Bridging Nuclear and Quark Matter: Quantum van der Waals Approach to Quarkyonic Transition

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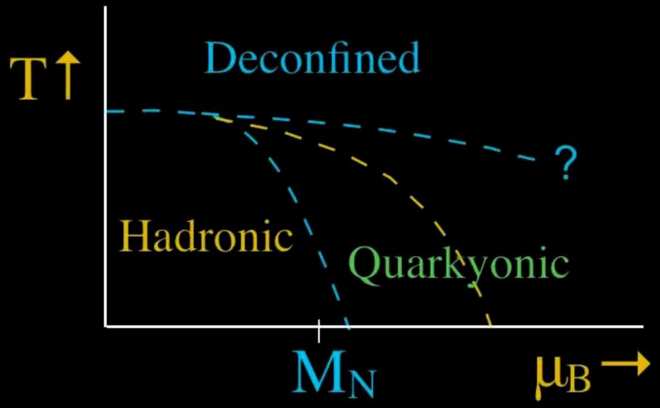
Plan

- Quarkyonic matter (quasiparticle, quark-hadron duality, baryquark)
- Theory of baryon-baryon interactions
- Results
- Applications to neutron star phenomenology

Quarkyonic phase

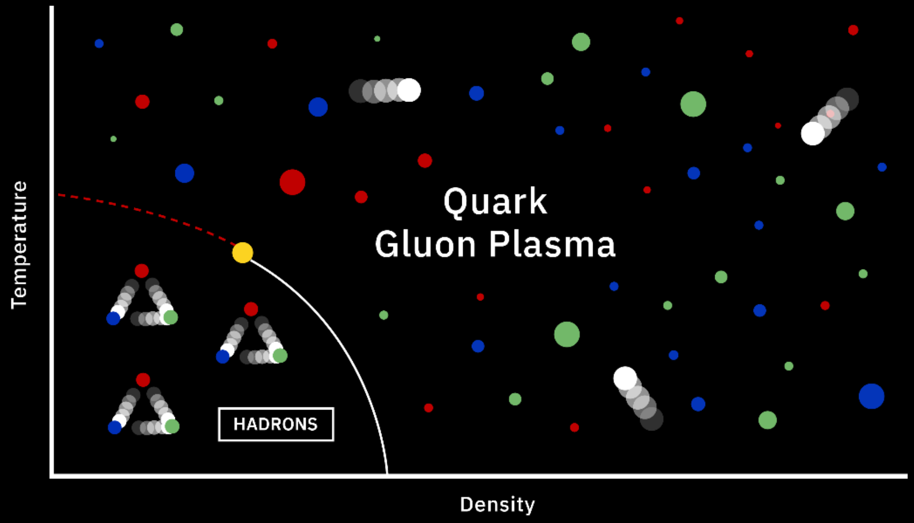


[McLerran, Pisarski, Nucl.Phys.A, 2007]

- Is still confined (color neutrality, quarks are not free) although very dense.
- Density of quarks exceed energy scale of confining interactions ($n_q \gg \Lambda^3_{QCD}$)
- (Confined) quarks become natural degrees of freedom.
- Pauli exclusion principle between individual quarks starts to play a significant role.
- Inspired by limit of large N_c of QCD where confinement persists.

Possible quarkyonic phase would have properties different from both hadronic matter and QGP

Deconfined phase (Quark matter)



[Big Think / Ben Gibson]

- Degrees of freedom are quarks, Screening of color charges, Polyakov loop expectation value in non-zero

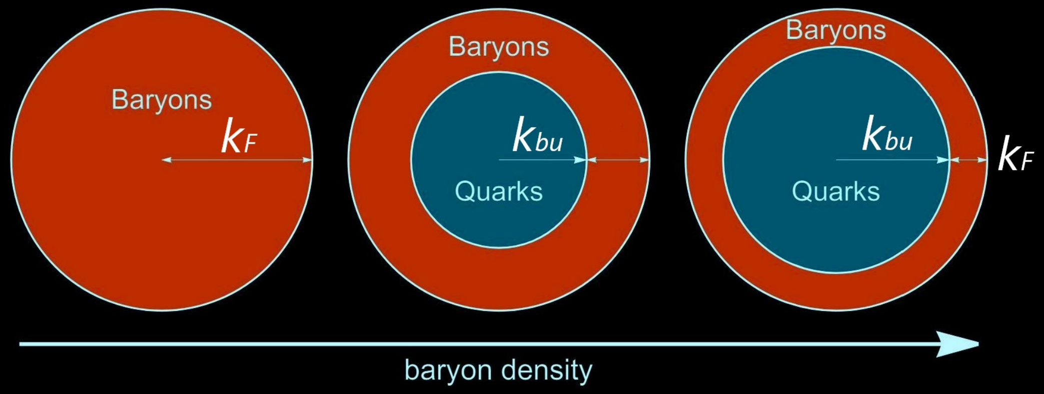
The density of the onset of deconfinement is unknown

Realization of quarkyonic matter at T=0

Quasiparticle picture

Postulates "free" quarks and triplets of "confined" quarks (nucleons) which Pauli exclude each other.
Nucleons consist of N_c quarks, $m_q = m_N/N_c$, $N_c=3$.

Shell structure in momentum space



$$n_N^{\rm id}(k_F) = \frac{g}{2\pi^2}\int_{k_{\rm bu}}^{k_F} k^2\, dk,$$

$$\varepsilon_N^{\rm id}(k_F) = \frac{g}{2\pi^2}\int_{k_{\rm bu}}^{k_F} k^2\, \epsilon_N(k) dk,$$

$$n_Q^{\rm id}(q_F) = \frac{g}{2\pi^2}\int_{0}^{k_{\rm bu}/N_c} q^2\, dq,$$

$$\varepsilon_Q^{\rm id}(q_F) = N_c\frac{g}{2\pi^2}\int_{0}^{k_{\rm bu}/N_c} q^2\, \epsilon_Q(q) dq.$$

$$g = g_Q = g_N = 4$$

$f_q = n_q/n_B$ at each n_B is found by minimizing the energy density.

