Modelling choices in multi-fluid-dynamical descriptions of low energy collisions

3 fluid code: MUFFIN

Cimerman, Karpenko, Tomàšik, Huovinen PRC 107 (2023) 044902

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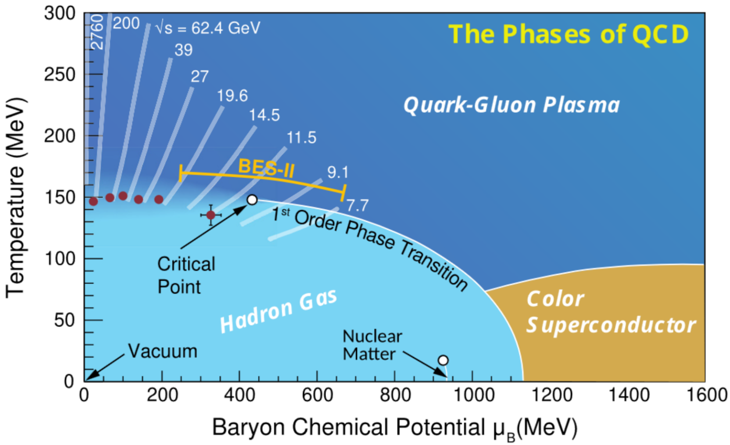
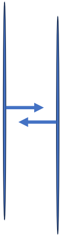
QCD Phase Diagram



- goal: probe QCD phase diagram via heavy ion collisions
- expect 1st Order transition line and critical end point
- to probe this region, need higher $\mu_B \Leftrightarrow$ lower $\sqrt{s_{NN}}$

Fluiddynamical simulations for low energy collisions

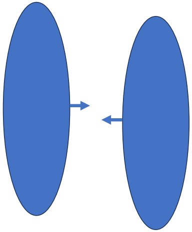
Hydrodynamics works well at high $\sqrt{s_{NN}}$.
What are the challenges at low $\sqrt{s_{NN}}$?



Lorentz contraction:

- at high energies, nuclei are flat
  $\Rightarrow$ timescale separation of:
  1. energy deposition
  2. pre-equilibrium
  3. hydrodynamic stage

  many early time descriptions rely on this!
- low energies: extended interpenetration; all 3 processes happen simultaneously!



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What are the challenges at low $\sqrt{s_{NN}}$?

Baryon stopping vs baryon transparency:

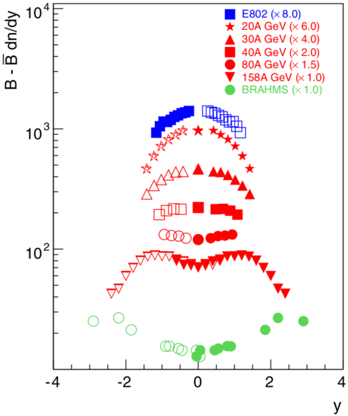
- high beam energies: baryons from nuclei escape collision region
  $\Rightarrow$ this is why $\mu_B \approx 0$!
- very low energies: nuclei completely stopped
- intermediate energies: partial baryon transparency causes double peak!

State of the art:

Start hydro at late time with a very complicated intial condition

Shen, Schenke PRC 105 (2022) 064905

net baryon rapidity distribution for low $\sqrt{s_{NN}}$'s:



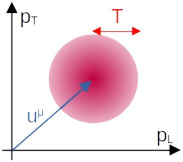
Multi-fluid Hydrodynamics

idea: Hydrodynamics can also describe cold nuclei ("liquid drop model")

⇒ can we use it throughout entire collision evolution?

problem: Hydrodynamics describes a system close to equilibrium, i.e. isotropic in momentum space

not true for colliding nuclei!



