Impact of Lattice QCD Data at Large Isospin Density on Baryon-Rich QCD Matter

Yuki Fujimoto
(University of Washington)

INSTITUTE for NUCLEAR THEORY

References:
Y. Fujimoto, S. Reddy, PRD 109 (2024) (Selected for Editors' suggestion) [2310.09427]
Y. Fujimoto, PRD109 (2024) [2312.11443]; in preparation [2405.?????]

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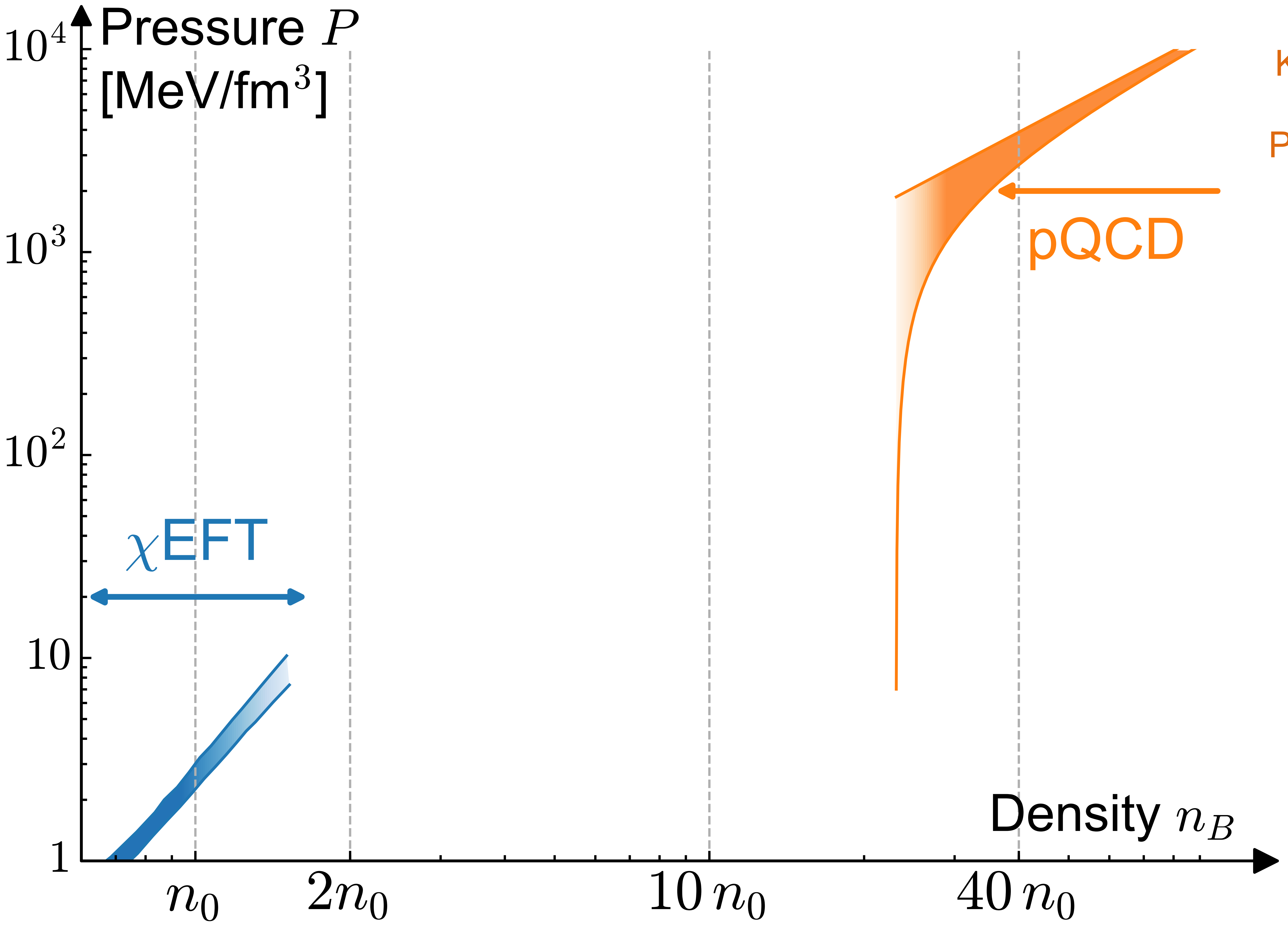
QCD equation of state at finite baryon density

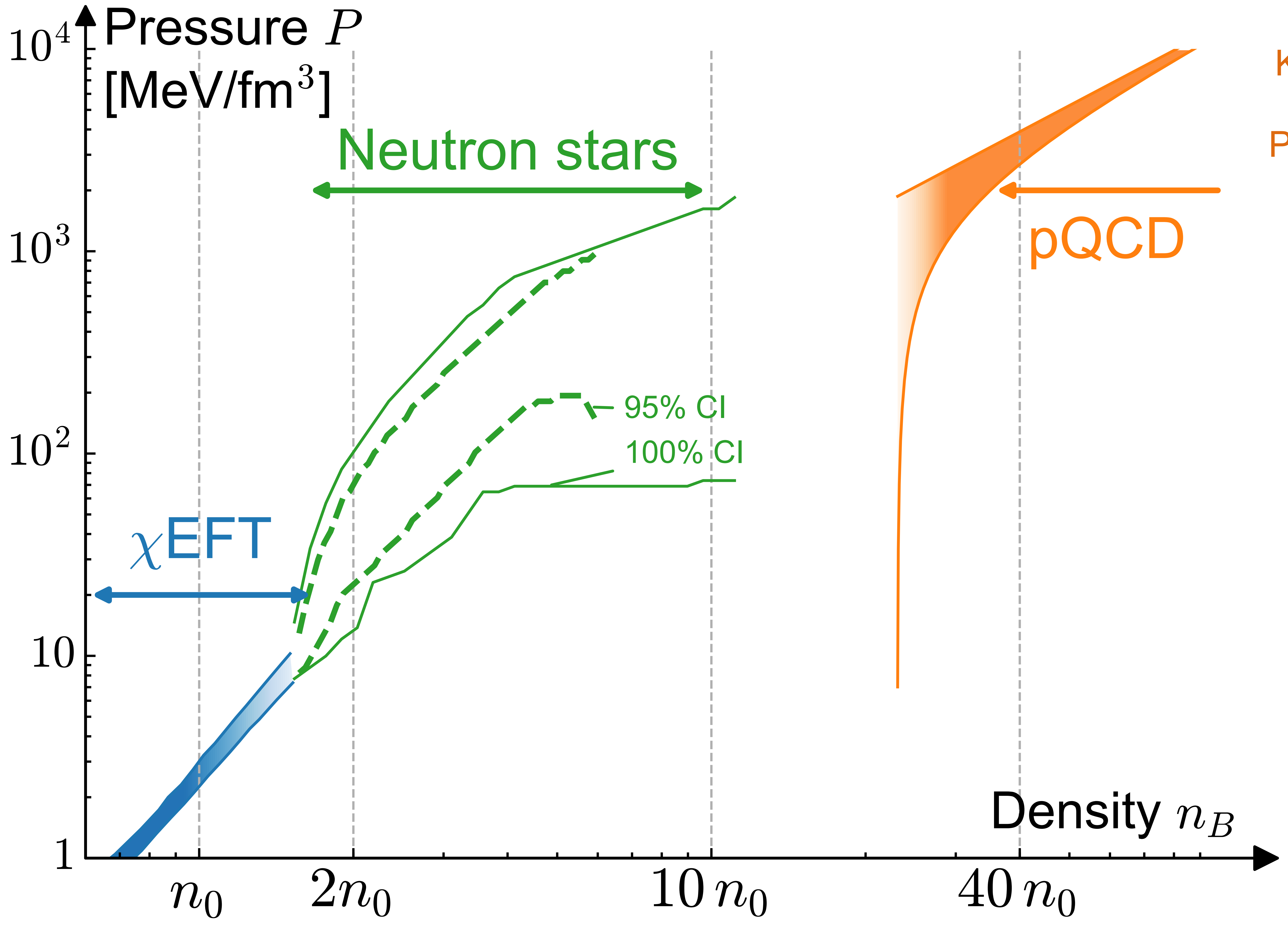
Freedman,McLerran(1978);
Baluni(1979);
Kurkela,Romatschke,Vuorinen,
Gorda,Säppi,
Paatelainen,Seppänen+(2009-)



