# 3D Ising Critical Point Mapped onto Lattice-based QCD Equation of State


## Micheal KAHANGIRWE


Based on M.K. et al. arXiv:2402.08636v1 PRD (2024)


**In collaboration with**:
Steffen A. Bass, Elena Bratkovskaya, Johannes Jahan, Pierre Moreau, Paolo Parotto, Damien Price, Claudia Ratti, Olga Soloveva and Mikhail Stephanov.

**University of Houston**


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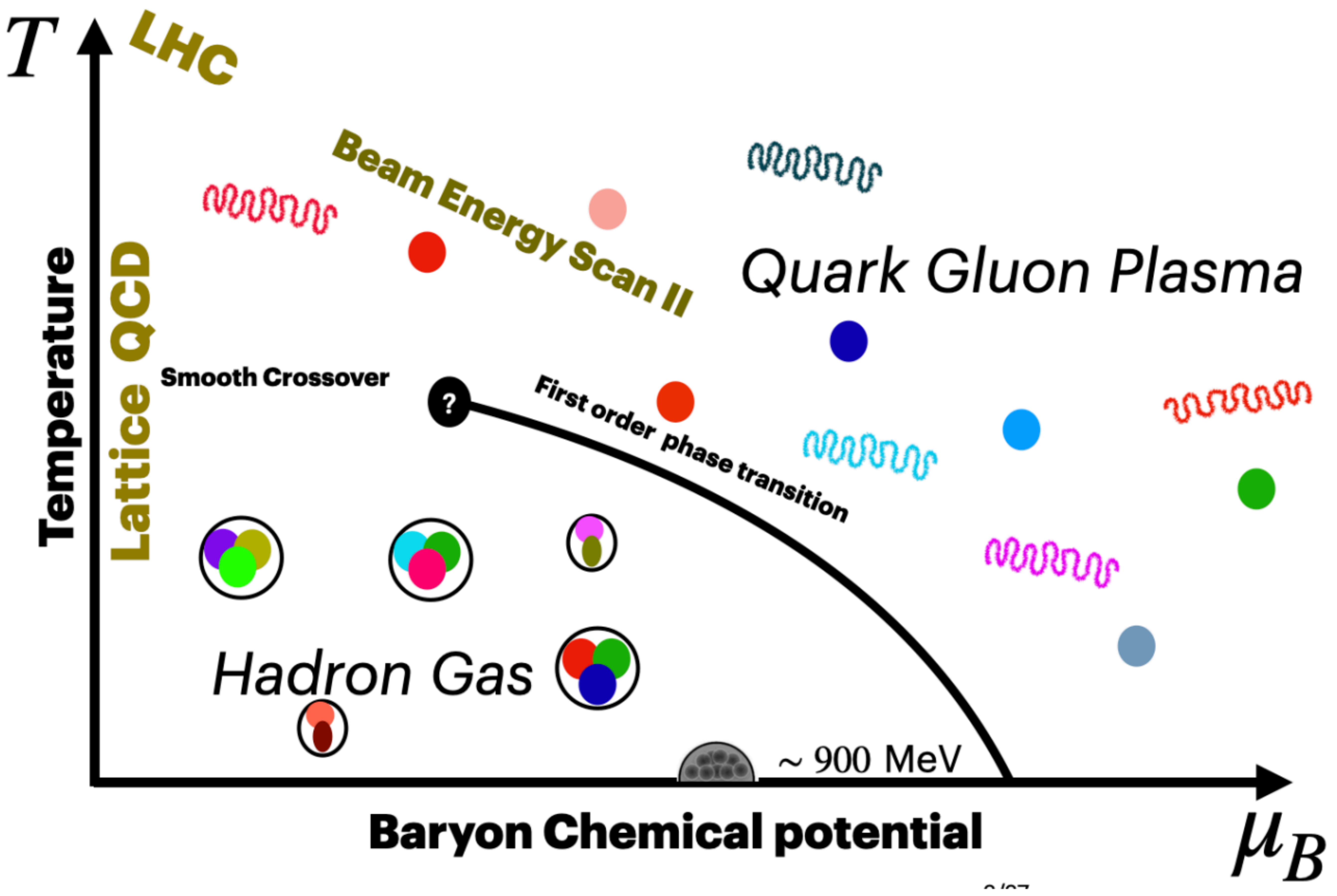



## What we know

- At $\mu_B = 0$, deconfinement transition is well established (**Smooth crossover**)
- At finite $\mu_B$, QCD **critical point** is expected but not yet seen
- Lattice simulations are challenging at finite density **(Fermi-sign problem)**

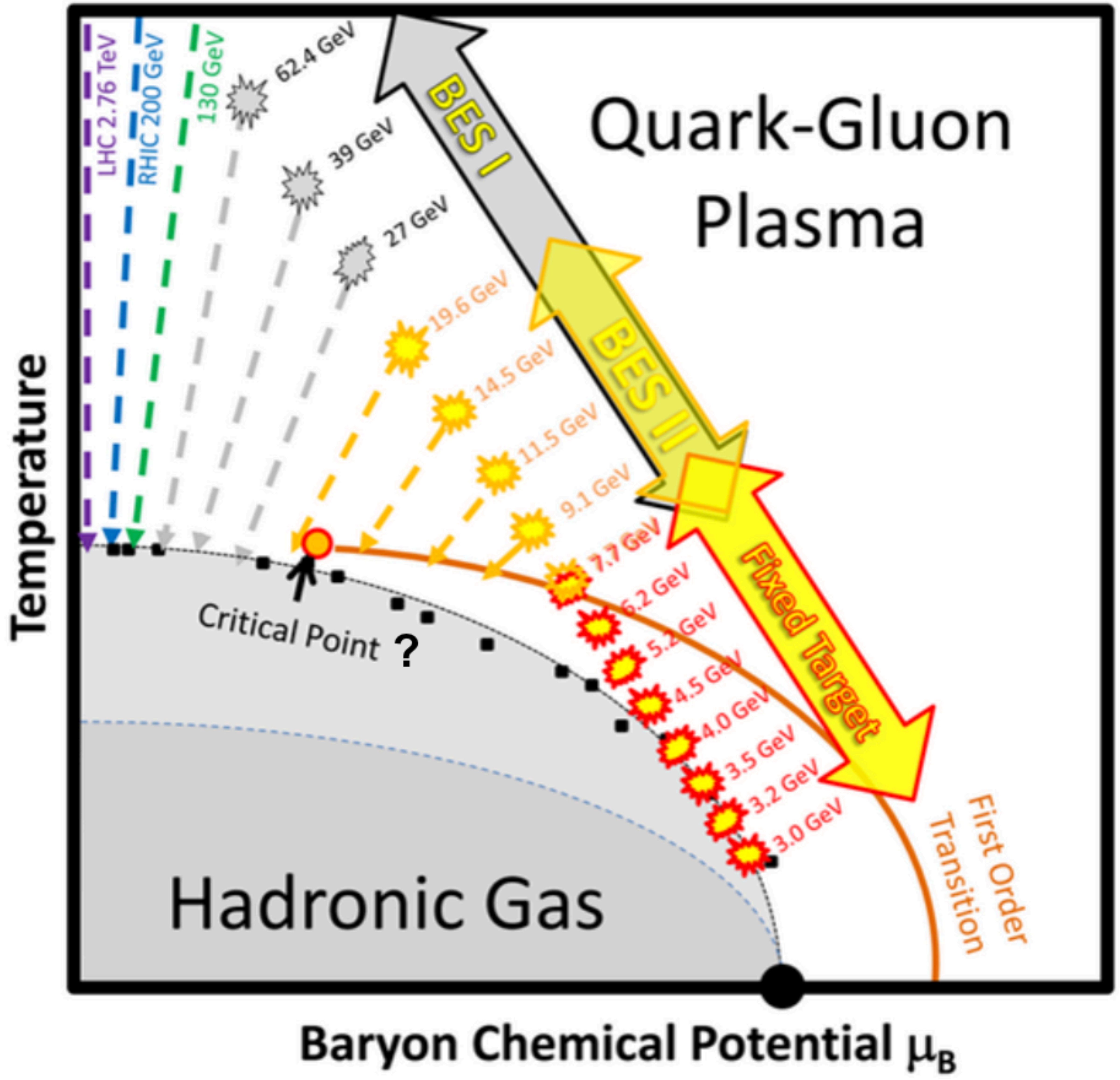
## Attempts

- Direct simulation at finite $\mu_B$ like **re-weighting** are employed but limited to small volumes lattices

*[ Giordano, M. et al JHEP.05 088(2020)]*
*[ Borsanyi, S. et al PhysRevD.105 014026(2022)]*
*[ Borsanyi, S et al PhysRevD.107, L091503 (2023)]*

- **Extrapolation schemes** are needed to describe finite density physics.

# QCD Phase Diagram




## Experiments

- Finite density physics is achieved by lowering the $\sqrt{S_{NN}}$ in **BES II program**

## Theoretical interpretation

- **Hydrodynamic simulations** describe the evolution of the fireball in heavy ion collisions and neutron star mergers
- An **Equation of state (EoS)** is required as an Input

**It is crucial that the EoS used encompasses all existing physics knowledge with adjustable parameters**

**Part 1: Taylor Expansion**

Part 2: T' Expansion Scheme (T ExS)

Part 3: Introducing Critical Point (3D-Ising)

Part 4: Merging 3D Ising with T' Expansion (Ising-TExS)

Part 5: Constraints on the EoS

# Taylor: Lattice QCD results

## Taylor Expansion around $\mu_B = 0$

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{2n!}\chi_{2n}(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^{2n}$$

*[ Borsanyi, S. et al High Energy Physics.9(8), 1-16.(2012)]*
*[ Bazavov, A et al PhysRevD.95, 054504 (2017)]*

$$\frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\bigg|_{\mu_B=0}$$

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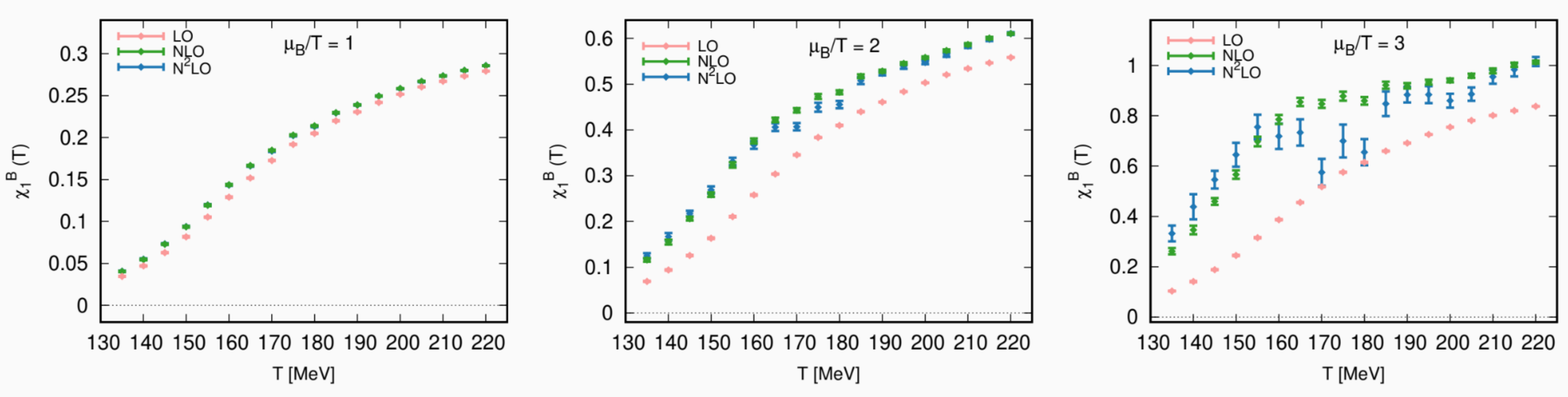
## Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 3$ despite great computational effort**
- **Including one more higher-order term does not remove unphysical behavior due to truncation of Taylor series**

*[Bollweg, D. et al Phys.Rev.D 108 (2023) 1, 014510]*

*[Borsanyi , S et al arXiV:2312.07528v1. (2023)]*




*[Borsányi, S et al PhysRev.L 108(1), 101.034901(2021)]*

# Taylor: merging of lattice QCD results and critical behavior

$$n_B(T,\mu_B) = T^3 \sum_{n=0}^{2} \frac{1}{(2n-1)!} \chi_{2n}^{non-Ising}(T) \left(\frac{\mu_B}{T}\right)^{2n-1} + \frac{T_C^4}{T} n_B^{Ising}(T,\mu_B)$$

BEST COLLABORATION

Taylor expansion up to $\mathcal{O}((\mu_B/T)^4)$

$$\chi_n^{lat}(T) = \chi_n^{non-Ising}(T) + \frac{T_C^4}{T^4} \chi_n^{Ising}(T)$$

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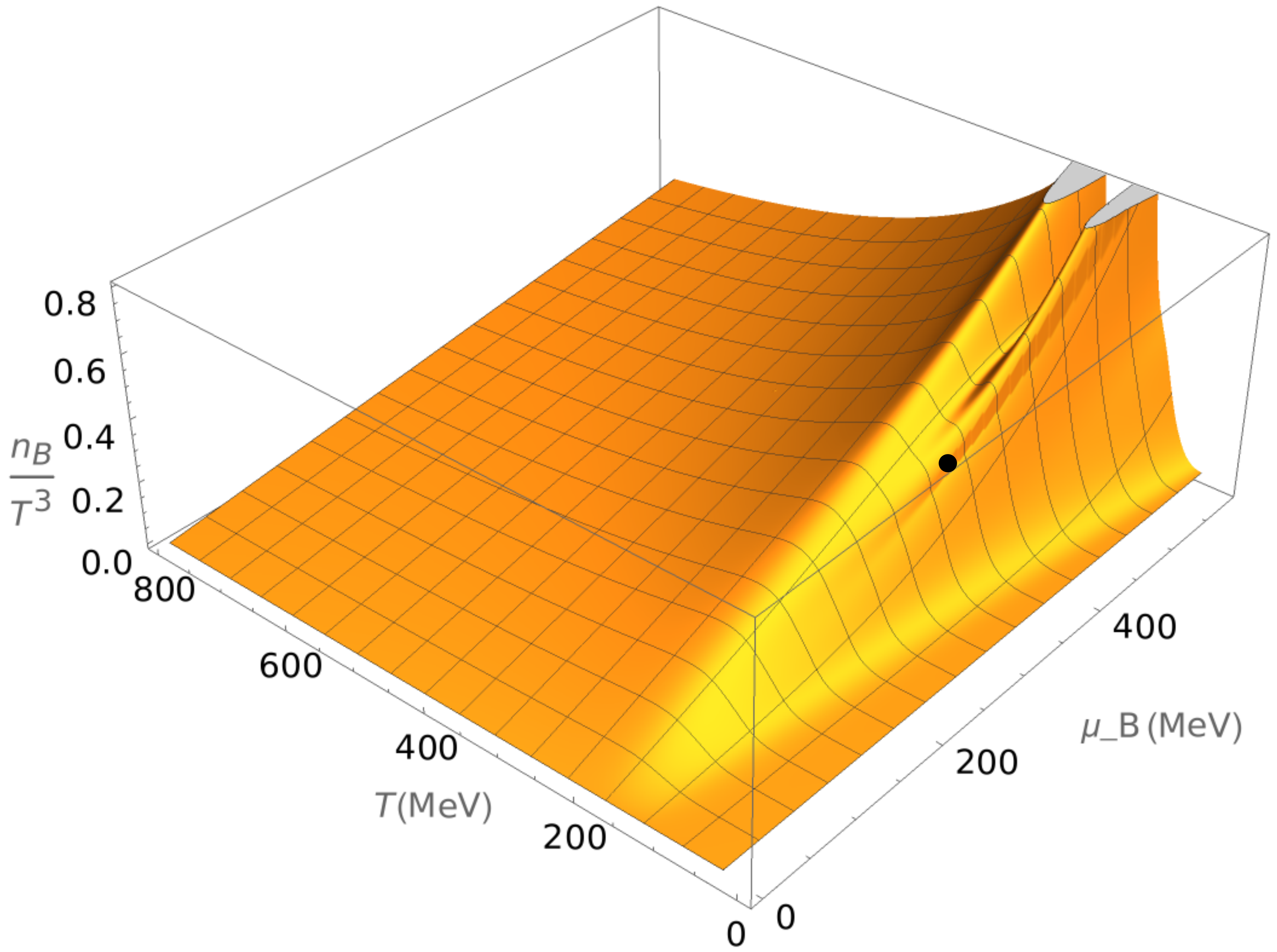


*[Parotto, P et al PhysRevC. 108(1), 101.034901(2020)]*
*[Karthein, J, et al Eur.Phys.J.Plus 136 (2021) 6, 621]*

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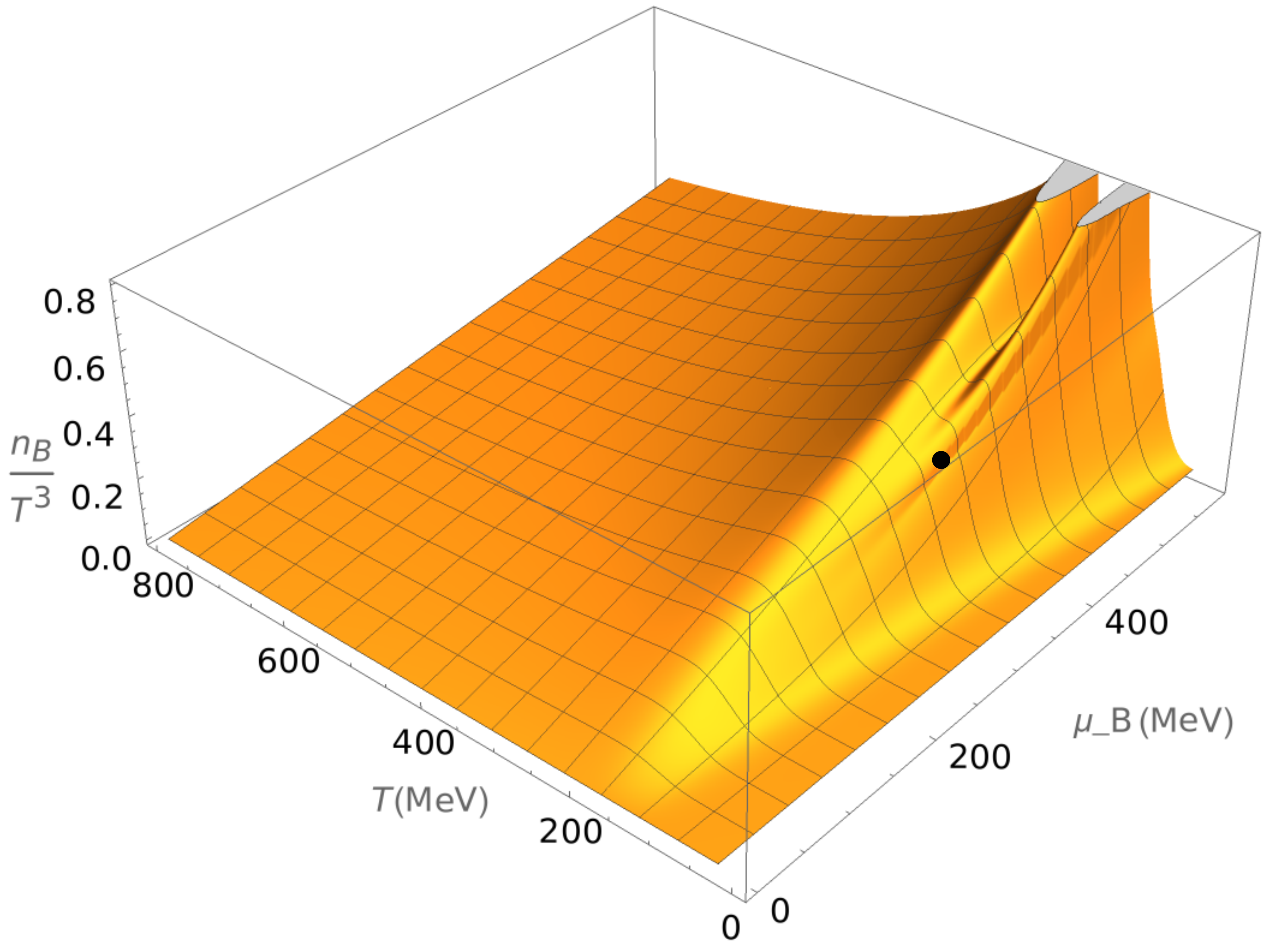