Infrared regulator limits the abruptness of quark onset:

$$\rho_Q(q) \to \rho_Q(q)\frac{\sqrt{q^2+\Lambda^2}}{q}$$

[McLerran, Pisarsky, Nucl.Phys.A, 2007]
[McLerran, Reddy, PRL, 2019]
[Kie Sang Jeong, McLerran, Sen, PRC, 2020]

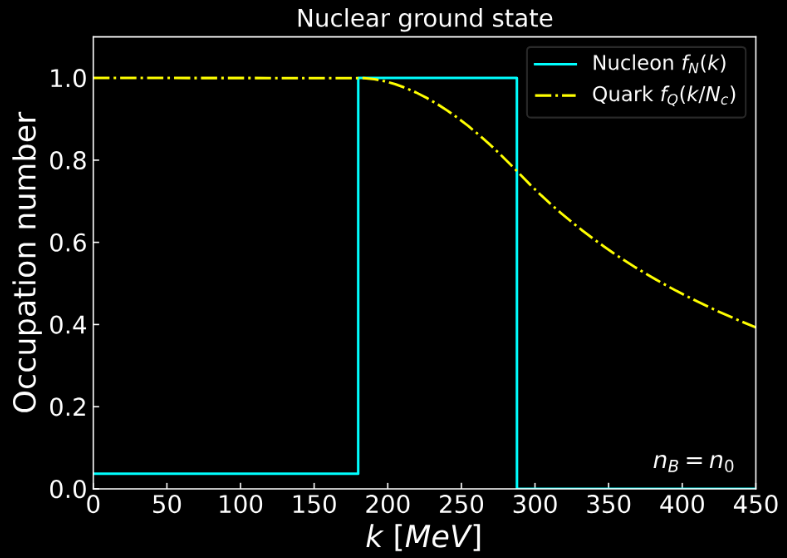
Realization of quarkyonic matter at T=0

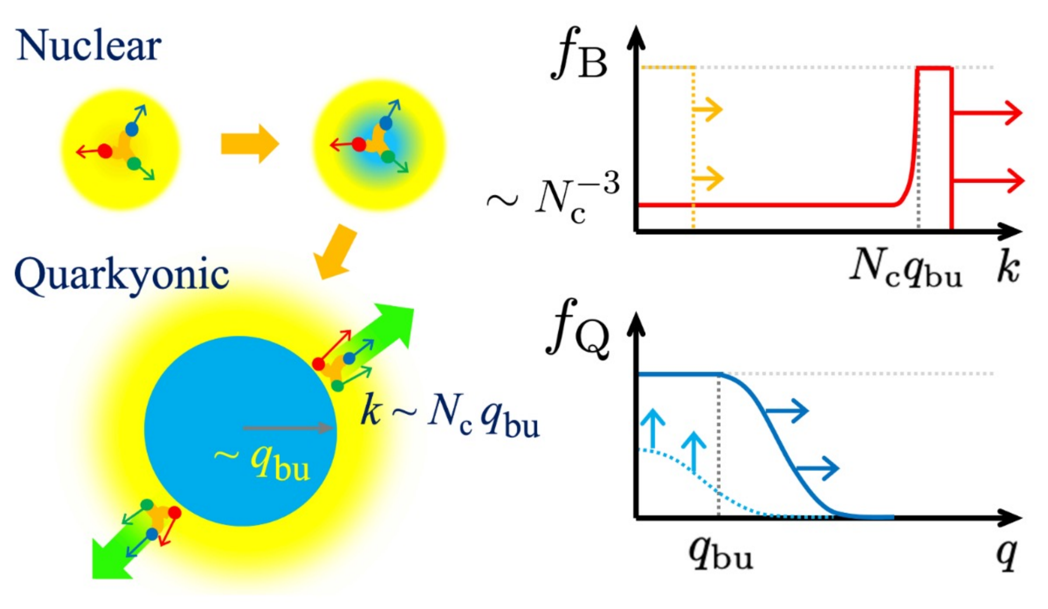
Dual description

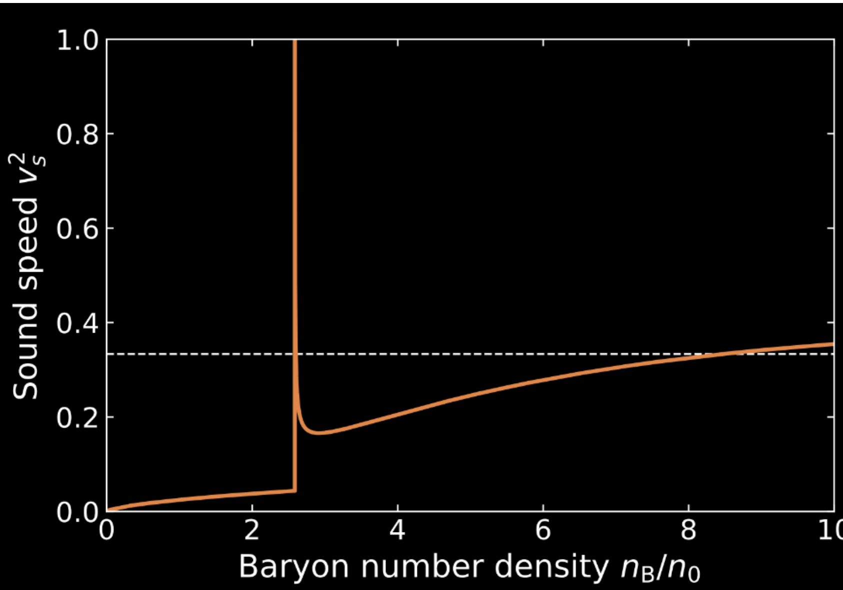
$$n_{\rm B} = 4\int_k f_{\rm B}(k) = 4\int_q f_{\rm Q}(q)$$

$$\varepsilon = \varepsilon_{\rm B}[f_{\rm B}]\big|_{n_{\rm B}} = \varepsilon_{\rm Q}[f_{\rm Q}]\big|_{n_{\rm B}}$$

$$[f_{\rm Q}(q)]_{f\sigma} = \sum_{i=n,p,\cdots}\sum_{\sigma'=\uparrow,\downarrow}\int_k \left[\varphi\left(\boldsymbol{q}-\frac{\boldsymbol{k}}{N_c}\right)\right]^{i\sigma'}_{f\sigma}[f_{\rm B}(k)]_{i\sigma'}$$