Tews et al.(2013);
Drischler et al. (2020);
Keller,Hebeler,Schwenk(2022);
& many others

Saturation density: $n_0 = 0.16$ fm^{-3}

QCD equation of state at finite baryon density



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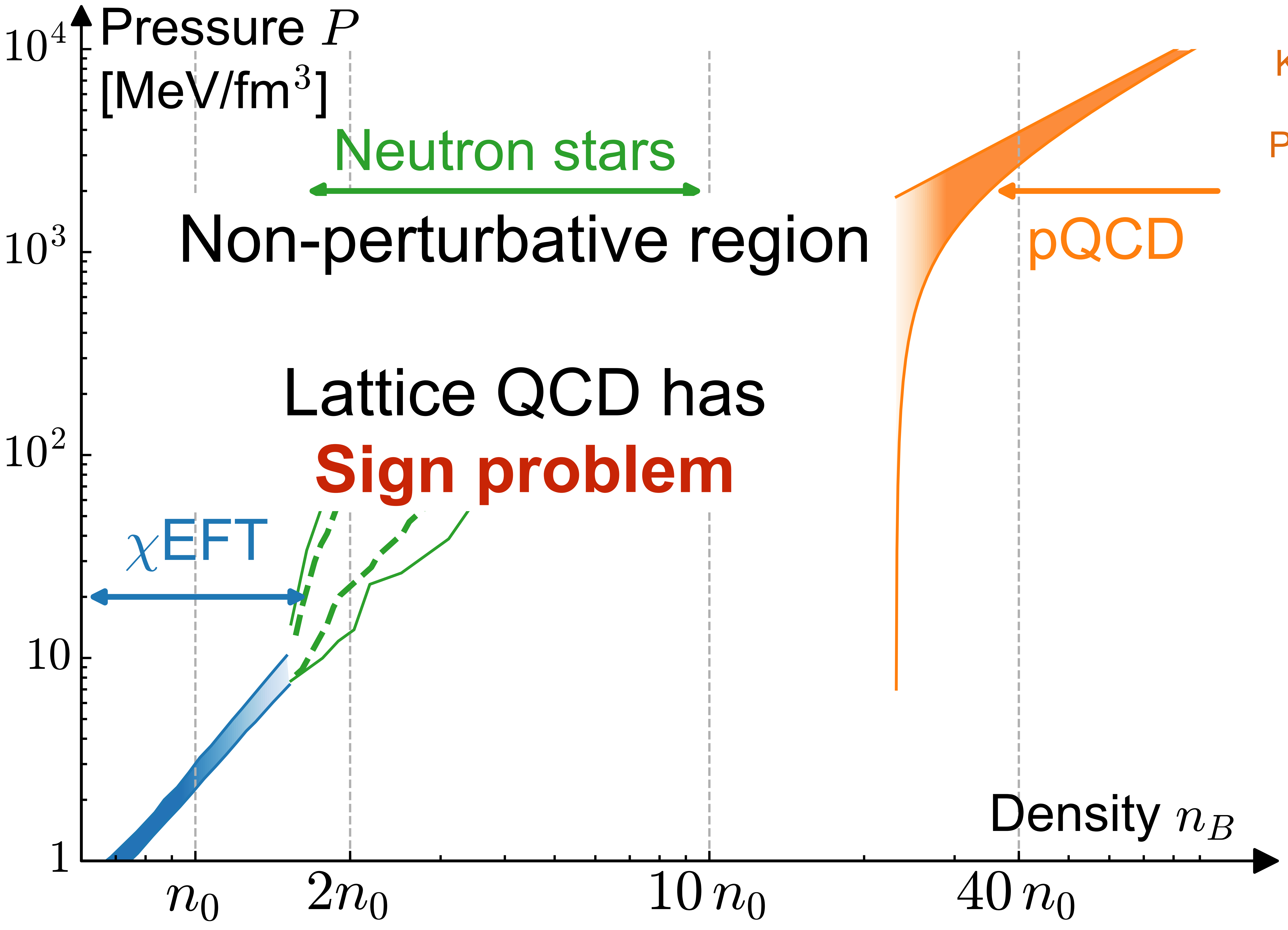

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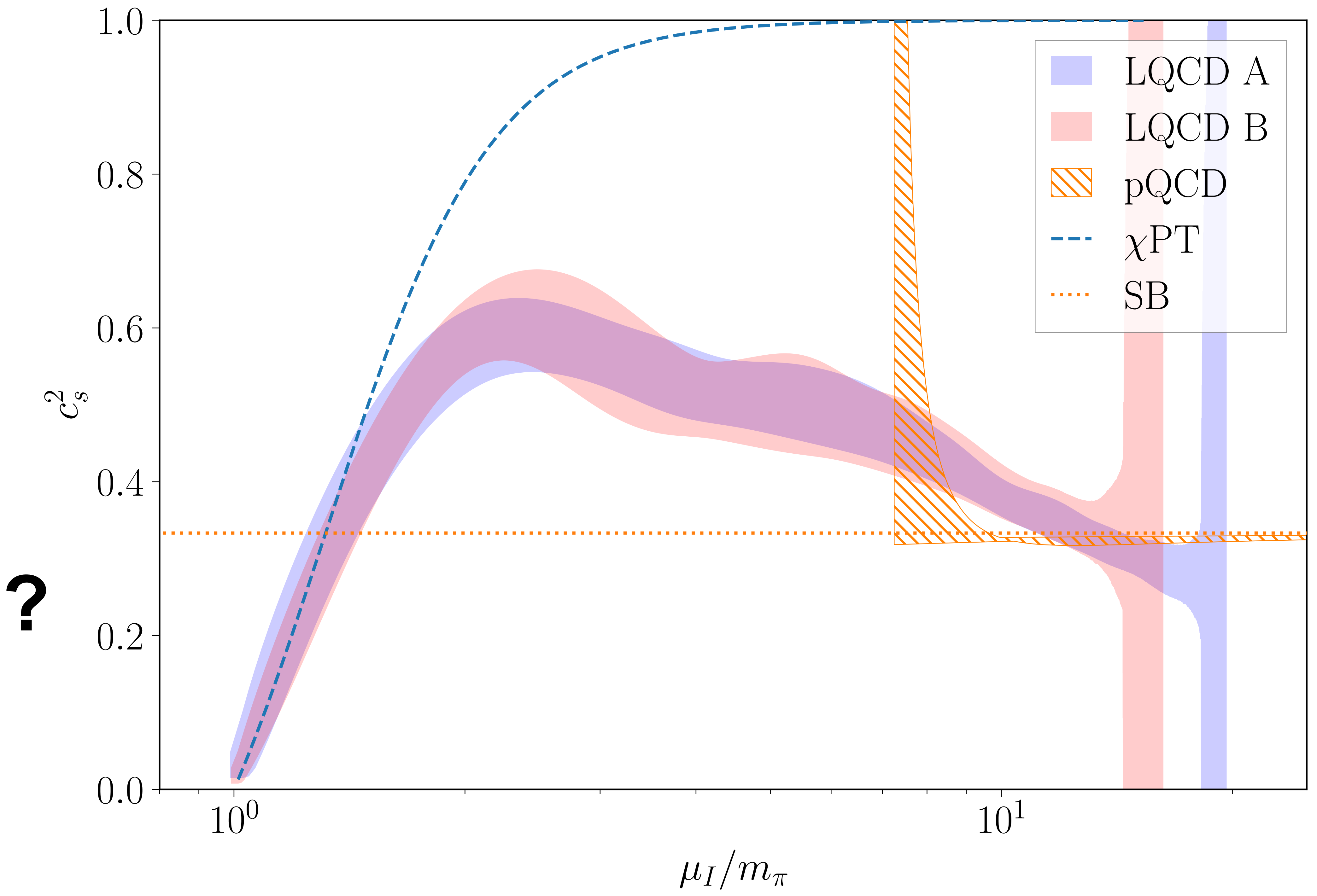
Exceptional case: QCD at finite *isospin* density

Alford,Kapustin,Wilczek (1999); Son,Stephanov (2000); Kogut,Sinclair (2002-);
Beane,Detmold,Savage et al. (NPLQCD) (2007-); Endrődi et al. (2014-)…

- **Isospin chemical potential:** $\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$ … Fermi surface of u & $\bar{d}$
  (conjugate to isospin density I_3)
  NO sign problem → can be simulated on the lattice!