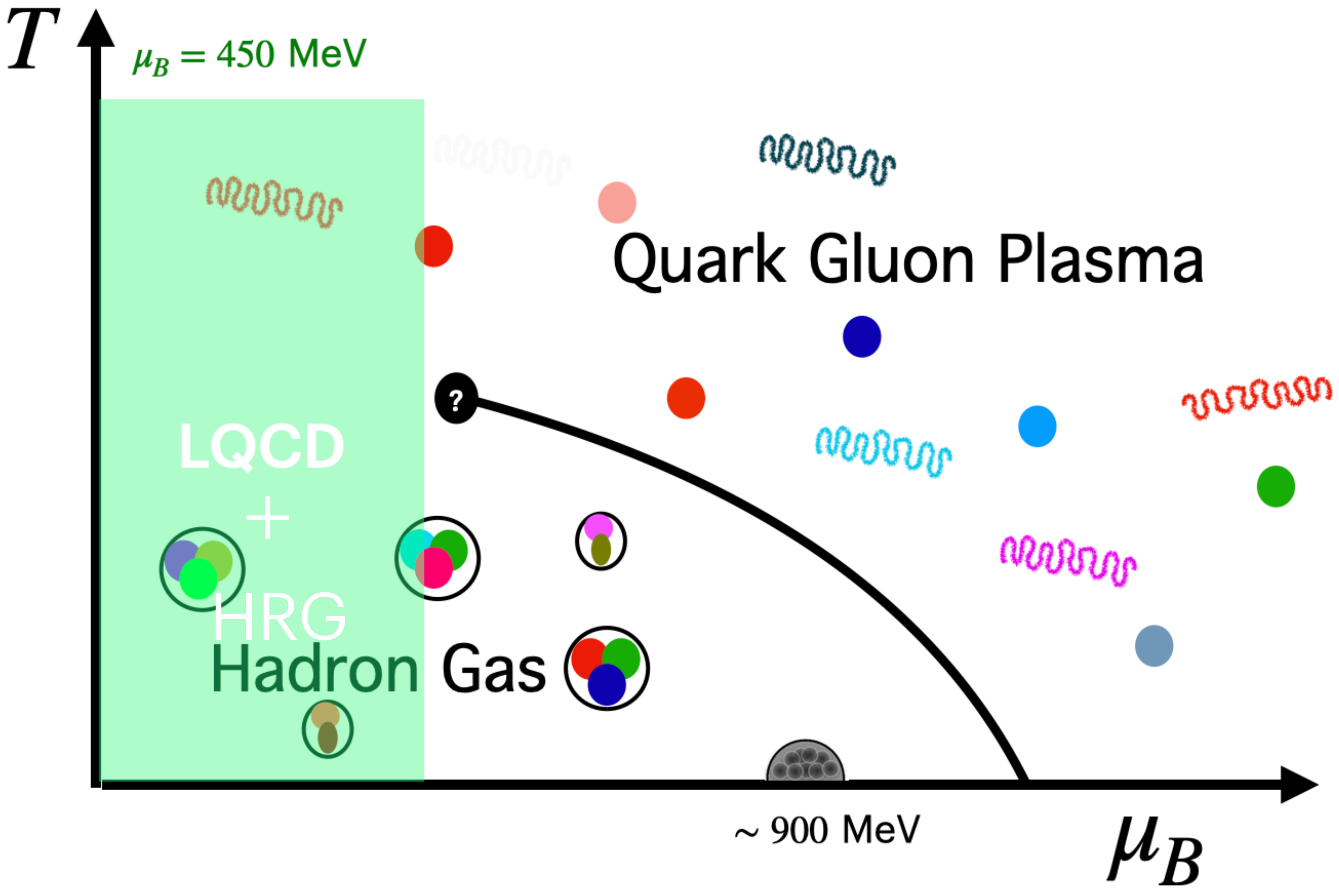


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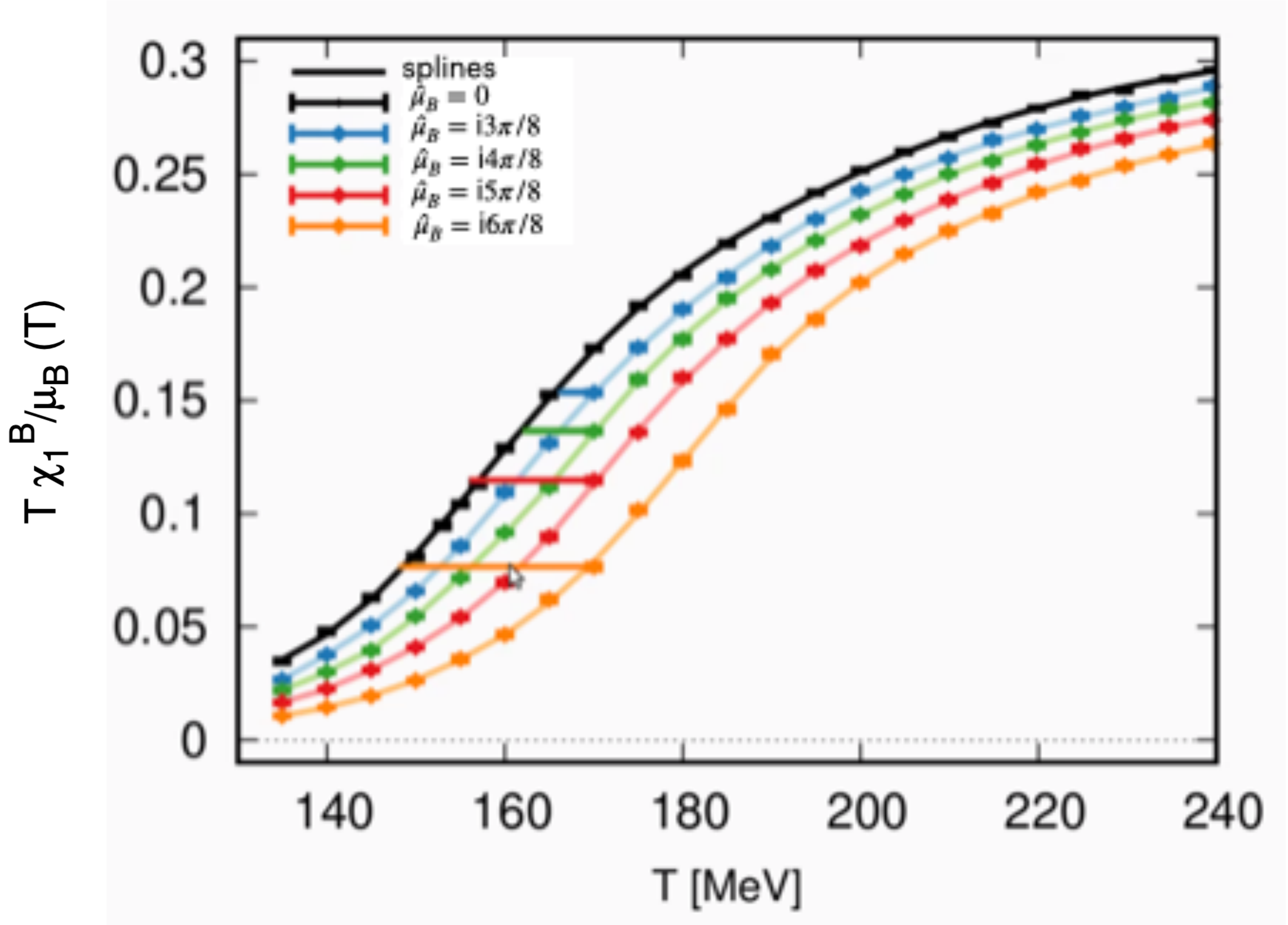
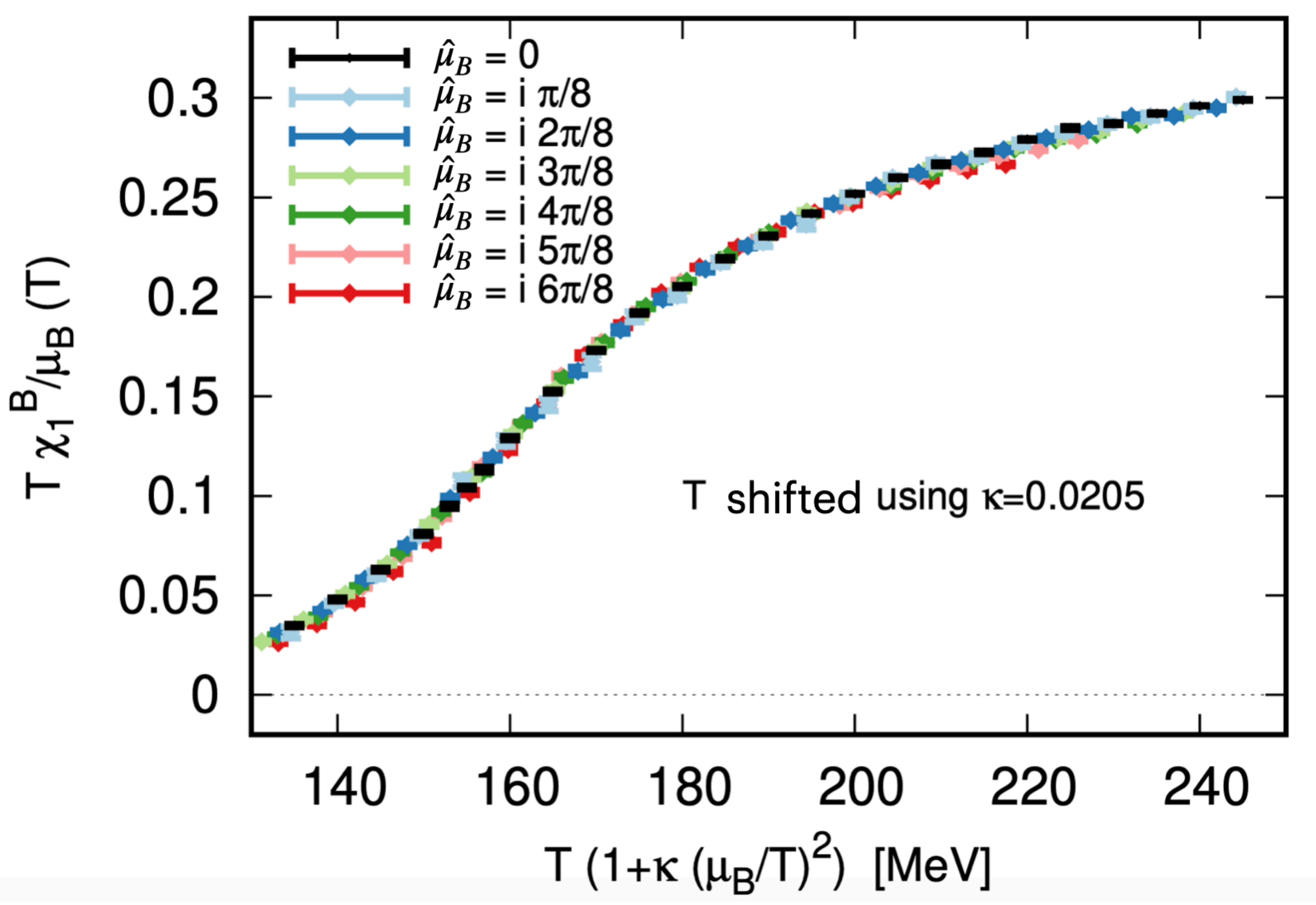
# T′ Expansion scheme (T ExS)

## Simulating at Imaginary $\mu_B$

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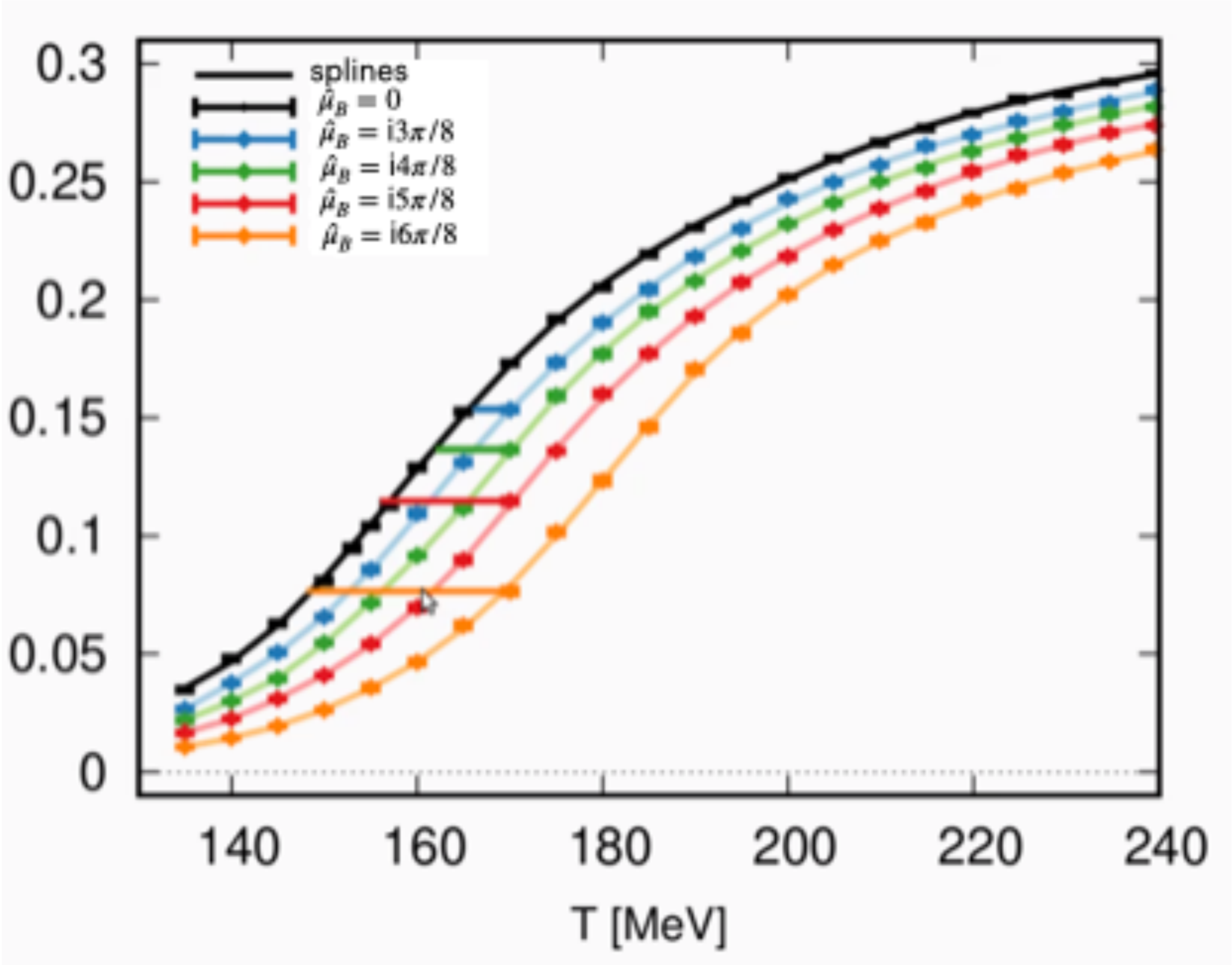


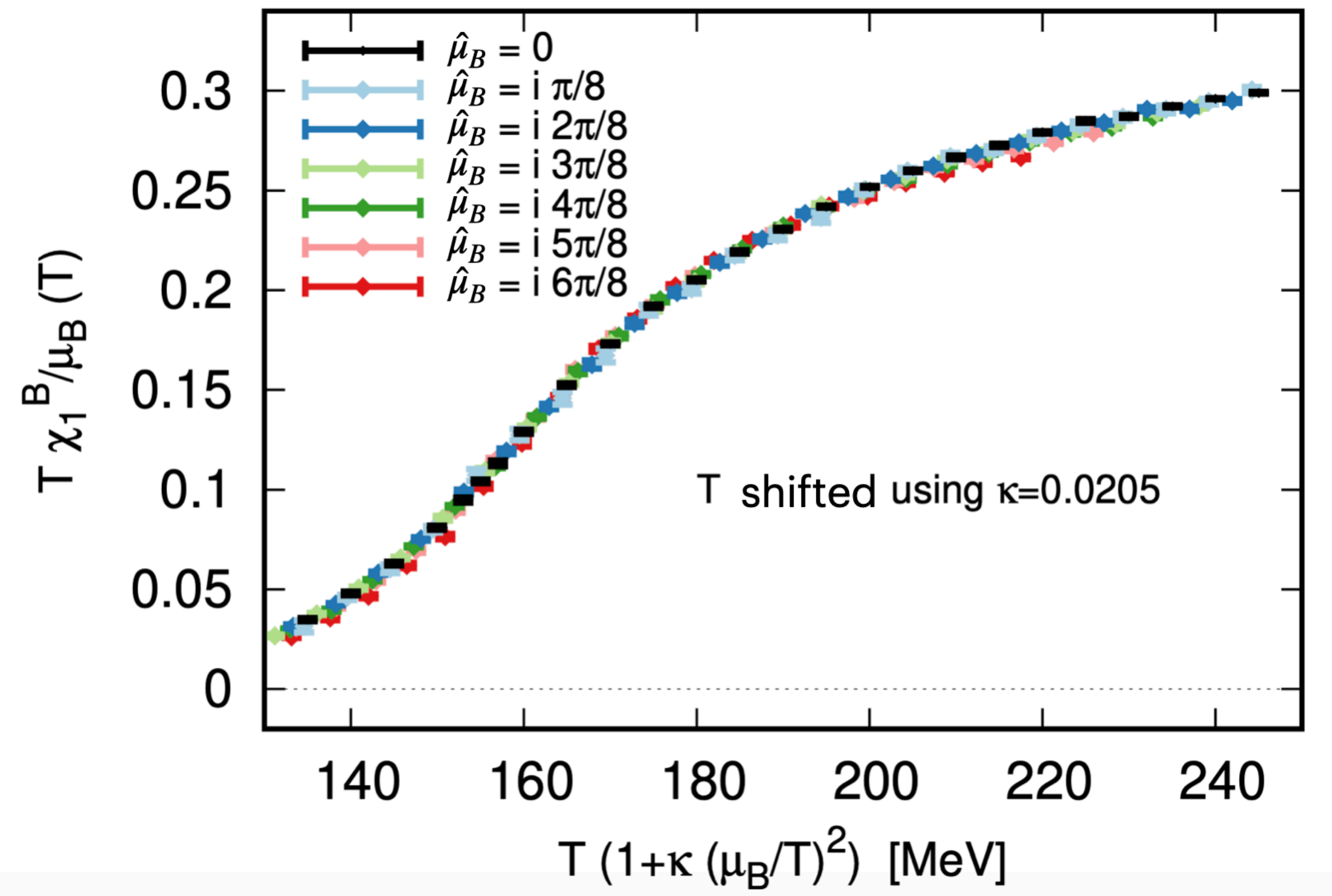


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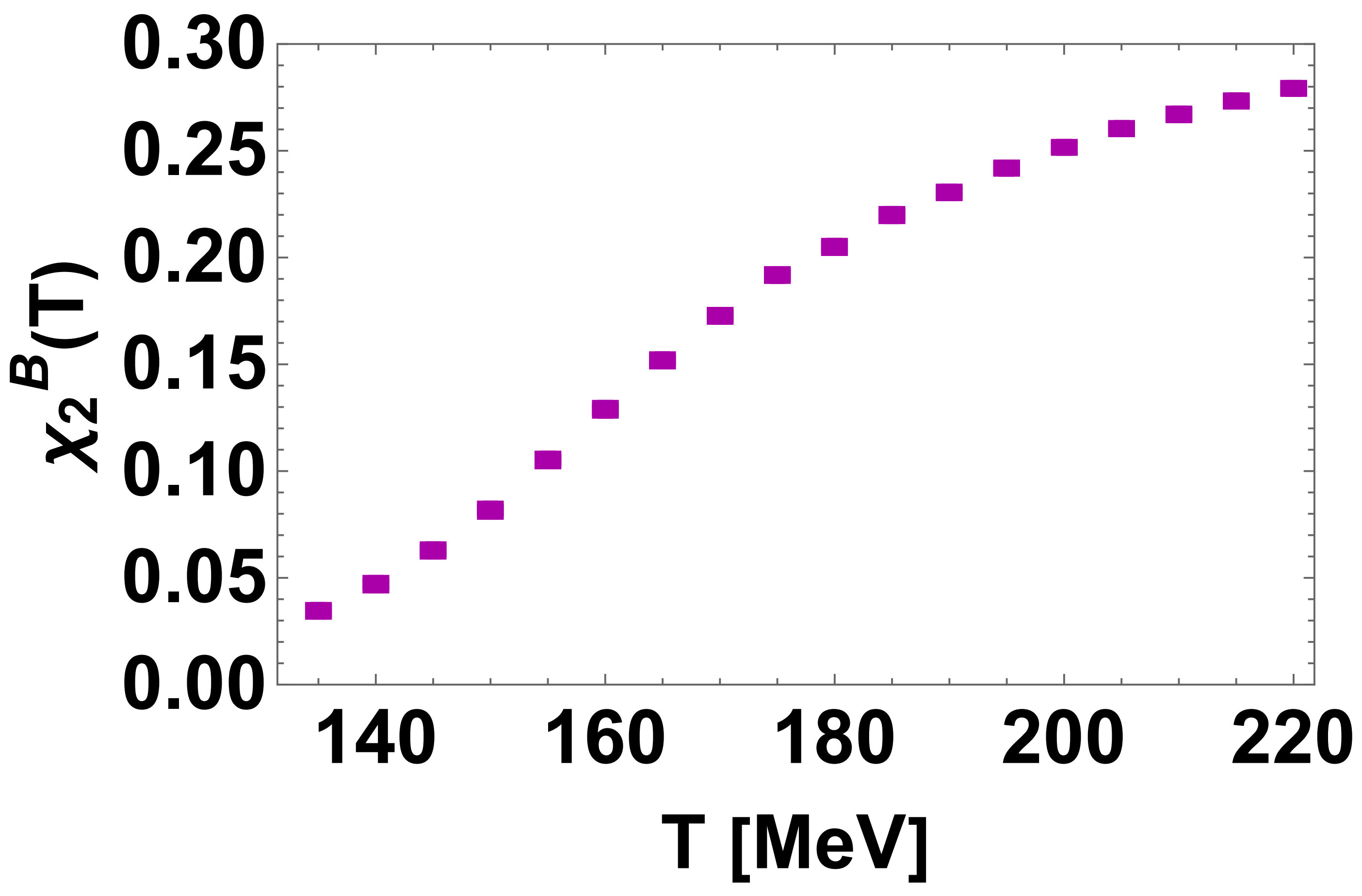


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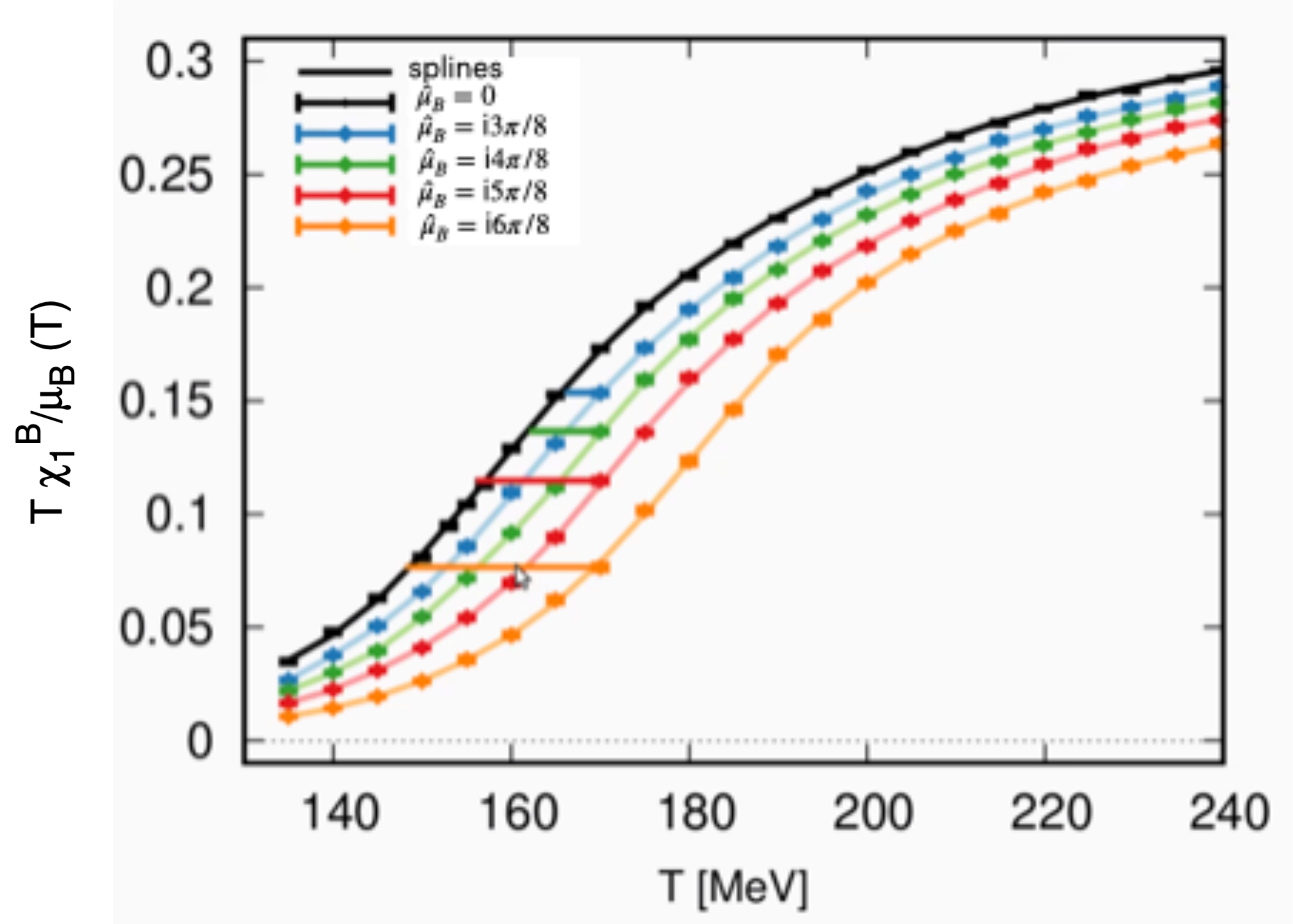


