arXiv:2405.12320

Chiral and deconfinement properties of the QCD crossover have a different volume and baryochemical potential dependence

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Properties of QCD crossover

Chiral

- $SU(2) \times SU(2)$ symmetry in limit $m_q \to 0$
- **order parameter:** chiral condensate $< \bar{\psi}\psi >$
- we study: chiral condensate and its chiral susceptibility χ

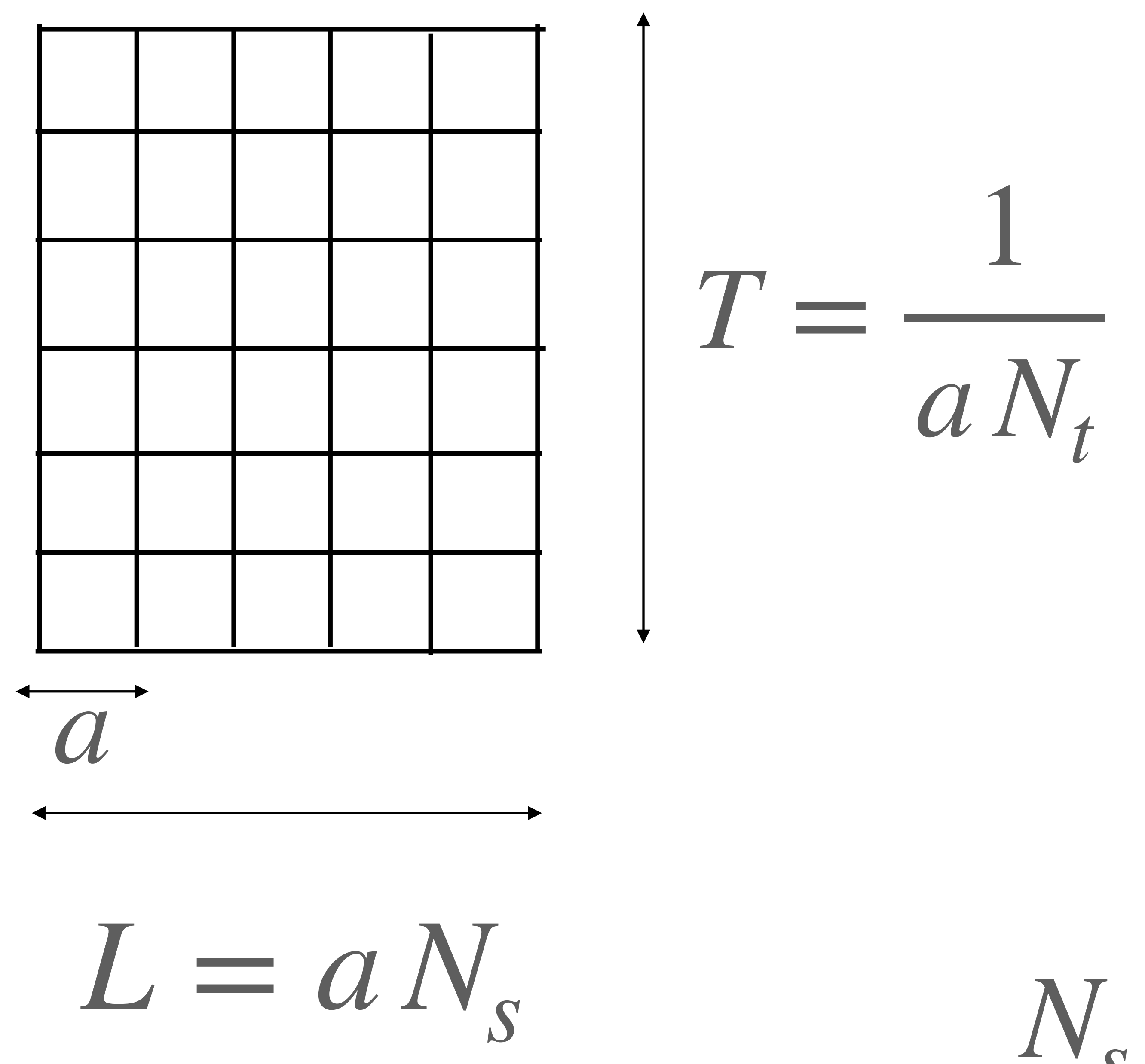
Deconfinement

- Z_3 symmetry in limit $m_q \to \infty$
- **order parameter**: Polyakov loop $P \sim e^{-F_Q/T}$
- we study: from P, the static quark free energy F_Q and the static quark entropy S_Q

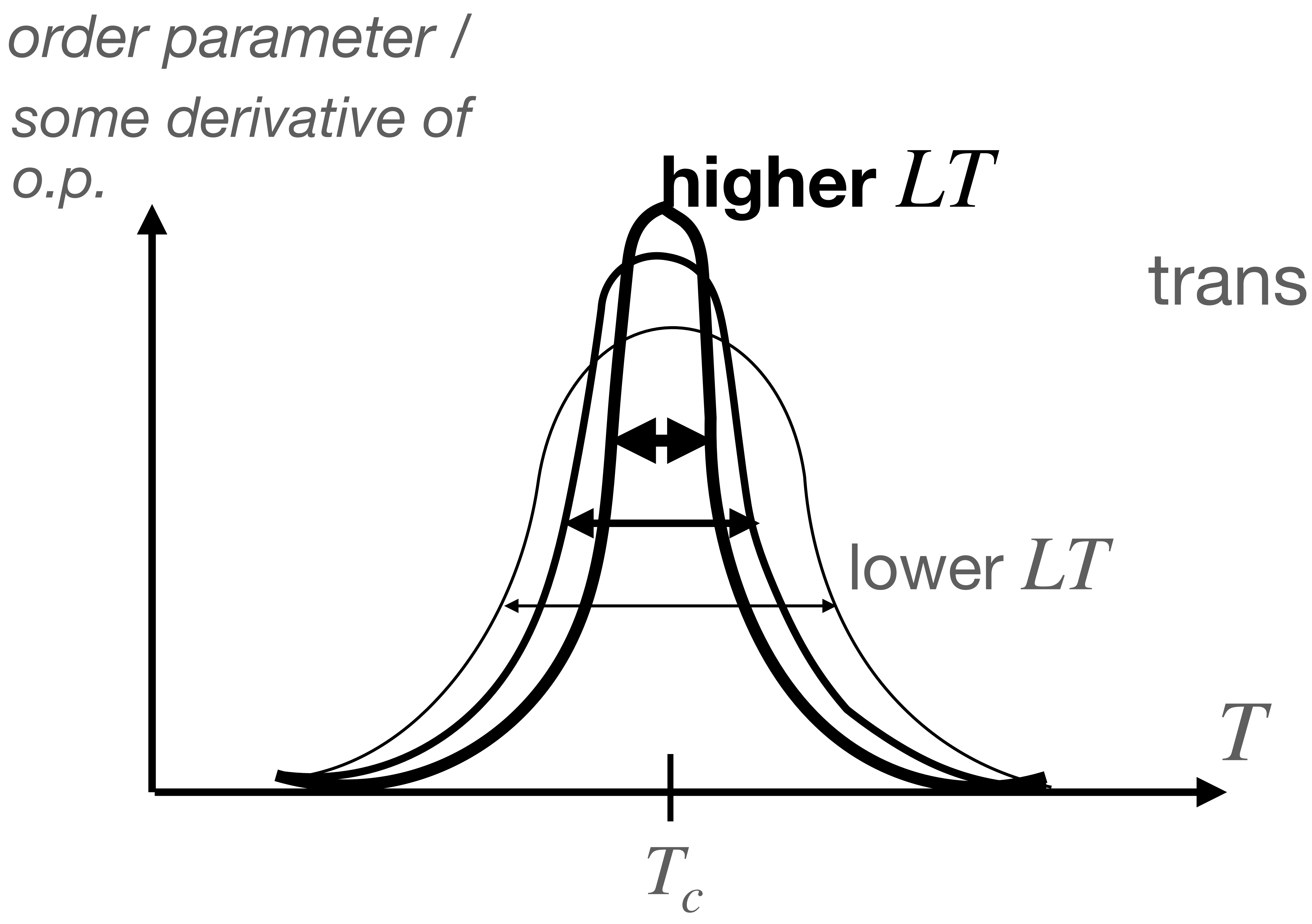
Plan:

- $\mu_B = 0$: volume dependence of T_c
- finite μ_B: μ_B dependence of the different definitions of T_c, strength of crossover

- $m_q \neq 0, \infty$ → both are approximate order parameters
- **Lattice** → we are at fixed volume $\mathbf{V}$ → we can't see transitions
- what happens in thermodynamic limit $\mathbf{LT} \to \infty$?



