Finite-size scaling analysis of proton cumulants

Agnieszka Sorensen

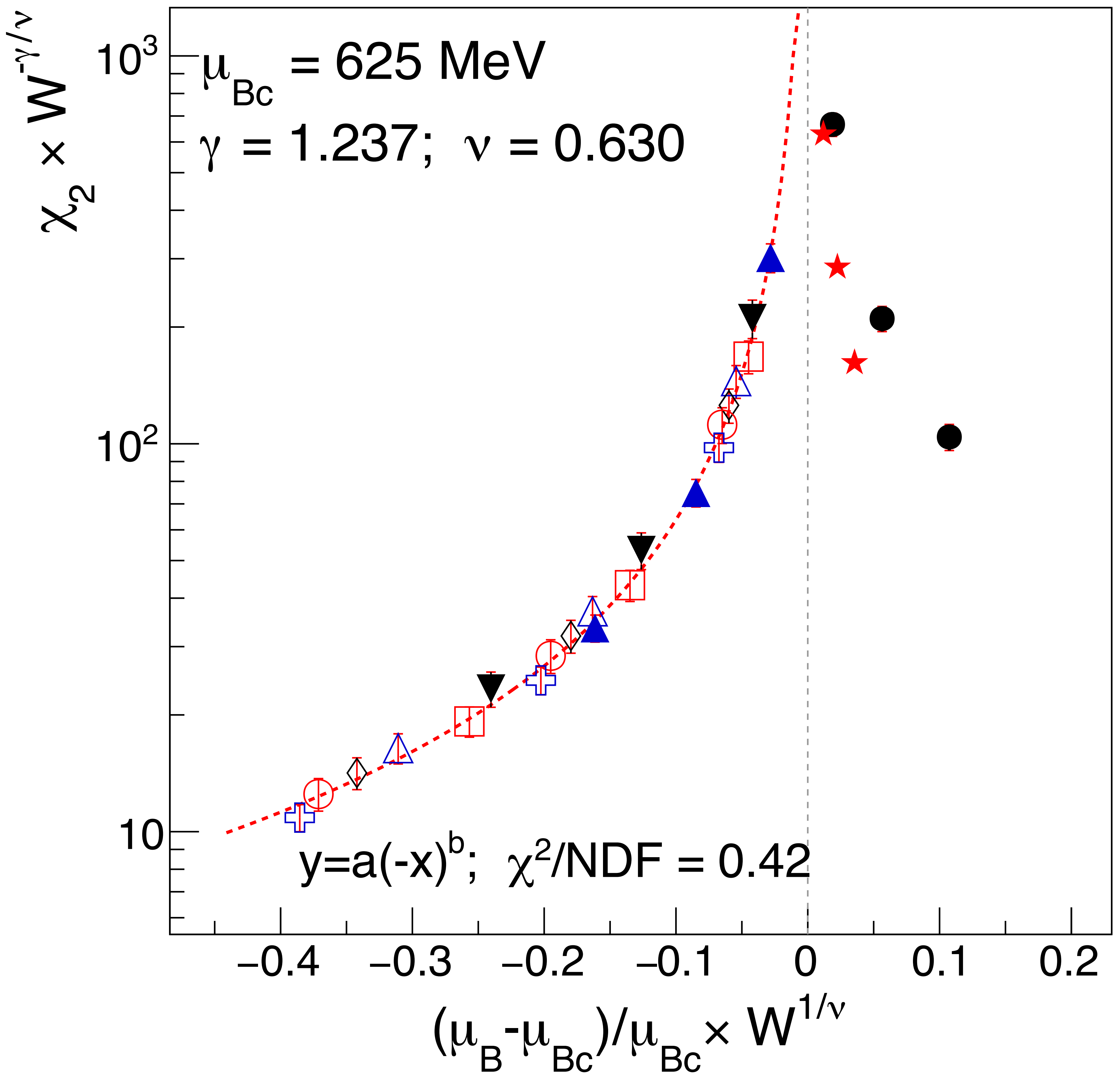
w/ Paul Sorensen

arxiv:2405.10278

INSTITUTE for NUCLEAR THEORY

Main result

- proton number susceptibilities scale with the size of the system: use finite-size scaling
- use rapidity bin widths as the system size
- extract the location of the CP



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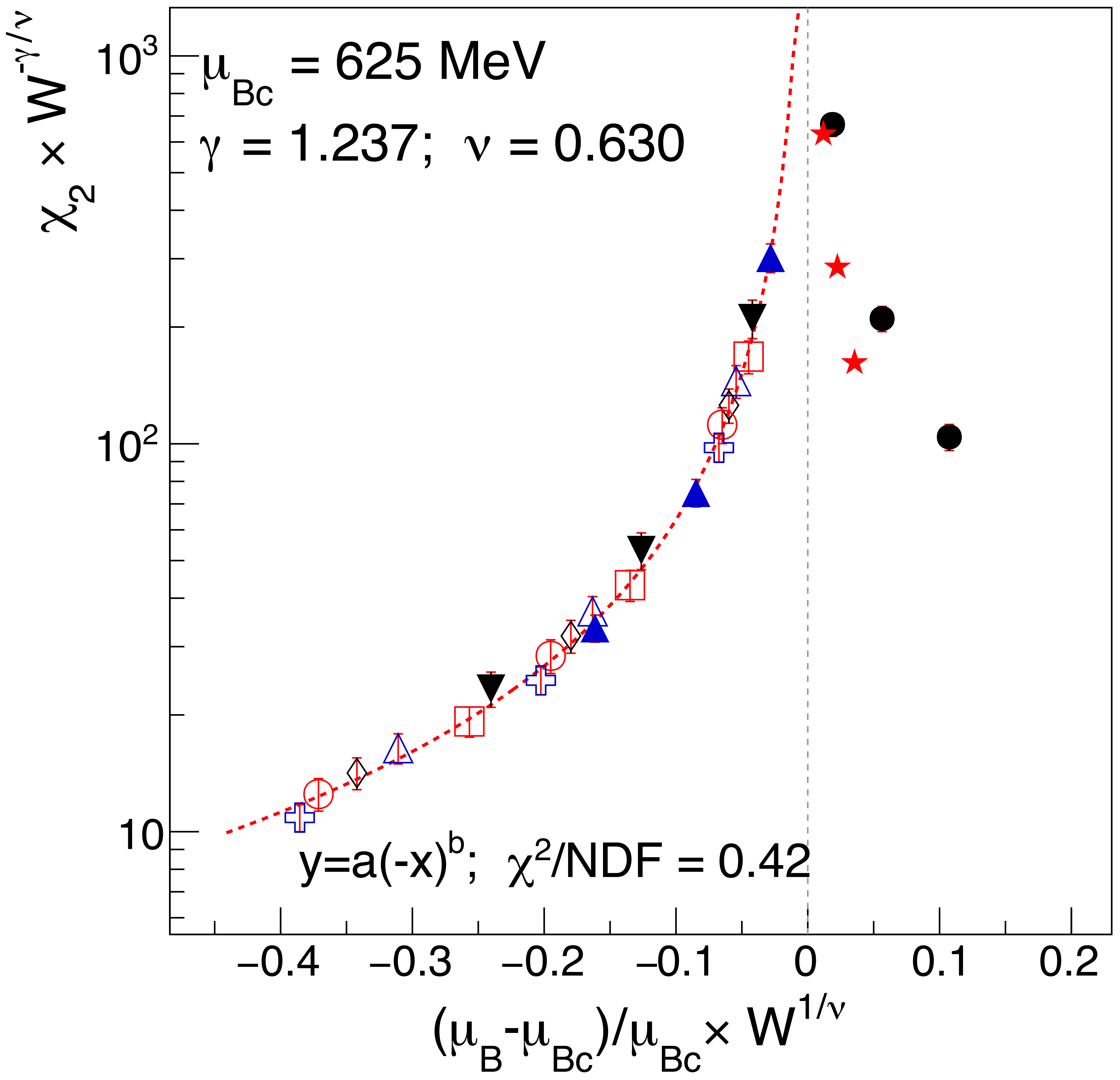
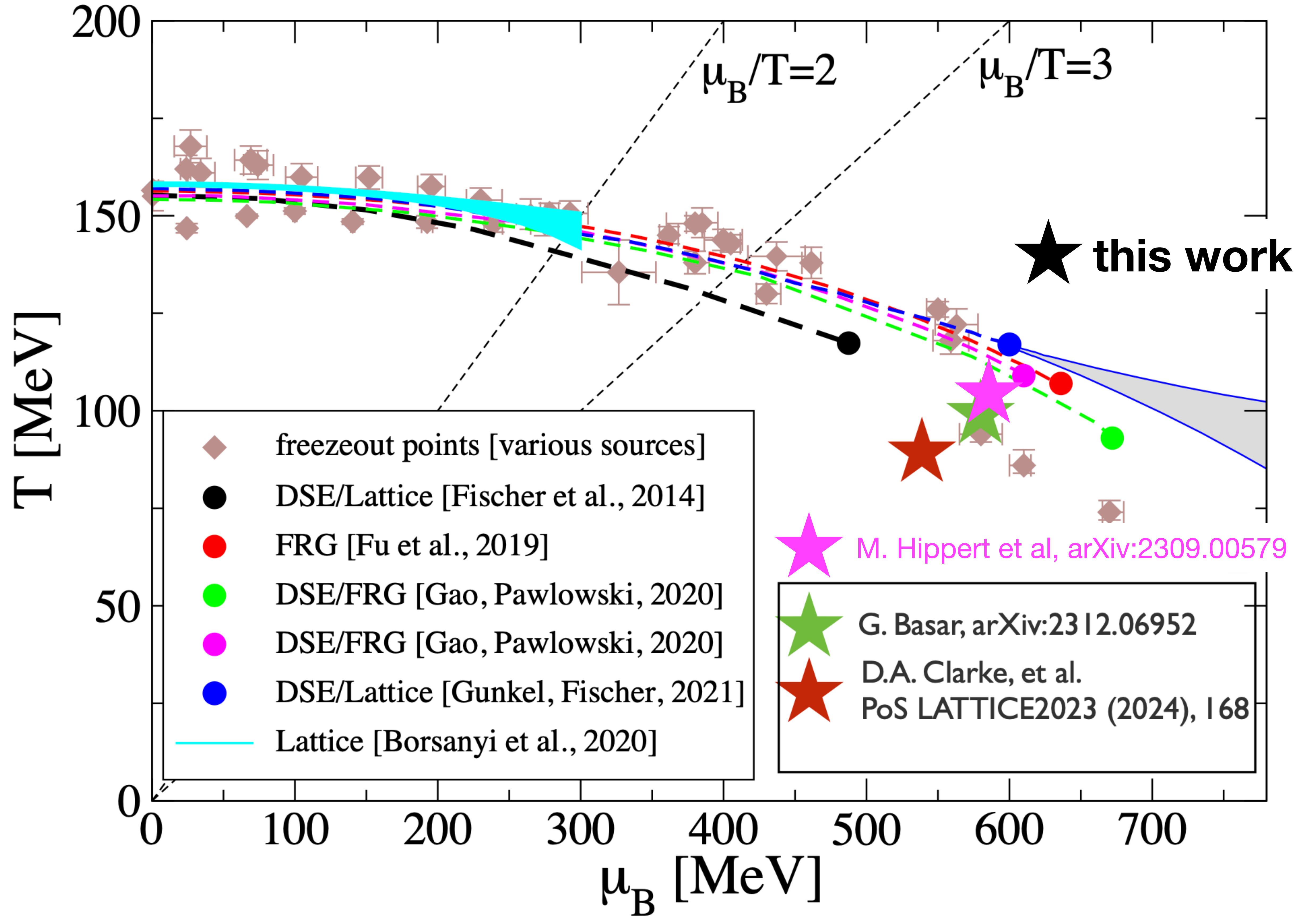


Christian Fischer's slides, CPOD 2024:



Introduction

Universal behavior

Near CP:

$$c_\infty(t,0) \sim |t|^{-\alpha}$$

$$\tilde{n}_\infty(t,0) \sim (-t)^{\beta}$$

$$\tilde{n}_\infty(0,m) \sim m^{\frac{1}{\delta}}$$

$$\chi_\infty(t,0) \sim |t|^{-\gamma}$$

$$\xi_\infty(t,0) \sim |t|^{-\nu}$$

$$\xi_\infty(0,m) \sim |m|^{-\nu_c}$$

$$t \equiv \frac{T - T_c}{T_c}$$

$$m \equiv \frac{\mu - \mu_c}{\mu_c}$$

For a thermodynamic quantity $X \sim |t|^{-\sigma}$: $$X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}}$$

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CP: infinite volume concept

In real world ξ does not go to infinity = thermodynamic functions do not exhibit singularities

ξ is bound by the size of the system L

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one can find CP by plotting

Finite size vs. window size

$$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

Finite-size scaling (original): change the size of the system, calculate $X_L(t_L)$, repeat

However: changing SIZE is not always possible or doesn't probe the same system (bird flocks, heavy-ions)

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D. Martin, T. Ribeiro, S. Cannas, *et al.*, Box scaling as a proxy of finite size correlations, Sci Rep 11, 15937 (2021)

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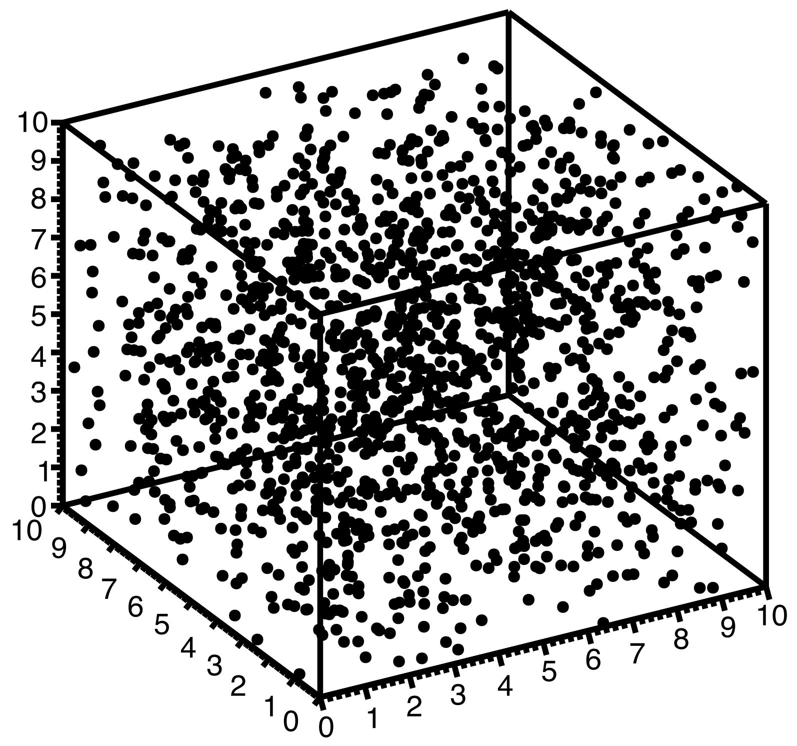
Does it work??

Tests in simulations

Finite-size scaling analysis of cumulants in a periodic box

- framework: hadronic transport with a *known* EOS (calculate κ_2/κ_1 "on paper")
- simulation: box with periodic boundary conditions at chosen (n_B, T)
- uniform initialization at $t = 0$, development of fluctuations in response to the EOS



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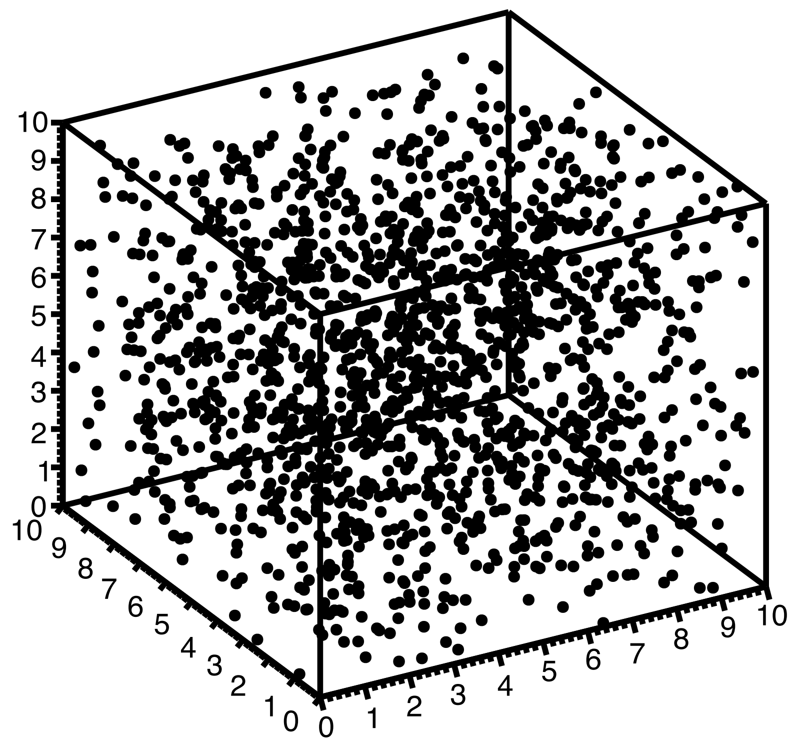
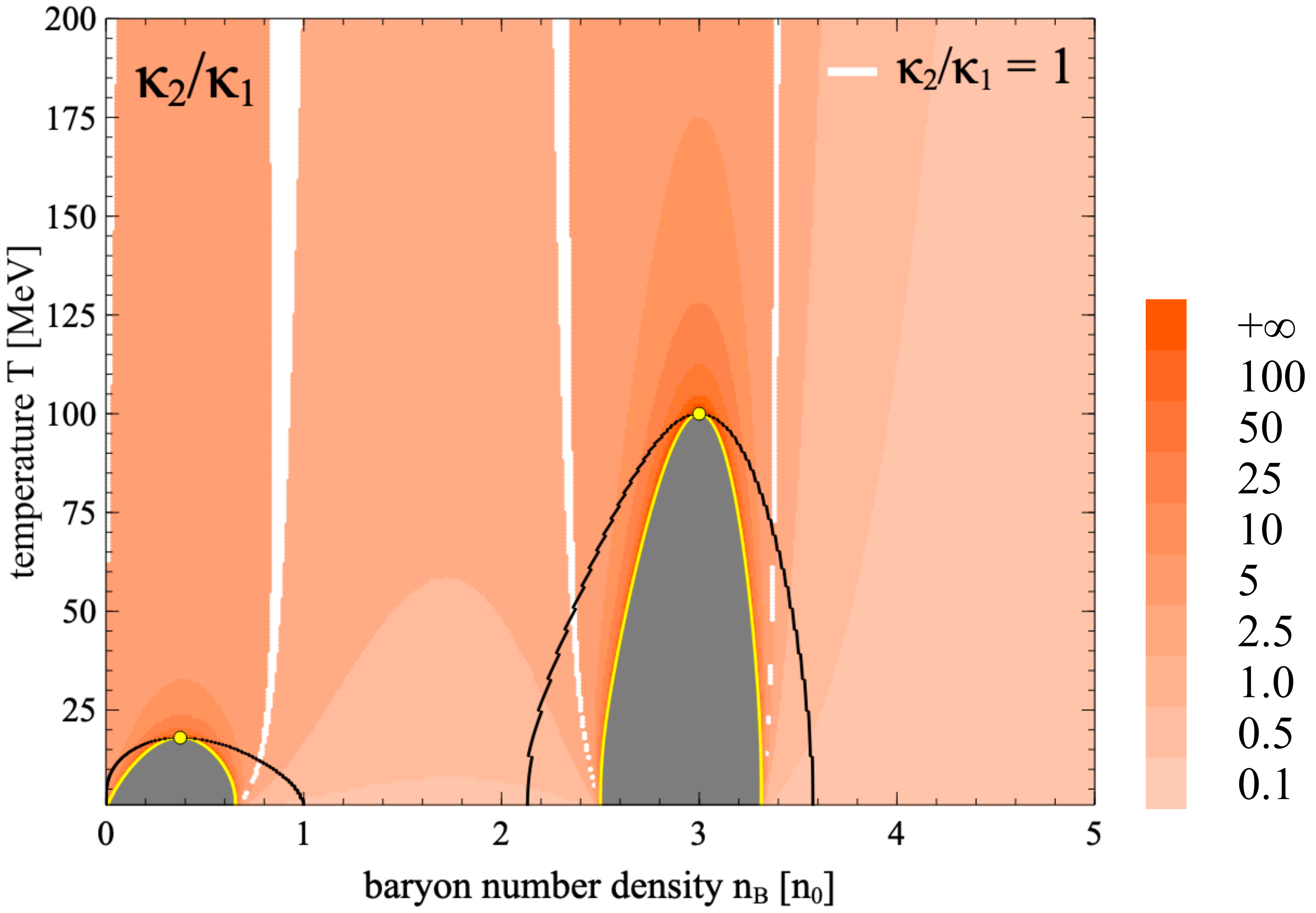


VDF EOS: relativistic polynomial w/ 2 phase transitions



AS and V. Koch, Phys. Rev. C **104**, 3, 034904 (2021), arXiv:2011.06635

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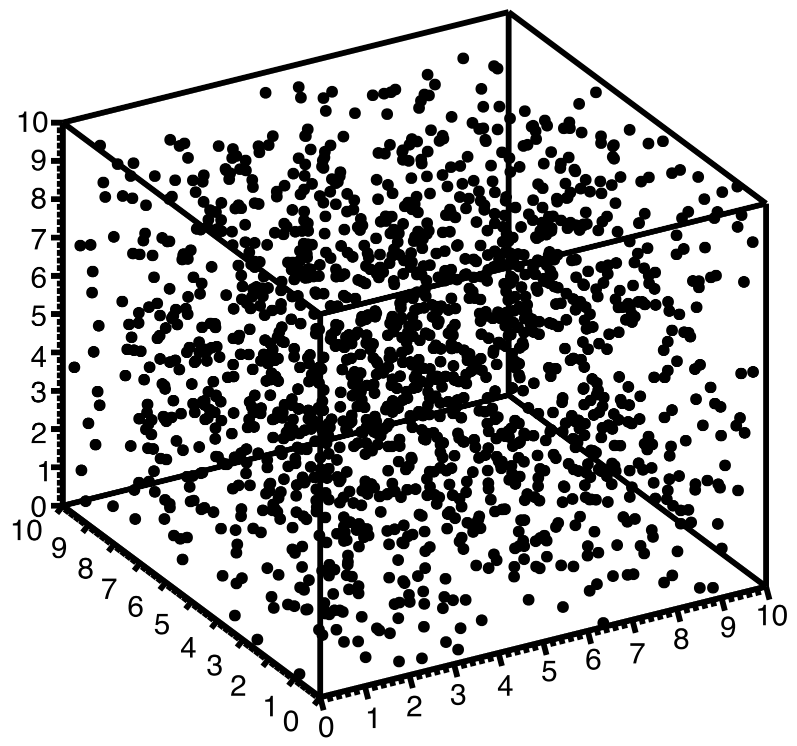

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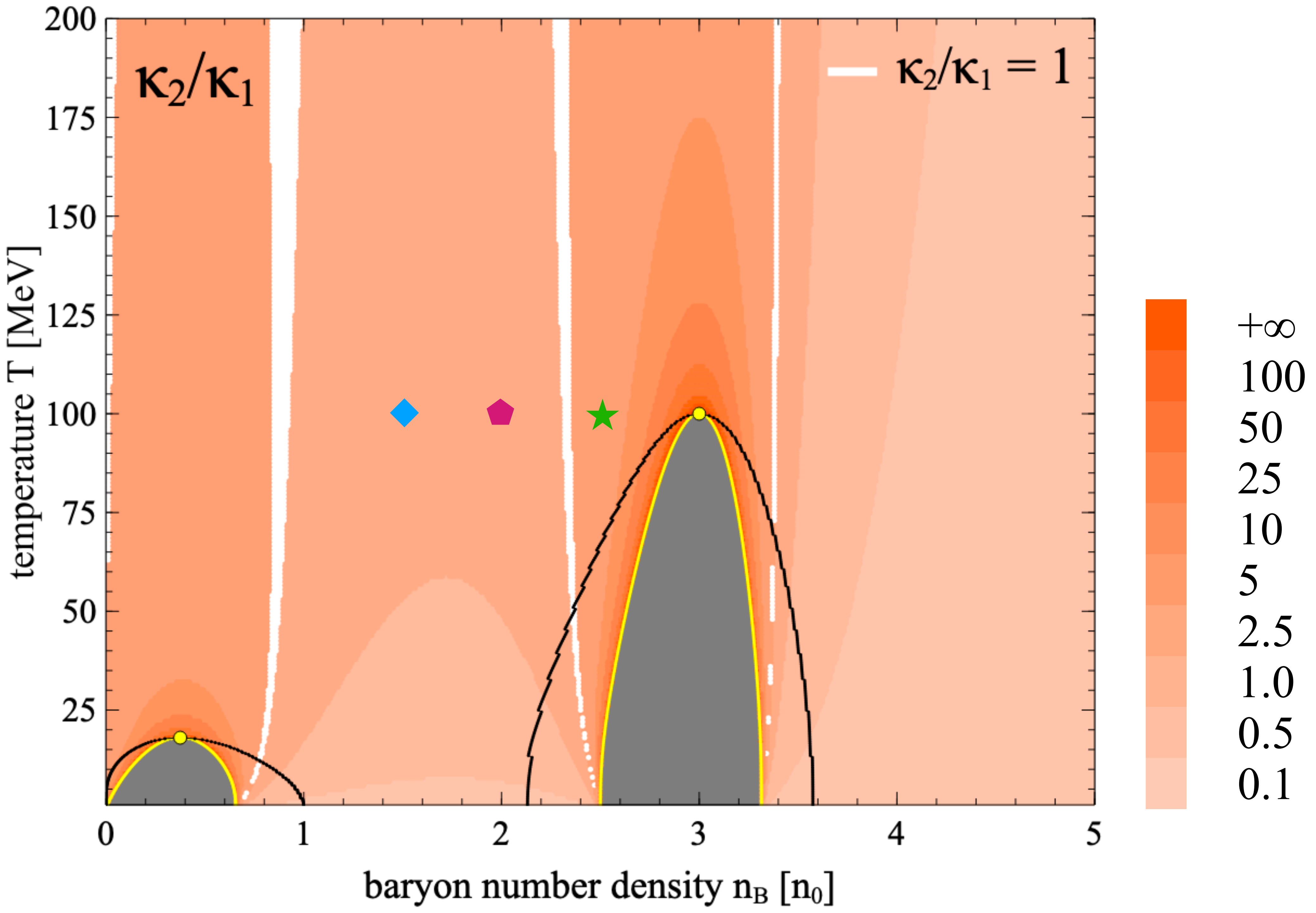

