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U.S. DEPARTMENT OF ENERGY
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A Bayesian Approach to Radiation Image Reconstruction and Uncertainty Quantification

NSD Meeting – Mar. 5, 2024

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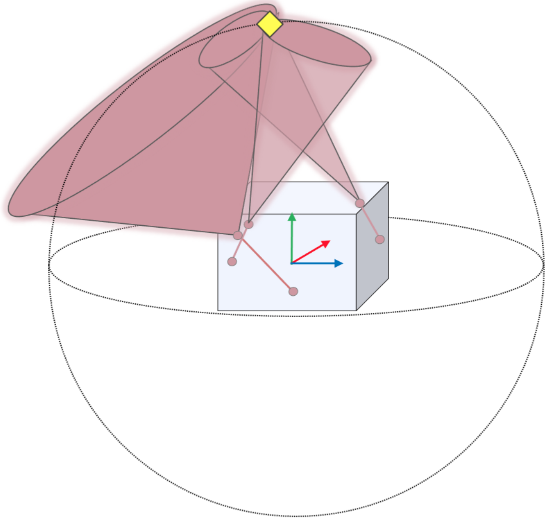
Outline

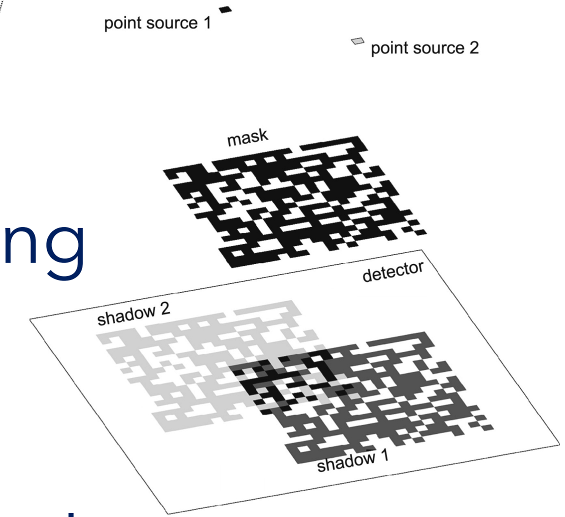
- Introduction – Scene Data Fusion / Single Detector Imaging
- Gaussian Process Prior (GPP) for Image Reconstruction
- Bayesian Uncertainty Quantification
- Simultaneous Full Spectral Imaging and Spectral Decomposition

Single Detector Imaging - Introduction

Radiation Imagers

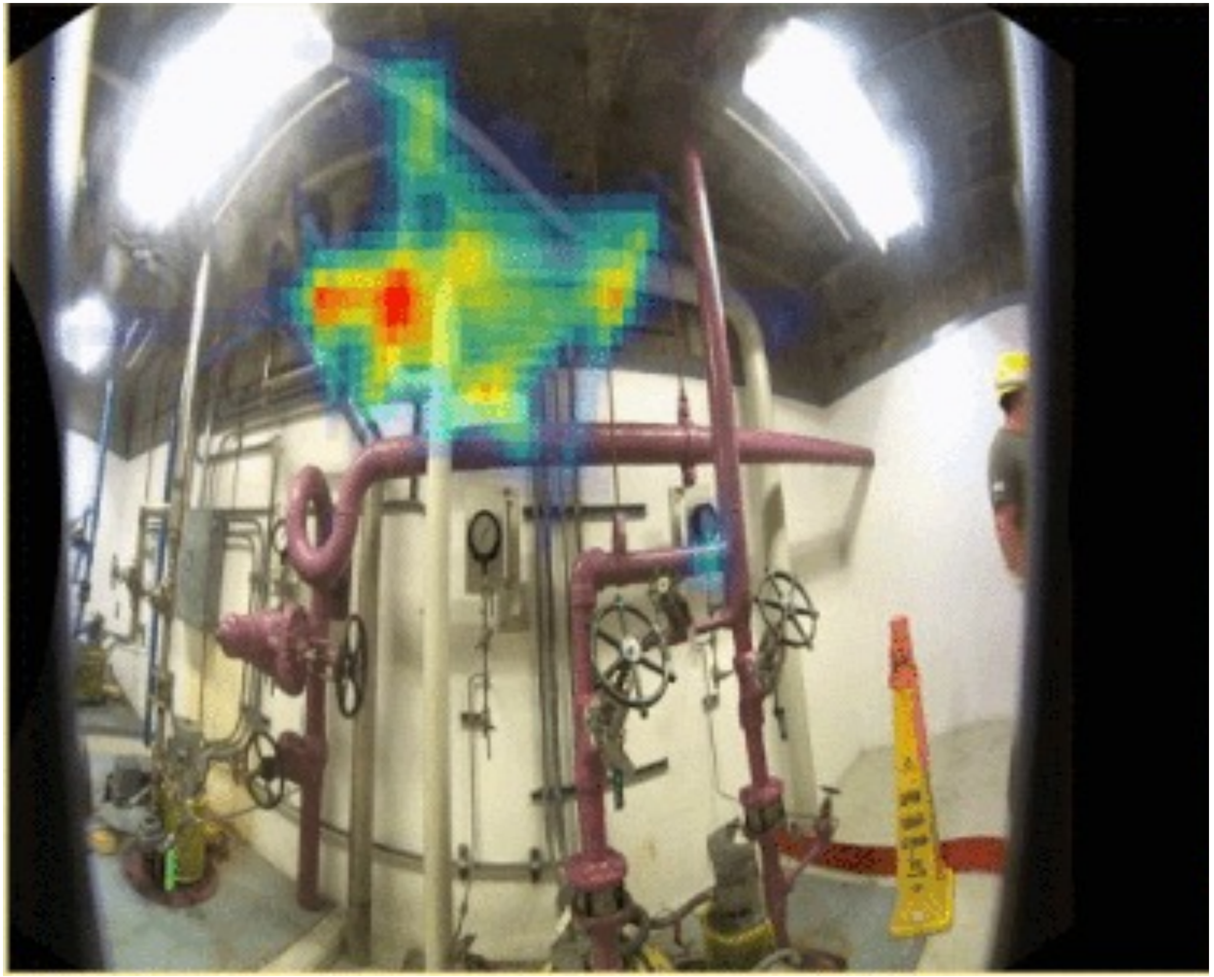


Compton imaging



Coded aperture imaging

†



Co-60 Imaging using a Compton camera

† https://h3dgamma.com/

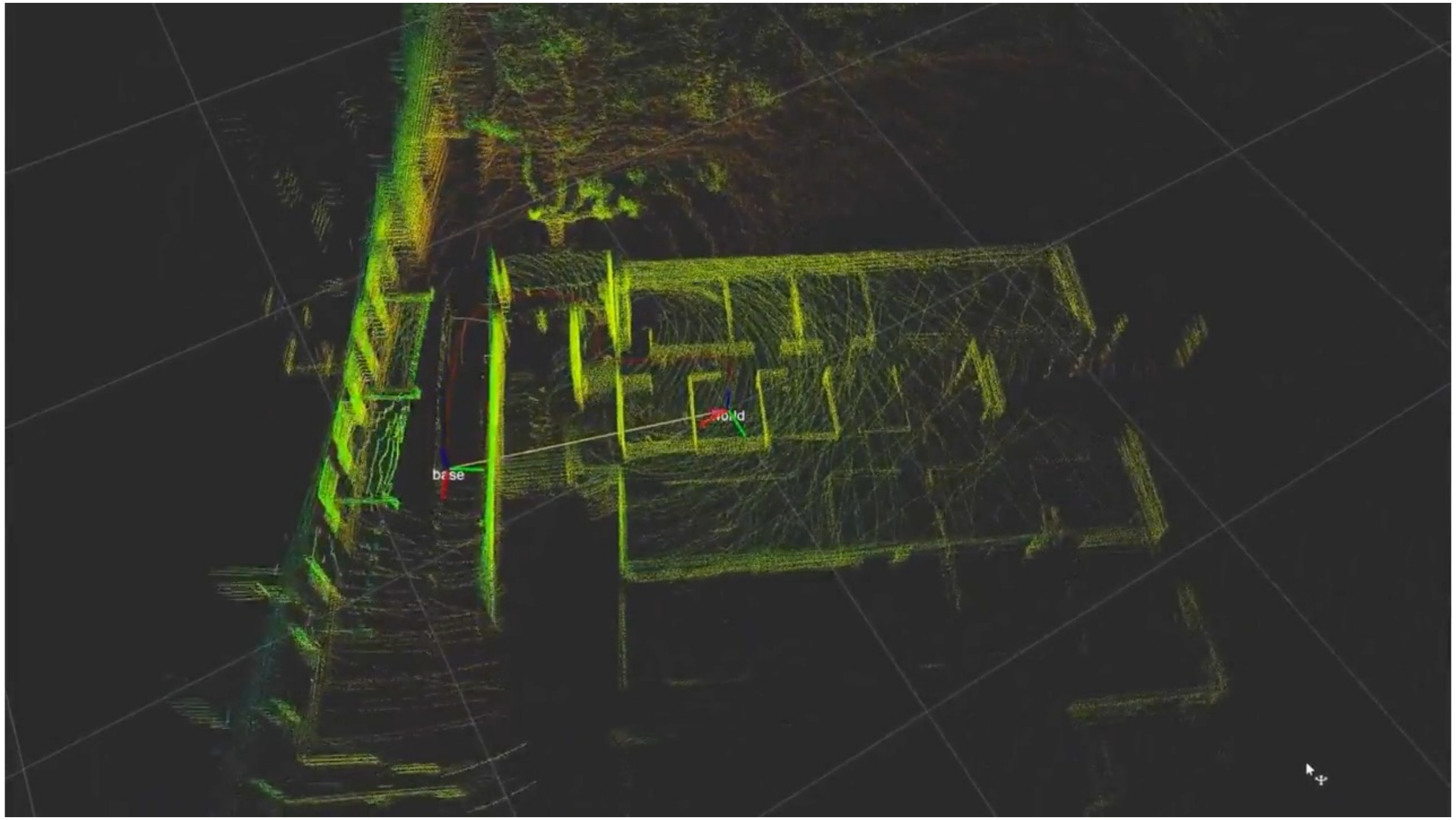
Single Detector Imaging - Introduction

Radiation Imagers

SLAM, Odometry

LiDAR

Stereo camera \ IMU

†


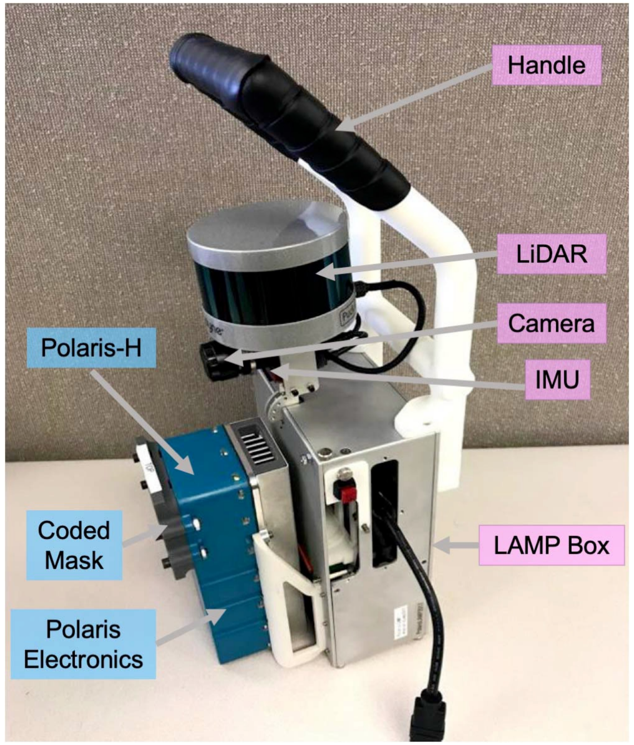
Simultaneous Localization and Mapping (SLAM)

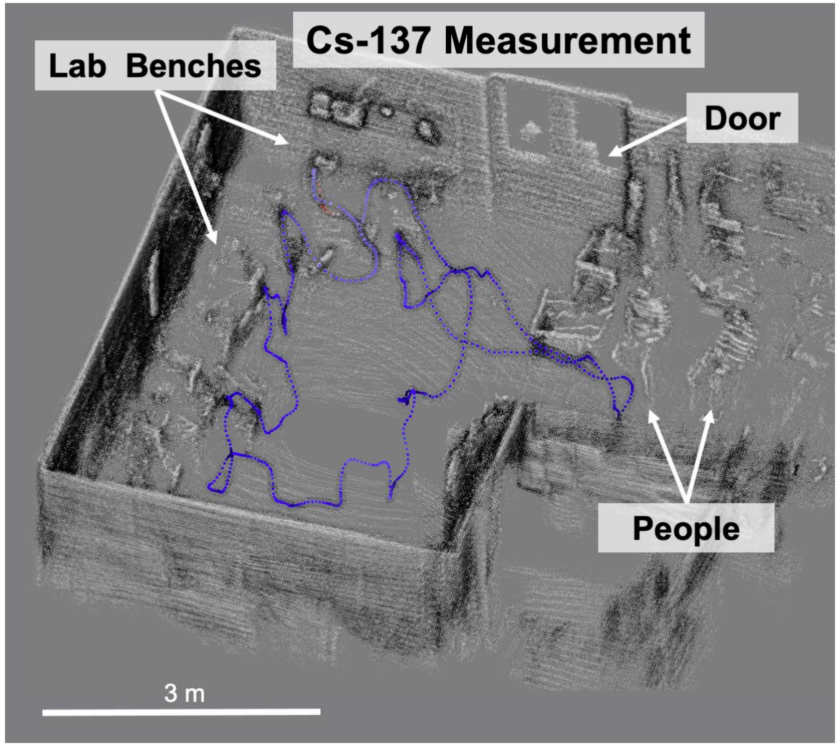
† BLAM, https://www.youtube.com/watch?v=08GTGfNneCI

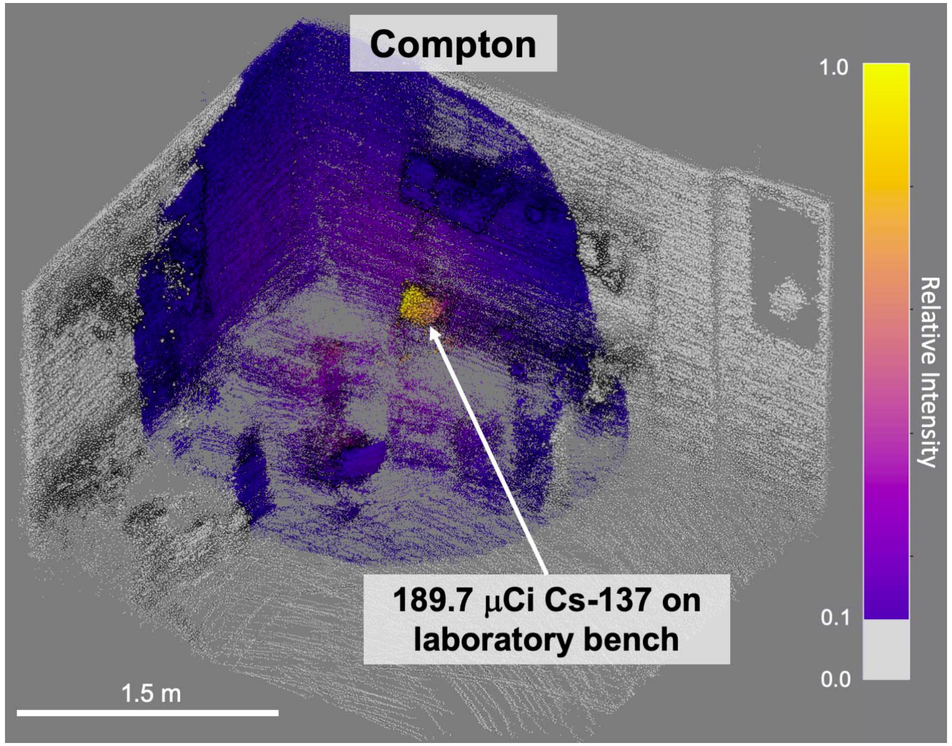
Single Detector Imaging - Introduction

Radiation Imagers + SLAM, Odometry = Scene Data Fusion (SDF)

†







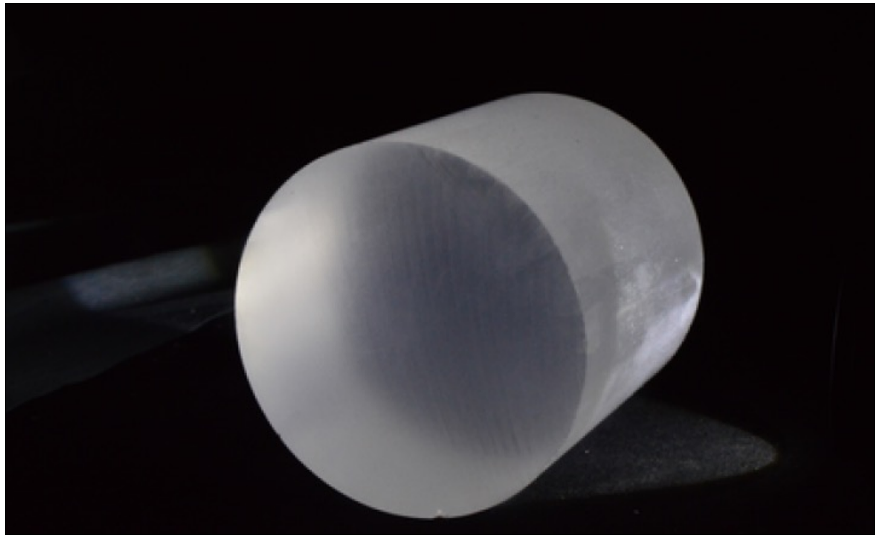
Polaris-LAMP

Free-moving, 3D radiation imaging

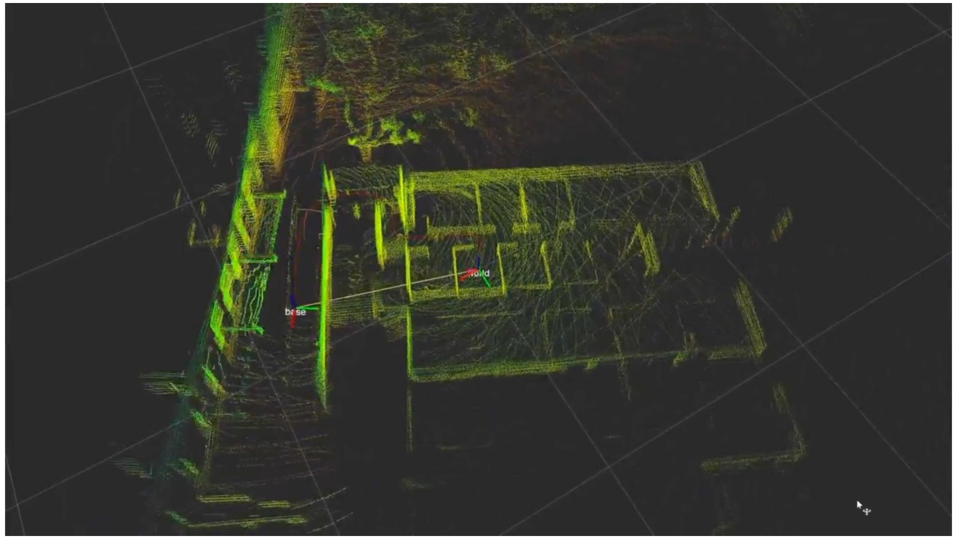
† J. Hecla *et al.*, "Polaris-LAMP: Multi-Modal 3-D Image Reconstruction With a Commercial Gamma-Ray Imager," *IEEE Transactions on Nuclear Science*

Single Detector Imaging - Introduction

Single detector	+	SLAM, Odometry	=	Scene Data Fusion (SDF)