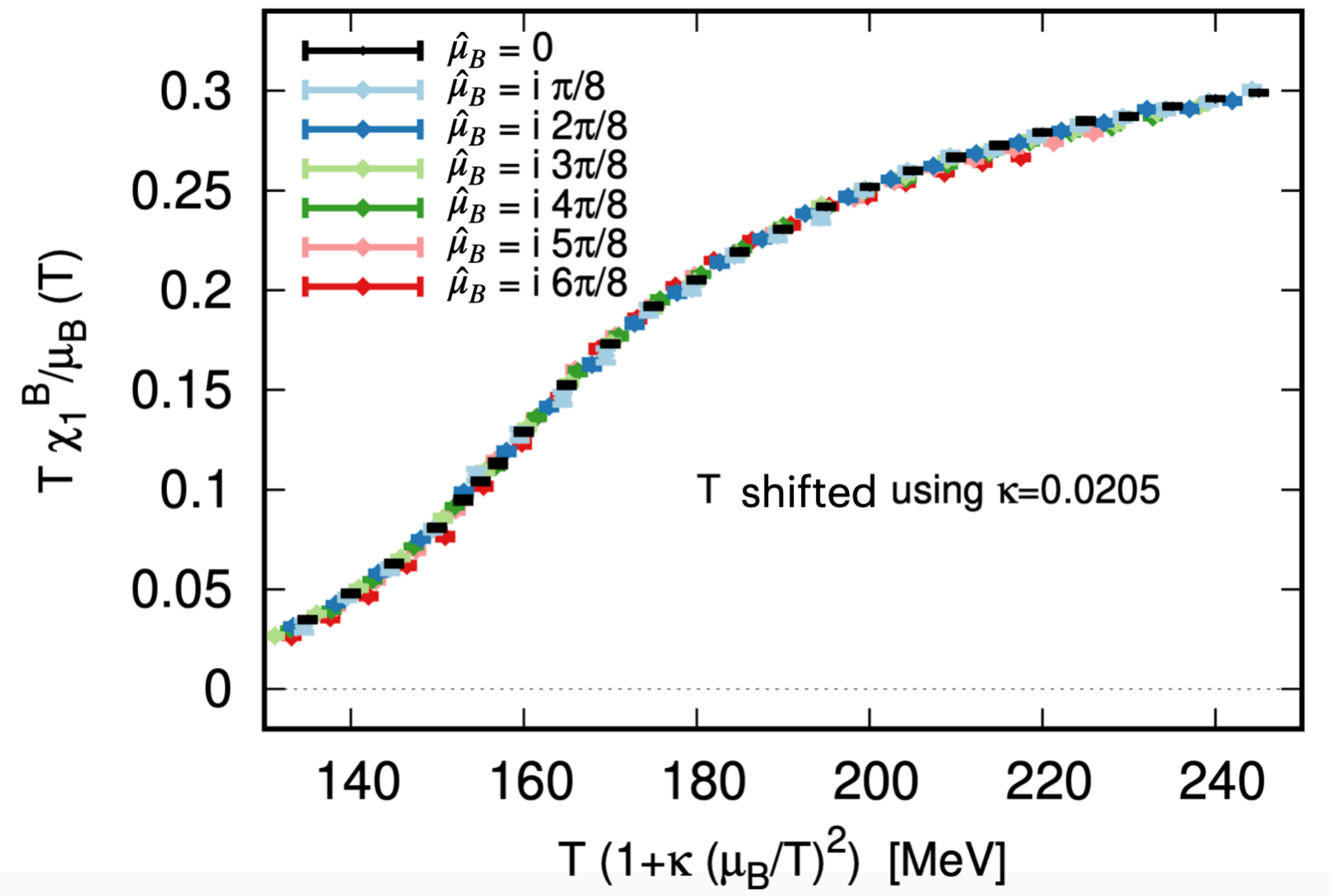


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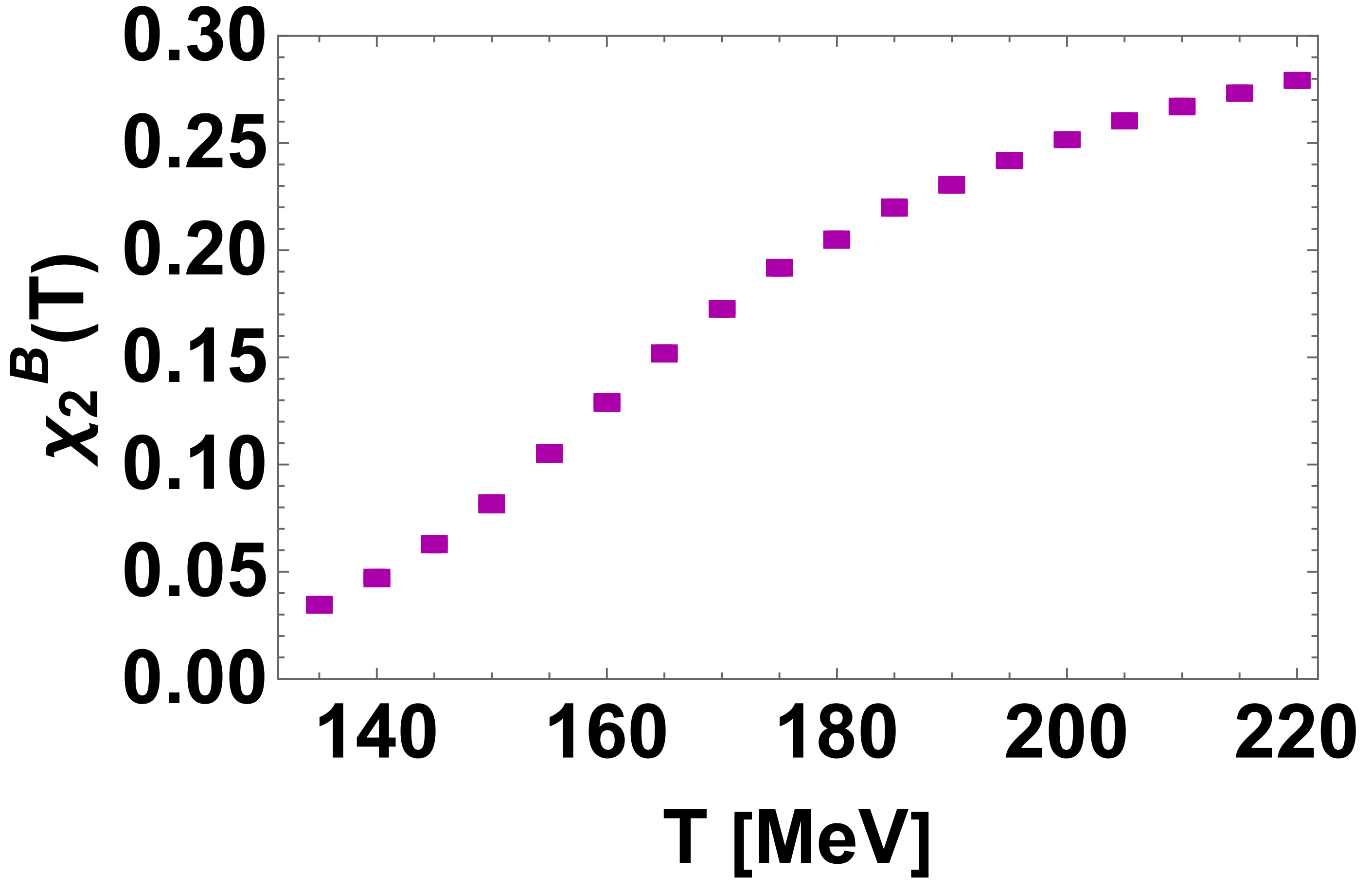


*[Borsányi, S et al PhysRev.Lett. 108(1), 101.034901(2021)]*

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$T'(T,\mu_B) = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

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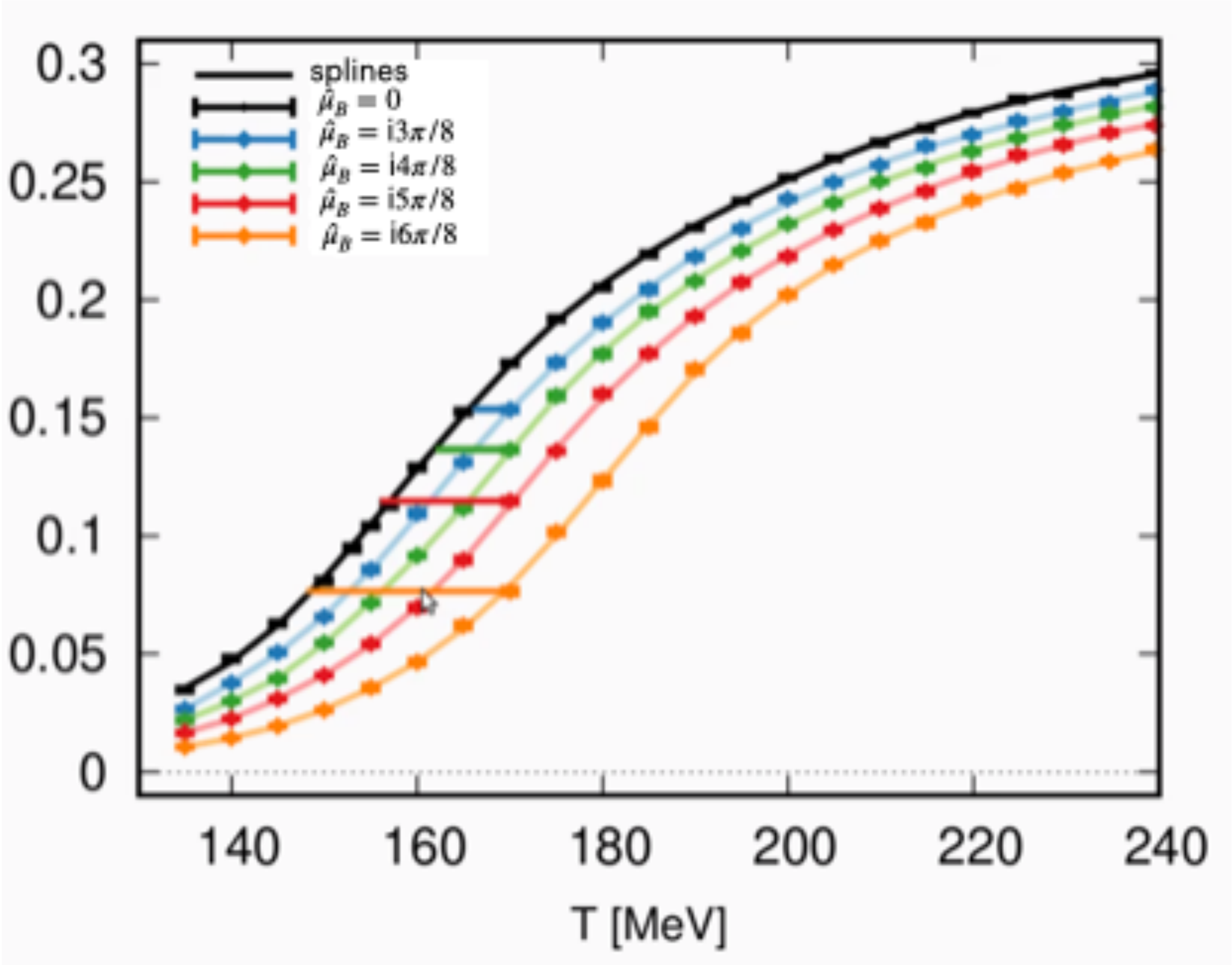


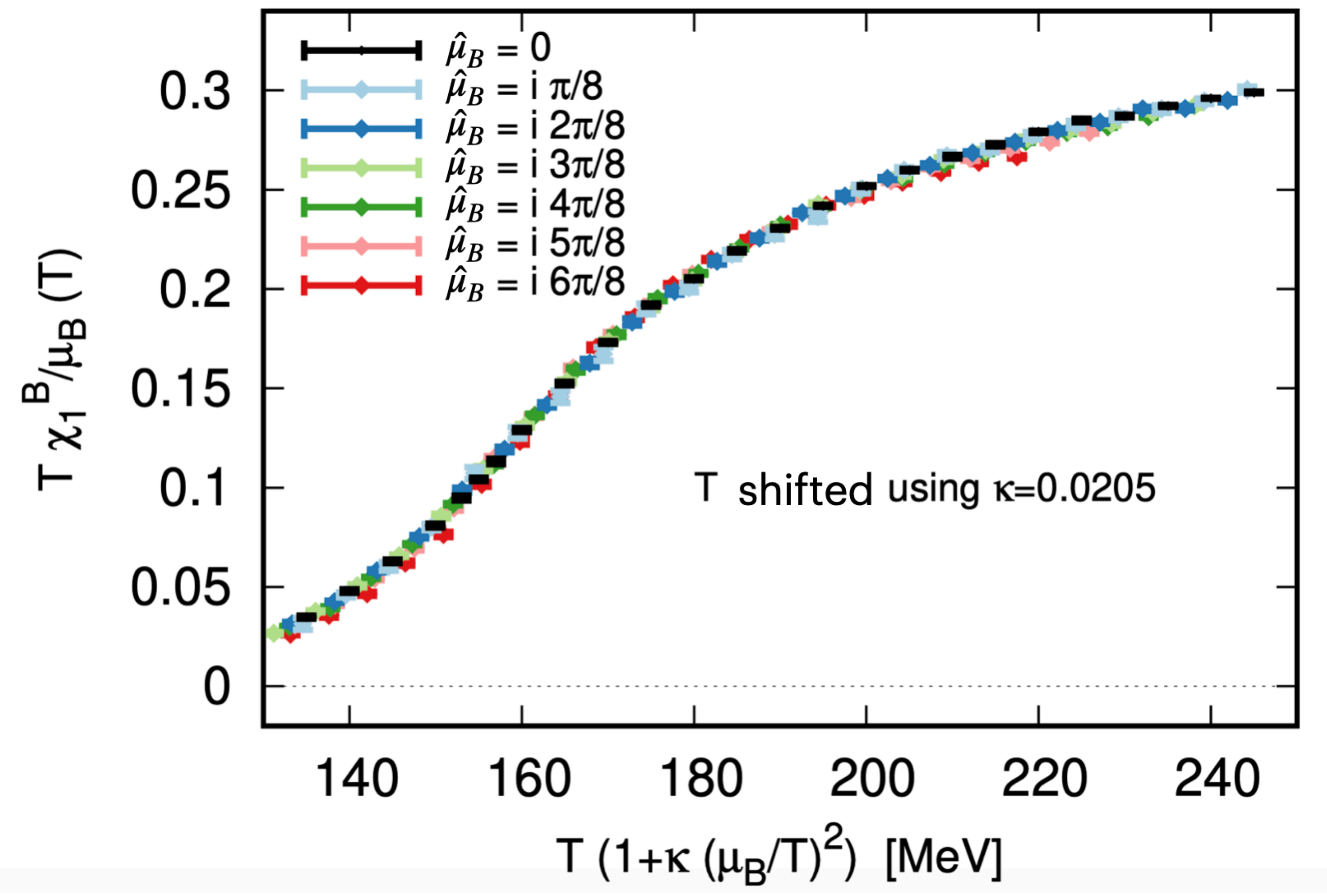


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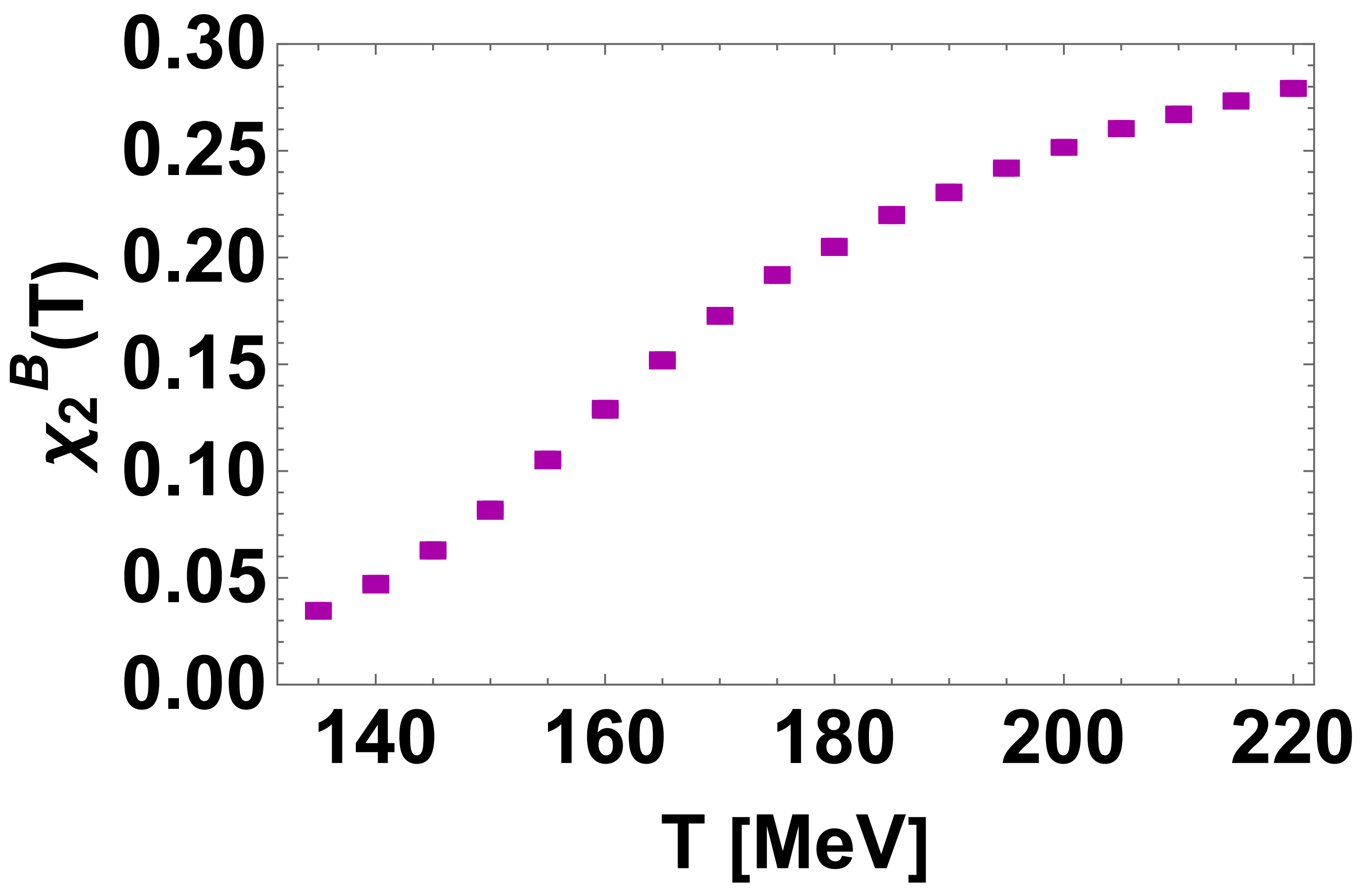


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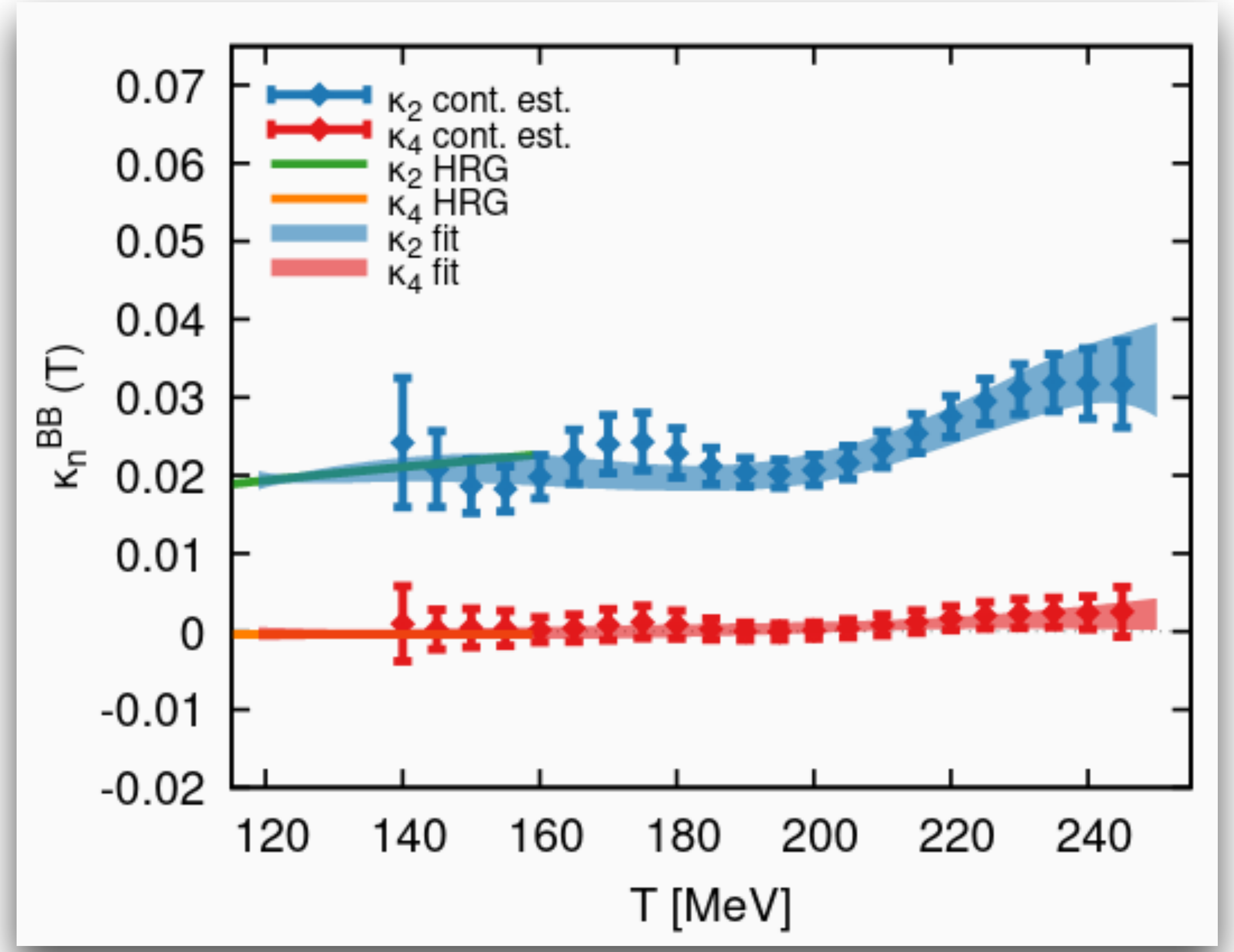
- Uses few expansion terms
- $\mu_B$ dependence is captured in T-rescaling.
- Trusted up to $\frac{\mu_B}{T} = 3.5$

# T′ Expansion scheme (T ExS)

Relationship between **Taylor expansion** and **T′ expansion**

- $\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\partial\chi_2'^B(T)}$

- $\kappa_4^{BB}(T) = \frac{1}{360T\chi_2'^B(T)^3}\left(3\chi_2'^{B2}\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$




[Borsányi, S et al PhysRev.Lett 108(1), 101.034901(2021)]

## Pros

- $\kappa_2(T)$ is fairly constant over a large T-Range
- There is a separation of scale between $\kappa_2(T)$ and $\kappa_4(T)$
- $\kappa_4(T)$ is almost zero $\rightarrow$ faster convergence
- A good agreement with HRG results at Low Temperature