	◆	⬟	★
T [MeV]	100	100	100
nB [n0]	1.5	2.0	2.5
(k2/k1)inf	0.67	0.70	1.46

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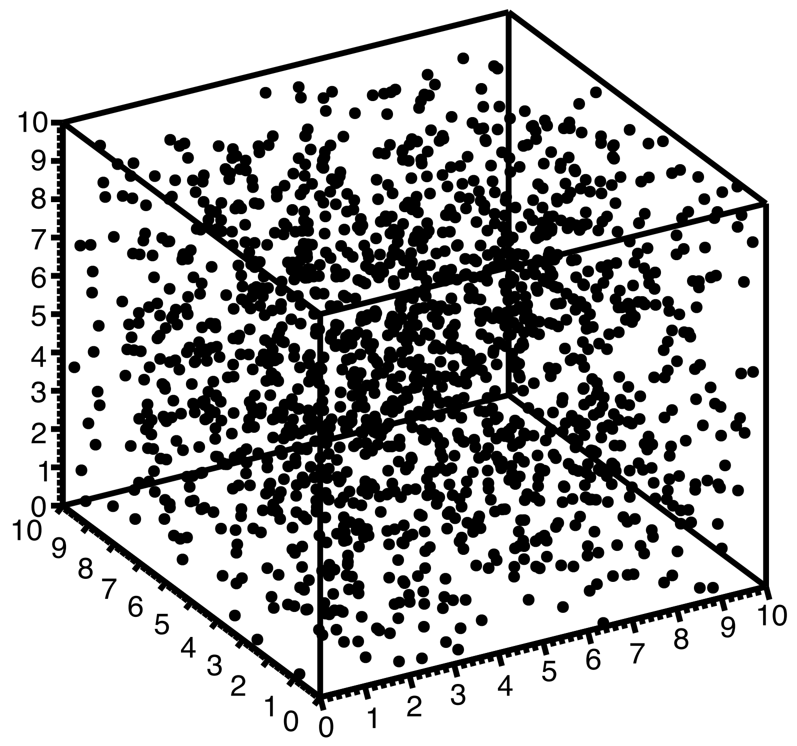

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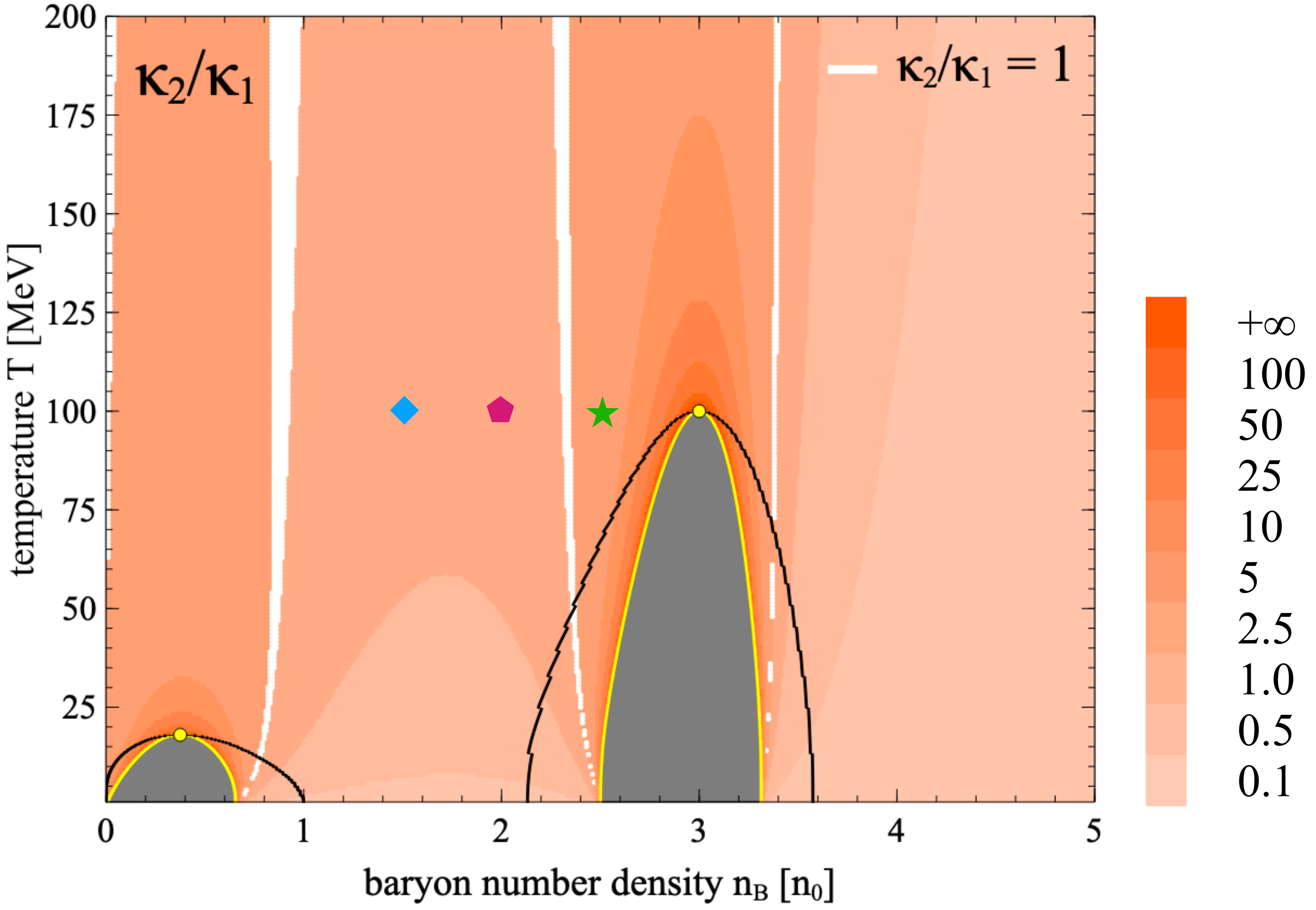

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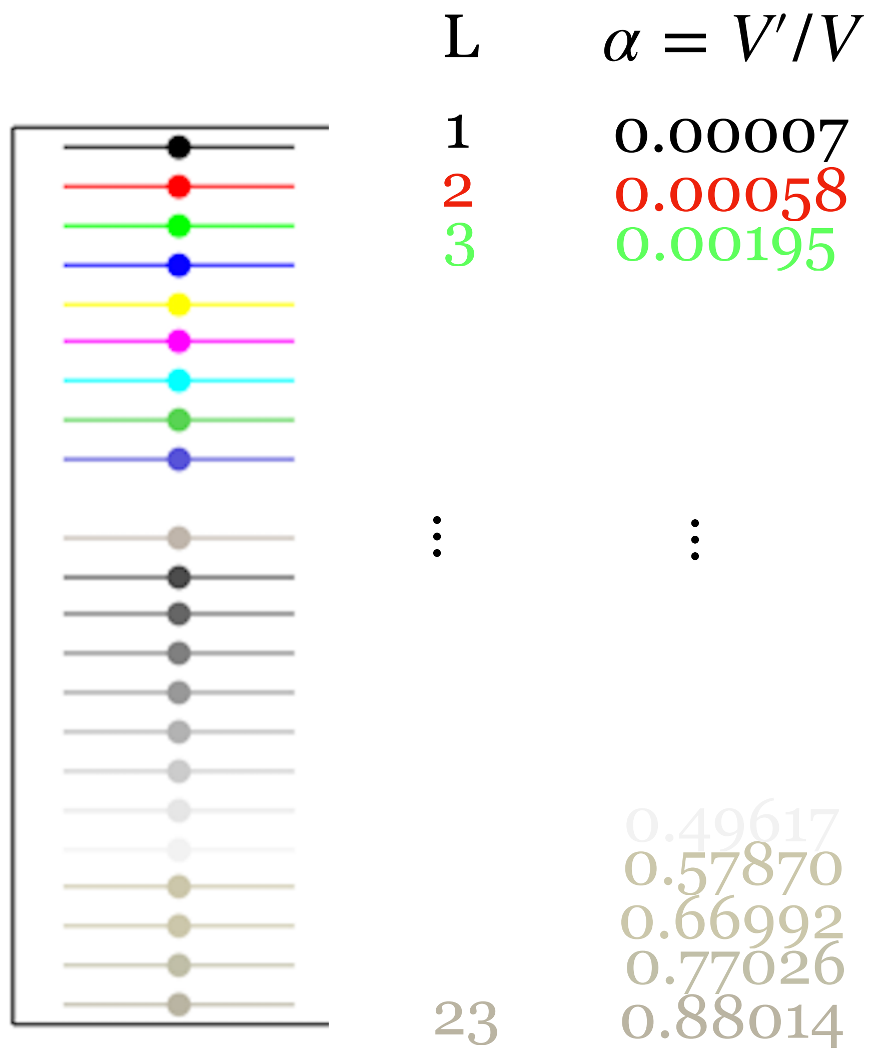

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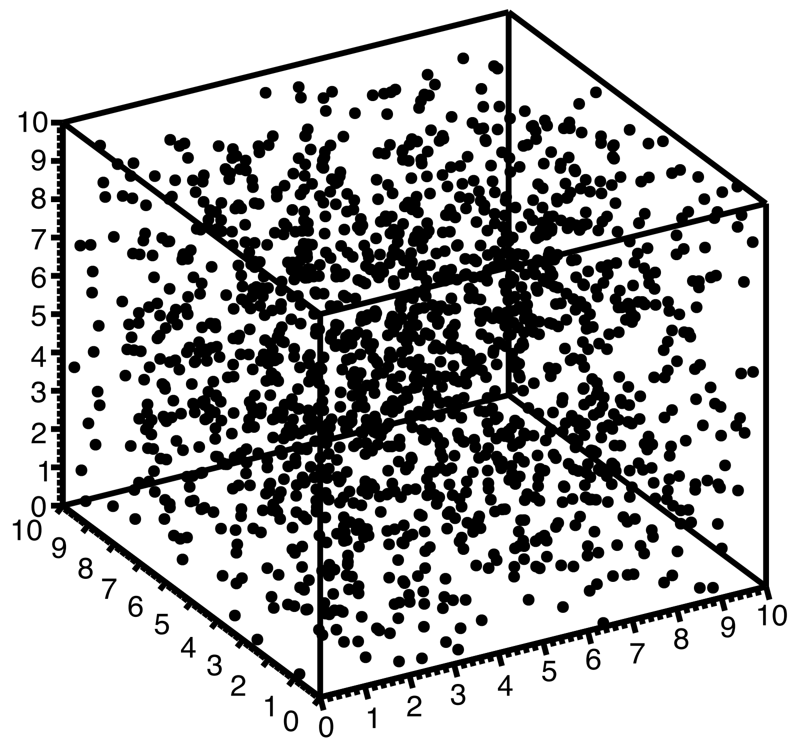

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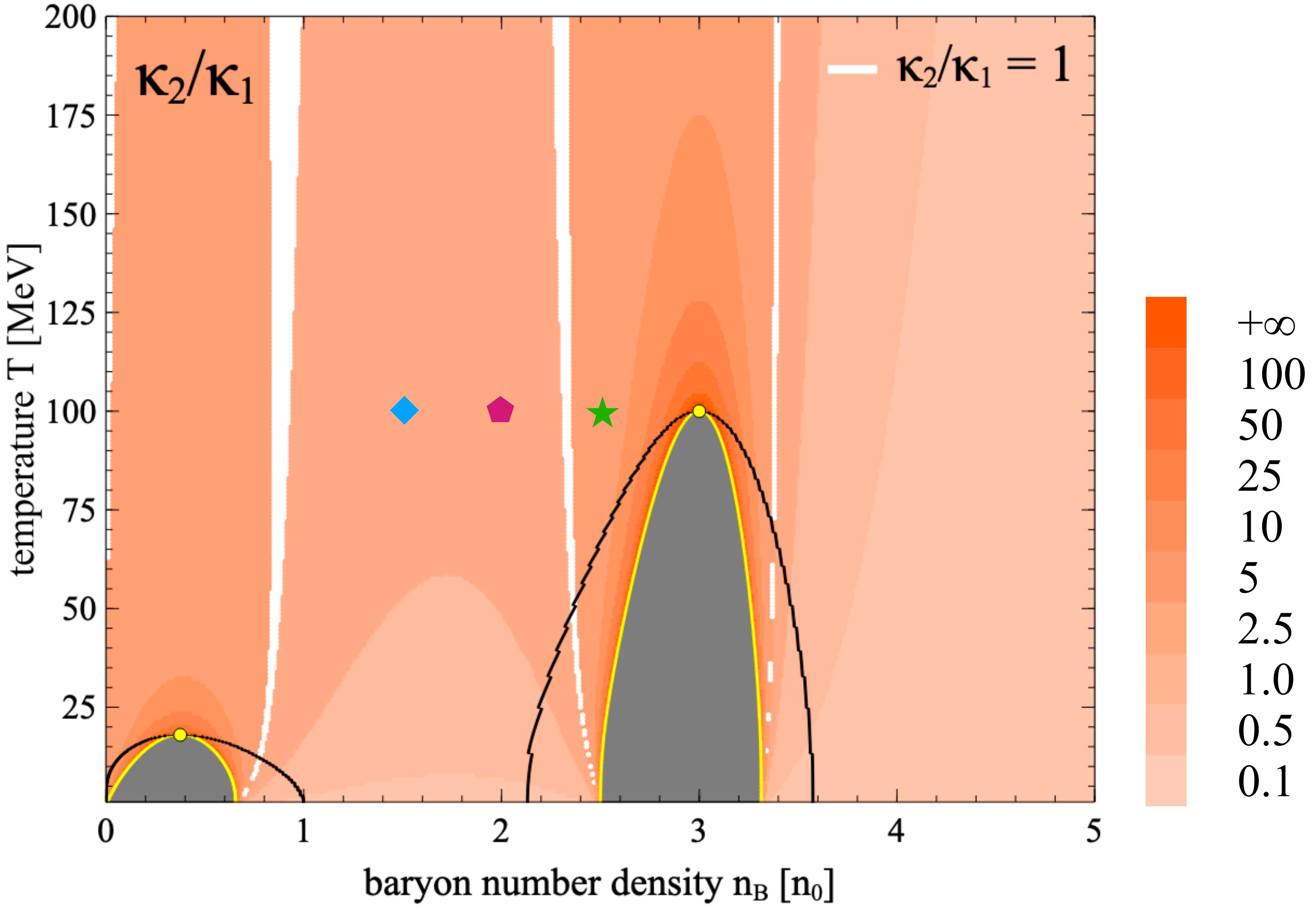

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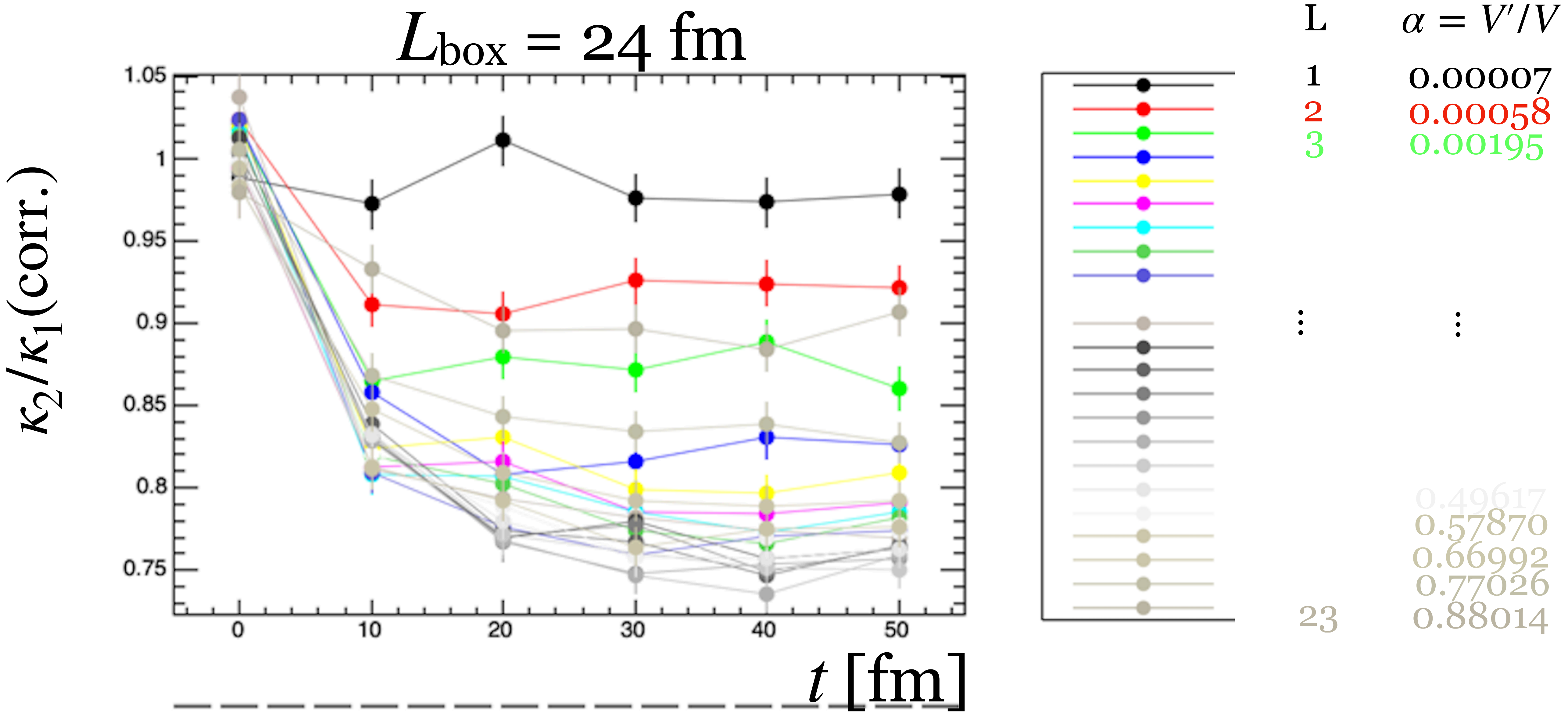

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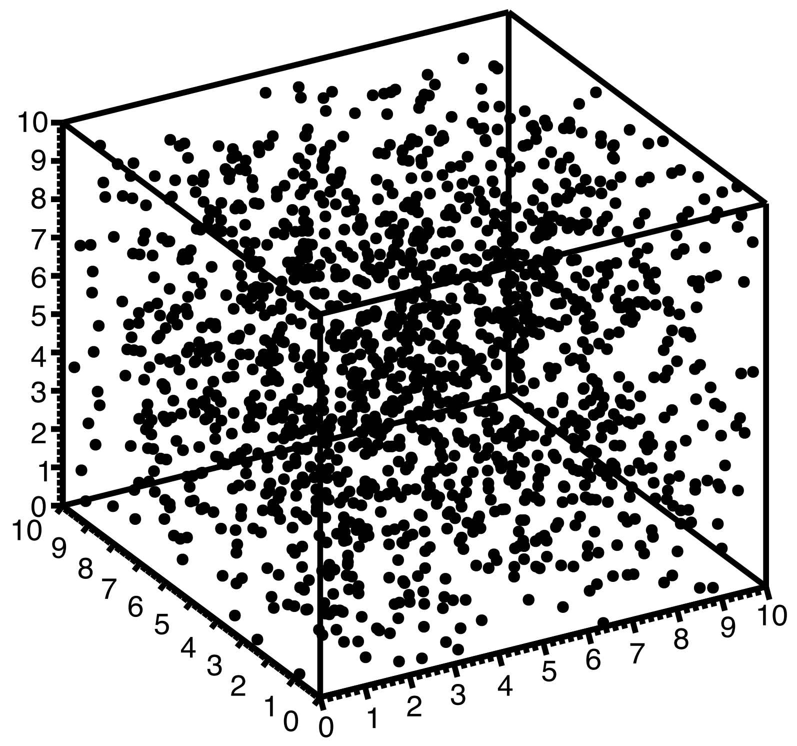

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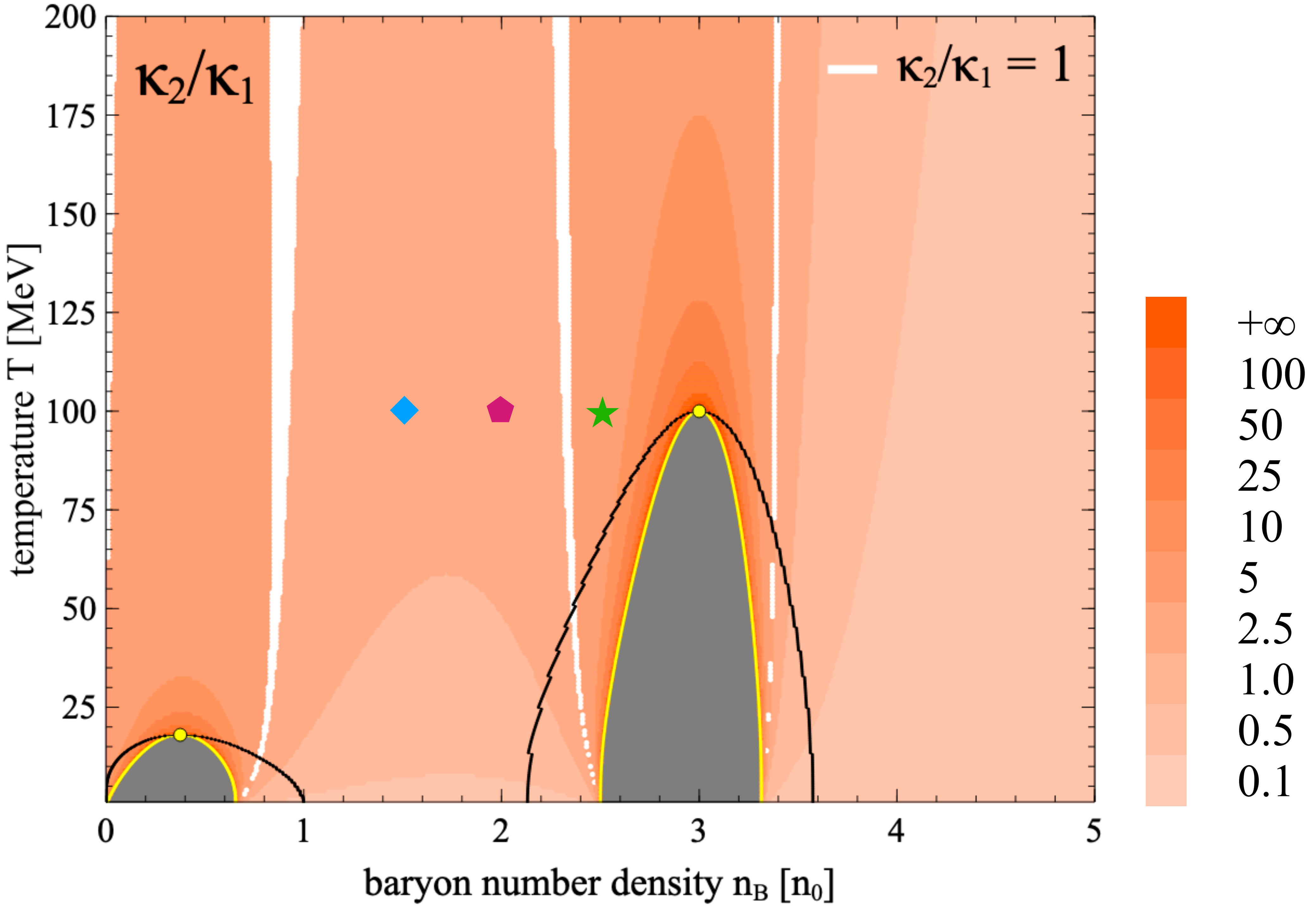

