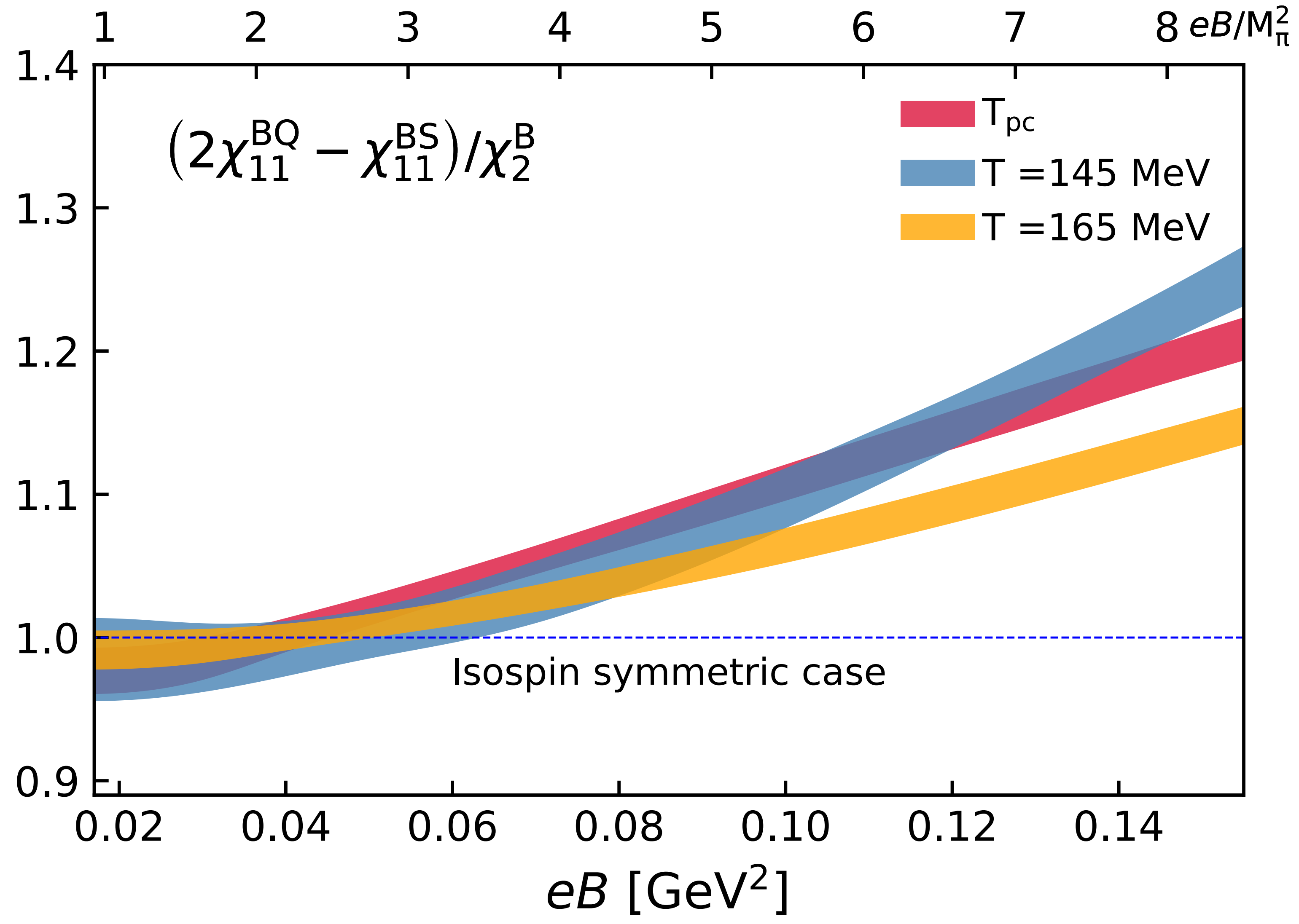
Isospin symmetry breaking at nonzero magnetic field



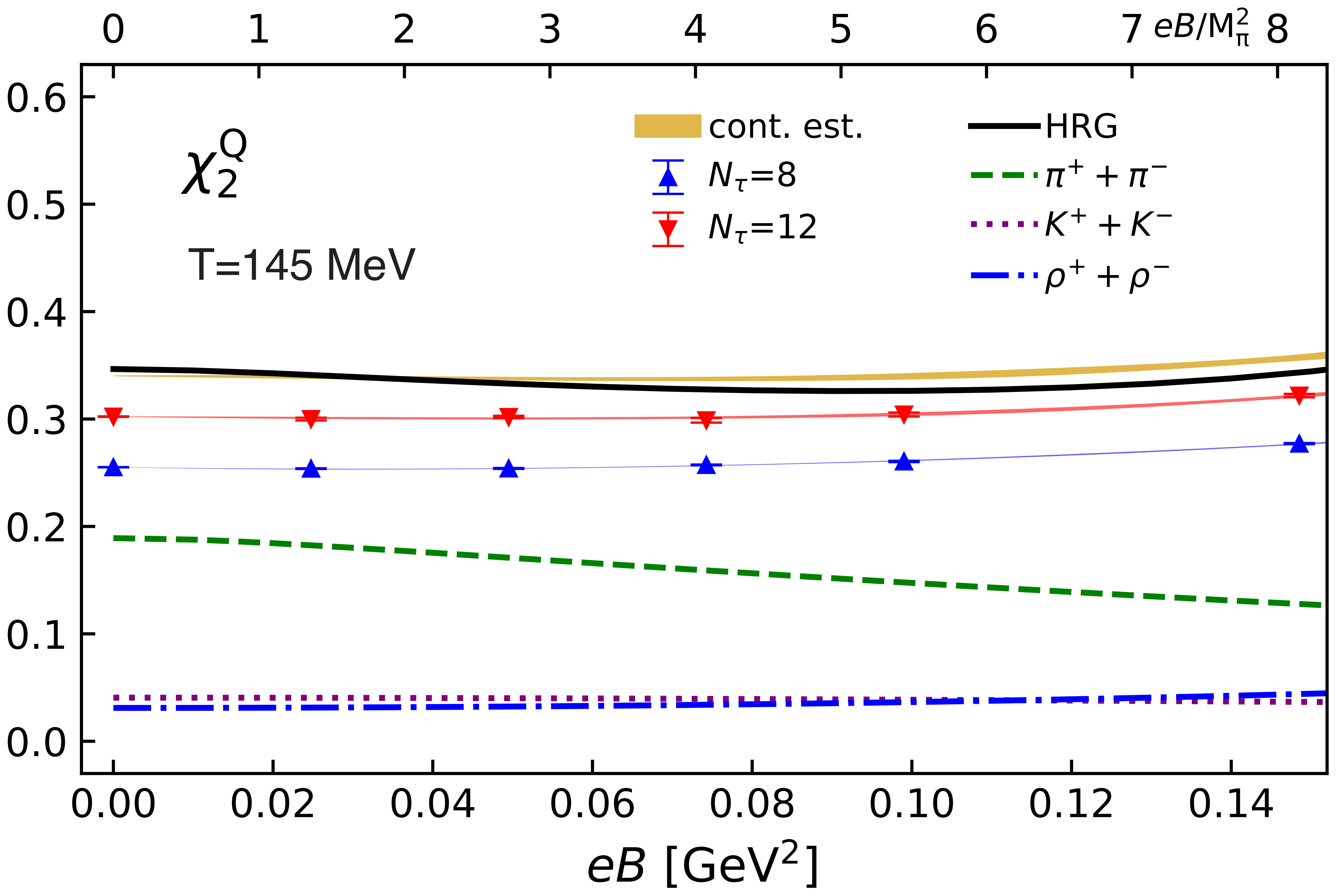


At eB=0, $\chi_2^u=\chi_2^d$,

$$2\chi_{11}^{QS}-\chi_{11}^{BS}=\chi_2^S\ ,\ 2\chi_{11}^{BQ}-\chi_{11}^{BS}=\chi_2^B$$



Net electric charge fluctuations at T=145 MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

- In HRG: Thermal pressure arising from charged hadrons in strong magnetic fields

Fukushima & Hidaka, PRL 16'

$$\frac{P_c}{T^4} = \frac{|q_i|B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} \mathrm{K}_1\left(\frac{n\varepsilon_0}{T}\right)$$

$$\varepsilon_0 = \sqrt{m_i^2 + 2|q_i|B(l + 1/2 - s_z)}$$

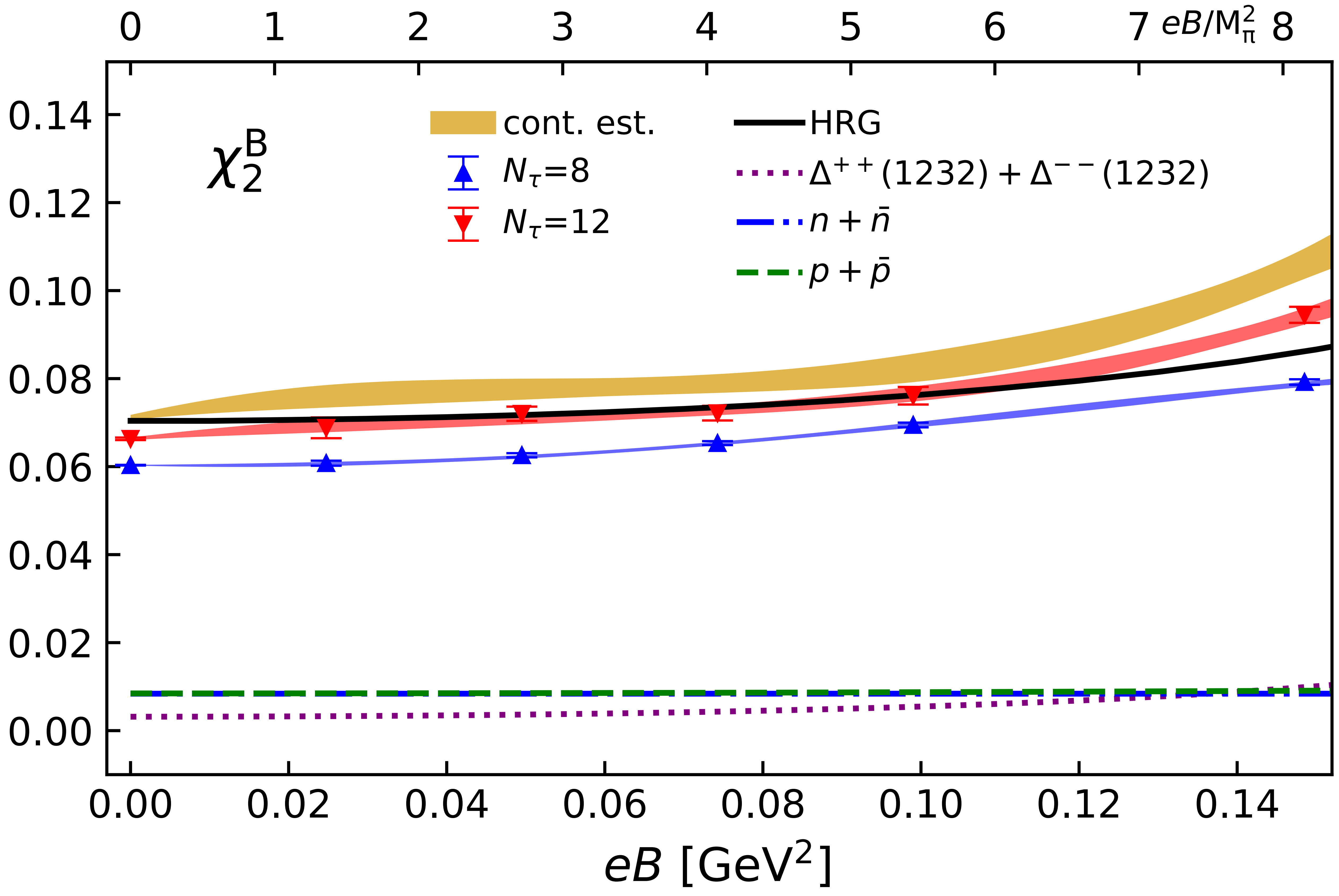
- Diagonal fluctuations from HRG

HTD, Li, Shi & Wang , EPJA 21'

$$\chi_2^X = \frac{B}{2\pi^2 T^3} \sum_i |q_i| X_i^2 \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} f(\varepsilon_0)$$

$$f(\varepsilon_0) = \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} n\, \mathrm{K}_1\left(\frac{n\varepsilon_0}{T}\right)$$

Net baryon number fluctuations at T=145 MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