Single detector

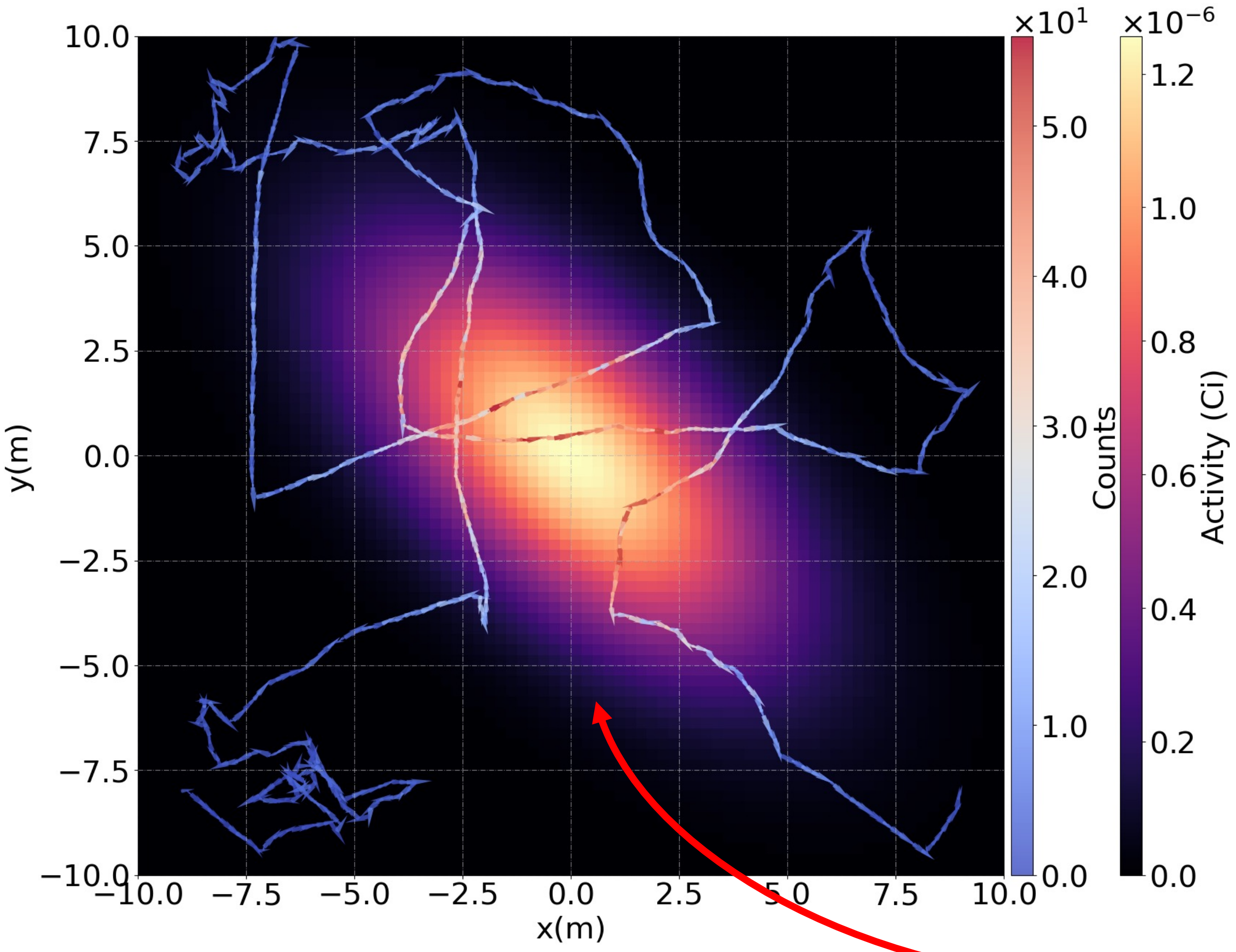


SLAM

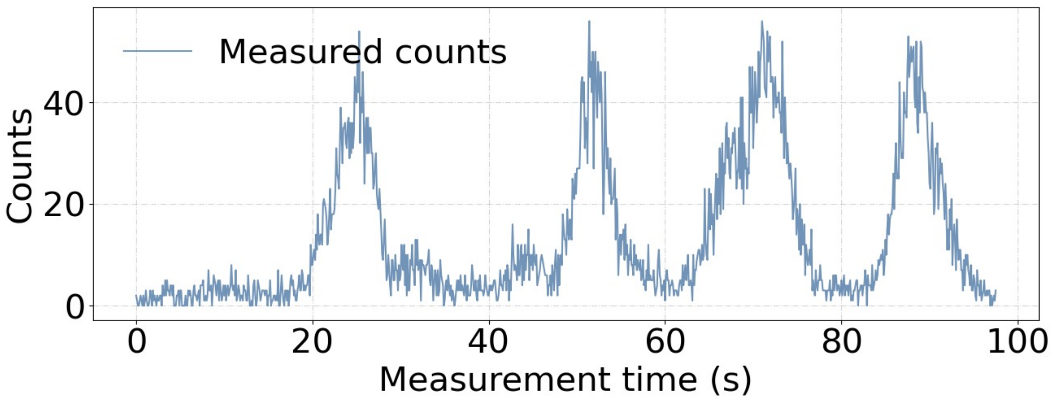
Single detector Free-moving 3D radiation imaging?

Inexpensive, less complicated electronics, rugged...

Single Detector Imaging - Introduction



- 100 μCi distributed source
- ~100 s total measurement time
- 10 cm radius spherical detector

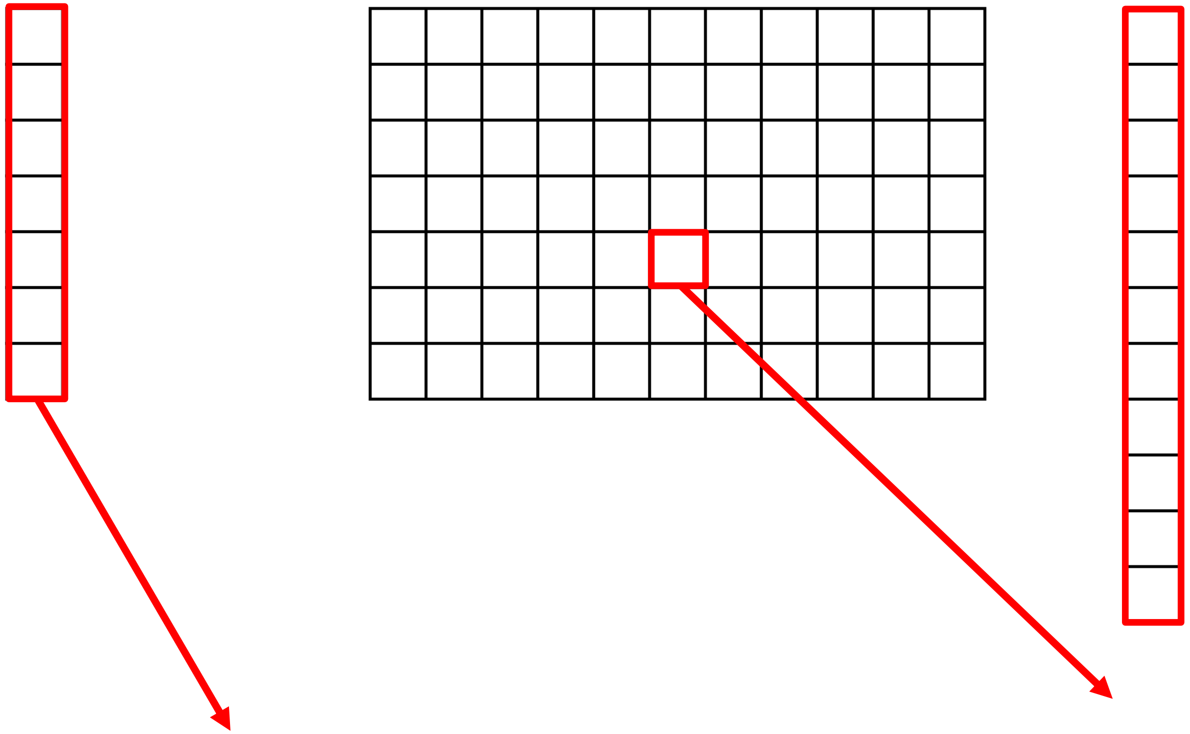


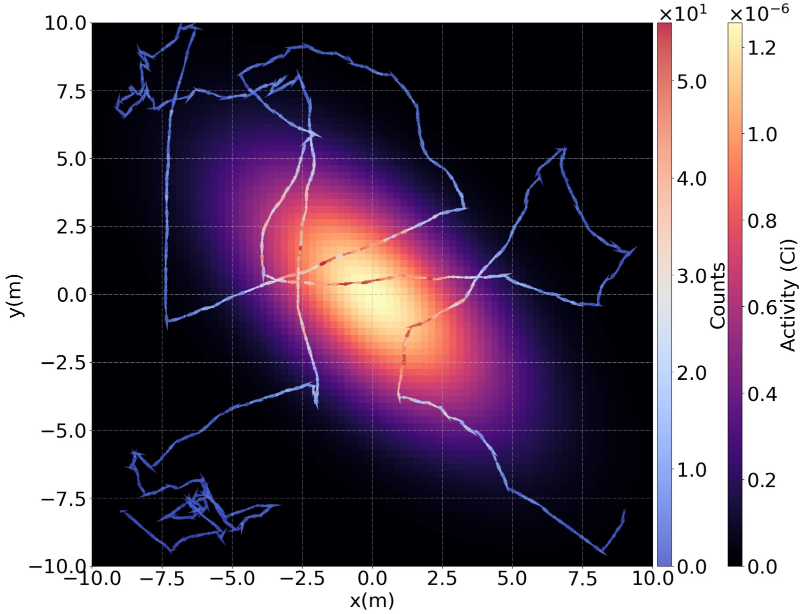
Can we reconstruct the source image just by looking at the measurement?

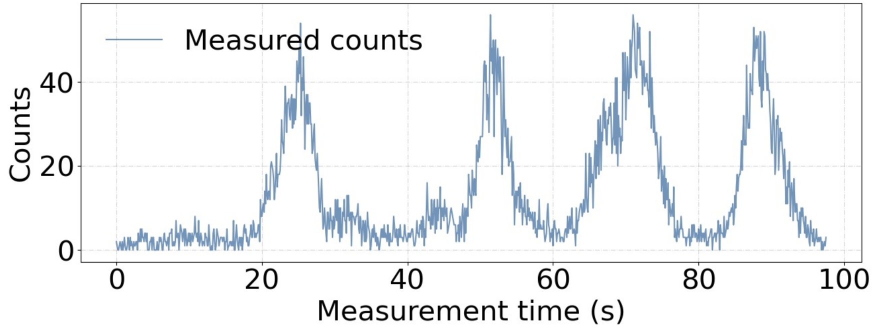
Single Detector Imaging - Introduction

$\mathbf{Y} \approx \mathbf{A} \; \mathbf{X}$

Data vector — System matrix — Image vector

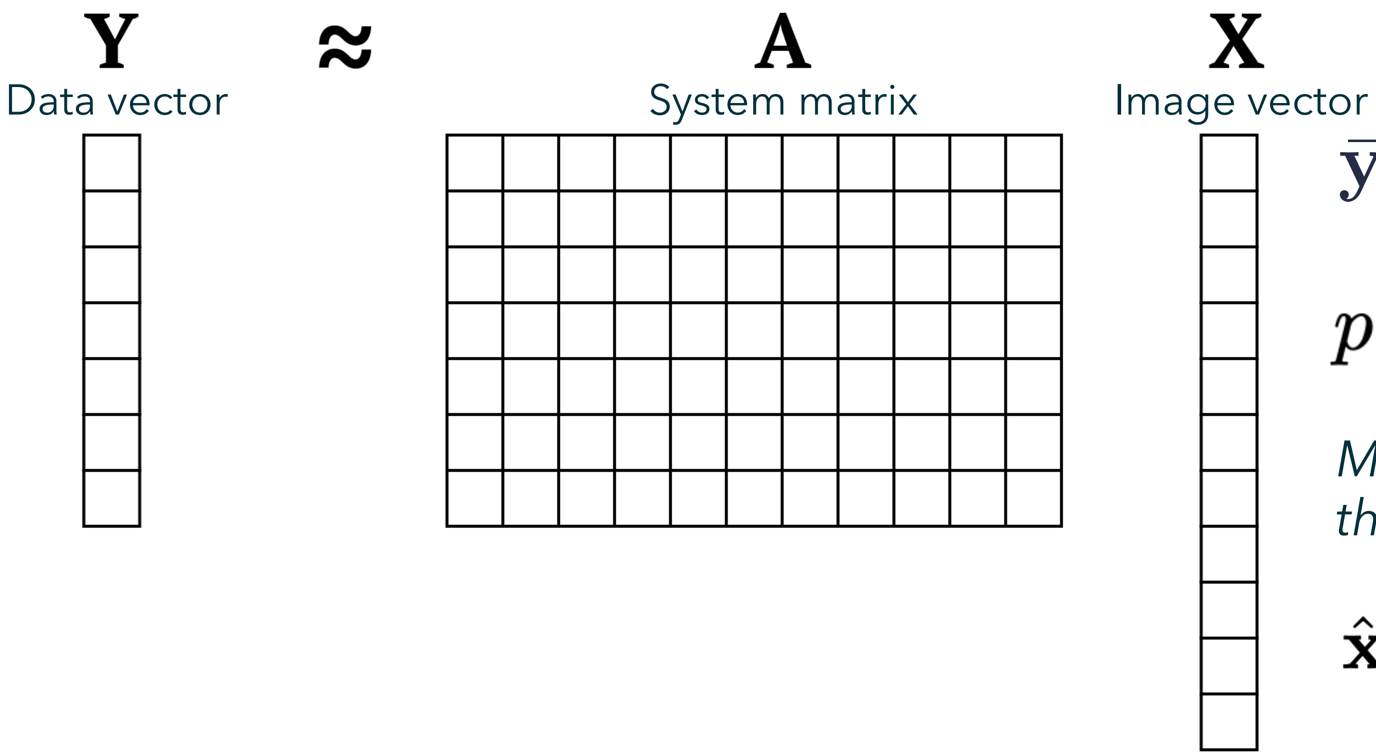






A_{ij} : Probability of a photon emitted from voxel x_j detected in the measurement bin y_i

Single Detector Imaging - Introduction



$$\bar{\mathbf{y}} = \mathbf{A}\mathbf{x} \qquad \mathbf{y} \sim \ Poisson\,(\bar{\mathbf{y}})$$

$$p(y_i; \bar{y}_i) = \frac{e^{-\bar{y}_i}\bar{y}_i^{y_i}}{y_i!}$$

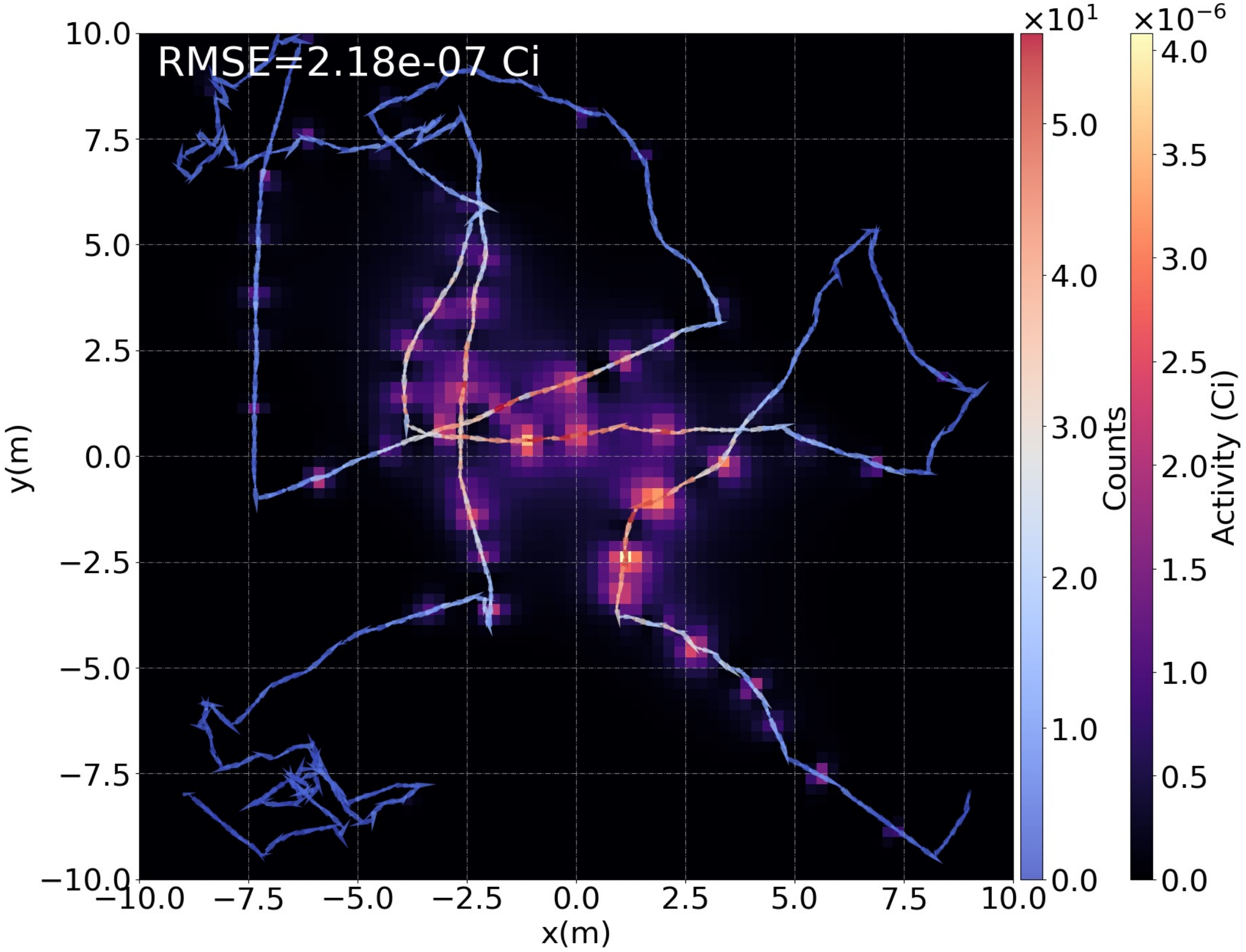
Maximum likelihood estimation (MLE) with the negative log-likelihood

$$\hat{\mathbf{x}} = \underset{\mathbf{x},\, \mathbf{x} \succcurlyeq 0}{\arg\min} \sum_{i=1}^{m} [\mathbf{A}\mathbf{x} - \mathbf{y} \odot \log \mathbf{A}\mathbf{x}]_i$$

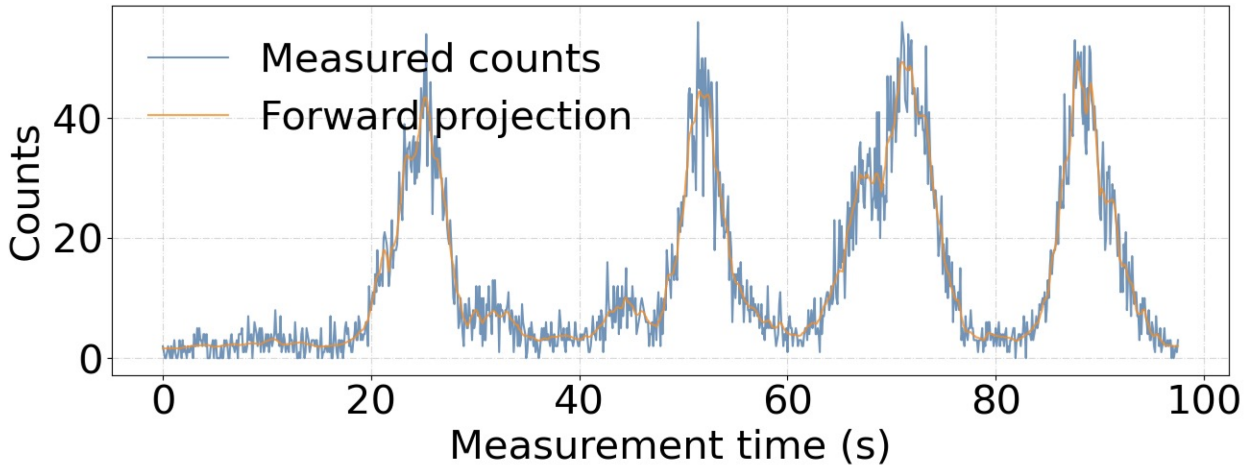
Maximum likelihood – Expectation Maximization (ML-EM) algorithm update rule

$$\hat{x}_j^{n+1} = \frac{\hat{x}_j^n}{\sum_{k=1}^{m} a_{kj}} \sum_{i=1}^{m} \frac{a_{ij} y_i}{[\mathbf{A}\hat{\mathbf{x}}^n]_i}$$

Failure of ML-EM



MLEM 200 iterations results.
A major failure!