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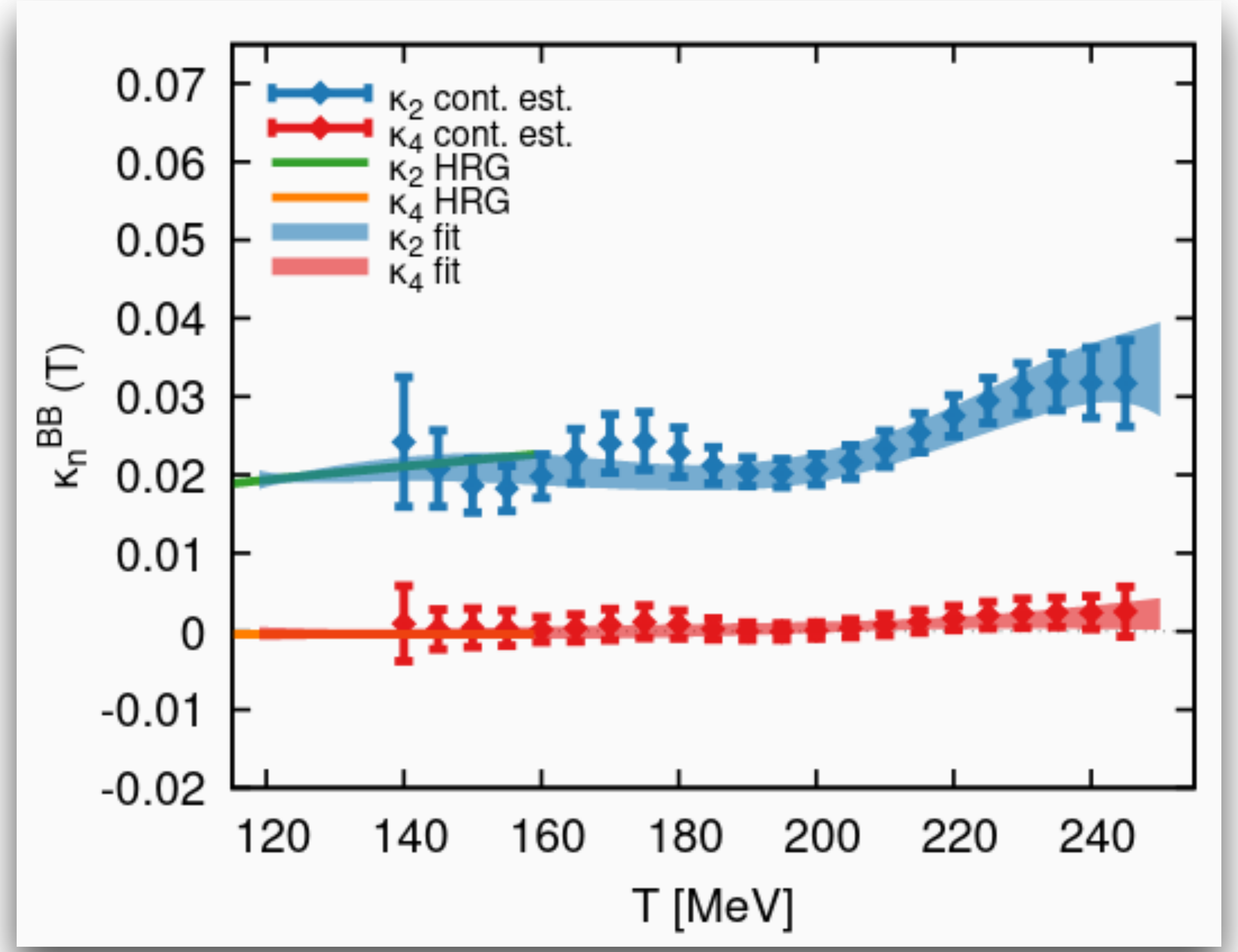


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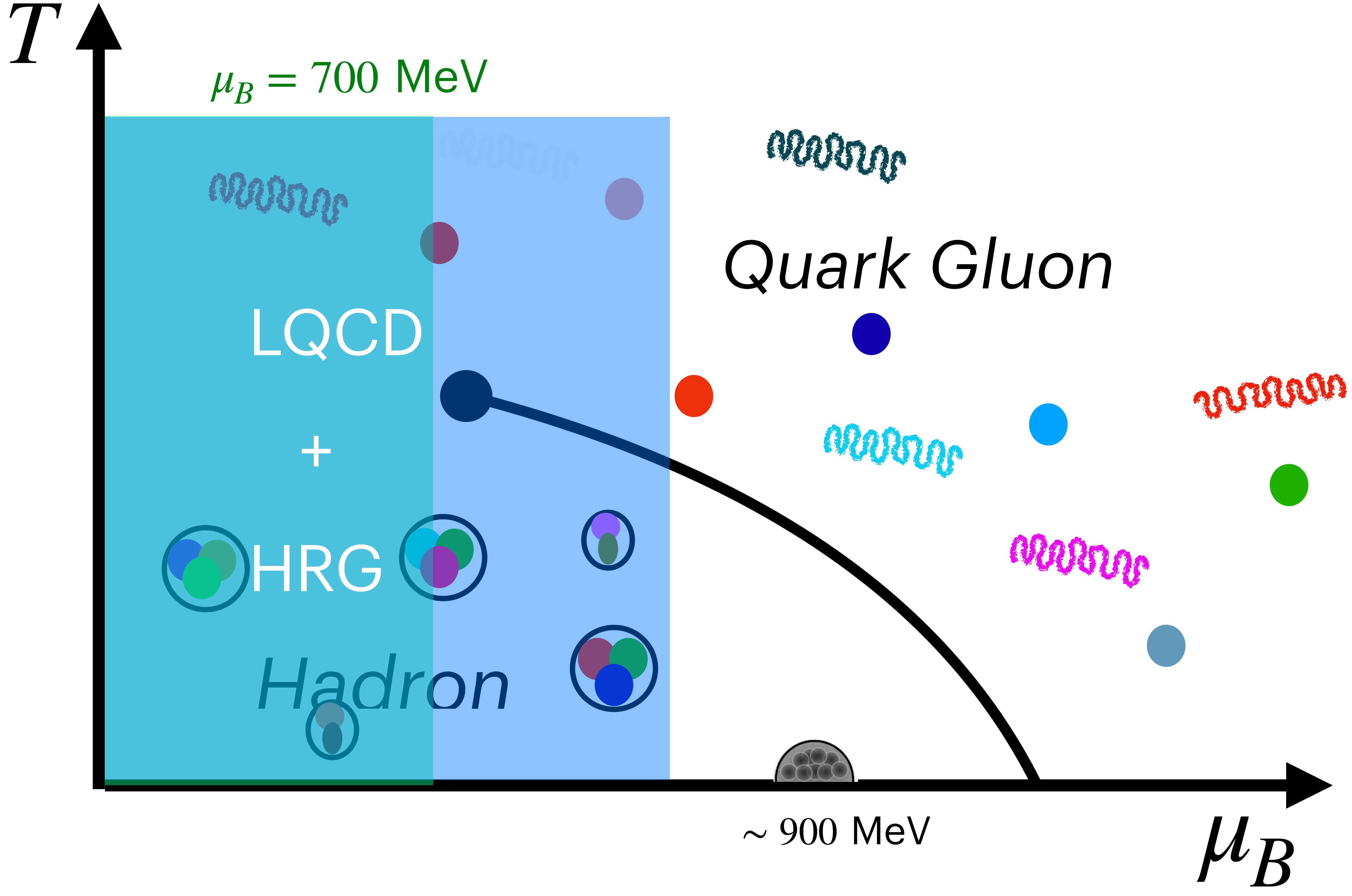

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# Introducing Critical Point

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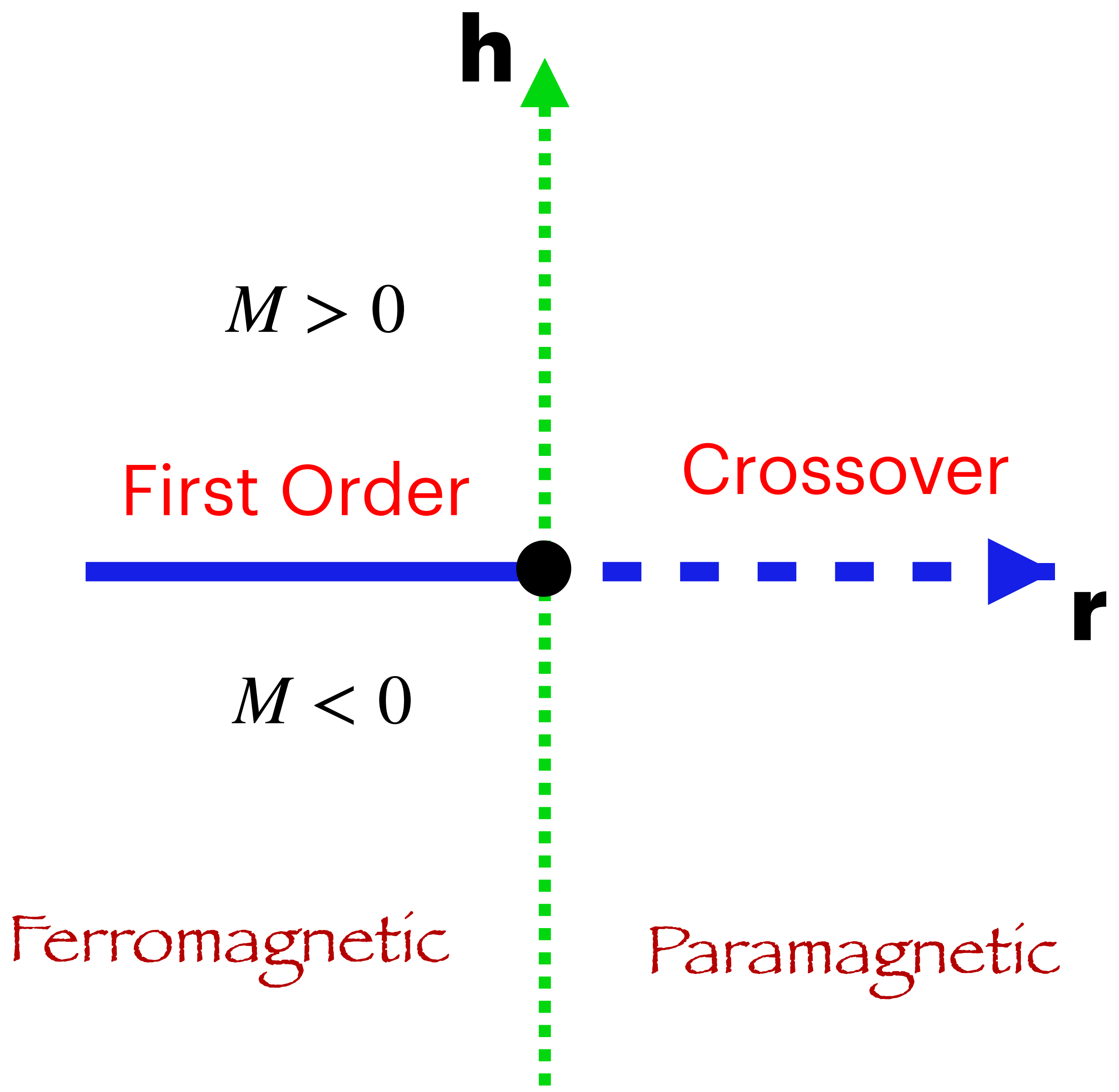
## Mapping 3D Ising to QCD




**3D Ising coordinates**

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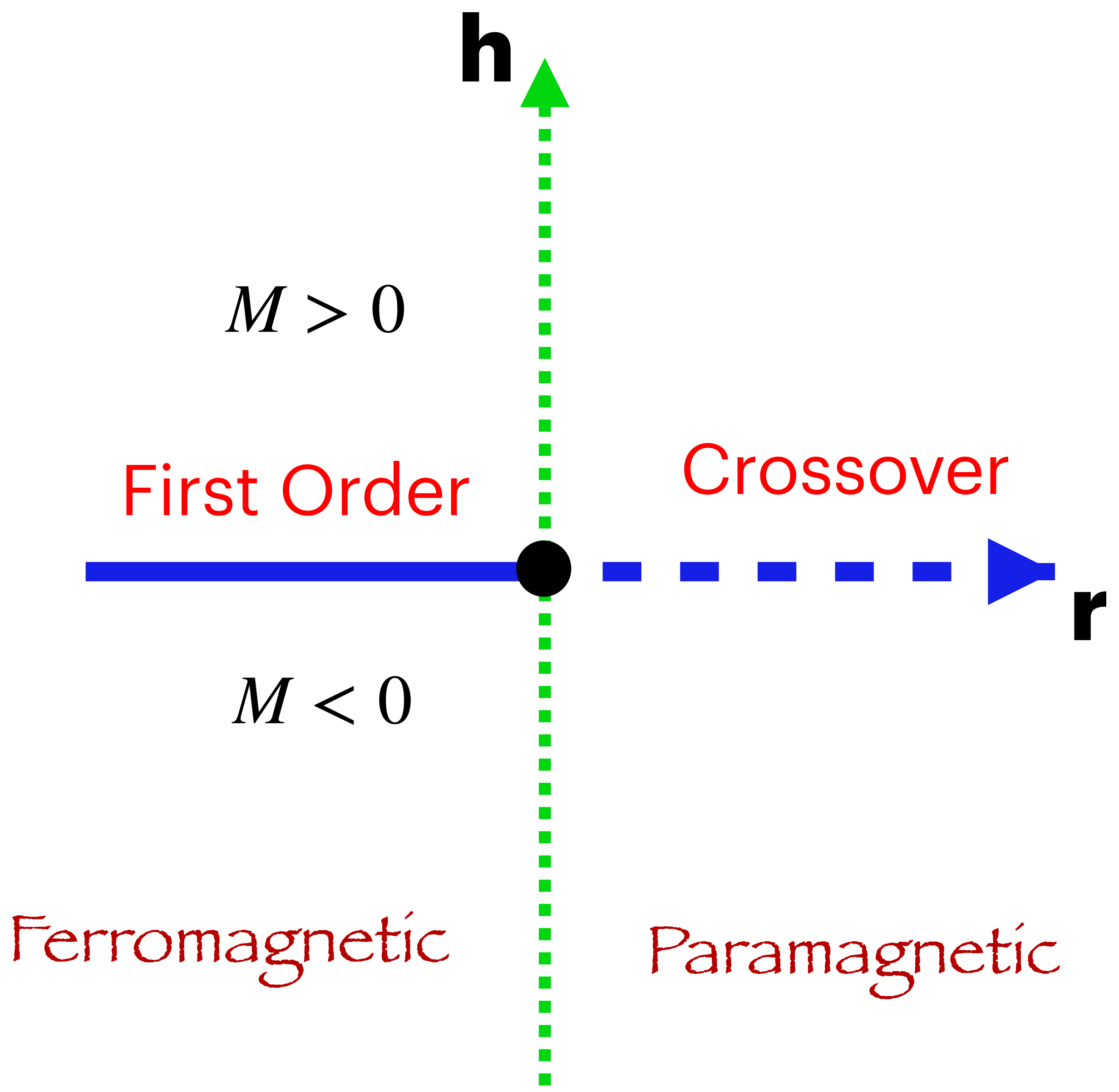
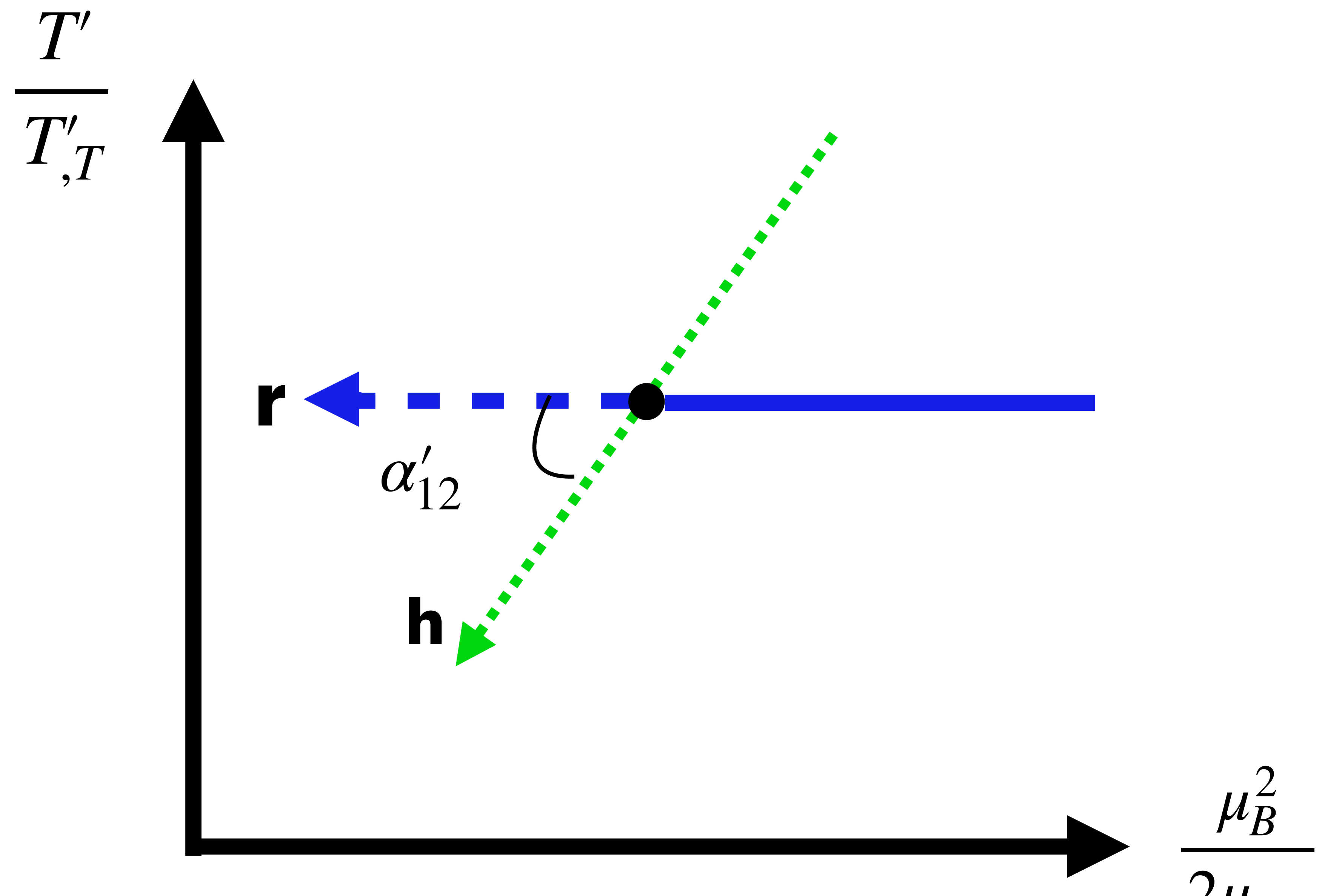



**3D Ising coordinates**

$$\frac{T'-T_0}{T_C T'_{,T}} = -w' h \sin \alpha'_{12}$$

$$\frac{\mu_B^2 - \mu_{BC}^2}{2\mu_{BC}T_C} = w'(-r\rho' - h \cos \alpha'_{12})$$




**T' expansion coordinates**

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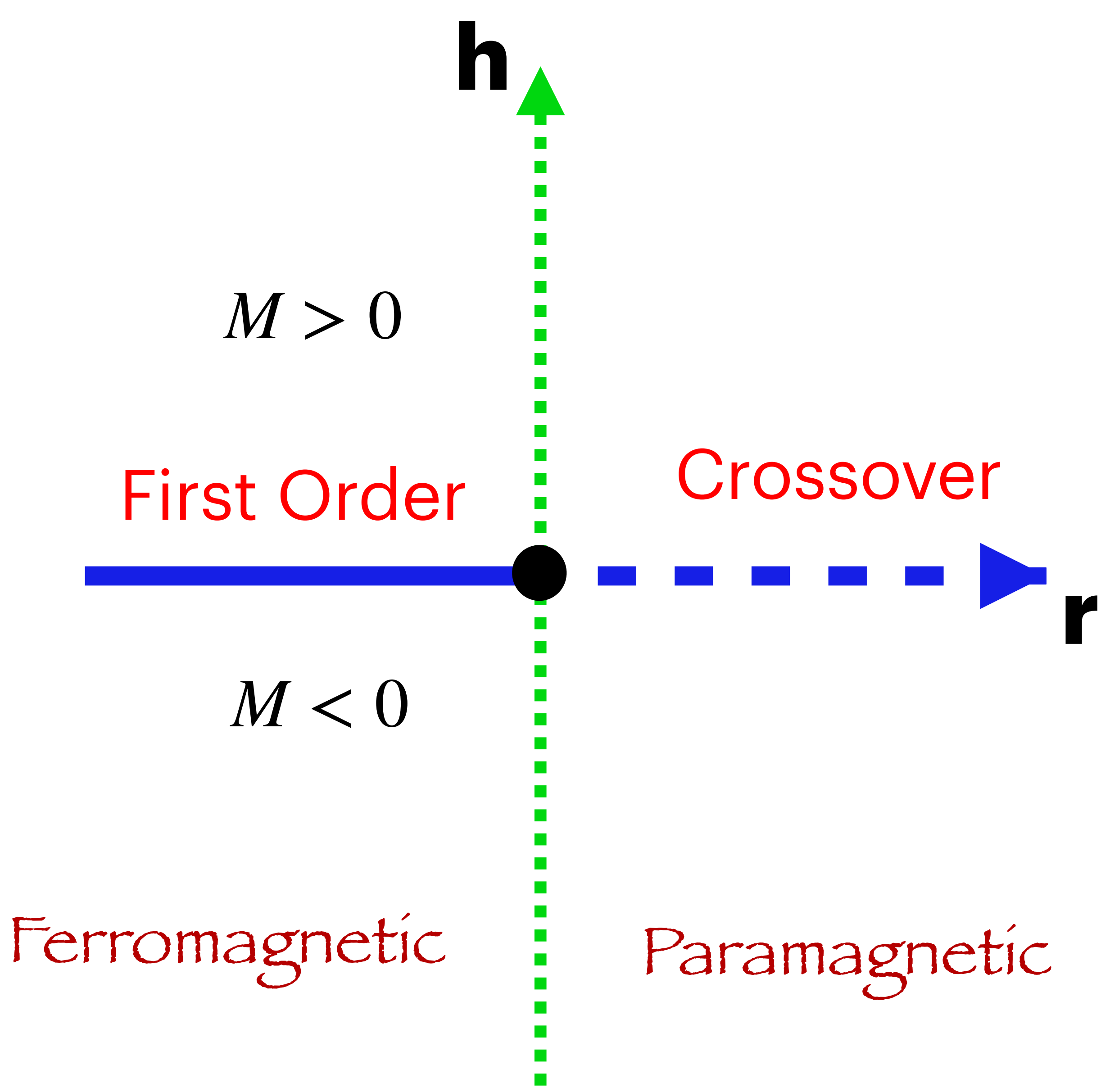