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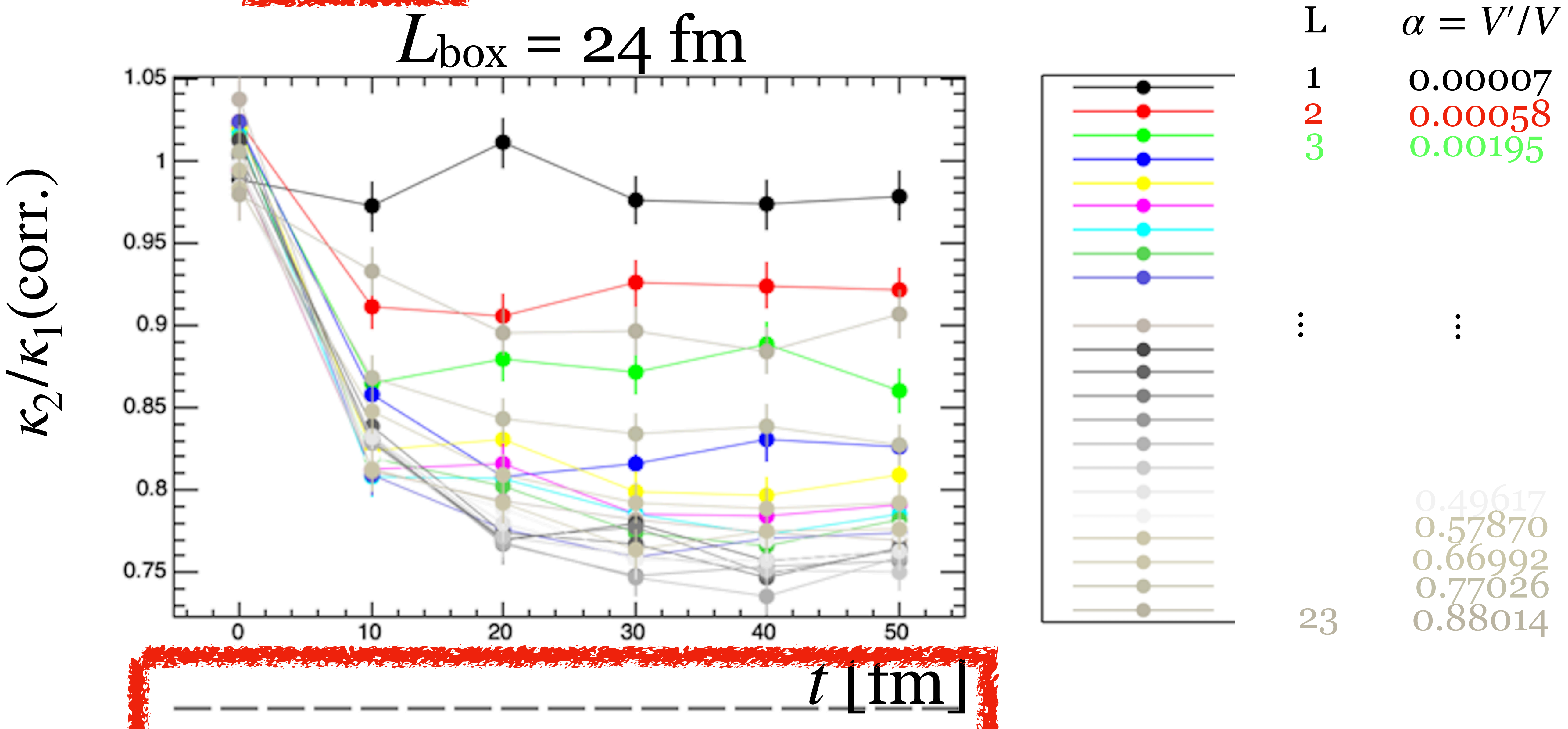
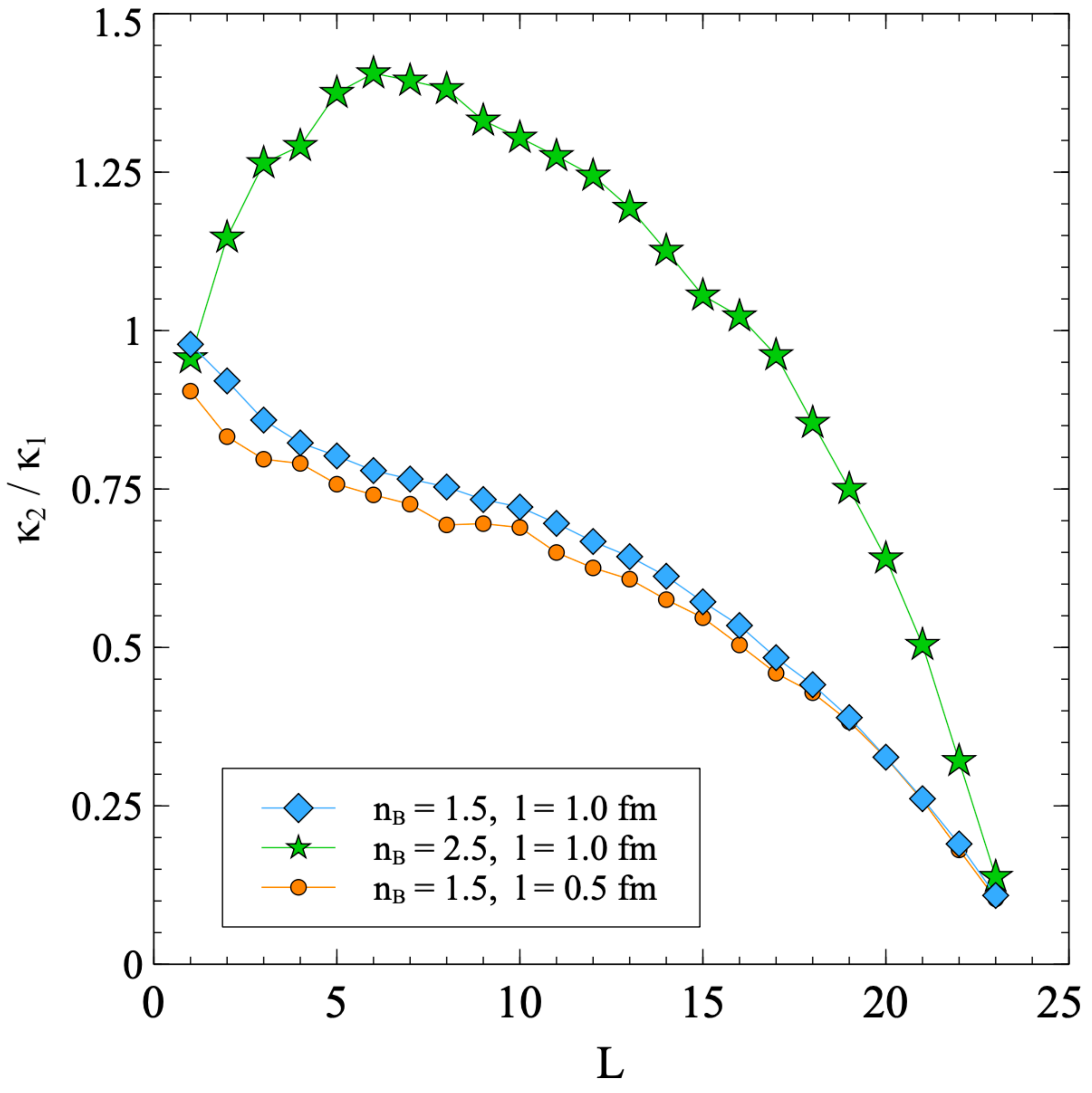
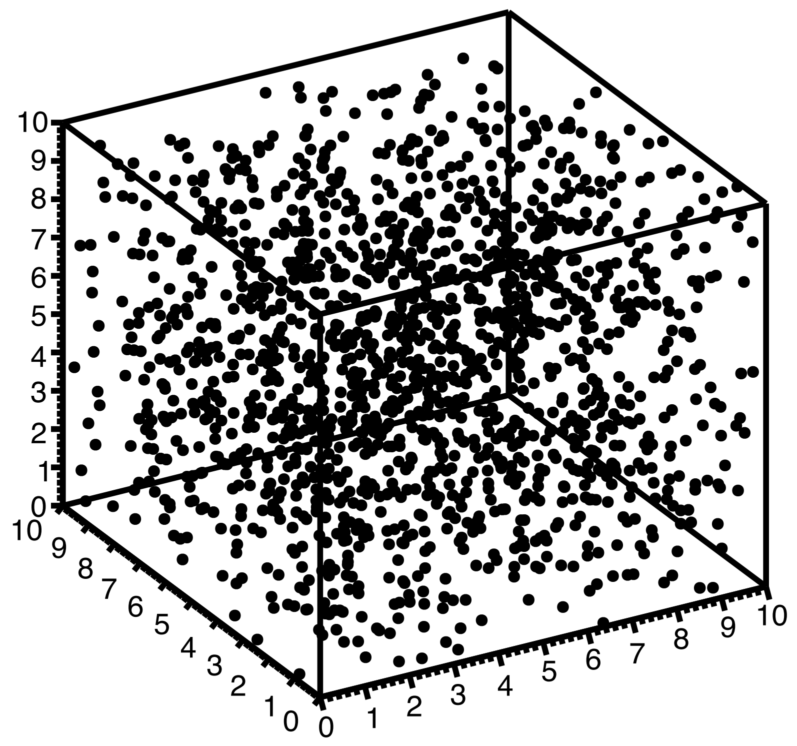


Finite-size scaling analysis of cumulants in a periodic box

$L_{\text{box}} = 24\ \text{fm},\ t_{\text{end}} = 50\ \text{fm}/c$





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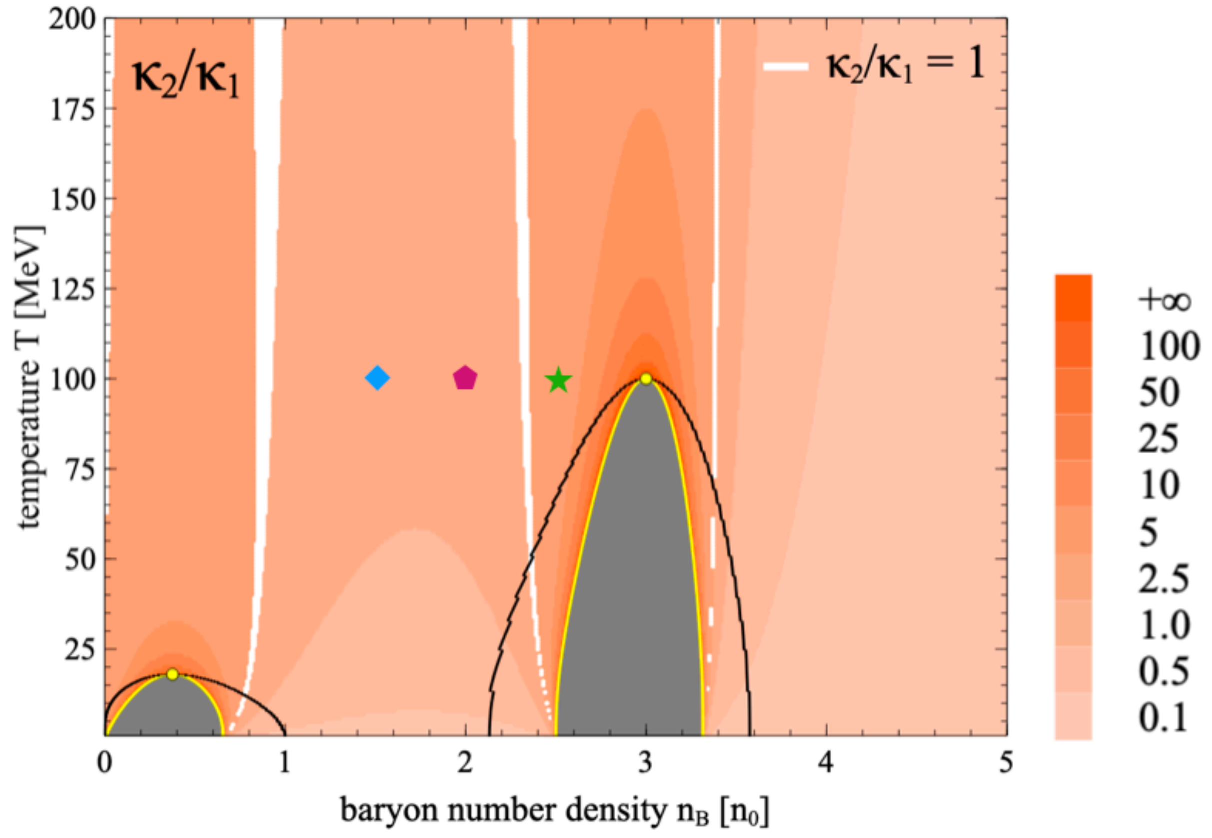

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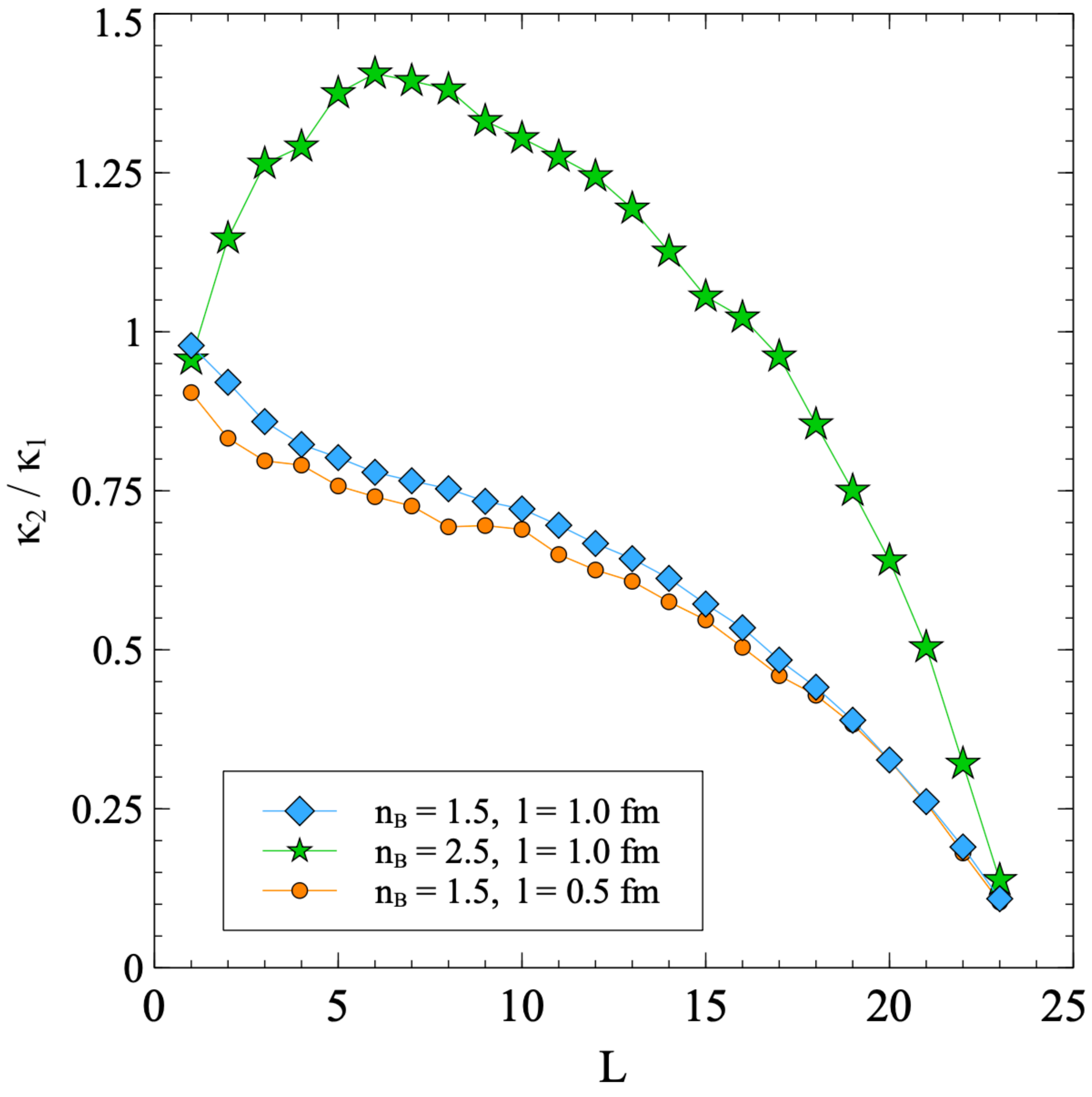
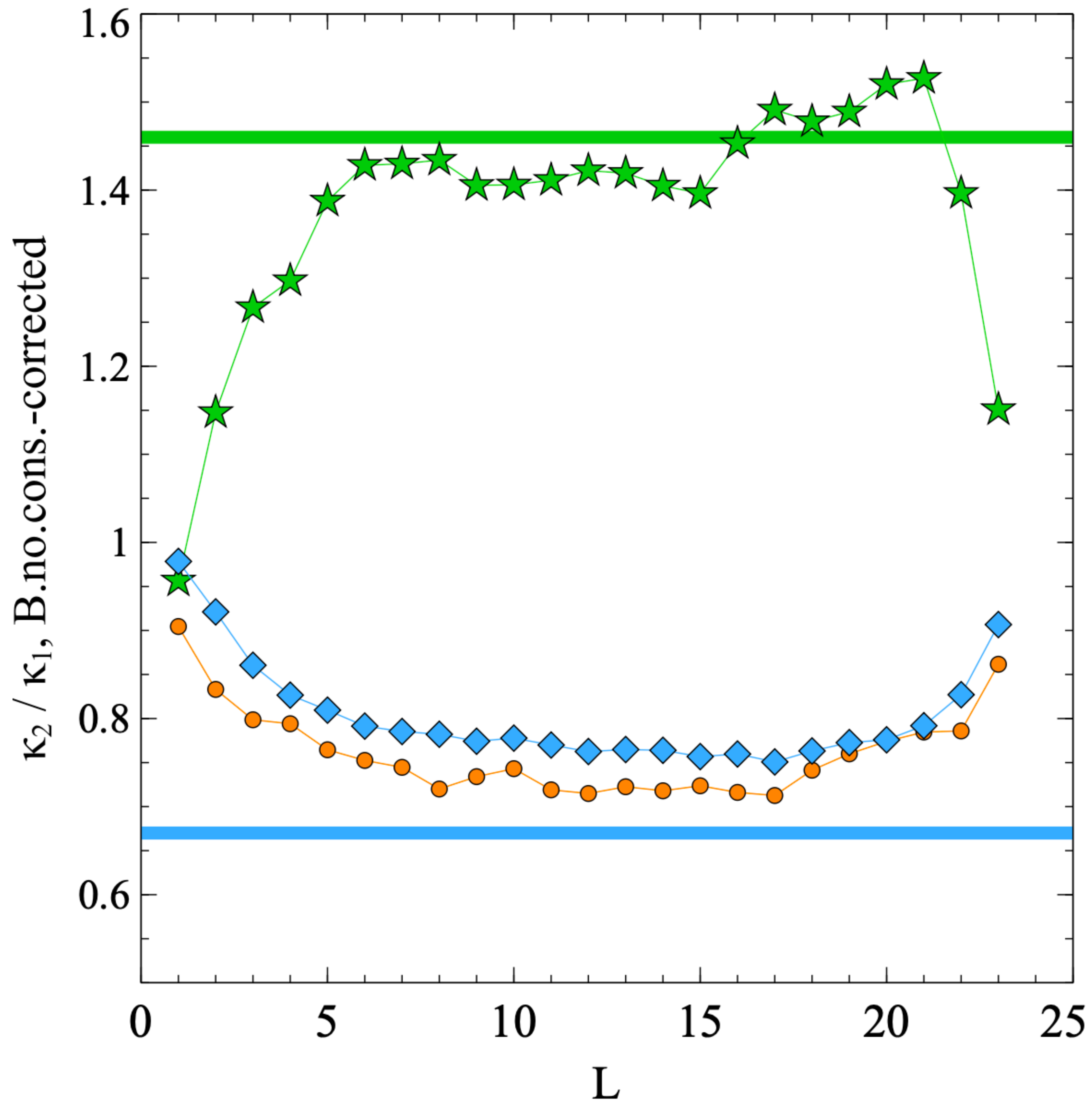
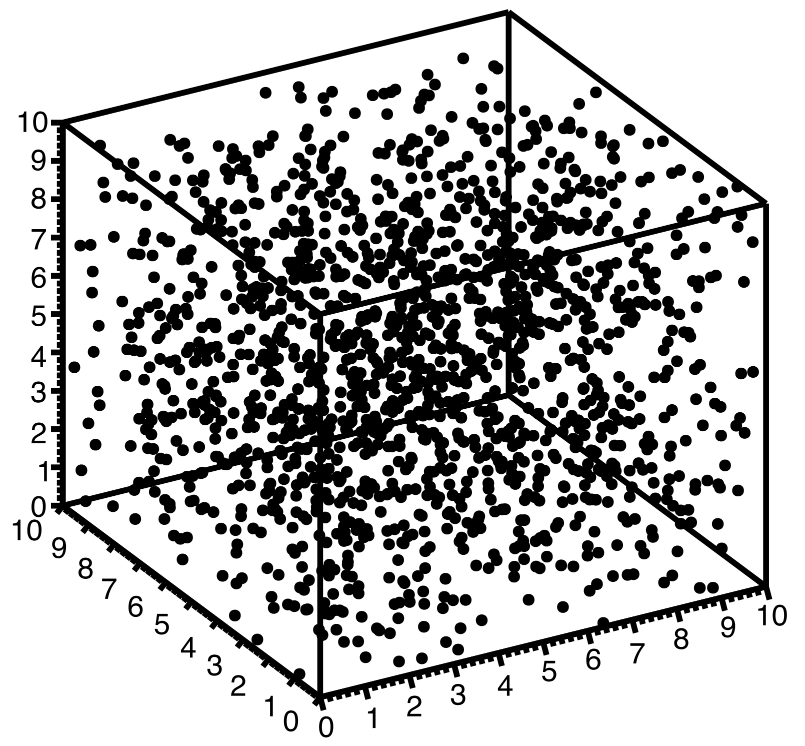
correcting for effects of baryon number conservation:

V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, V. Vovchenko, Phys. Rev. C **105** no.4, 044903 (2022), arXiv:2201.08486







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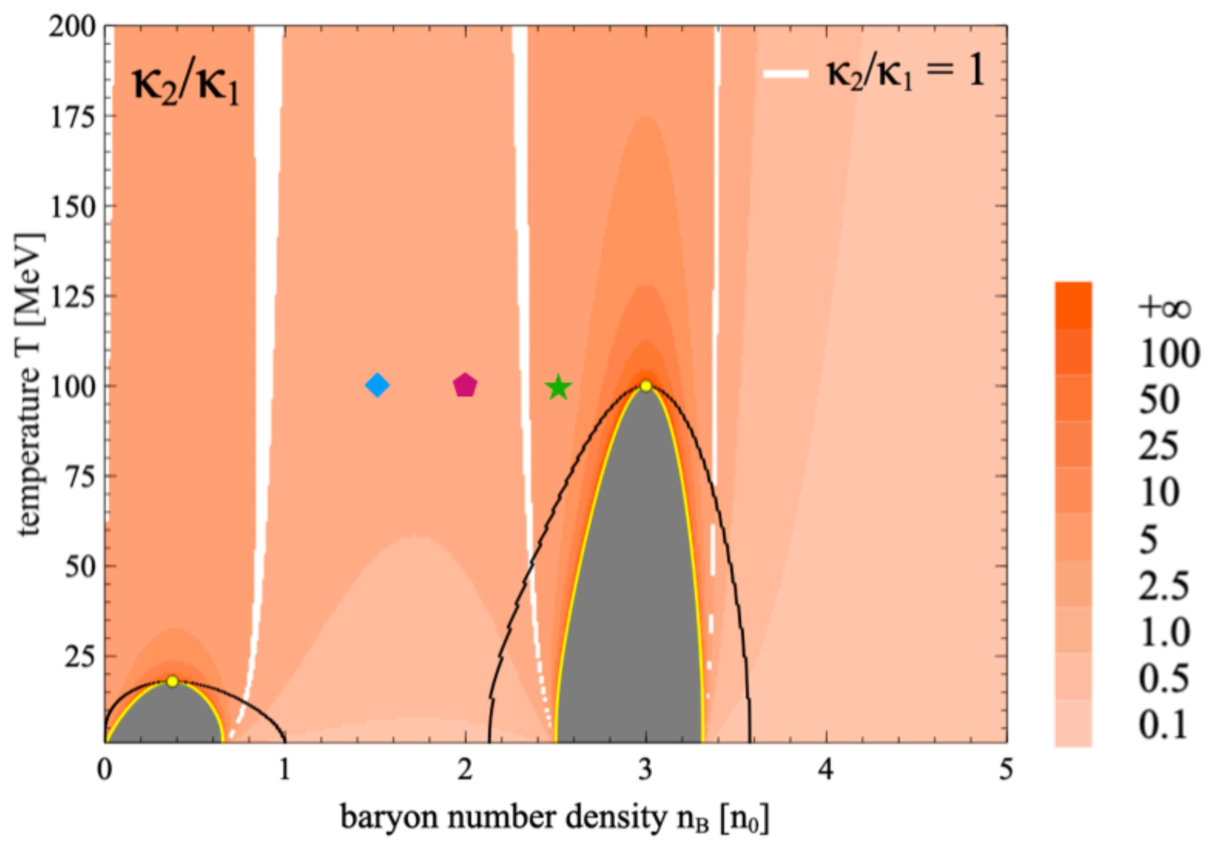