$LT = \frac{N_s}{N_t}$

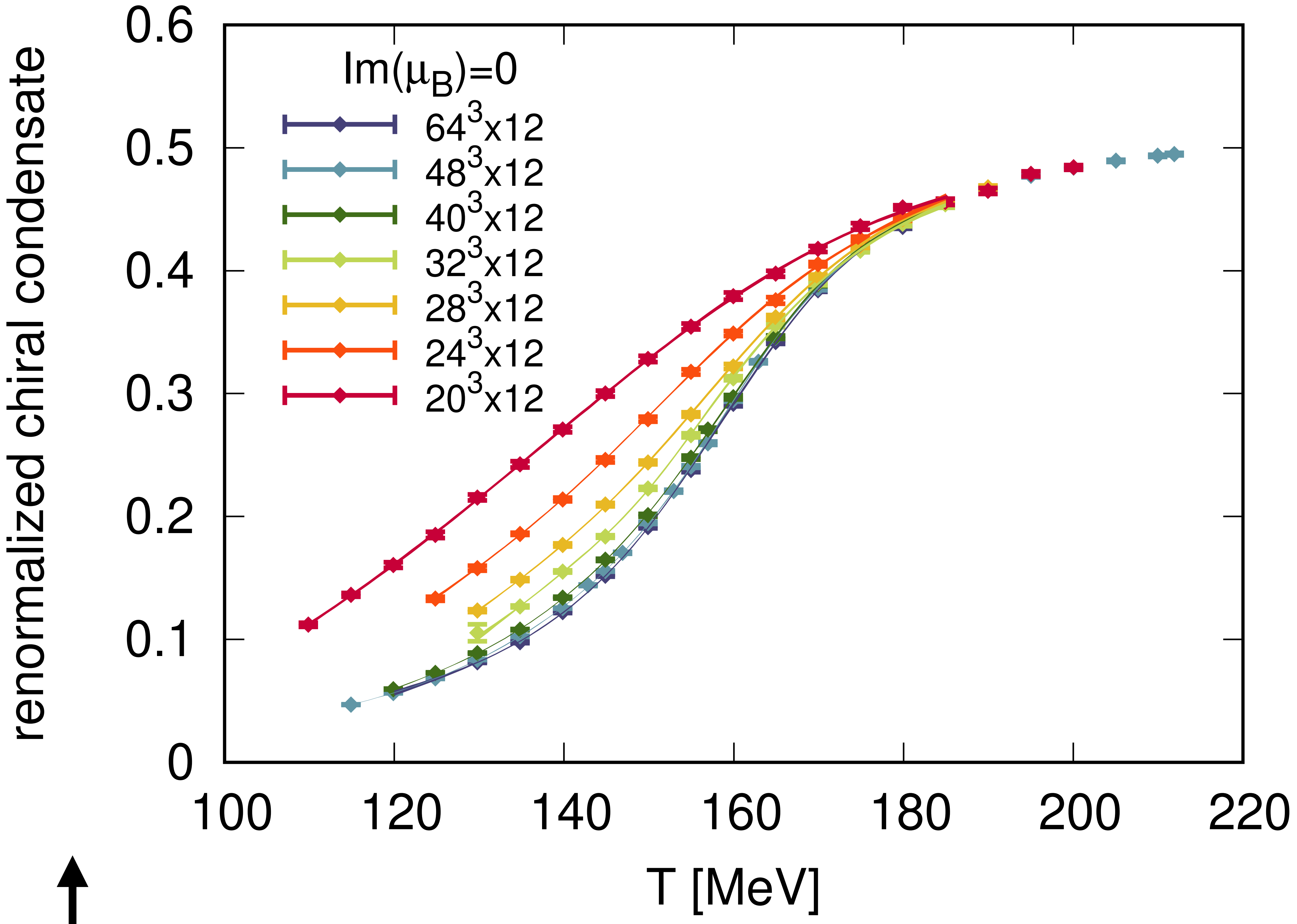


transition

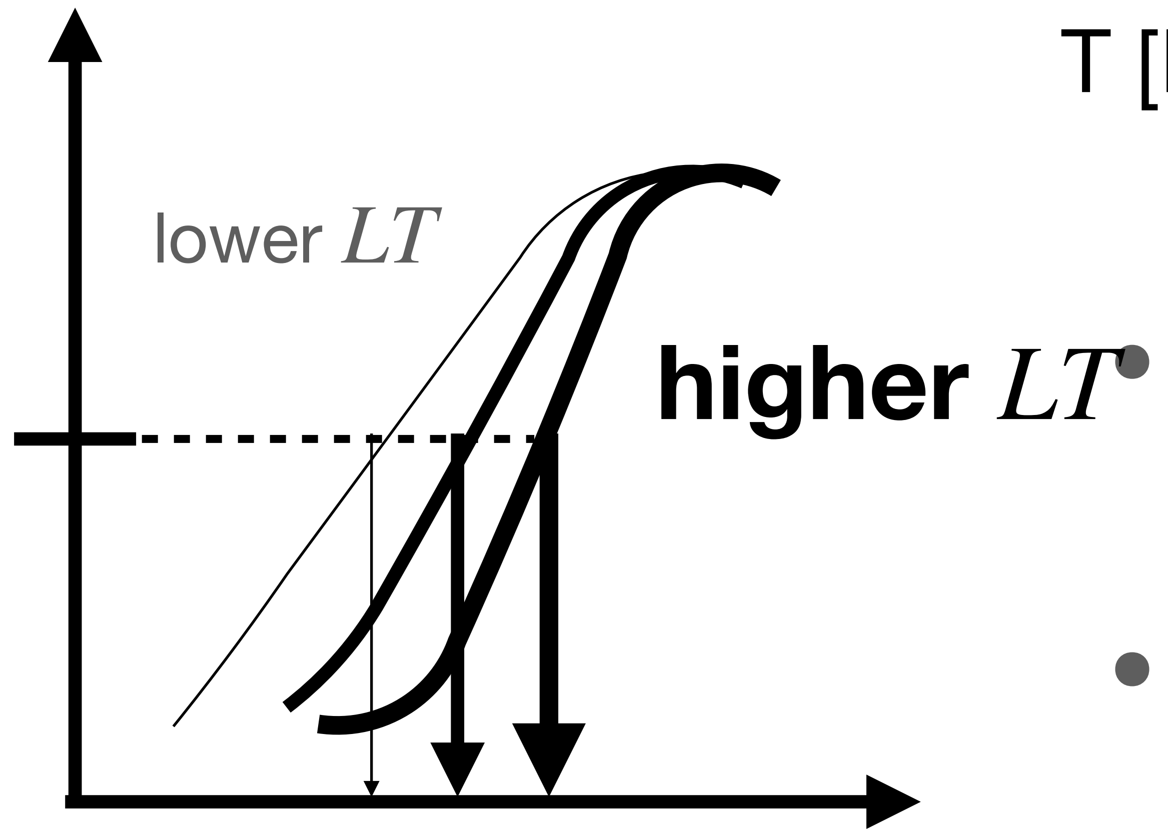
- how do chiral and deconfinement observables behave under $LT \to \infty$?