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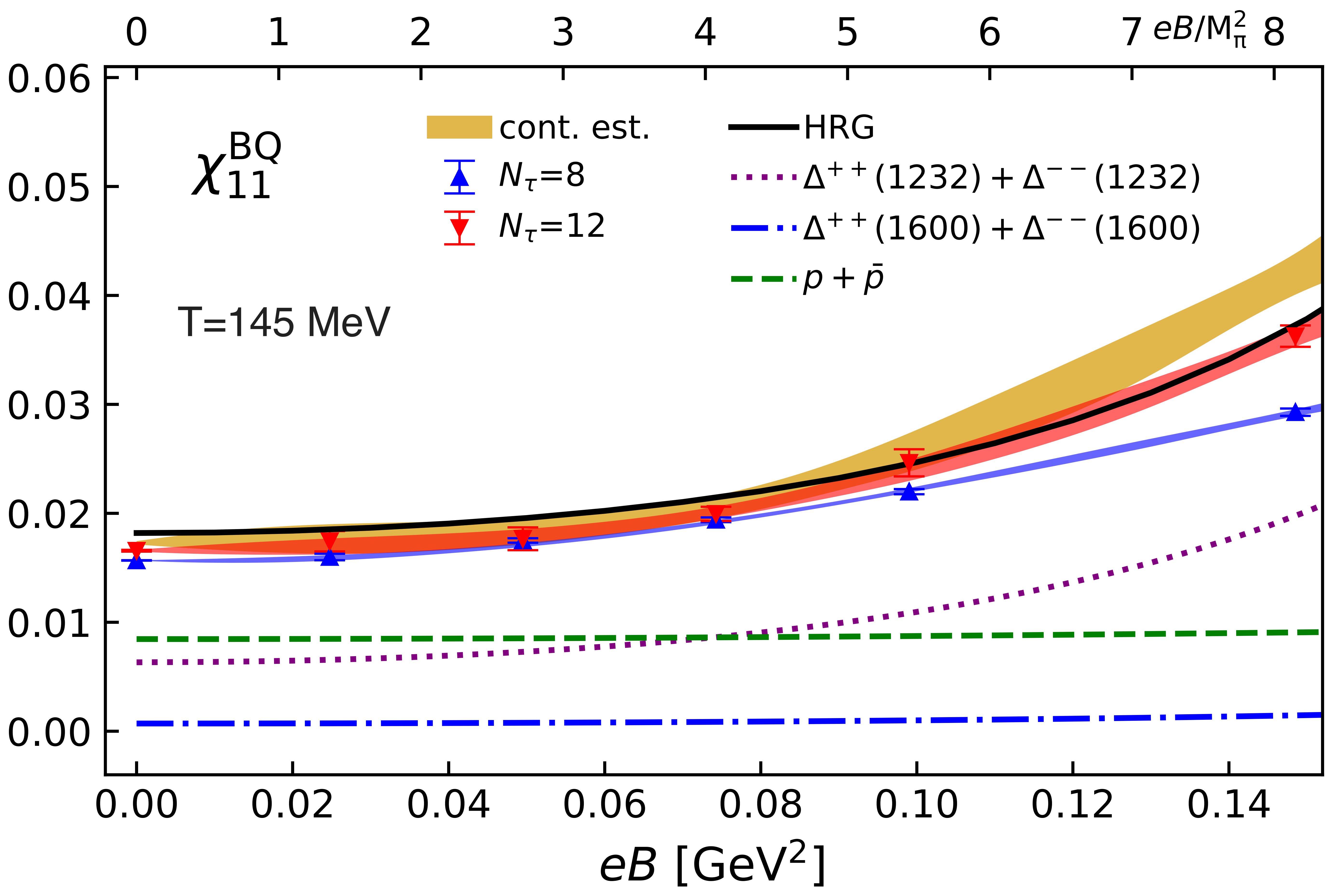
- χ_2^B receives contributions also from neutral baryons

- HRG, where neutral baryons are assumed to be unaffected by eB, undershoots the LQCD results

Baryon electric charge correlation at T=145 MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