Gaussian Process Prior (GPP) Image Reconstruction

Gaussian Process Prior - Introduction

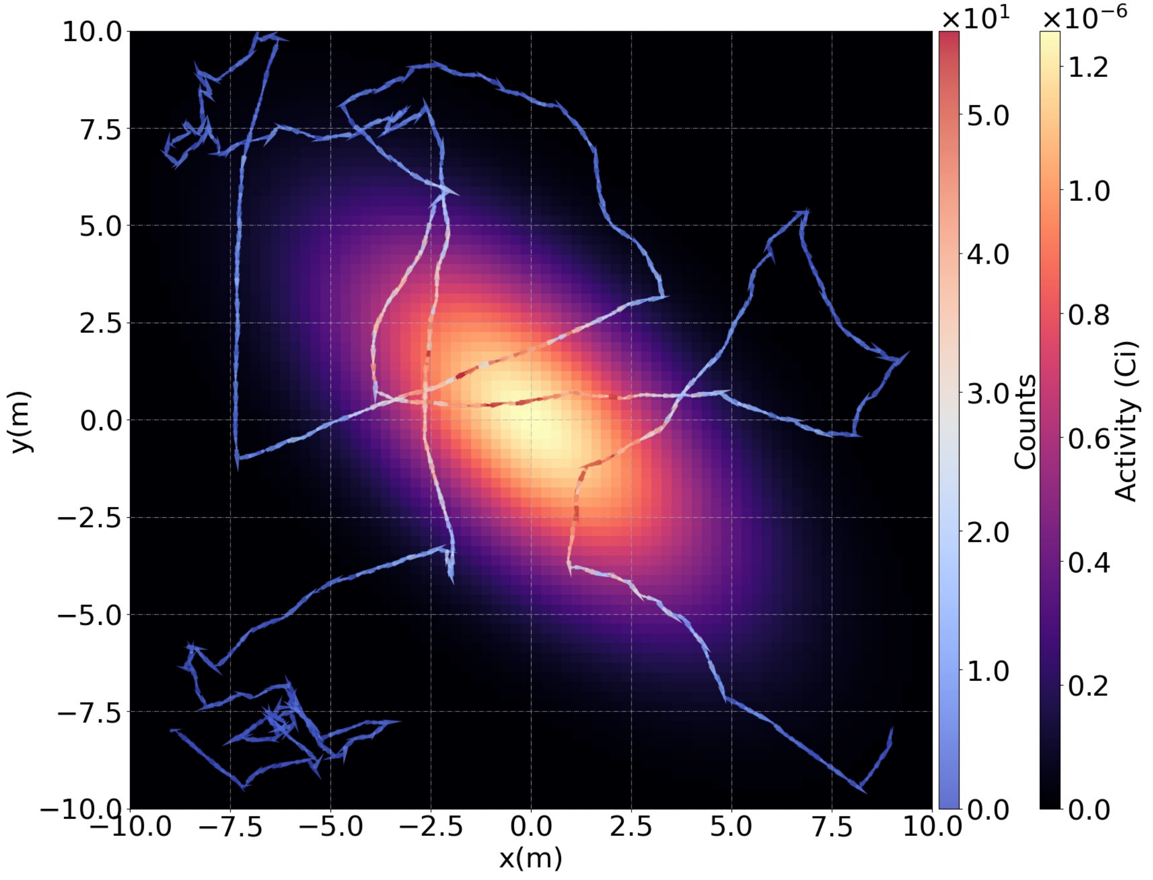
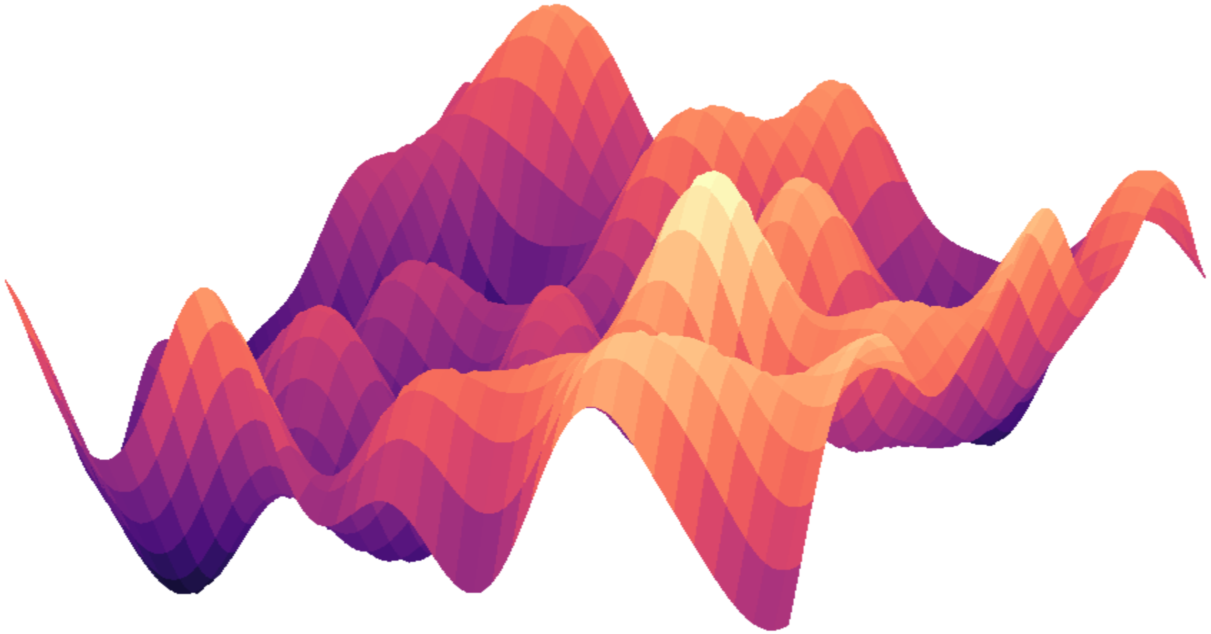


Image pixels are not independent!

Pixels close to each other are more correlated than pixels afar.

Gaussian Process Prior - Introduction



A Gaussian random field with local correlation

Image pixels are not independent!

Pixels close to each other are more correlated than pixels afar.

Using a ***Gaussian random field*** to model spatial correlation.

→ ***Gaussian process, Kriging***

Constructing a Gaussian Process Prior

Image vector $\mathbf{x}$ is governed by a zero-mean multivariate Gaussian distribution $\boldsymbol{\xi}$

$$\boldsymbol{\xi} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_{1,1} & \dots & \Sigma_{1,N} \\ \vdots & \ddots & \vdots \\ \Sigma_{N,1} & \dots & \Sigma_{N,N} \end{bmatrix}$$

Hyperparameter σ determines "characteristic correlation length"

Squared exponential kernel

$$\Sigma_{ij} = \exp\left(-\frac{\|\mathbf{r}_i - \mathbf{r}_j\|_2^2}{2\sigma^2}\right)$$

Hyperparameter λ determines the "mean pixel intensity"

Gaussian-to-exponential link function

$$x_i = f^{-1}(\xi_i) = -\frac{1}{\lambda}\log\left(\frac{1}{2} - \frac{1}{2}\Phi\left(\frac{\xi_i}{\sqrt{2}\Sigma_{ii}}\right)\right)$$

Constructing a Gaussian Process Prior

Image vector $\mathbf{x}$ is governed by a zero-mean multivariate Gaussian distribution $\boldsymbol{\xi}$

$$p(\boldsymbol{\xi}|\mathbf{y}) = \frac{p(\mathbf{y}|\boldsymbol{\xi})p(\boldsymbol{\xi})}{p(\mathbf{y})}$$

Bayes' rule in terms of the underlying variable $\boldsymbol{\xi}$.

$$p(\boldsymbol{\xi}) = \frac{1}{\sqrt{(2\pi)^n \det(\boldsymbol{\Sigma})}} \exp\left(\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \boldsymbol{\xi}\right)$$

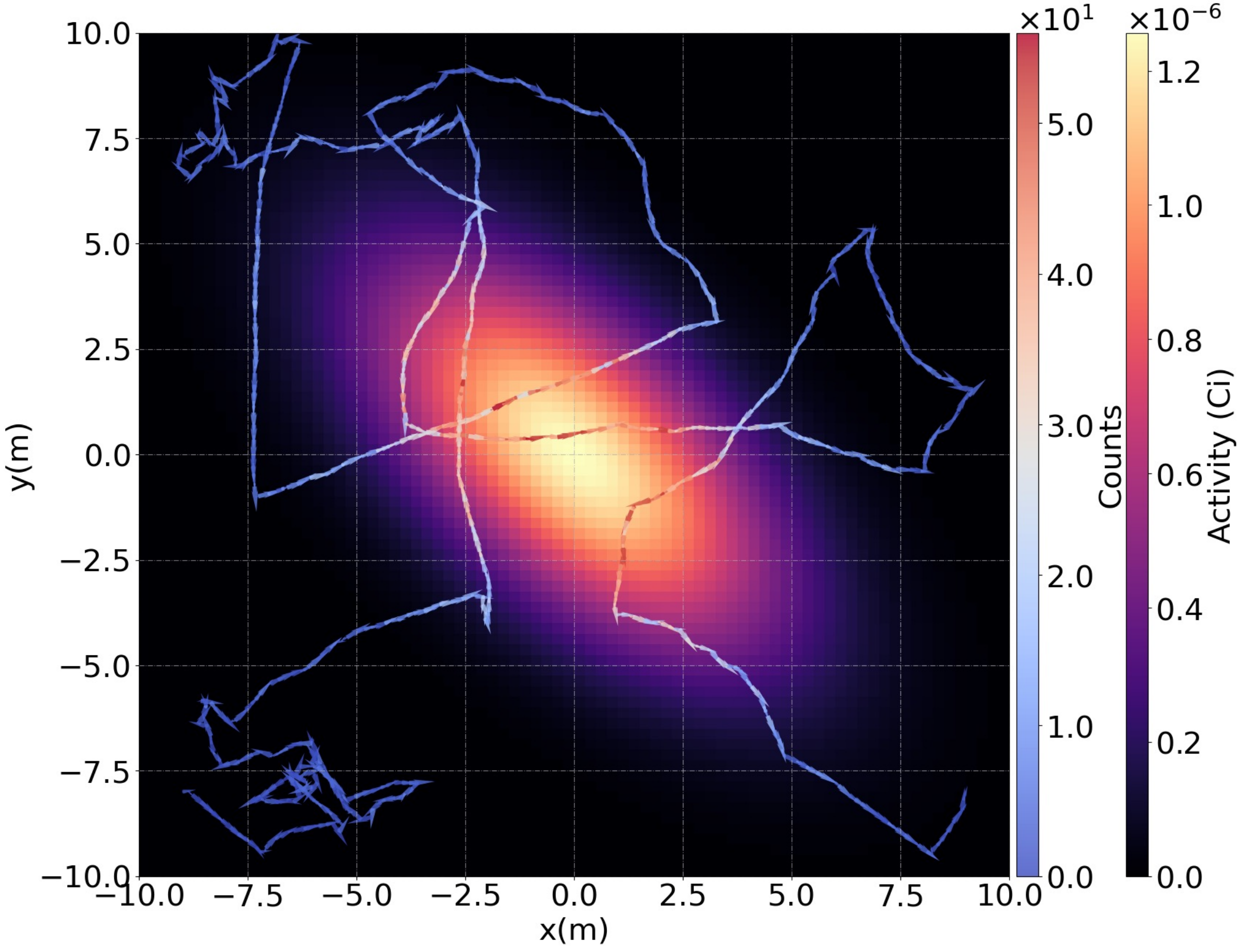
Prior distribution

$$\hat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \sum_{i}^{N} \left[\mathbf{A} f^{-1}(\boldsymbol{\xi}) - \mathbf{y} \odot \log\left(\mathbf{A} f^{-1}(\boldsymbol{\xi})\right)\right]_i + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \boldsymbol{\xi}$$

$$\hat{\mathbf{x}} = \mathbf{f}^{-1}(\hat{\boldsymbol{\xi}})$$

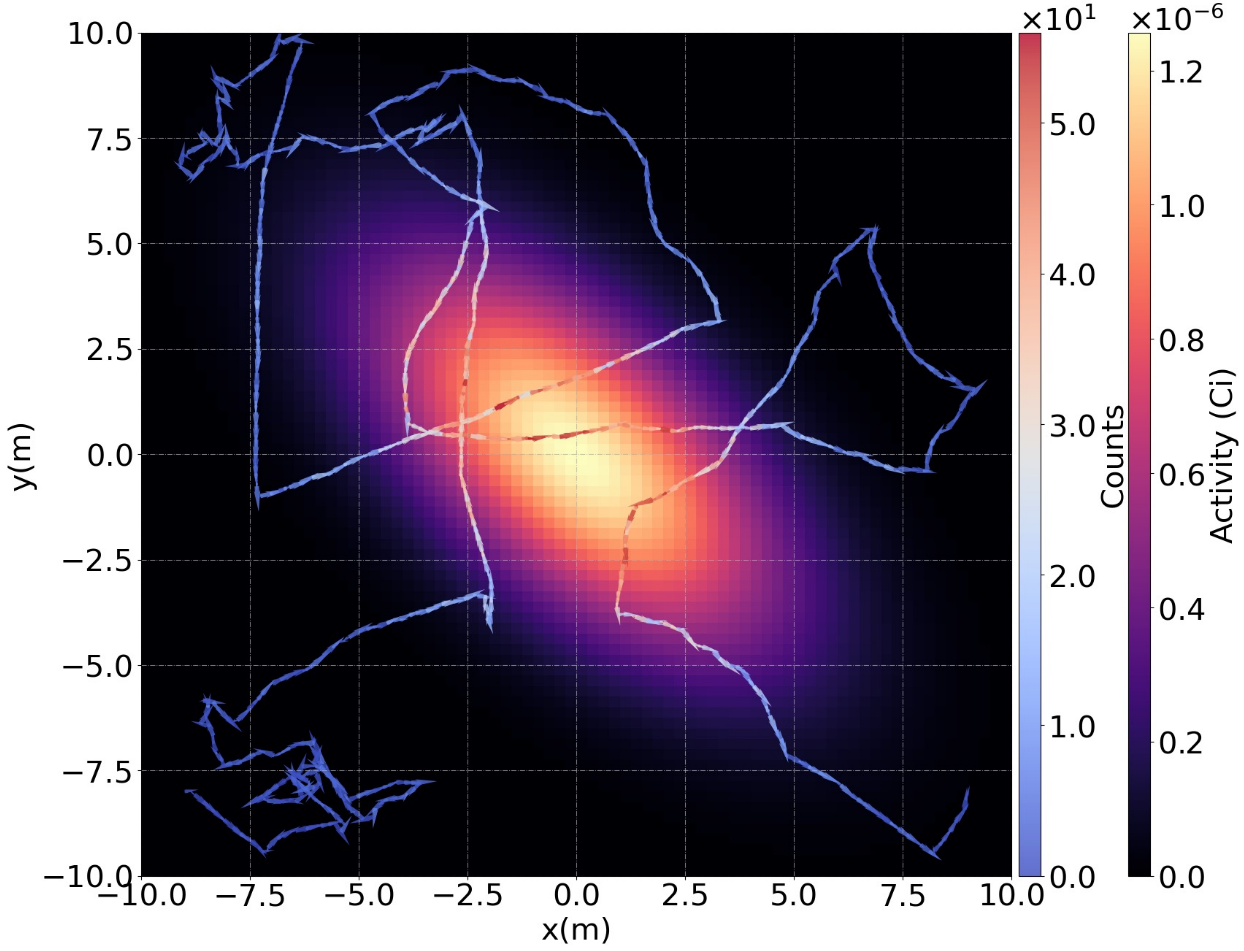
Maximum *a posteriori (MAP)* formulation