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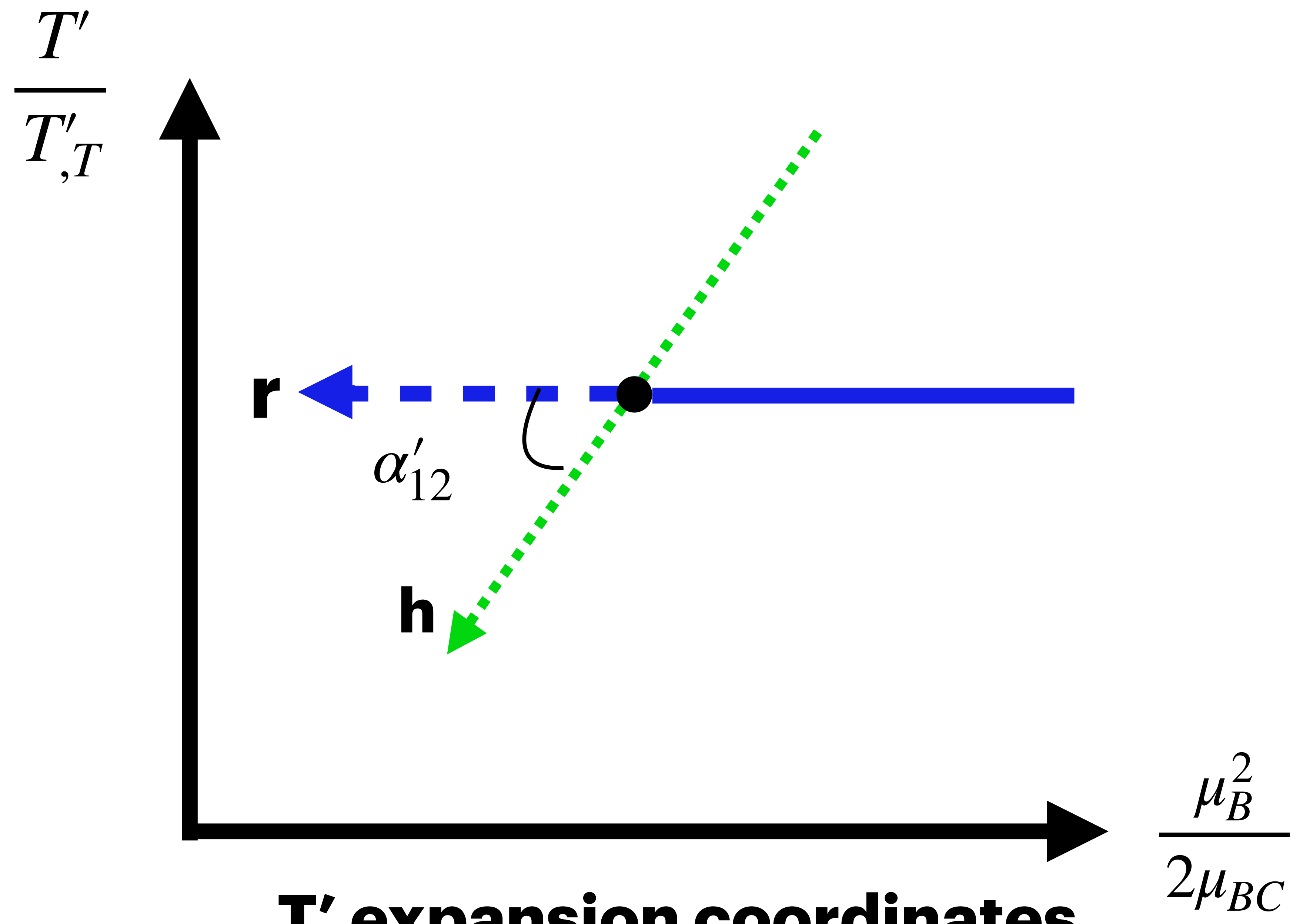
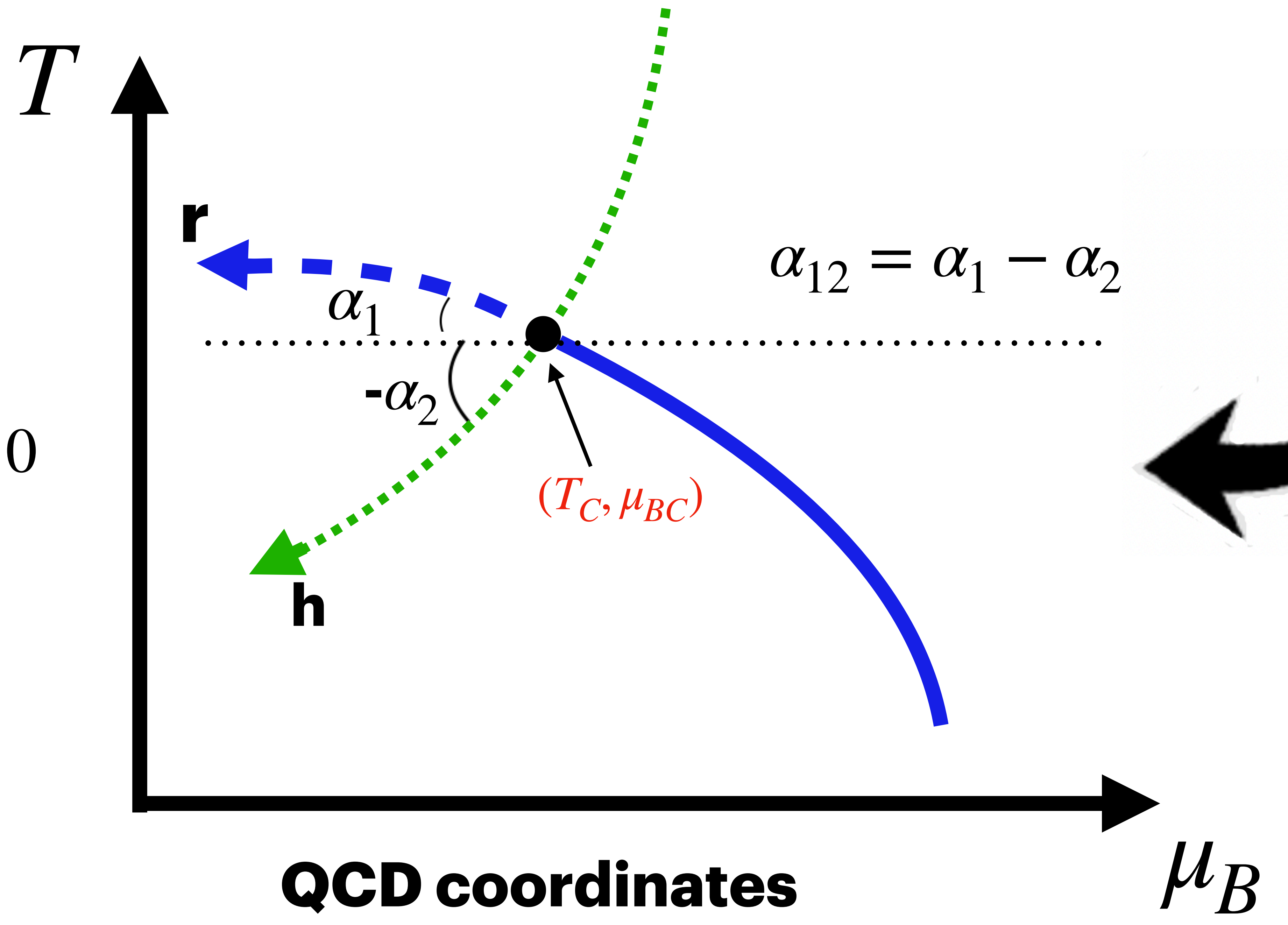



**T′ expansion coordinates**

$T'_{,T} = (\partial T'/\partial T)_\mu$ at the critical point

$T_0$ - Transition temperature at $\mu_B = 0$

$\mu_{BC}, T_C, w', \rho', \alpha'_{12}$ - Free parameters




**QCD coordinates**

$$T' = T\left[1 + \left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4\right]$$

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

# Important relations

## Relationship with BEST collaboration EoS

- The mapping is not universal
- Quadratic mapping is related to BEST Collaboration (linear) mapping

$\mu_{BC}, T_C, \alpha'_{12}, w', \rho'$ → $\mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$

6 parameters

## Transition Line

$T_0 = 158$ MeV - crossover temperature at $\mu_B = 0$

$$T'[T_C, \mu_{BC}] = T_0$$

Slope

Choosing $\mu_{BC}$ fixes $T_C$ and $\alpha_1$

$$\alpha_1 = \tan^{-1}\left(\frac{2\kappa_2(T_C)\mu_{BC}}{T_C T'_{,T}}\right)$$

Examples

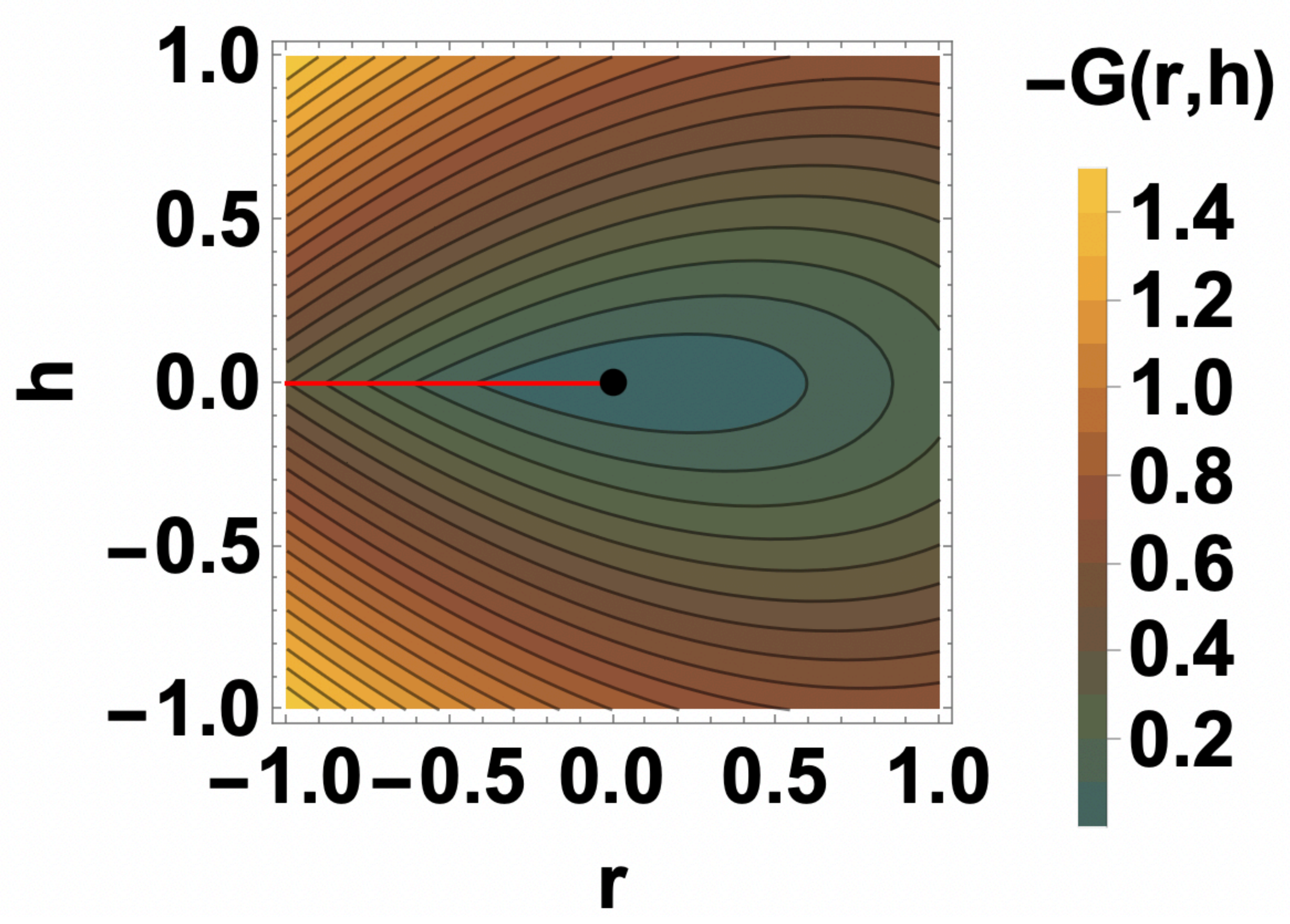
- $\mu_{BC} = 350$ MeV, $T_C = 140$ MeV and $\alpha_1 = 6.6^0$
- $\mu_{BC} = 600$ MeV, $T_C = 94.3$ MeV and $\alpha_1 = 14^0$

BEST COLLABORATION

TExS

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

# Introducing Critical Point




**3D Ising Coordinates**

**Parameters Choice**

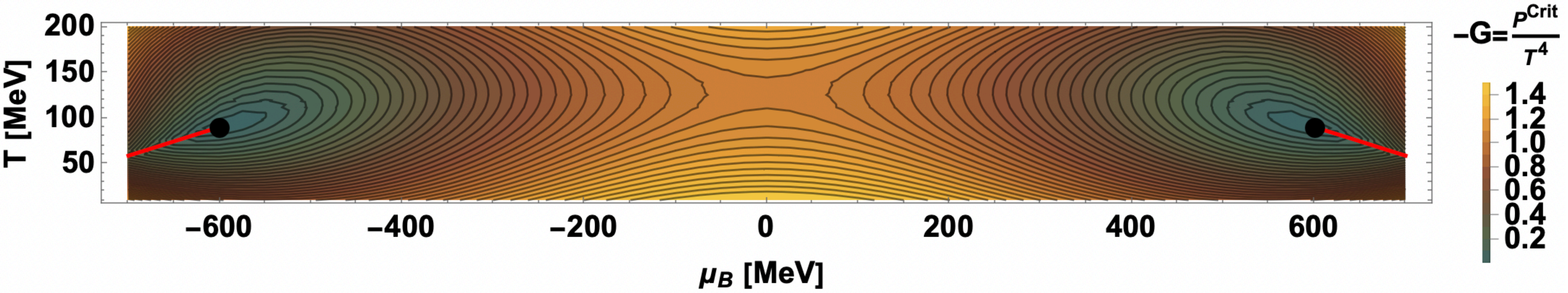
$$\mu_{BC} = 600 \text{ MeV}, \quad T_C = 94 \text{ MeV}$$

$$T_0 = 158 \text{ MeV}, \quad \alpha_1 = 14^0$$

$$\alpha_{12} = 90^0, \quad \alpha_2 = \alpha_1 - \alpha_{12}$$

$$w = 10, \quad \rho = 0.5$$

$$T_C \left[ 1 + \kappa(T_C) \left( \frac{\mu_{BC}}{T_C} \right)^2 \right] = T_0$$

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[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

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# Merging Ising with Lattice (Ising-T ExS)

## Full Baryon Density

$$\chi_1^B(T,\mu_B) = \frac{n_B(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi_{2,lat}^B(T',0)$$

$$T' = \underbrace{T'_{lat}(T,\mu_B)}_{\text{lower order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

**Lattice Term**

**Ising Term**

*[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]*

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**Lattice Term** **Ising Term**

## Introducing a Critical Point

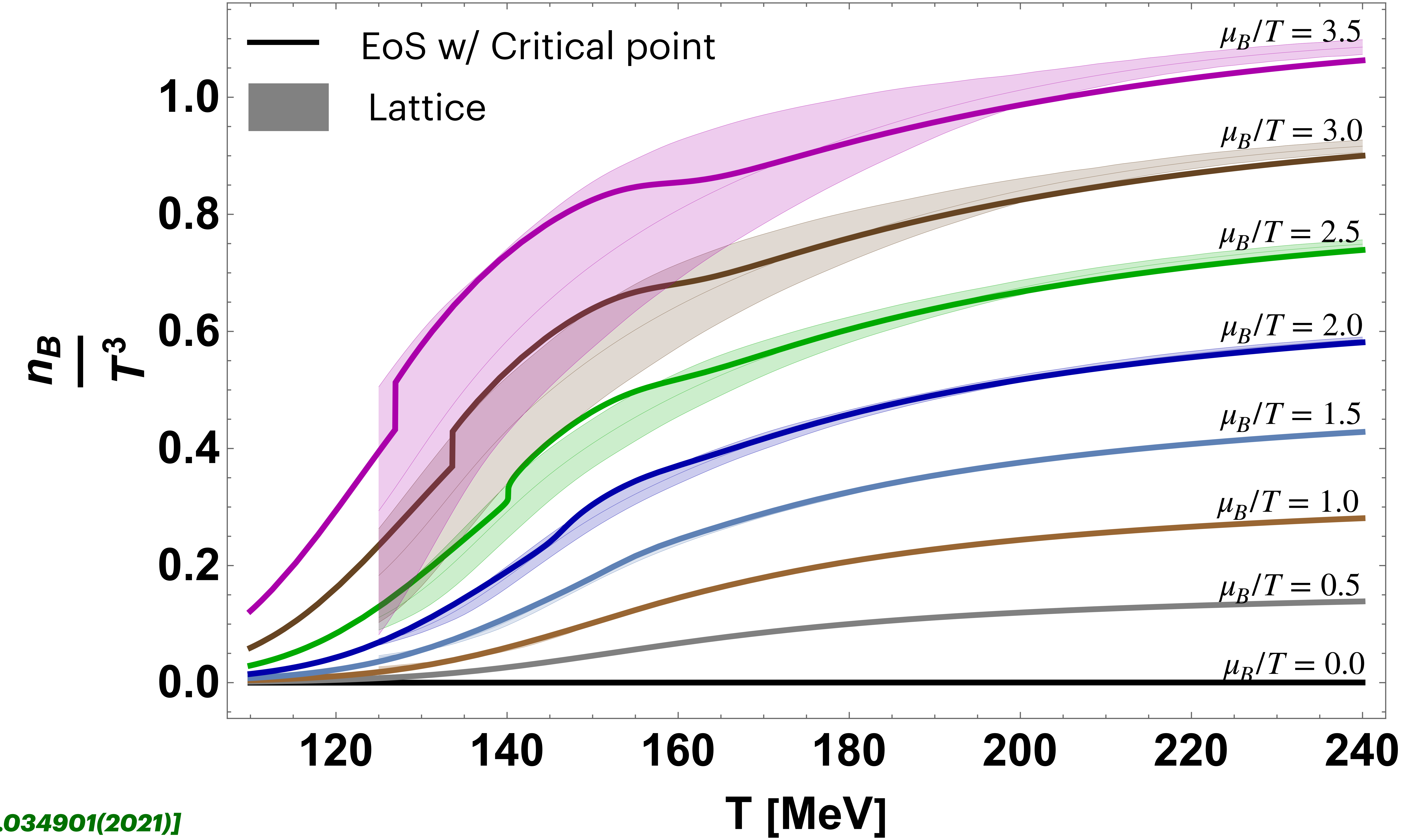
$$T'_{crit}(T,\mu_B) \approx \left(\left.\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\right|_{T=T_0}\right)^{-1} \frac{n_B^{crit}(T,\mu_B)/T^3}{(\mu_B/T)} + \dots$$

$$Taylor[T'_{crit}, n=2] \approx \left(\left.\frac{\partial\chi_{2,lat}^B(T,0)}{\partial T}\right|_{T=T_0}\right)^{-1} \left[\left.\frac{\partial(n_B^{crit}/T^3)}{\partial(\mu_B/T)}\right|_{\mu_B/T=0} + \frac{1}{3!}\left.\frac{\partial^3(n_B^{crit}/T^3)}{\partial(\mu_B/T)^3}\right|_{\mu_B/T=0}\left(\frac{\mu_B}{T}\right)^2 + \dots\right]$$

***[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]***

# Baryon density results

**Full Baryon Density at a constant $\frac{\mu_B}{T}$ compared with Lattice**




## Parameter choice

$\mu_B = 350$ [MeV]