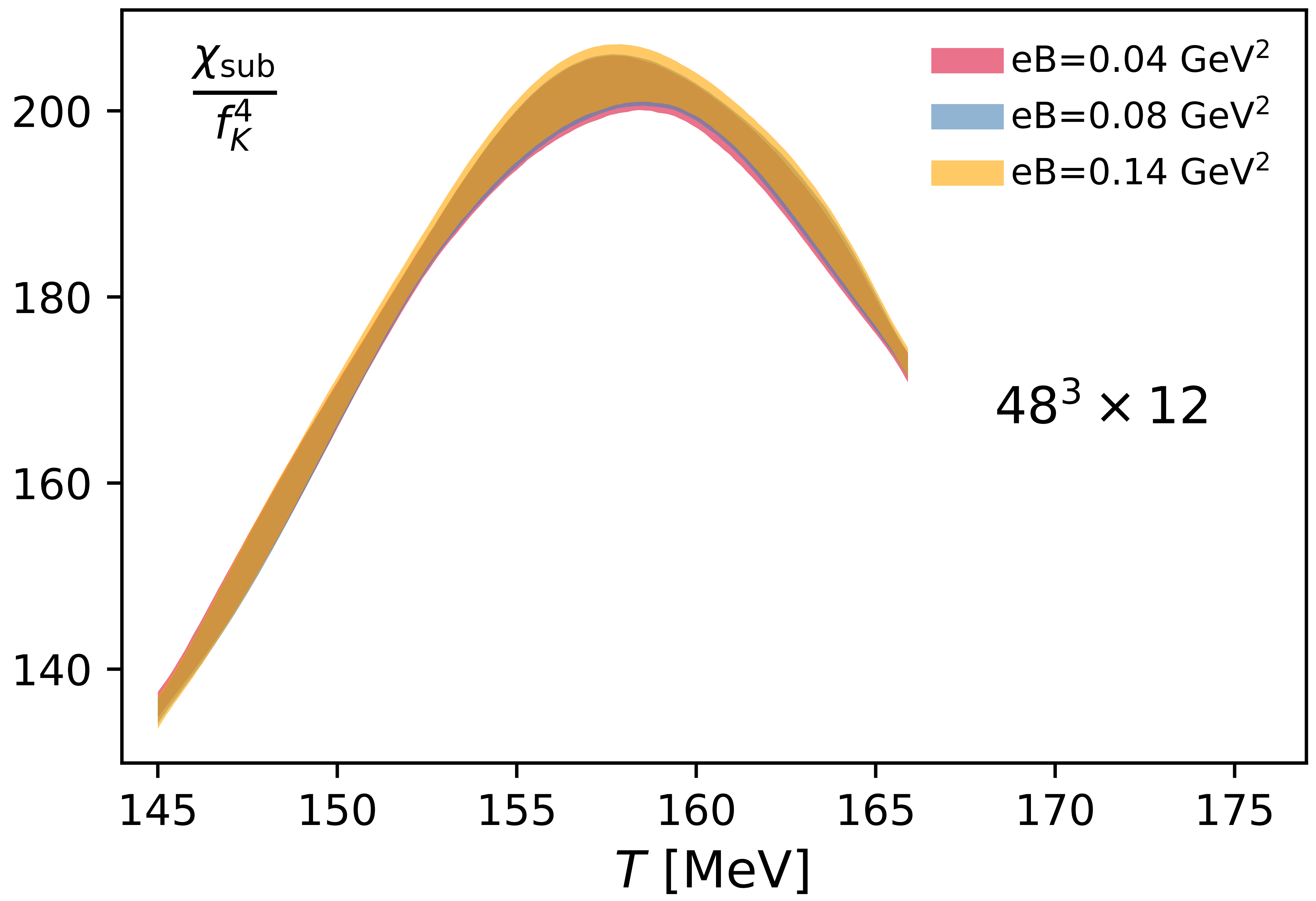
χ_{11}^{BQ}: Magnetometer of QCD

- Most of the eB-dependence comes from doubly charged Delta baryons

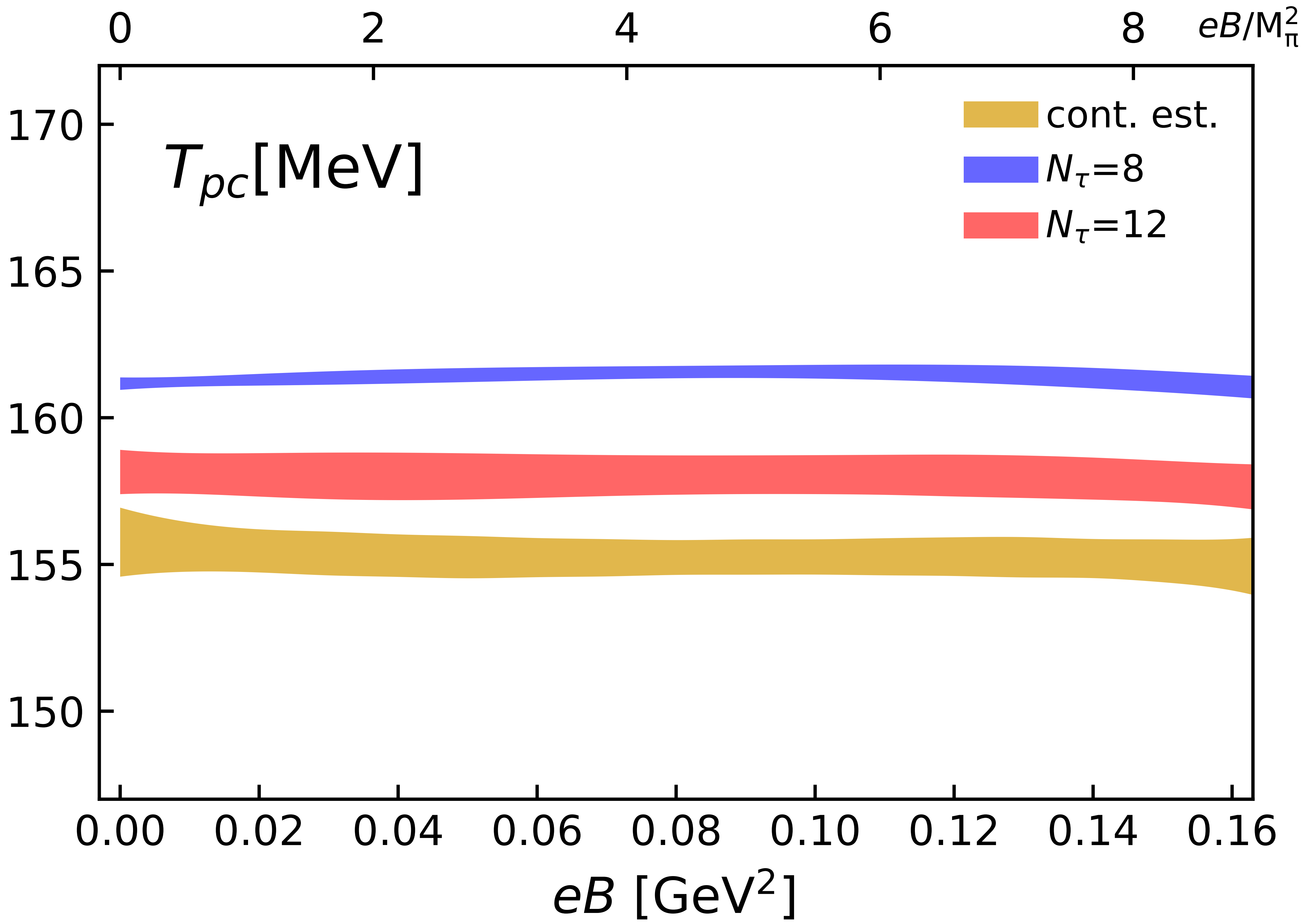
$$\Delta^{++} \to p + \pi^+$$

- Doubly charged Delta baryons: not-measurable in HIC experiments

QCD transition temperature in nonzero magnetic fields

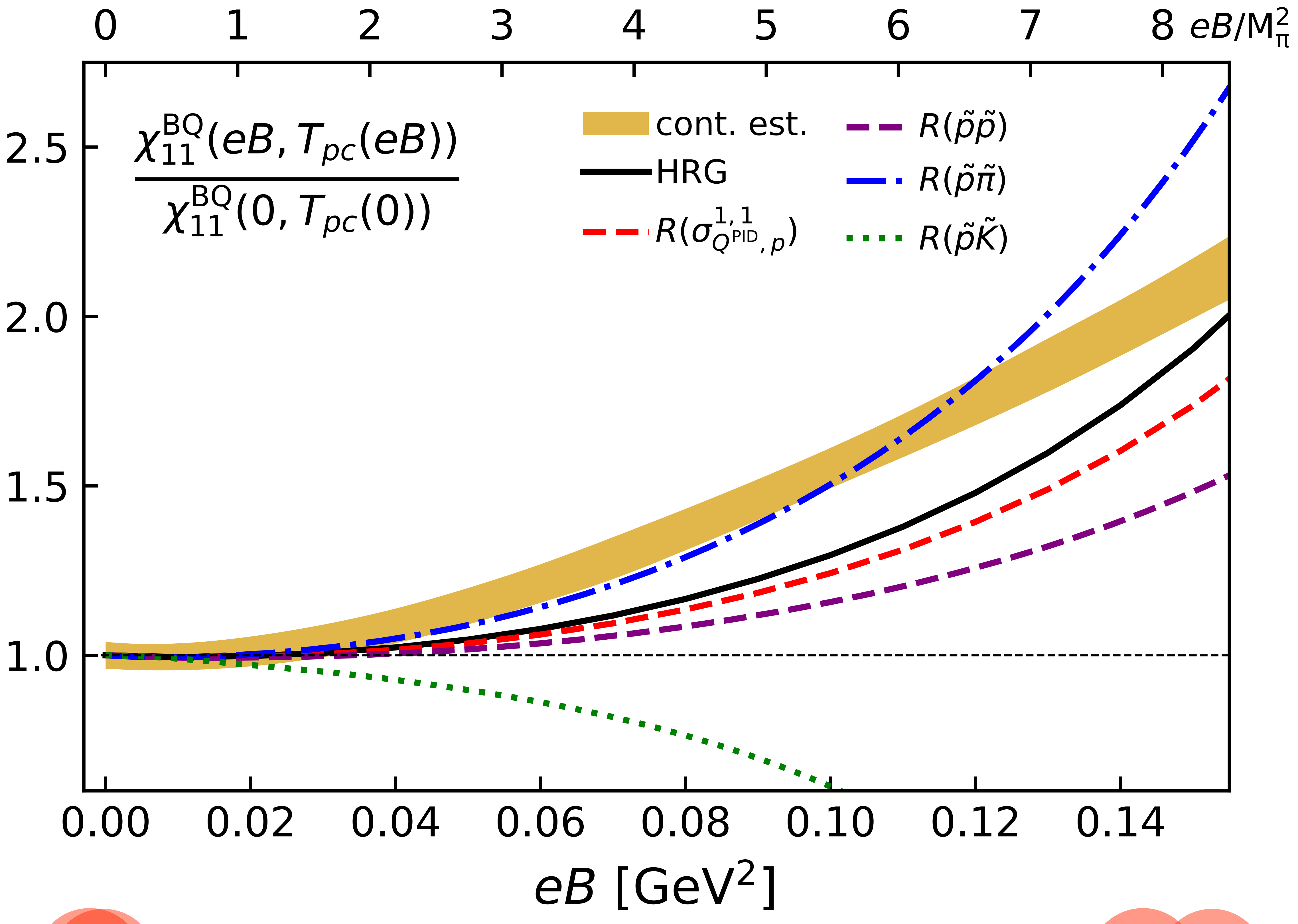


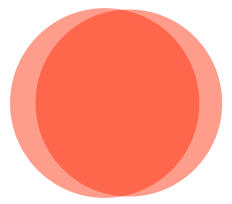
T_{pc} determined as the peak location of chiral susceptibility



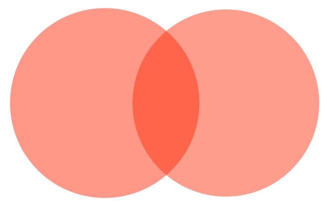
Negligible eB-independence at $eB \lesssim 0.16$ GeV^2

Baryon electric charge correlation along the transition line





Central collision



Peripheral collision

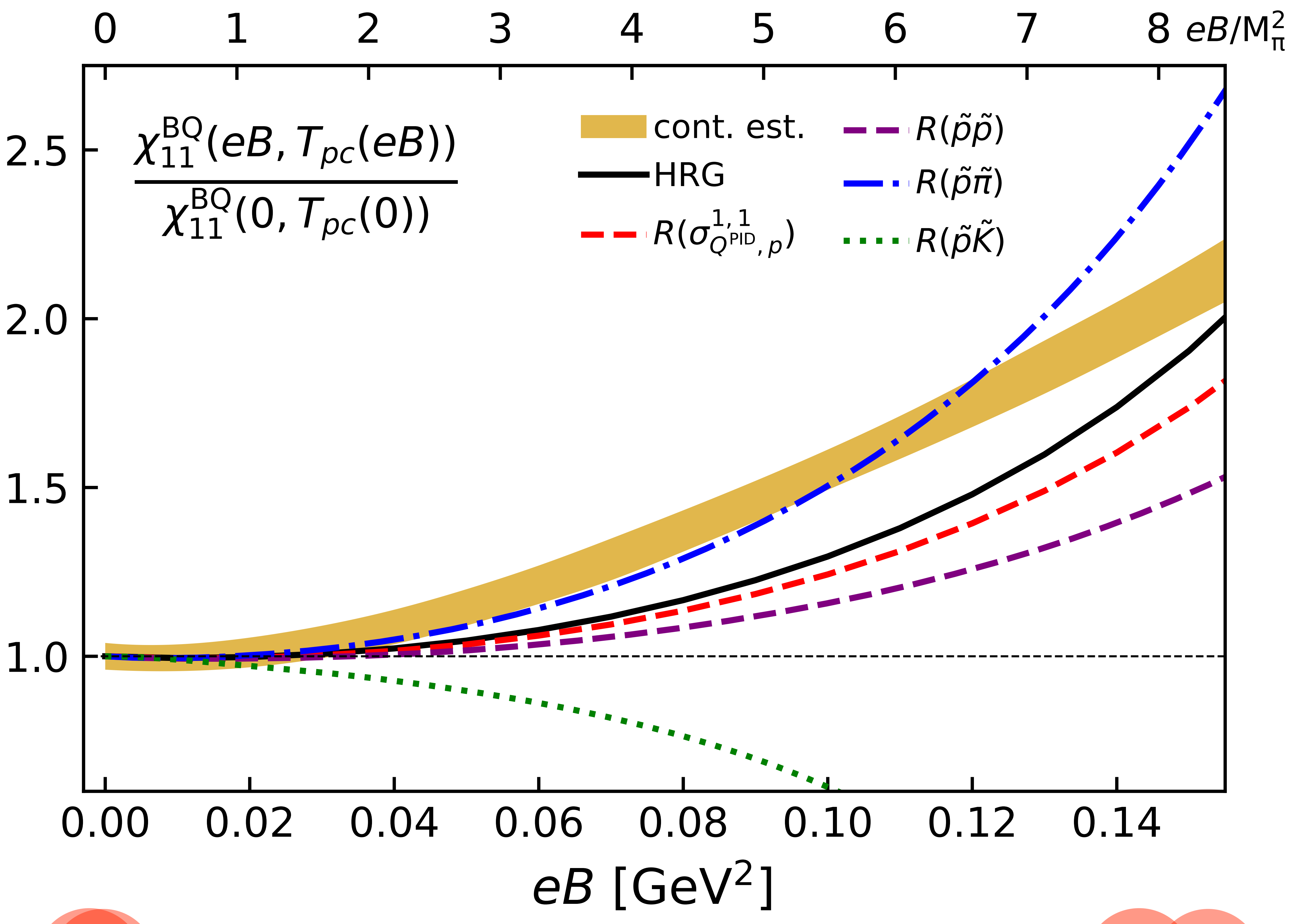
At $eB \lesssim M_\pi^2$: consistent with unity

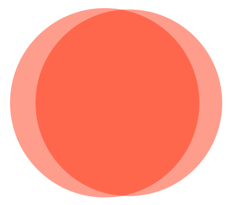
At $eB \simeq 8M_\pi^2$: ~2 !

X(eB)/X(eB=0) : Rcp like observable

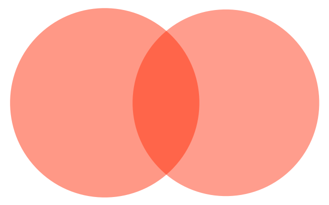
HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

Baryon electric charge correlation along the transition line





Central collision



Peripheral collision

Memory carried by the decays of Δ^{++}:

$$\Delta^{++} \to p + \pi^+$$

$$\sum_R B_R^l Q_R^m S_R^n I_p^R \to \sum_{i \in \text{stable}} \sum_R (P_{R\to i})^p B_i^l Q_i^m S_i^n I_p^R,$$

- net-B approximated by $\tilde{p}$
- net-Q approximated by Q^{PID}: $\tilde{\pi}^+, \tilde{K}^+, \tilde{p}$

See procedures to construct the proxy using HRG at eB=0, Bellwied et al., Phys. Rev. D 101 (2020) 034506

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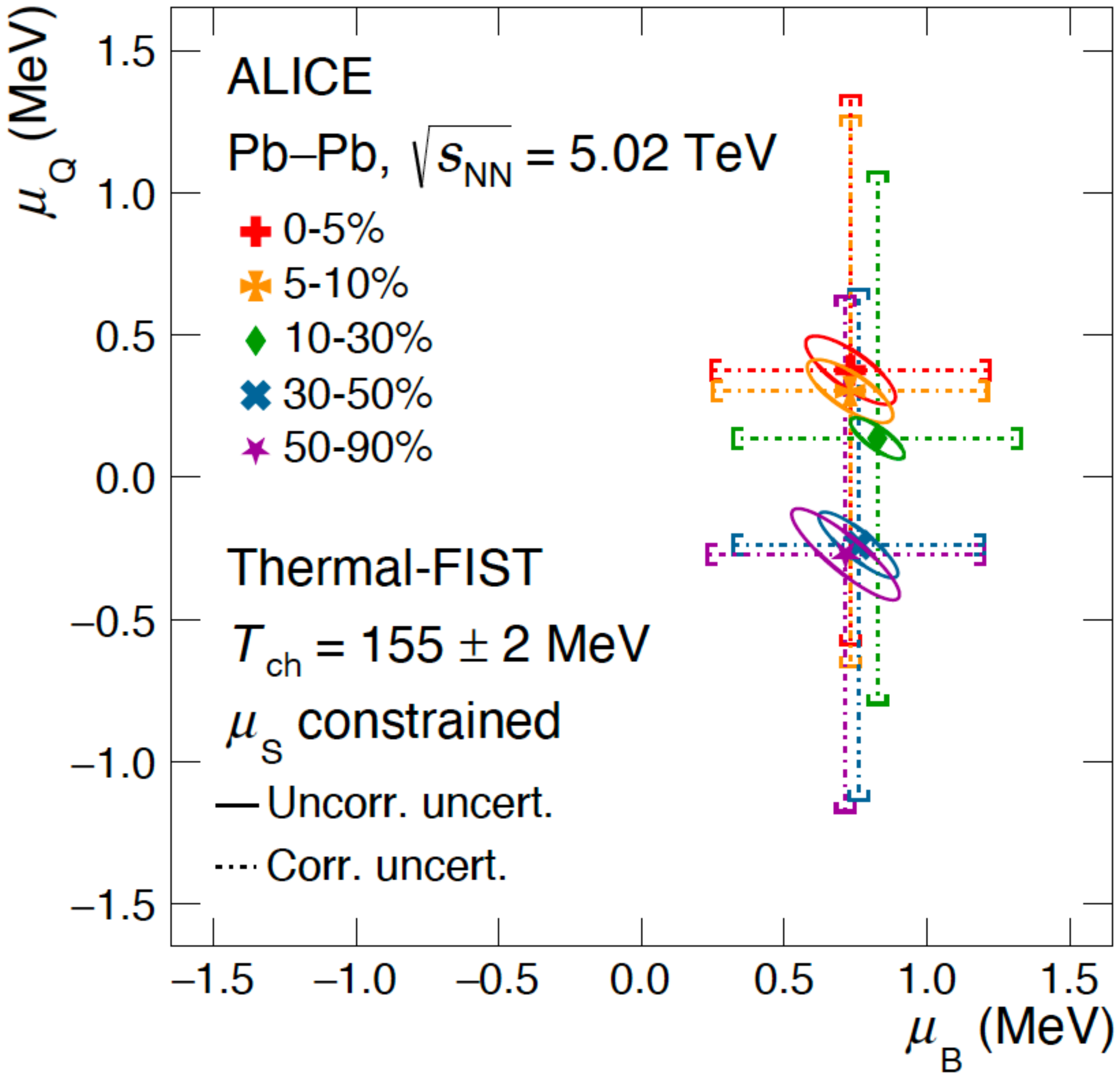
Electric charge chemical potential over baryon chemical potential