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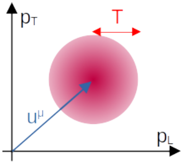

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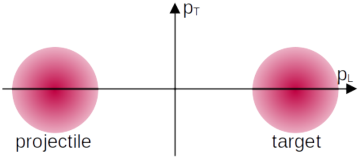

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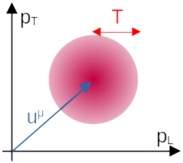

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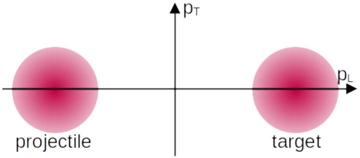

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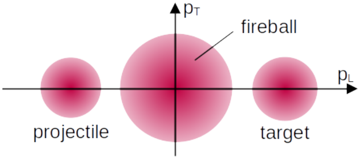

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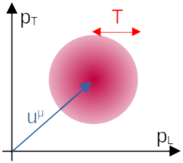

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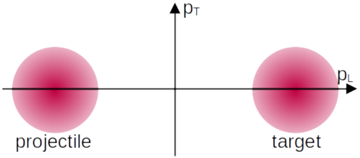

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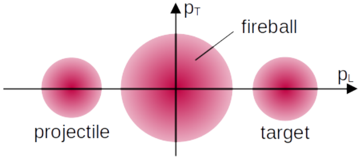

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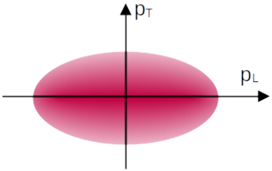

anisotropic hydro

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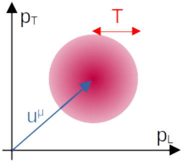

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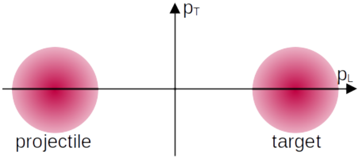

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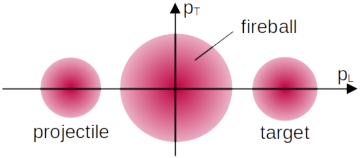

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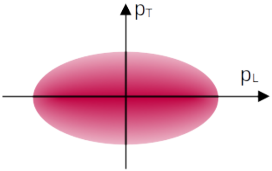

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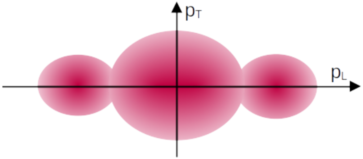

somewhat realistic distribution

Equations governing multi-fluid hydro

3 versions of energy-momentum tensor

$$T_p^{\mu\nu}\,,\ T_t^{\mu\nu}\,,\ T_f^{\mu\nu}$$

and 3 versions of baryon current

$$J^\mu_{B,p}\,,\ J^\mu_{B,t}\,,\ J^\mu_{B,f}$$

conservation equations hold only globally:

$$\partial_\mu(T_p^{\mu\nu}+T_t^{\mu\nu}+T_f^{\mu\nu})=0\,,\quad \partial_\mu(J^\mu_{B,p}+J^\mu_{B,t}+J^\mu_{B,f})=0$$

$\Rightarrow$ fluids can exchange energy, momentum and baryon charge via "friction"

$$\partial_\mu T_\alpha^{\mu\nu}=F_\alpha^\nu\,,\quad \partial_\mu J^\mu_{B,\alpha}=R_{B,\alpha}\qquad \text{where}\qquad \sum_\alpha F_\alpha^\nu=0\,,\ \sum_\alpha R_{B,\alpha}=0$$

a priori, any friction is allowed! but:

- system should not deviate too much from modelling assumptions
- needs to reproduce observed behaviour

Example: Csernai's model

"next-to-trivial" model: all that scatters is dumped into the fireball

Csernai, Lovas, Maruhn, Rosenhauer, Zimányi, Greiner PRC 26 (1982) 149

⇒ nuclei stay cold!

projectile/target:

$$\partial_\mu T^{\mu\nu}_{p/t} = F^\nu_{p/t} = u^\nu_{p/t} m_N R_{B,p/t}\,, \quad \partial_\mu J^\mu_{B,p/t} = R_{B,p/t}$$

fireball:

$$\partial_\mu T^{\mu\nu}_f = -F^\nu_p - F^\nu_t\,, \quad \partial_\mu J^\mu_{B,p/t} = -R_{B,p} - R_{B,t}$$

Only need to model $R_{B,p/t}$: how nuclei loose baryon charge

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Only need to model $R_{B,p/t}$: how nuclei loose baryon charge

problem: no double peak!