$\alpha_{12} = 90^0$

$w = 2$

$\rho = 2$

*[Borsányi, S et al PRL. 108(1), 101.034901(2021)]*

*[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]*

# Thermodynamic Observables

**Parameter choice**

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

$\alpha_{12} = \alpha_1 = 14^0$

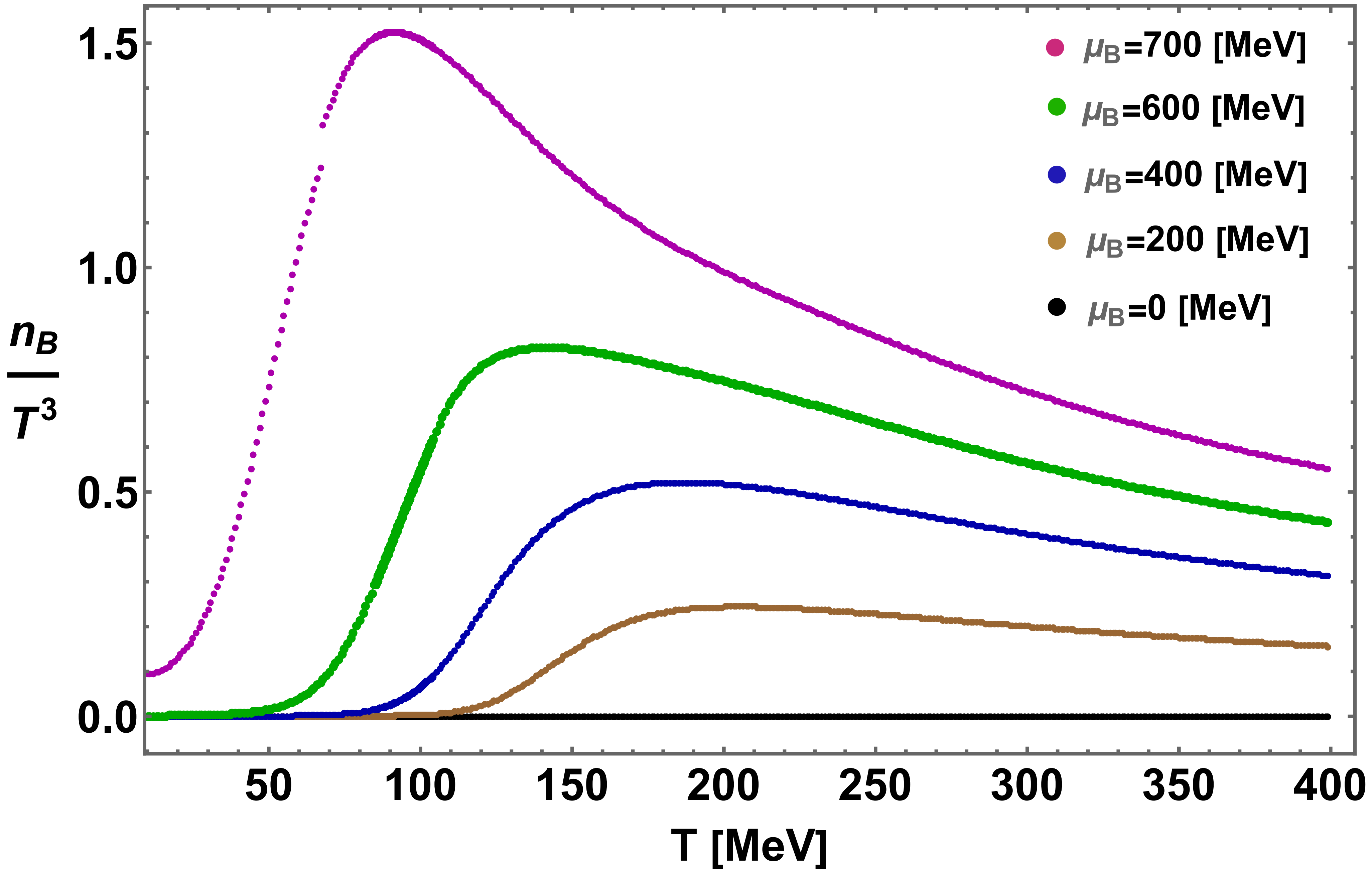
$\alpha_2 = 0^0$

$w = 15$

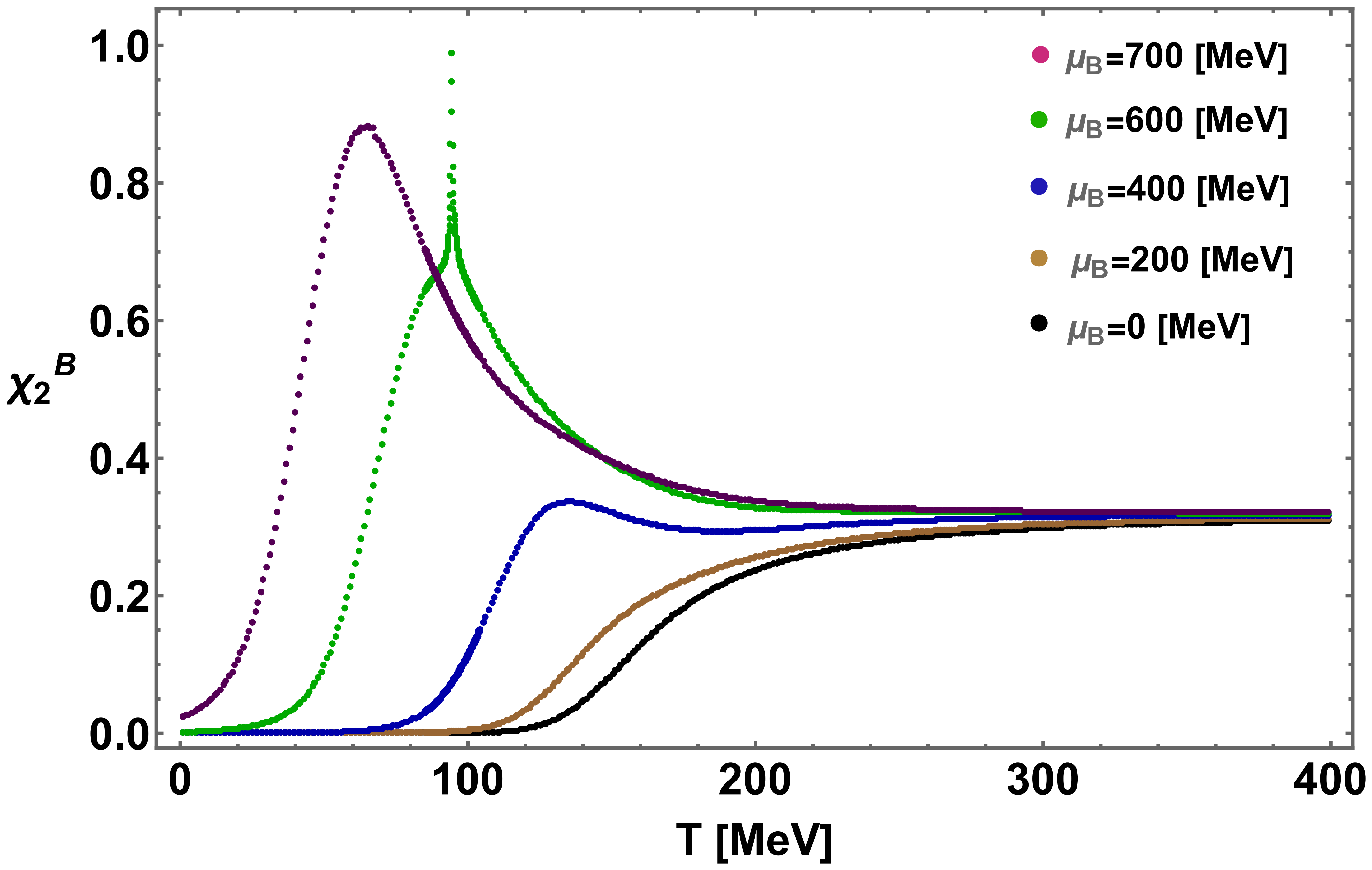
$\rho = 0.3$

$$\chi_2(T, \mu_B) = \frac{\partial(n_B/T^3)}{\partial(\mu_B/T)}\bigg|_T$$

**Baryon Density**




**Baryon number susceptibility**




*[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]*

# Other Observables

**Parameter choice**

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

$\alpha_{12} = \alpha_1 = 14^0$

$\alpha_2 = 0^0$

$w = 15$

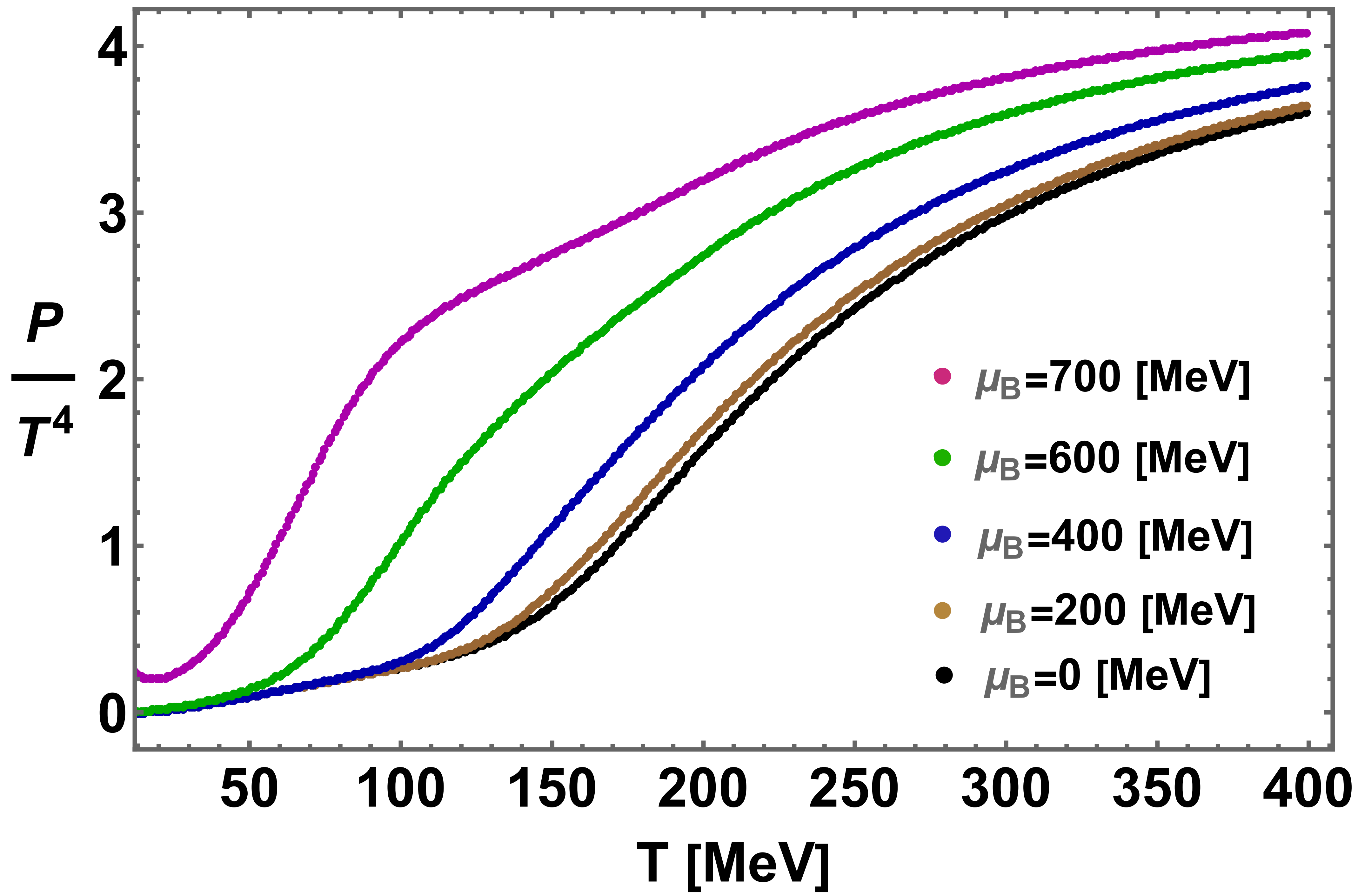
$\rho = 0.3$

$$\frac{P(T,\mu_B)}{T^4} = \chi^B_{0,lat}(T,0) + \frac{1}{T}\int_0^{\mu_B} d\hat{\mu}'_B \frac{n_B(T,\hat{\mu}'_B)}{T^3}$$

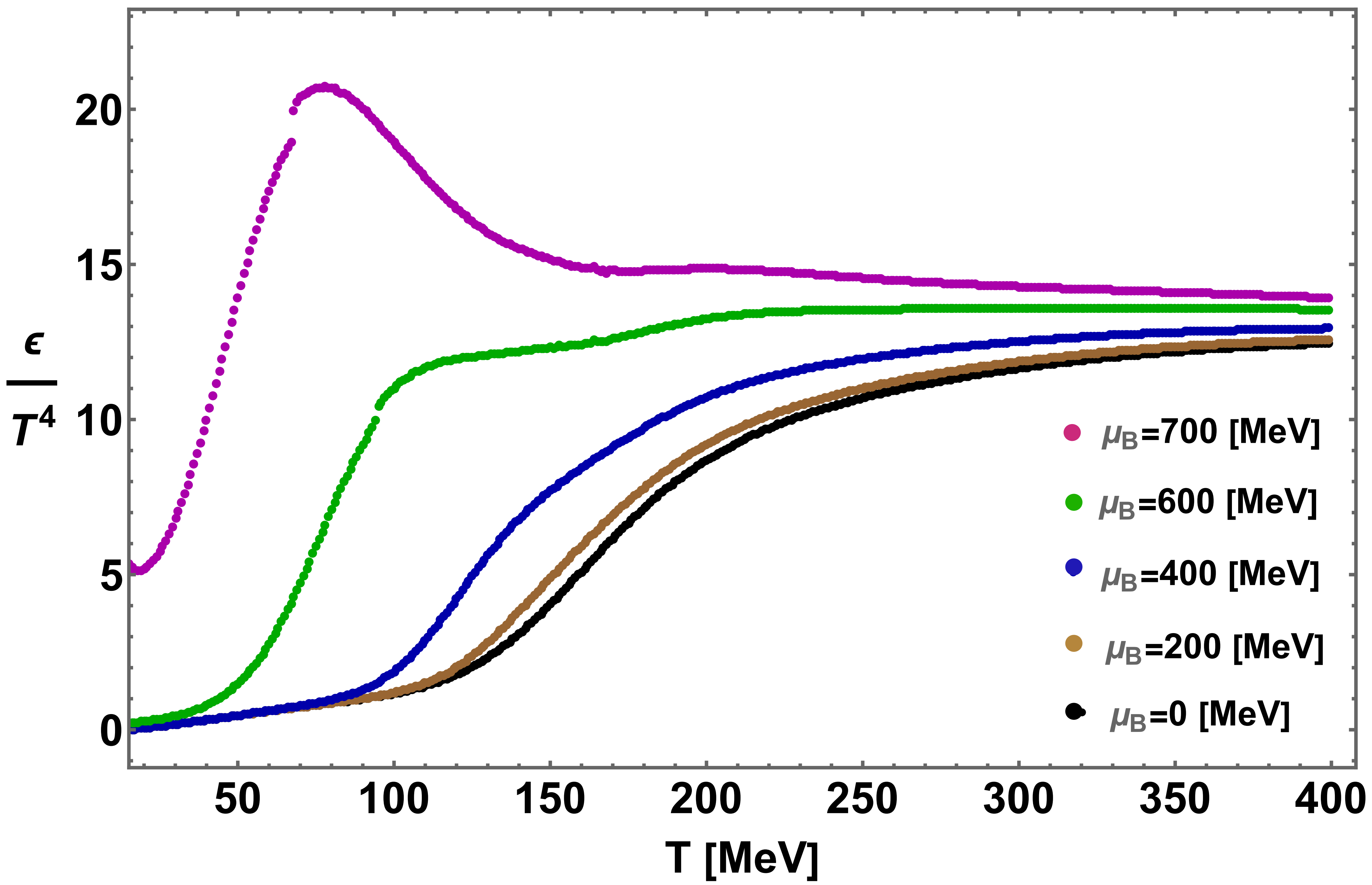
$$\frac{s(T,\mu_B)}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial T}\right)\Bigg|_{\mu_B}$$

$$\frac{\epsilon(T,\mu_B)}{T^3} = \frac{s}{T^3} - \frac{P}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3}$$

**Pressure**



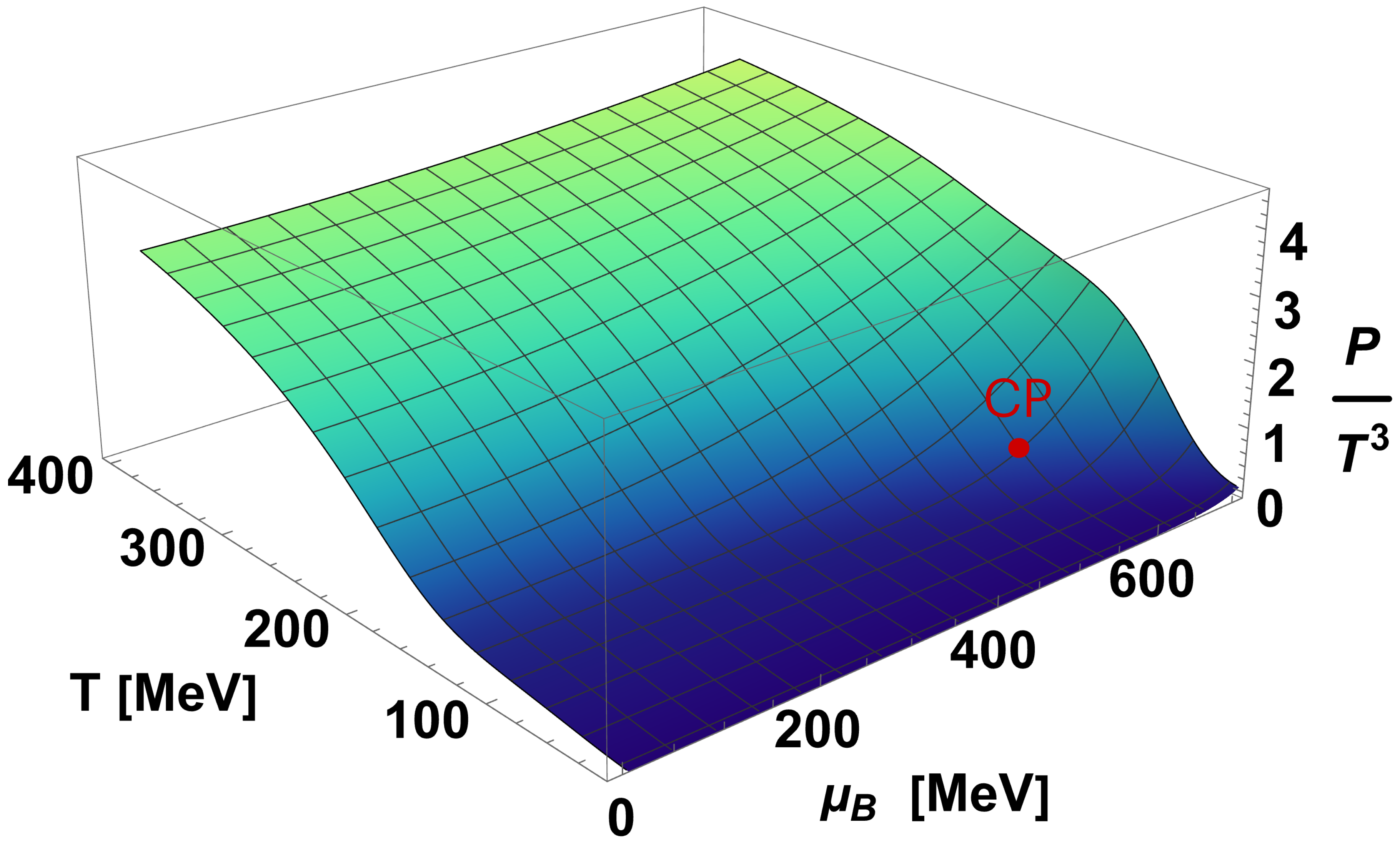

**Energy density**



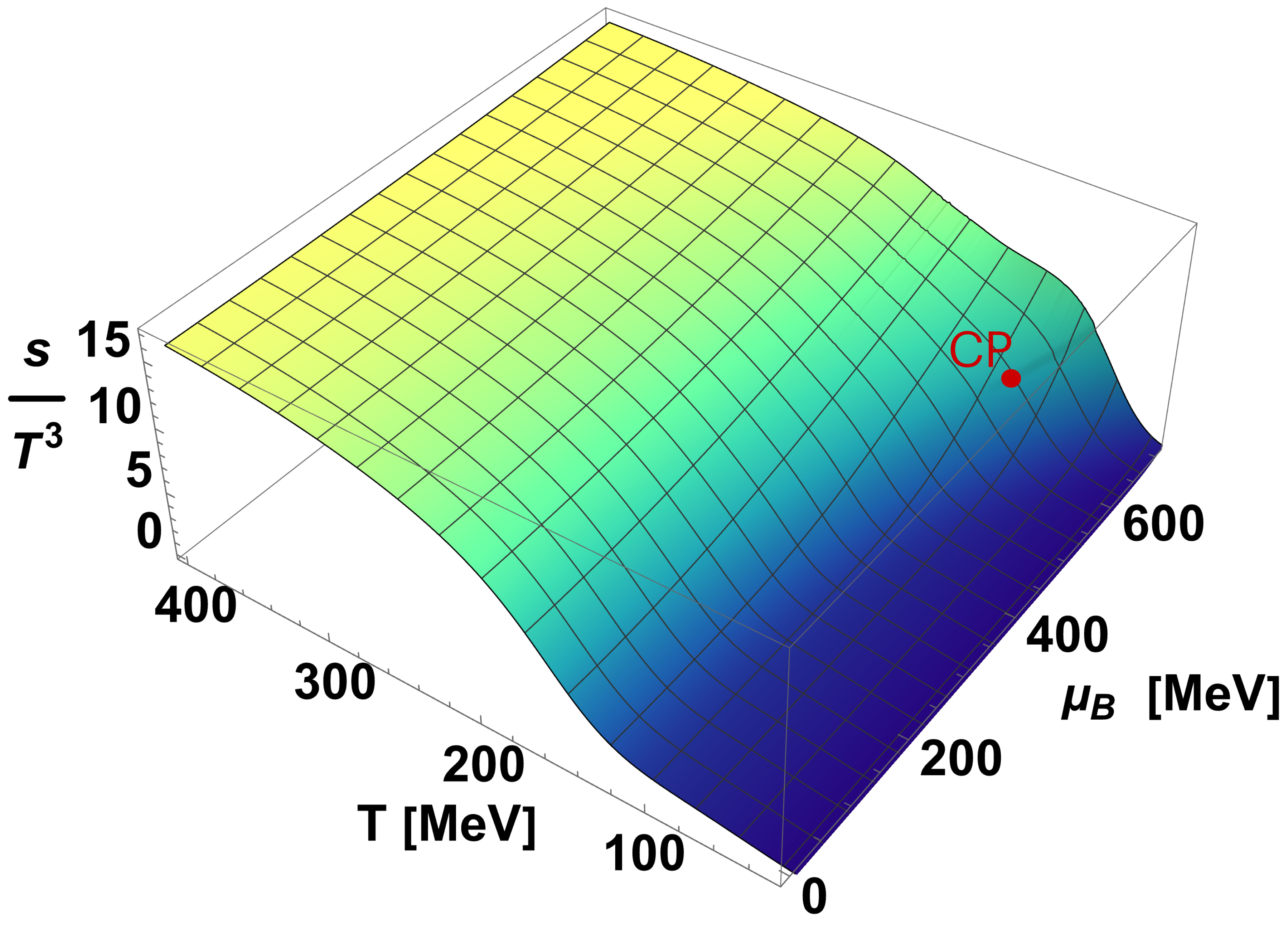

*[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]*

# Thermodynamic Observables

## Pressure




## Entropy density




## Parameter choice

$\mu_{BC} = 600$ MeV

$T_C = 94.3$ MeV

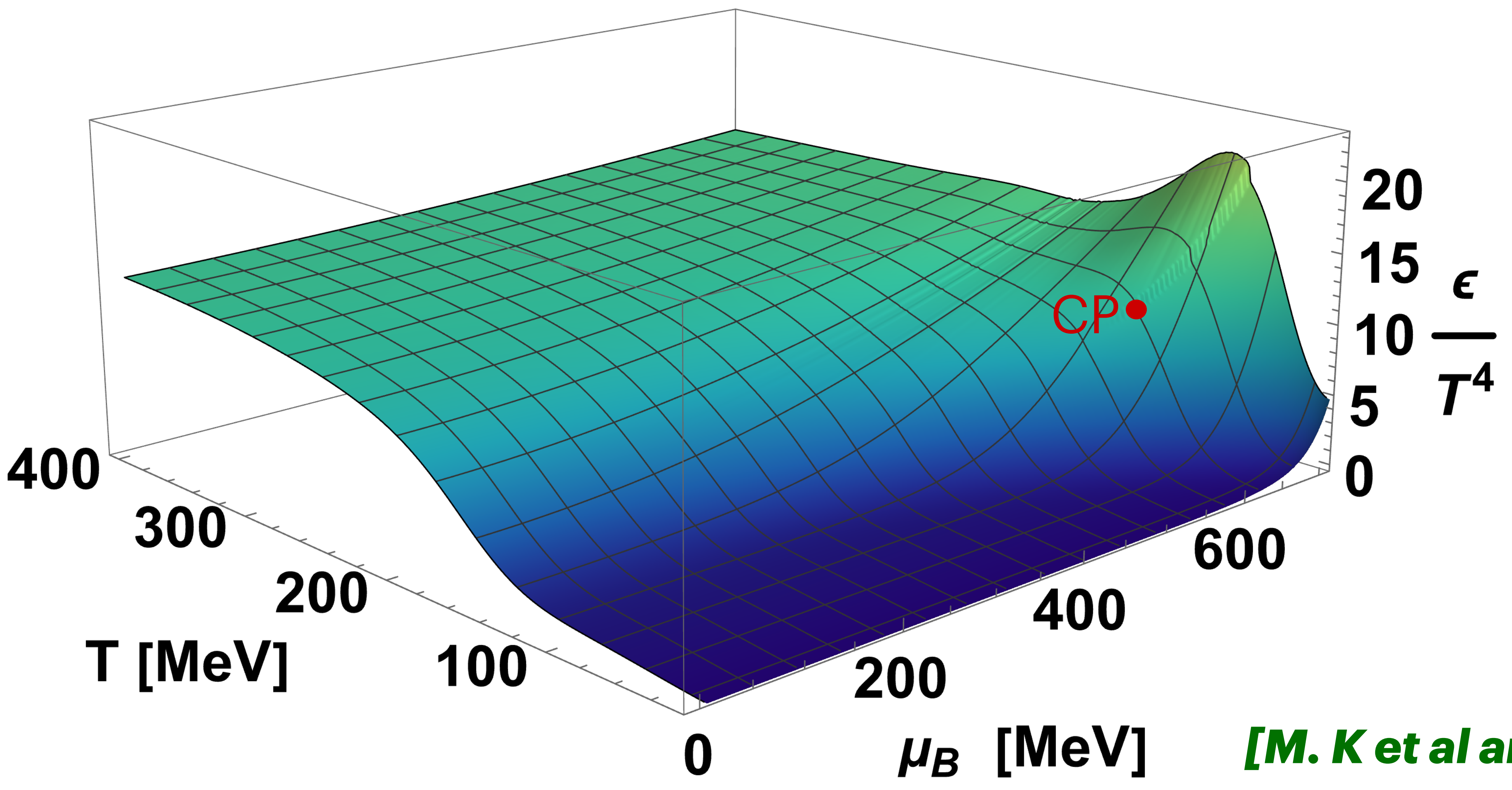
$\alpha_{12} = \alpha_1 = 14^0$

$\alpha_2 = 0^0$

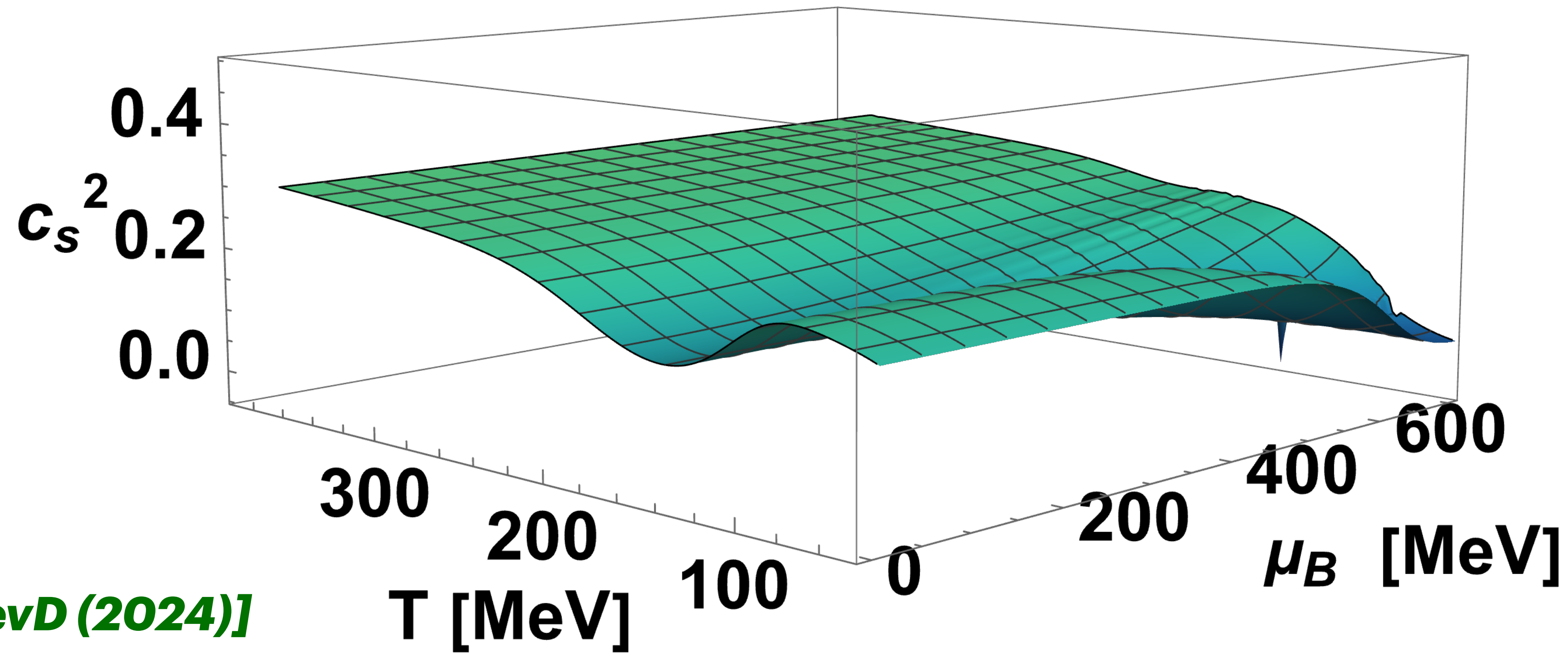
$w = 15$

$\rho = 0.3$

## Energy Density




## Speed of Sound




*[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]*

Part 1: Taylor Expansion

Part 2: T' Expansion Scheme (T ExS)