ALICE, arXiv: 2311.13332

- μ_Q/μ_B can be obtained from the thermal fits to particle yields

- μ_Q/μ_B is also connected to fluctuations of B,Q,S

$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

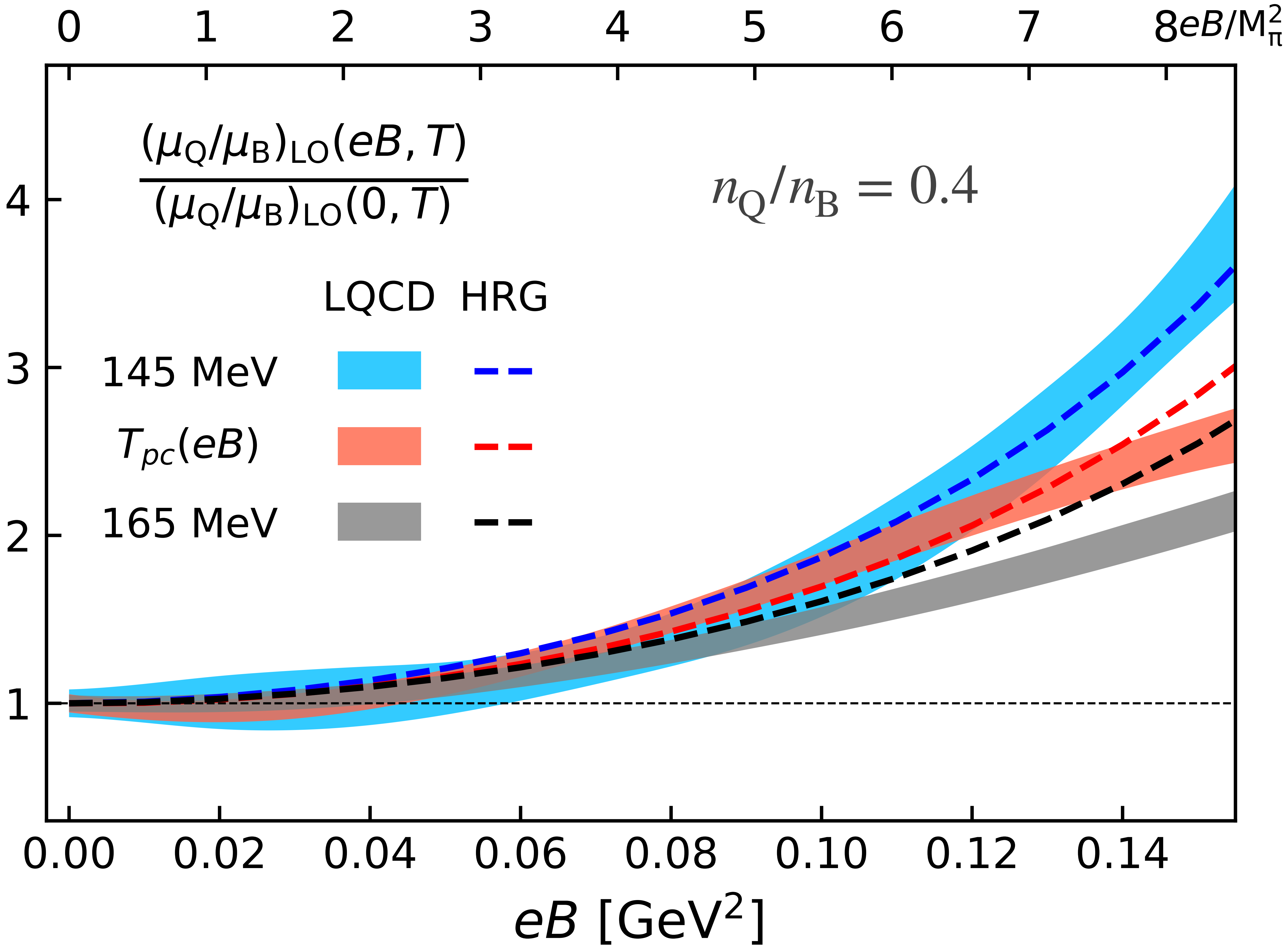
$$q_1 = \frac{r(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}) - (\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}{(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}) - r(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}$$

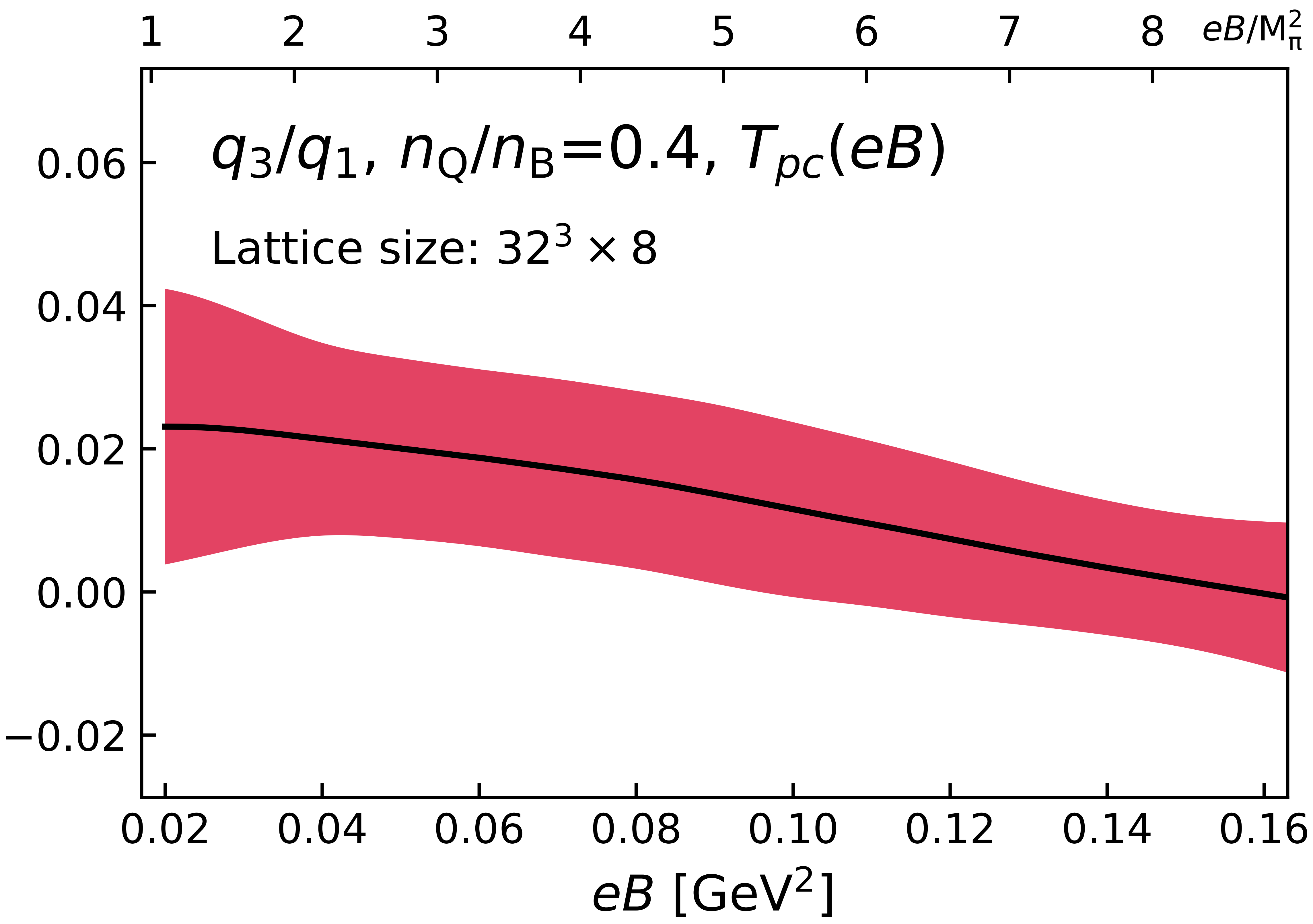
$$r = n_Q/n_B$$

HotQCD, PRL 109 (2012) 192302

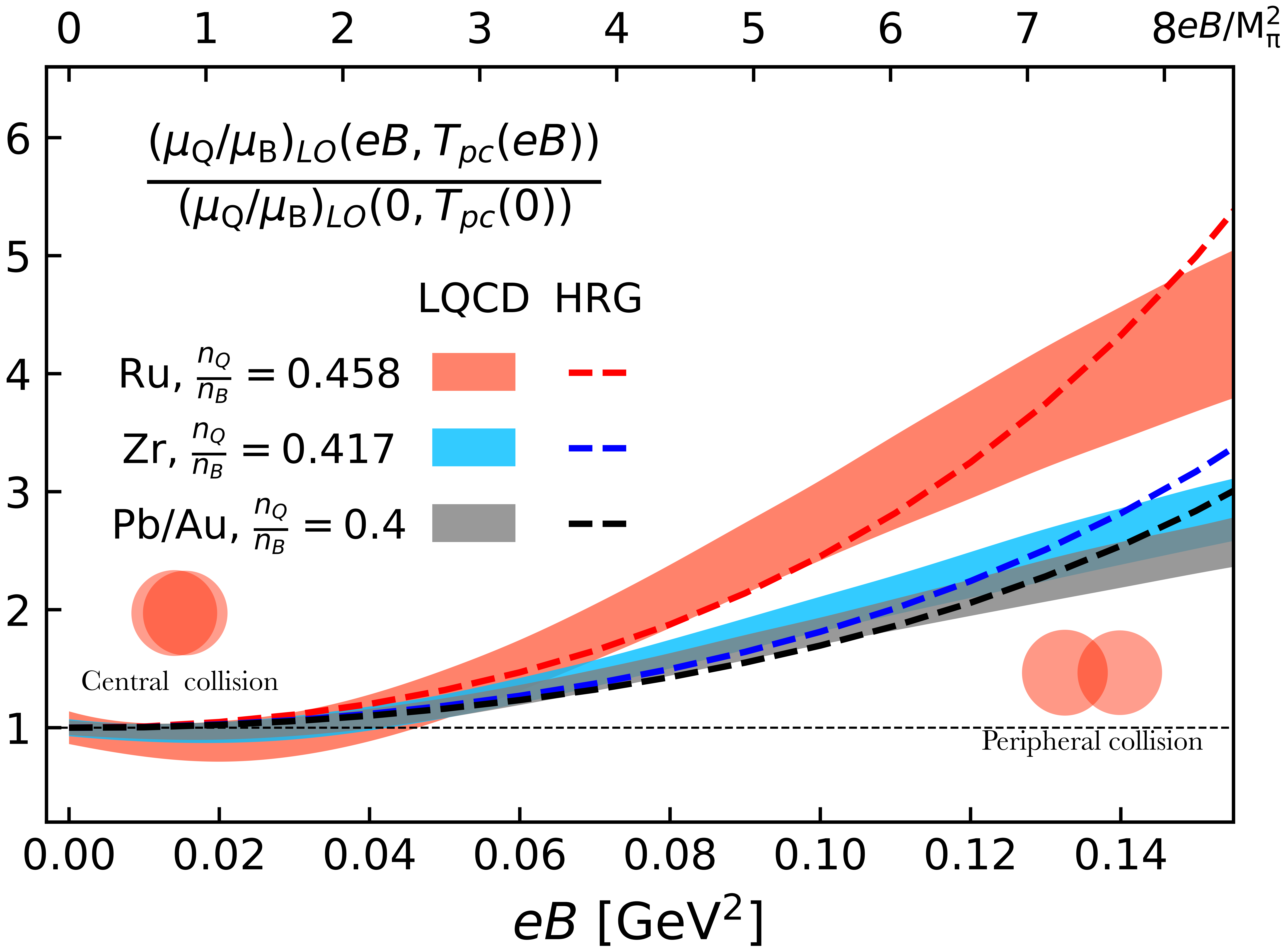
μ_Q/μ_B in nonzero magnetic fields

$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$





μ_Q/μ_B in different collision systems



$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

$$q_1 = \frac{r(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}) - (\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}{(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}) - r(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}$$