[Kojo,PRD,2021]

[Fujimoto, Kojo, McLerran, PRL, 2024]

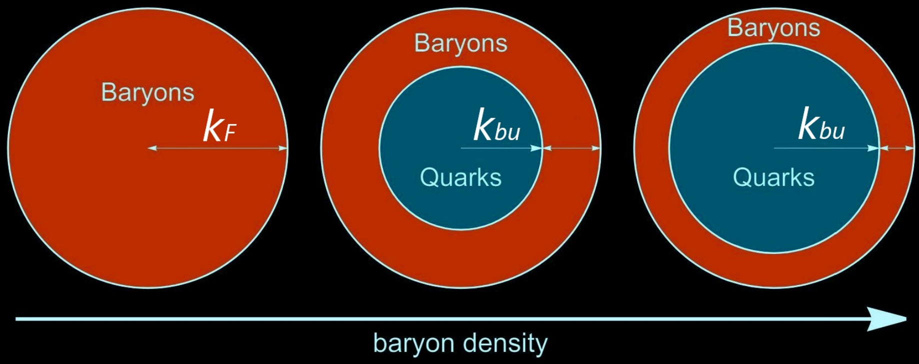
[Koch, McLerran, Miller, Vovchenko, 2403.15375, 2024]

Realization of quarkyonic matter at T=0

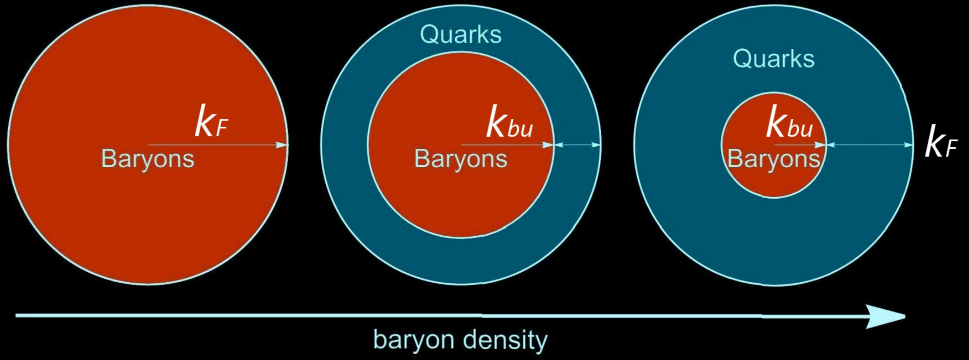
Baryquark matter

Shell structure in momentum space

(a) Quarkyonic



(b) Baryquark



$f_q = n_q/n_B$ at each n_B is found by minimizing the energy density.

$$n_N^{\mathrm{id}}(k_F) = \frac{g}{2\pi^2}\int_0^{k_{\mathrm{bu}}} k^2\, dk,$$

$$\varepsilon_N^{\mathrm{id}}(k_F) = \frac{g}{2\pi^2}\int_0^{k_{\mathrm{bu}}} k^2\, \epsilon_N(k) dk,$$

$$n_Q^{\mathrm{id}}(q_F) = \frac{g}{2\pi^2}\int_{k_{\mathrm{bu}}/N_c}^{k_{\mathrm{F}}/N_c} q^2\, dq,$$

$$\varepsilon_Q^{\mathrm{id}}(q_F) = N_c\frac{g}{2\pi^2}\int_{k_{\mathrm{bu}}/N_c}^{k_{\mathrm{F}}/N_c} q^2\, \epsilon_Q(q) dq.$$

Does not require infrared regulator

The most energetically favorable configuration

However, not yet consistent with dual approach

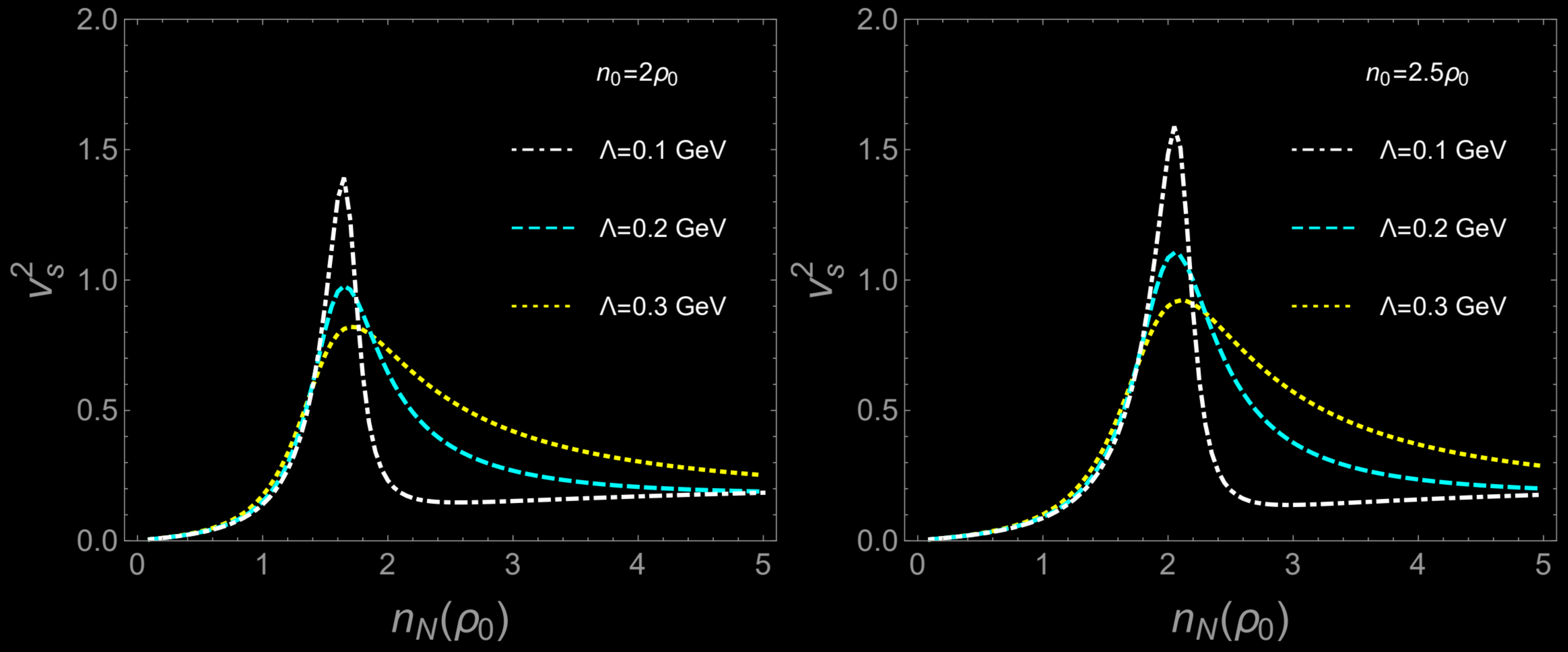
[Koch, Vovchenko, PLB, 2023]