$\mu_B = 0$

Chiral observables



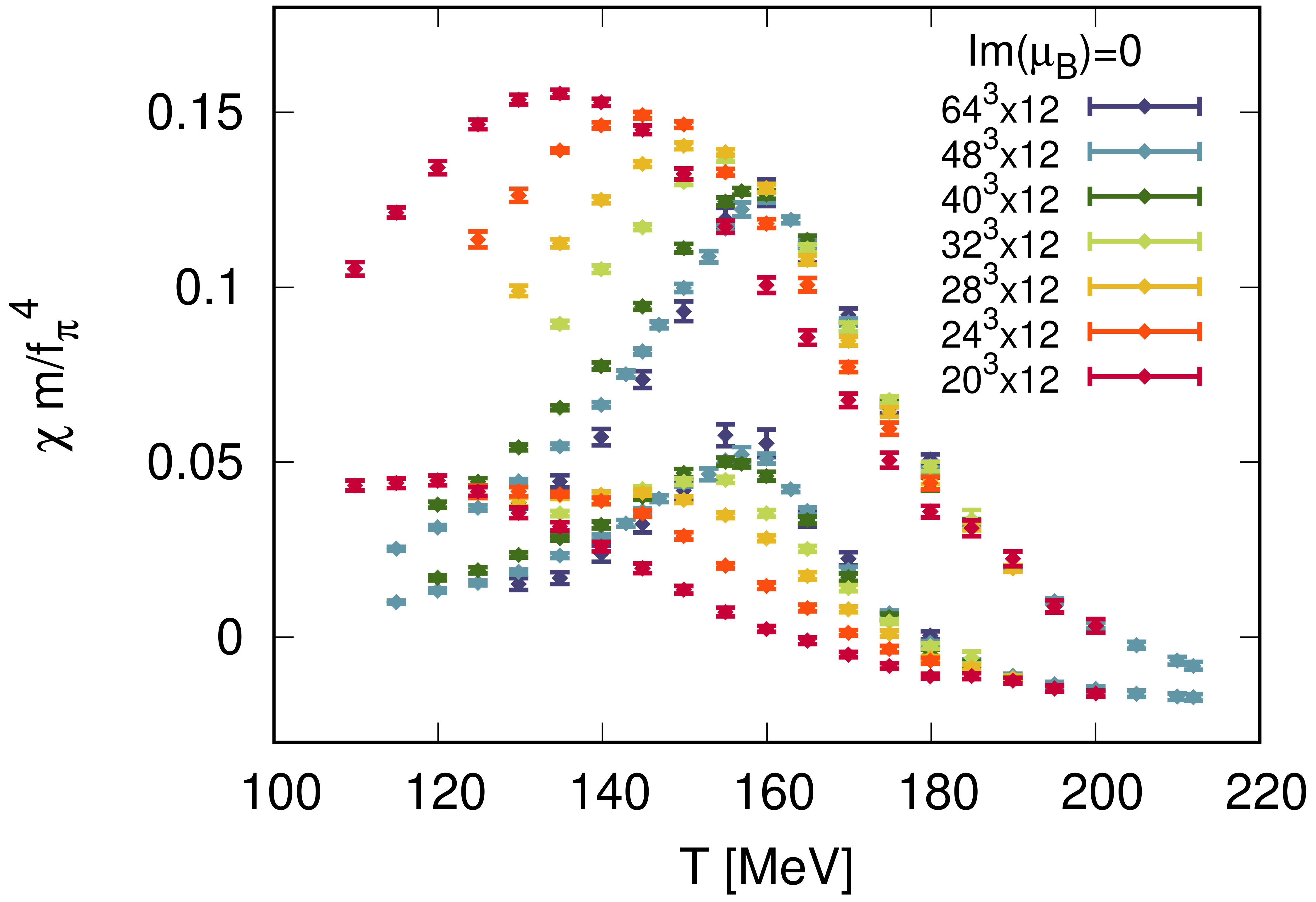
- chiral condensate $<\bar{\psi}\psi> = \frac{T}{V}\frac{\partial \log Z}{\partial m_{ud}}$
- At **higher temperatures** finite **volume effects** tend to **decrease**
- inflection point difficult to compute (especially at low volumes)



- You can see how T changes along curves of fixed $<\bar{\psi}\psi>$
- we can expect $\mathbf{T_c}$ to **increase** with the **volume**

$\mu_B = 0$

Chiral observables

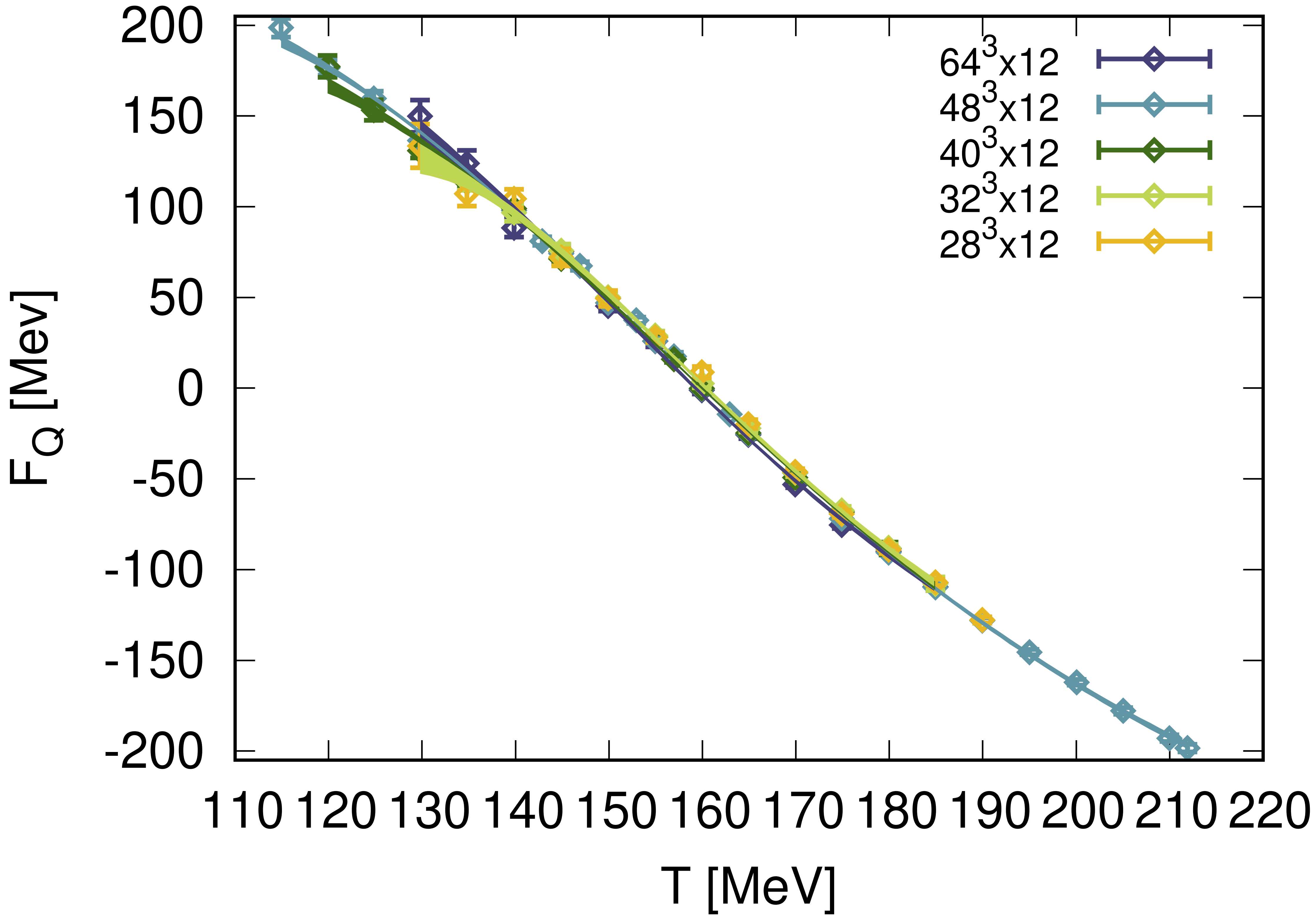


- chiral susceptibility $\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_{ud}^2}$
- disconnected chiral susceptibility
$$\chi_{\text{disc}} = \frac{T}{V}\left(\frac{\partial^2 \log Z}{\partial m_u \partial m_d}\right)_{m_u=m_d}$$
- At **higher temperatures** finite **volume effects** tend to **decrease**

- Peak position of χ, χ_{disc} as a measure of T_c
- **Increase** of $\mathbf{T_c}$ with the **volume**