- Recent impact:
  EoS up to $n_I \sim 180\, n_{\rm sat}$
  by lattice QCD

What can we learn from this data?



Abbott et al. (NPLQCD) (2023)

Outline

1. **Bounds on isospin symmetric EoS from QCD inequality**

   Y. Fujimoto, S. Reddy, PRD109 (2024)

2. **Comparison with weak-coupling results**

   Y. Fujimoto, PRD109 (2024);
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Notation

- QCD_I: QCD at finite μ_I (and zero μ_B)
- QCD_B: QCD at finite μ_B (and zero μ_I)

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QCD inequality

Inequality among observables from path integrals Weingarten (1983); Witten (1983)

e.g. earlier application to QCD$_I$: constraint on the QCD critical point location in large N_c

Hidaka,Yamamoto (2011)

Inequality considered here:

QCD inequality for pressure $P \propto \log Z$:

$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Pressure of dense QCD$_B$ matter
(what we want to know)

Pressure of dense QCD$_I$ matter
(what we already know from lattice QCD)

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

QCD inequality: derivation

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA]\ \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_I}{2})\right|^2 e^{-S_G}$

u quark (first determinant) d quark (second determinant)

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA]\ \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c})\, e^{-S_G} = \int [dA] \,\mathrm{Re} \left[\det \mathscr{D}(\frac{\mu_B}{N_c})\right]^2 e^{-S_G}$

charge conjugation symmetry $\mu_B \to -\mu_B$

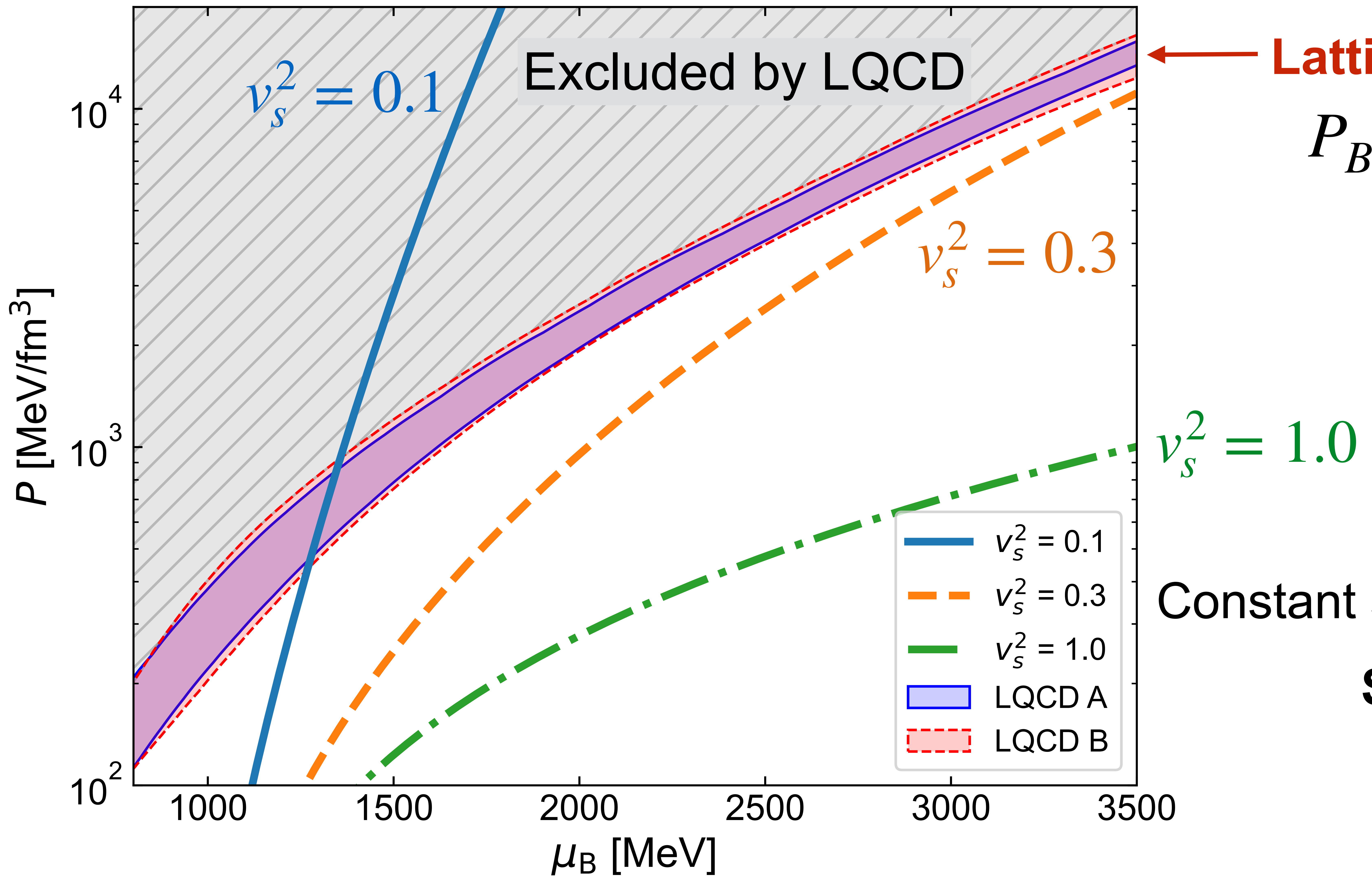
Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\mathrm{Re}\, z^2 \leq |z^2| = |z|^2$:

$$Z_B(\mu_B) \leq \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_B}{N_c})\right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound P_I

$$P_B(\mu_B) \le P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller v_s^2) is excluded

Robust bounds on $P(\varepsilon)$

Fujimoto, Reddy (2023)

From

Kurkela, Komoltsev (2021)