Quarkyonic matter: baryon excluded volume repulsion

$$n_{ex}^{N} = \frac{n_N^N}{1 - n_N^N/n_0}$$

b ≡ 1/n₀ is a free parameter

The role of attraction between baryons is unclear



[Kie Sang Jeong, McLerran, Sen, PRC, 2020]

Including nucleon-nucleon interactions: Quantum van der Waals theory

QvdW interactions

particle density: $n = f(n)\, n^*_{\rm id}$

energy density: $\varepsilon = f(n)\, \varepsilon^*_{\rm id} + n\, u(n)$

excl. volume factor: $f(n) = 1 - bn$

attractive mean-field: $u(n) = -an$

b – repulsion, *a* – attraction

Ground state of nuclear matter

$$p = 0,\ \varepsilon / n_B \approx 922\ \text{MeV},$$

$$n_B = \rho_0 \approx 0.16\ \text{fm}^{-3}$$

$$\Rightarrow a, b$$

[Vovchenko, Anchishkin, Gorenstein, PRC, 2015]

Modifications to the excluded volume

Carnahan-Starling (CS):

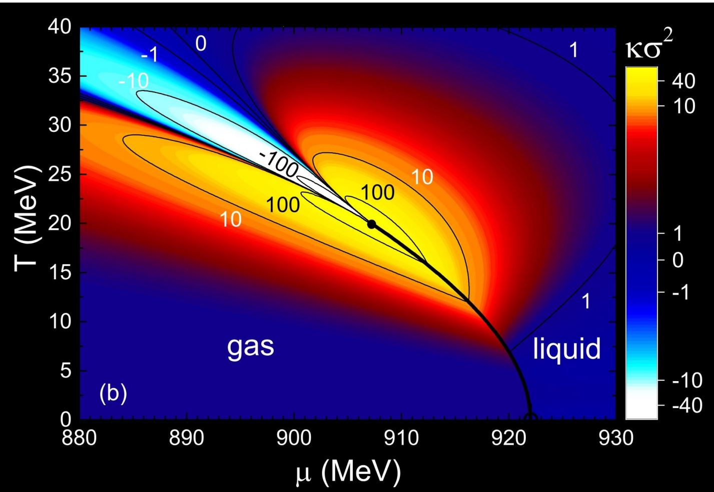
$$f_{\rm CS}(n) = exp\left[-\frac{3bn}{4-bn} - \frac{4bn}{(4-bn)^2}\right]$$

[Vovchenko, PRC, 2017]

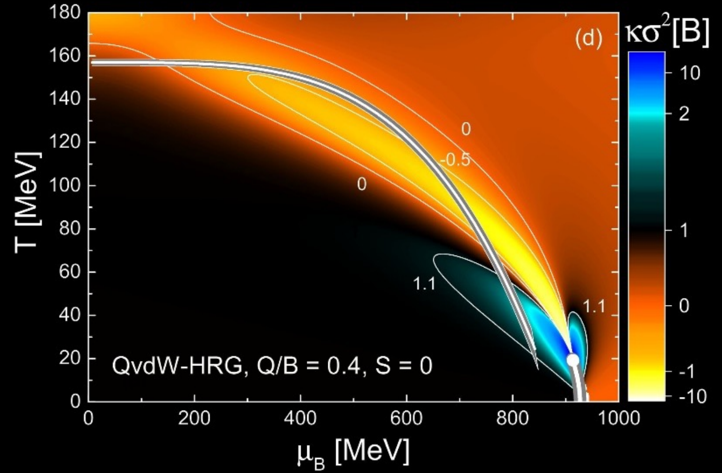
Trivirial Model (TVM):