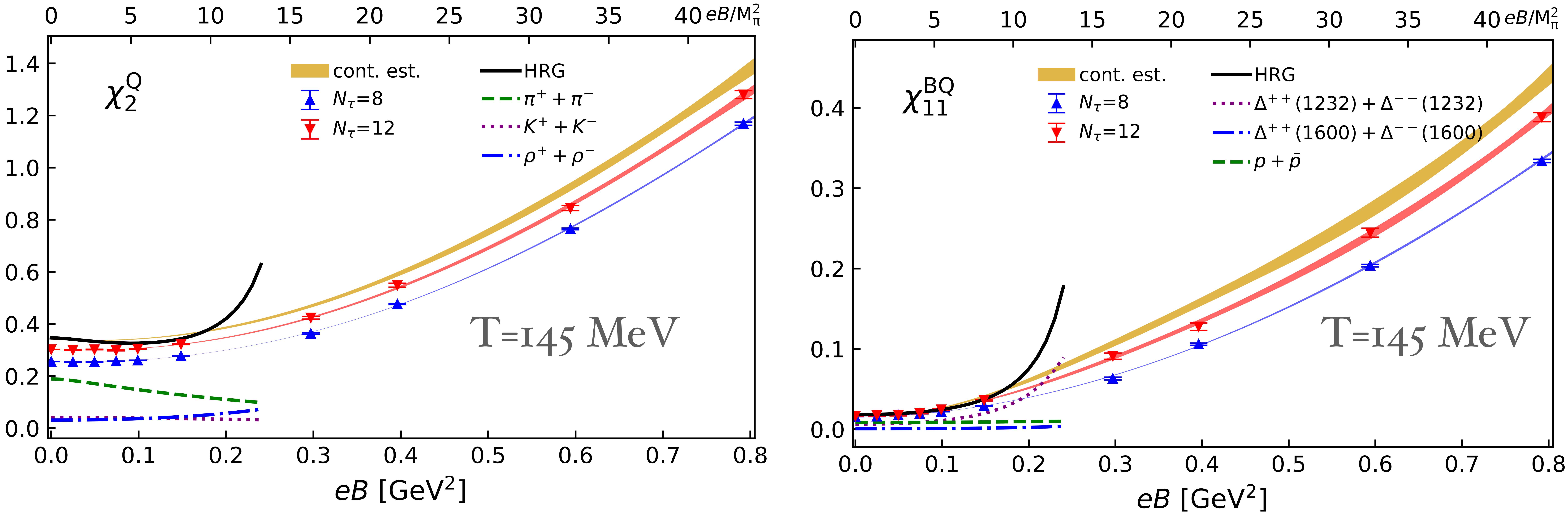
$$r = n_Q/n_B$$

${}^{96}_{44}\mathrm{Ru} + {}^{96}_{44}\mathrm{Ru}$: r=0.458

${}^{96}_{40}\mathrm{Zr} + {}^{96}_{40}\mathrm{Zr}$: r=0.417

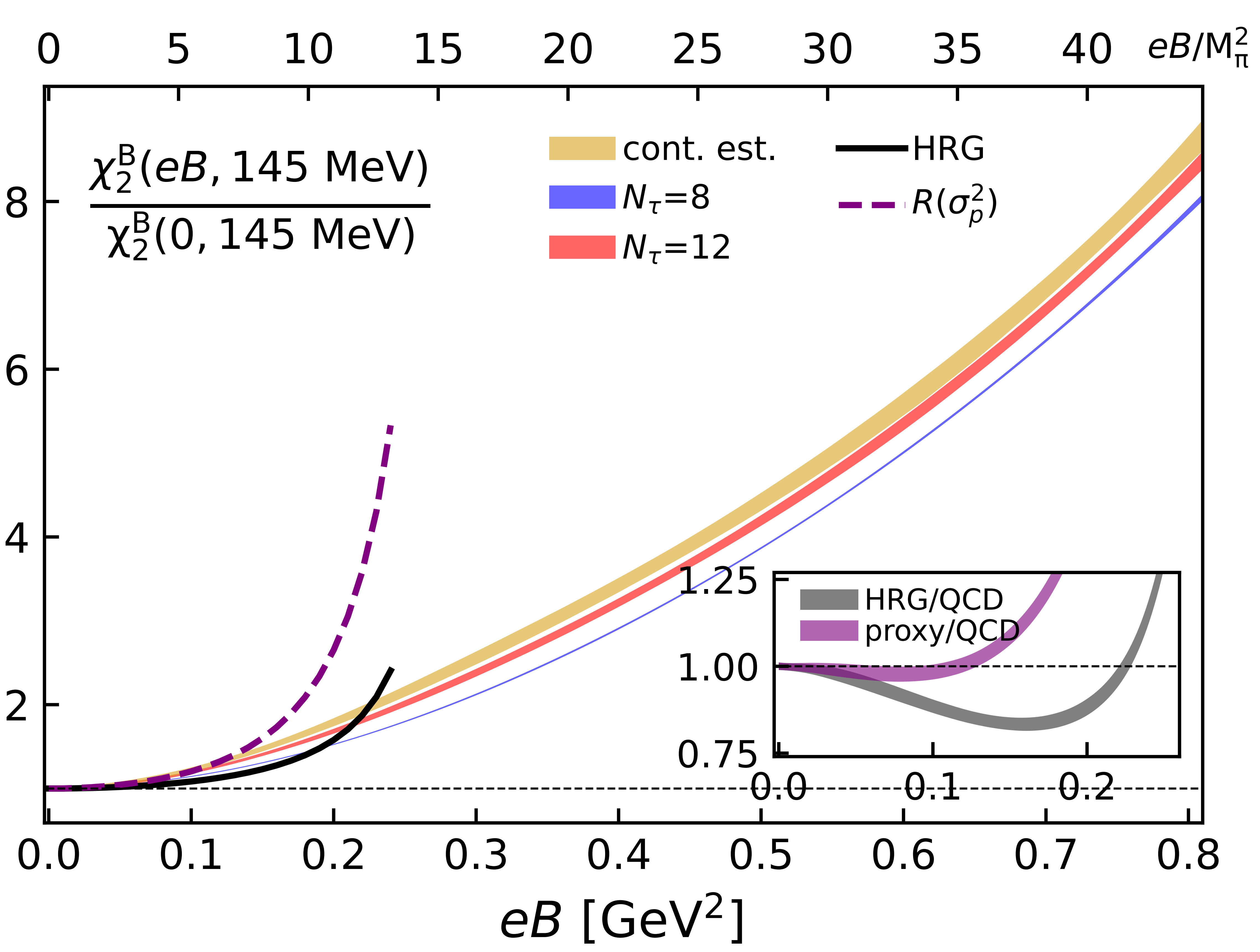
${}^{208}_{82}\mathrm{Pb} + {}^{208}_{82}\mathrm{Pb}$: r=0.4

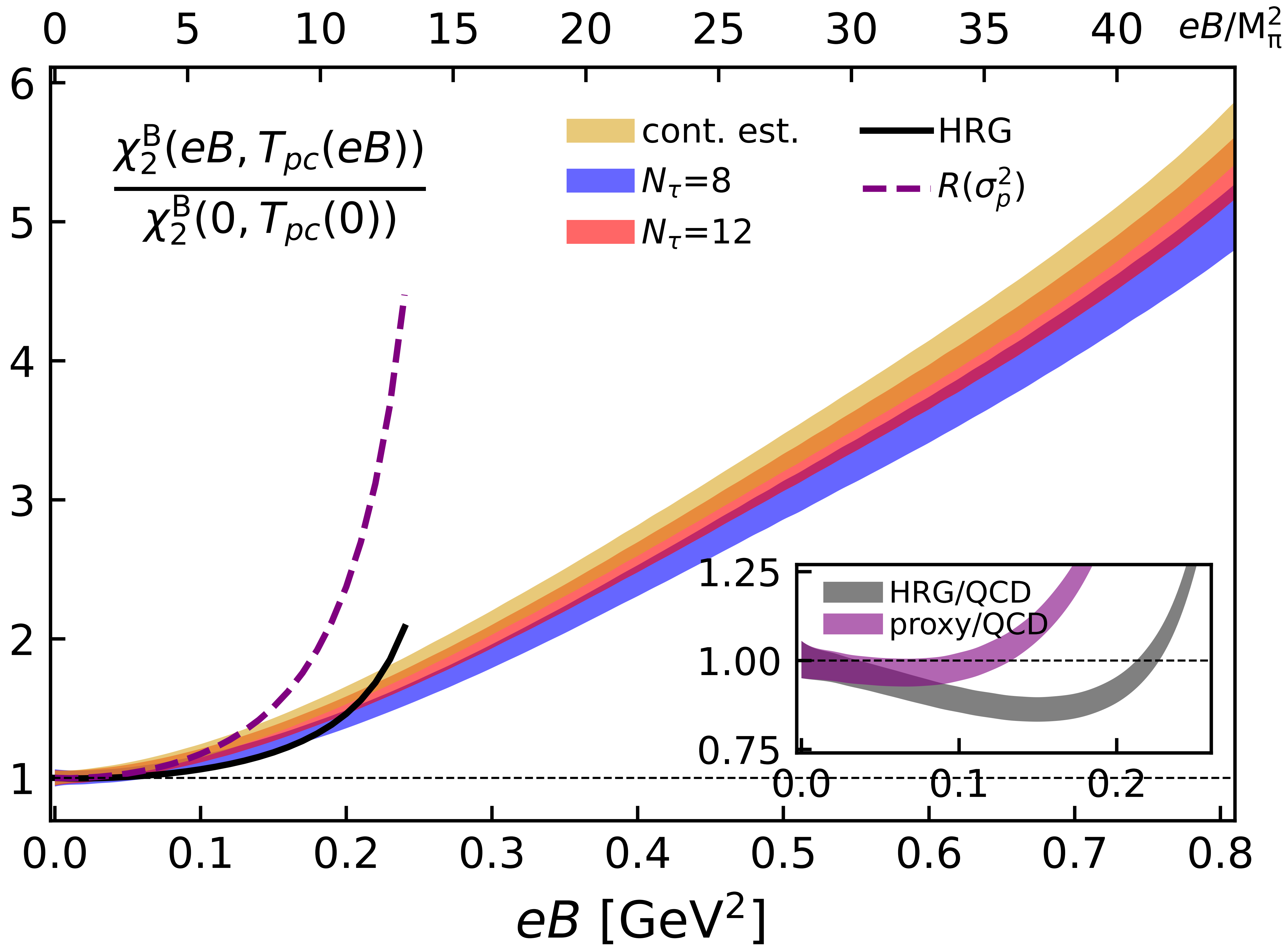
The breaking down of HRG in very strong magnetic fields



HTD, J.-B. Gu et al., work in progress

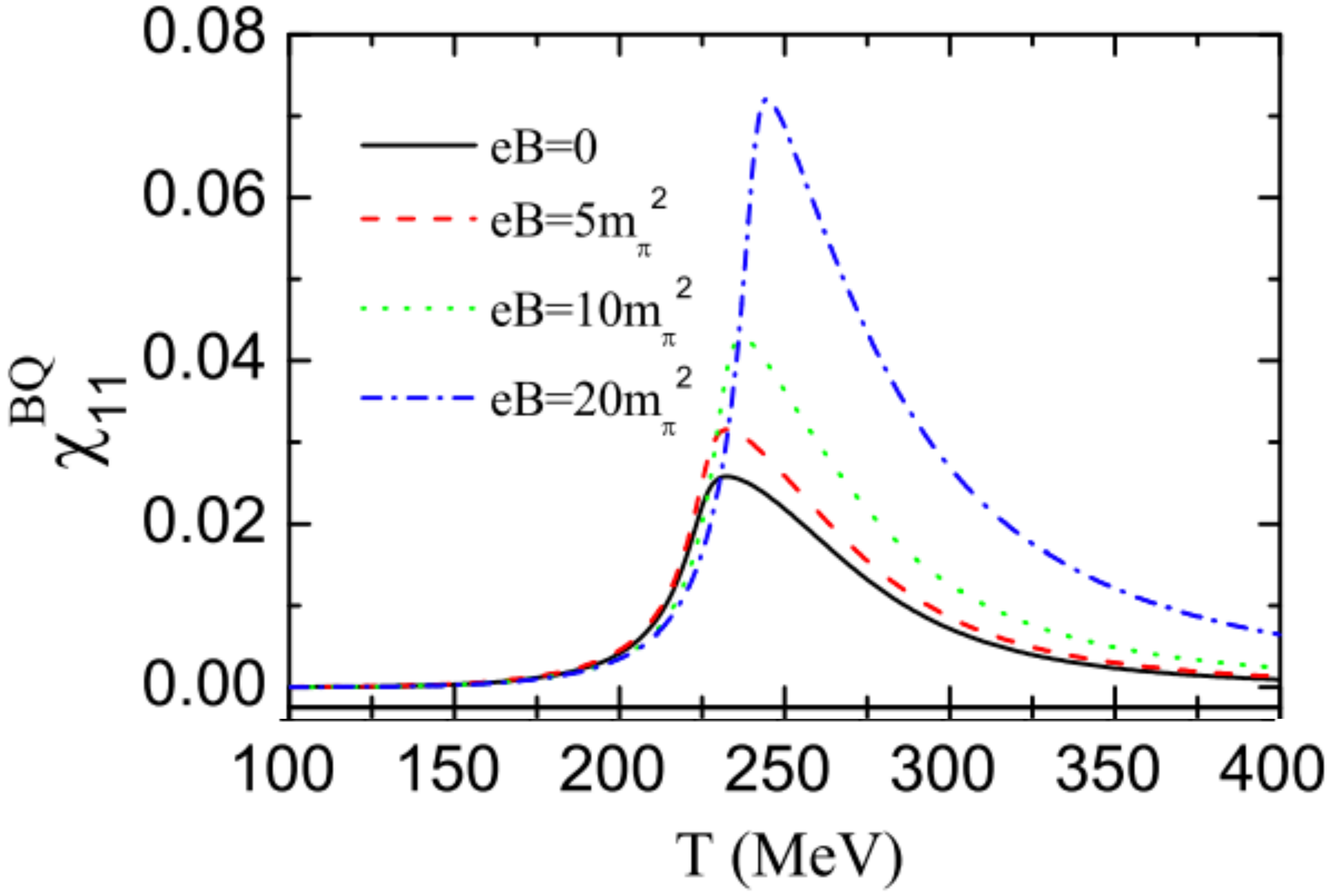
Baryon number fluctuations in very strong magnetic fields