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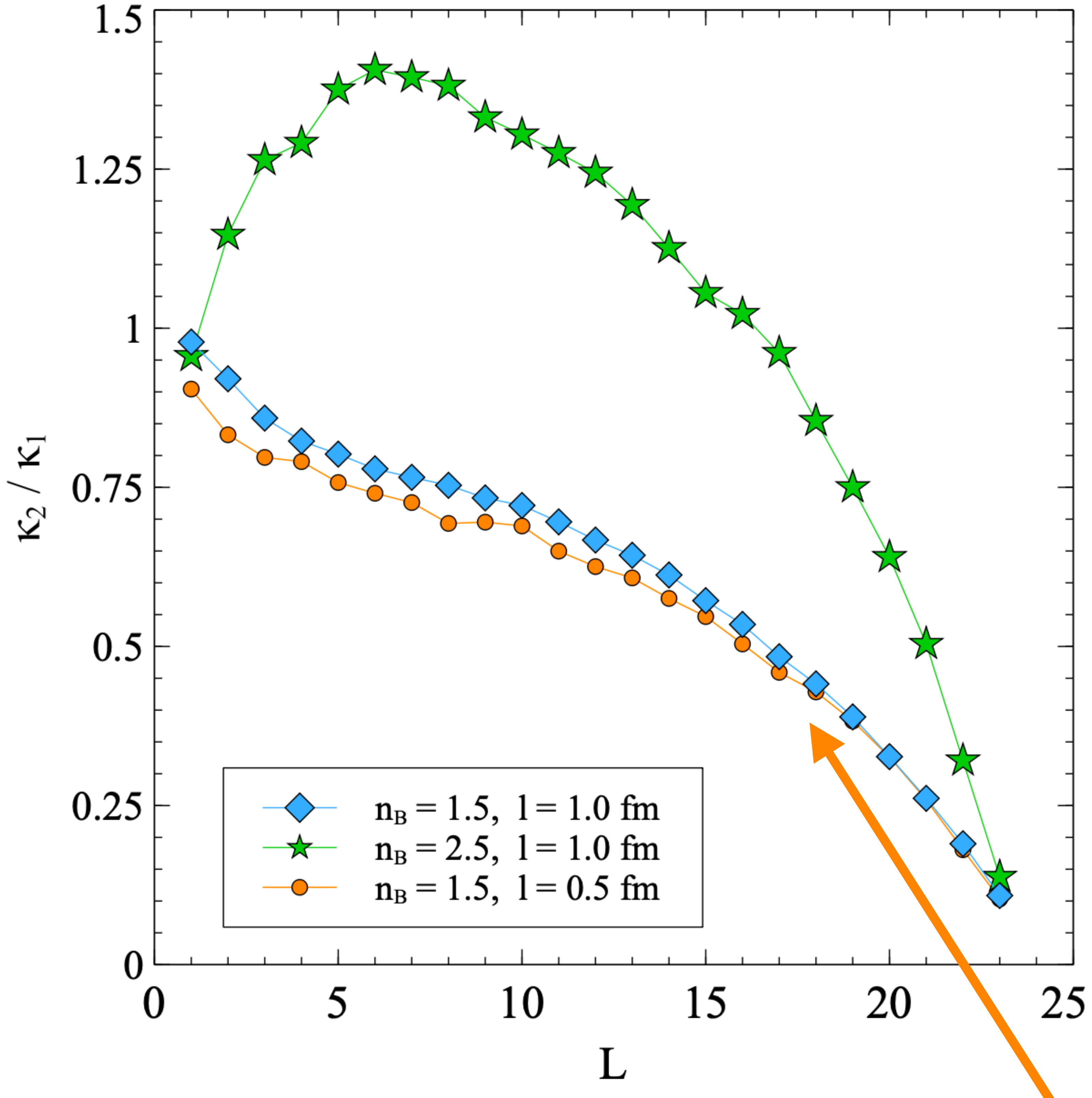

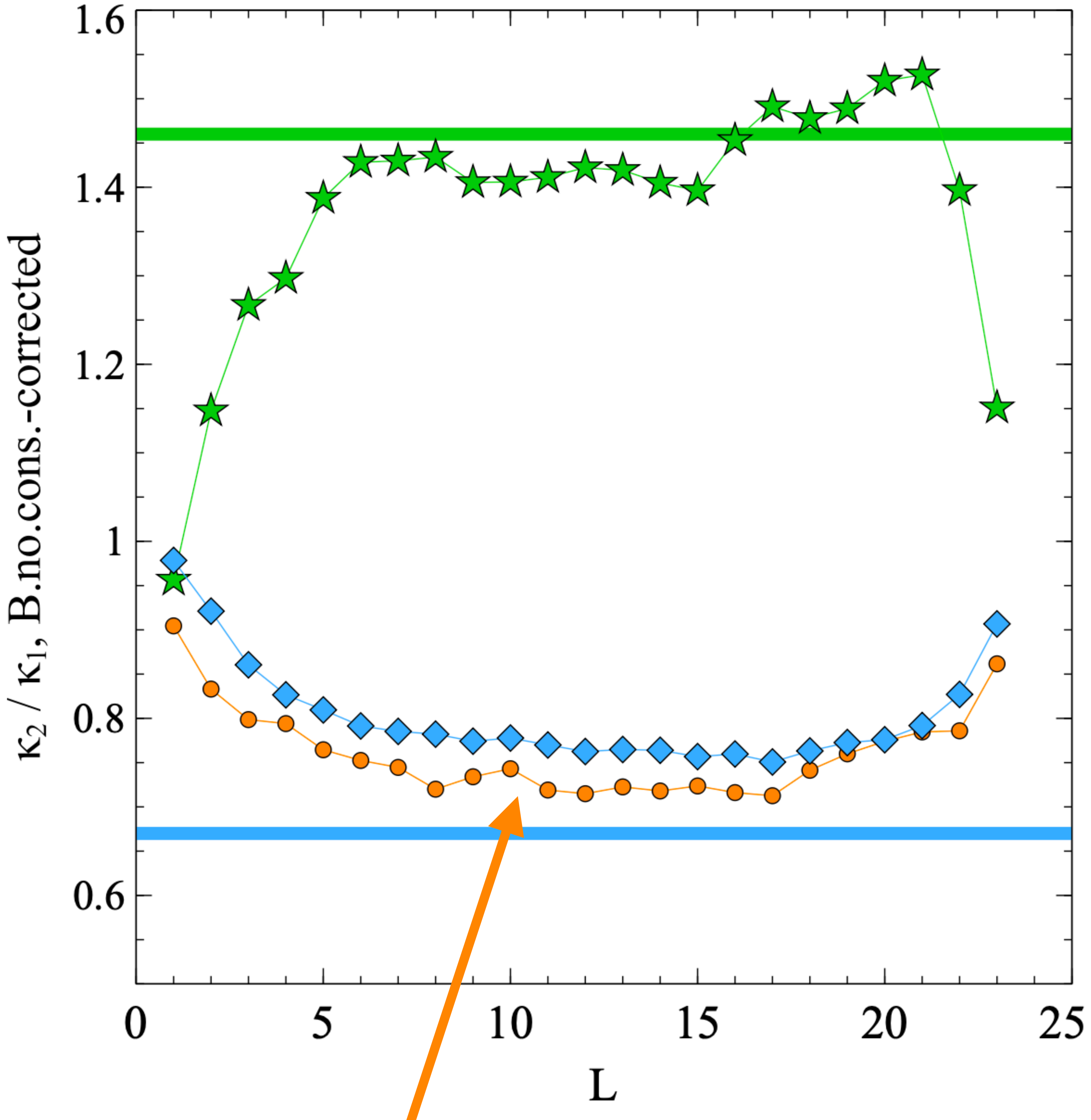

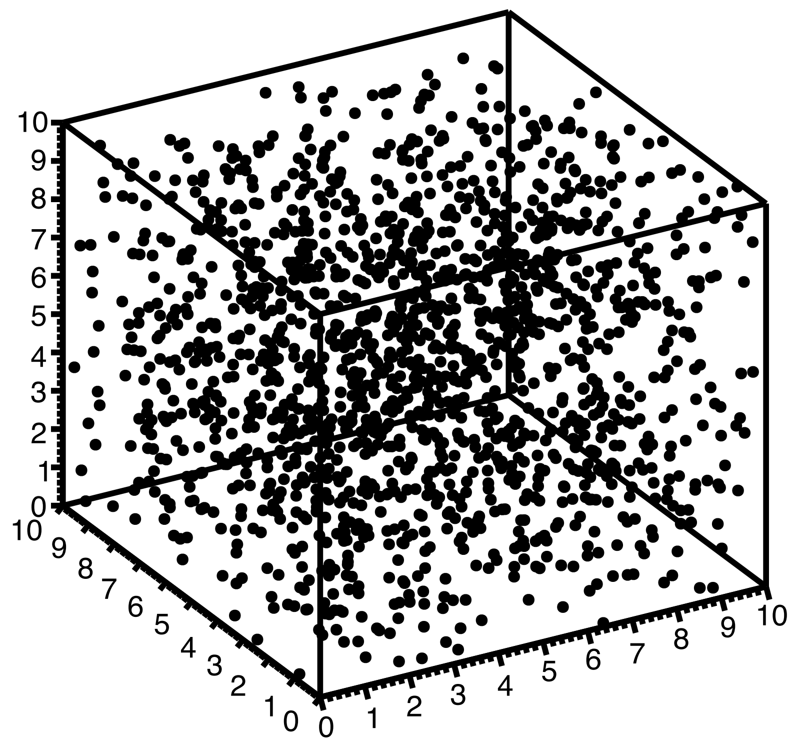

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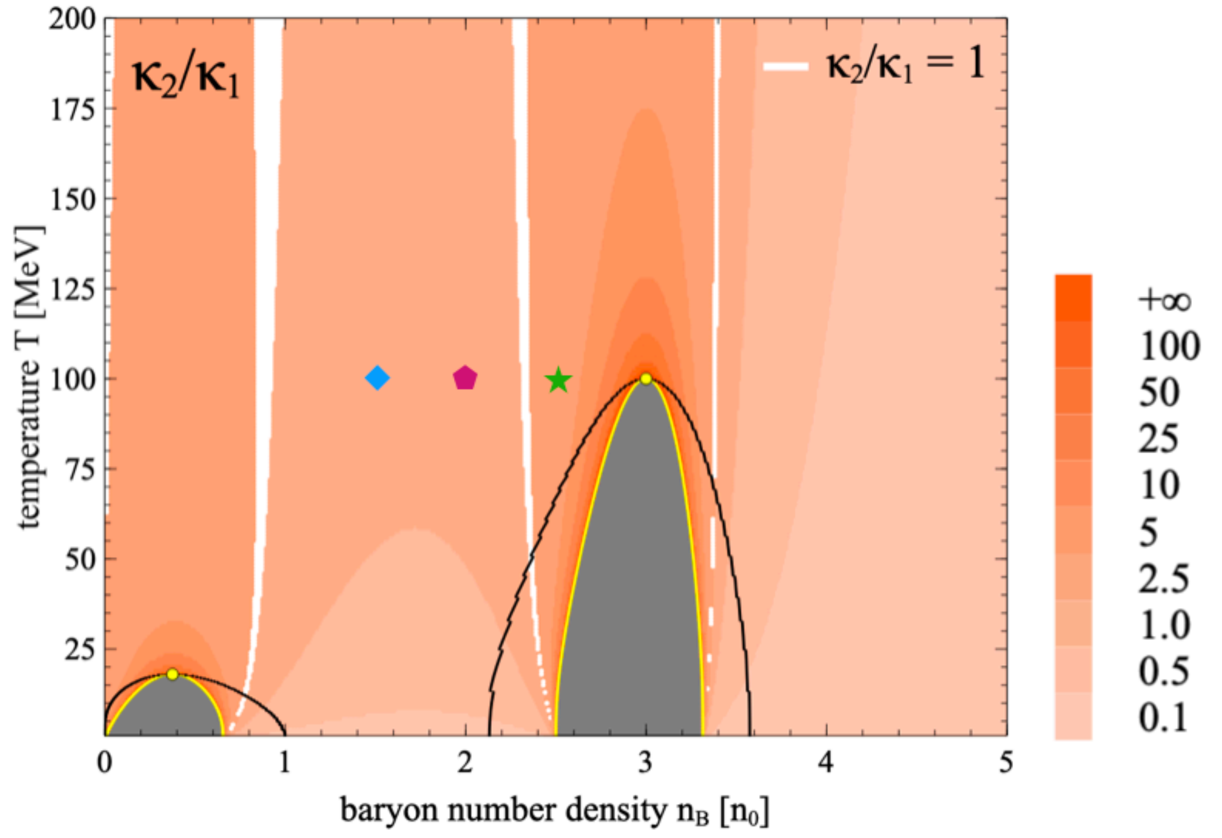

small dependence on the microscopic scale

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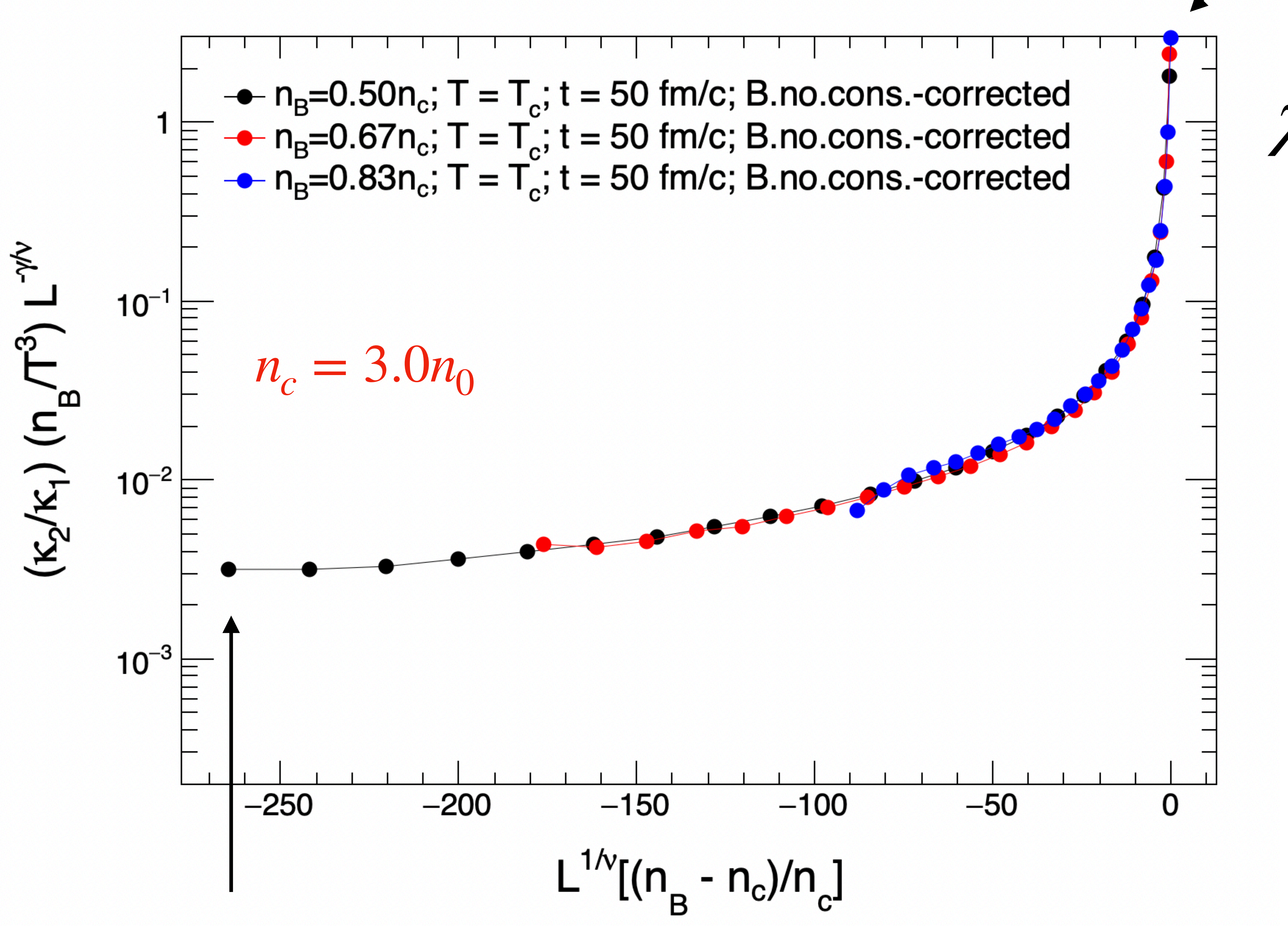

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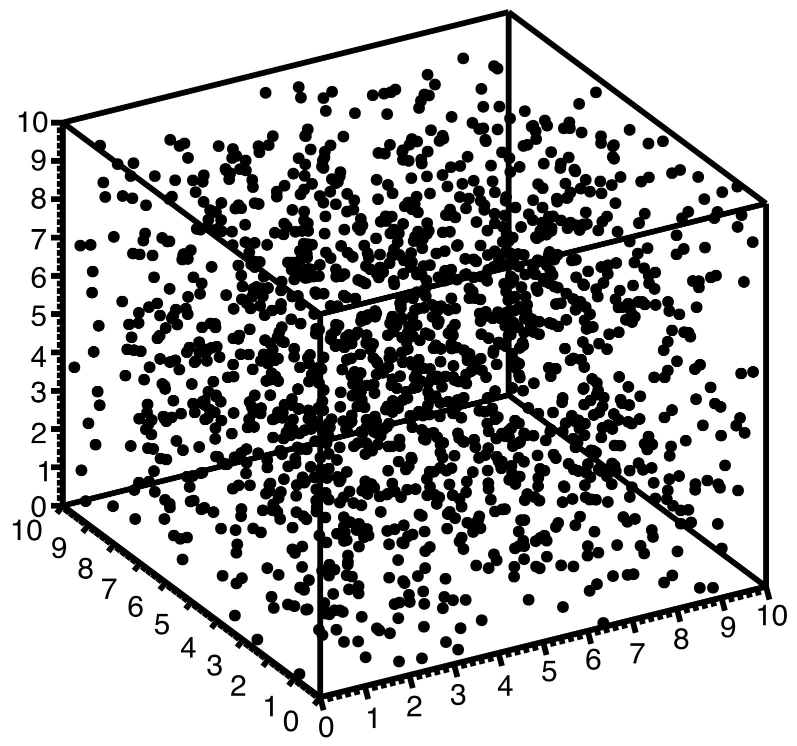

Finite-size scaling analysis of cumulants in a periodic box

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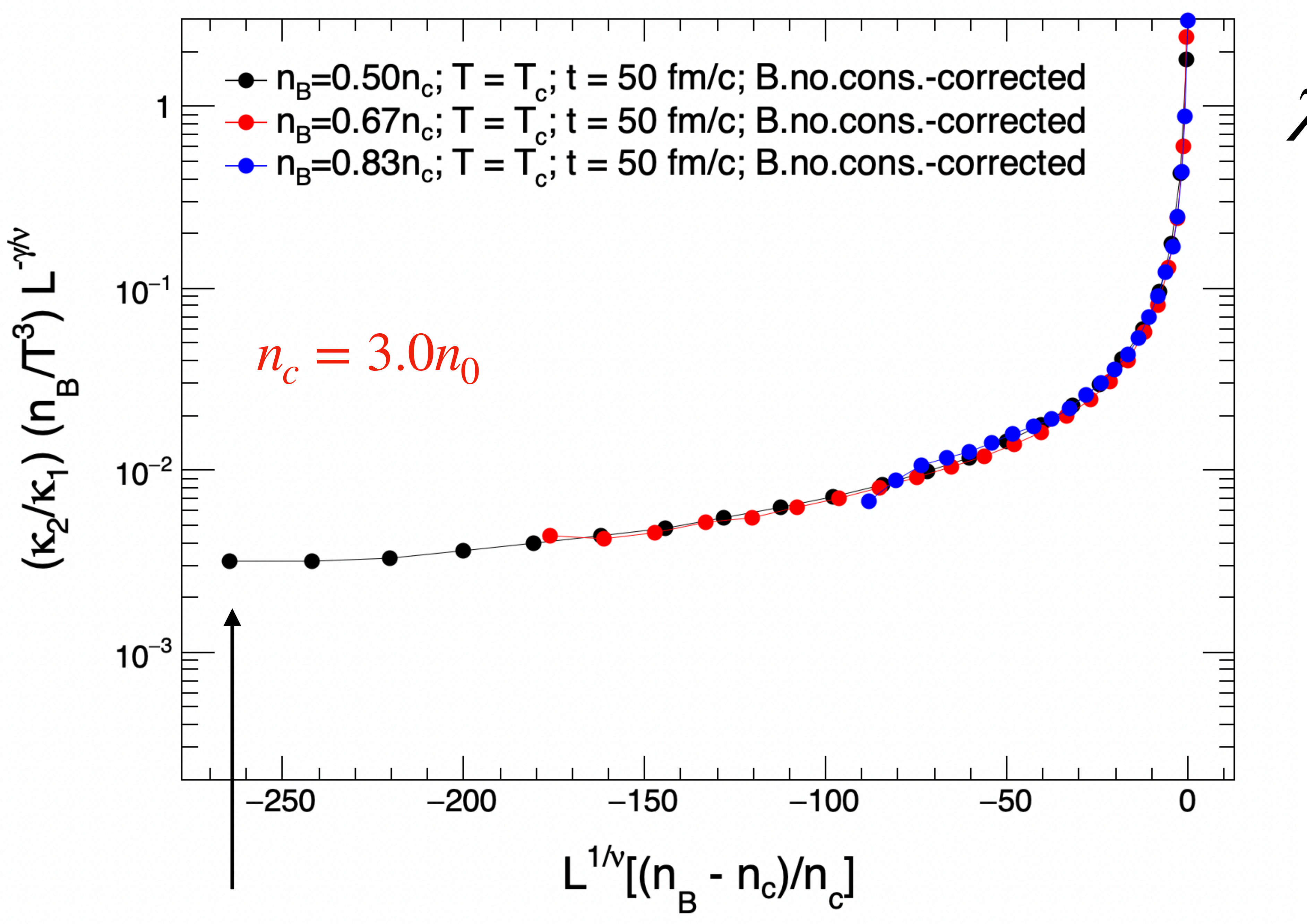

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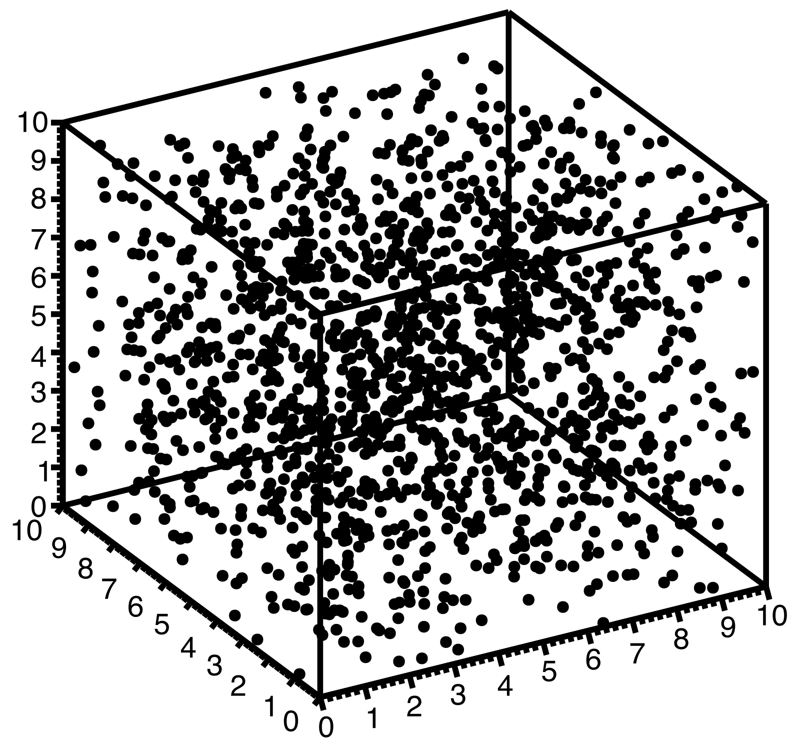