$$f_{\rm TVM}(n) = exp\left[-bn - \frac{b^2 n^2}{2}\right]$$

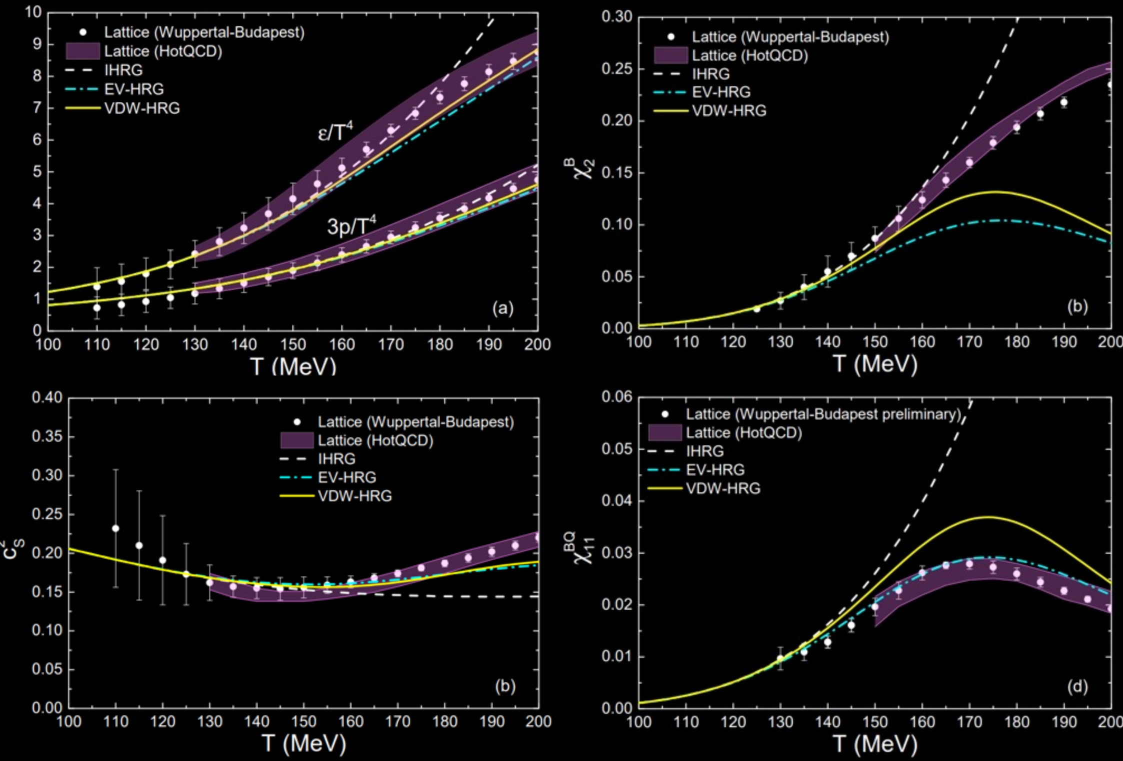
[Vovchenko, Greiner, Koch, Stoecker, PRD, 2020]



[Vovchenko, Anchishkin, Gorenstein, RP, PRC, 2015]

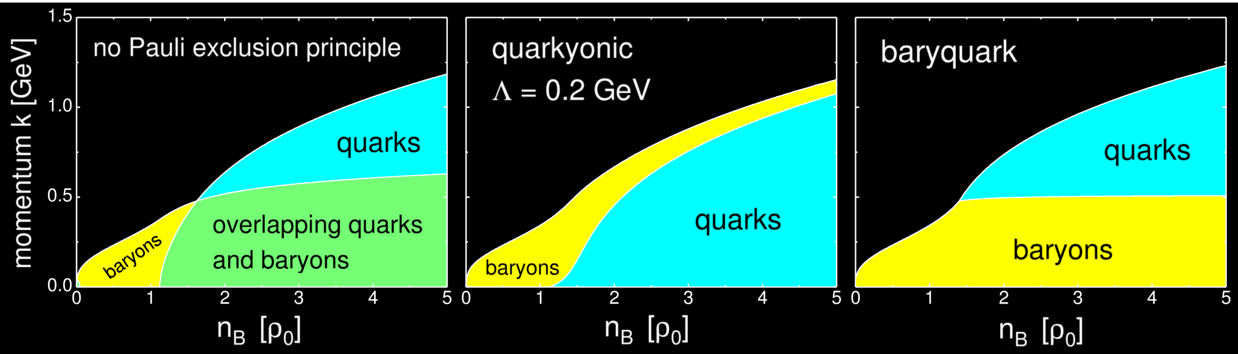


[RP,Vovchenko, Motornenko, Gorenstein, Stoecker, PRC, 2019]



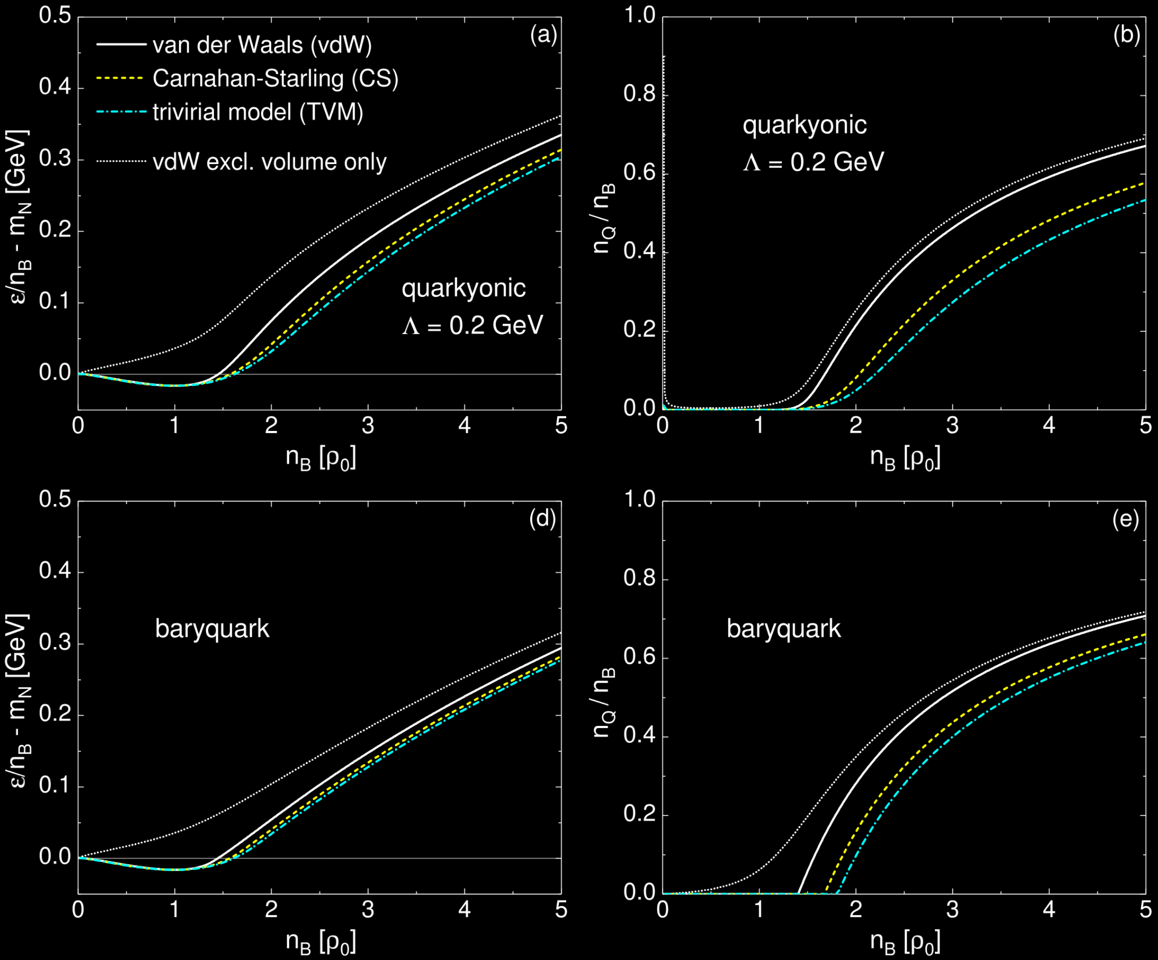
[Vovchenko, Gorenstein, Stoecker, PRL, 2017]