Deriving friction from kinetic theory

consider fluids as phase space distributions: $f_\alpha = \frac{\mathrm{d}N_\alpha}{\mathrm{d}^3x d^3p}$,
can obtain hydro d.o.f. as:

$$T^{\mu\nu}_\alpha = \int \frac{\mathrm{d}^3p}{p^0} p^\mu p^\nu f_\alpha \,, \quad J^\mu_{B,\alpha} = B_\alpha \int \frac{\mathrm{d}^3p}{p^0} p^\mu f_\alpha$$

time evolution is described via Boltzmann equation

$$p^\mu \partial_\mu f_\alpha = C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} C^{\beta\gamma}_\alpha [f_\beta, f_\gamma]$$

for given $C^{\beta\gamma}_\alpha$, friction equations obtained as

$$\partial_\mu T^{\mu\nu}_\alpha = \int \frac{\mathrm{d}^3p}{p^0} p^\nu C_\alpha = F^\nu_\alpha \,, \quad \partial_\mu J^\mu_{B,\alpha} = B_\alpha \int \frac{\mathrm{d}^3p}{p^0} C_\alpha = R_{B,\alpha}$$

Microscopic input

collision integrals are given in terms of scattering crosssections

$\alpha\beta \to \underline{\underline{\alpha}}X$: Modelling goes <u>here</u>!

$$C_\alpha^{\alpha\beta}[f_\alpha, f_\beta](p_\alpha) = \int \mathrm{d}^3 p_\beta \, p_\alpha^0 \Big[\underbrace{-f_\alpha(p_\alpha) f_\beta(p_\beta) v_{\mathrm{rel}} \sigma_{\alpha\beta\to X}}_{\text{loss}} + \underbrace{\int d^3 q_\alpha f_\alpha(q_\alpha) f_\beta(p_\beta) v_{\mathrm{rel}} \frac{\mathrm{d}\sigma_{\alpha\beta\to\alpha X}}{\mathrm{d}^3 p_\alpha}}_{\text{gain}} \Big]$$

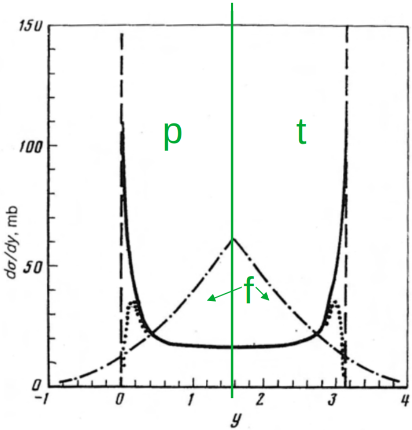
from these, approximative friction formulae are derived

problems:

- crosssections may not be fully measured in experiment
- d.o.f. change in deconfinement transition

Satarov/Ivanov model

- N+N scattering: N strongly peaked at ingoing rapidities, π at midrapitidy
  $\Rightarrow$ in p-t friction: N stay in p/t, π go to f
- $\pi + N$ mostly resonance formation
  $\Rightarrow$ all outgoing particles from p-f friction go to p
- uncertainty in deconfined phase: densities multiplied with $\sqrt{s}$-dependent prefactor

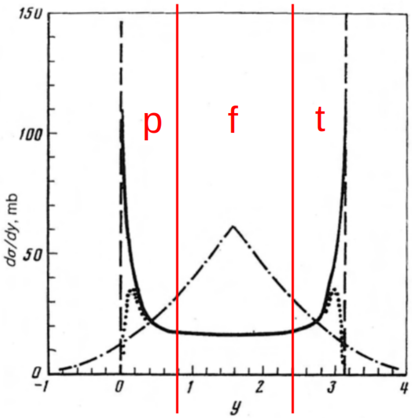


pros: only need total crosssections.
can describe the double peak!

Ivanov, Russkikh, Toneev PRC 73 (2006) 044904

cons: $\mu_B = 0$ in fireball

- for our purposes:
  need high μ_B also in fireball!
- idea: divide outgoing N from N+N into 3 regions
  $\Rightarrow$ modified p+t friction moves B to fireball
  but: need doubly differential crosssections! (y, E)