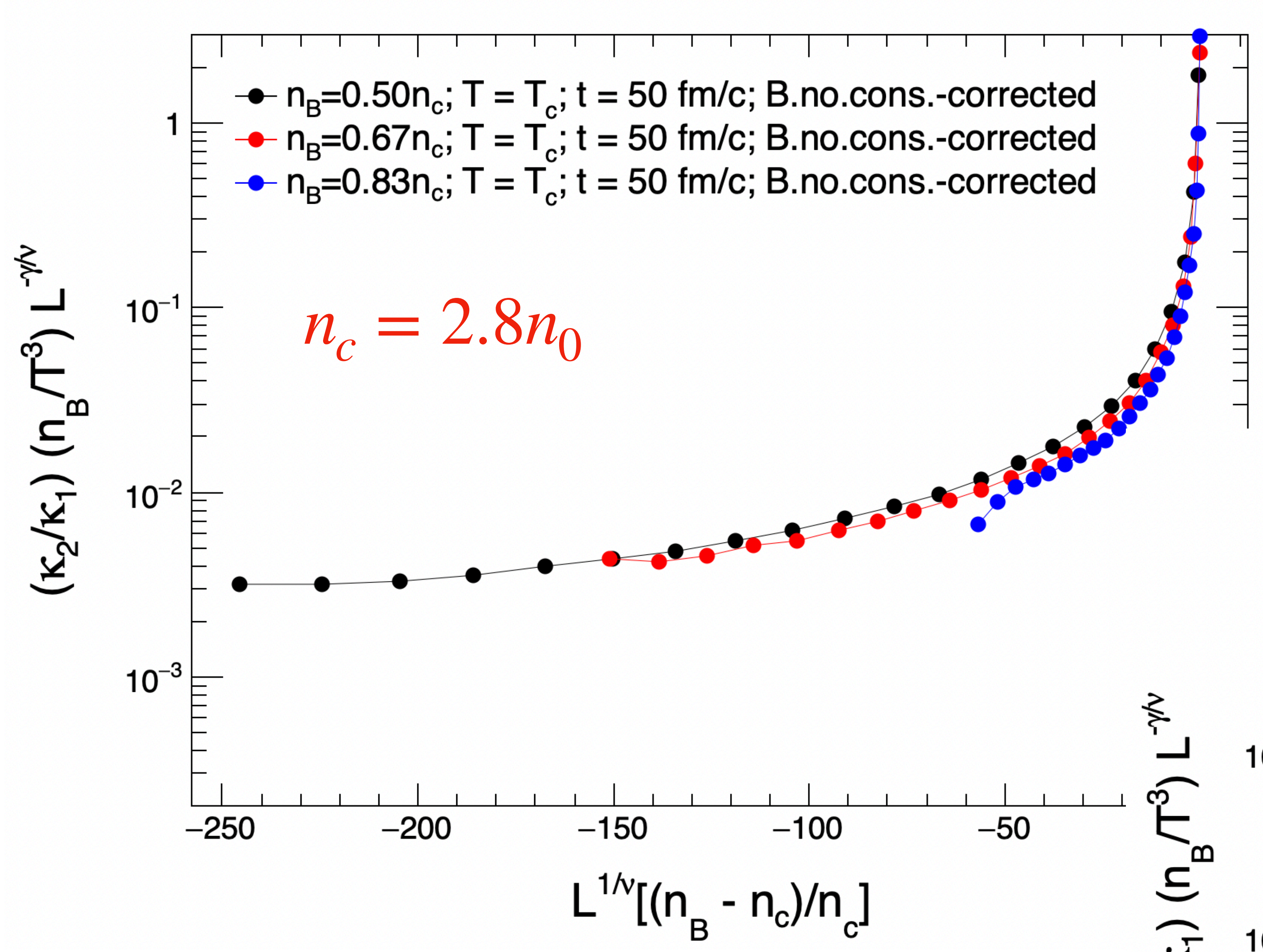

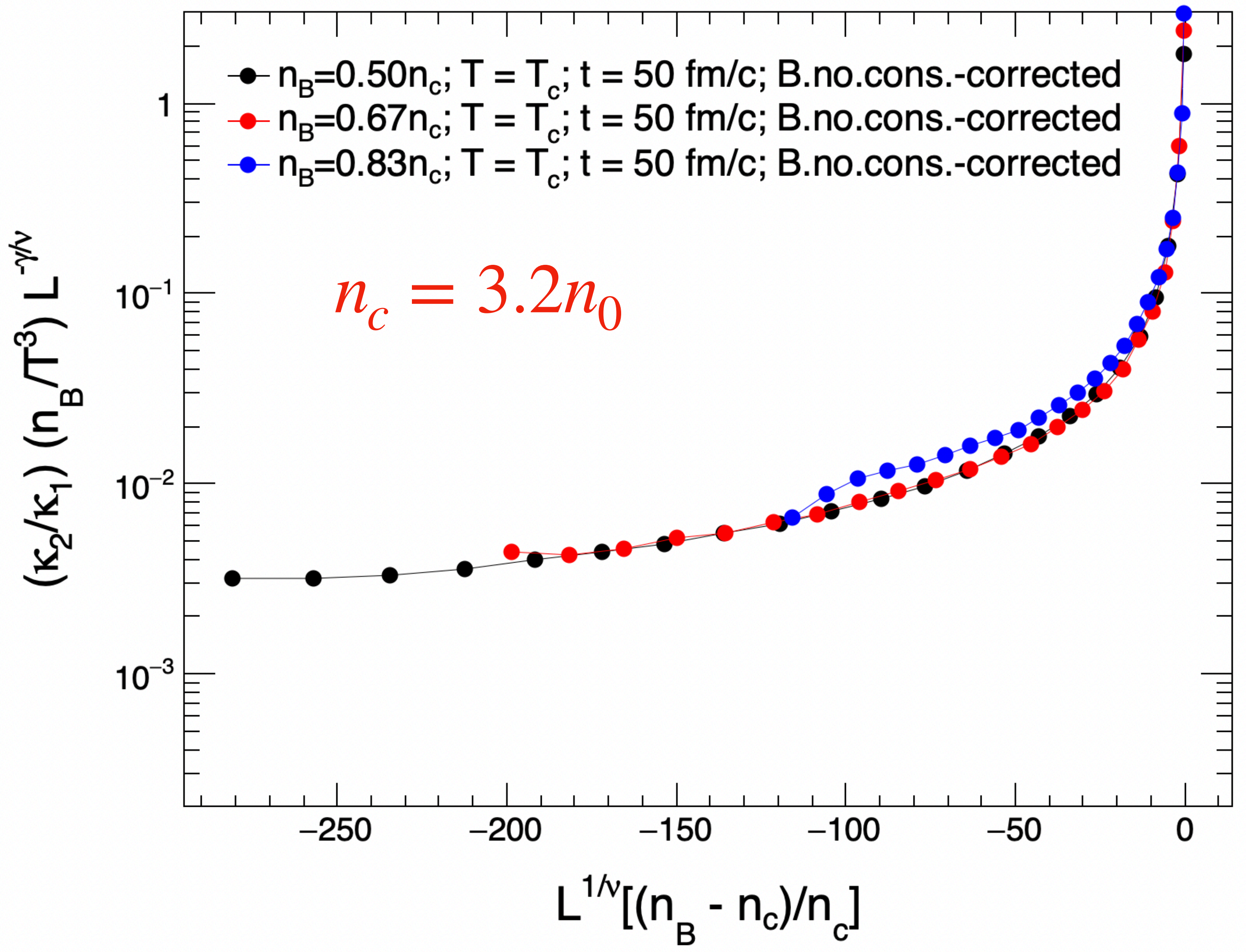

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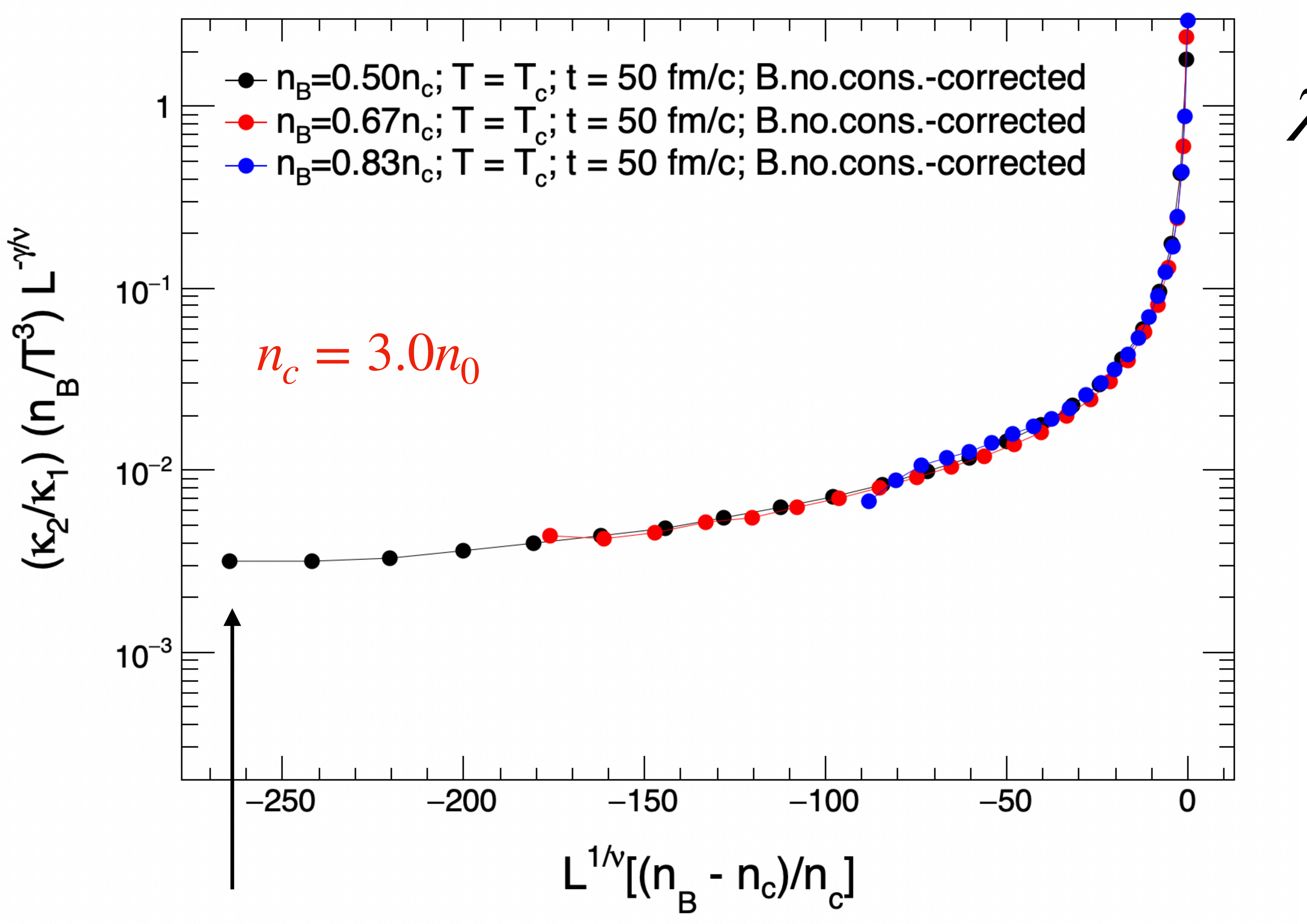
$L_{\rm box} = 24$ fm

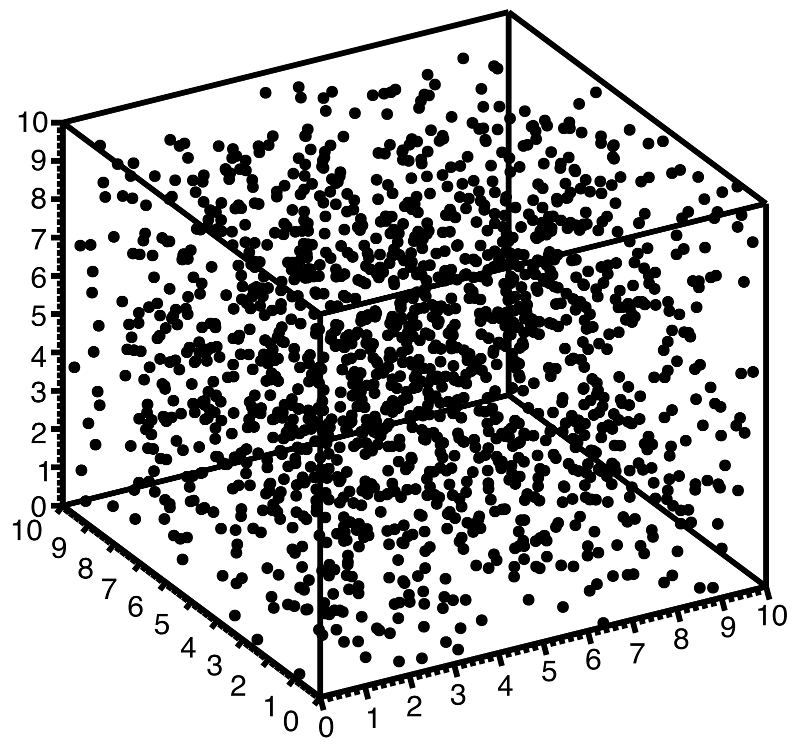
smallest L



largest L

$$\chi_2 = \frac{C_2}{C_1}\frac{n_B}{T^3}$$

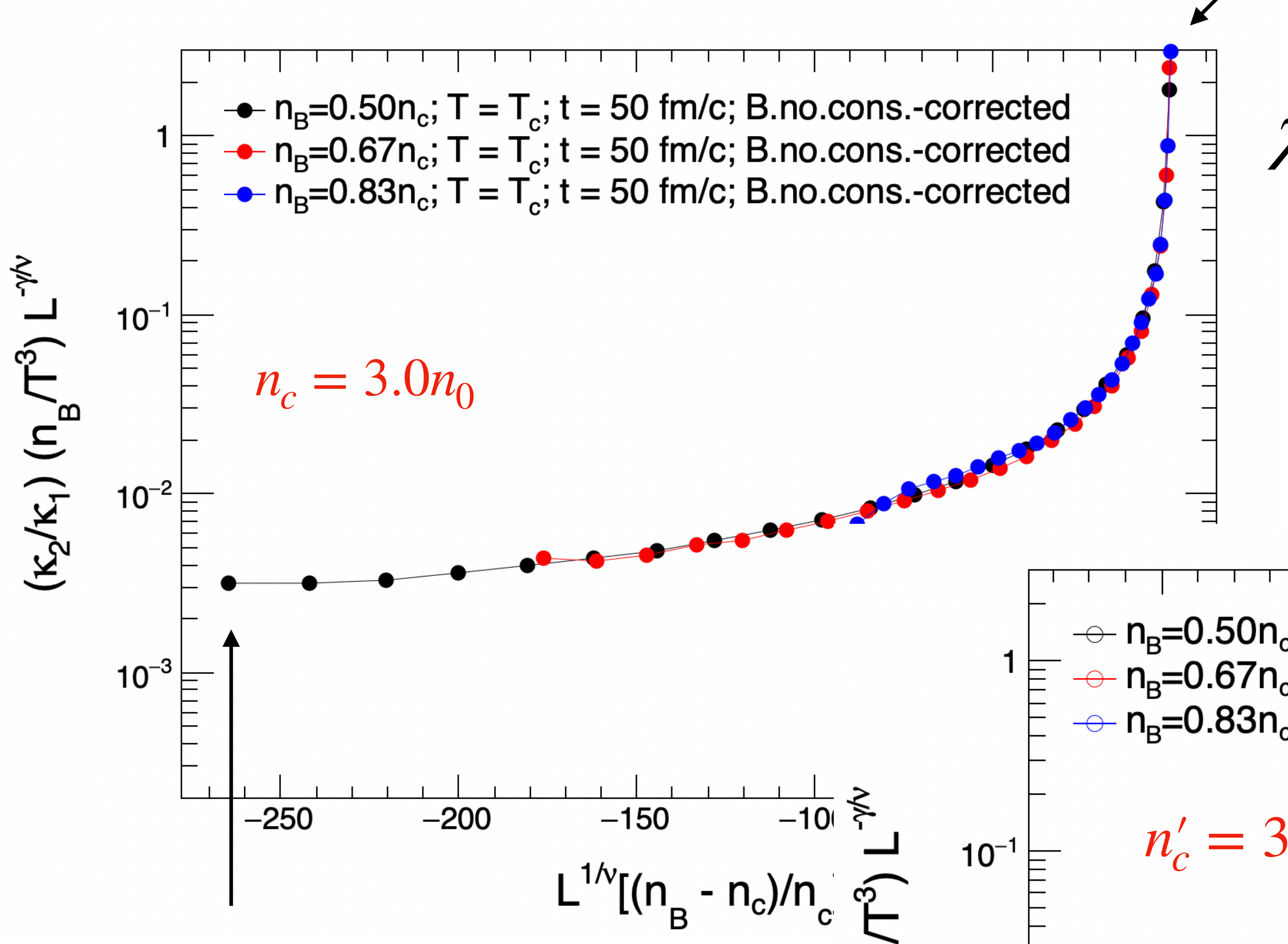
	◆	⬟	★
T [MeV]	100	100	100
nB [n0]	1.5	2.0	2.5
k2/k1	0.67	0.70	1.46



Finite-size scaling analysis of cumulants in a periodic box

$L_{\text{box}} = 24\ \text{fm}$

smallest L



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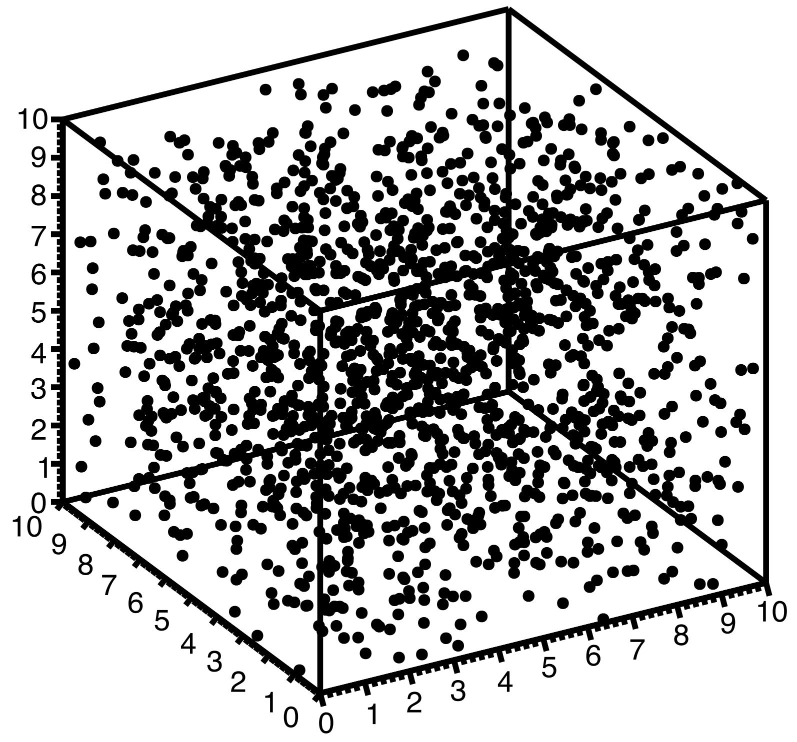

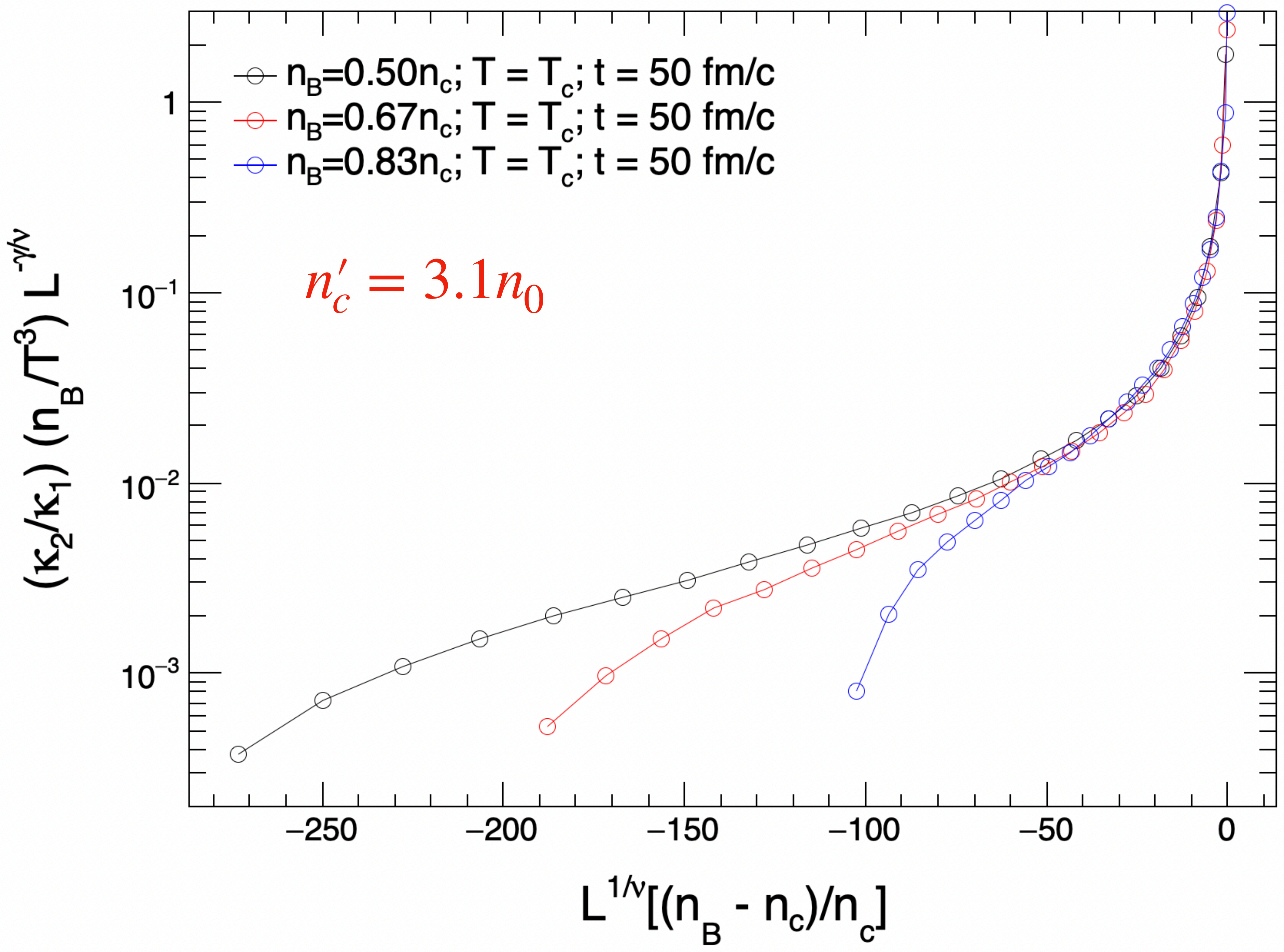

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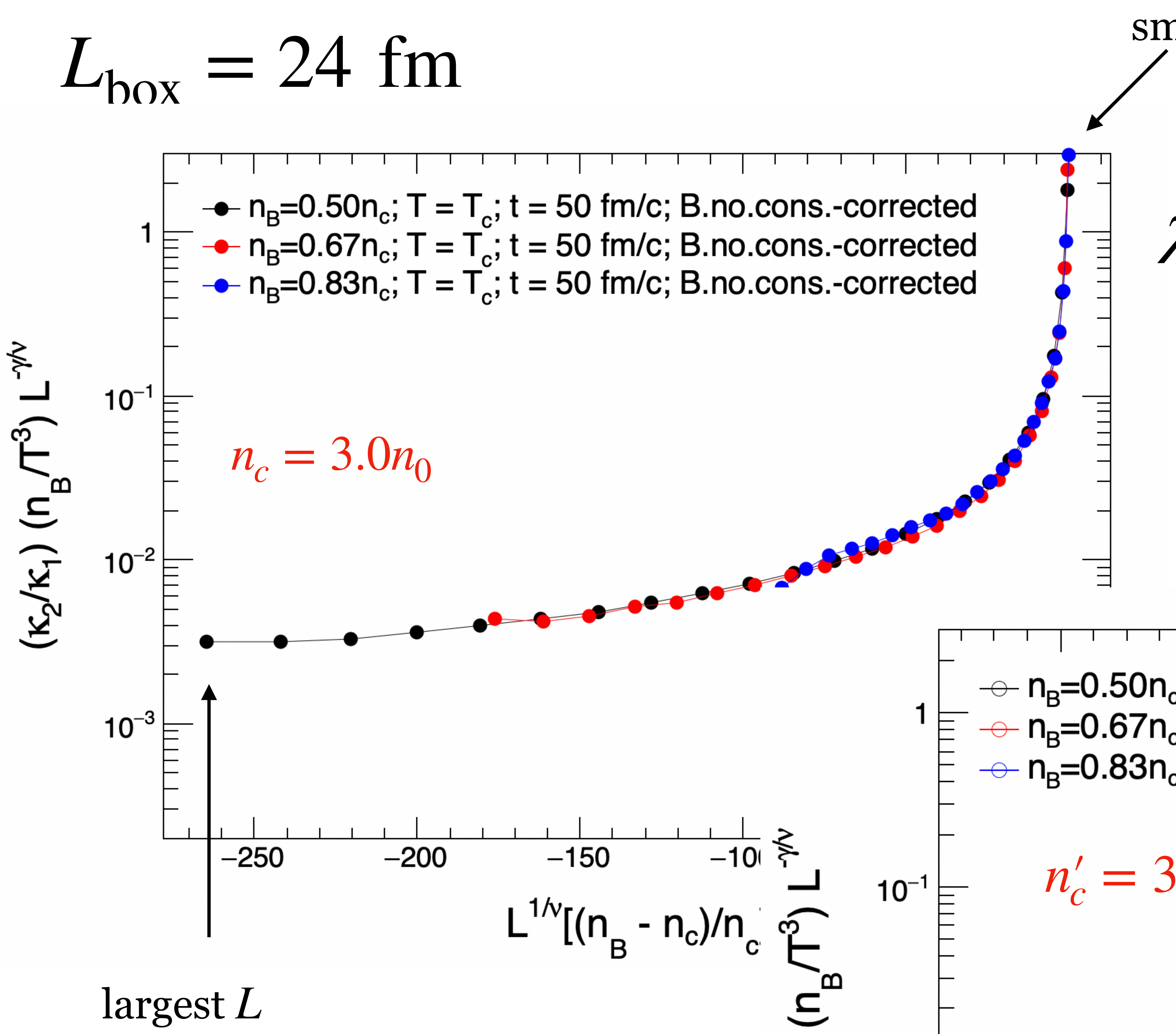