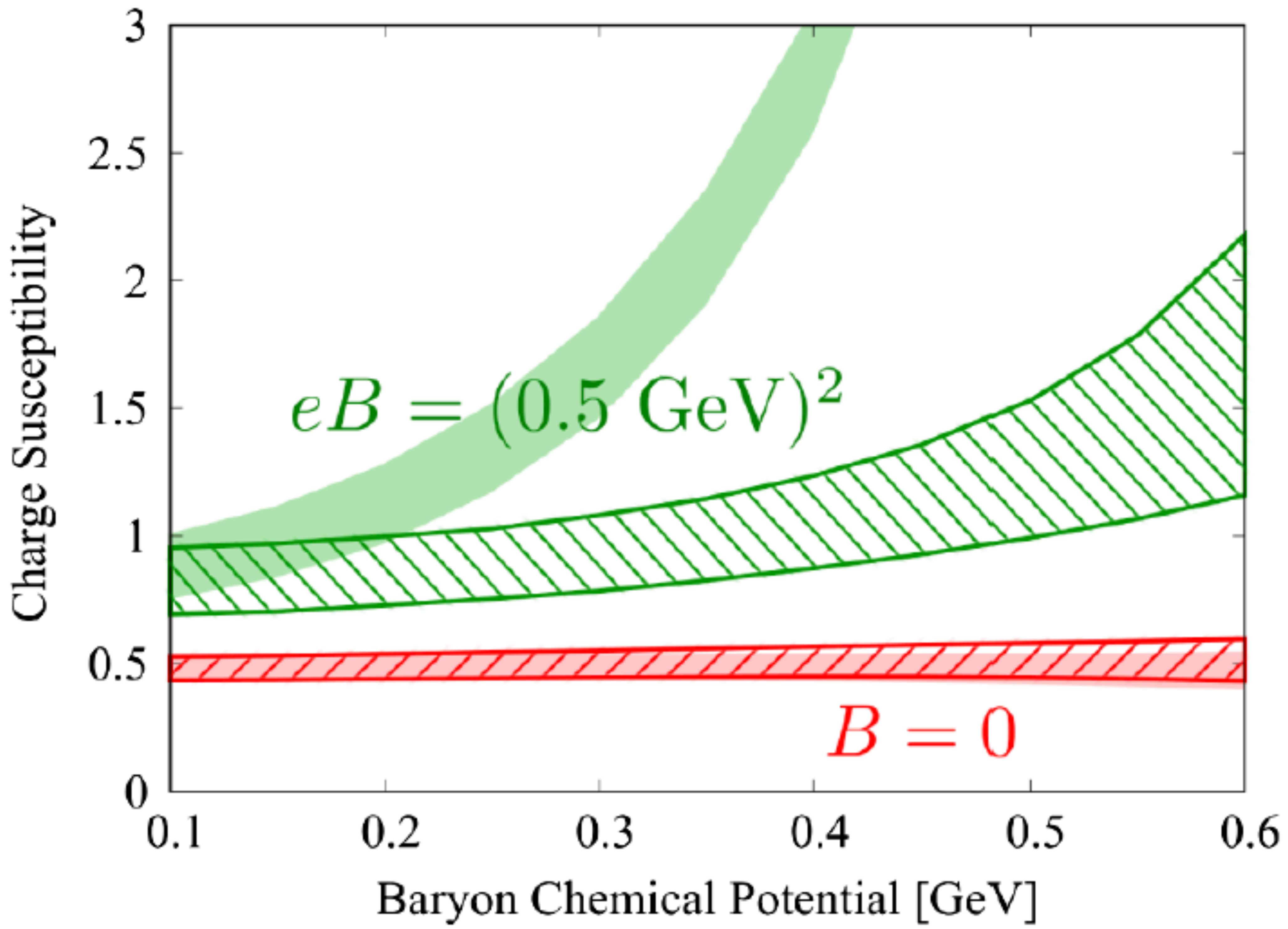


HTD, J.-B. Gu et al., work in progress

Results from effective theory and model studies



W.-J. Fu, Phys. Rev. D 88 (2013) 014009

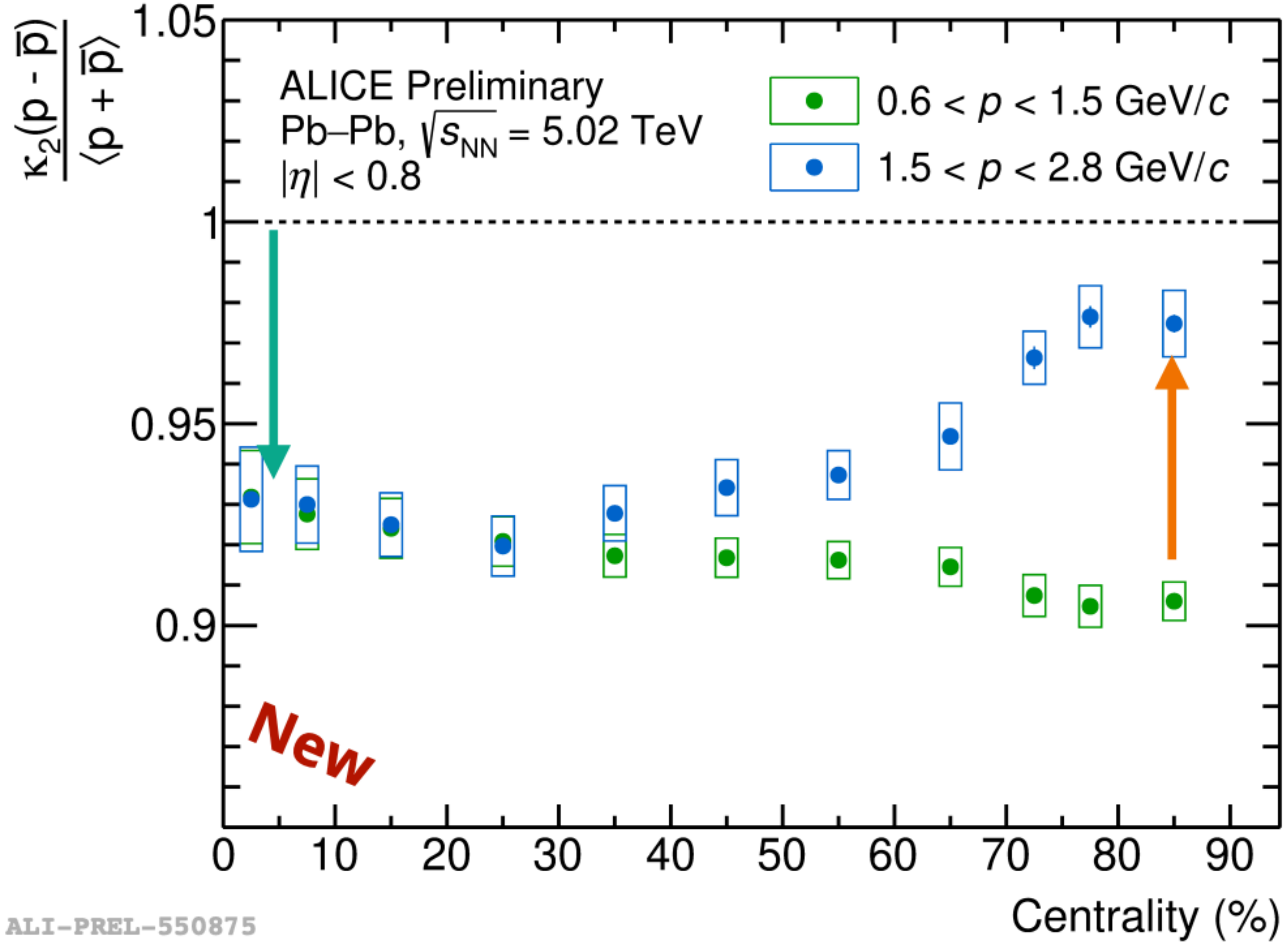


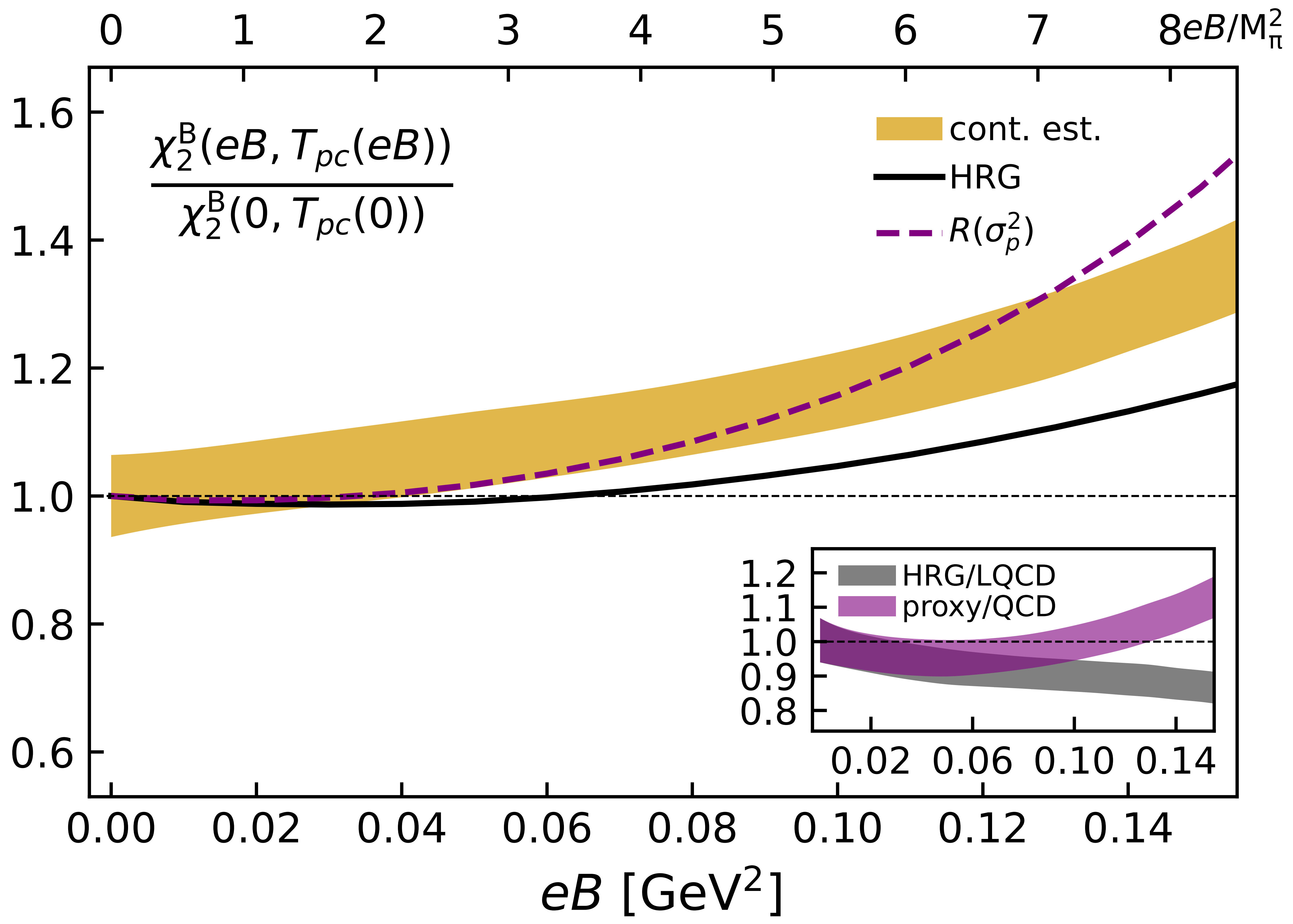
K. Fukushima and Y. Hidaka , Phys. Rev. Lett. 117 (2016) 102301