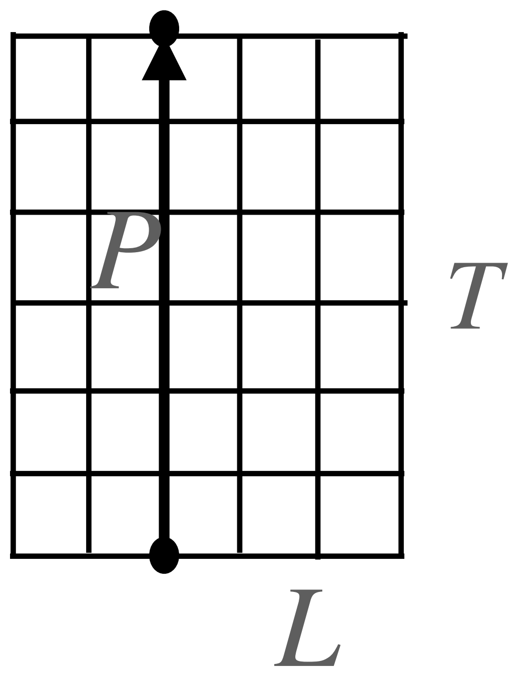
$\mu_B = 0$

Deconfinement observables



- from $P(\vec{x}) = \prod_{x_4=0}^{N_t-1} U_4(\vec{x}, x_4)$ compute
- static quark free energy F_Q=

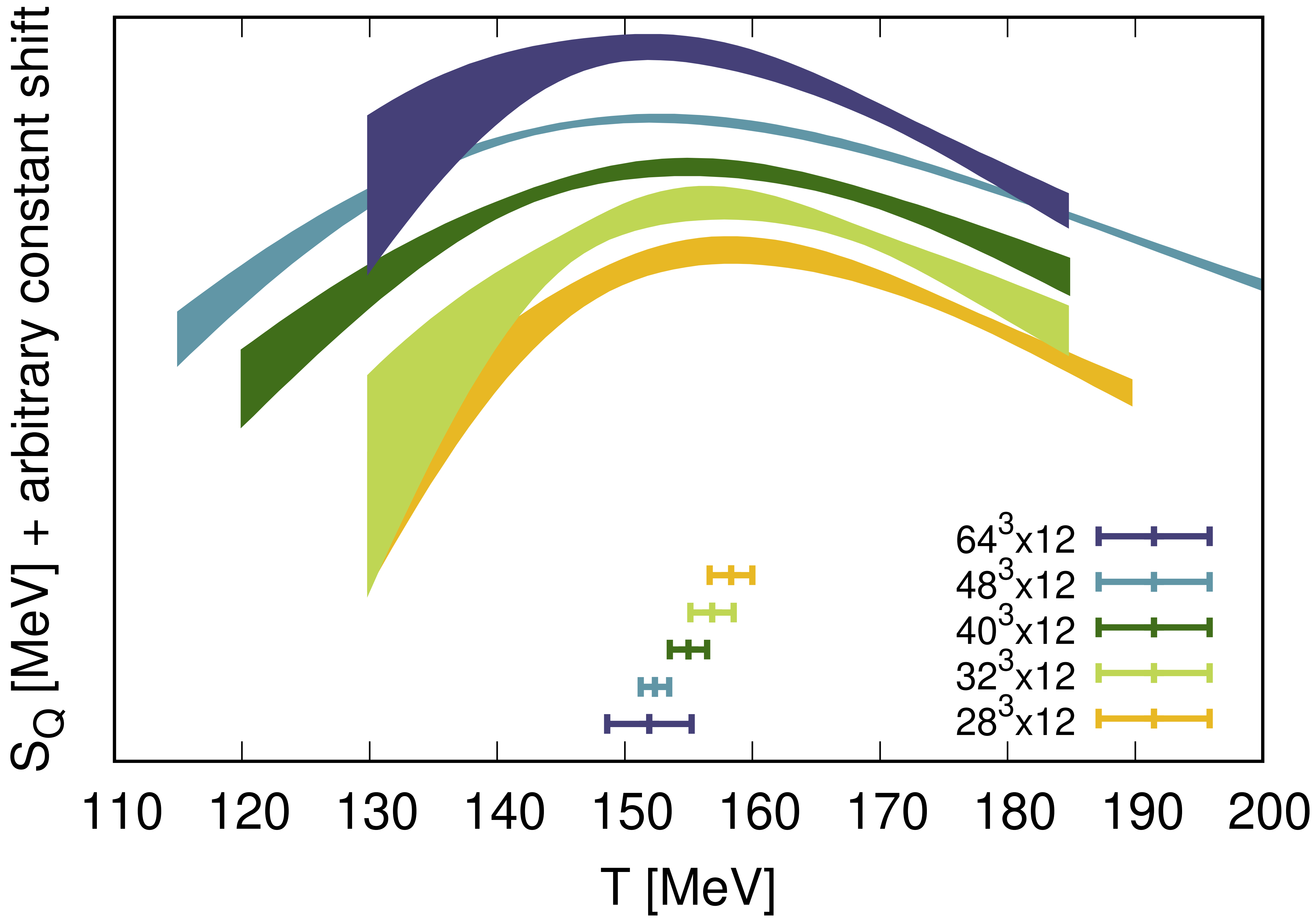
$$-T \log\left(\frac{1}{V}\sum_{\vec{x}} |< P(\vec{x}) >_T |\right) + T_0 \log\left(\frac{1}{V}\sum_{\vec{x}} |< P(\vec{x}) >_{T_0} |\right)$$



- You can see how T changes along curves of fixed F_Q
- **Mild volume effects**

$\mu_B = 0$

Deconfinement observables



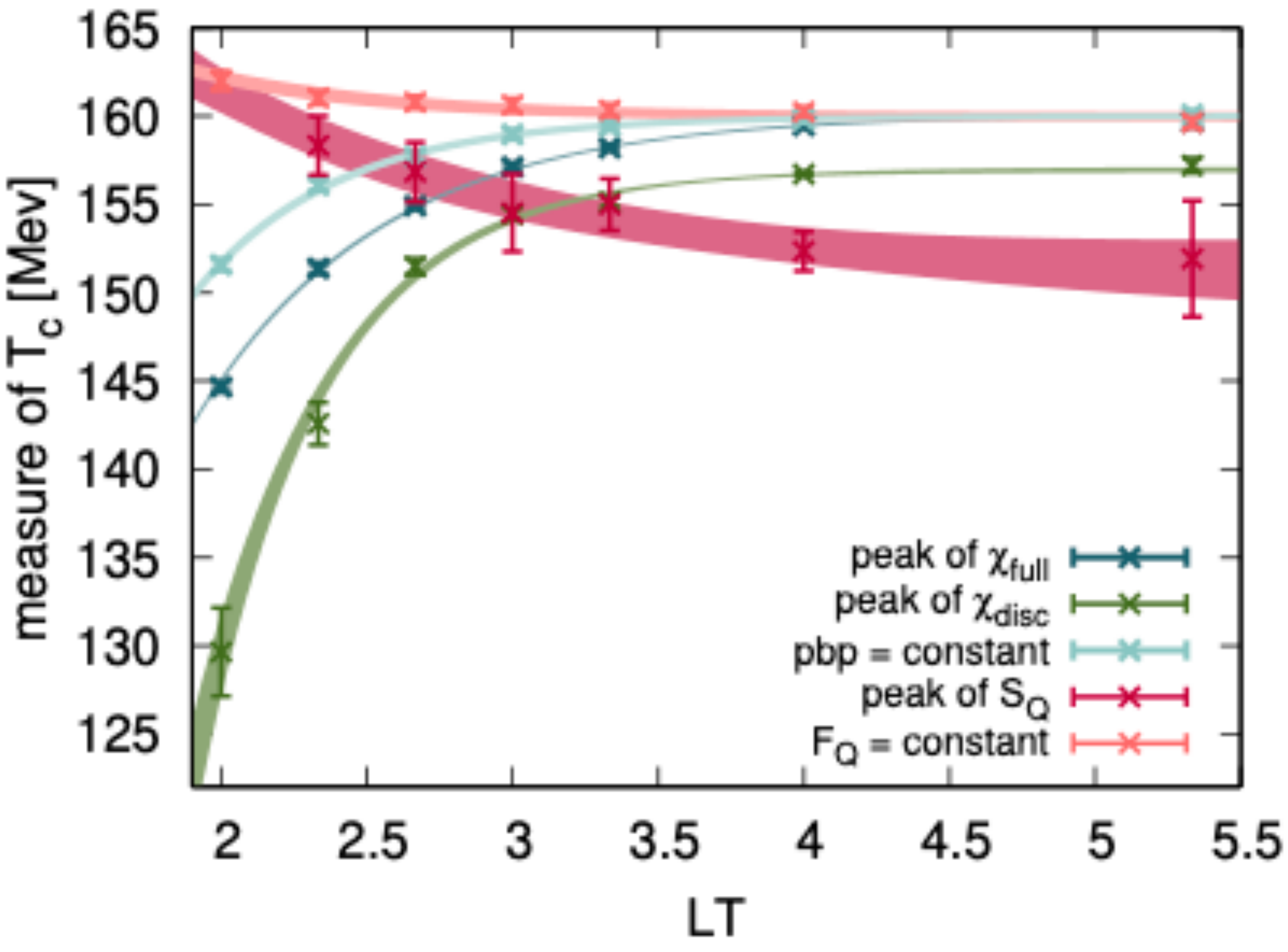
- static quark entropy $S_Q = -\frac{\partial F_Q}{\partial T}$
- interpolate lattice results for F_Q with Padé fits and derive the fitted functions
- Peak position of S_Q as a measure of T_c
- **Decrease** of $\mathbf{T_c}$ with the **volume**