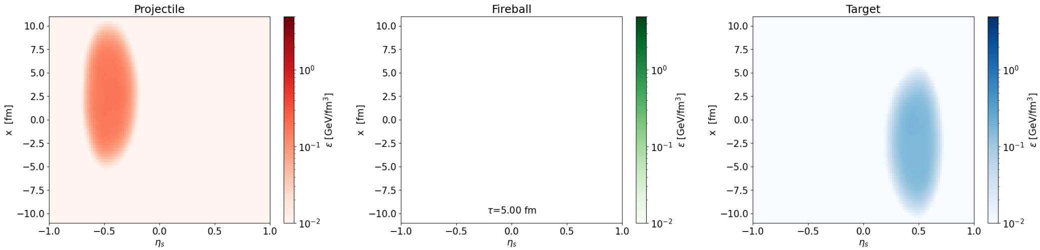


MUFFIN

- implementation of 3-fluid (ideal) hydrodynamics based on vHLLE
  - hybrid with SMASH

Cimerman, Karpenko, Tomàšik, Huovinen PRC 107 (2023) 044902

- allows easy modification of e.o.s.
- all 3 fluids heat up, p and t slow down a lot



Au+Au 7.7 GeV $b = 5$ fm

MUFFIN

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  - hybrid with SMASH

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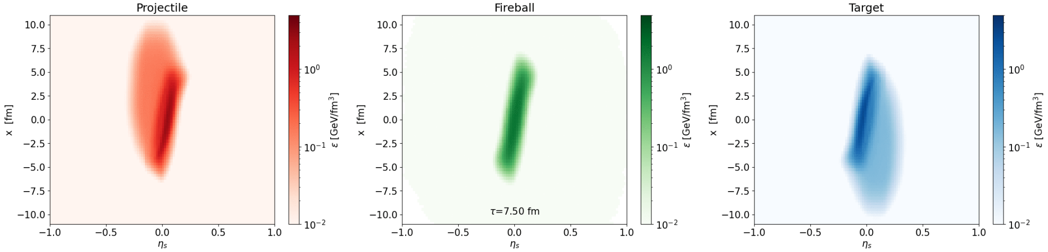

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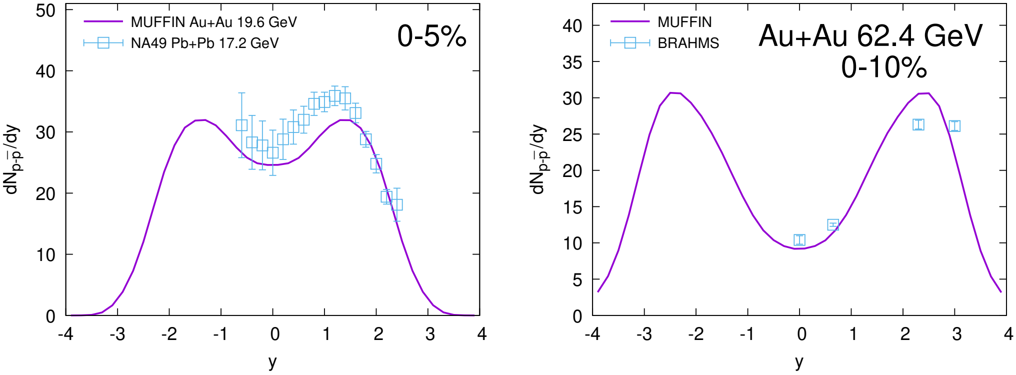
Baryon transparency/stopping

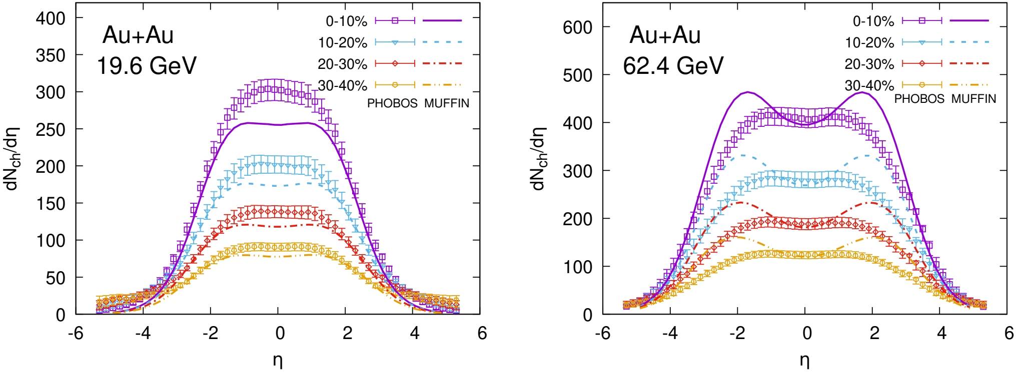


net proton rapidity distributions:

simulated results agree well with experiment

$\Rightarrow$ correct baryon stopping!