The inverse magnetic catalysis is missing

At eB=0.25 GeV^2, HRG overshoots the LQCD data

Centrality dependences in HIC



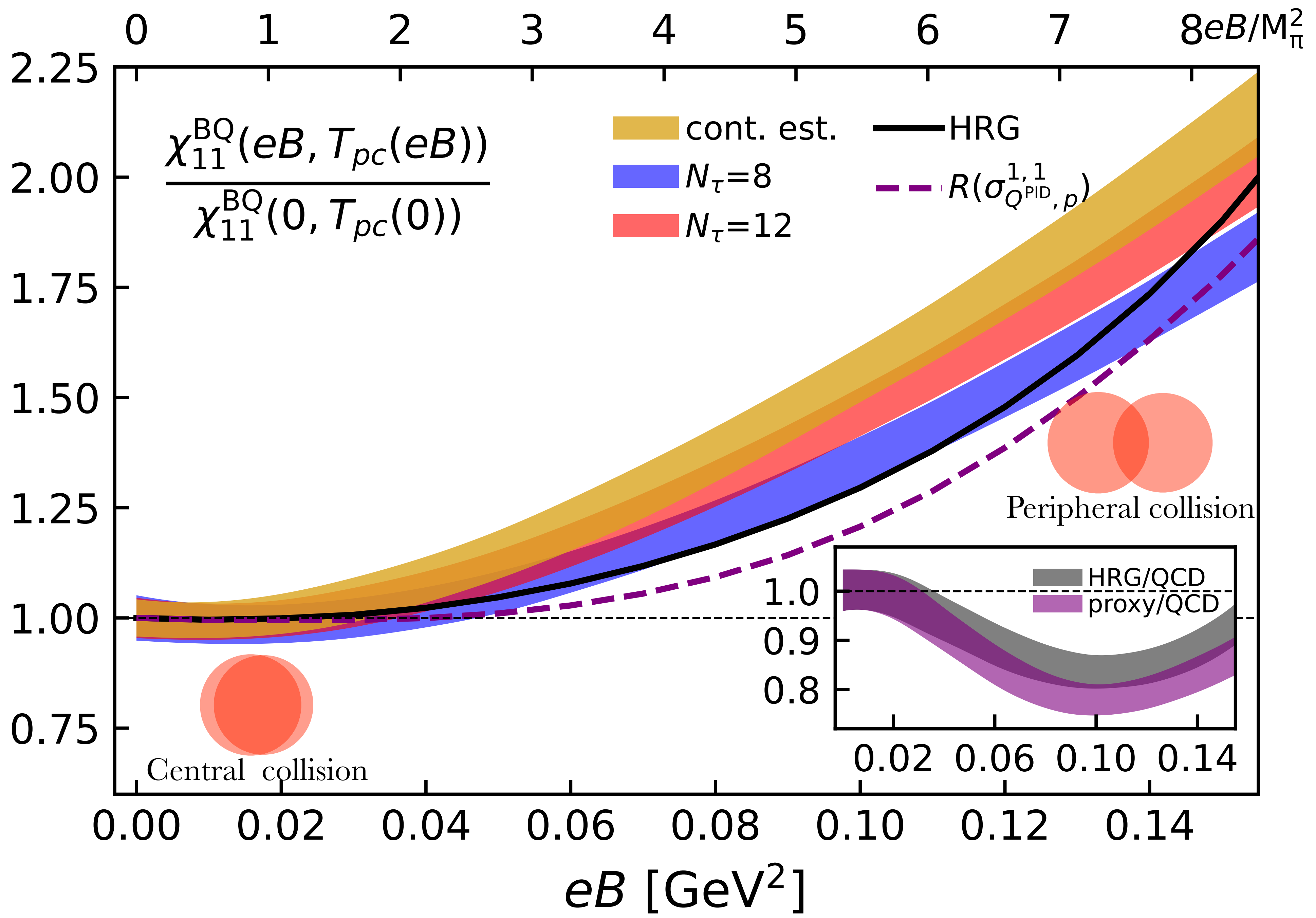


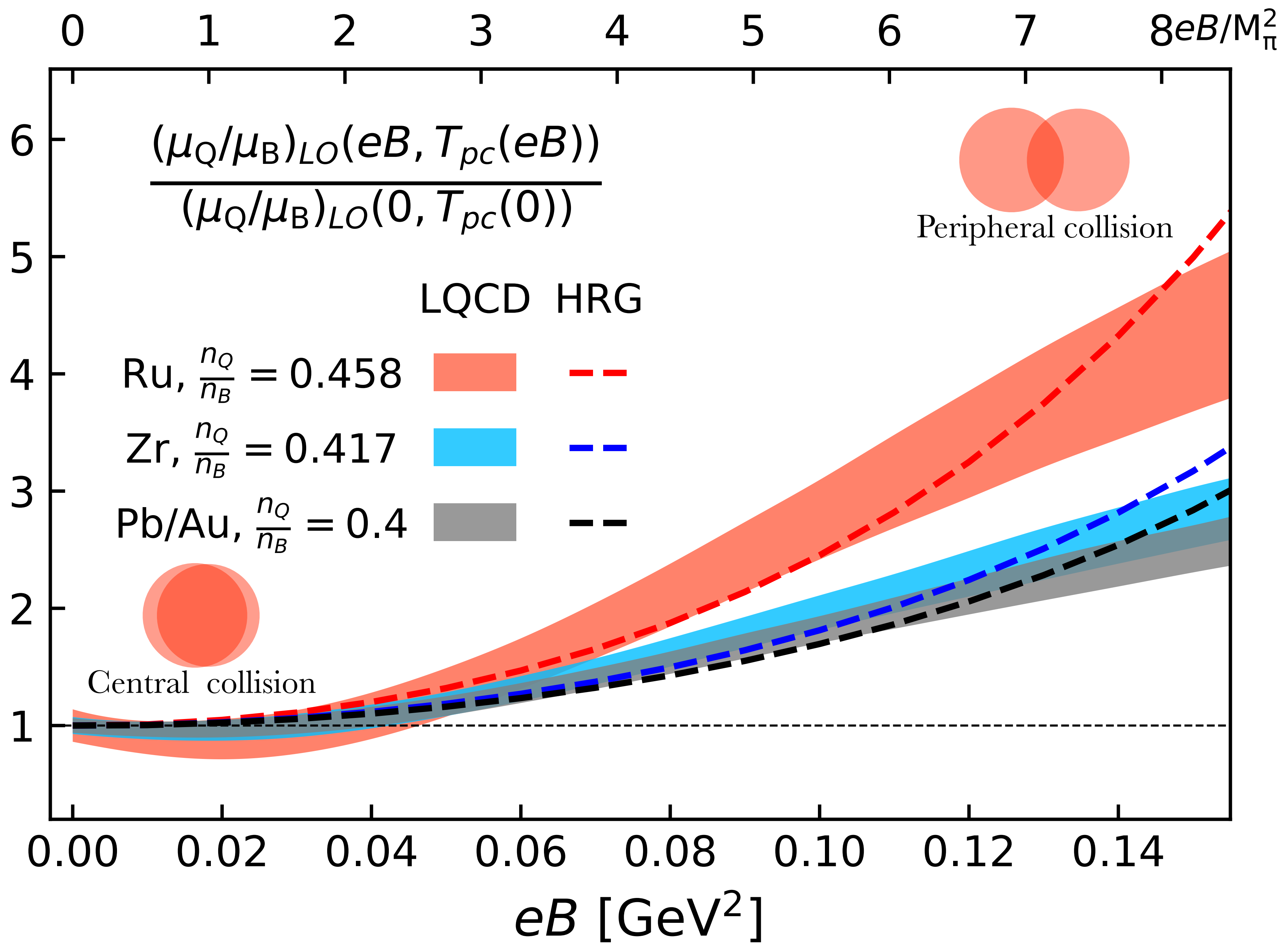
Ilya Fokin, ALICE, Quark Matter 2023

HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

Summary

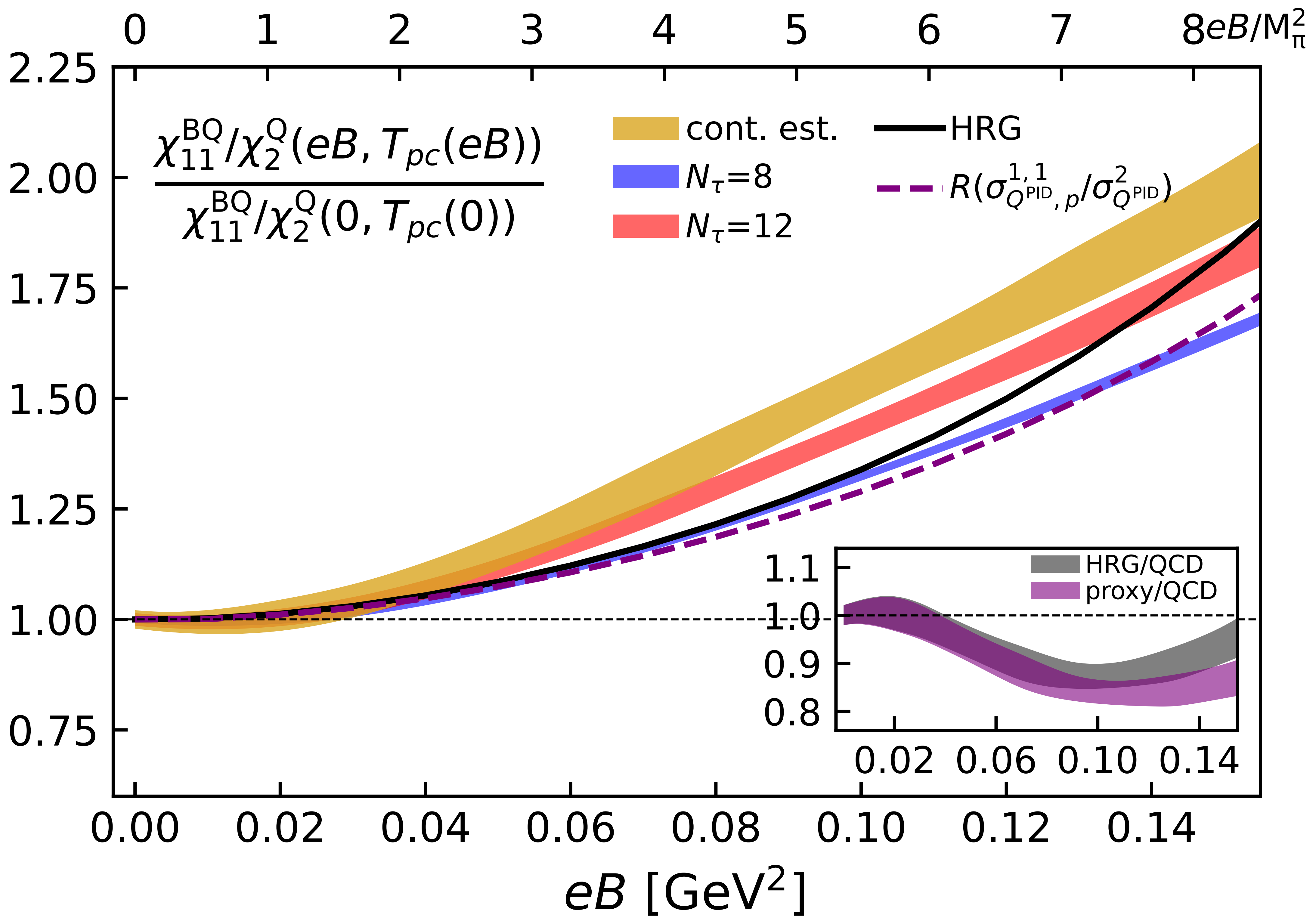
- A first lattice QCD computation of fluctuations of B, Q, S in nonzero magnetic fields
- QCD baselines for effective theories and model studies
- Probes to detect imprints of magnetic fields in HIC: χ_{11}^{BQ} measured from proxy and $\mu_{\mathrm{Q}}/\mu_{\mathrm{B}}$ obtained from thermal fits to particle yields in HIC