TUMQCD Collaboration
arXiv:1603.06637

T_c vs LT $(\mu_B = 0)$

- $T_c^{(S_Q)} < T_c^{(\chi^R_{\rm disc})} < T_c^{(\chi^R)}$ for $LT \geq 3$

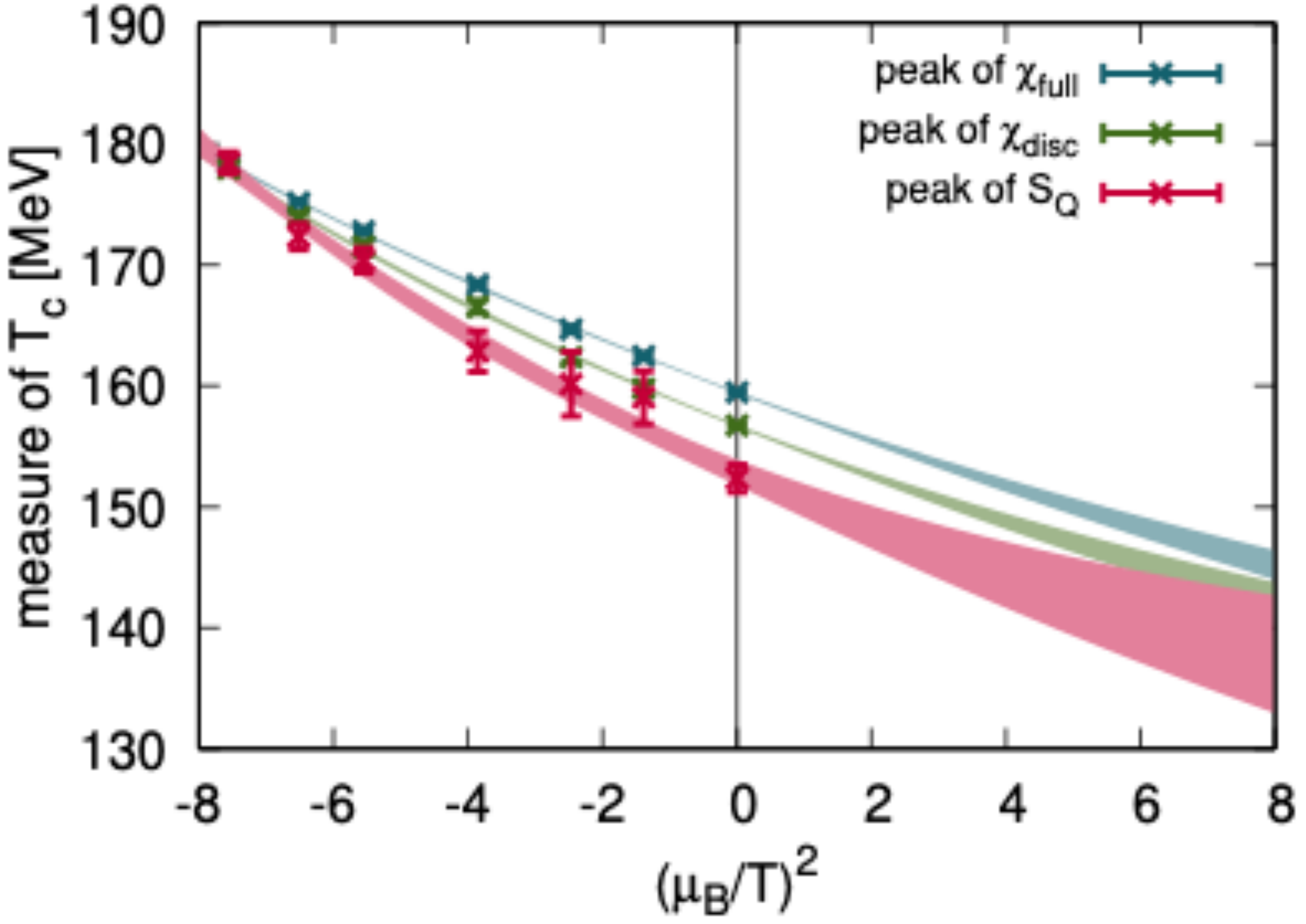
Previous result of TUMQCD Collaboration [1603.06637]: agreement within errors



- $T_c^{(\chi^R_{\rm disc})}$, $T_c^{(\chi^R)}$, $T_c^{(<\bar\psi\psi>)}$ **increase** with the volume
- $T_c^{(S_Q)}$, $T_c^{(F_Q)}$ **decrease** with the volume
- $T_c^{(S_Q)}$, $T_c^{(F_Q)}$ have **milder volume effects** as $T_c^{(\chi^R_{\rm disc})}$, $T_c^{(\chi^R)}$, $T_c^{(<\bar\psi\psi>)}$

Finite μ_B

$48^3 \times 12$ volume



- **simulations** at **imaginary** μ_B → extrapolations to real values
- simulations only for the $32^3 \times 12, 40^3 \times 12, 48^3 \times 12$ lattices, at $\mathrm{Im}\frac{\mu_\mathrm{B}}{\mathrm{T}}\frac{\pi}{8} = 0, 3, 4, 5, 6, 6.5, 7$ $(\mu_B/T < 3)$
- strangeness neutrality setting: $< N_S > = 0$