Gaussian Process Prior – Results

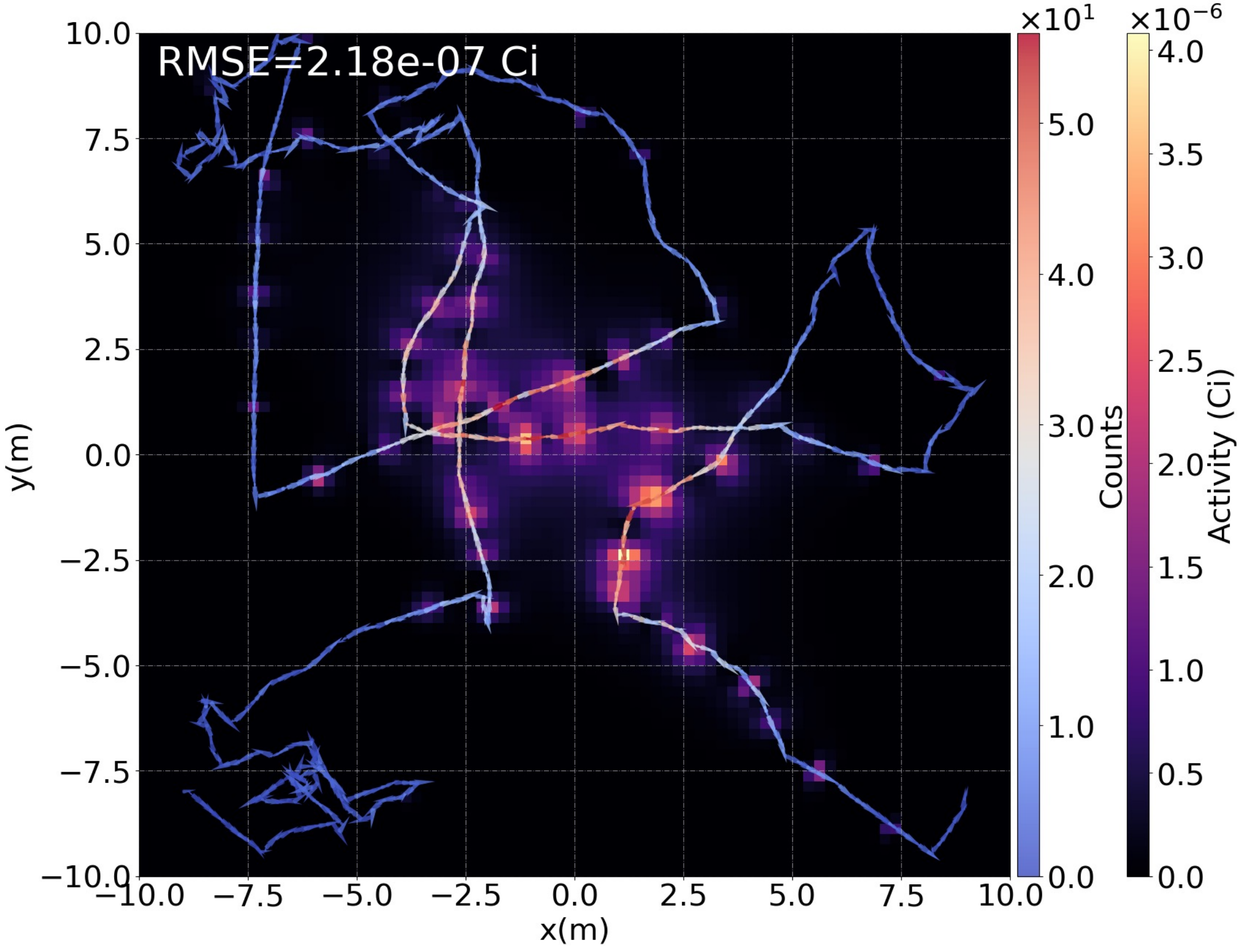


Ground truth

Gaussian Process Prior – Results

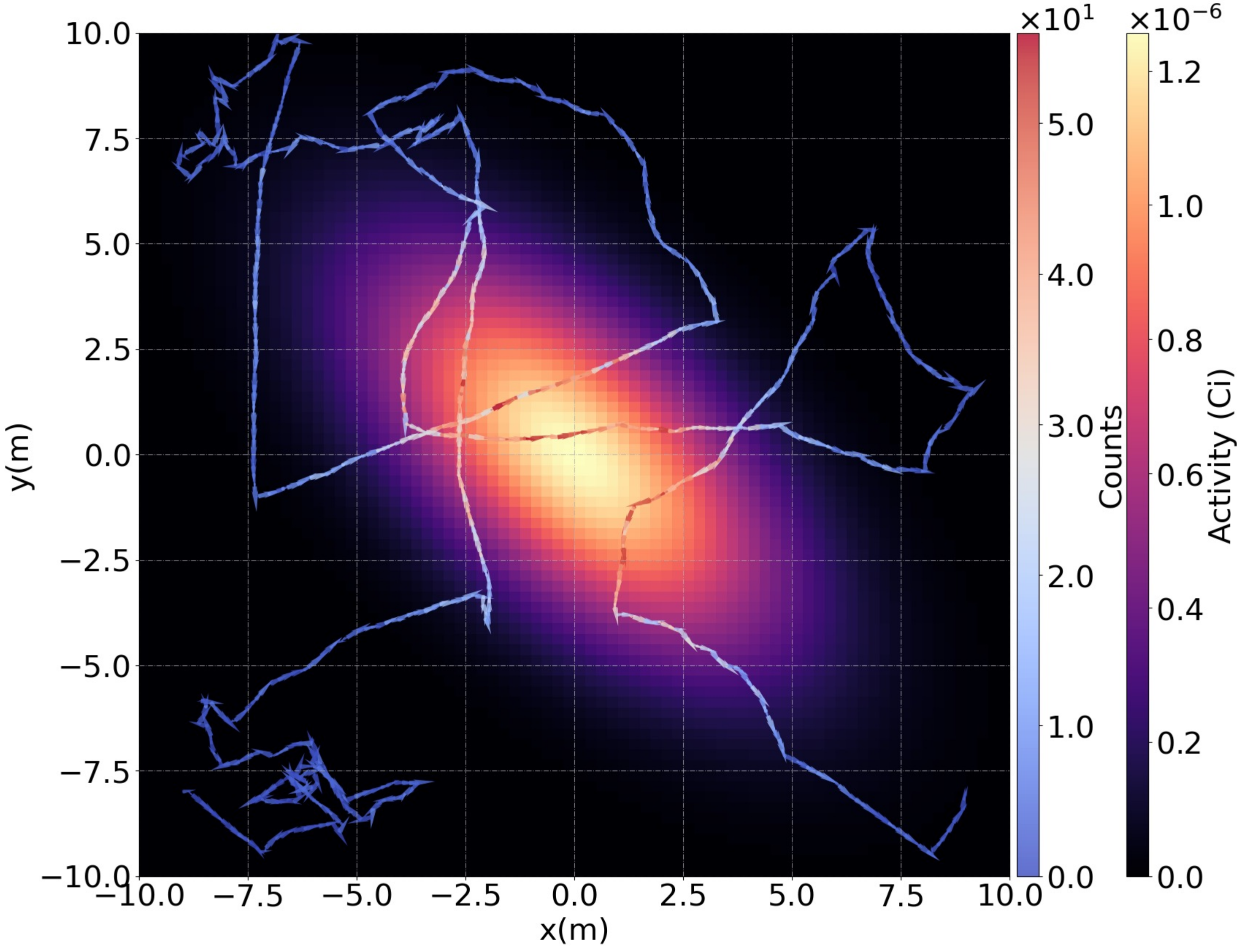


Ground truth

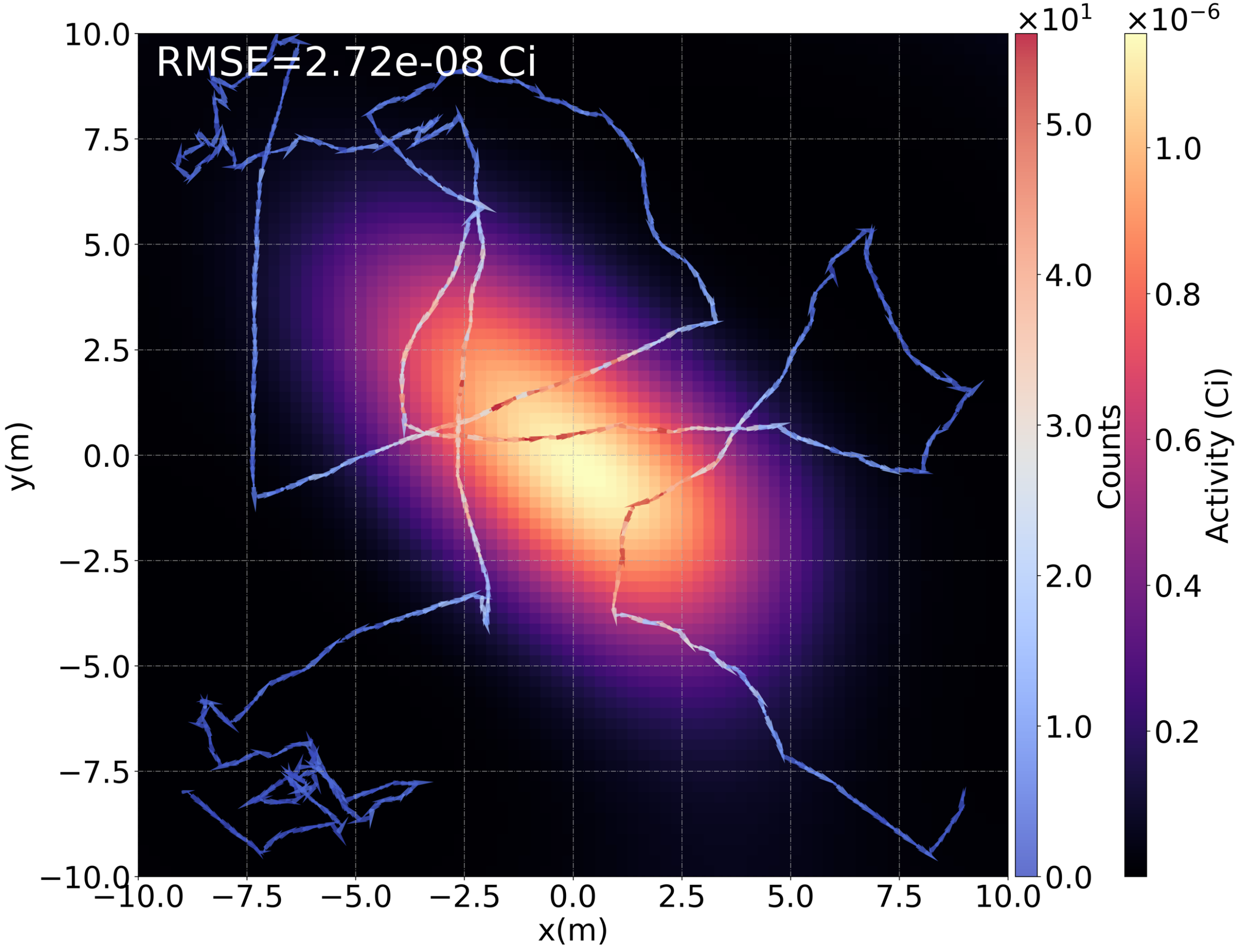


MLEM reconstruction 200 iterations

Gaussian Process Prior – Results



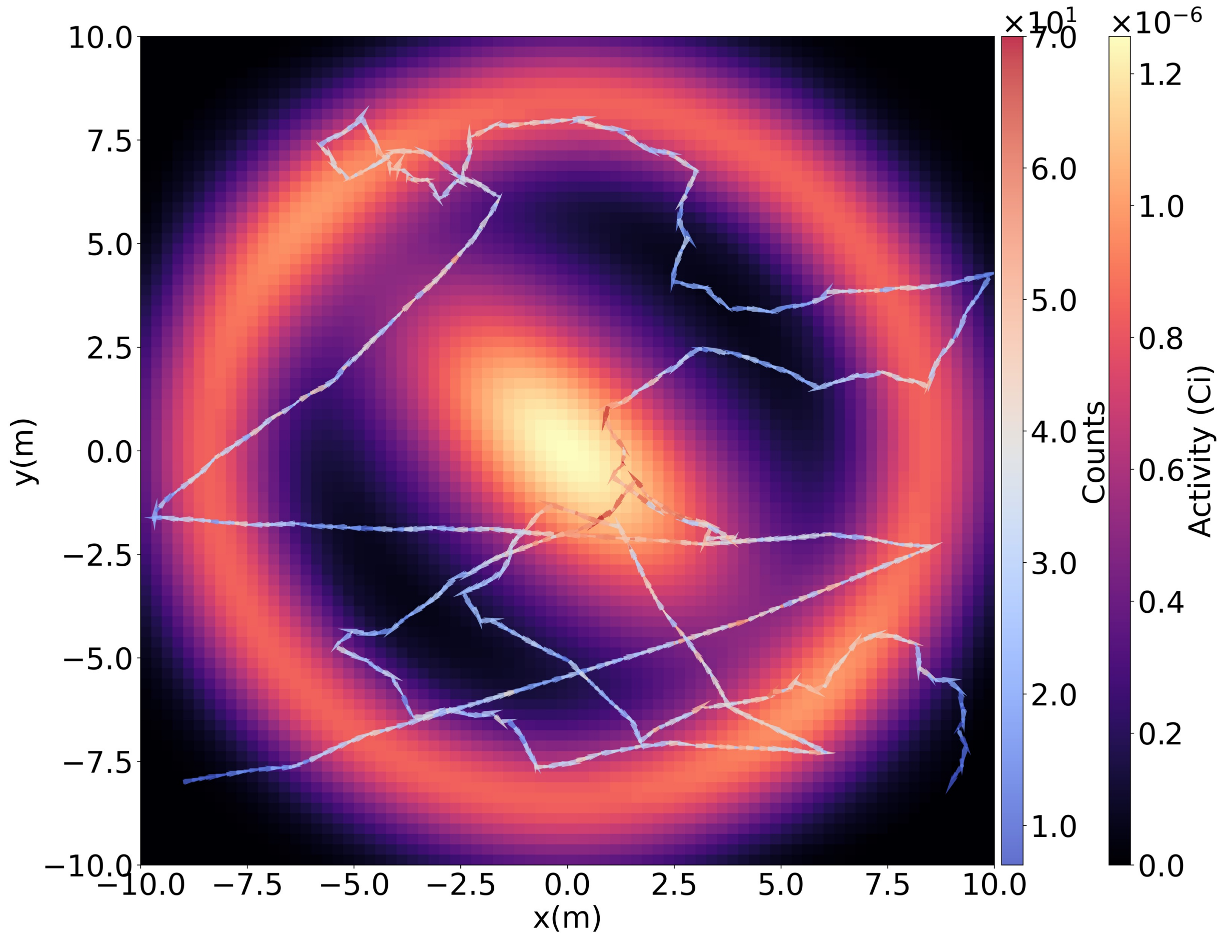
Ground truth



GPP reconstruction

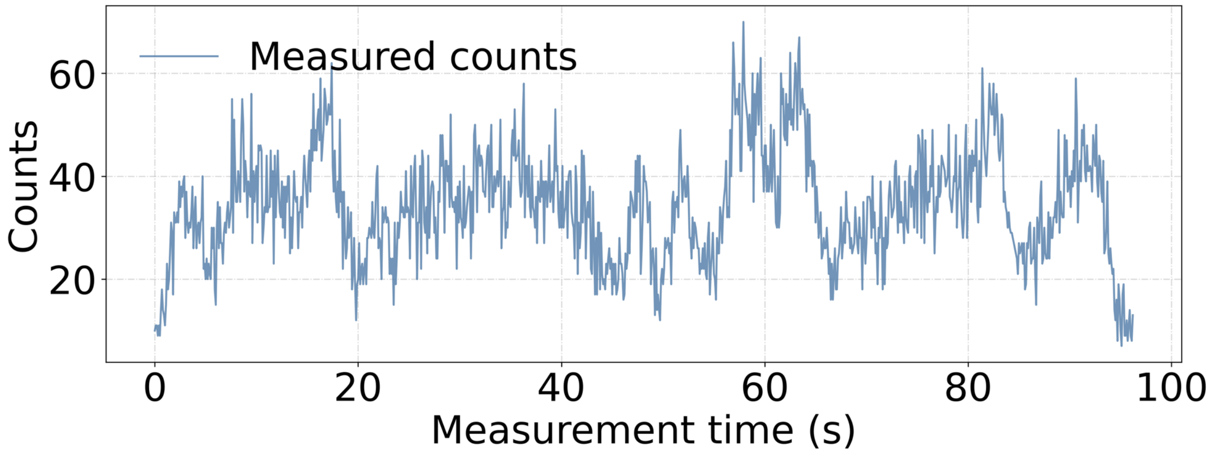
$\sigma = 4.1\,m,\ \lambda = 1.4\times10^{-4} Bq^{-1}$

Gaussian Process Prior – Results

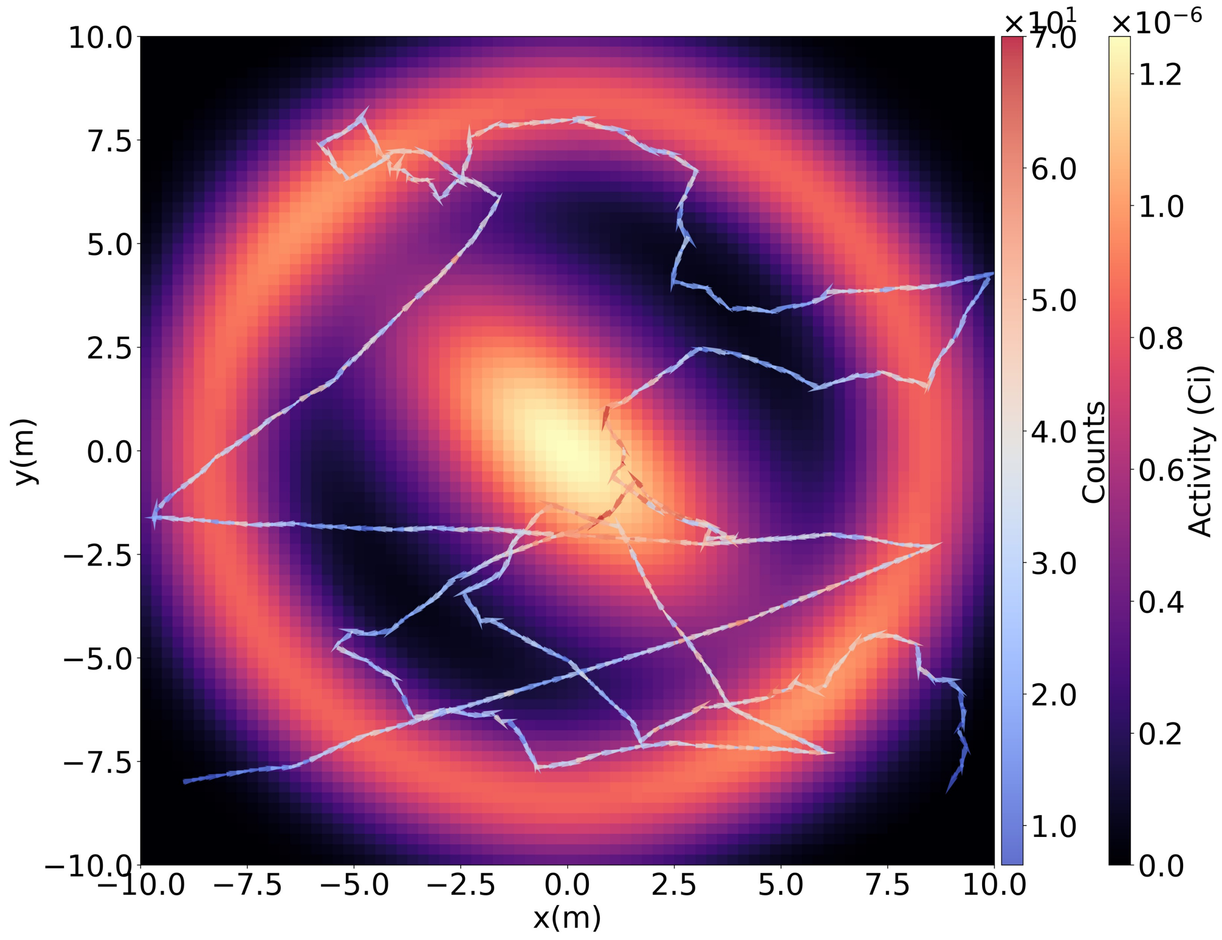


Ground truth

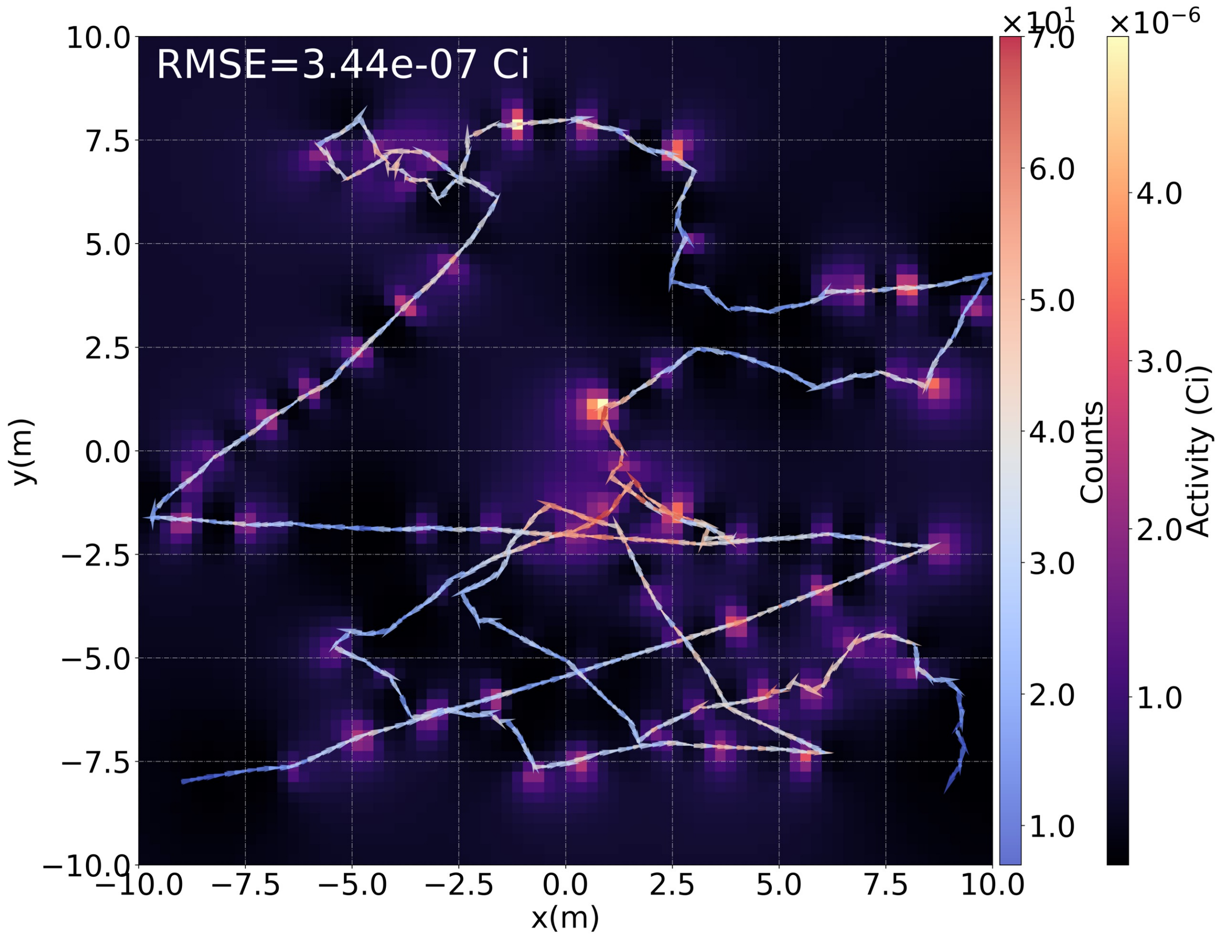
300 μCi distributed source



Gaussian Process Prior – Results

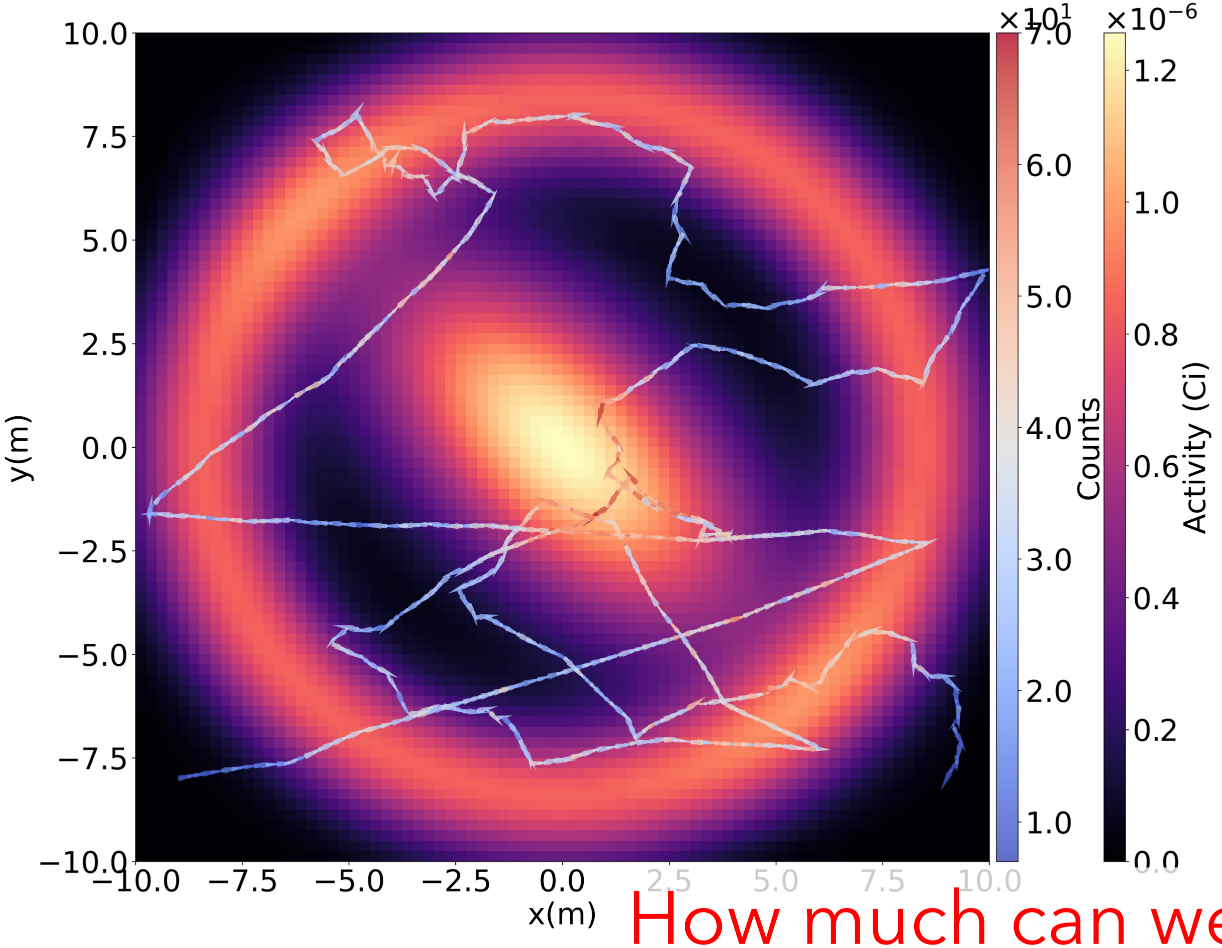


Ground truth

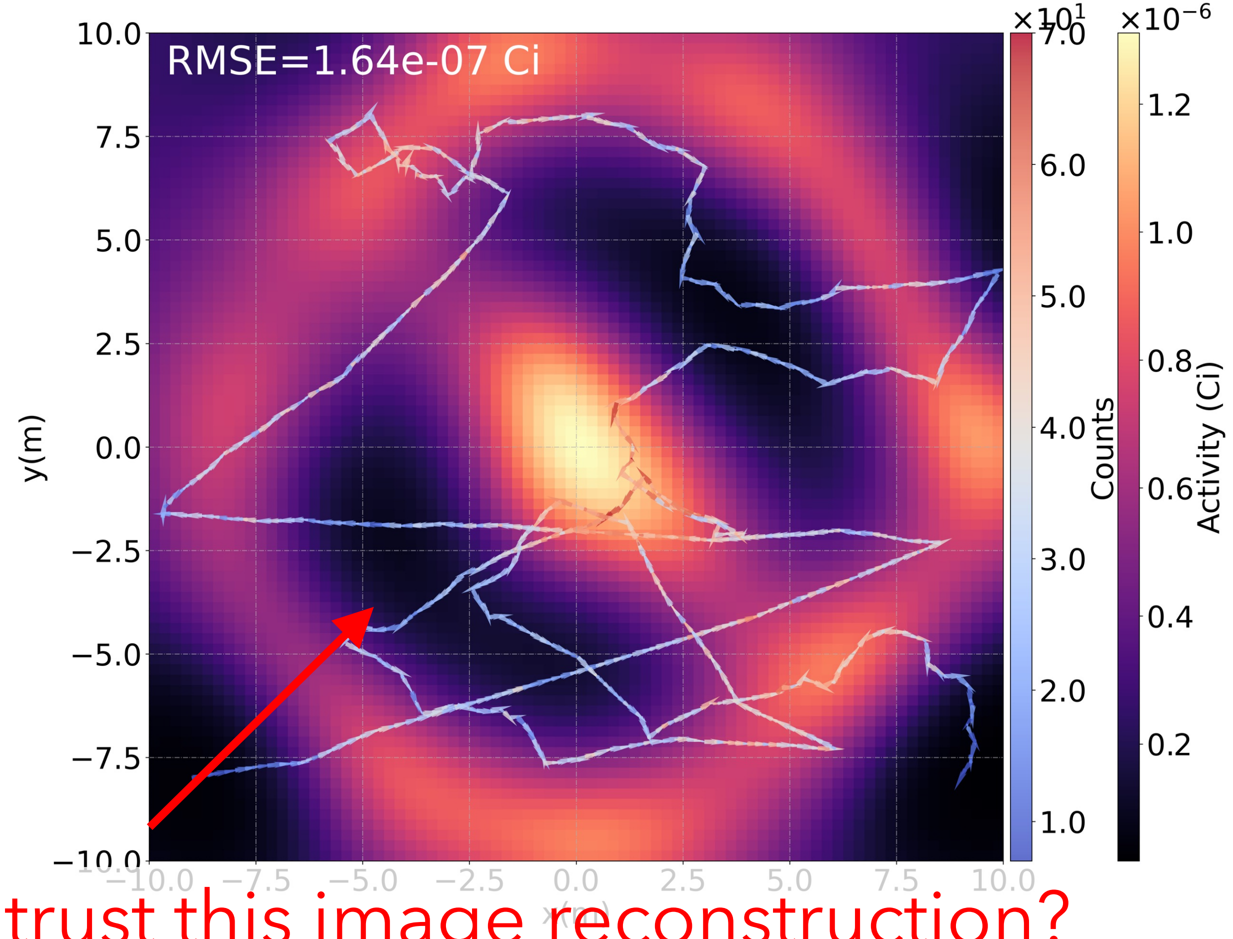


MLEM reconstruction 200 iterations

Gaussian Process Prior – Results



Ground truth



How much can we trust this image reconstruction?