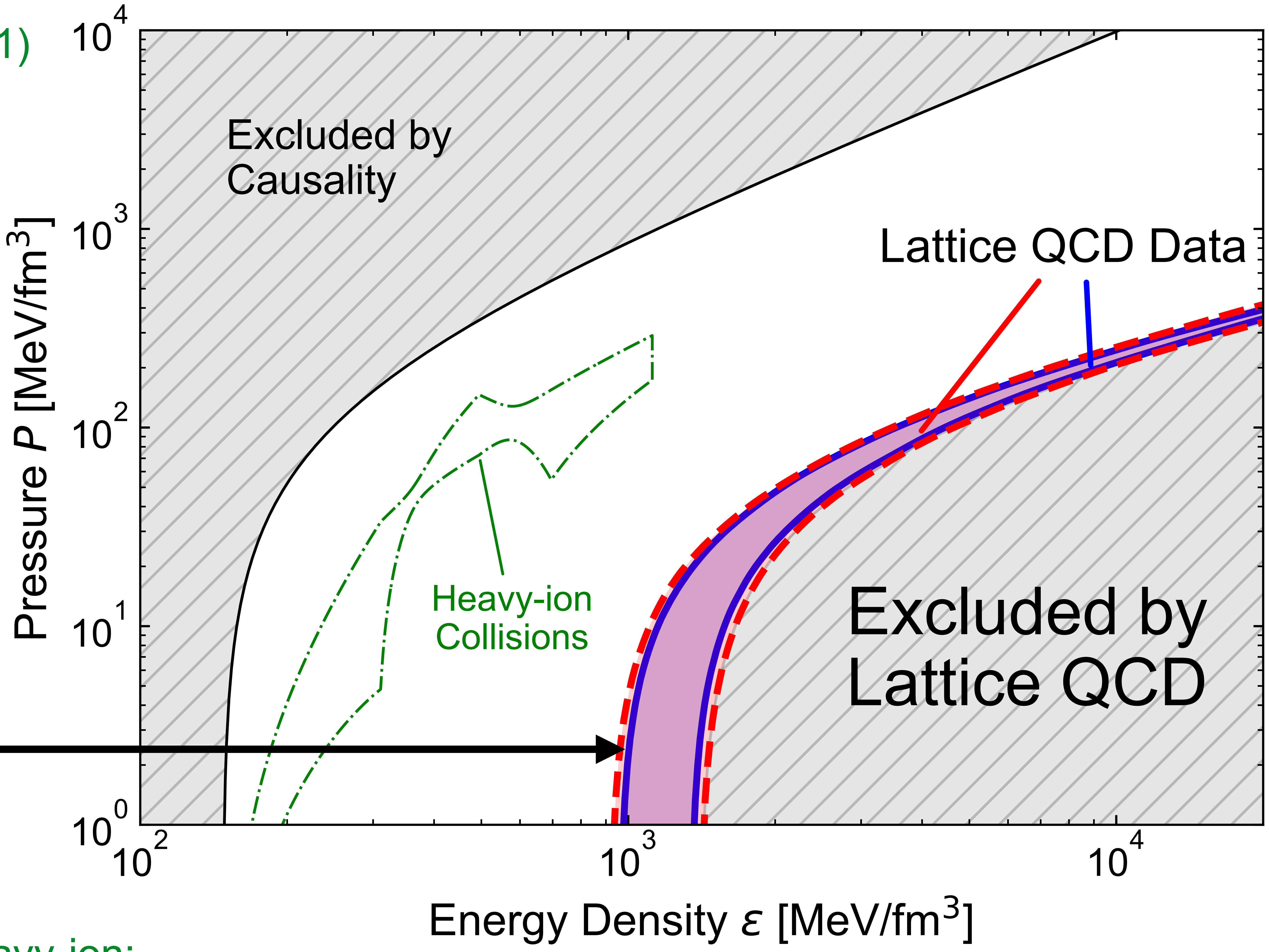
- **Causality:** $dn_B/d\mu_B > n_B/\mu_B$
- **Integral version of inequality:**

$$\int_{\mu_0}^{\mu_B} d\mu\, n_B(\mu) \le P_I(2\mu_B/N_c),$$

$n_B(\mu_B)$ can be constrained

Then, from the relation

$\varepsilon = -P + \mu_B n_B$



Heavy-ion:
Oliinychenko et al.(2022)

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Weak-coupling results in high-density QCD

Freedman,McLerran (1978); Kurkela et al. (2009-); Fujimoto (2023)
Review: Alford,Rajagopal,Schafer,Schmitt (2008);

QCD EoS in weak-coupling α_s expansion:

$$P_{\mathrm{QCD}}(\mu) = \frac{3\mu^4}{4\pi^2}\left(1 - a_1\frac{\alpha_s}{\pi} - a_2\frac{\alpha_s^2}{\pi^2}\right) + \frac{3\mu^2\Delta^2}{2\pi^2}\left(1 + c_1\sqrt{\frac{\alpha_s}{\pi}}\right)$$

where

$$\ln\left(\frac{\Delta}{\mu}\right) = -b_{-1}\left(\frac{\alpha_s}{\pi}\right)^{-1/2} - b_0$$

$(N_c = 3,\ N_f = 3)$

Pros and cons for the applicability at low μ:

Cons 👎: (e.g. $\mu_{\mathrm{limit}} \sim 10^5\ \mathrm{GeV}$) [Rajagopal,Shuster (2000)]

- Folklore — only applicable at very large μ
- Coupling constant is large:
  $\alpha_s = g^2/4\pi \sim 0.1 \to g \sim 1$

Pros 👍 $(\mu_{\mathrm{limit}} \sim 0.8\ \mathrm{GeV})$

- Convergence is good
- Standard pQCD (e.g. in high-energy QCD) is used down to $\mu \sim 1\ \mathrm{GeV}$
- Derivation of the gap eqn. for Δ is valid as long as $\Delta \ll m_{\mathrm{D}} \ll \mu$

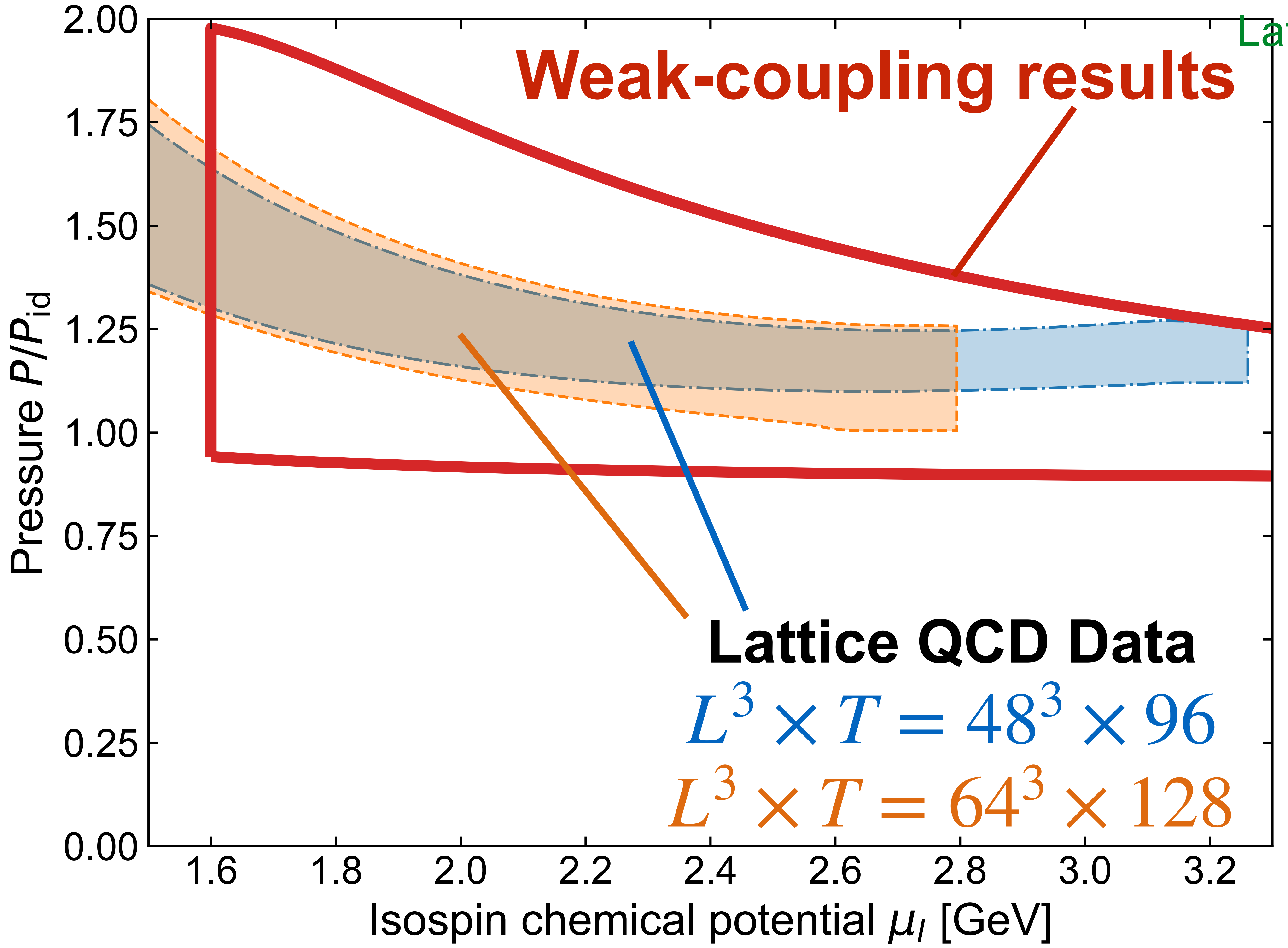
Weak-coupling formula is **universal** for QCD$_B$ and QCD$_I$ up to $\mathcal{O}(\alpha_s^2)$