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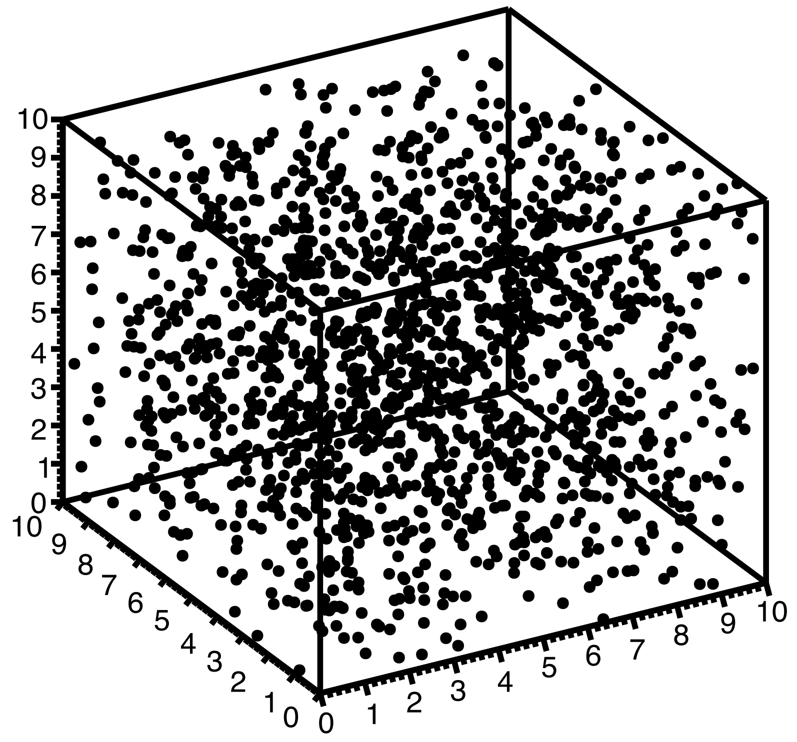
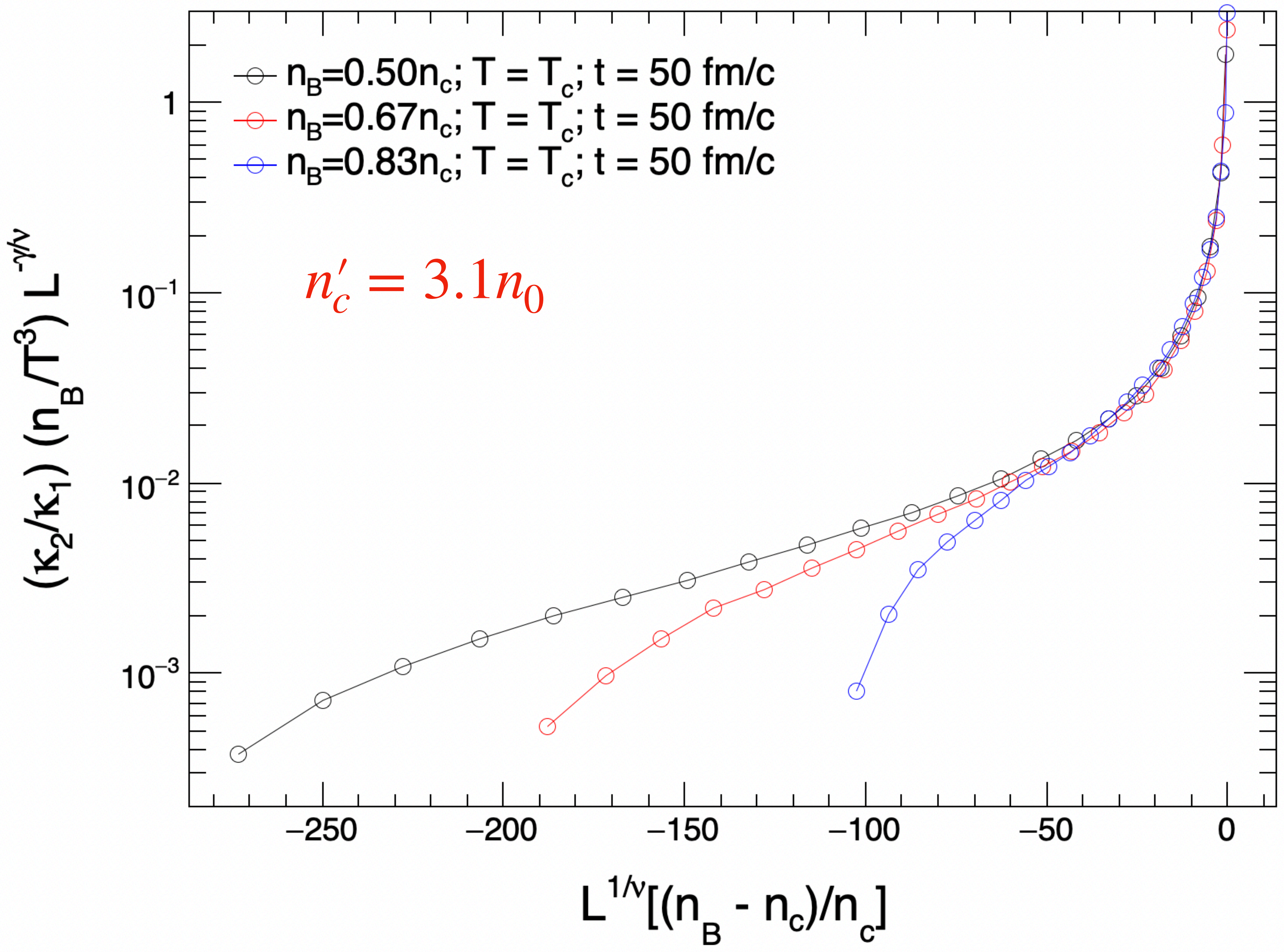
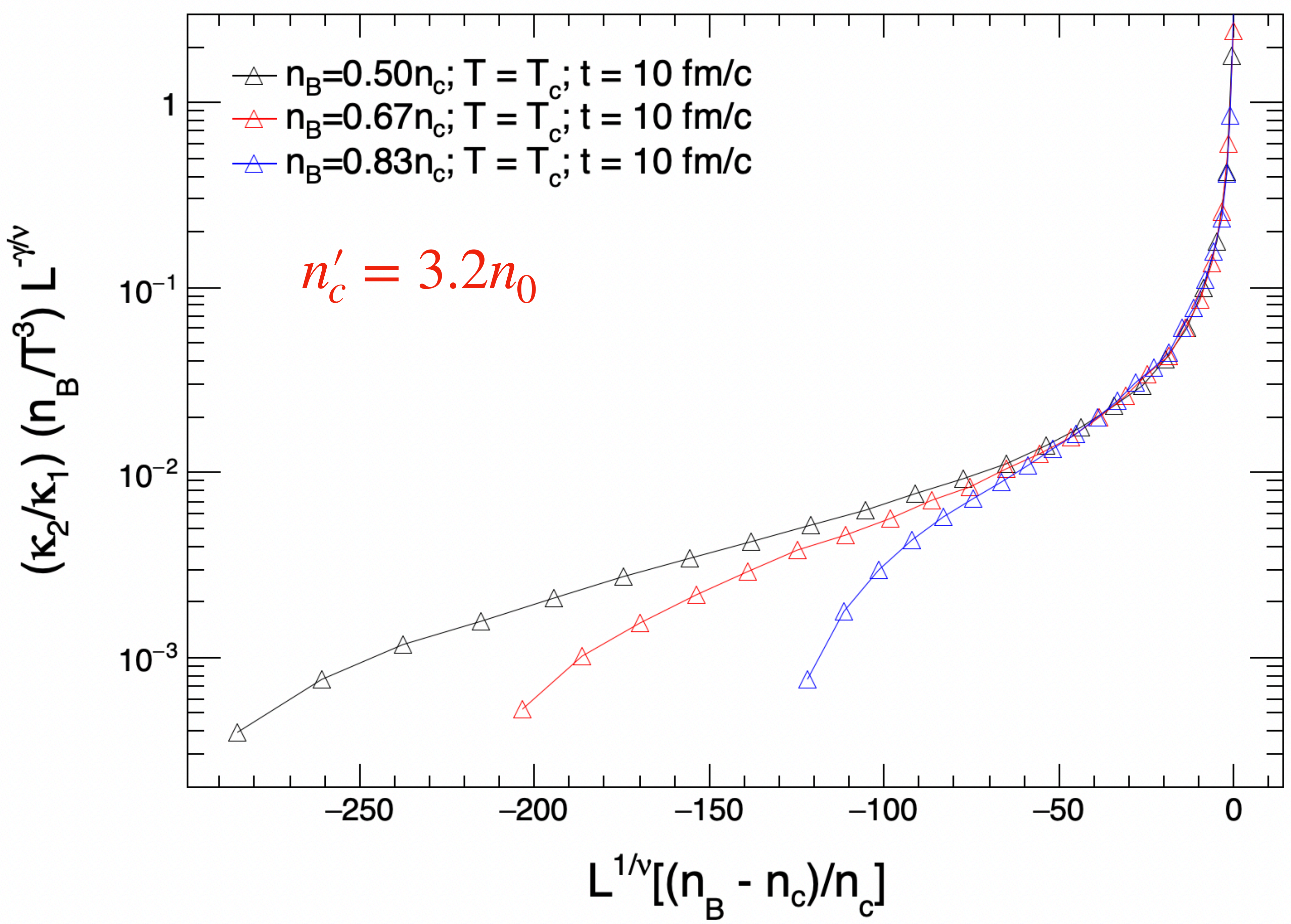


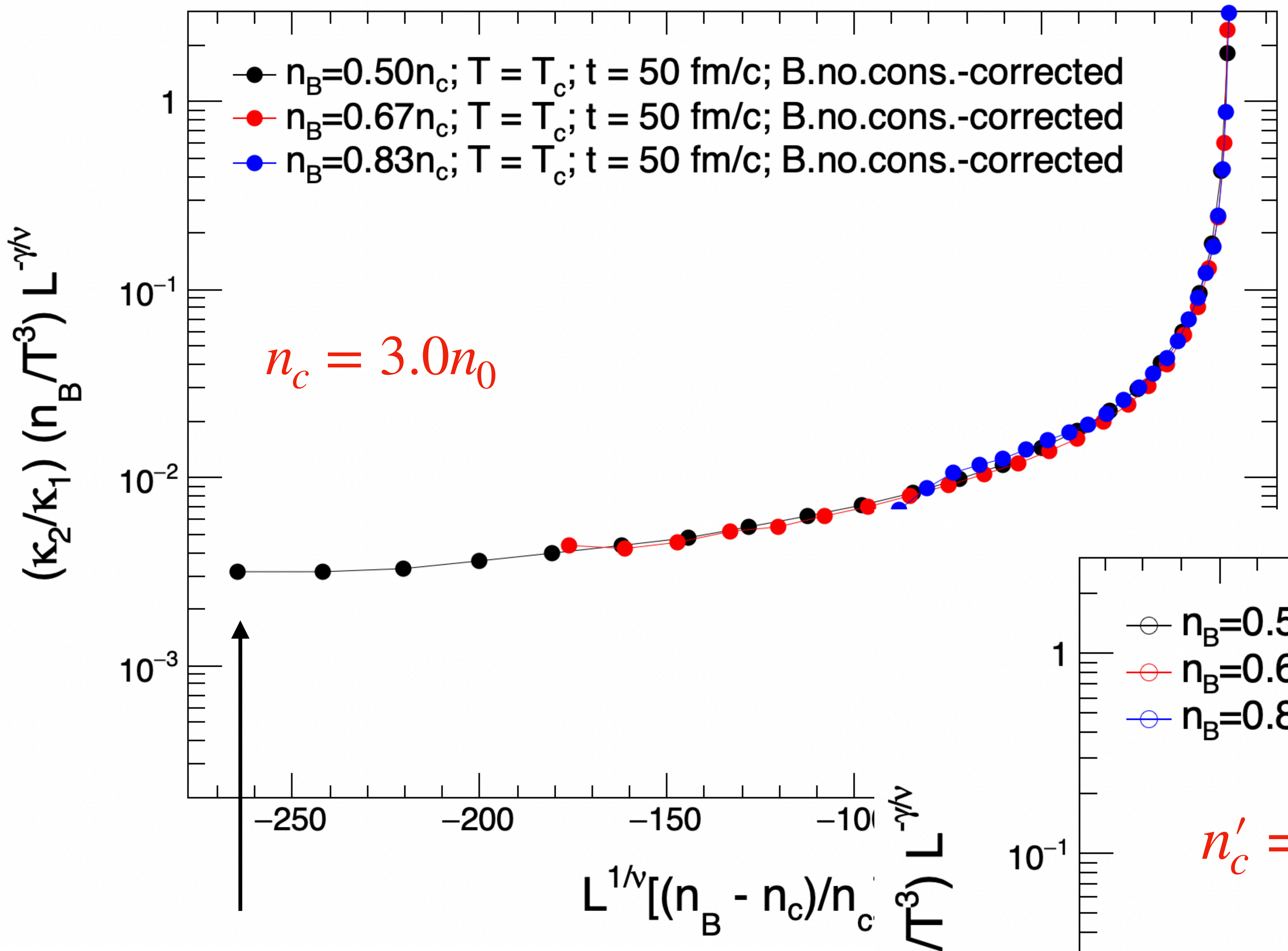




Finite-size scaling analysis of cumulants in a periodic box

$L_{\text{box}} = 24$ fm

smallest L



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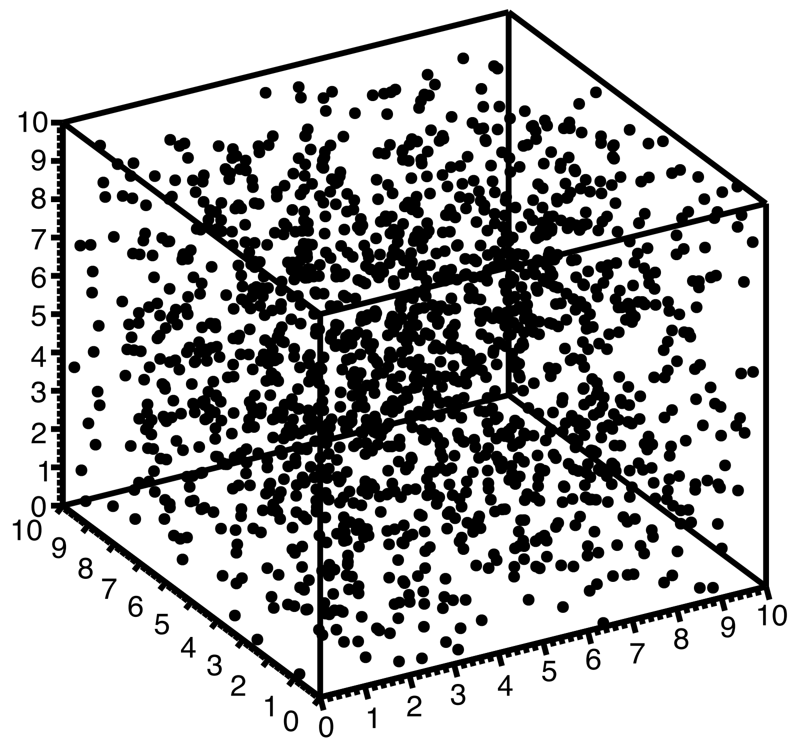
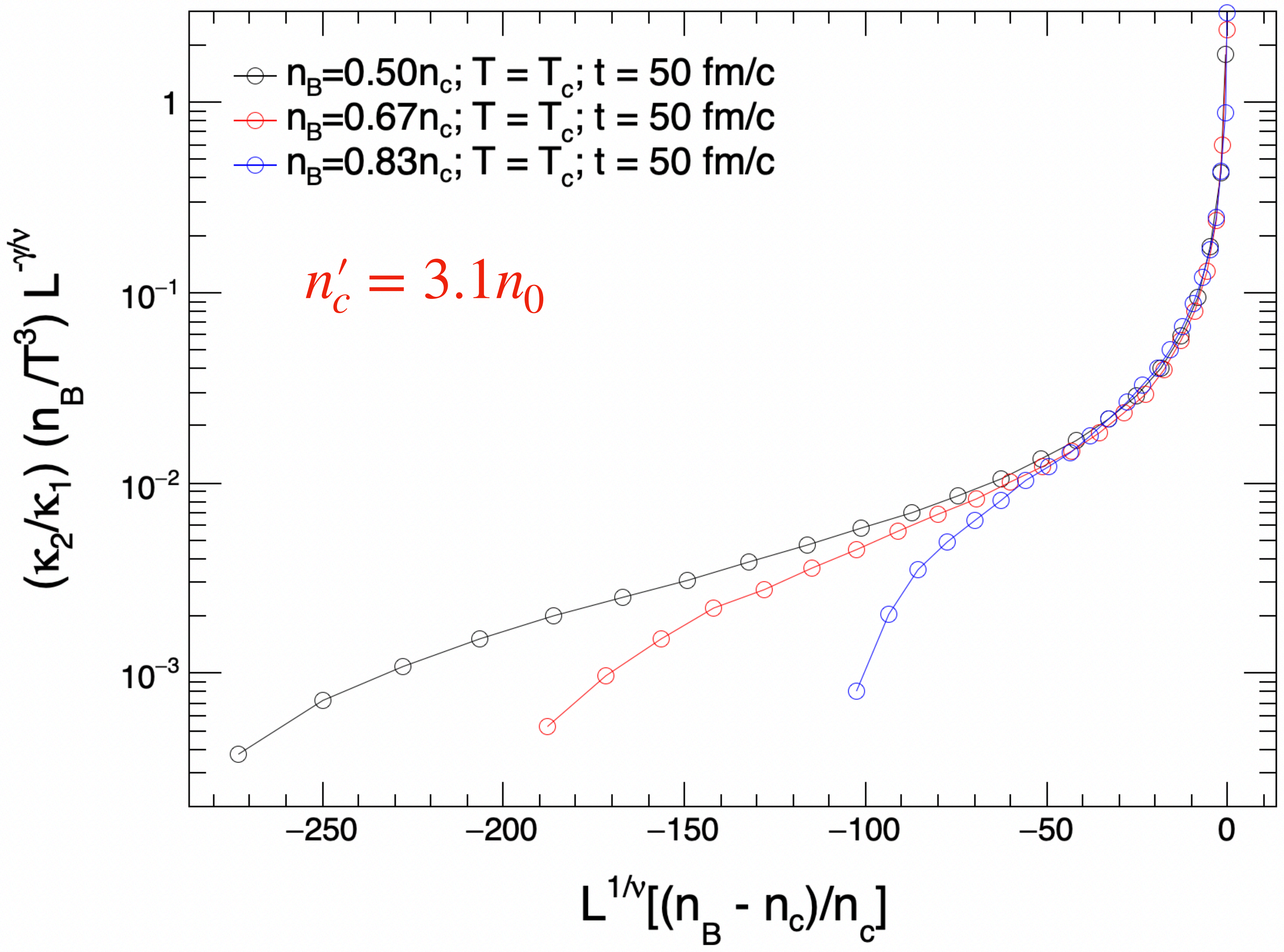
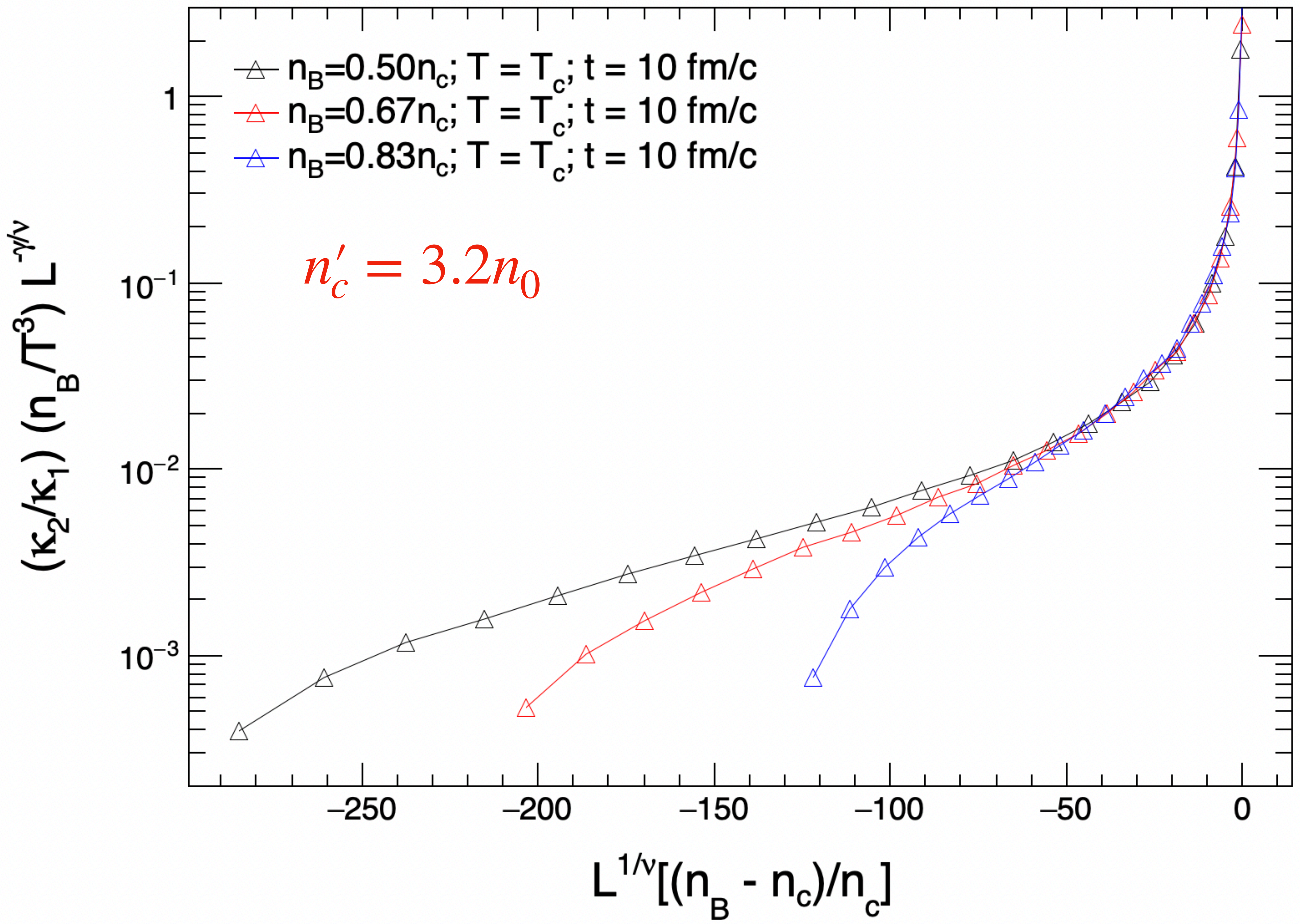


Baryon number conservation & finite time effects suppressed at $V'/V < 1/4$





Results using data from BES-I

Thermal model

$$\chi_2 = \frac{C_2}{T^3 V} \quad \Rightarrow \quad \chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$$

- We use rapidity bin width W as the subsystem size
- We used published thermal model fits for $T_{\rm fo}$ and $\mu_{B,\rm fo}$
- We parameterize $dV_{\rm fo}/dy$ from several publications. For 2.4 GeV, $T_{\rm fo}^3 V$ is highly uncertain, ranging from about 65 to 650
- Experiments can improve results by publishing $dV_{\rm fo}/dy$, $T_{\rm fo}$ and $\mu_{B,\rm fo}$ from thermal model fits for specific W

$\sqrt{s_{\rm NN}}$ (GeV)	$y_{\rm beam}$	$\mu_{\rm fo}$ (GeV)	$T_{\rm fo}$ (GeV)	$dV_{\rm fo}/dy$ (fm^3)
2.4	0.73	0.776	0.050	17157
3.0	1.05	0.720	0.080	4850
7.7	2.09	0.398	0.144	1044
11.5	2.50	0.287	0.149	1047
14.5	2.73	0.264	0.152	1080
19.6	3.04	0.188	0.154	1137
27	3.36	0.144	0.155	1218
39	3.73	0.103	0.156	1341
54.4	4.06	0.083	0.160	1487