GPP reconstruction

$\sigma = 2.4\ m,\ \lambda = 6.0\times10^{-5}\,Bq^{-1}$

Bayesian Uncertainty Quantification

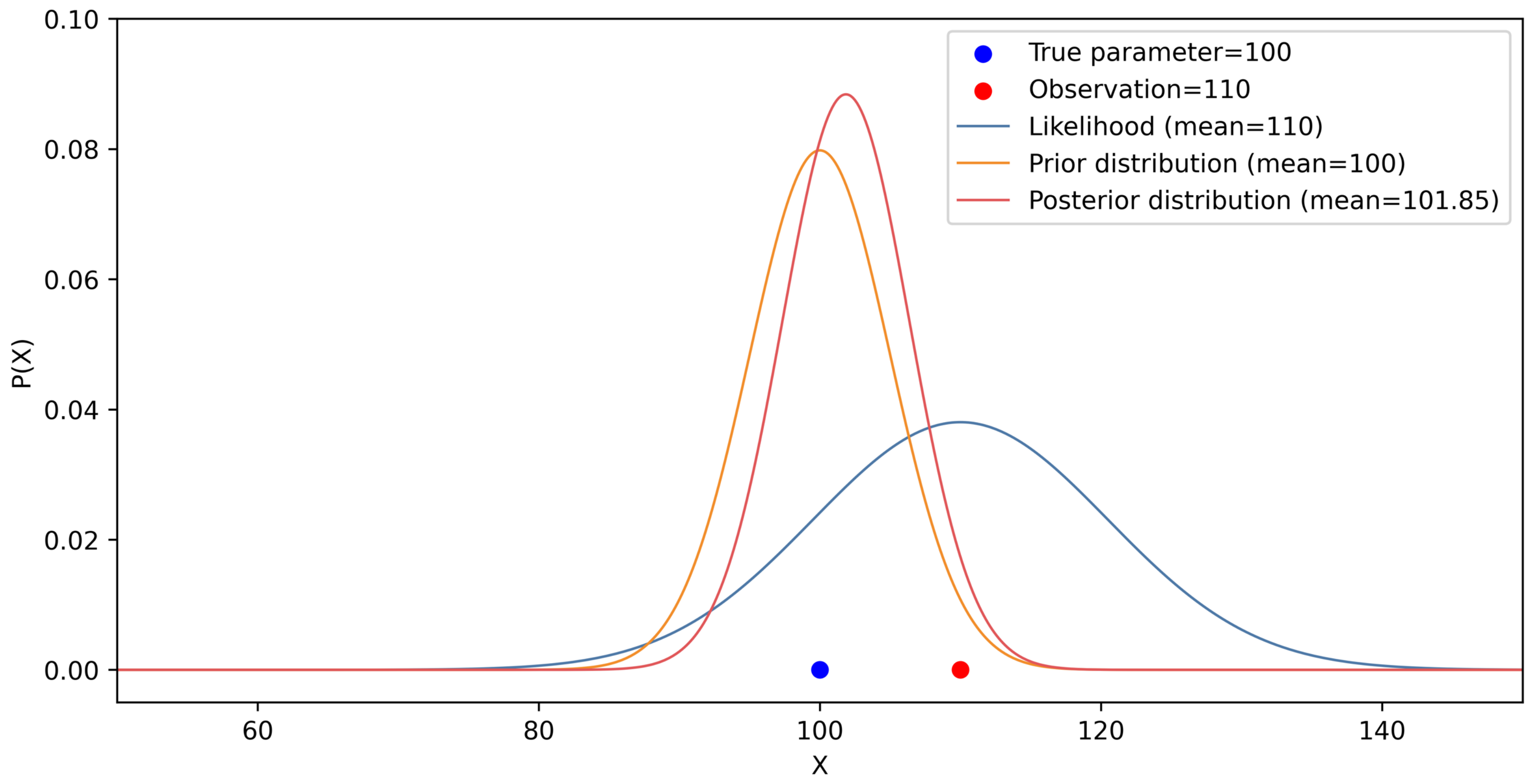
Uncertainty – Bayesian Credible Interval

Bayes' rule $$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})}$$

Prior distribution

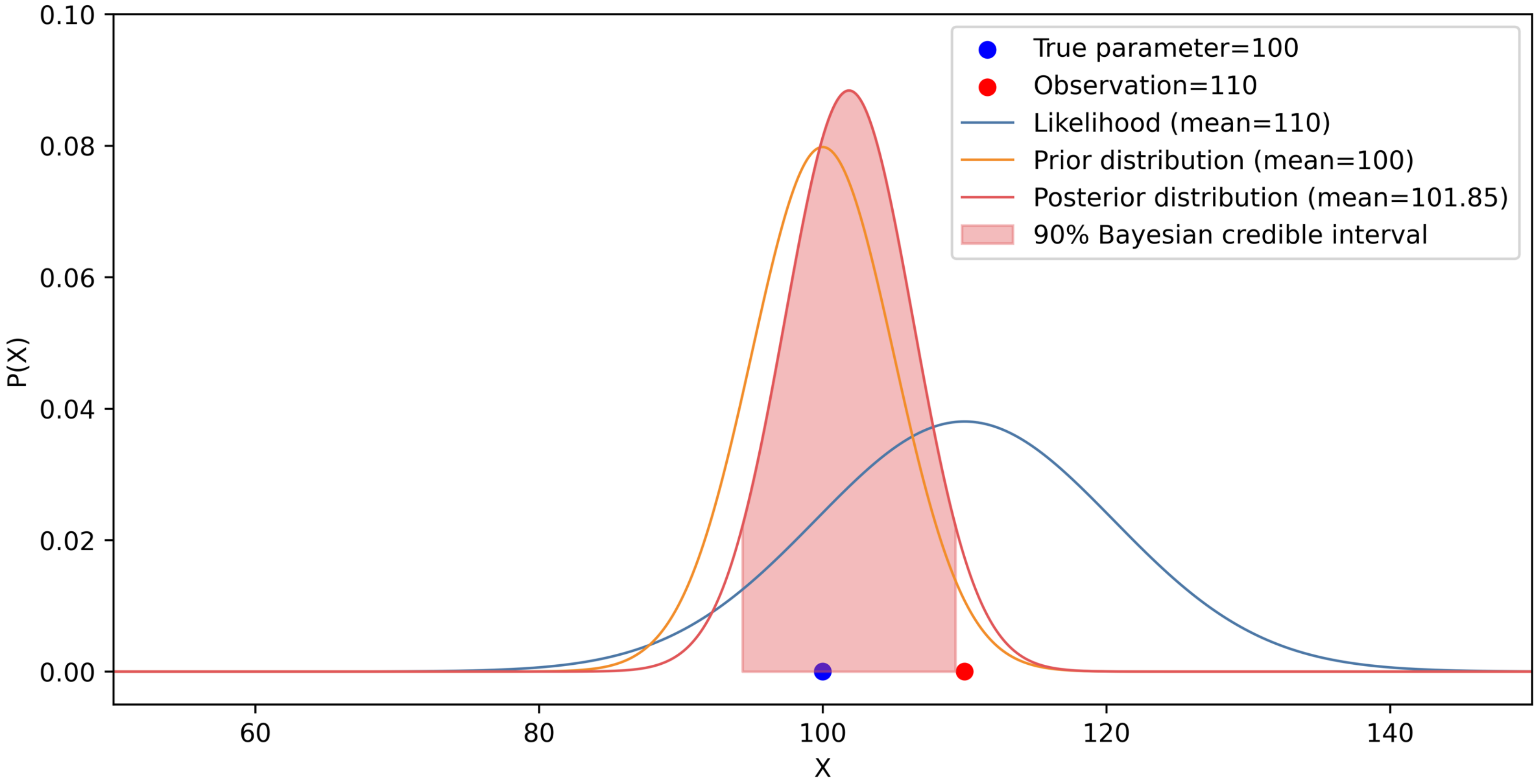
Posterior distribution

Likelihood



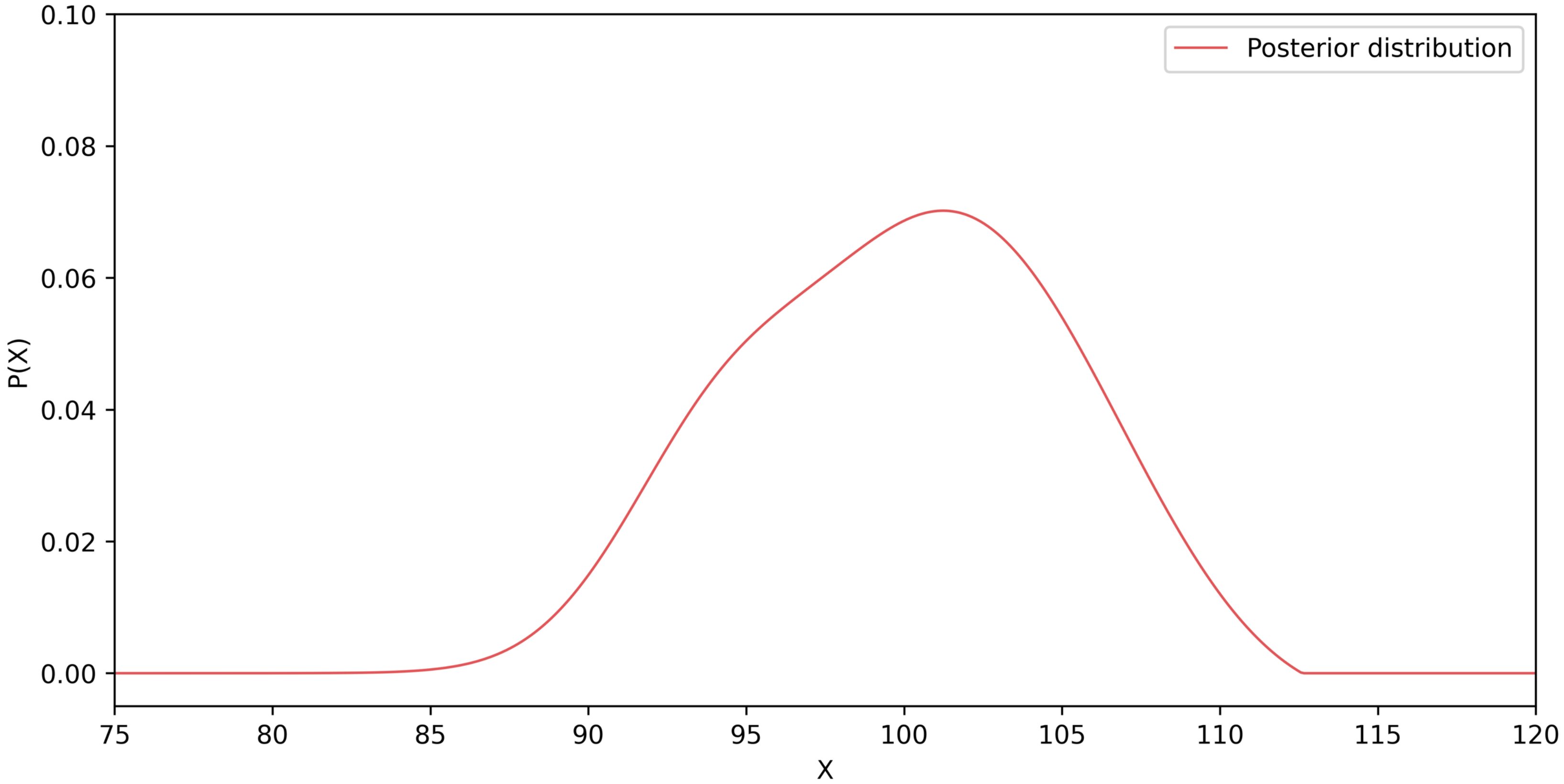
Uncertainty – Bayesian Credible Interval

In Bayesian statistics, ***credible intervals*** are often quoted to represent uncertainties.



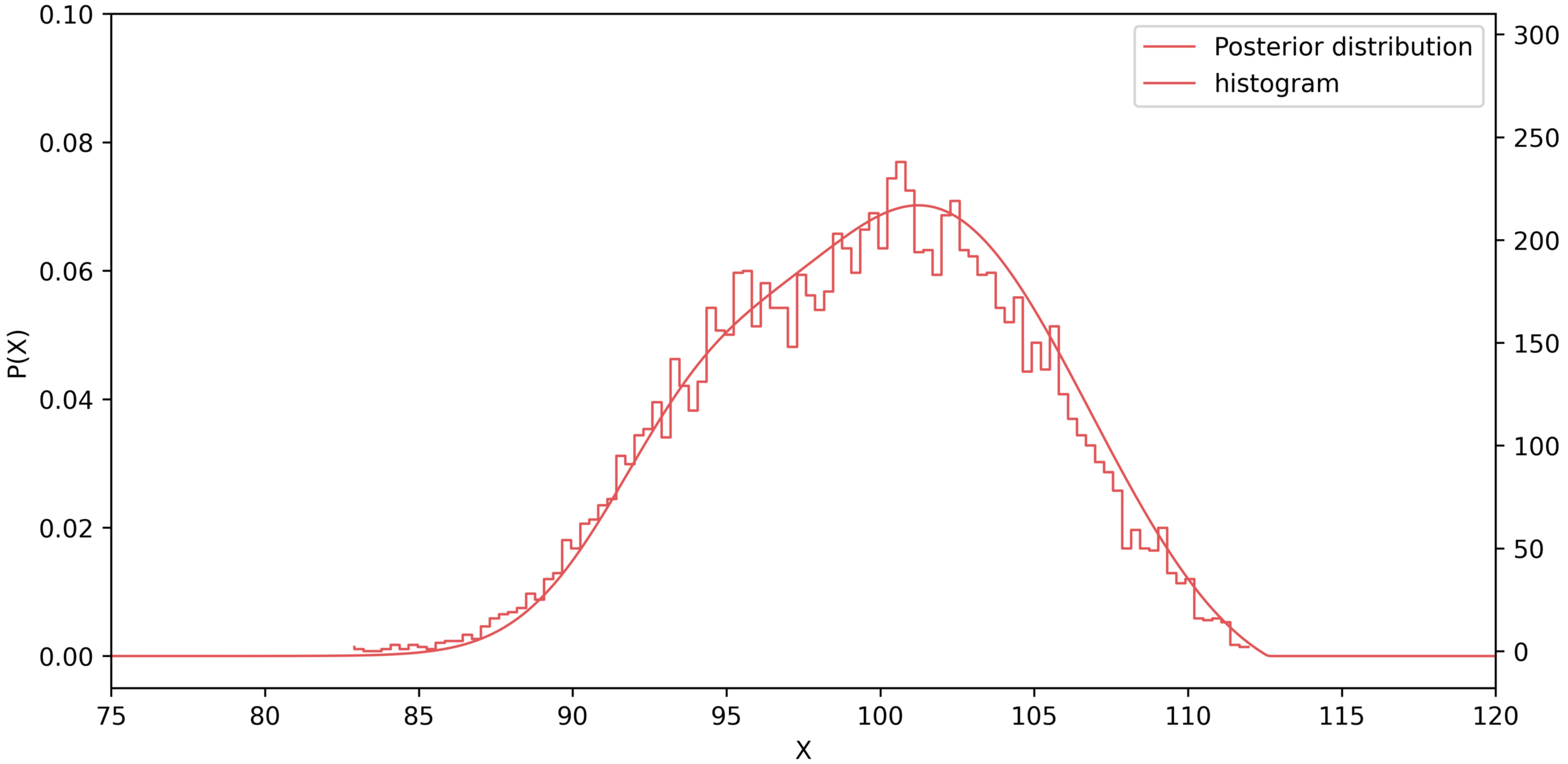
Uncertainty – Bayesian Credible Interval

How can we find the credible interval, when **the posterior is not analytically available?**



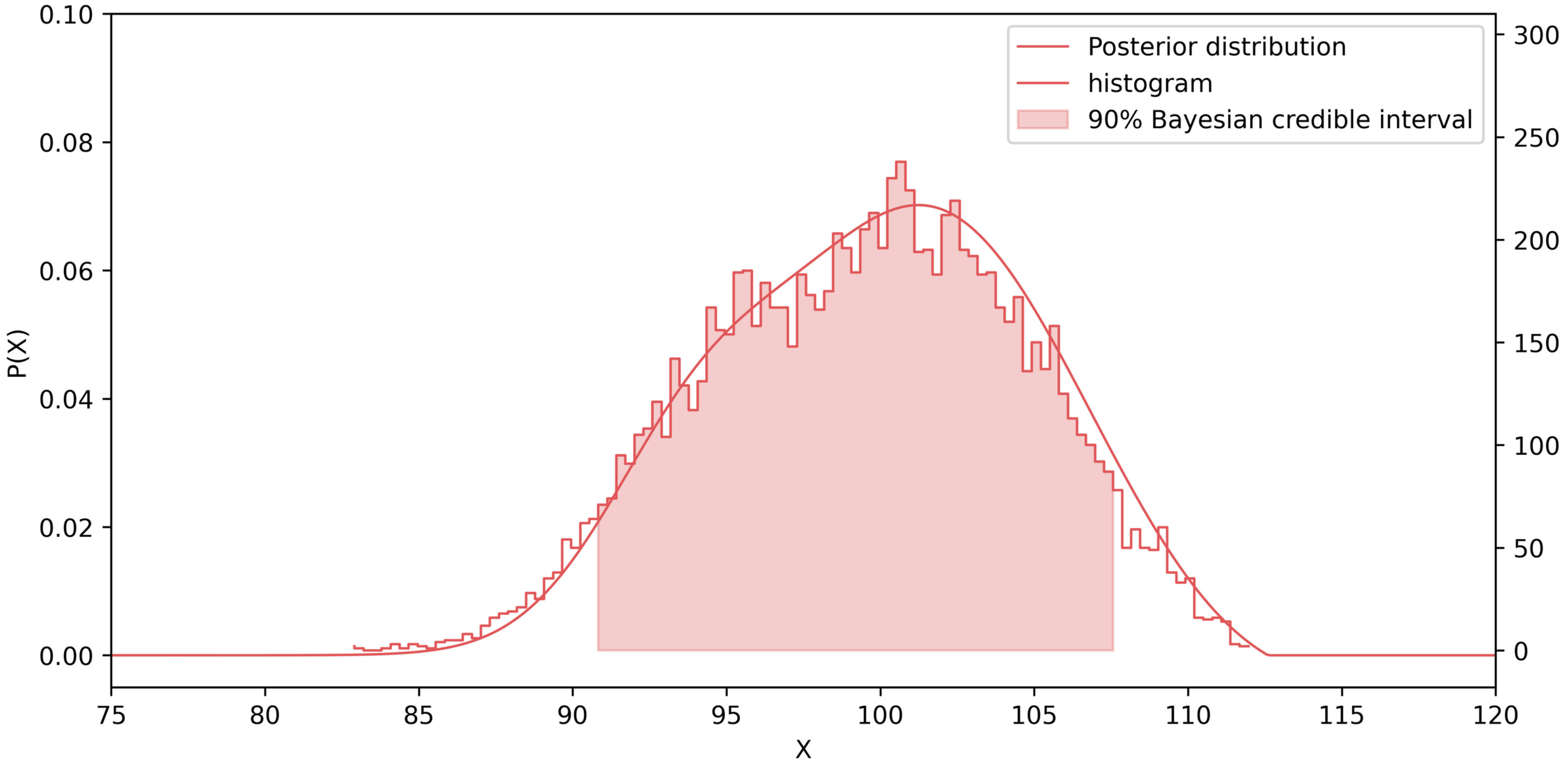
Uncertainty – Bayesian Credible Interval