Part 3: Introducing Critical Point (3D-Ising)

Part 4: Merging 3D Ising with T' Expansion (Ising-TExS)

**Part 5: Constraints on the EoS**

# Known constraints on the EoS

$\mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$

6 free parameters

- Lattice QCD disfavors $\mu_{BC} < 300$ MeV

[Borsányi, S et al PhysRevL. 125, 052001(2020)]

- Choosing $\mu_{BC}$ fixes $T_C$ and $\alpha_1$
- $\alpha_{12}$ is fixed by physical quark mass requirement $(\alpha_{12} = \alpha_1)$

[ Pradeep, M. S., & Stephanov, M PhysRevD 100(5), 056003.(2019)]

## Stability and causality

- $w$ and $\rho$ are imposing fixed stability and causality

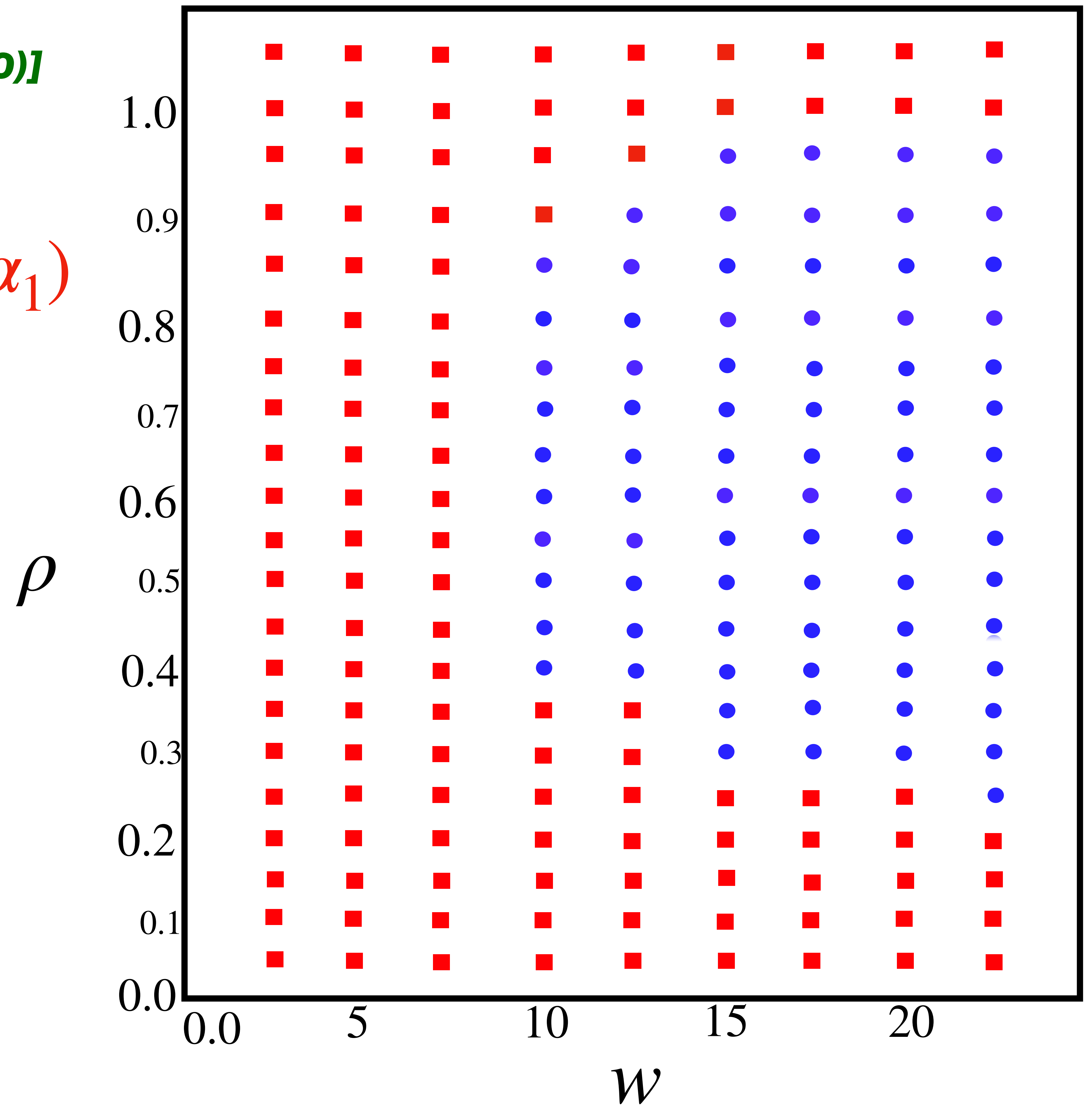
$$c_v = \left(\frac{\partial s}{\partial T}\right)\Bigg|_{n_B} > 0$$

$$\chi_2(T, \mu_B) = \left(\frac{\partial n_B}{\partial \mu_B}\right)\Bigg|_{T} > 0$$

$$0 < c_s^2(T, \mu_B) < 1$$

$\mu_{BC} = 600$ MeV

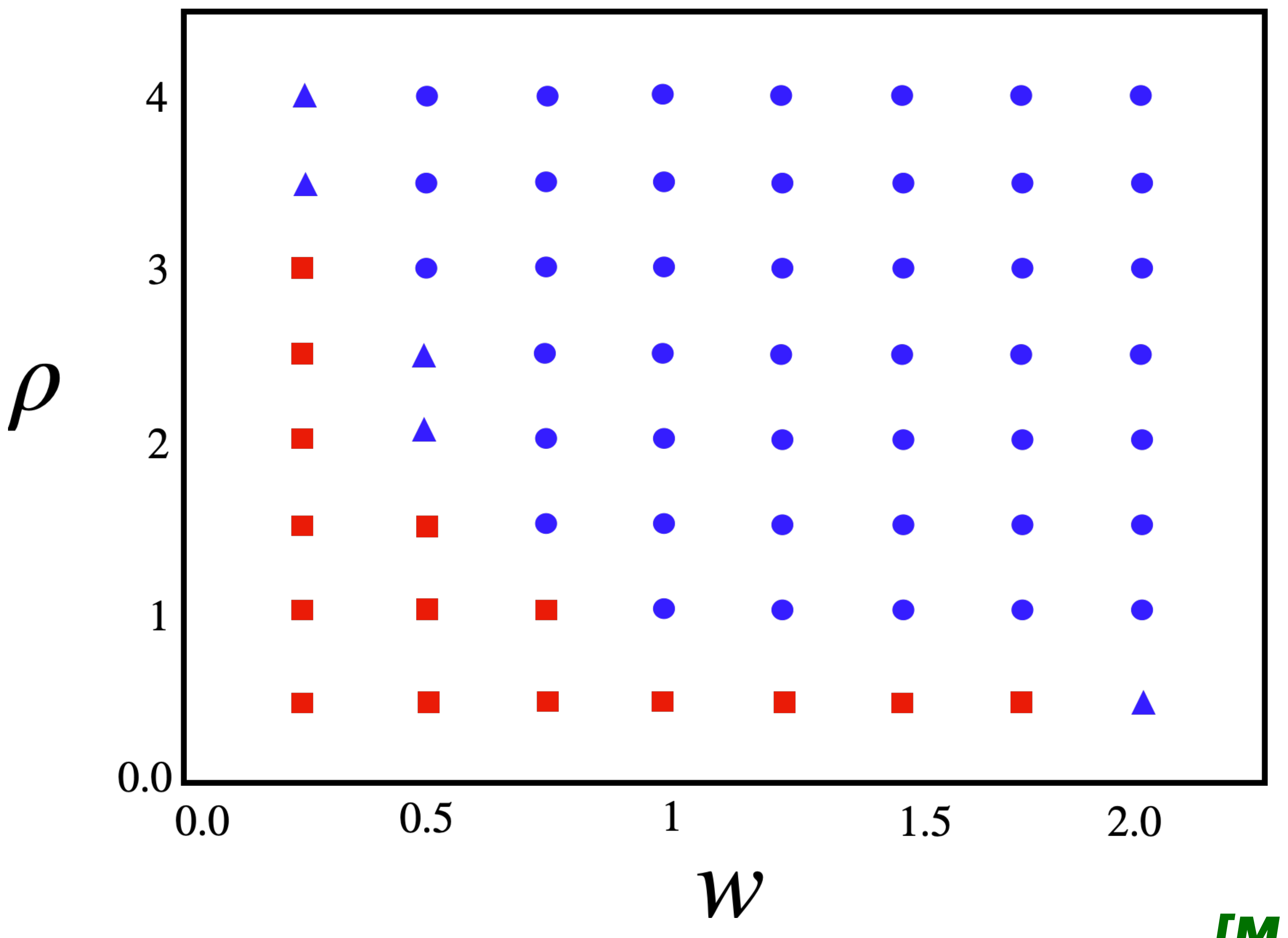
$\alpha_{12} = \alpha_1$



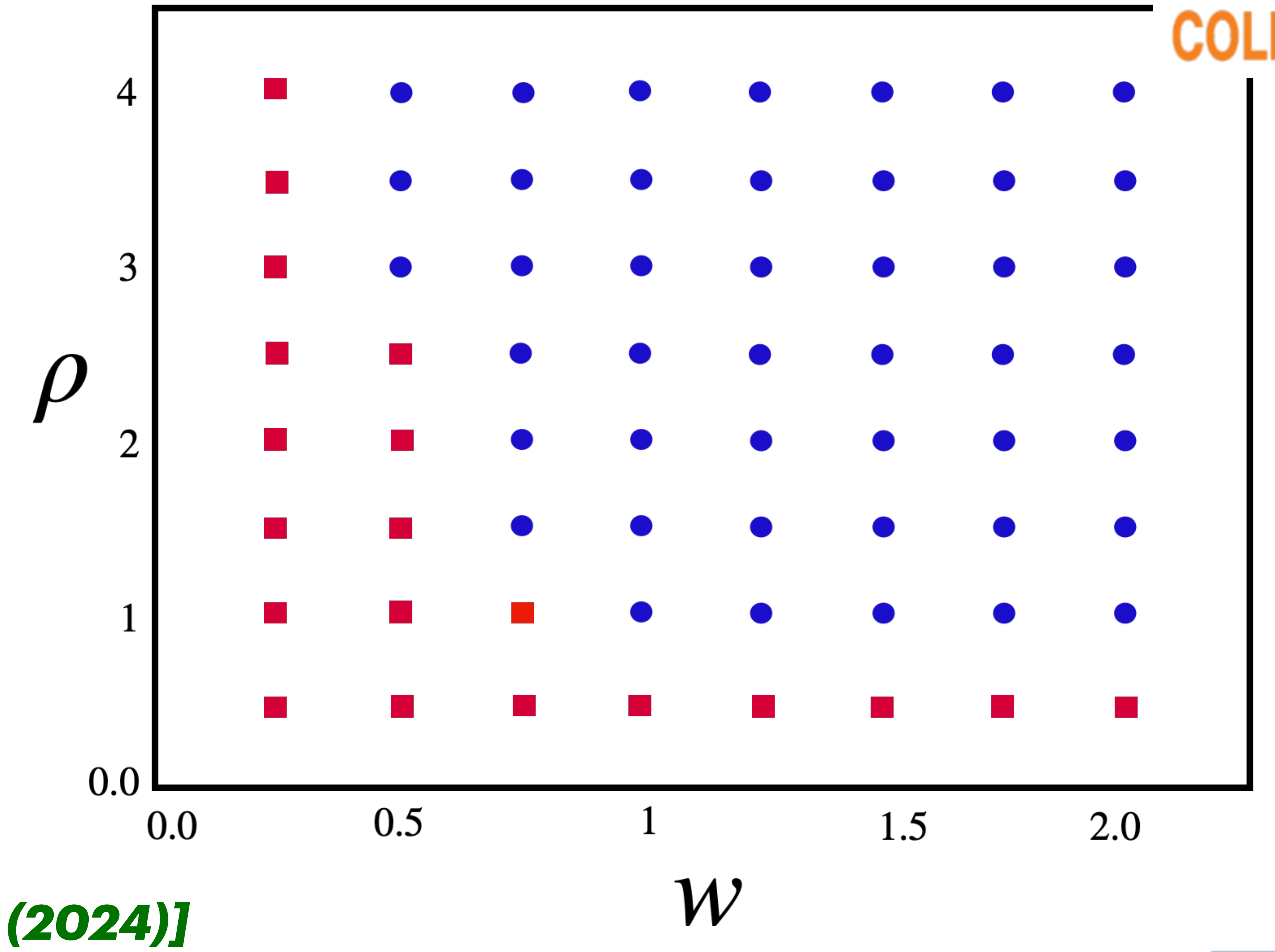

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

# Comparison of Ising-TEXS with BEST EoS

## Ising-T ExS




## Taylor Expansion

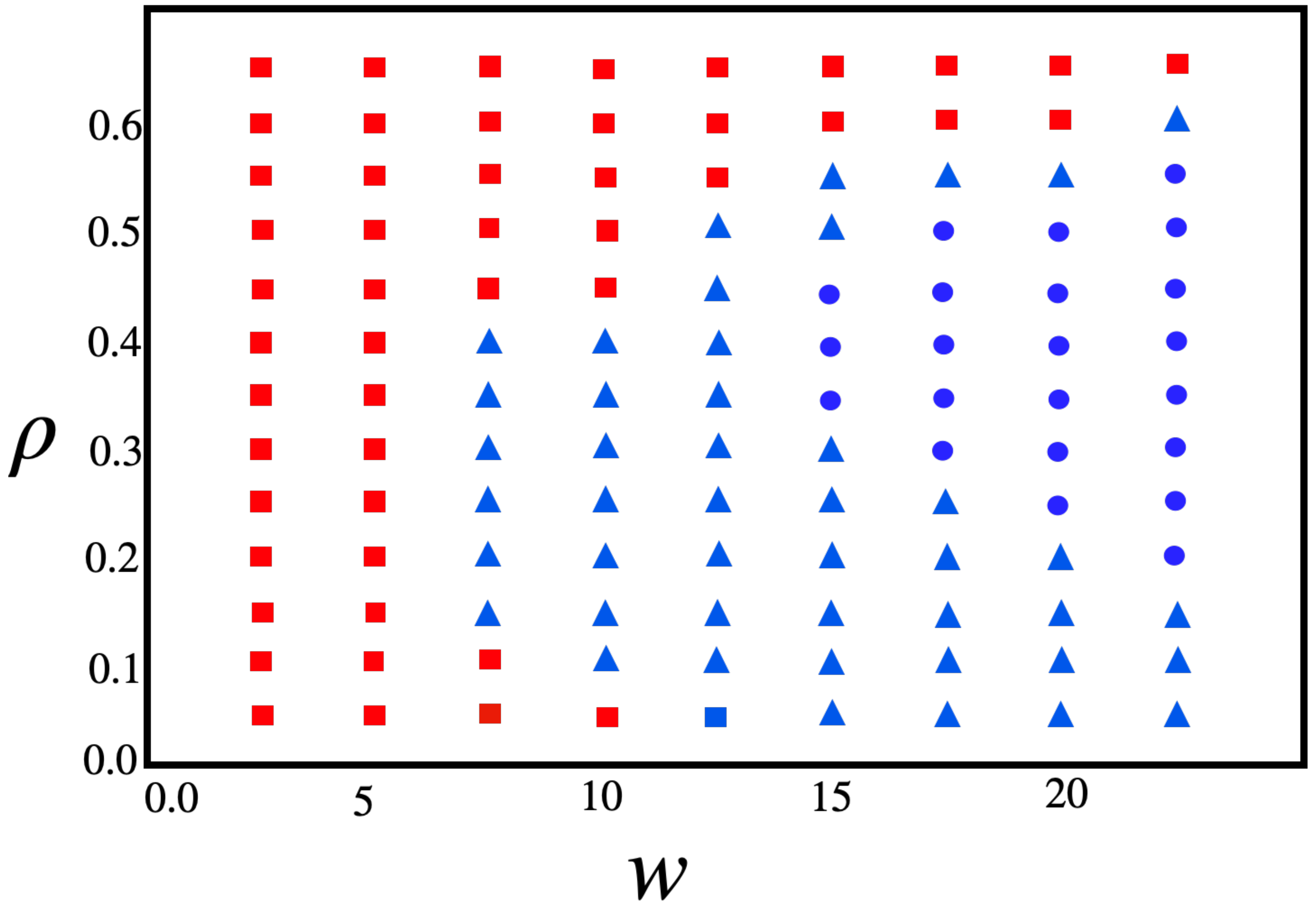



$\mu_{BC} = 350$ MeV

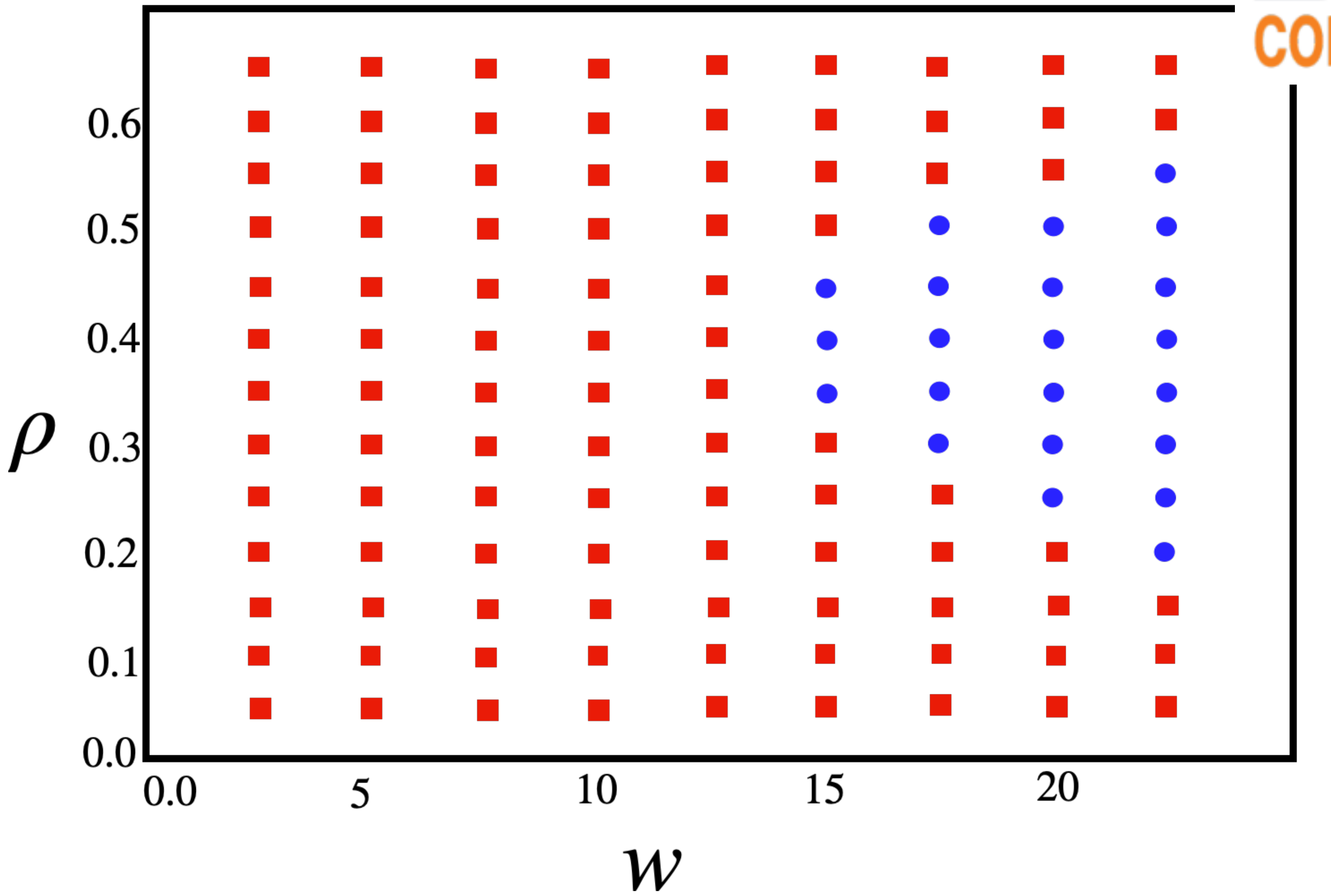
$\alpha_{12} = 90$

[M. K et al arXiv:2402.08636v1, PhysRevD (2024)]

## Ising-T ExS




## Taylor Expansion




$\mu_{BC} = 350$ MeV

$\alpha_{12} = \alpha_1$

# Summary and conclusions

- We provide an **enhanced coverage** for family of EoS with a 3D Ising critical point up to $\mu_B = 700\ MeV$ and matching lattice at low $\mu_B$.
- Ising TExS EoS incorporates **charge conjugation** symmetry inbuilt directly from the Ising -QCD mapping.
- Ising TExS mapping can be constrained to reproduce expectations based on **physical quark masses.**
- Ising TExS has adjustable parameters and can be used as input in hydrodynamical simulations to compare with the data from the Experiment. (Beam Energy Scan II)

**Disclaimer! : We do not predict the location of the critical point**

# Thank you for listening !

# Back up!

# Important relations

## Relationship of TExS with BEST Mapping

BEST COLLABORATION

$$\mu_{BC}, T_C, \alpha'_{12}, w', \rho' \longrightarrow \mu_{BC}, T_C, \alpha_1, \alpha_2, w, \rho$$

6 parameters

$$\tan \alpha'_{12} = \tan \alpha_1 - \tan \alpha_2, \qquad \rho' = \rho \frac{\cos^2 \alpha_1}{\sqrt{(\cos \alpha_1 \cos \alpha_2)^2 + (\sin \alpha_{12})^2}} \qquad w' = w \frac{1}{\cos \alpha_1} \sqrt{(\cos \alpha_1 \cos \alpha_2)^2 + (\sin \alpha_{12})^2}$$

*[M. K et al arXiv:2402.08636v1]*

*[ Parotto et al PhysRevC.101.034901(2020)]*

## Strength of the discontinuity

leading singular behavior of specific heat at constant pressure $cp$

$$cp = T^3 \left( \frac{(s_c/n_c)\sin \alpha_1 - \cos \alpha_1}{w \sin \alpha_{12}} \right)^2 G_{hh} \left(1 + \mathcal{O}(r^{\beta\delta-1})\right)$$

$w \sin \alpha_{12}$ -Controls the strength of the jump

$G_{hh}$ – order parameter in Ising Model

# Lattice data: Parametrization

- To have a smooth temperature description from $25$ MeV $< T < 800$ MeV,
  We parameterize lattice data and merge with HRG