→ Lattice QCD$_I$ can be used as a benchmark

Weak-coupling results vs lattice QCD_I data

Lattice data: Abbott et al. (2023); Fujimoto (2023)

Uncertainty in weak-coupling results: quantified by varying the renormalization scale $\bar{\Lambda}$ by a factor 2 around its typical scale $\bar{\Lambda} = 2\mu$



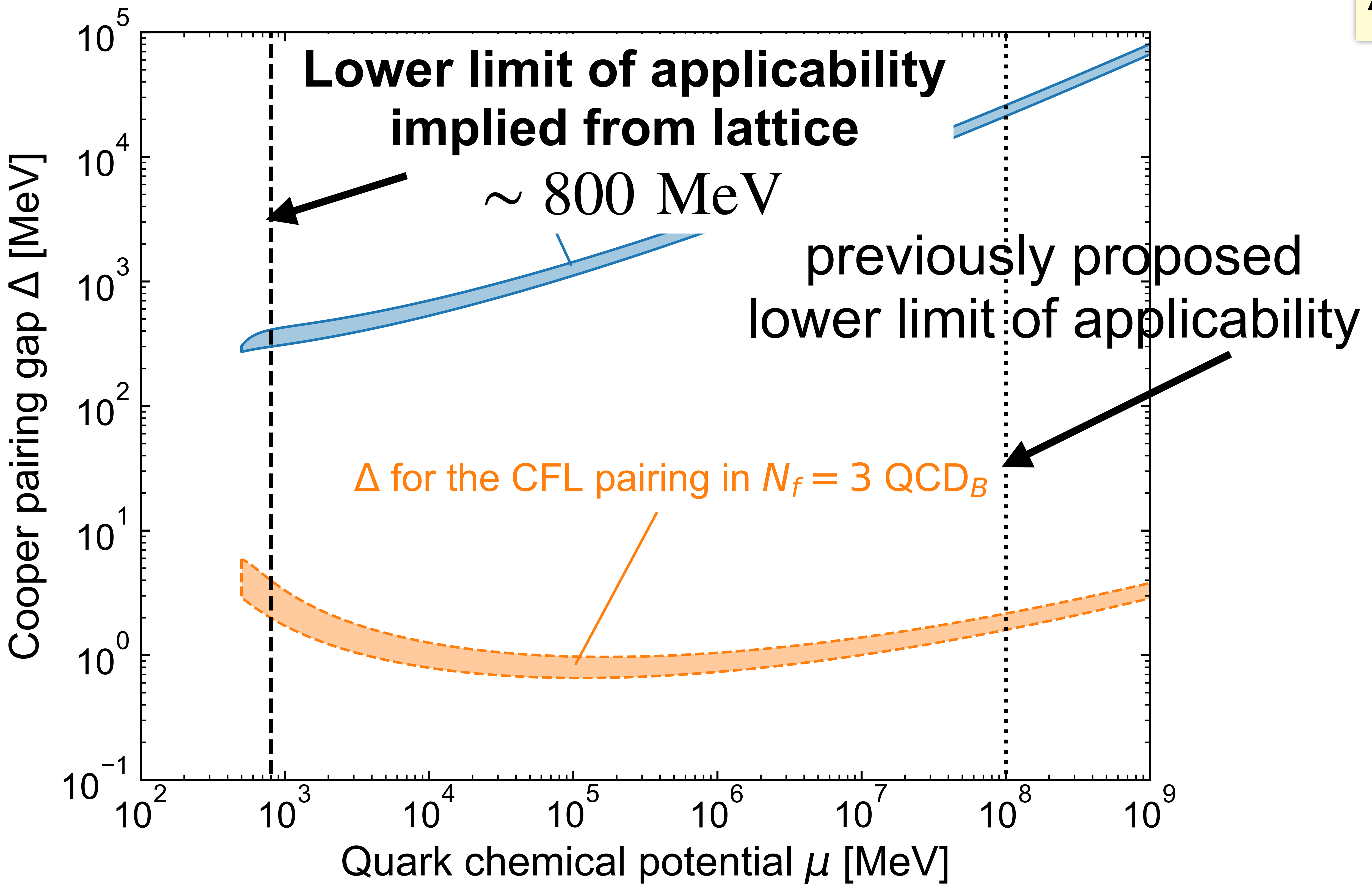
Empirical evidence for the dense-QCD weak-coupling results to be applicable down to $\mu \sim 800$ MeV

At least the magnitude is correct

Impact on color superconductivity

Fujimoto, in preparation

Plot of weak-coupling Cooper pairing gap:



$\Delta_{\rm CFL} \sim 2-3\ \text{MeV}$ at $\mu = 800\ \text{MeV}$

cf. Kurkela,Rajagopal,Steinhorst (2024)

- A negligibly **small** contribution to bulk thermodynamics

- Comparable to the stress by strange quark mass: $\Delta_{\rm CFL} \sim m_s^2/4\mu$