How can we find the credible interval, when **the posterior is not analytically available? : Collect samples.**



Uncertainty – Bayesian Credible Interval

How can we find the credible interval, when **the posterior is not analytically available? : Collect samples.**

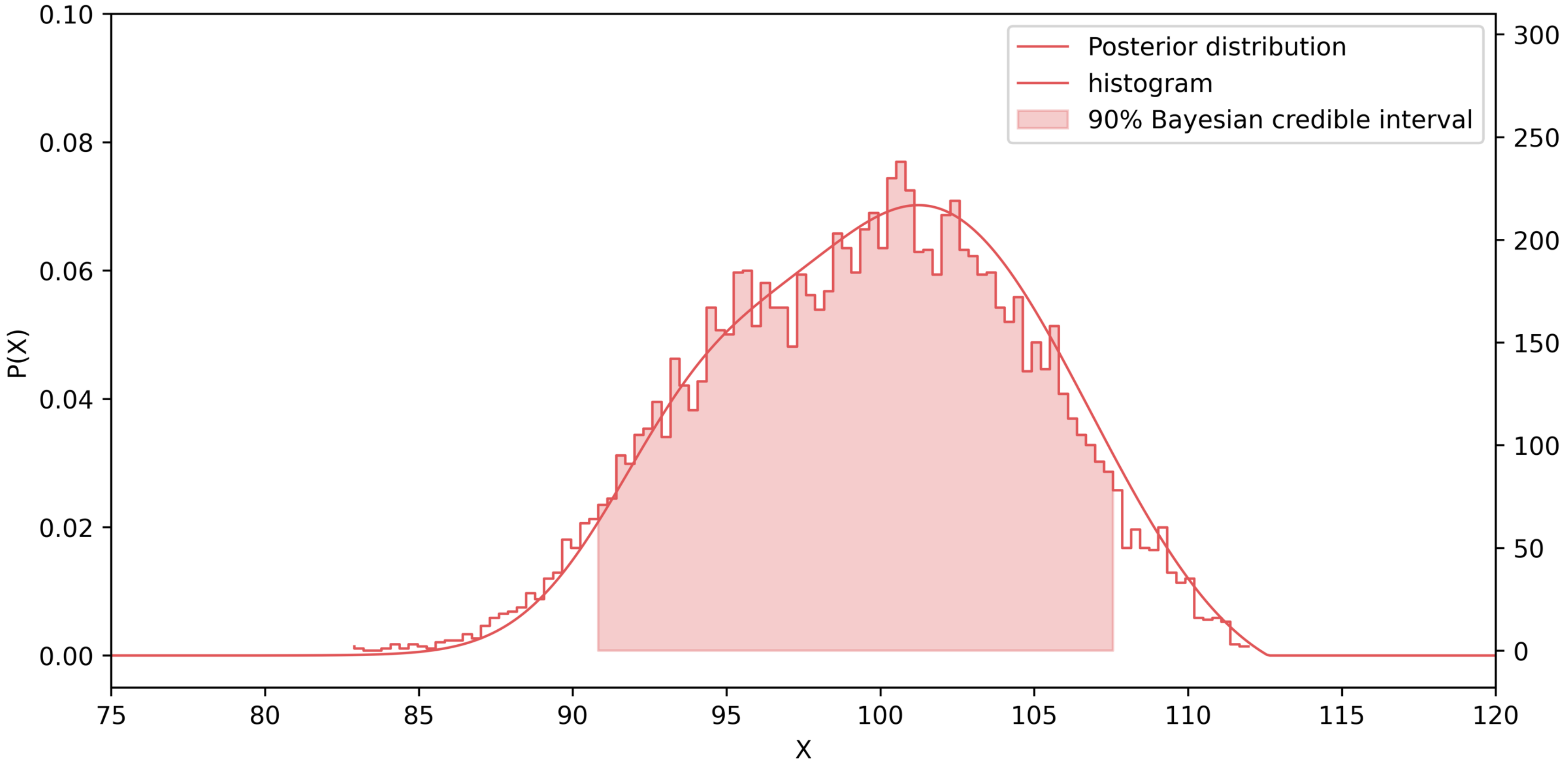


Uncertainty – Bayesian Credible Interval

Posterior from the GPP algorithm :

$$P(\vec{\xi}_w|\vec{y}) = \frac{P(\vec{y}|\vec{\xi}_w)P(\vec{\xi}_w)}{\int P(\vec{y}|\vec{\xi}_w)P(\vec{\xi}_w)} = \frac{\exp\left(\sum_i \left((Af^{-1}(C_\Sigma^T\vec{\xi}_w))_i + y_i \log\left(Af^{-1}(C_\Sigma^T\vec{\xi}_w)_i\right)\right) + \vec{\xi}_w^T\vec{\xi}_w\right)}{\int \exp\left(\sum_i \left((Af^{-1}(C_\Sigma^T\vec{\xi}_w))_i + y_i \log\left(Af^{-1}(C_\Sigma^T\vec{\xi}_w)_i\right)\right) + \vec{\xi}_w^T\vec{\xi}_w\right) d\vec{\xi}_w}$$

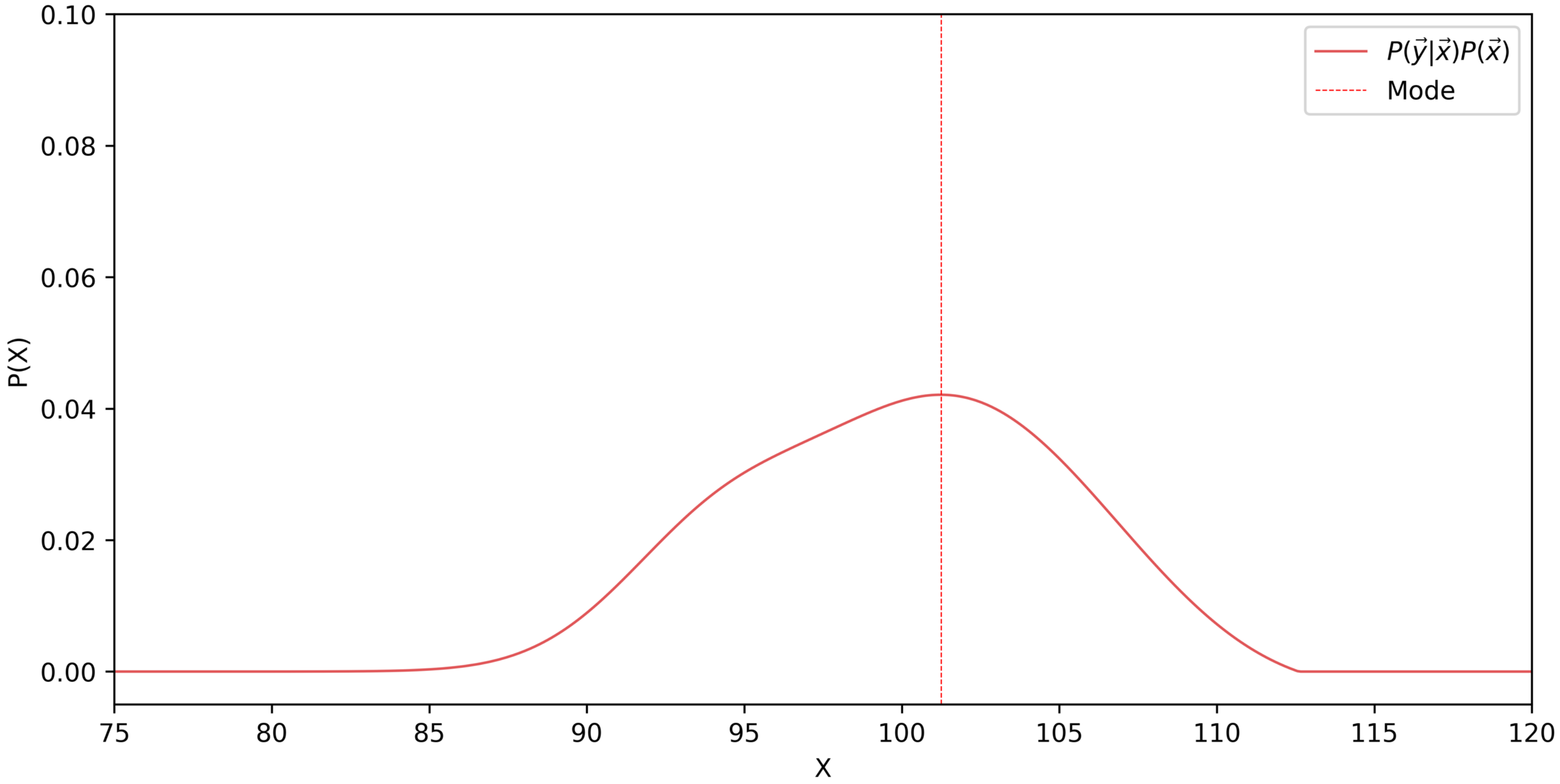
Markov Chain Monte Carlo (MCMC) to sample from the intractable posterior



Laplace Approximation

Instead of sampling the posterior, **<u>we approximate the posterior</u>**

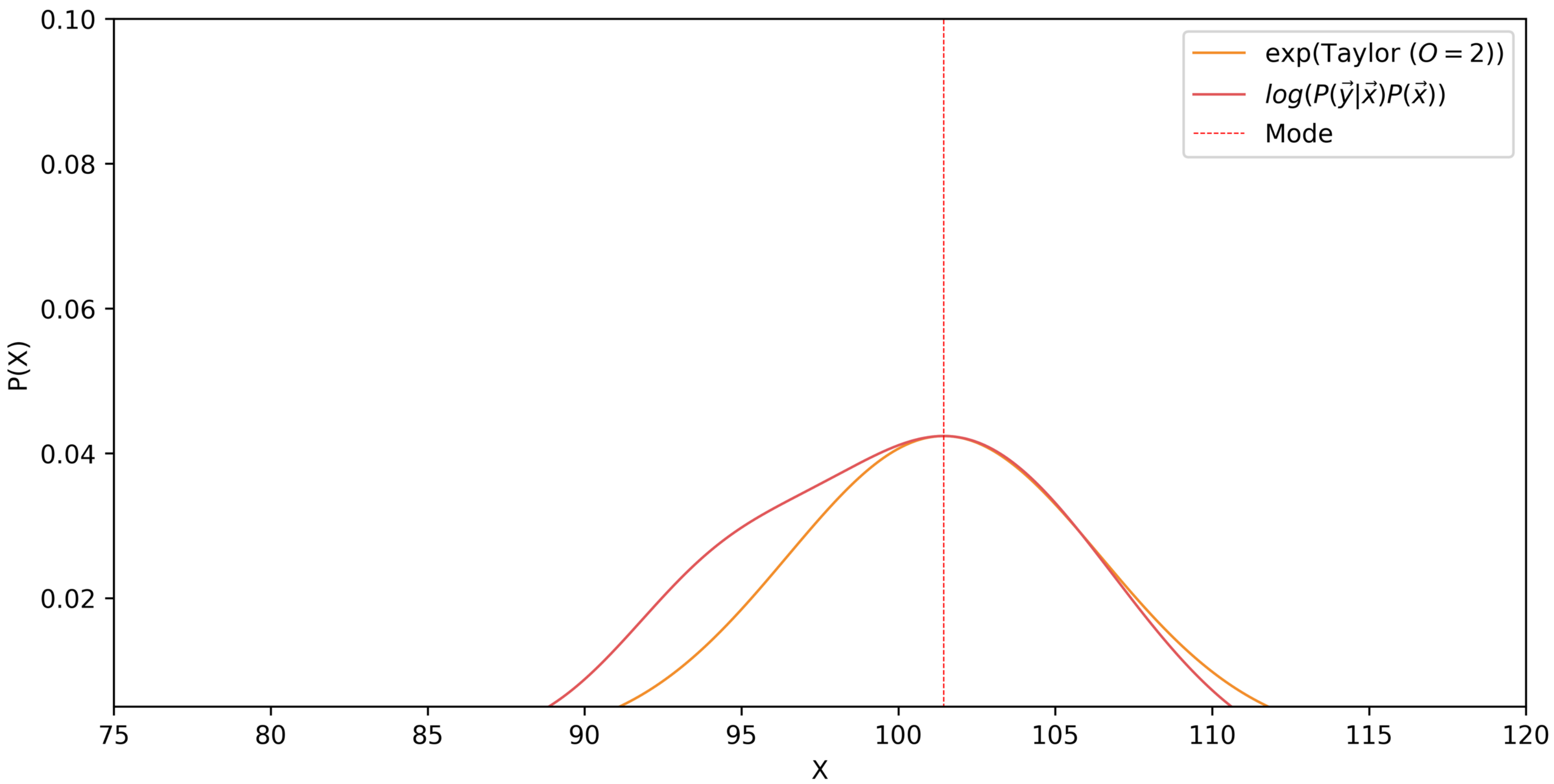
$$
\begin{aligned}
P(\xi_w|\mathbf{y}) &= \frac{P(\mathbf{y}|\xi_w)P(\xi_w)}{\int P(\mathbf{y}|\xi_w)P(\xi_w)} \quad = \frac{\sum_i \left((Af^{-1}(C_\Sigma^T\xi_w))_i + y_i \log\left(Af^{-1}(C_\Sigma^T\xi_w)_i\right)\right) + \xi_w^T\xi_w}{\int \sum_i \left((Af^{-1}(C_\Sigma^T\xi_w))_i + y_i \log\left(Af^{-1}(C_\Sigma^T\xi_w)_i\right)\right) + \xi_w^T\xi_w \; d\xi_w} \\
&= \frac{h(\xi_w)}{Z}
\end{aligned}
$$



Laplace Approximation

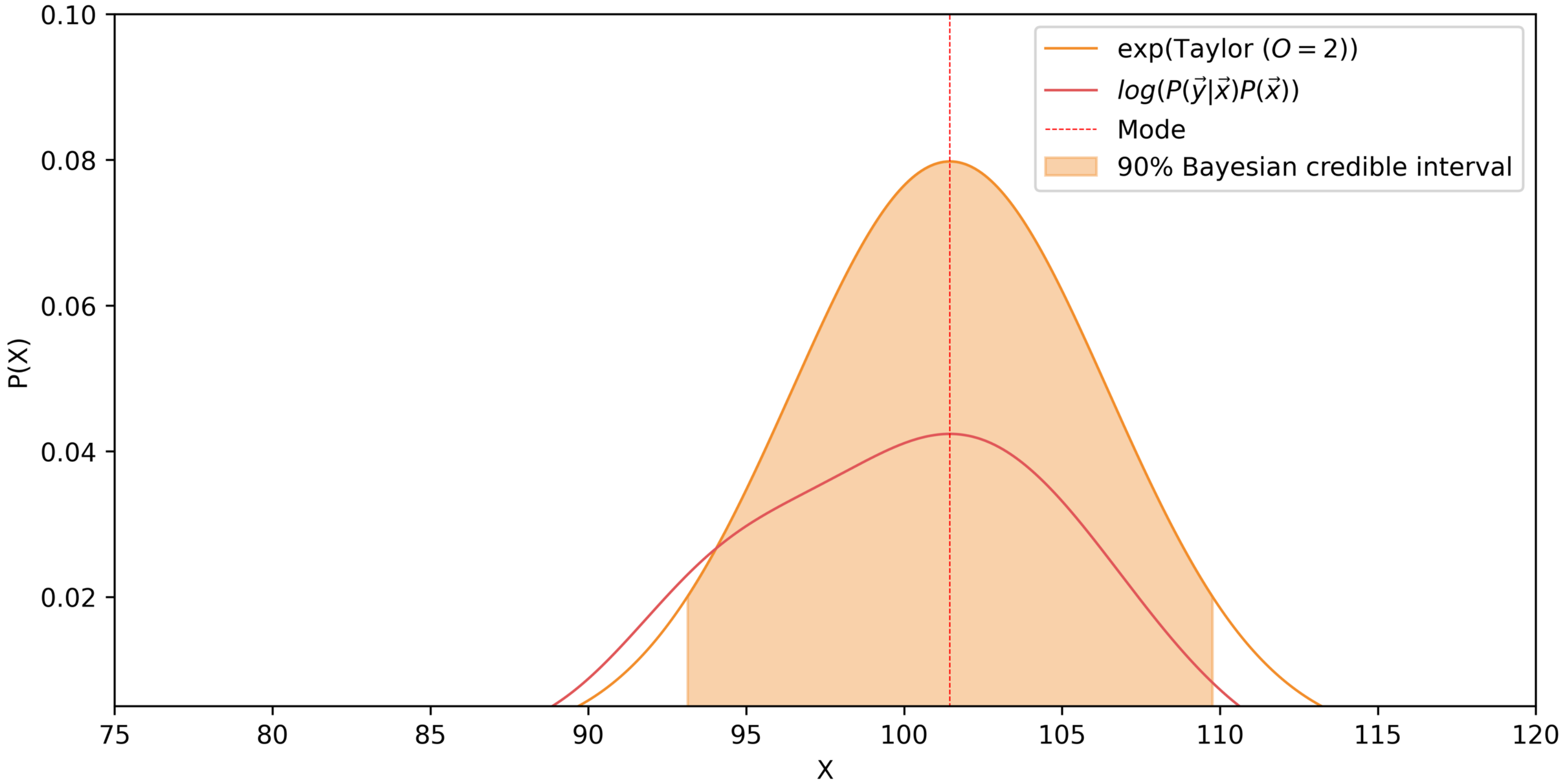
Approximate the posterior with a multivariate Gaussian distribution

$$P(\boldsymbol{\xi}|\boldsymbol{y}) \approx \frac{1}{Z} h(\hat{\boldsymbol{\xi}}) \exp\left(-\frac{1}{2}(\boldsymbol{\xi} - \hat{\boldsymbol{\xi}})^\intercal \left(-\frac{d^2 \log h(\hat{\boldsymbol{\xi}})}{d\boldsymbol{\xi}^2}\right)(\boldsymbol{\xi} - \hat{\boldsymbol{\xi}})\right)$$



Laplace Approximation

$$P(\boldsymbol{\xi}|\boldsymbol{y}) \approx \frac{1}{Z} h(\hat{\boldsymbol{\xi}}) \exp\left(-\frac{1}{2}(\boldsymbol{\xi}-\hat{\boldsymbol{\xi}})^\intercal \left(-\frac{d^2 \log h(\hat{\boldsymbol{\xi}})}{d\boldsymbol{\xi}^2}\right)(\boldsymbol{\xi}-\hat{\boldsymbol{\xi}})\right) = \mathcal{N}\left(\hat{\boldsymbol{\xi}}, -\frac{d^2 \log h(\hat{\boldsymbol{\xi}})}{d\boldsymbol{\xi}^2}\right)$$