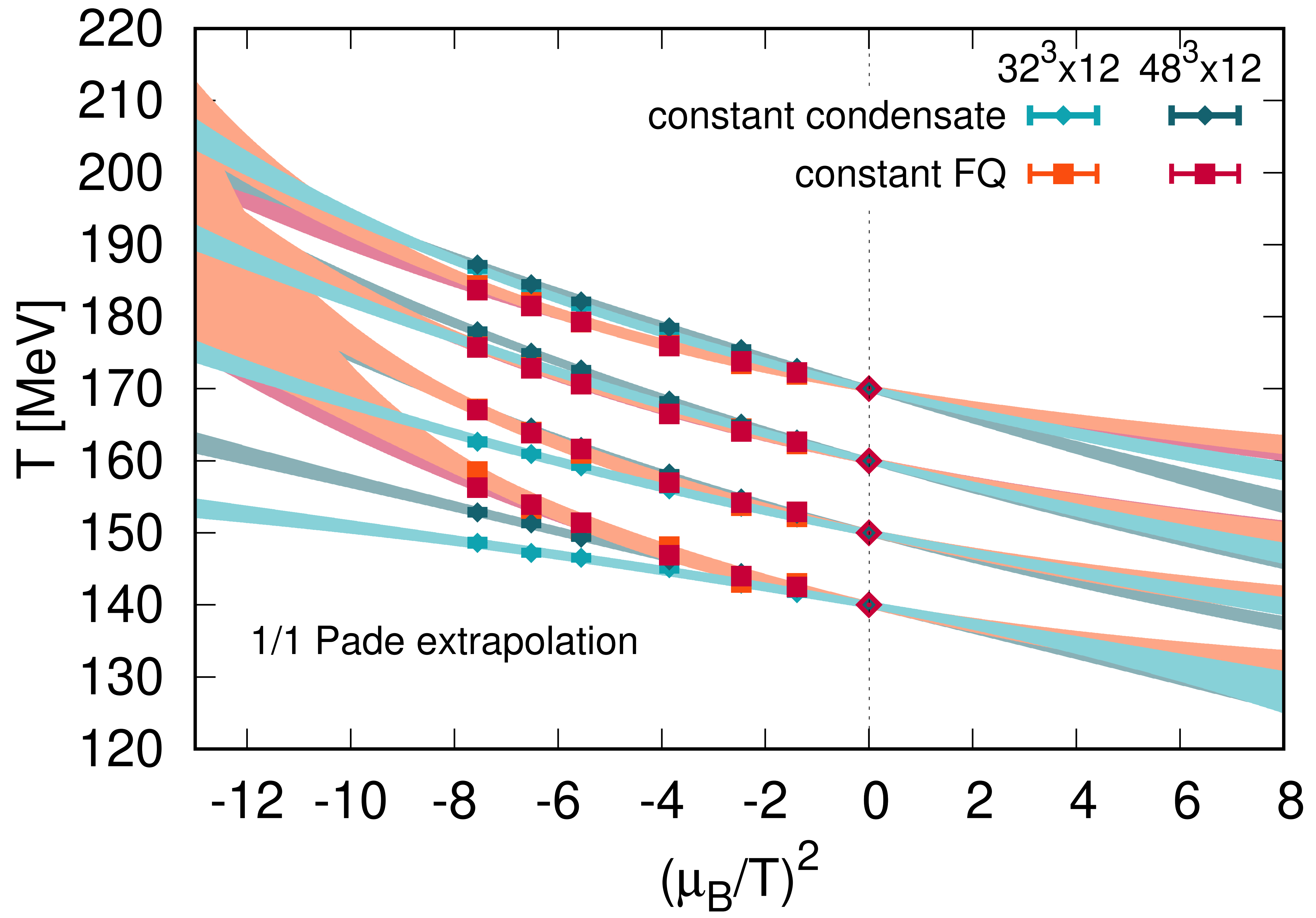
- **near** $\mu_B = 0$ similar slopes in $(\mu_B/T)^2$ for $T_c^{(\chi^R_{\mathrm{disc}})}$, $T_c^{(\chi^R)}$, $T_c^{(S_Q)}$
- $(\mu_B/T)^2$**<<0:** the three definitions tend to converge

Let's explore a bit $\mathbf{T}(\mu_\mathbf{B})$ along **curves of fixed values** for $<\bar{\psi}\psi>$, F_Q

- for a fixed T^* in we solve

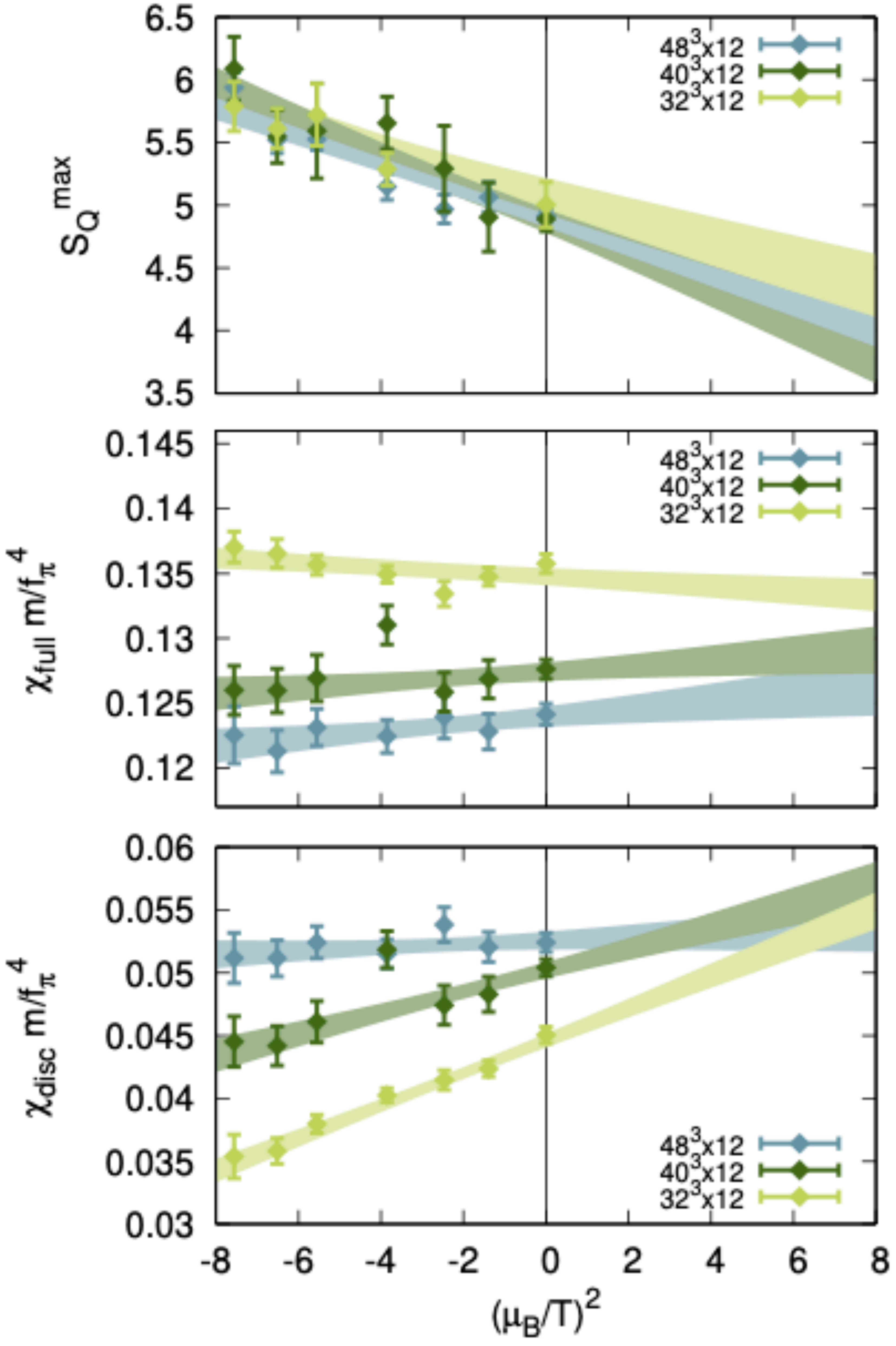
$$<\bar{\psi}\psi>(\underline{\mathbf{T}(\mu_\mathbf{B})}, \mu_B) = [<\bar{\psi}\psi>(T^*,0)]$$

(repeat for different T^* , analogous for F_Q)



- F_Q **: milder volume effects** for all T^*
- If we were near to a Roberge-Weiss transition region, we would have expected F_Q to be more sensitive to volume effects
- Bonati et al. [1507.03571]: $\mu_S = 0 \to$ RW expected at $\mu_B/(\pi T) \sim 3 * 0.45$
- $<\bar{\psi}\psi>$: volume effects decrease also for larger imaginary μ_B/T at larger T

Height of peaks of $\chi^{\mathbf{R}}, \chi^{\mathbf{R}}_{\mathrm{disc}}, \mathbf{S_Q}$



- The **only** quantity that shows a **rise for large** μ_B^2 is $\chi^{\mathbf{R}}_{\mathrm{disc}}$
- peak of χ^R is constant in μ_B^2 for all the volumes, that of S_Q decreases
- Can $\chi^{\mathbf{R}}_{\mathrm{disc}}$ **promise something?** We don't know
- S_Q **has mild volume effects**, also at large imaginary μ_B (again, we are far enough from RG transition region)

Trying to summarize: volume effects

Chiral observables

Deconfinement observables

$\mu_{\mathbf{B}} = \mathbf{0}$

- $T_c(LT)$ increase with the volume
- $T_c(LT)$
- $T_c(LT)$ decrease with the volume
- **mild** volume effects