→ CFL may not be the ground state even at $\mu_B = 2.4\ \text{GeV}$

Outlook: Finite-μ lattice simulations

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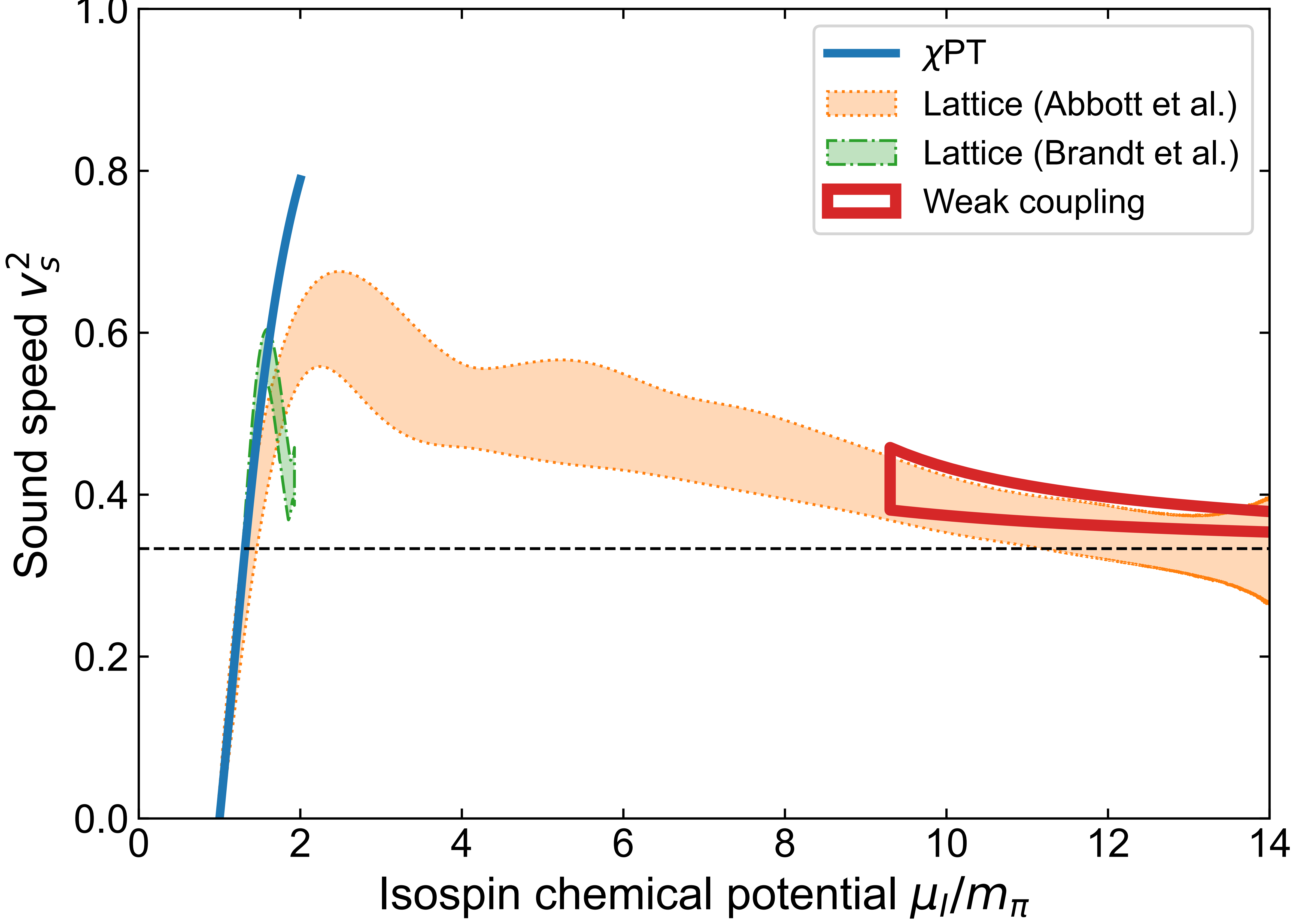
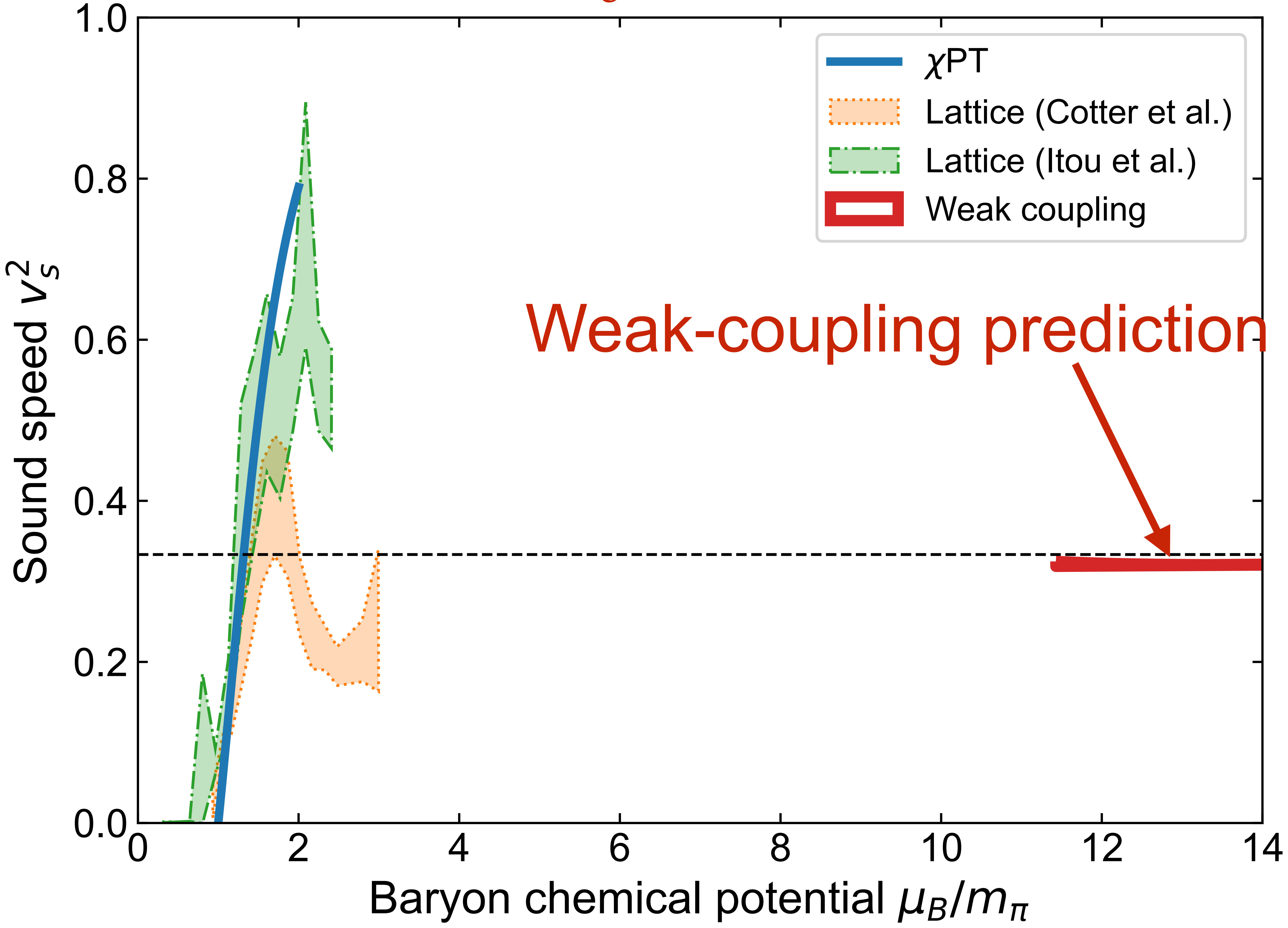
Already calculated

QCD$_I$



Calculable w/o the sign problem

$N_c = 2$ **QCD**



Summary

QCD at finite μ_I: a clean lab for QCD at finite μ_B; lattice simulation feasible

QCD inequality: Robust constraints on the symmetric nuclear matter EoS from lattice QCD & saturation property

Weak-coupling results: Matches well with lattice QCD$_I$

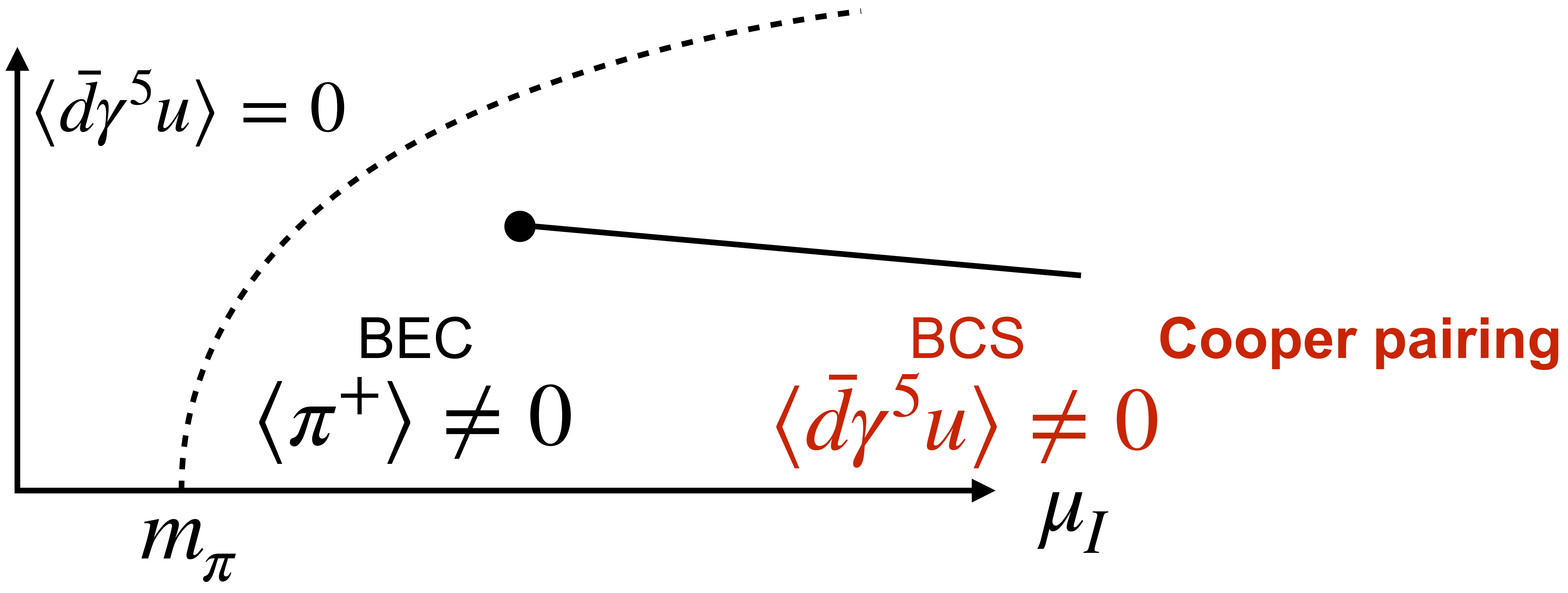
- Empirical evidence for the validity down to $\mu \simeq 800\ \mathrm{MeV}$
- Color-superconducting gap around $\mu \sim 800\ \mathrm{MeV}$:
  - * negligible contribution in EoS
  - * CFL phase may not be the ground state
- Crosscheck with lattice-QCD can be provided in $N_c = 2$
- Impacts the neutron-star EoS; excludes some stiff EoSs