Charged hadron yield

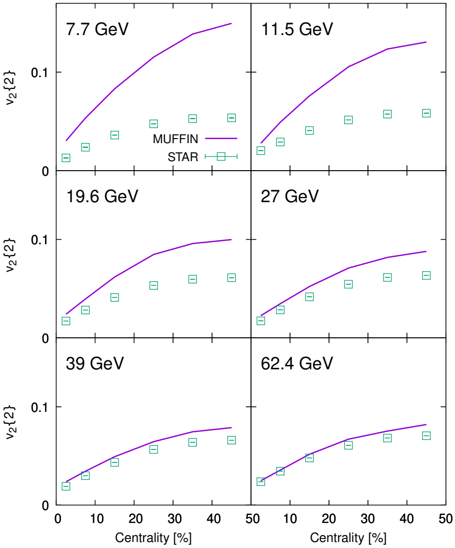


reasonable agreement also in $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$

$\Rightarrow$ correct entropy production

Collective Flow

v_2 is strongly overestimated at small $\sqrt{s_{NN}}$
$\Rightarrow$ probably need viscosity



Outlook: Dissipation in multi fluid dynamics

New effects to consider:

- shear stress ($\pi^{\mu\nu}$) evolution equation:
  friction introduces two new terms! (similarly also for Π,V^{μ})

$$D_{\text{fric}}\pi^{\langle\mu\nu\rangle} = \underbrace{T_{\alpha}^{\rho\sigma} D_{\text{fric}} \Delta^{\mu\nu}_{\rho\sigma}}_{\text{friction changes restframe}} + \underbrace{\int \frac{\mathrm{d}^3 p}{(2\pi)^3 E(p)^2} p^{\langle\mu} p^{\nu\rangle} C_{\alpha}^{\beta,\gamma}[f_\beta, f_\gamma]}_{\text{momentum distribution of scattering particles}}$$

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- dissipative quantities modify f_α:

$$f_\alpha(p) = f_{\alpha,\text{eq}}(p)\left[1 - \Pi \frac{3}{m^2}\mathcal{H}_0^{(0)}(p) + V^\mu p_\mu \mathcal{H}_0^{(1)}(p) + \pi^{\mu\nu} p_\mu p_\nu \mathcal{H}_0^{(2)}(p)\right]$$

$\Rightarrow$ affects all terms involving $C_\alpha^{\beta\gamma}[f_\beta, f_\gamma]$, even energy & momentum transfer!

Summary

- 3 fluid hydrodynamics models 3 momentum space components of the collision: projectile, target, fireball
- modelling freedom in friction $\Rightarrow$ wide range of phenomenological behaviour
- fluid friction obtained from kinetic theory; requires crosssection input
- can describe full time evolution,
  correct results for baryon stopping and entropy production

Backup

Wait, how many different terms do we need to model now?

$$\partial_\mu T^{\mu\nu}_\alpha = \int \frac{\mathrm{d}^3 p}{p^0} p^\nu C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} \int \frac{\mathrm{d}^3 p}{p^0} p^\nu C^{\beta,\gamma}_\alpha[f_\beta, f_\gamma] = \sum_{\beta,\gamma} F^{\beta\gamma,\nu}_\alpha$$

$$C_\alpha[f_p, f_t, f_f] = \sum_{\beta,\gamma} C^{\beta\gamma}_\alpha[f_\beta, f_\gamma] \longrightarrow F^{\beta\gamma,\nu}_\alpha$$

How many unique $F^{\beta\gamma}_\alpha$ are there?

a priori	$3 \cdot 3 \cdot 3 = 27$
$\beta = \gamma$ is hydro	$3 \cdot 3 \cdot 2 = 18$
symmetry $\beta \Leftrightarrow \gamma$	$3 \cdot 3 = 9$
symmetry $\alpha = t \leftrightarrow \alpha = p$	$2 \cdot 3 = 6$
symmetry $\beta = f$: $\gamma = t \leftrightarrow \gamma = p$	$2 \cdot 2 = 4$
conservation	$4 - 2 = 2$

have to model

- how p-t interaction affects p/f
- how p-f interaction affects p/f