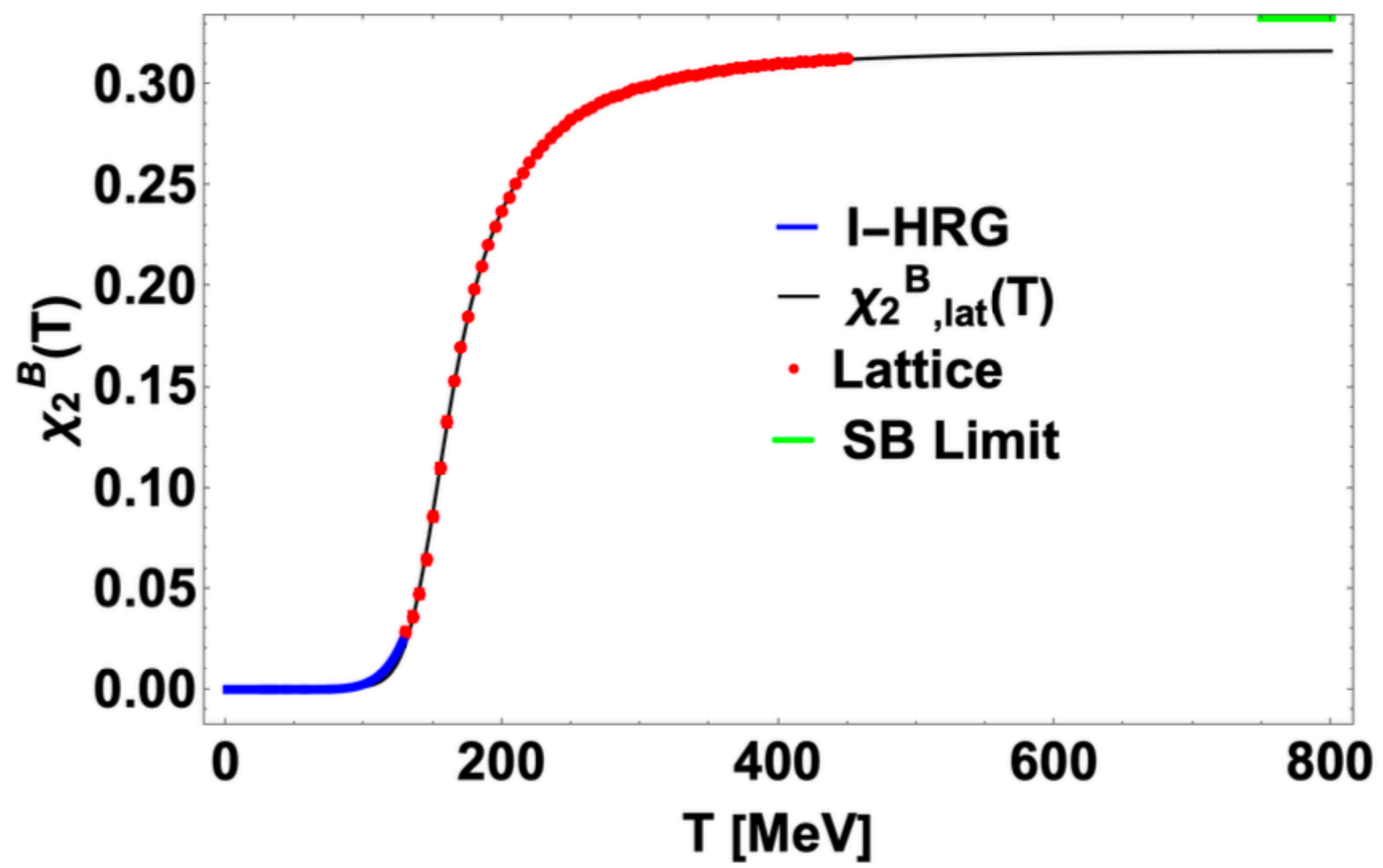
$$\chi^B_{2,\mathrm{lat}}(T) = \left(\frac{2m_p}{\pi x}\right)^{3/2} \frac{e^{-m_p/x}}{1+\left(\frac{x}{d_1}\right)^{d_2}} + d_3 \frac{e^{-d_4^2/x^2 - d_5^4/x^4}}{1+\left(\frac{x}{d_1}\right)^{-d_2}}$$

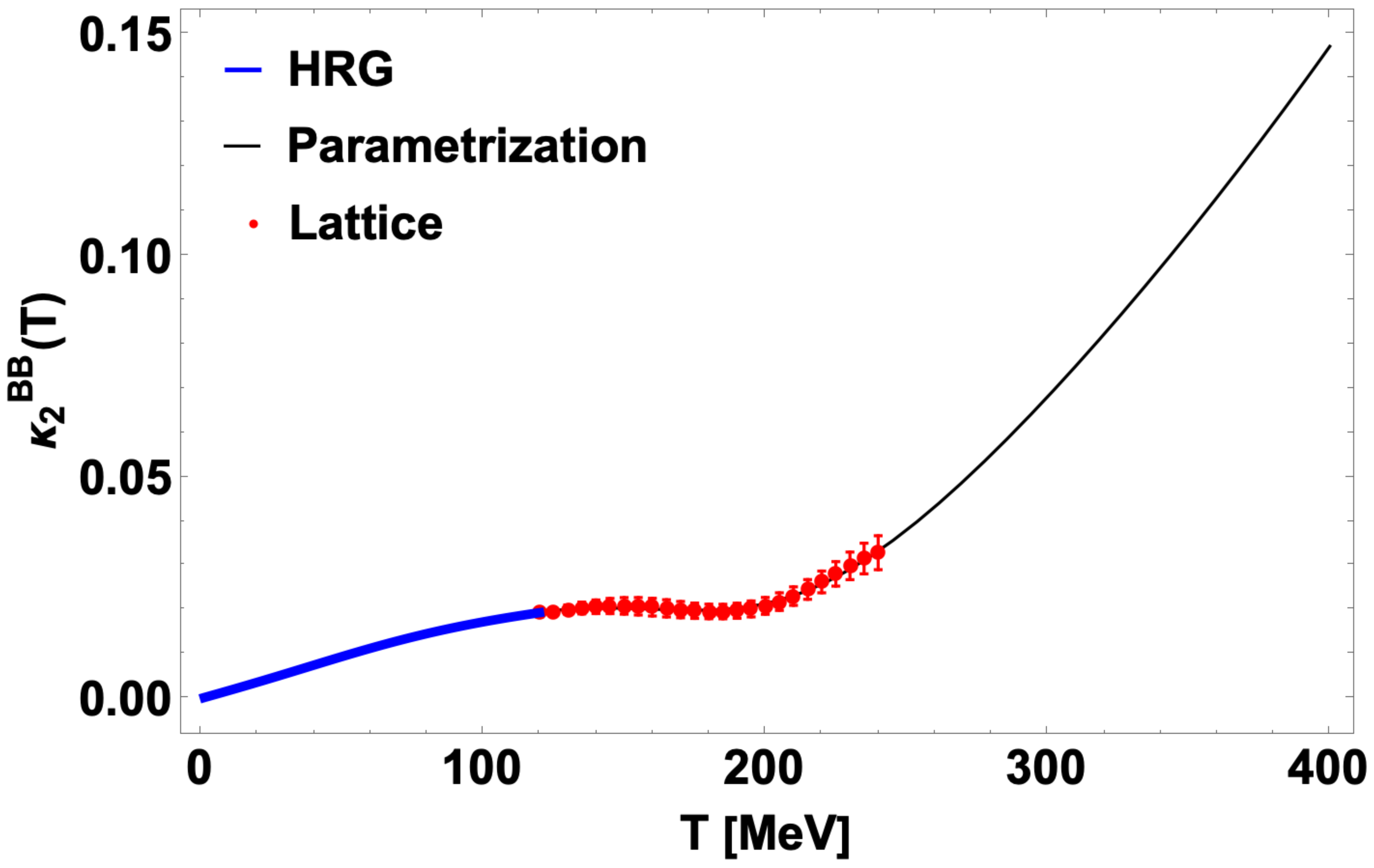
$x = \frac{T}{200 \text{ MeV}}$ $d_i$ - fitting parameters

$m_p$ - proton mass (in units of 200 MeV)

$$\kappa_2^{BB}(T) = \frac{a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5}{b_0 + b_1 x + b_2 x^2 + a_5/A\, x^3}$$

$x = \frac{T}{200 \text{ MeV}}$ $d_i$ - fitting parameters

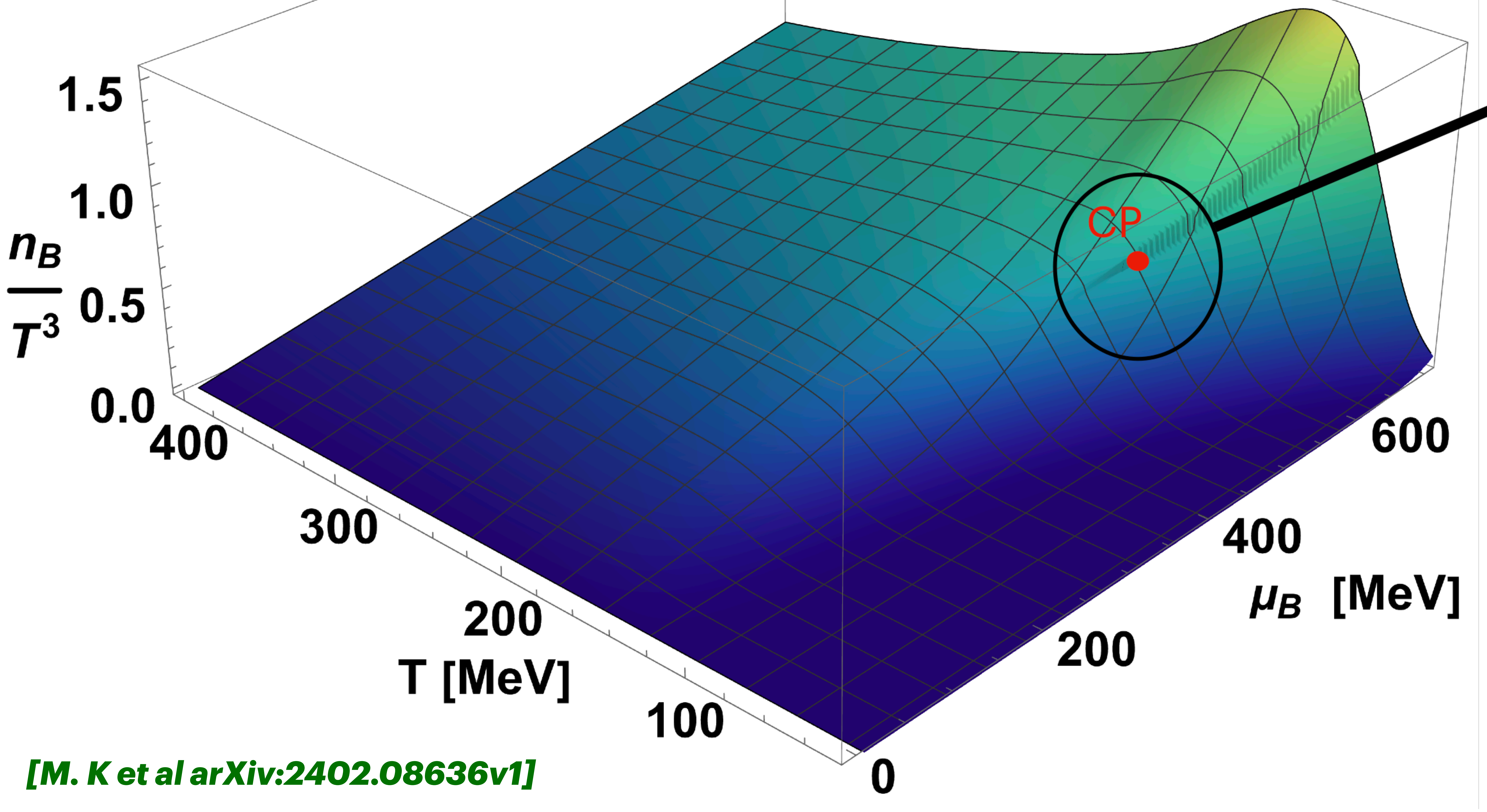


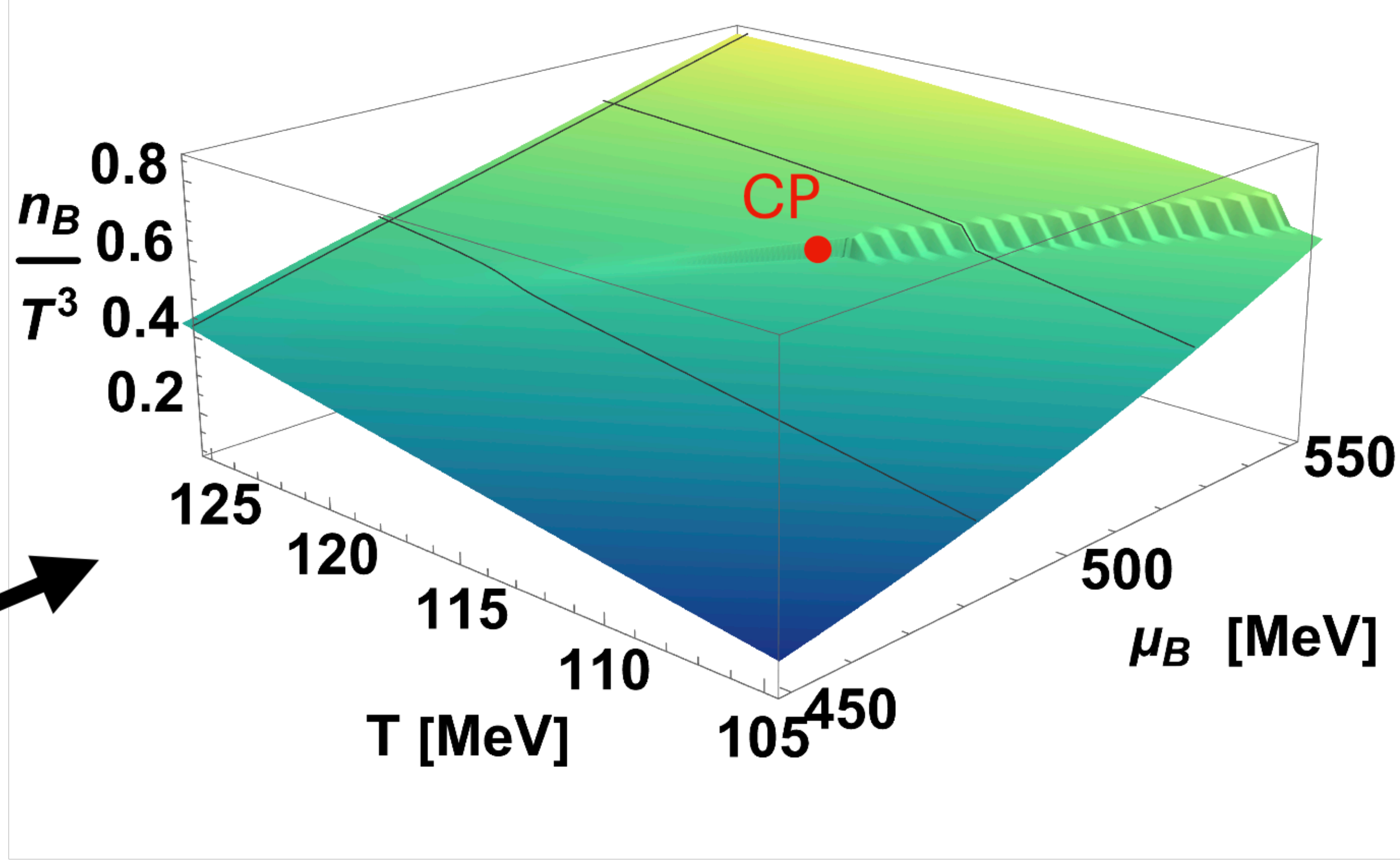




[M. K et al arXiv:2402.08636v1]

# Thermodynamic Observables

## Baryon Density $n_B/T^3$




**Parameter choice**

$\mu_{BC} = 500$ MeV

$T_C = 117$ MeV

$\alpha_{12} = \alpha_1 = 11^0$

$\alpha_2 = 0^0$

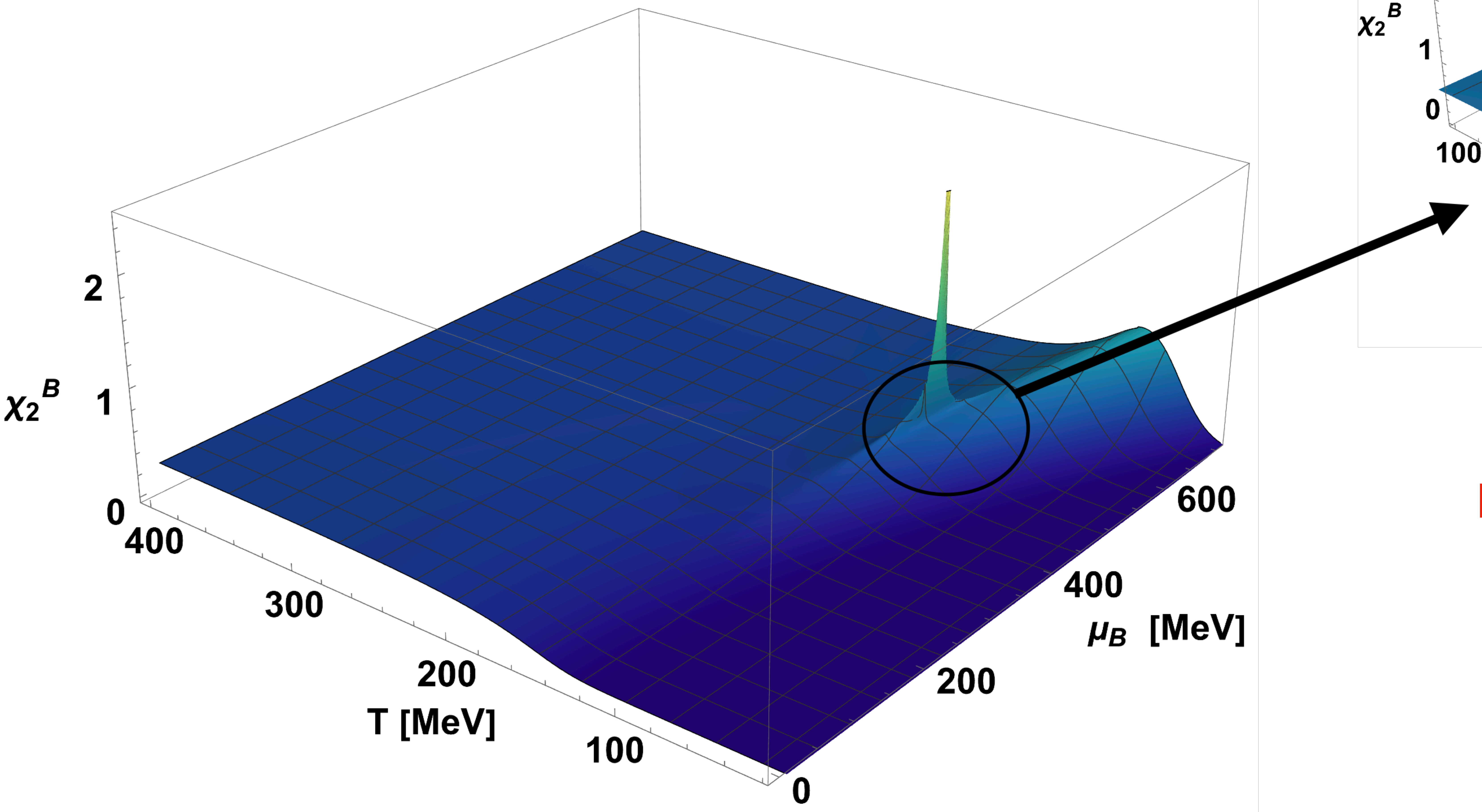
$w = 15$

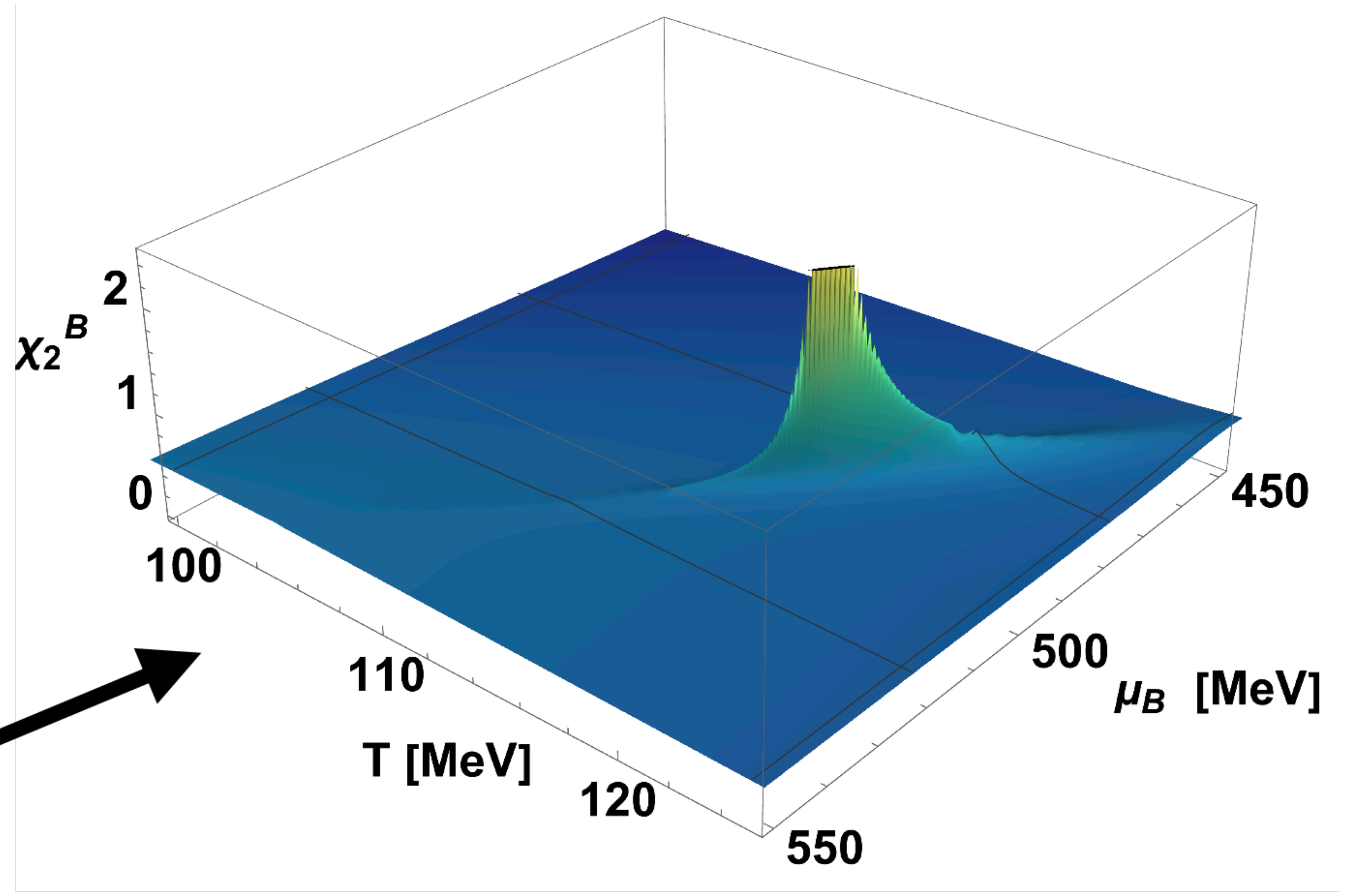
$\rho = 0.3$

*[M. K et al arXiv:2402.08636v1]*

# Thermodynamic Observables

## Baryon number susceptibility $\chi_2^B$

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