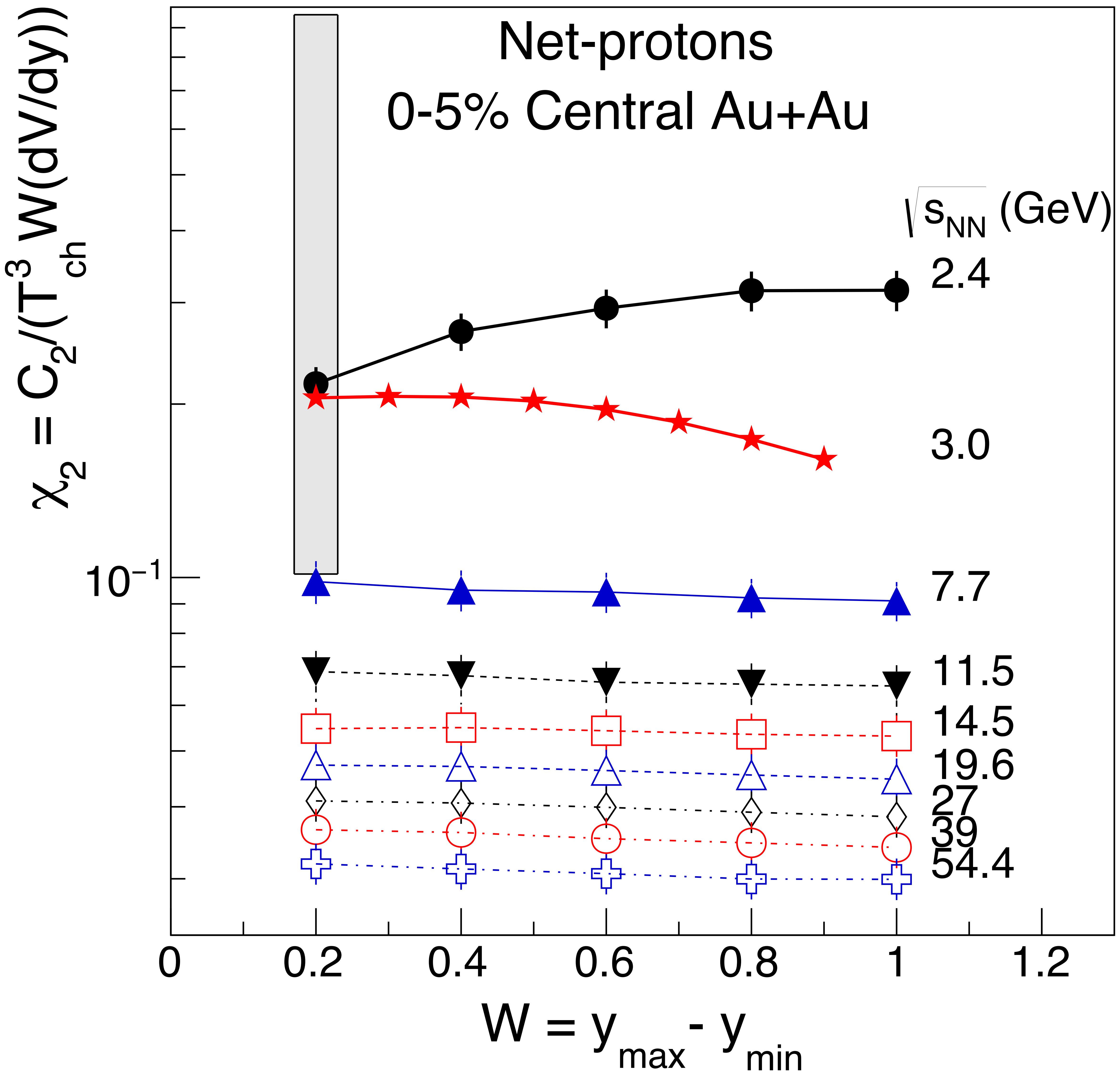
J. Adamczewski-Musch et al. (HADES), Phys. Rev. C 102, 024914 (2020)
M. Abdallah *et al.* (STAR), Phys. Rev. C 104, 024902 (2021)
M. Abdallah *et al.* (STAR), Phys. Rev. C 107, 024908 (2023)
A. Andronic, P. Braun-Munzinger, J. Stachel, Acta Phys. Polon. B 40, 1005-1012 (2009)
A. Motornenko *et al.*, Phys. Lett. B 822, 136703 (2021)
S. Chatterjee *et al.*, Adv. High Energy Phys. 2015, 349013 (2015)

Susceptibility

$$\chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$$

- Grey band shows uncertainty from freeze-out ambiguities for the 2.4 GeV data. Uncertainty precludes any conclusion about observing a maximum in χ_2

- Data do indicate a change in slope at higher μ_B and at small W: χ_2 decreases with increasing W for 7.7-54.4 GeV but changes slope at 2.4 GeV (3.0 GeV is ~flat)

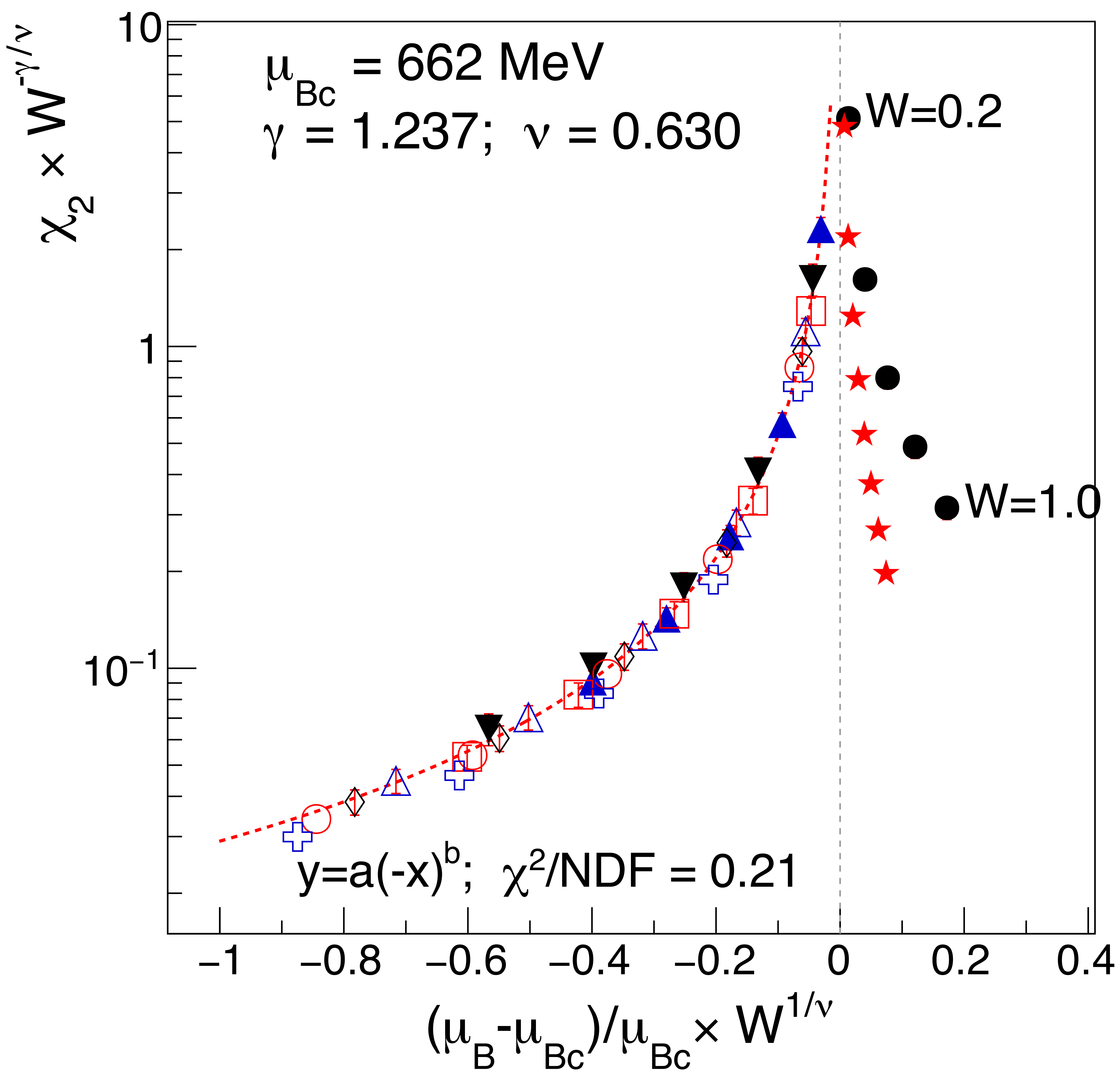


Scaled susceptibility

$$\chi_2(W,m) = W^{\gamma/\nu}\Phi(mW^{1/\nu})$$
$$m = (\mu_B - \mu_{B,c})/\mu_{B,c}$$

- Good scaling for negative m
- Low energy points do not scale well
- Scaling function Φ is well described by a power law
- This scaling neglects variation of $t = (T - T_c)/T_c$; not a bad approximation for 7.7 GeV and above, but worse for 2.4 and 3.0 GeV

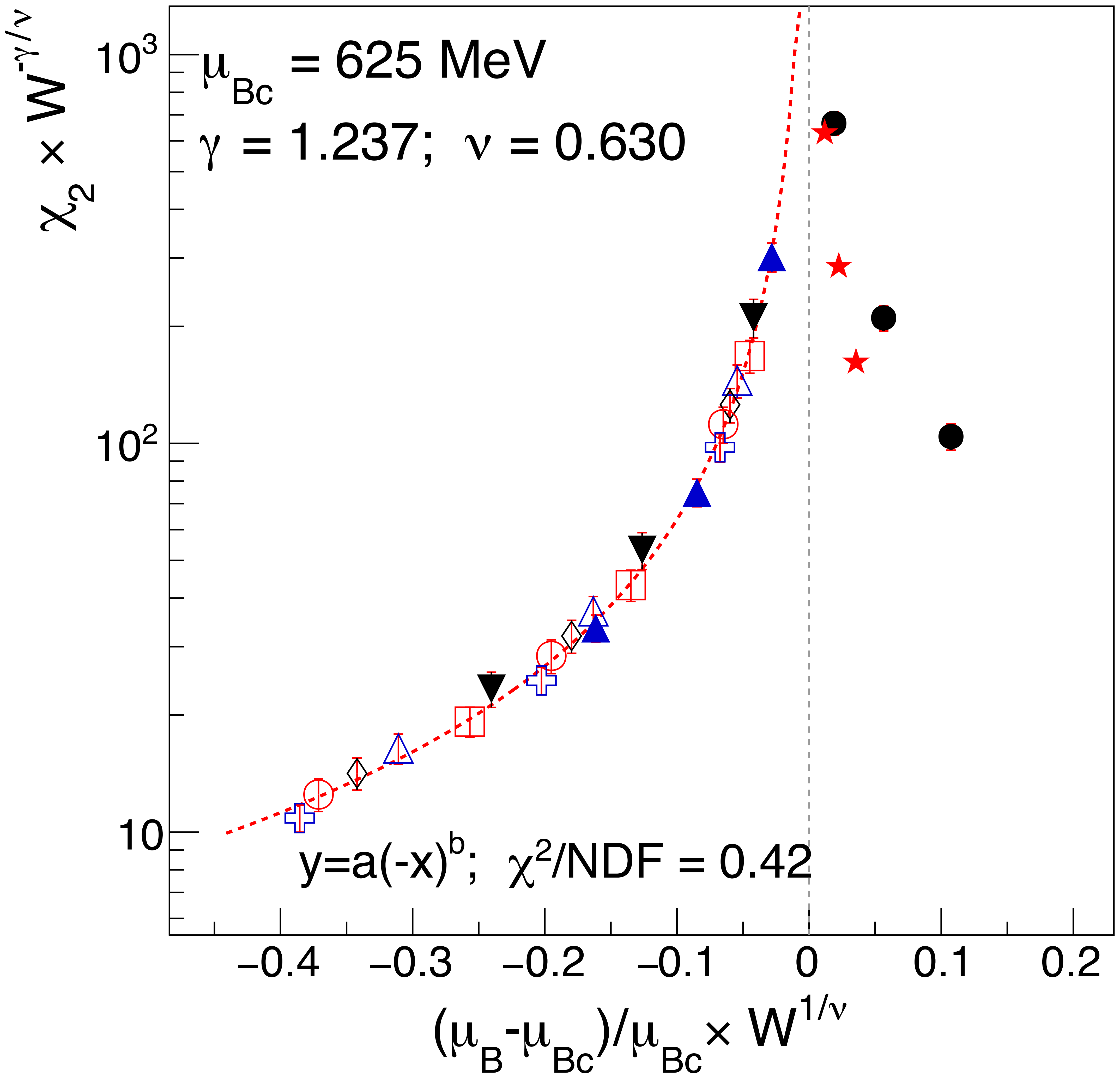
$$\mu_{B,c} = 662 \pm 22 \text{ MeV}$$



Scaled susceptibility: excluding widest bins

- Our simulations showed that baryon number conservation may spoil the scaling for larger values of *W*
- Excluding W=0.8 and 1.0 reduces the $\mu_{B,c}$ (as was expected from simulations)
- The fraction of measured baryons to total baryons is likely well below 25% for all these points except the 2.4 and 3.0 GeV data (not in the fit)

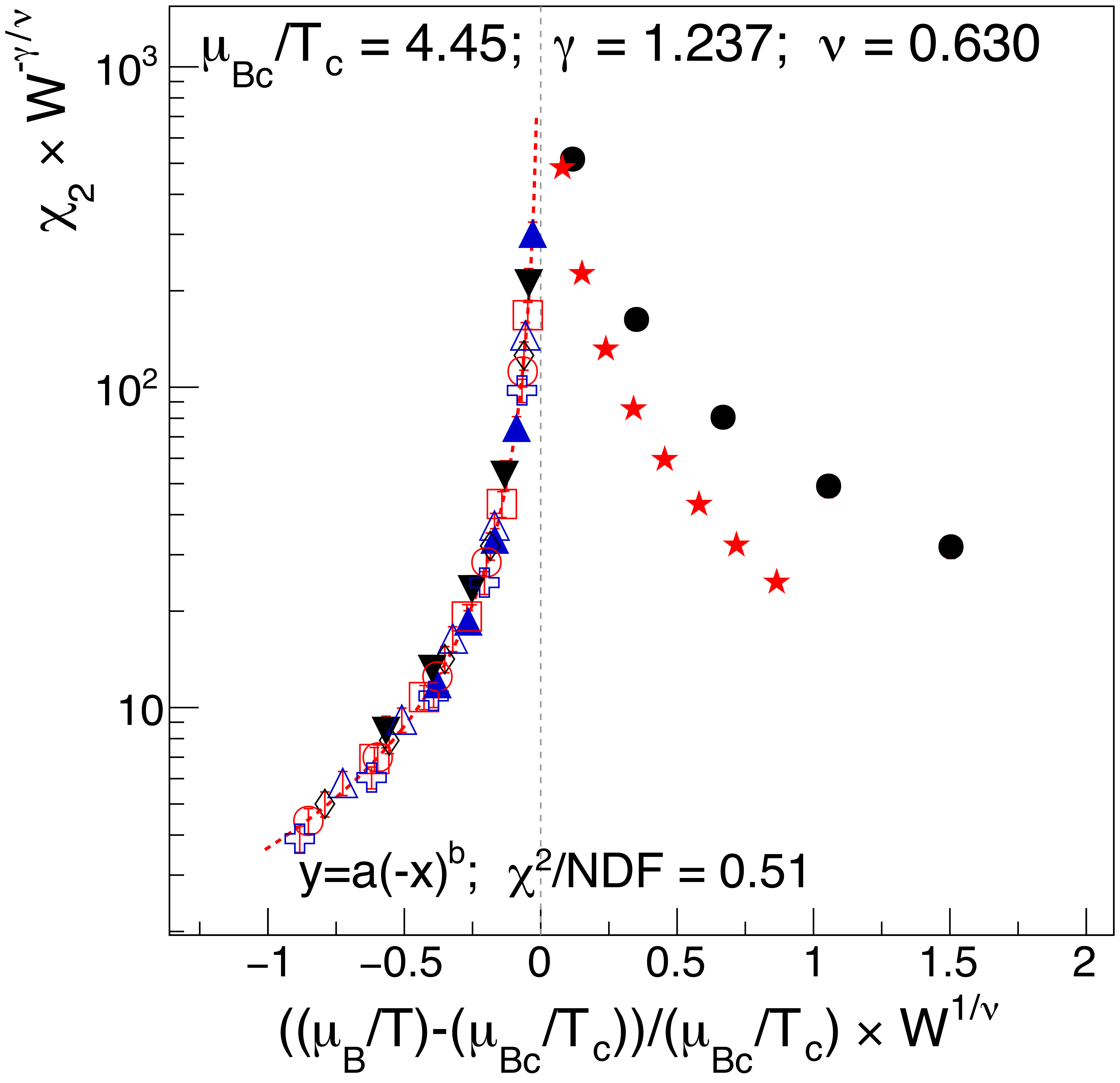
$\mu_{B,c} = 625 \pm 20$ MeV



Scaled susceptibility: μ_B/T fit

- To explore factoring in the temperature dependence used $(r - r_c)/r_c$ where $r = \mu/T$
- From that, we extract $T_c = 140 \pm 13$ MeV

$\mu_{B,c}/T_c = 4.45 \pm 0.12$

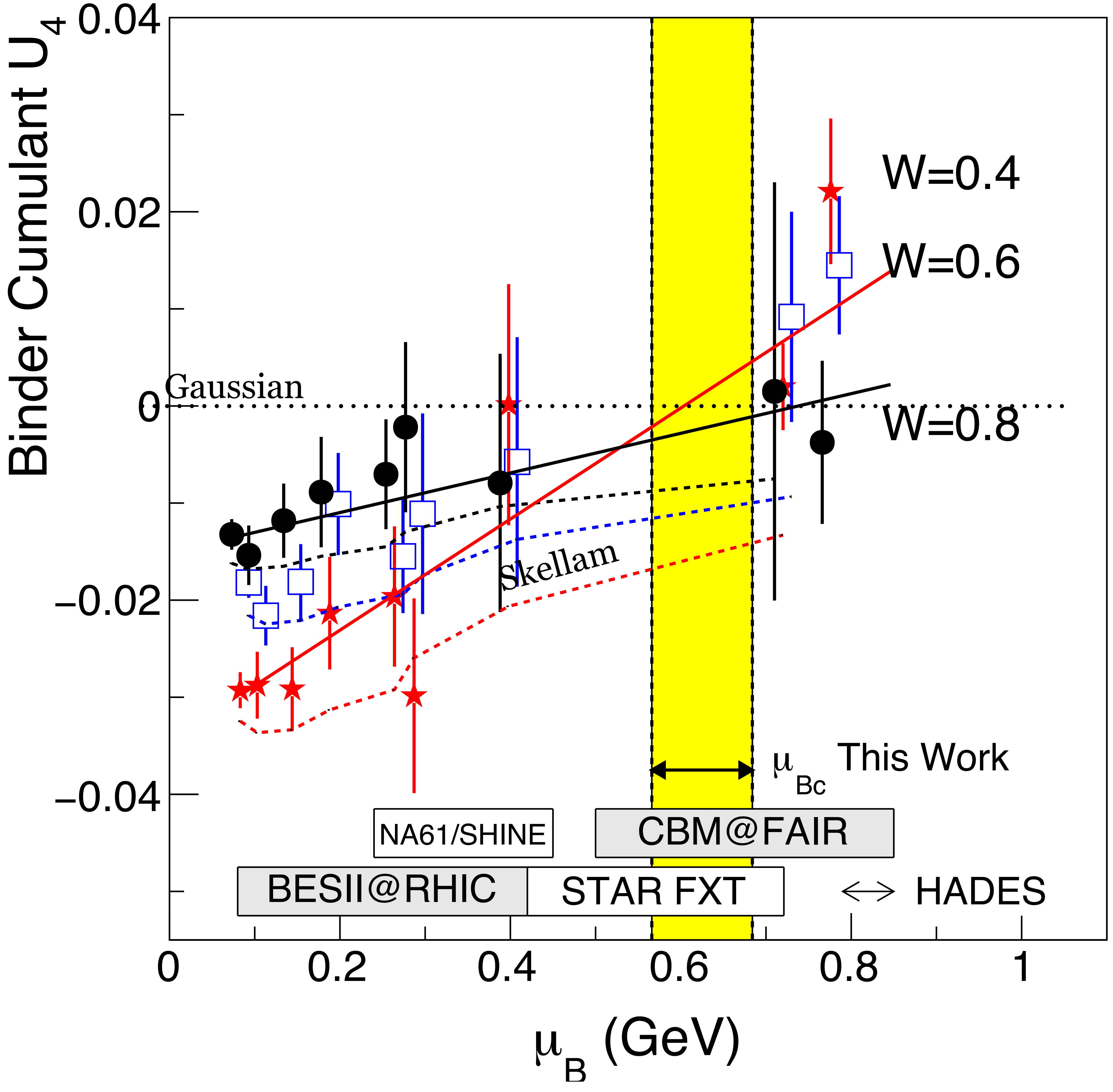


Binder cumulants

K. Binder, Z. Phys. B 43, 119 (1981).

$$U_4 = -C_4/(3C_2^2)$$

- Expectation: $U_4 = 0$ (Gaussian), 2/3 (bimodal), crosses at the critical point
$$U_4 \approx a_1 + a_2(\mu_B - \mu_{B,c})W^{1/\nu}$$
- At low μ_B, U_4 follows Skellam with $U_4(W=0.8) > U_4(W=0.6) > U_4(W=0.4)$
- At $\mu_B > 400$ MeV, the ordering appears to reverse
- Data are consistent with a critical point between μ_B of 400 and 800 MeV



Summary

- Simulations show that window-size analysis works: effects due to finite time, baryon number conservation can be controlled by considering less than ~25% of the total volume
- We observe finite-size scaling for χ_2 extracted from 7.7-54.4 GeV data:
  we obtain $\mu_B \approx 625 \pm 60$ MeV and $T_c = 140 \pm 13$ MeV
- We explored a variety of fit ansaetze: μ_B , μ_B/T , (μ_B, T), different critical exponents…