Finite $\mu_{\mathbf{B}}$

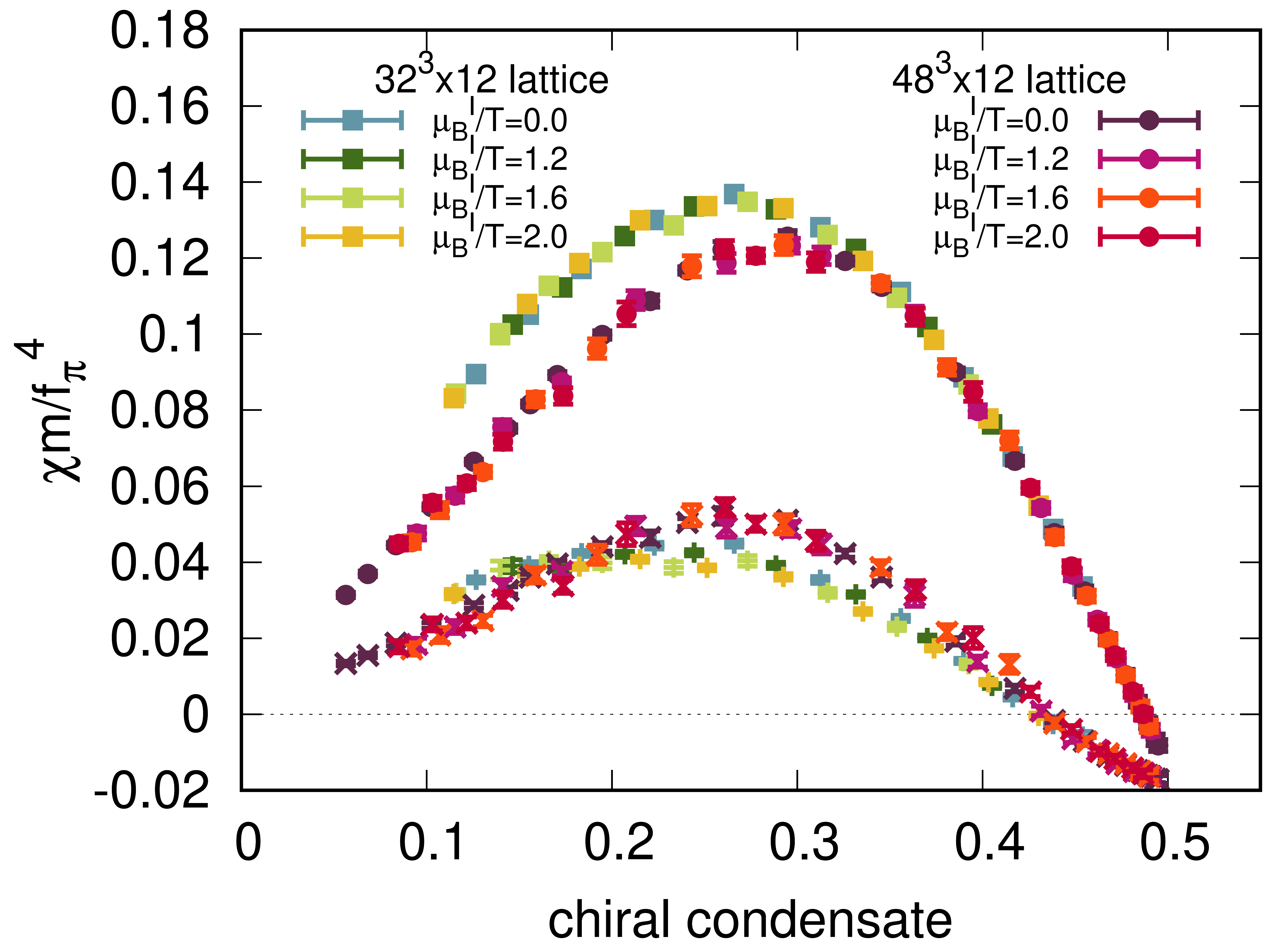
- curves of fixed values for $< \bar{\psi}\psi >$, F_Q in T-μ_B plane
- **mild** volume effects

- peak of χ_{disc} increase for the 2 smallest volumes
- peaks of $S_Q, \chi,$ χ_{disc}
- **mild** volume effects