Quarkyonic matter with QvdW interactions for nucleons at T = 0



← matter composition in the momentum space

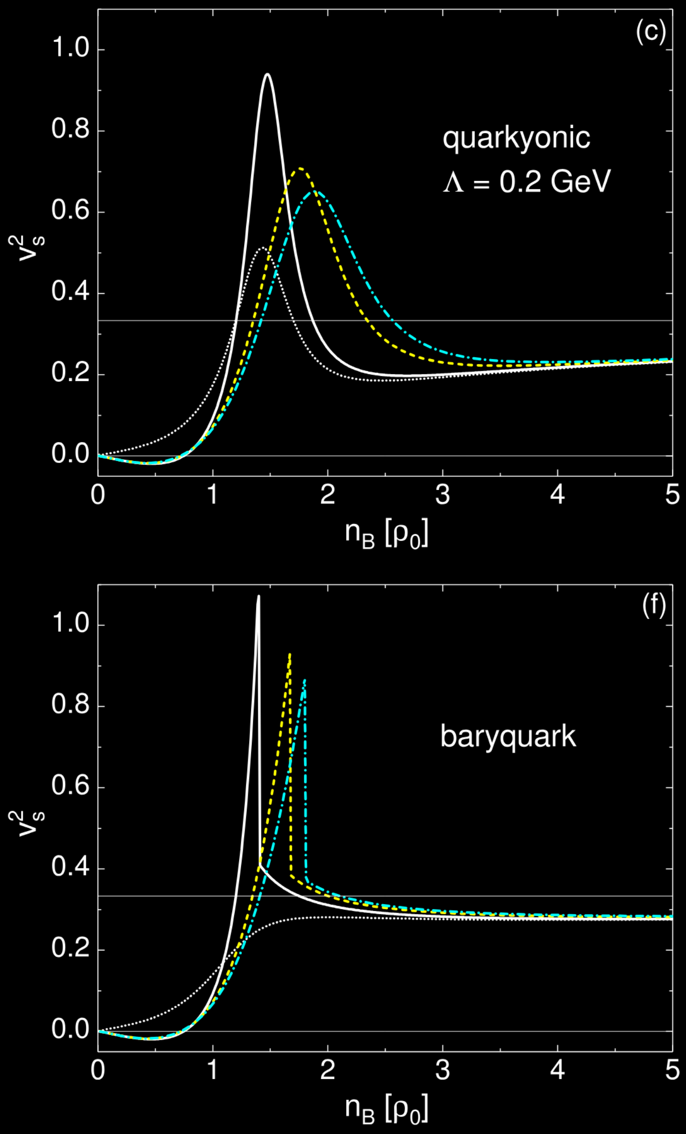
$$n_N = f(n_N)\, n_N^{\mathrm{id}}(k_F),$$
$$\varepsilon_N = f(n_N)\, \varepsilon_N^{\mathrm{id}}(k_F) + n_N\, u(n_N)$$



- The transition to quarkyonic regime takes place at $n_B^{tr} = 1.5 - 2\ \rho_0$
- In contrast to the earlier works, n_B^{tr} does not depend on any free parameter but stems from empirical properties of the nuclear ground state.
- The obtained estimates suggest that quarkyonic matter can be reachable in intermediate energy HIC.
- The transition is qualitatively similar for all considered modifications of EV regardless of whether they have limiting density.
- Baryquark configuration is the most energetically favorable in all considered scenarios. The quark onset is sudden with a kink in $f_q(n_B)$. At the same time, the resulting EoS are very similar.