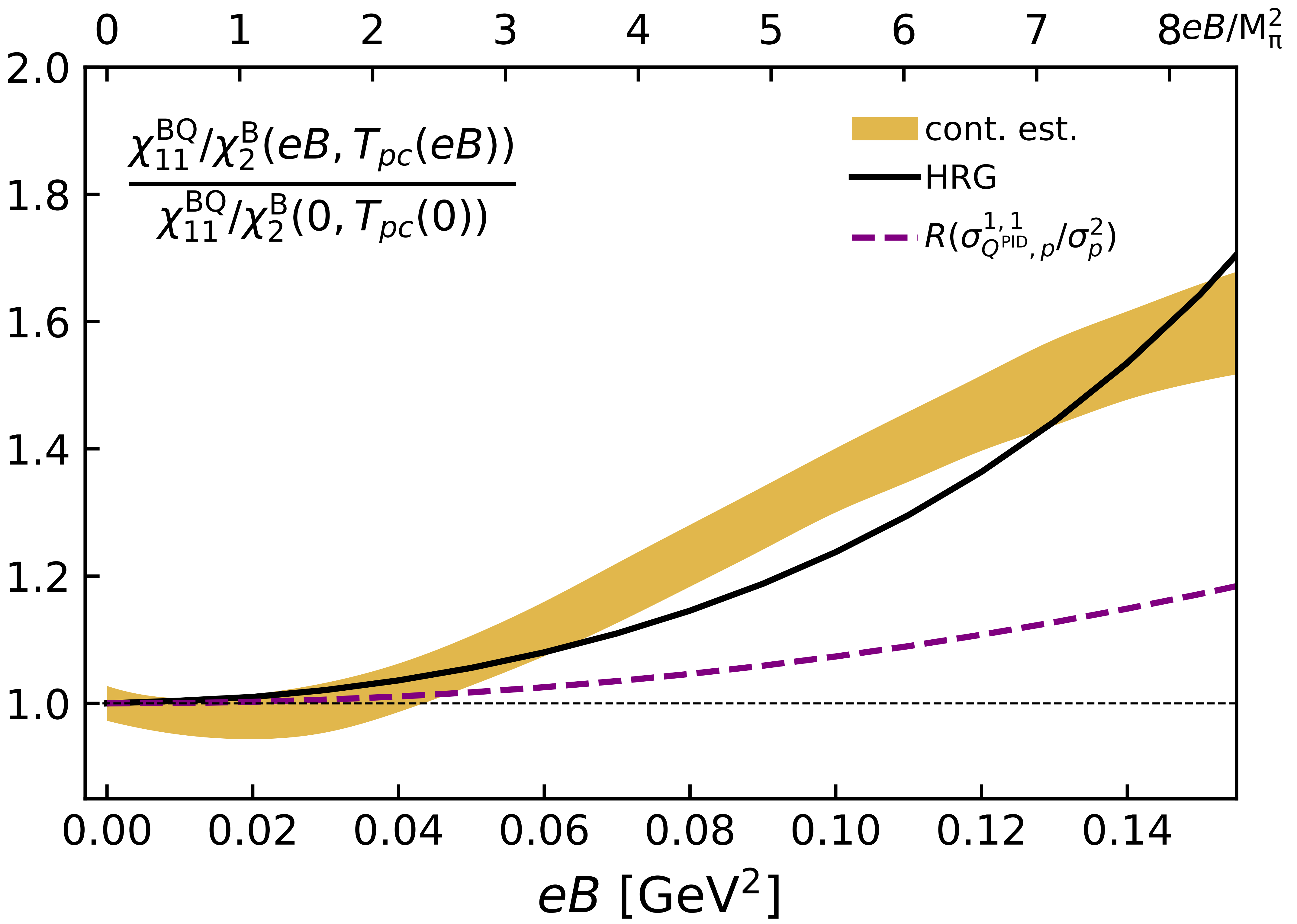




Backup

Double ratios





Construction of the proxy $\sigma^{1,1}_{Q^{\mathrm{PID}},p}$ for χ^{BQ}_{11}

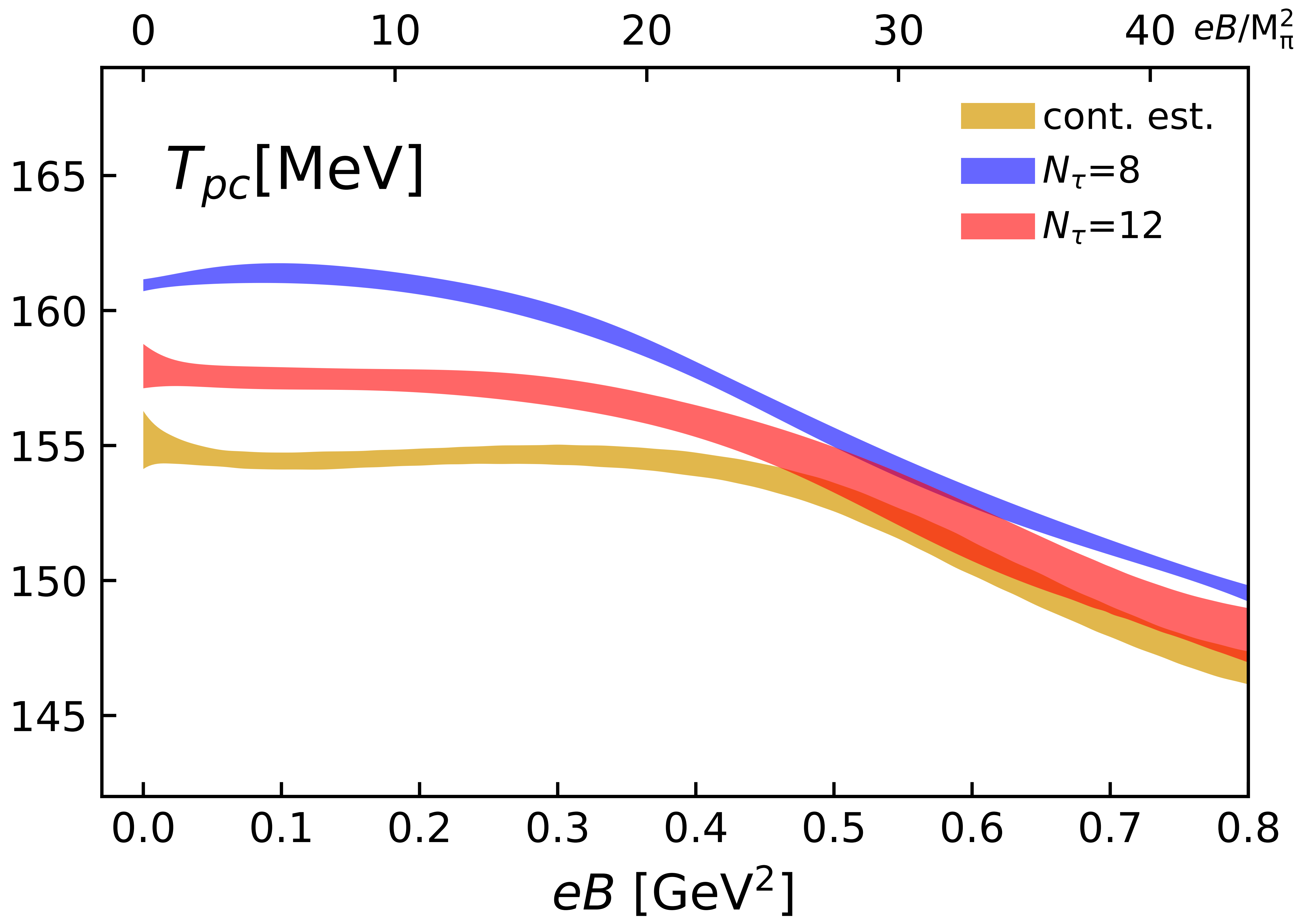
arXiv: 2312.08860

$$\sigma^{1,1}_{Q^{\mathrm{PID}},p} = \sum_R \left(P_{R\to\tilde{p}}\right)\left(P_{R\to Q^{\mathrm{PID}}}\right) I^{\mathrm{BQ}}_R + I^{\mathrm{BQ}}_{\tilde{p}}$$

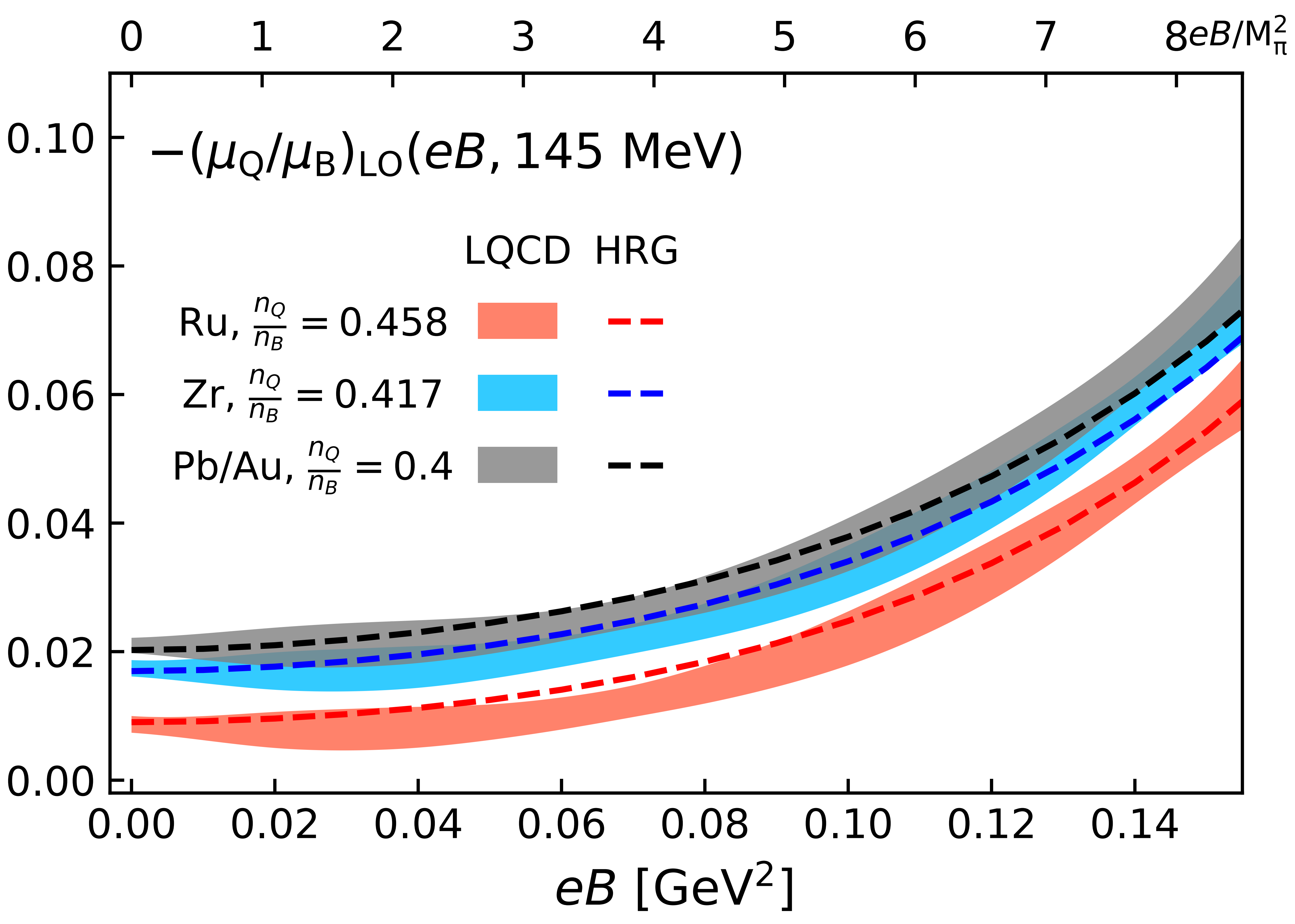
$$\chi^{XY}_{11} = \frac{B}{2\pi^2 T^3}\sum_i \left|q_i\right| X_i Y_i \sum_{s_z=-s_i}^{s_i}\sum_{l=0}^{\infty} f\left(\varepsilon_0\right) \equiv I^{\mathrm{BQ}}_i, \quad f\left(\varepsilon_0\right) = \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} n \; \mathrm{K}_1\left(\frac{n\varepsilon_0}{T}\right)$$

- $P_{R\to\tilde{j}} = P_{R\to j} - P_{R\to\bar{j}}$: difference of P between the particle j and its antiparticle $\bar{j}$
- $P_{R\to Q^{\mathrm{PID}}} = P_{R\to\tilde{p}} + P_{R\to\tilde{K}} + P_{R\to\tilde{\pi}}$
- $P_{R\to j} = \sum_\alpha \mathrm{N}^{\alpha}_{R\to j} n^R_{j,\alpha}$: the average number of particle j produced by each particle R after the entire decay chain
    - $\mathrm{N}^{\alpha}_{R\to j}$: the branching ratio of decay channel α
    - $n^R_{j,\alpha}$: the number of stable particle j produced from this decay channel

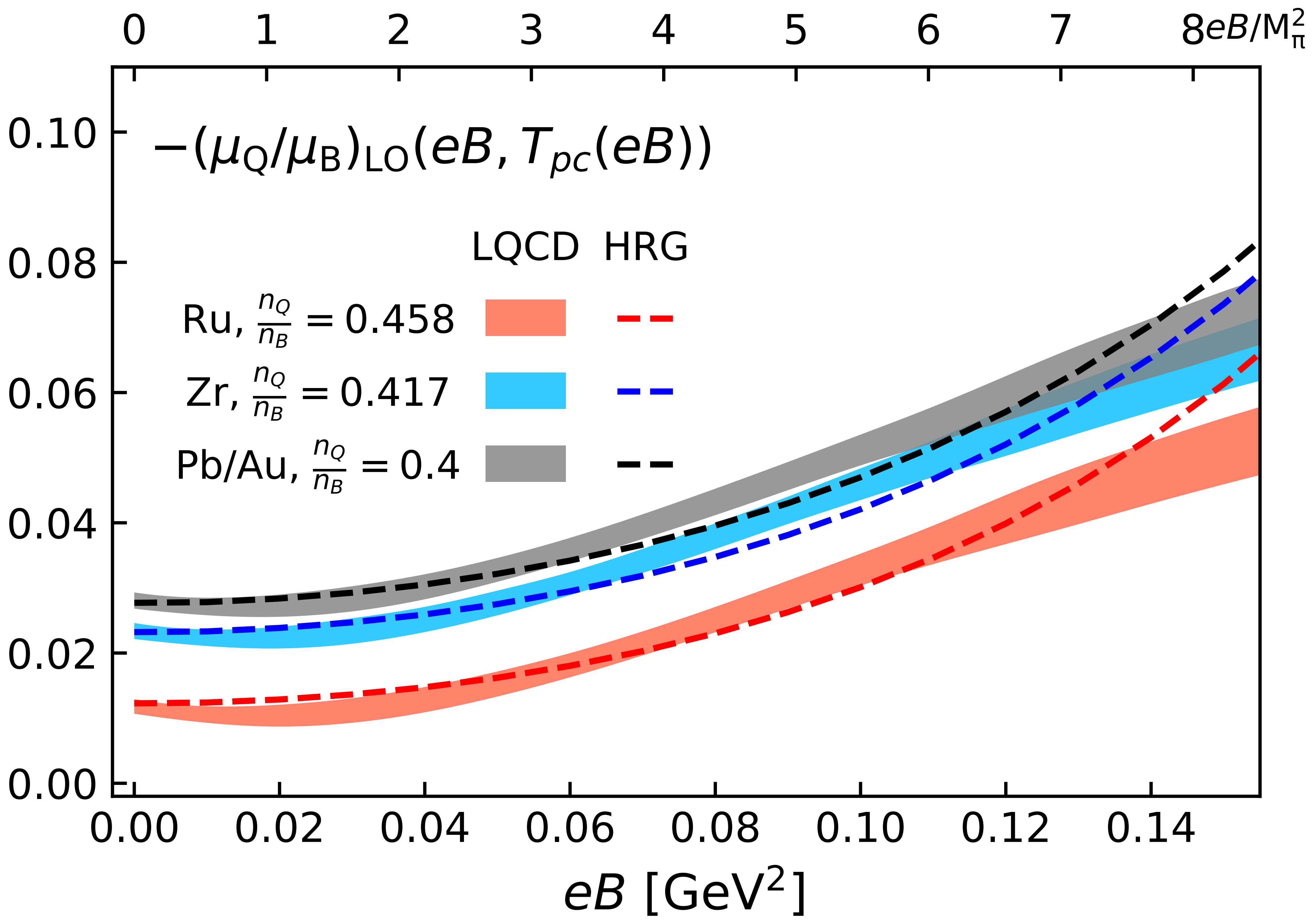
T_{pc} in strong magnetic fields



μ_Q/μ_B



$-(\mu_Q/\mu_B)_{LO}(eB, 145\ \text{MeV})$
LQCD HRG
Ru, $\frac{n_Q}{n_B} = 0.458$
Zr, $\frac{n_Q}{n_B} = 0.417$
Pb/Au, $\frac{n_Q}{n_B} = 0.4$
eB/M_π^2
eB [GeV^2]



$-(\mu_Q/\mu_B)_{LO}(eB, T_{pc}(eB))$
LQCD HRG
Ru, $\frac{n_Q}{n_B} = 0.458$
Zr, $\frac{n_Q}{n_B} = 0.417$
Pb/Au, $\frac{n_Q}{n_B} = 0.4$
eB/M_π^2
eB [GeV^2]