Bayesian UQ Can Be Done in Two Ways…

1. Sampling method

Slow, but more accurate

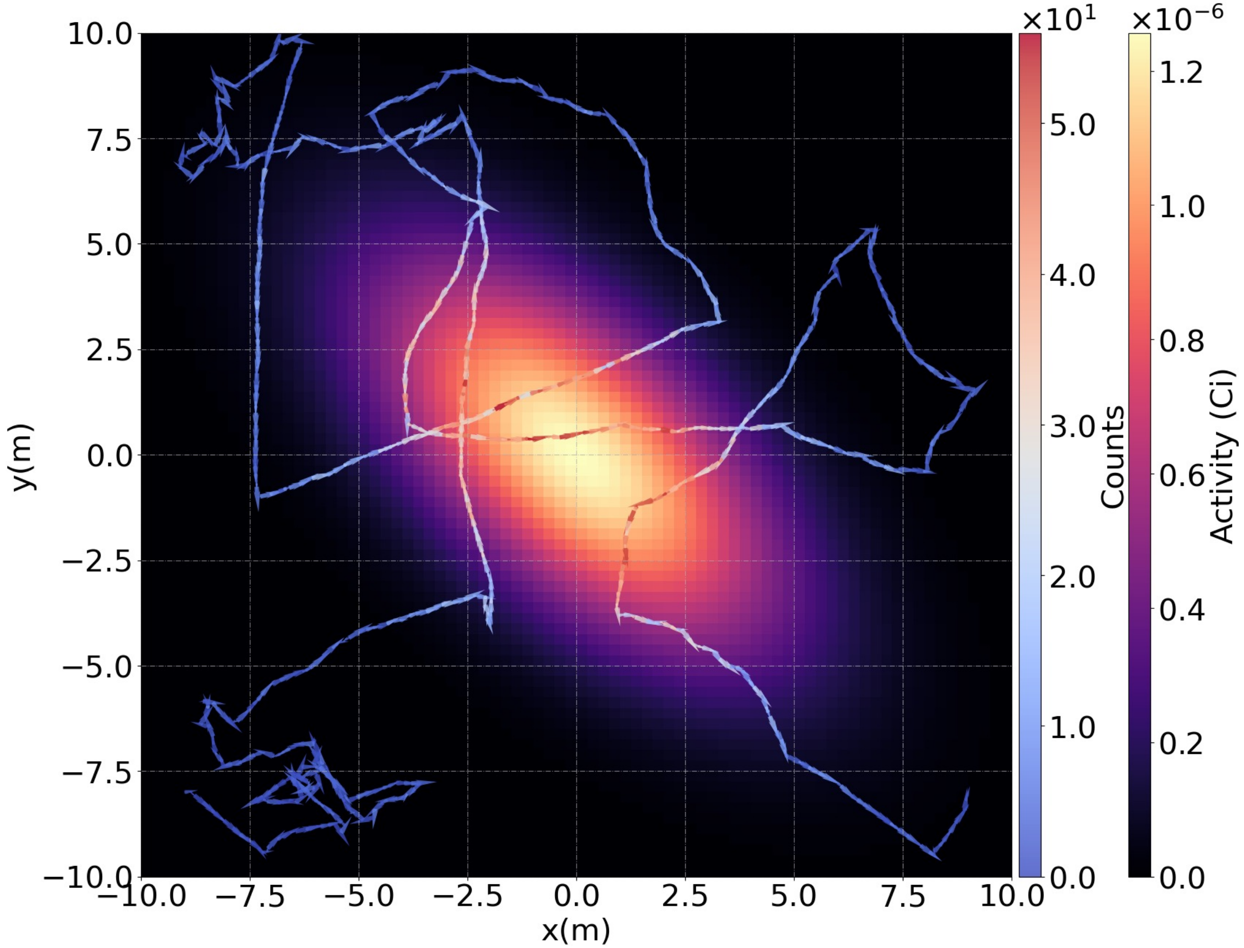
– Preconditioned Crank-Nicolson Markov Chain Monte Carlo (pCN MCMC)

2. Approximation method

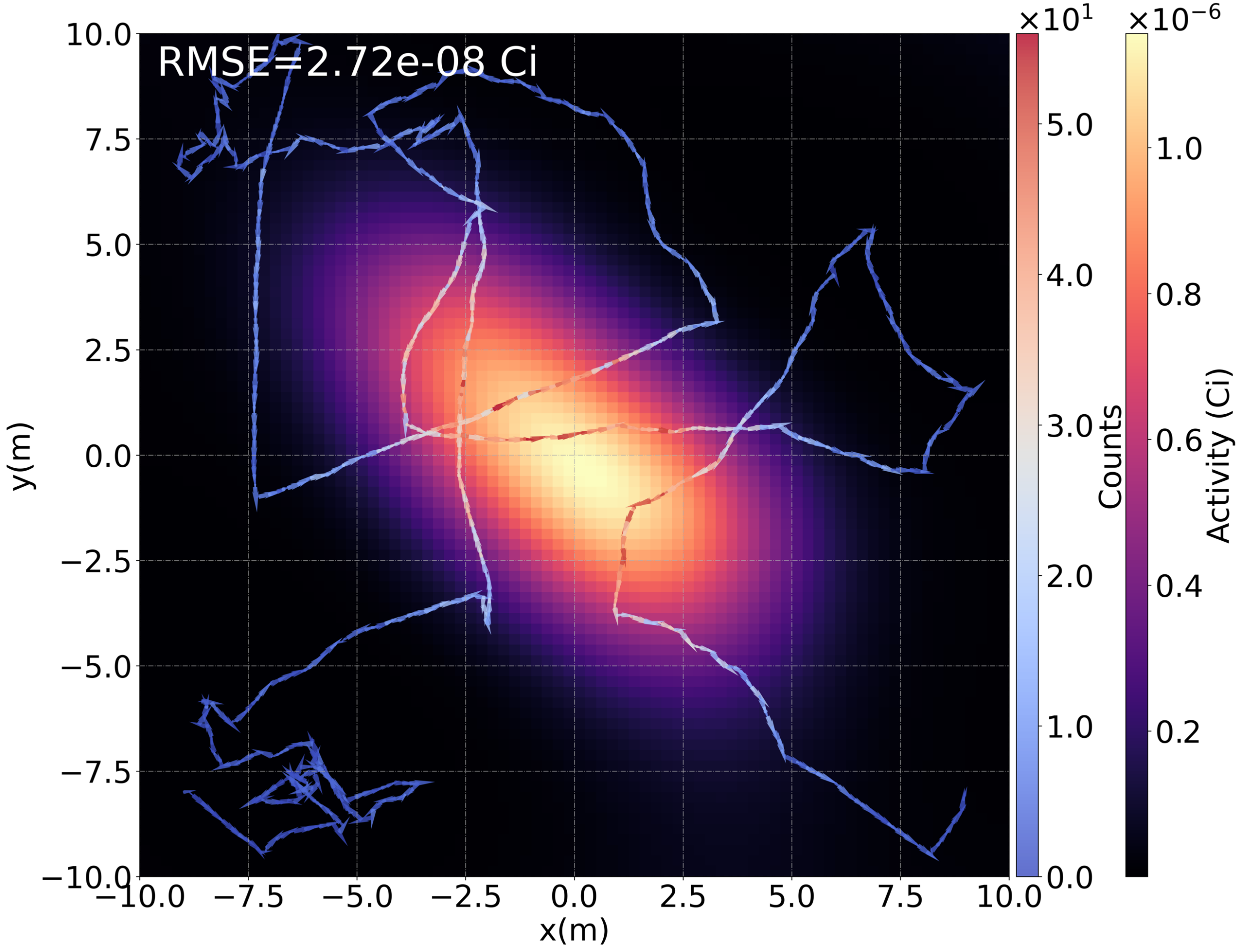
Fast, but less accurate

– Laplace approximation

pCN MCMC Uncertainty Quantification



Ground truth

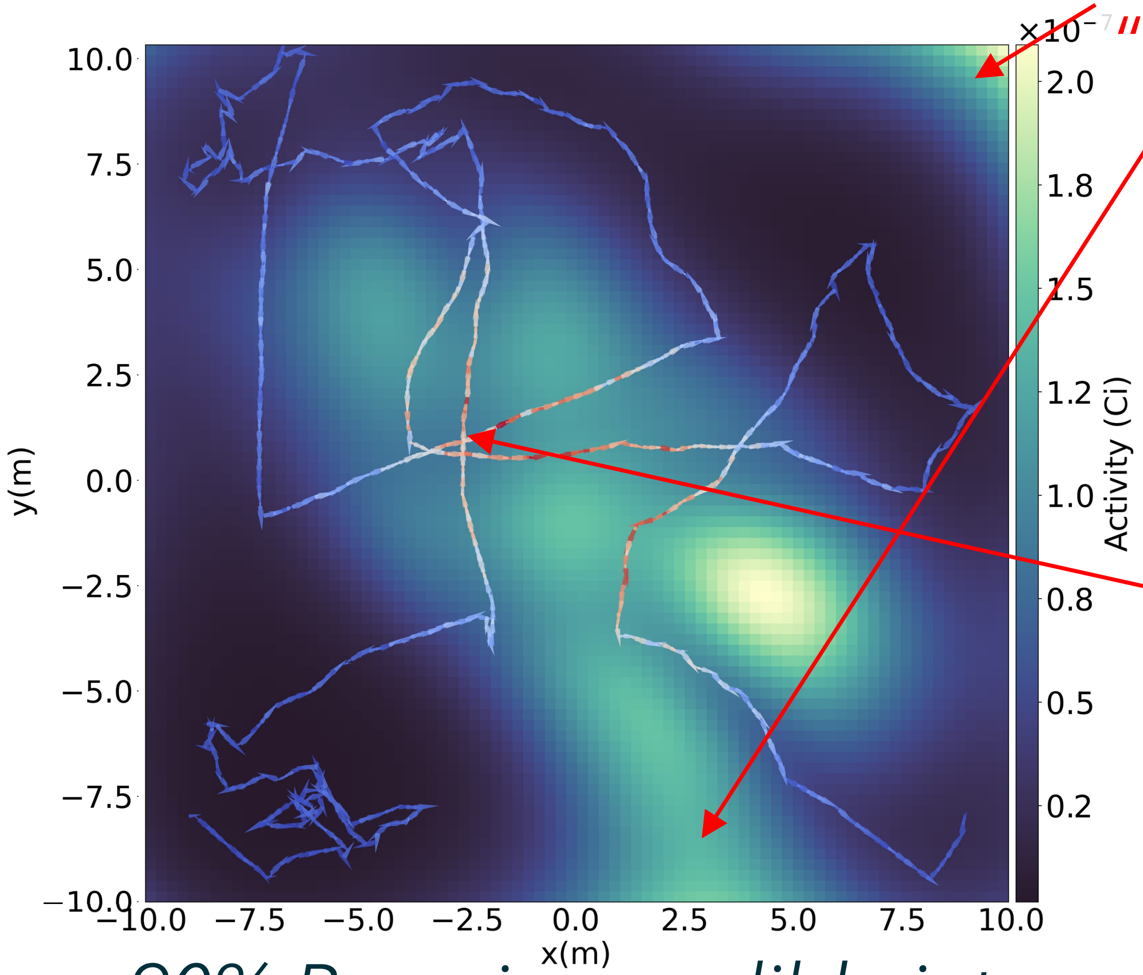


GPP reconstruction

$\sigma = 4.1\ m,\ \lambda = 1.4{\times}10^{-4} Bq^{-1}$

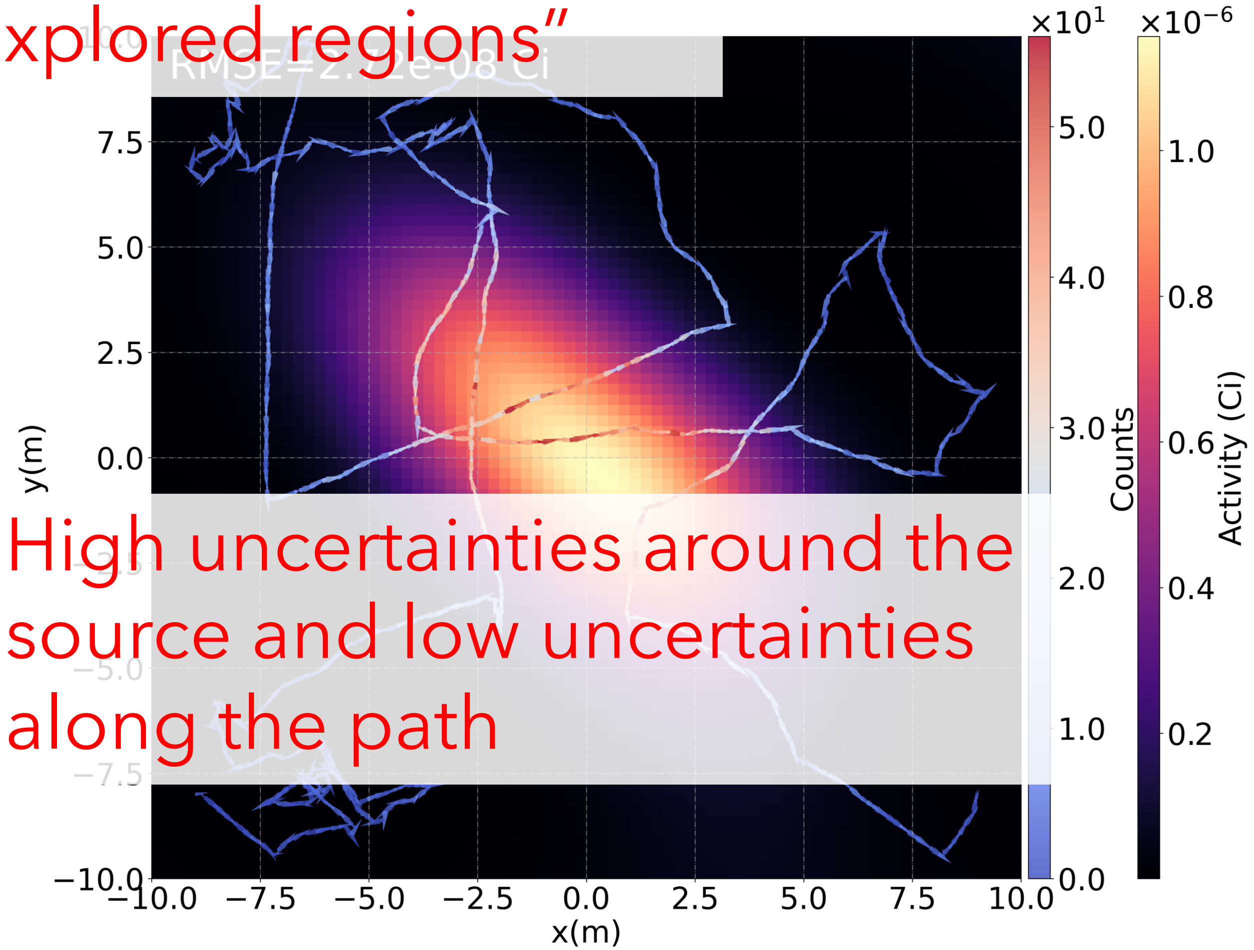
pCN MCMC Uncertainty Quantification

High uncertainties in the "unexplored regions"



High uncertainties around the source and low uncertainties along the path

90% Bayesian credible interval (pCN MCMC, 10^6 samples, ~45 min)

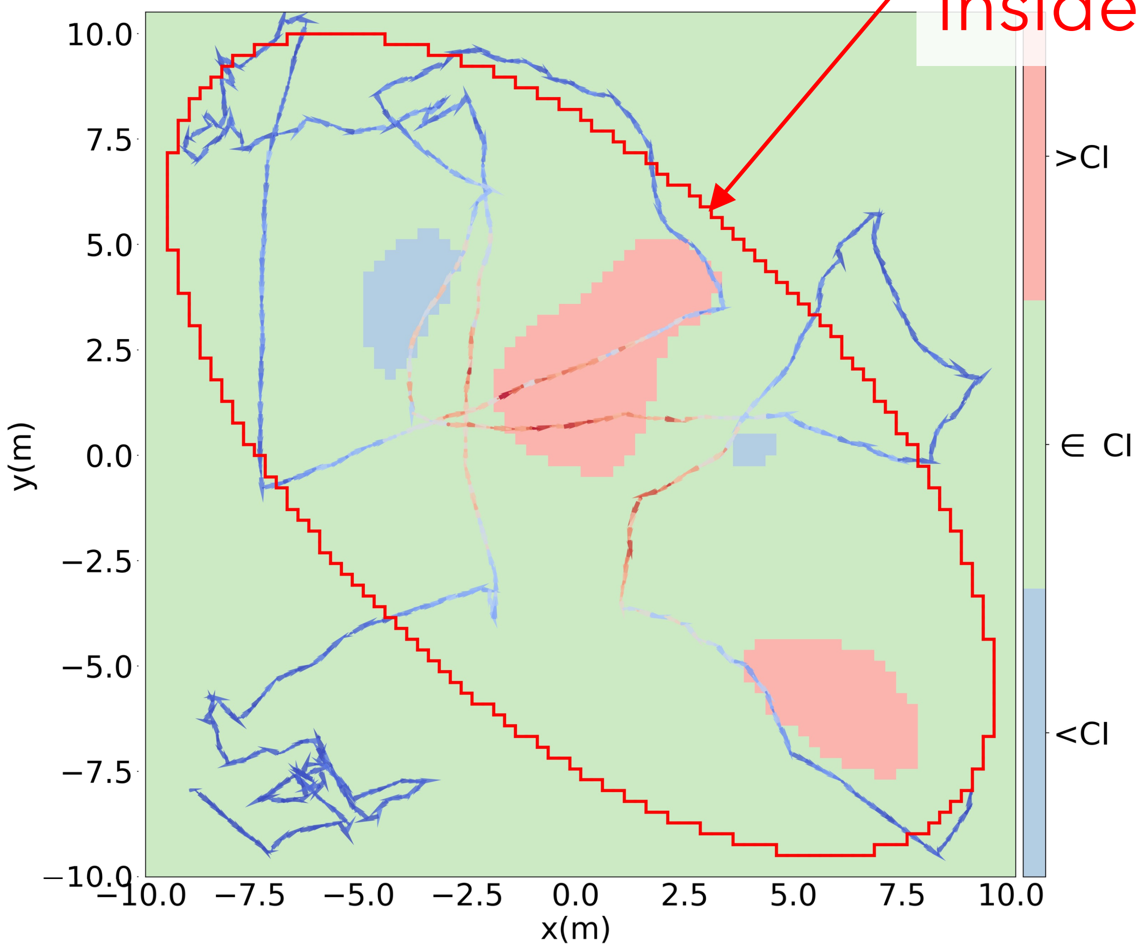


GPP reconstruction

$\sigma = 4.1\ m,\ \lambda = 1.4\times10^{-4}Bq^{-1}$

pCN MCMC Uncertainty Quantification

Non-zero true source activities inside the red boundary (ROI)

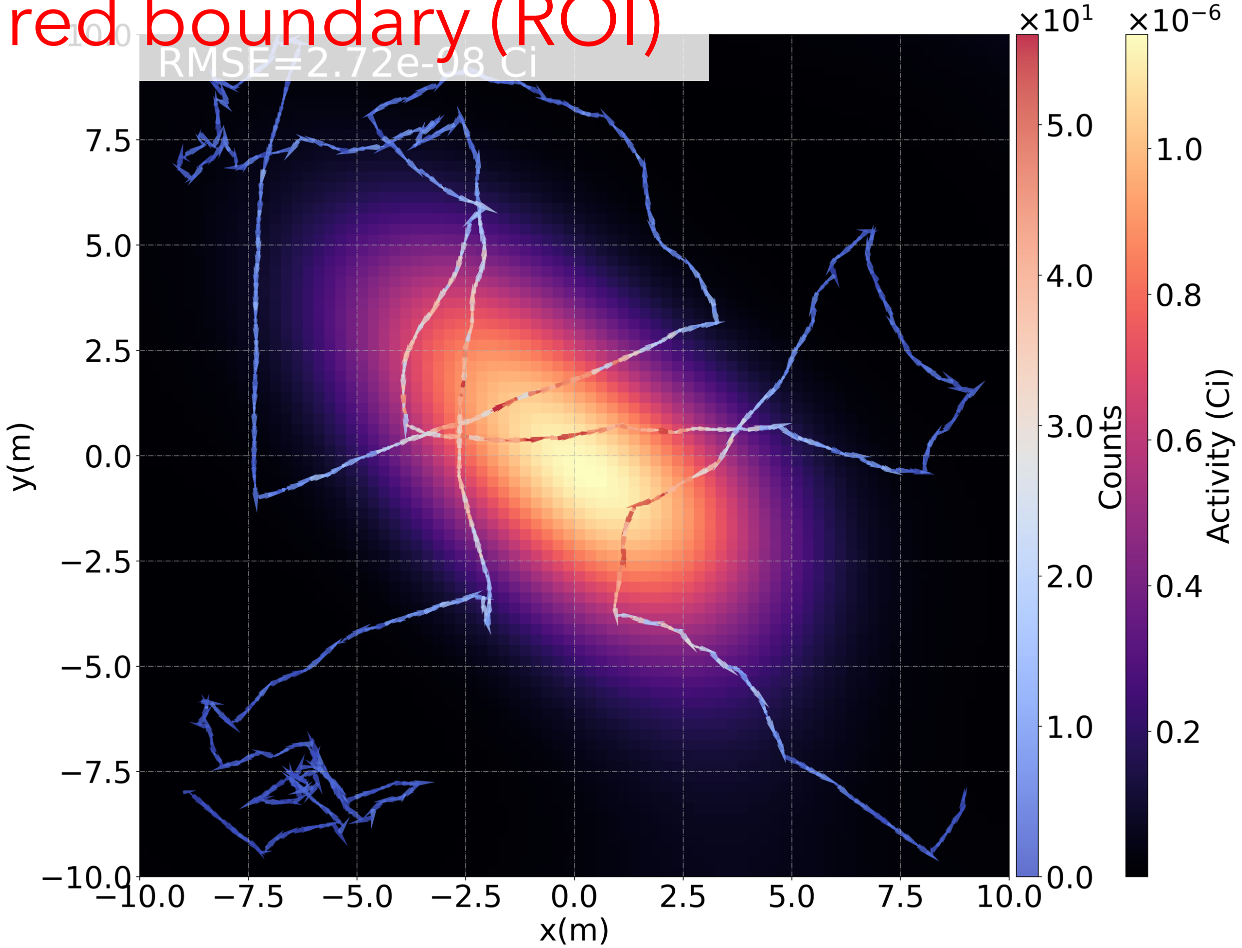


Boolean map (pCN MCMC ~45 min)