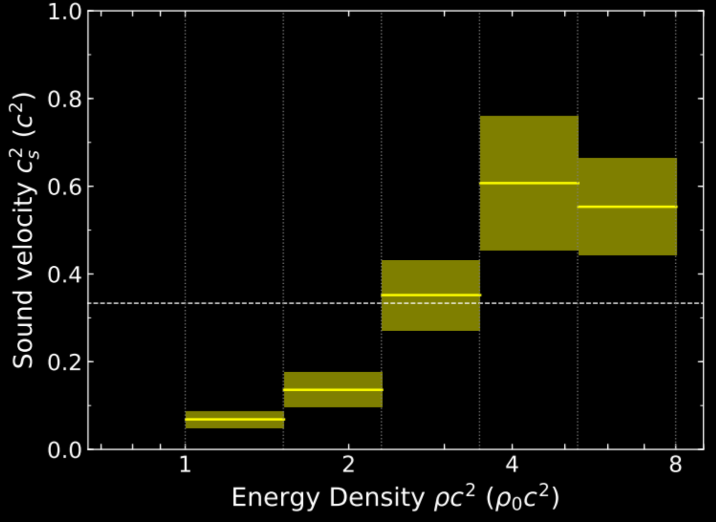
[RP, Stoecker, Vovchenko, PRC, 2023]

Quarkyonic matter with QvdW interactions for nucleons: sound velocity

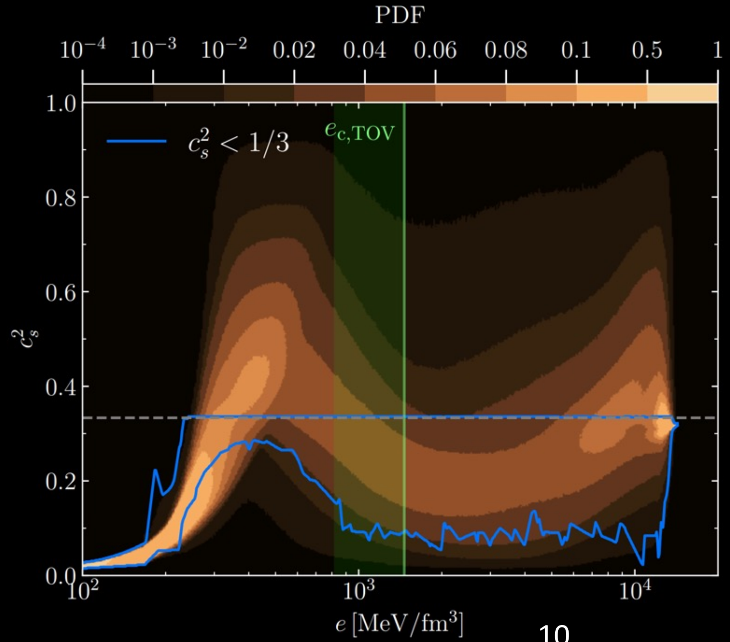


- In all considered cases the transition to quarkyonic regime results in the pronounced peak in sound velocity v_s^2 which rises well above the conformal limit.
- The presence of nuclear attraction and the associated GS makes the peak in v_s^2 considerably sharper.
- We confirm that quarkyonic matter with the excluded volume mechanism requires the introduction of a regulator (Λ) to avoid the singular behavior in v_s^2.
- For baryquark matter
  - no singular behavior is observed even in the absence of the regulator.
  - v_s^2 exhibits a discontinuous drop at the quark onset as a result of a kink.
  - v_s^2 is causal for all considered cases except of vdW-EV where it slightly overshoots unity.

[RP, Stoecker, Vovchenko, PRC, 2023]



[Fujimoto, Fukushima, Murase, PRD, 2020]

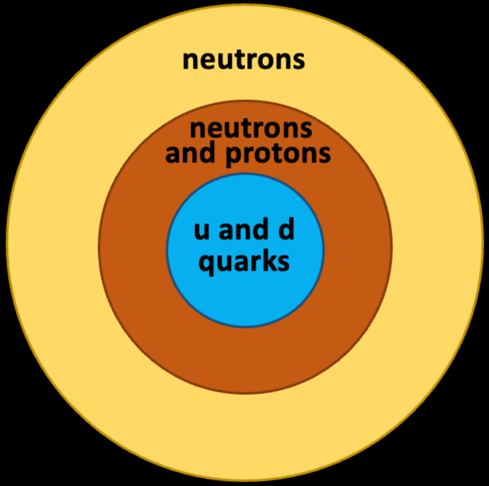


[Altiparmak, Ecker, Rezzolla, Astrophys.J.Lett., 2022]

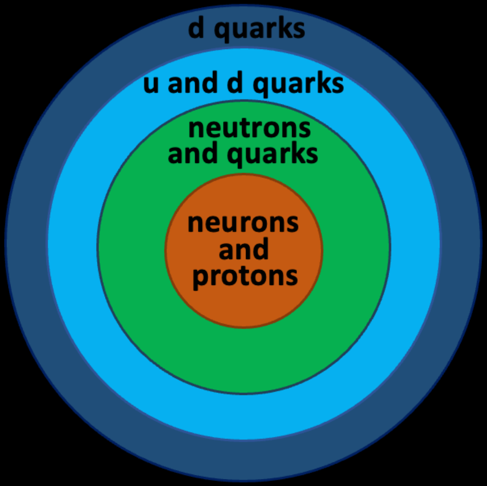
Further directions

- Application to neutron stars [Tripp, RP, Vovchenko, in progress]
- Quark-baryon interactions (Kie Sang Jeong)
- Dual description of quarkyonic matter
- Inclusion of heavier baryons and quark flavors
- Chiral symmetry restoration
- Extension to finite temperatures, see e.g. [Sen, Warrington, Nucl.Phys.A,2021]

Application to neutron stars: asymmetric matter (in progress)



quarkyonic



baryquark

Assumptions:

- In the case of QY matter u and d quarks share the same Fermi surface, $q^u_{\rm bu} = q^d_{\rm bu} = q_{\rm bu}$
- Asymmetry parameter:

$$y = \frac{\rho_Q}{\rho_B} = \frac{n_p}{n_p + n_n} = \frac{\frac{2}{3}n_u - \frac{1}{3}n_d}{\frac{1}{3}n_u + \frac{1}{3}n_d}$$

$$\boxed{\begin{gathered}\varepsilon_N = \sum_{i\in p,n} f_{\rm vdW}(x_i)\varepsilon_i^{\rm id} - a_n(n_p^2 + n_n^2) - 2a_{pn}n_pn_n \\ n_{p,n} = f_{\rm vdW}(x_{p,n})n_{p,n}^{\rm id}\end{gathered}}$$