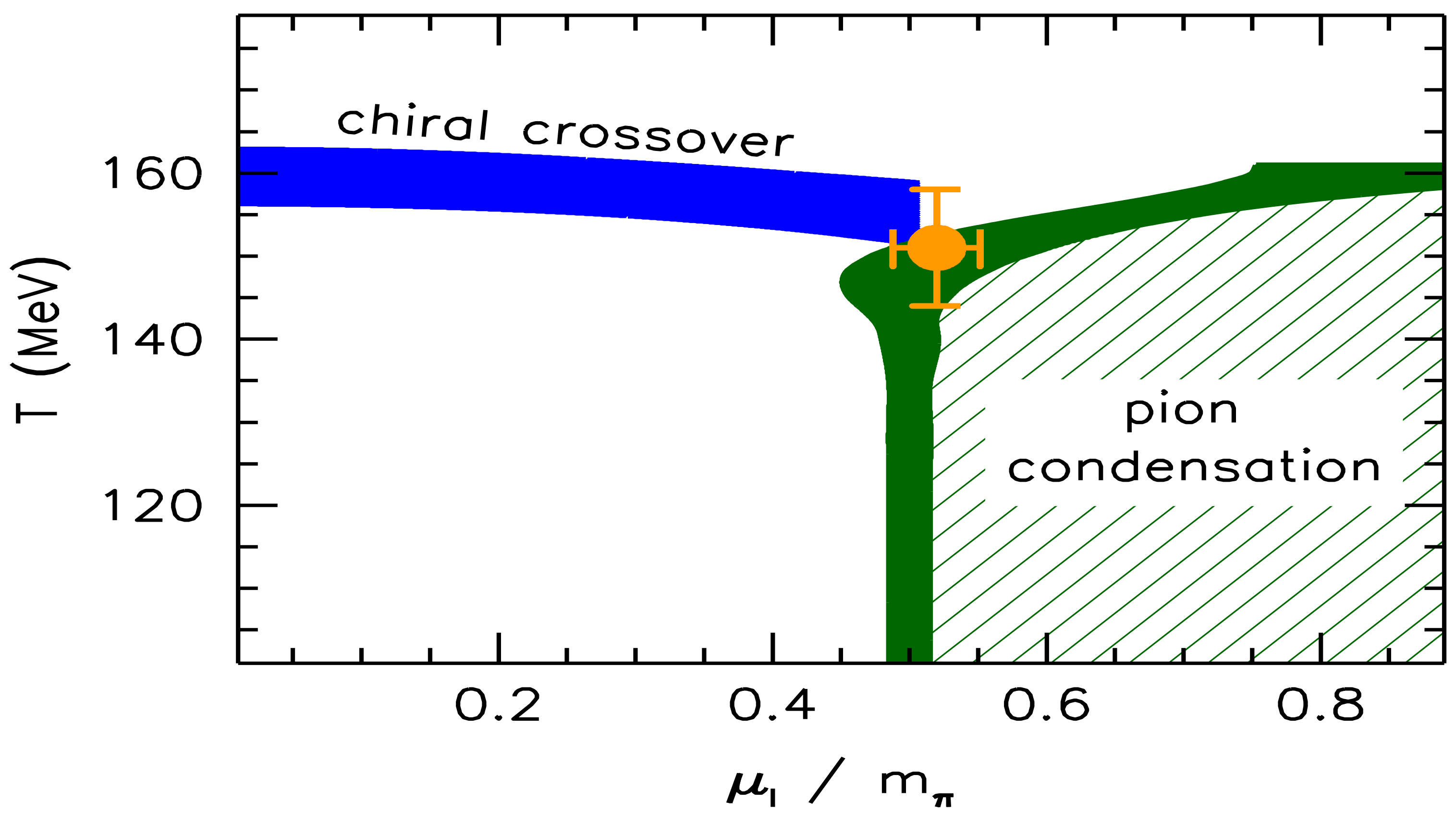
Supplemental materials

QCD at finite isospin density

- Phase structure:

Son,Stephanov (2000)





Brandt,Endrődi,Schmalzbauer (2018)

QCD inequality: multi-flavor/isospin imbalanced extension?

Cohen (2003); Moore,Gorda (2023)

$$Z(\{\mu_f\}) = \int [dA] \prod_f \det \mathscr{D}(\mu_f) e^{-S_{\mathrm{G}}}$$

$$\leq Z_{\mathrm{PQ}}(\{\mu_f\}) = \int [dA] \prod_f \left| \det \mathscr{D}(\mu_f) \right| e^{-S_{\mathrm{G}}}$$

$$\det \mathscr{D} = |\det \mathscr{D}| e^{i\theta}$$

Phase quenching

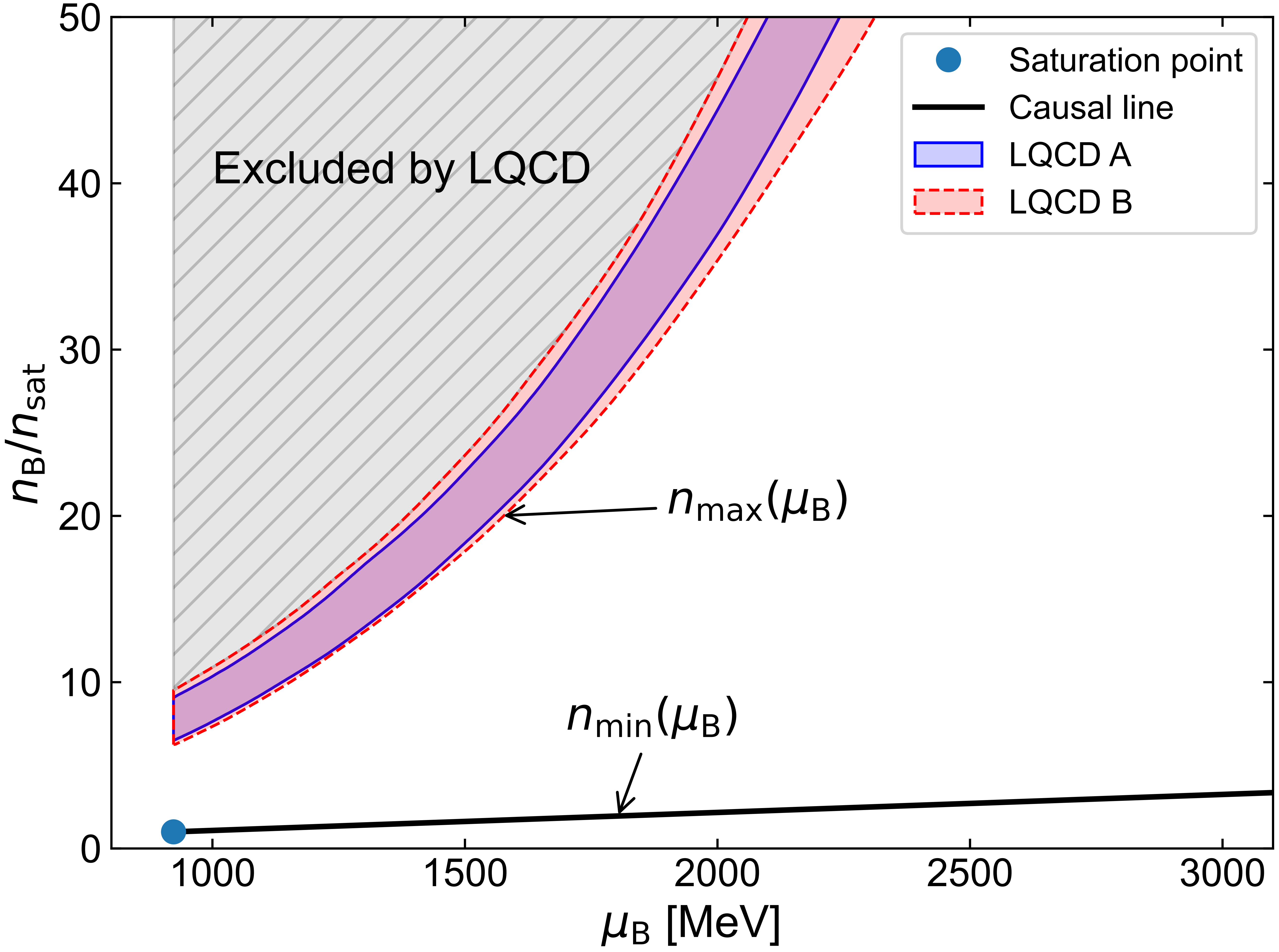
Challenges:

- Only works for symmetric matter: beta-equilibrium/charge neutrality cannot be implemented in the rigorous way

- Computationally demanding: efficient algorithm only for canonical ensemble $\mu_I \neq 0$ case, absent in general case?

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\mu_{sat}}^{\mu_B} d\mu'\, n_B(\mu') \;\leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Lower bound of the integral must be specified
fix it to the **empirical saturation property**

Is the gap Δ the only correction?

Alford,Braby,Paris,Reddy (2004)

$$P = a_4\mu^4 + a_2\mu^2 - B$$

- a_4: Ideal gas behavior + pQCD correction (Dominant)

- a_2: **Gap correction $a_2 \propto \Delta^2$ (large, ~20-200%),**
  Quark mass $a_2 \propto -m_f^2$ (small, ~1%)
  Temperature $a_2 \propto T^2$ (small, ~1%)

- B: Bag constant, typically $B^{1/4} \simeq 200\ \mathrm{MeV}$ (small, ~0.5%)
  Instantons, suppressed by $\dfrac{m_f}{\Lambda_{\mathrm{QCD}}} \sim 10^{-3}$

Shuryak; Kallman; Abrikosov; de Carvalho; Chemtob; Baluni (late 70s - early 80s)

“Uncertainty” in pQCD

Fraga,Pisarski,Schaffner-Bielich(2001)

$$P_{\rm pQCD}(\mu;\bar\Lambda) = \frac{3\mu^4}{4\pi^2}\left[1 - 2\frac{\alpha_s(\bar\Lambda)}{\pi} - \left(2\ln\frac{\alpha_s(\bar\Lambda)}{\pi} + \frac{29}{6}\ln\frac{\bar\Lambda^2}{(2\mu)^2} + 17.39\right)\left(\frac{\alpha_s(\bar\Lambda)}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$$

- **$\bar\Lambda$: renormalization scale**

  … only ambiguity in pQCD from perturbative series truncation

- “Uncertainty” quantified by varying by a factor 2 around typical scale $\bar\Lambda = 2\mu$

  i.e. $X \in [1/2,\ 2]$ with $X \equiv \bar\Lambda/(2\mu)$

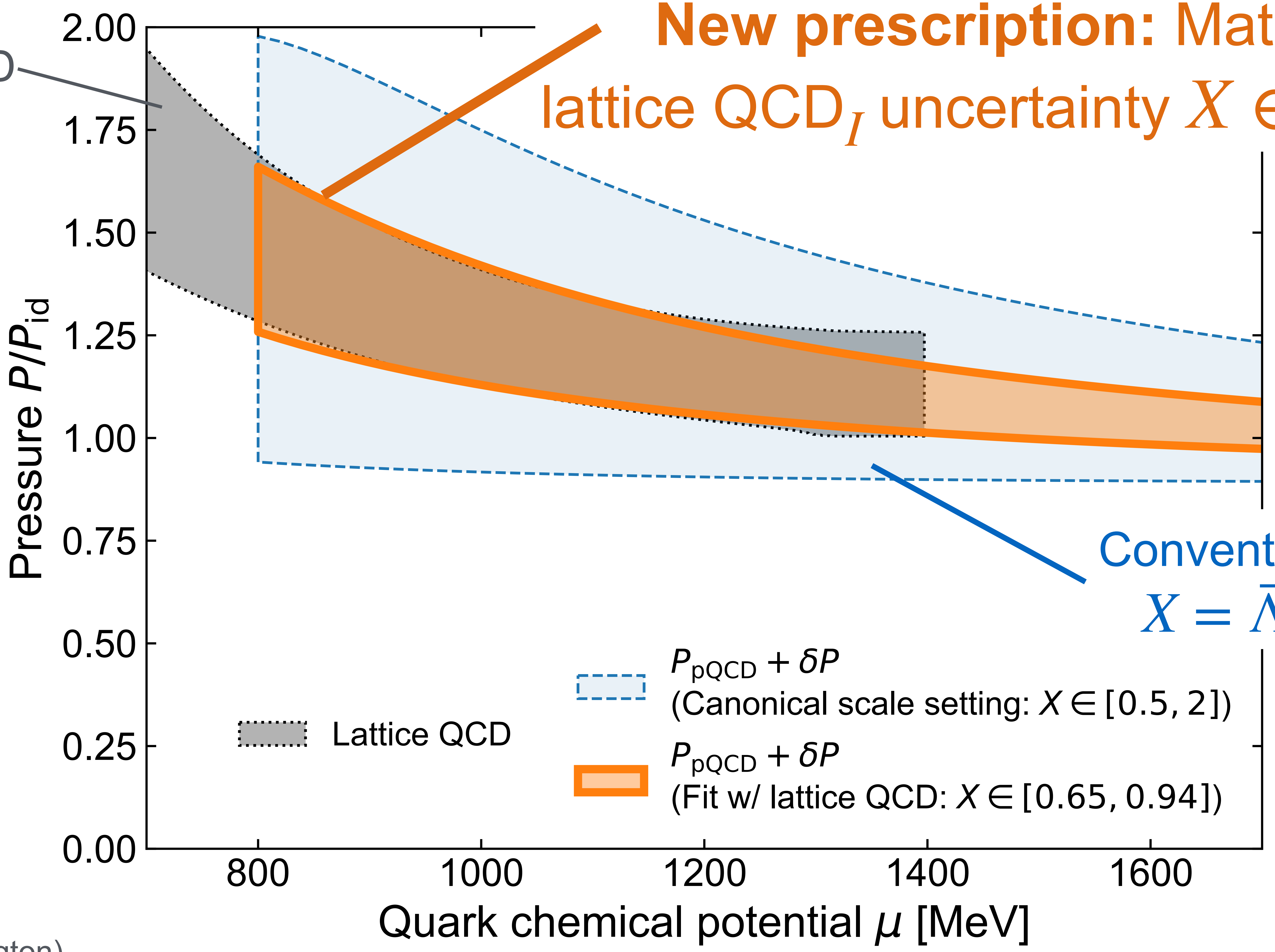
  … ad hoc procedure, purely based on historical practice

  → size of uncertainty does not mean any meaning

Prescription for $\bar{\Lambda}$ determination

Fujimoto, in preparation

New prescription: Matching $\bar{\Lambda}$ to lattice QCD_I uncertainty $X \in [0.64,\ 0.95]$



Conventional choice for $\bar{\Lambda}$
$X = \bar{\Lambda}/2\mu \in [1/2, 2]$

Neutron-star phenomenology

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