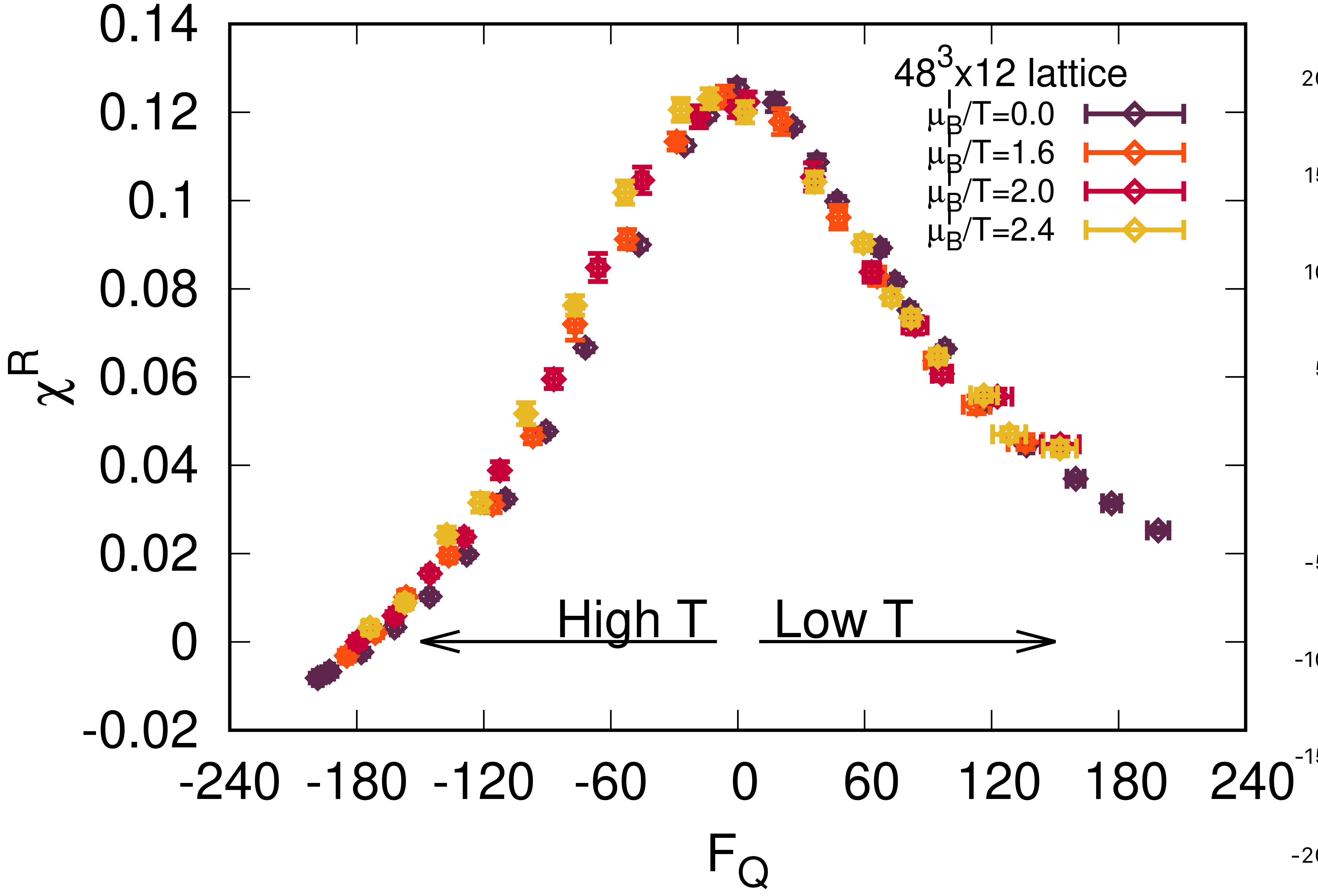
Conclusions

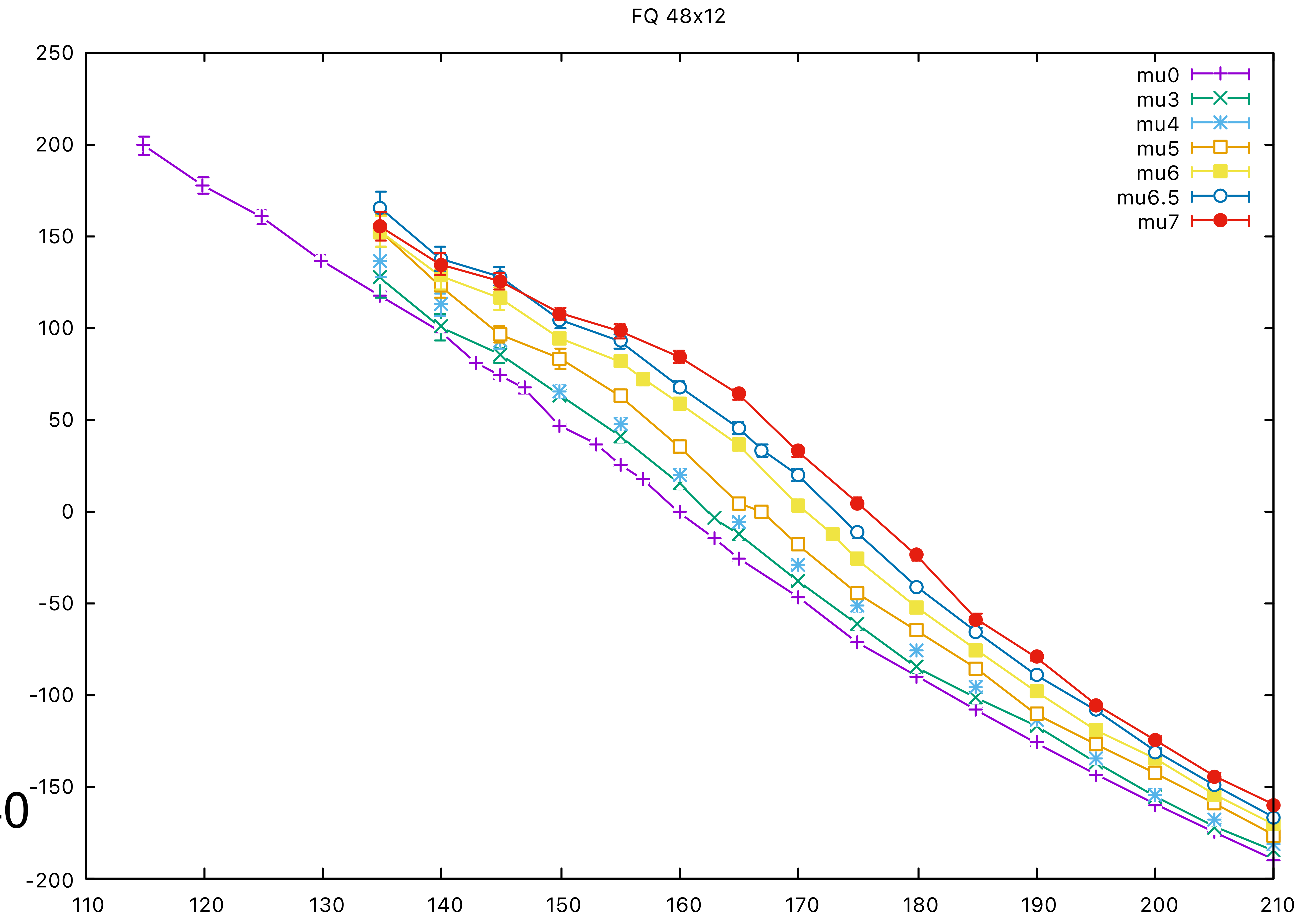
- In the thermodynamic limit $T_c^{(S_Q)} < T_c^{(\chi^R_{\rm disc})} < T_c^{(\chi^R)}$
- They also have **different volume dependence**
- **Only** the peak of $\chi^{\mathbf{R}}_{\rm disc}$ shows a **rise for increasing** μ_B^2 $\rightarrow$ picture is unclear but interesting
- **Deconfinement observables** have always **milder volume** effects w.r.t. chiral ones $\rightarrow$ useful also at small volume

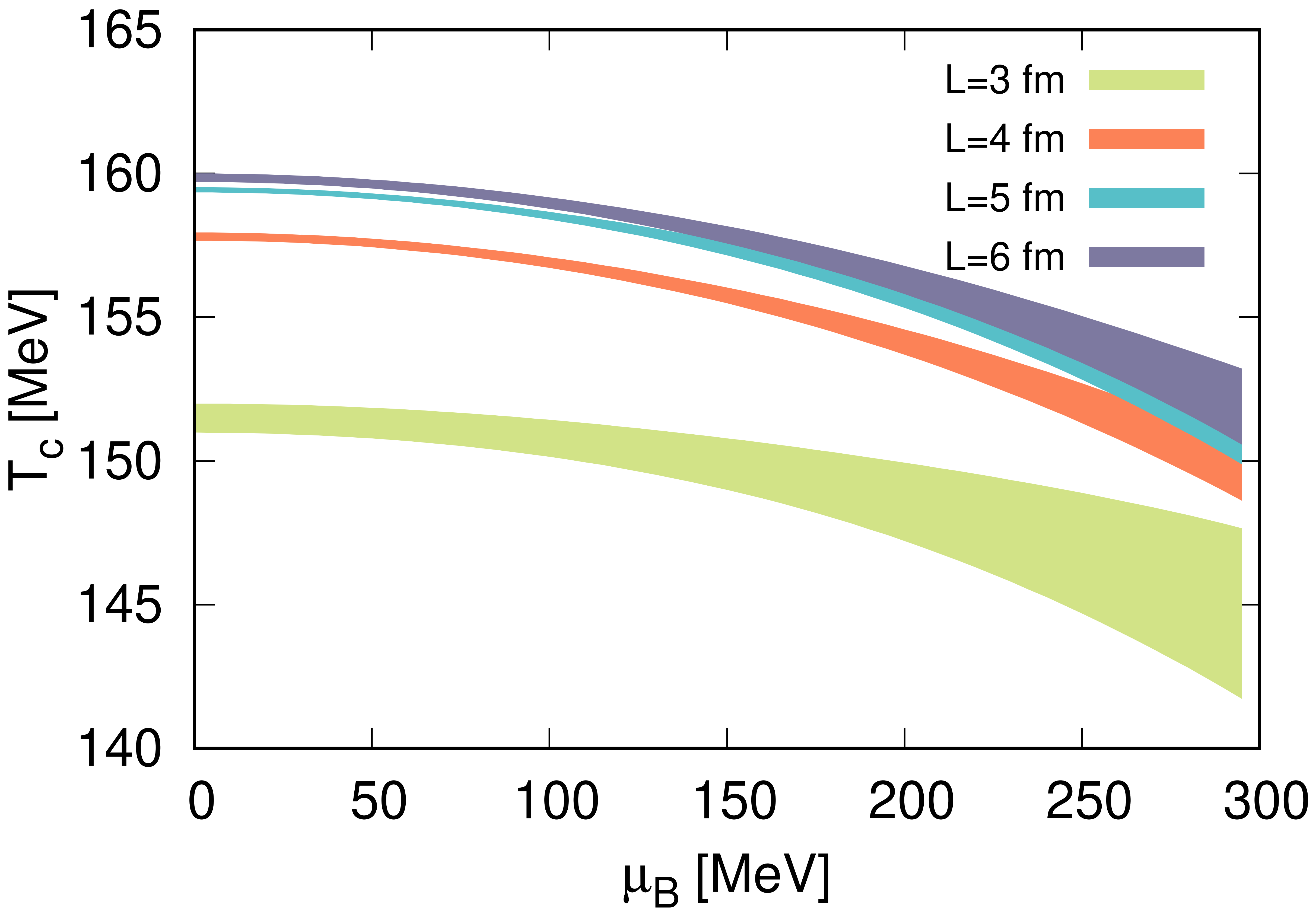
Backup slides



0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
$\chi m/f_\pi^4$
0
0.1
0.2
0.3
0.4
0.5
chiral condensate
32^3x12 lattice
μ_B^I/T=0.0
μ_B^I/T=1.2
μ_B^I/T=1.6
μ_B^I/T=2.0
48^3x12 lattice
μ_B^I/T=0.0
μ_B^I/T=1.2
μ_B^I/T=1.6
μ_B^I/T=2.0


48^3x12 lattice
μB/T=0.0
μB/T=1.6
μB/T=2.0
μB/T=2.4
High T
Low T
χR
FQ


FQ 48x12
mu0
mu3
mu4
mu5
mu6
mu6.5
mu7


165
160
155
150
145
140
0
50
100
150
200
250
300
T_c [MeV]
μ_B [MeV]
L=3 fm
L=4 fm
L=5 fm
L=6 fm