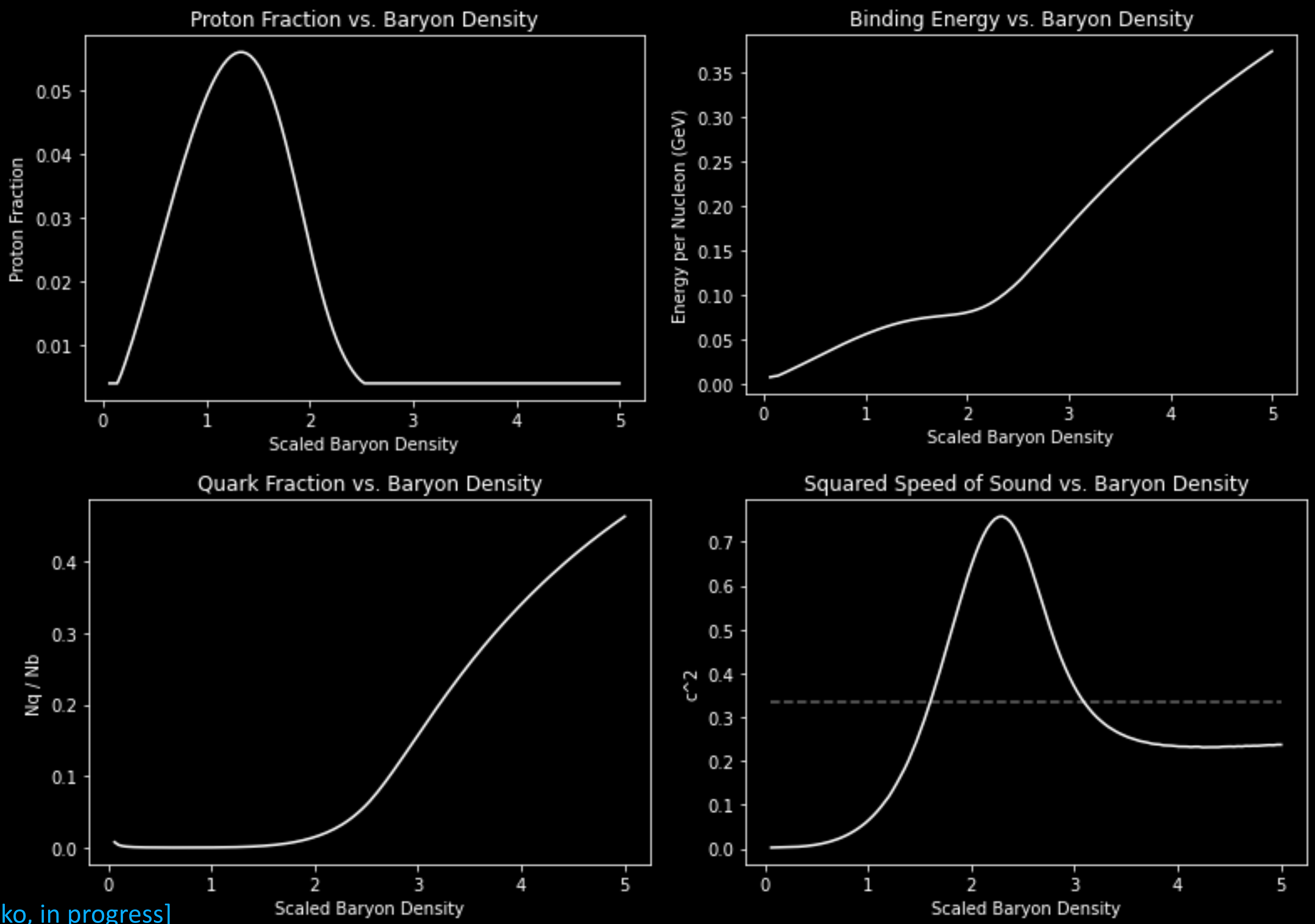
$$x_p = b_nn_p + b_{pn}n_n$$
$$x_n = b_{pn}n_p + b_nn_n$$

$$\boxed{\begin{aligned} f_{\rm vdW}(x) &= 1 - x \\ f_{\rm CS}(x) &= \exp\left[-\frac{3x}{4-x} - \frac{4x}{(4-x)^2}\right] \\ f_{\rm TVM}(x) &= \exp\left[-x - \frac{x^2}{2}\right].\end{aligned}}$$

$$\boxed{a_{pn}, a_n, b_{pn}, b_n} \Leftarrow \boxed{\begin{gathered}\frac{a_{pn} + a_n}{2} = a, \quad \frac{b_{pn} + b_n}{2} = b \\ J \approx 32 \text{ MeV}, \; L \approx 59 \text{ MeV}\end{gathered}}$$

[Tripp, RP, Vovchenko, in progress]

Application to neutron stars (preliminary)



Proton Fraction vs. Baryon Density

Binding Energy vs. Baryon Density

Quark Fraction vs. Baryon Density

Squared Speed of Sound vs. Baryon Density

[Tripp, RP, Vovchenko, in progress]

Summary

- Quarkyonic matter theory has been extended to include QvdW nucleon-nucleon interactions
- In contrast to previous works we incorporate the empirical properties of the nuclear matter
- The model predicts a transition to quarkyonic regime at densities around 1.5-2ρ_0 with speed of sound rising significantly above the conformal limit. It suggests that QY matter is reachable in HICs.
- Baryquark matter is more energetically favorable then quarkyonic matter for all considered scenarios and does not need to rely on free parameters
- There is a number of topics that can be studied in future works based on the present study (e.g. application to neutron star phenomenology)

	vdW	CS	TVM	Experiment
a [MeV fm^3]	329	347	349	–
b [fm^3]	3.42	4.43	4.28	–
ρ_0 [fm^{-3}]	0.16	0.16	0.16	0.15 ± 0.01
K_0 [MeV]	763	601	564	250 - 315
T_c [MeV]	19.7	18.6	18.3	17.9 ± 0.4
n_c [fm^{-3}]	0.072	0.070	0.069	0.06 ± 0.01