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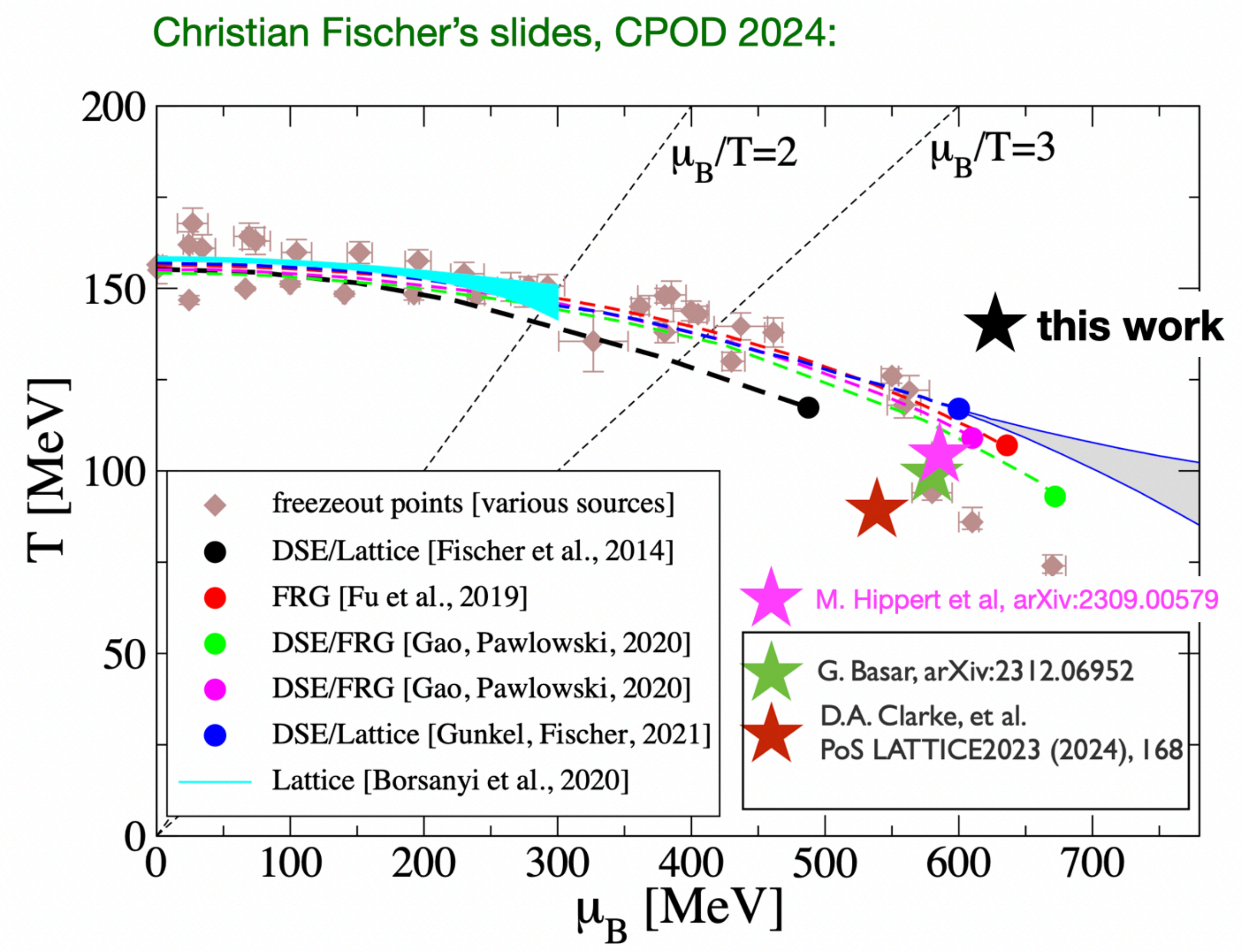
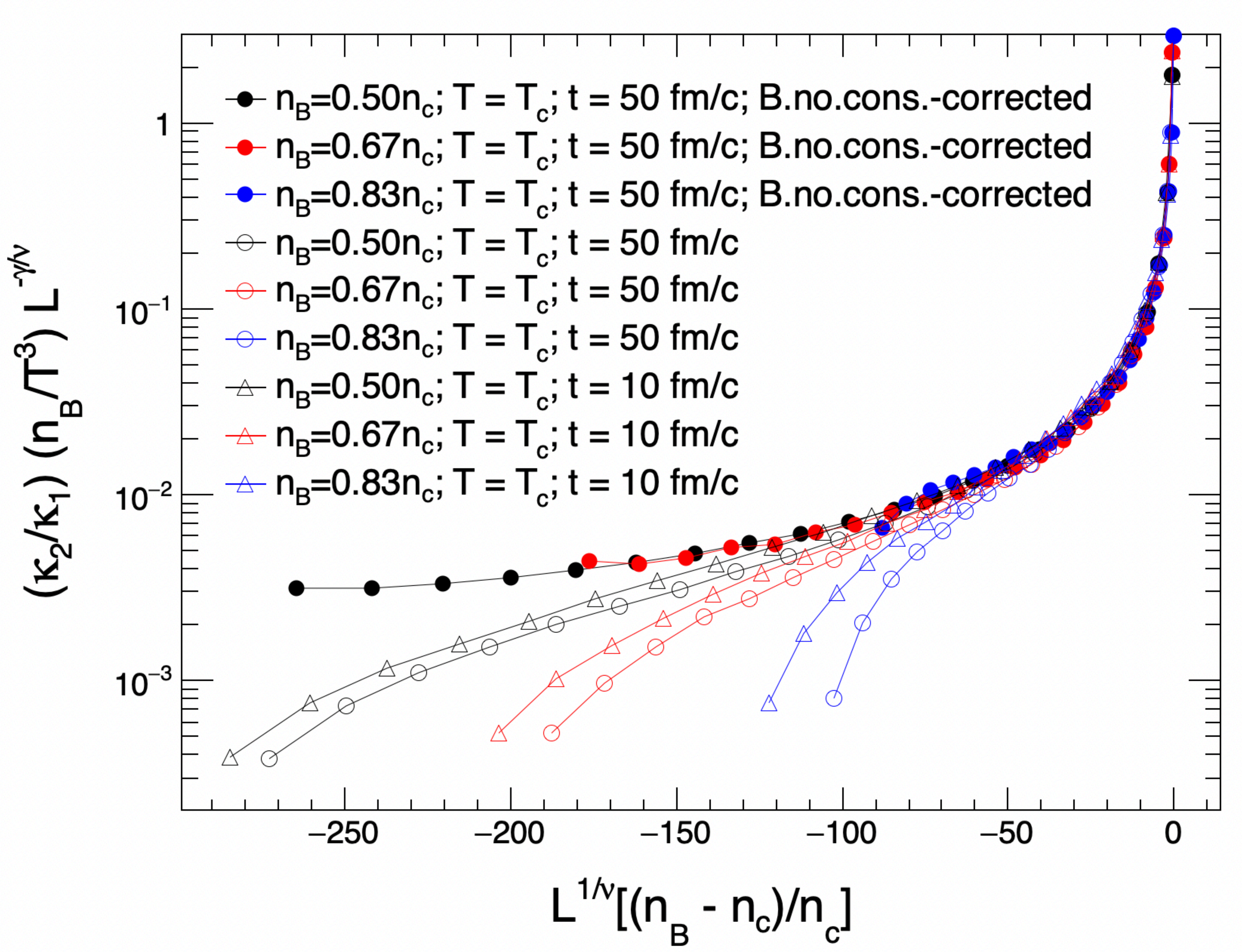
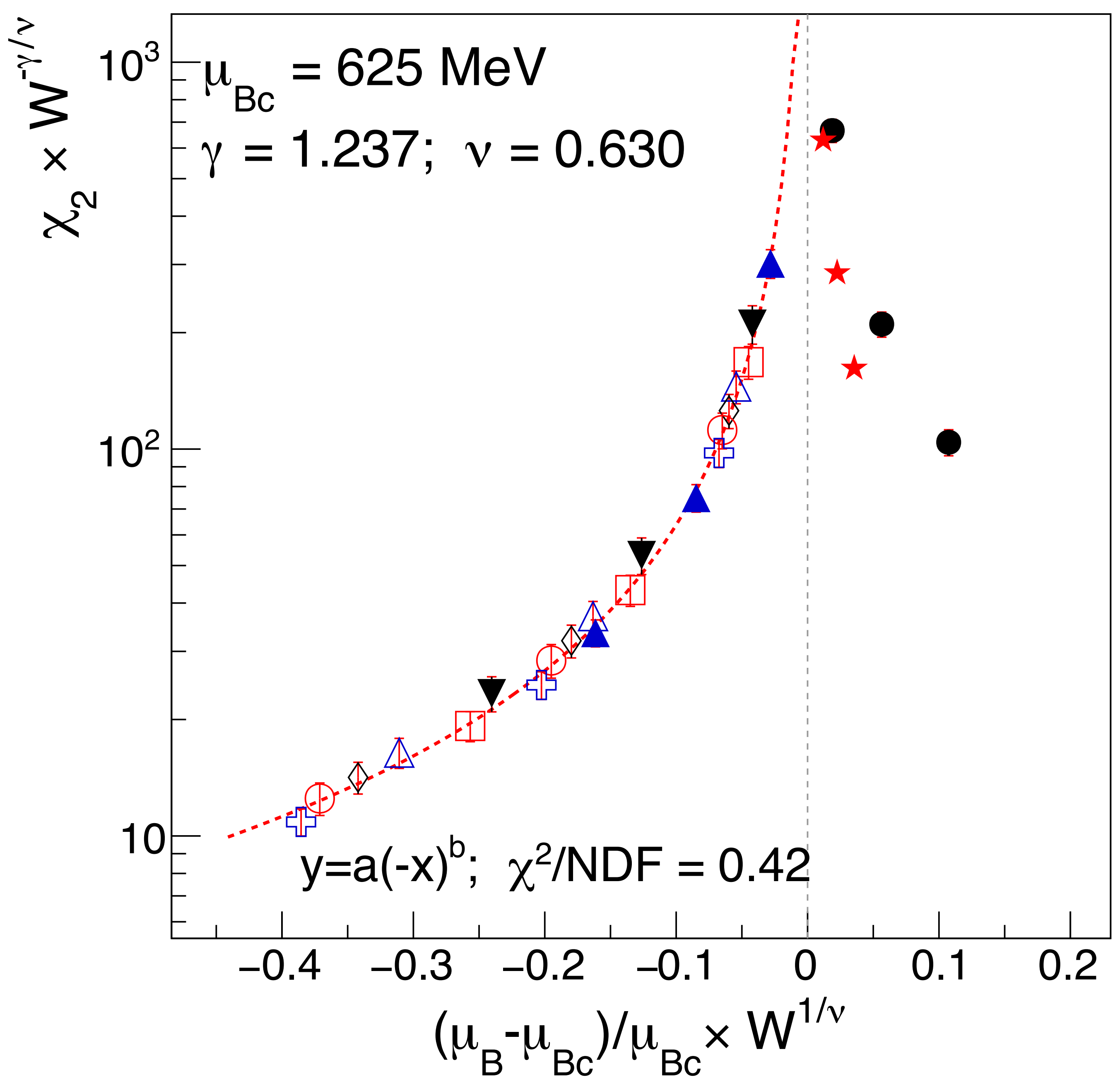


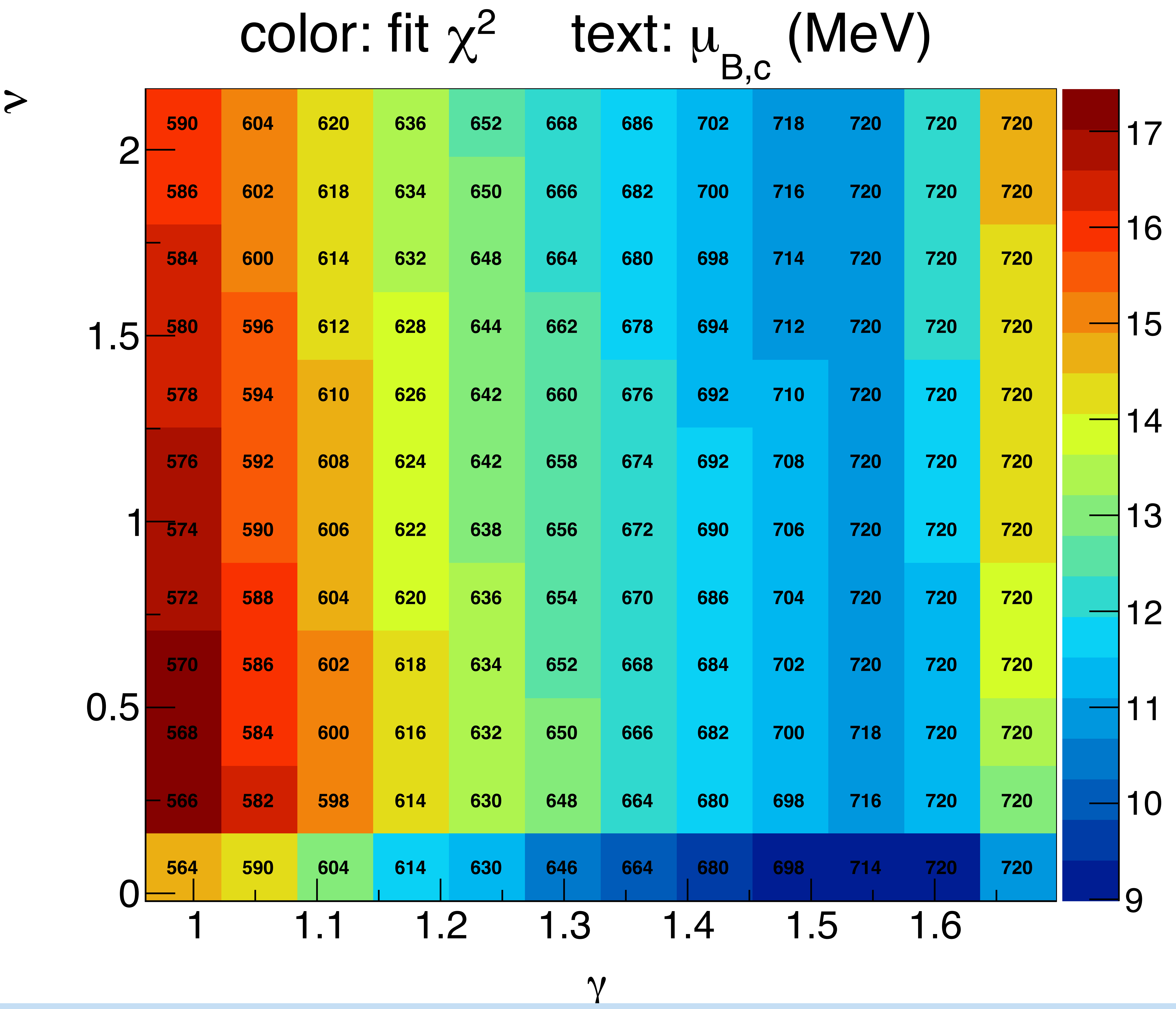




Thank you
for your attention

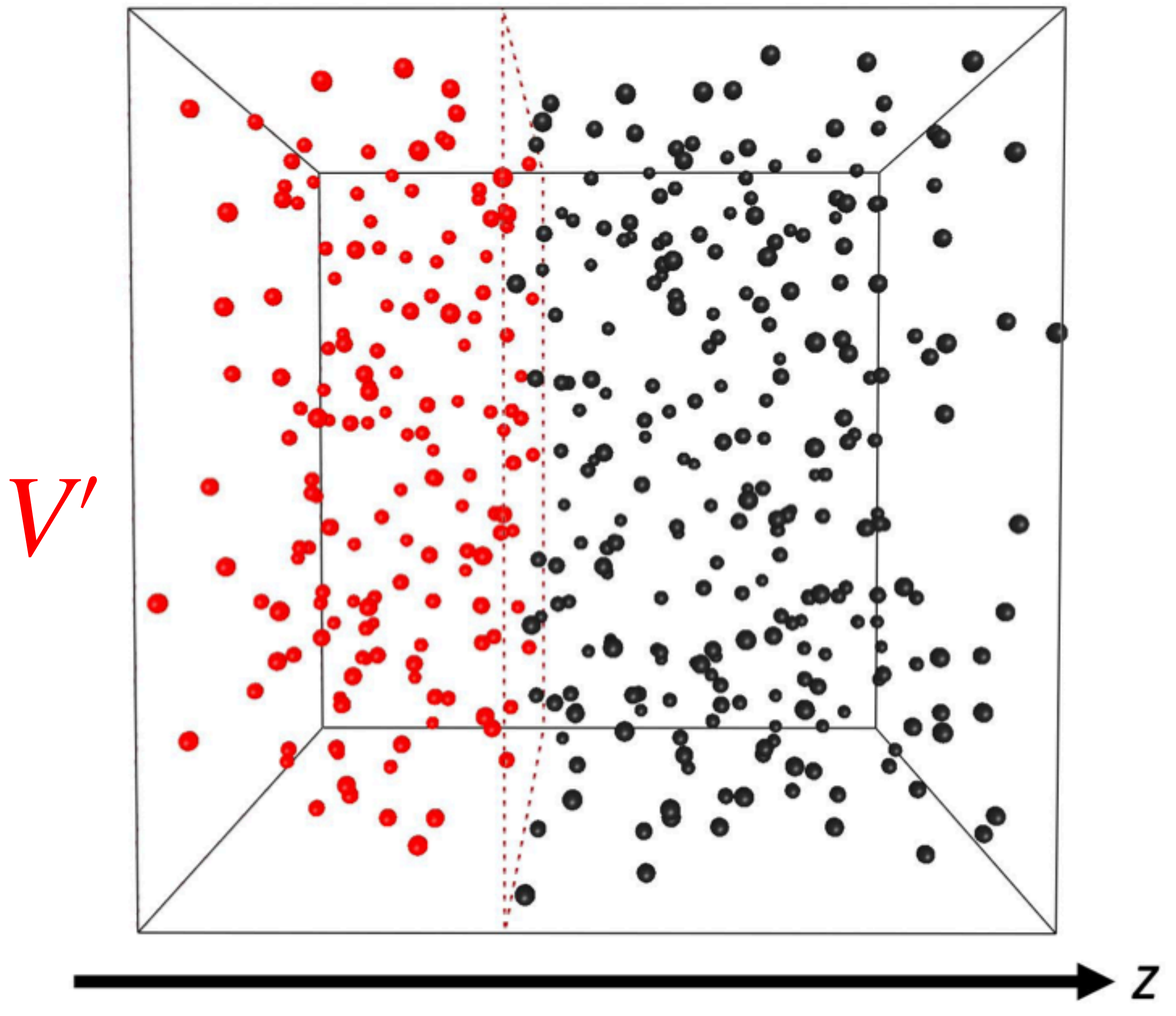
Different critical exponents

- We explored a broad range of critical exponents including mean-field (1.0, 0.5)
- For each selected critical exponent pair, we find the temperature that minimizes the Chi-square
- Chi-square is shown in color and $\mu_{B,c}$ as text
- Most results are satisfactory Chi-square values so we do not interpret the Chi-square valley as necessarily providing the correct exponents

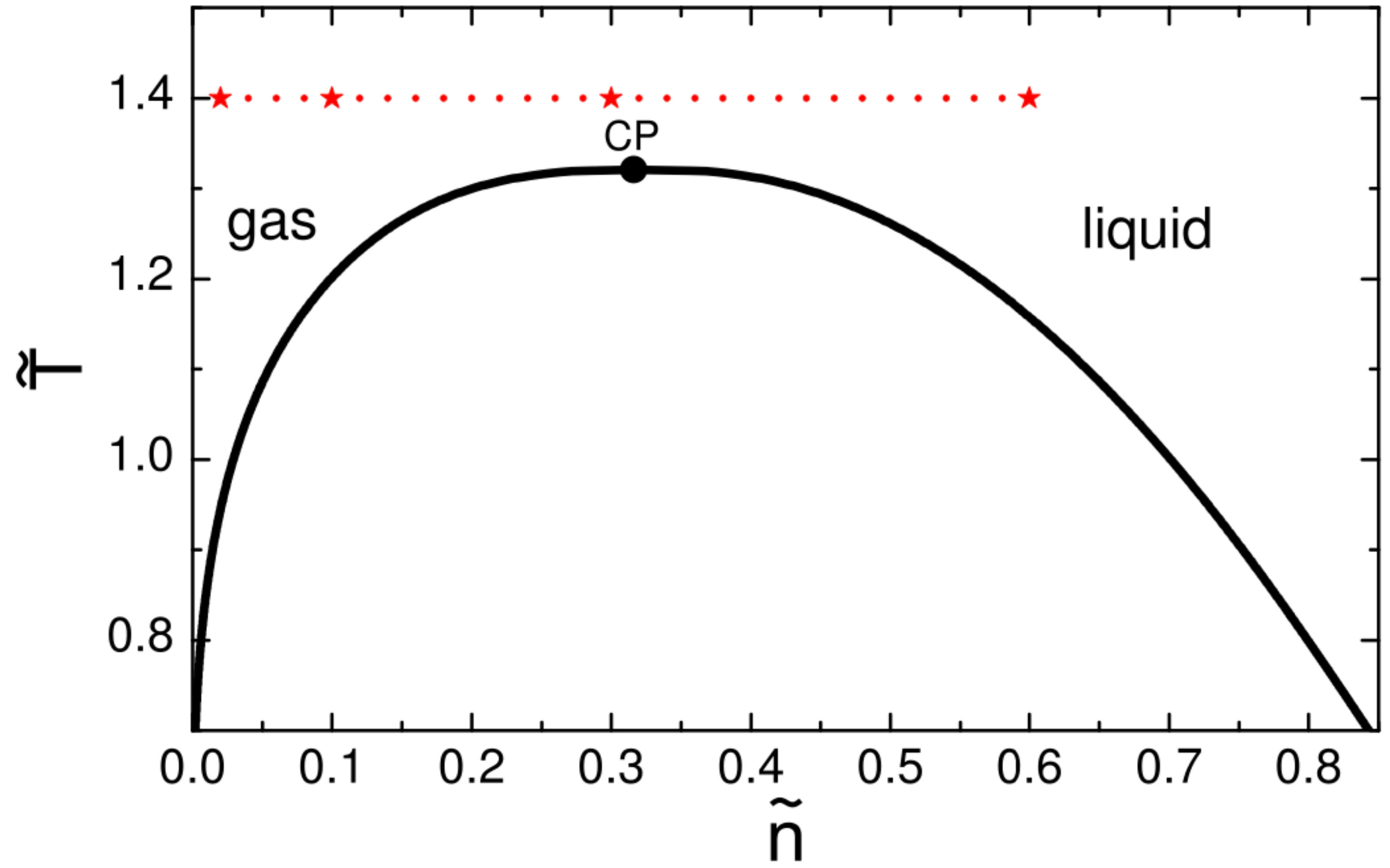


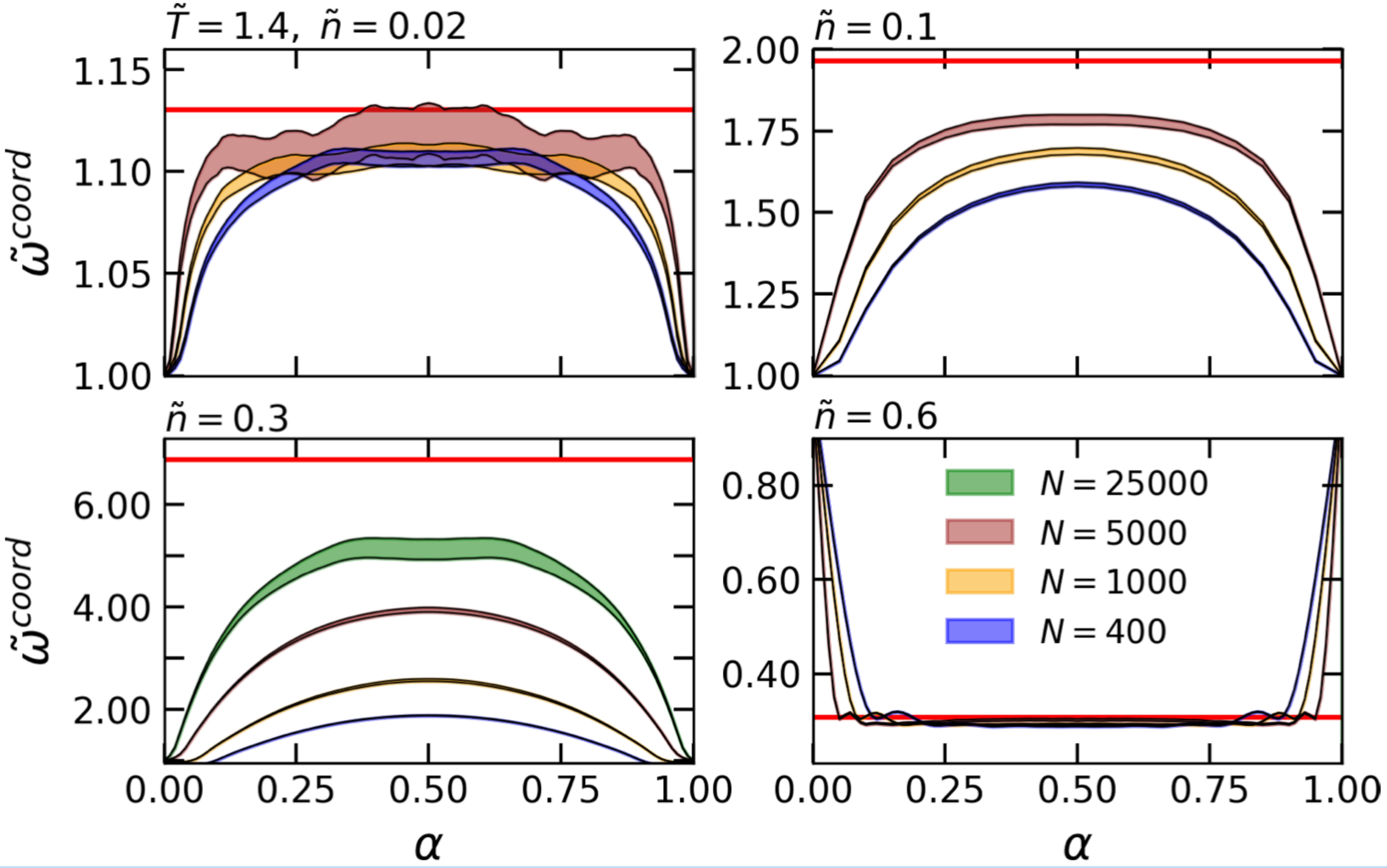
Motivation for VDF studies: cumulants in molecular dynamics

We will use some insights from V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, V. Vovchenko, Phys. Rev. C **105** no.4, 044903 (2022), arXiv:2201.08486



$$\omega = \frac{\kappa_2}{\kappa_1} \qquad \alpha = V'/V \qquad \tilde{\omega}^{coord} = \frac{\omega^{coord}}{1-\alpha} = \omega^{gce}$$





Behavior near a critical point

- Critical point (CP):
  a single point in the phase diagram where change from an ordered to disordered phase occurs

- The endpoint of a 1st order phase transition

  As systems approach the CP, latent heat decreases
  ⇒ it costs little energy for components of one phase to form a local "bubble" of the other phase
  ⇒ as CP is approached, correlation length ξ increases = large fluctuations (large bubbles)
  ⇒ critical opalescence phenomenon:
  → "bubbles" grow to sizes comparable with visible light wavelengths ($\xi \approx \lambda$)
  → light can be scattered and a translucent system becomes cloudy (like fog)
  ⇒ at CP, correlation length formally diverges;
  system experiences correlations of all sizes
  (proof: critical opalescence in methanol+cyclohexane persists at CP where ξ ~ 1 cm)