Red : True value above the interval

Green : True value within the interval

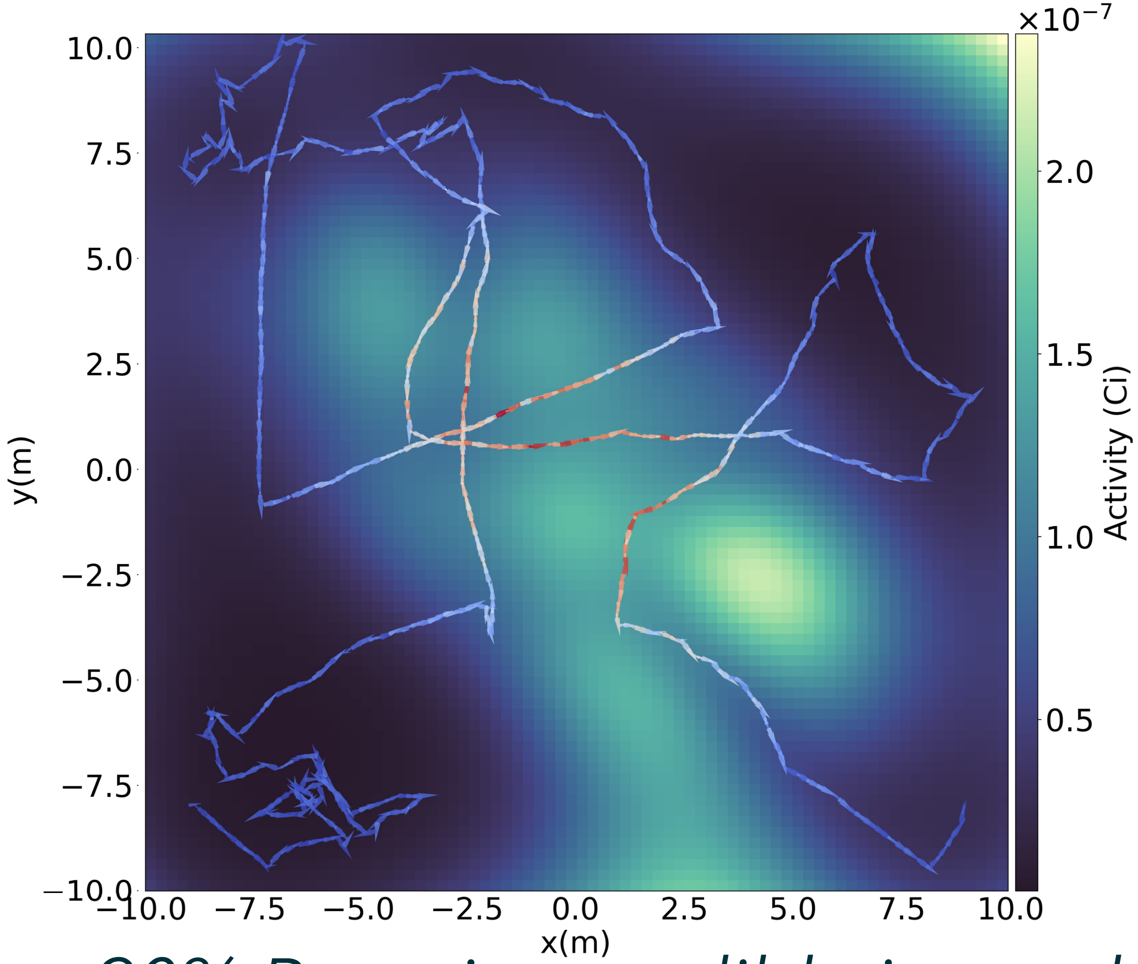
Blue : True value below the interval



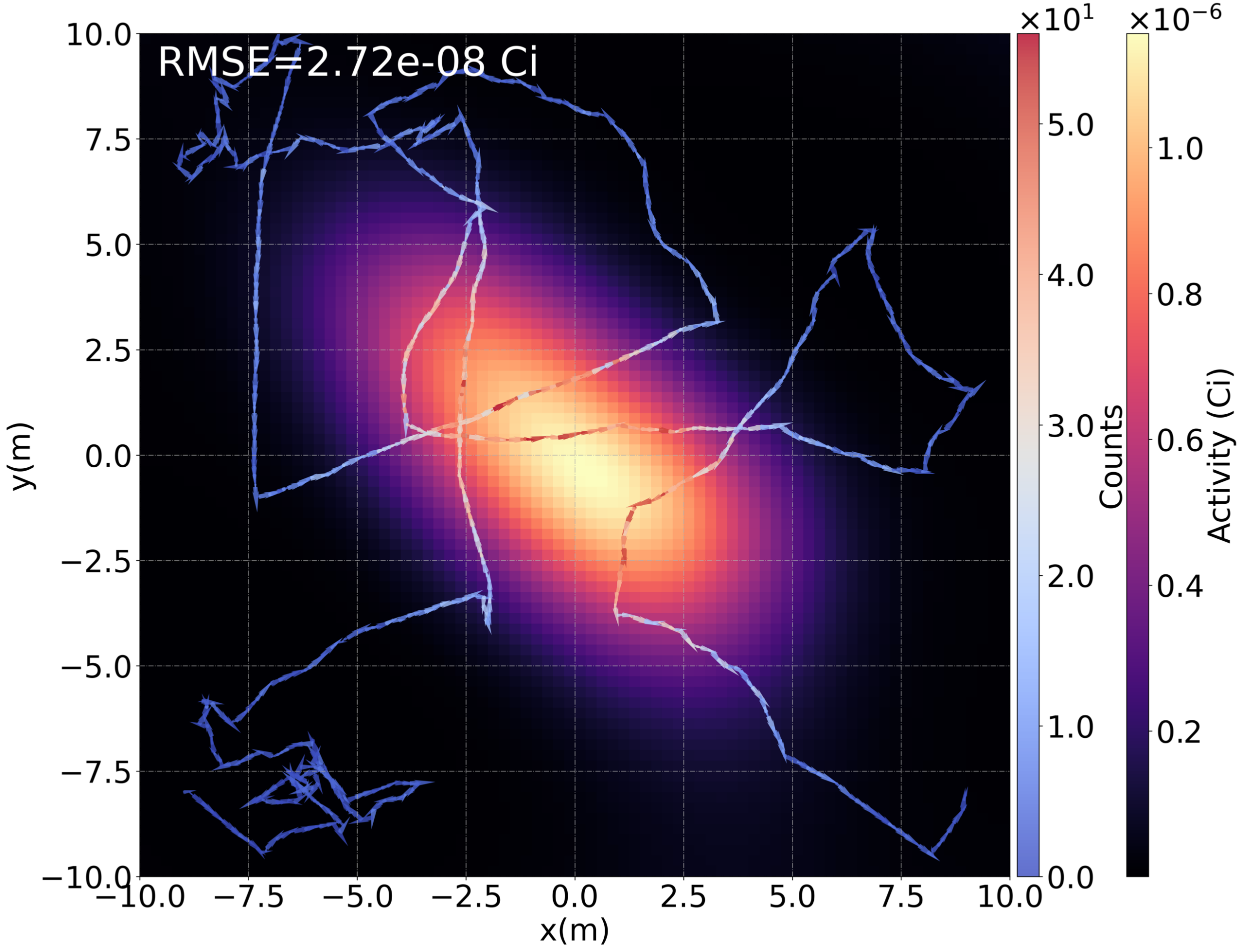
GPP reconstruction

$\sigma = 4.1\ m,\ \lambda = 1.4{\times}10^{-4} Bq^{-1}$

Laplace Approximation Uncertainty Quantification



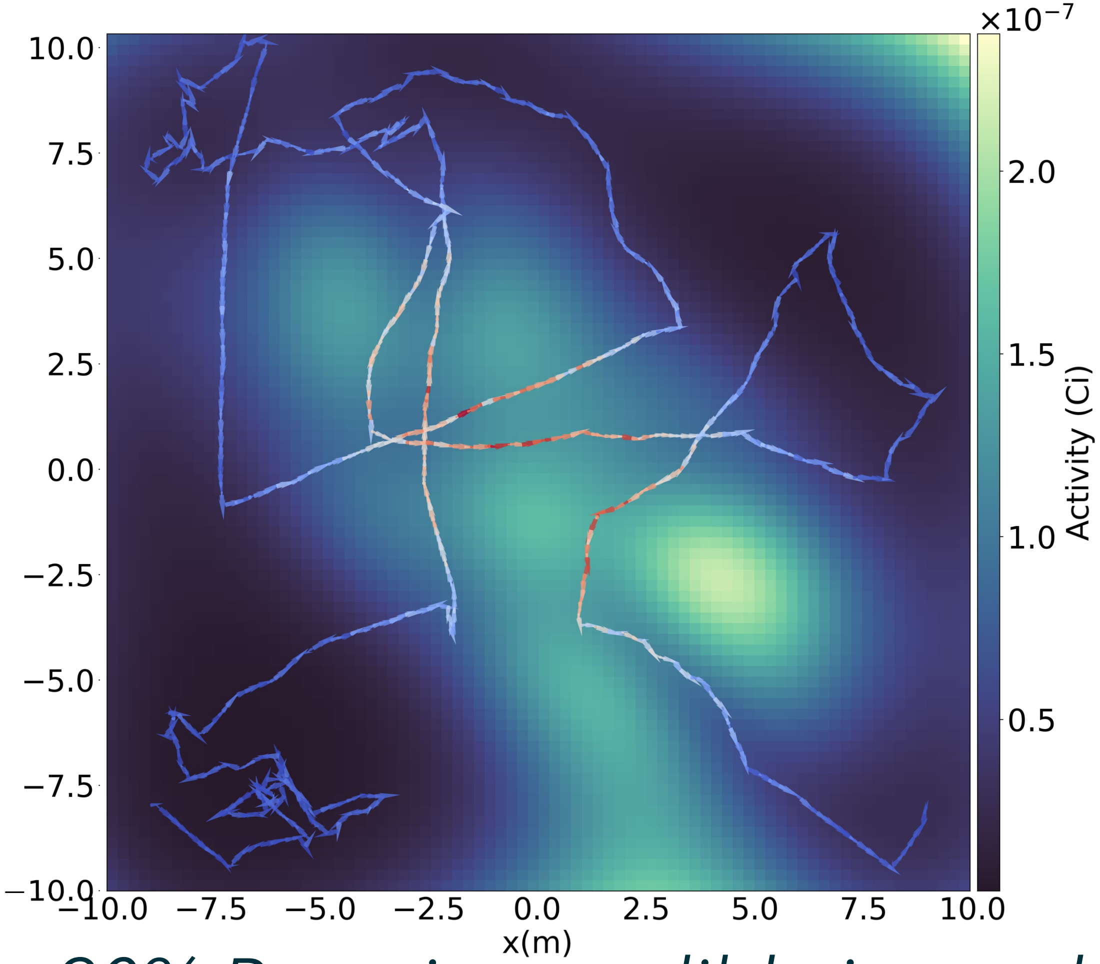
90% Bayesian credible interval (Laplace approx., ~0.5 s)



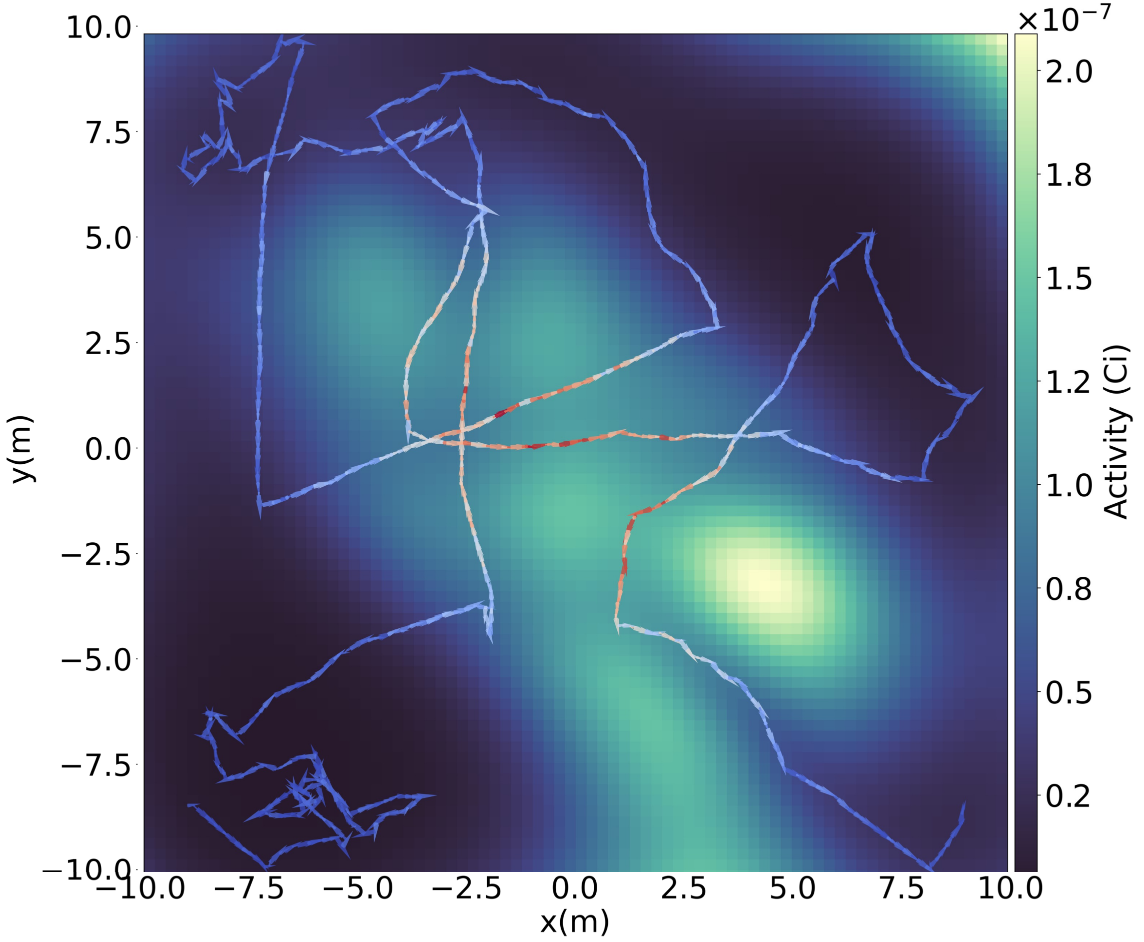
GPP reconstruction

$\sigma = 4.1\, m, \lambda = 1.4{\times}10^{-4} Bq^{-1}$

Laplace Approximation Uncertainty Quantification

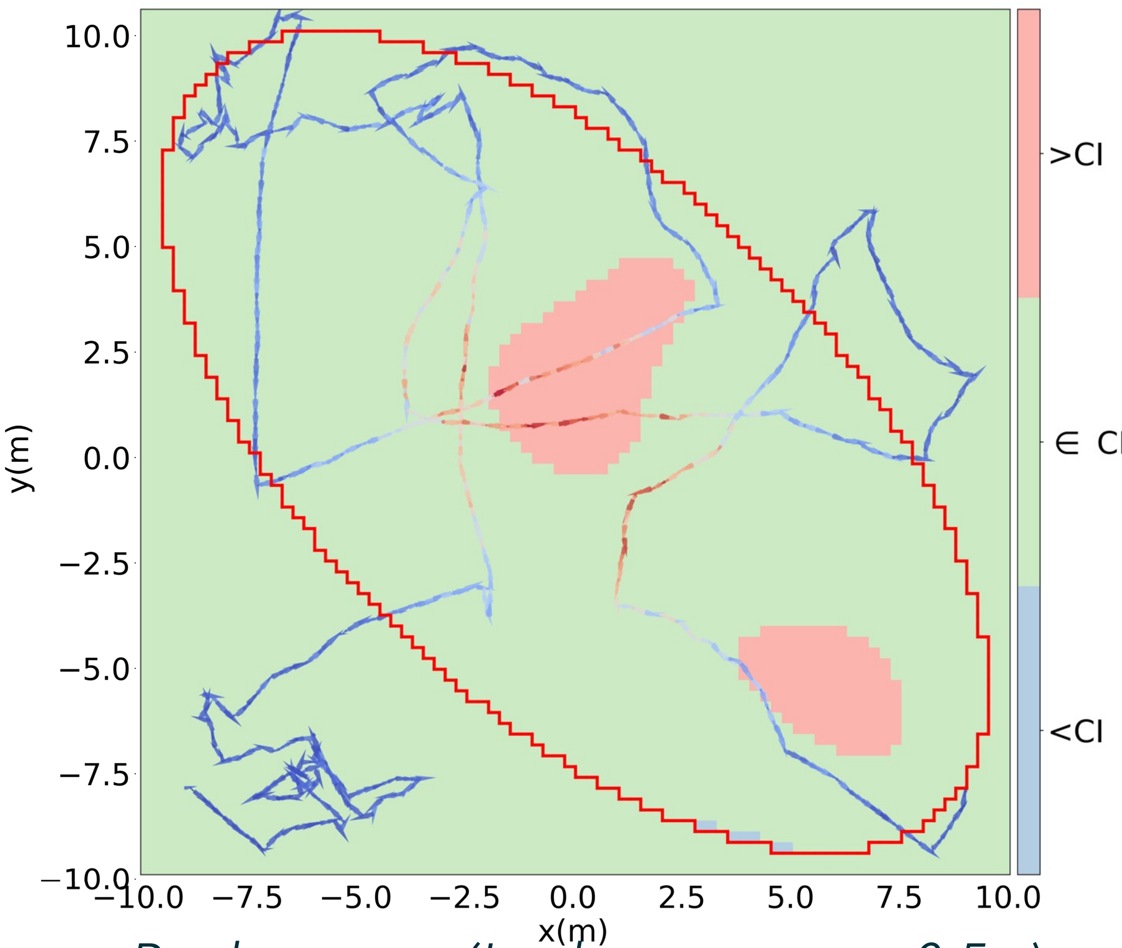


90% Bayesian credible interval (Laplace approx., ~0.5 s)



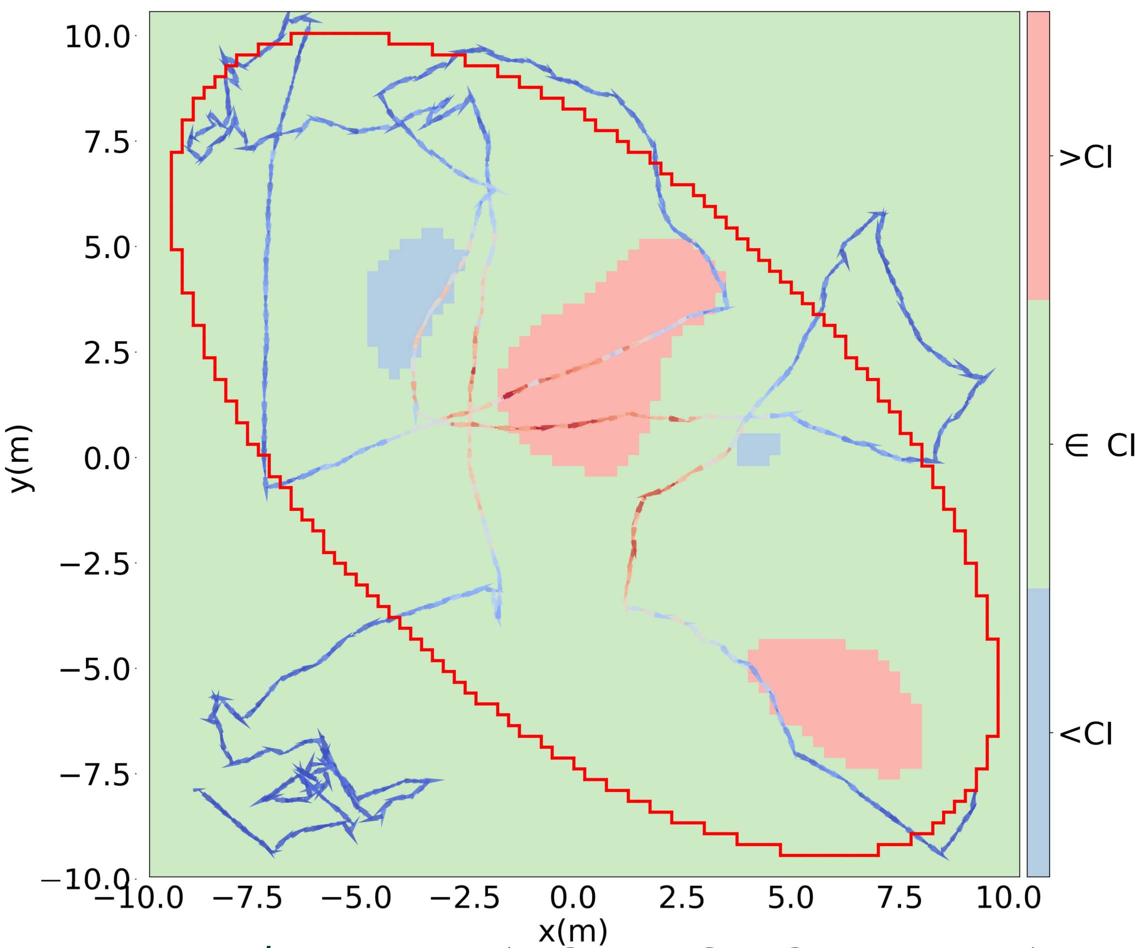
90% Bayesian credible interval (pCN MCMC, 10^6 samples, ~45 min)

Laplace Approximation Uncertainty Quantification



Boolean map (Laplace approx. ~0.5 s)

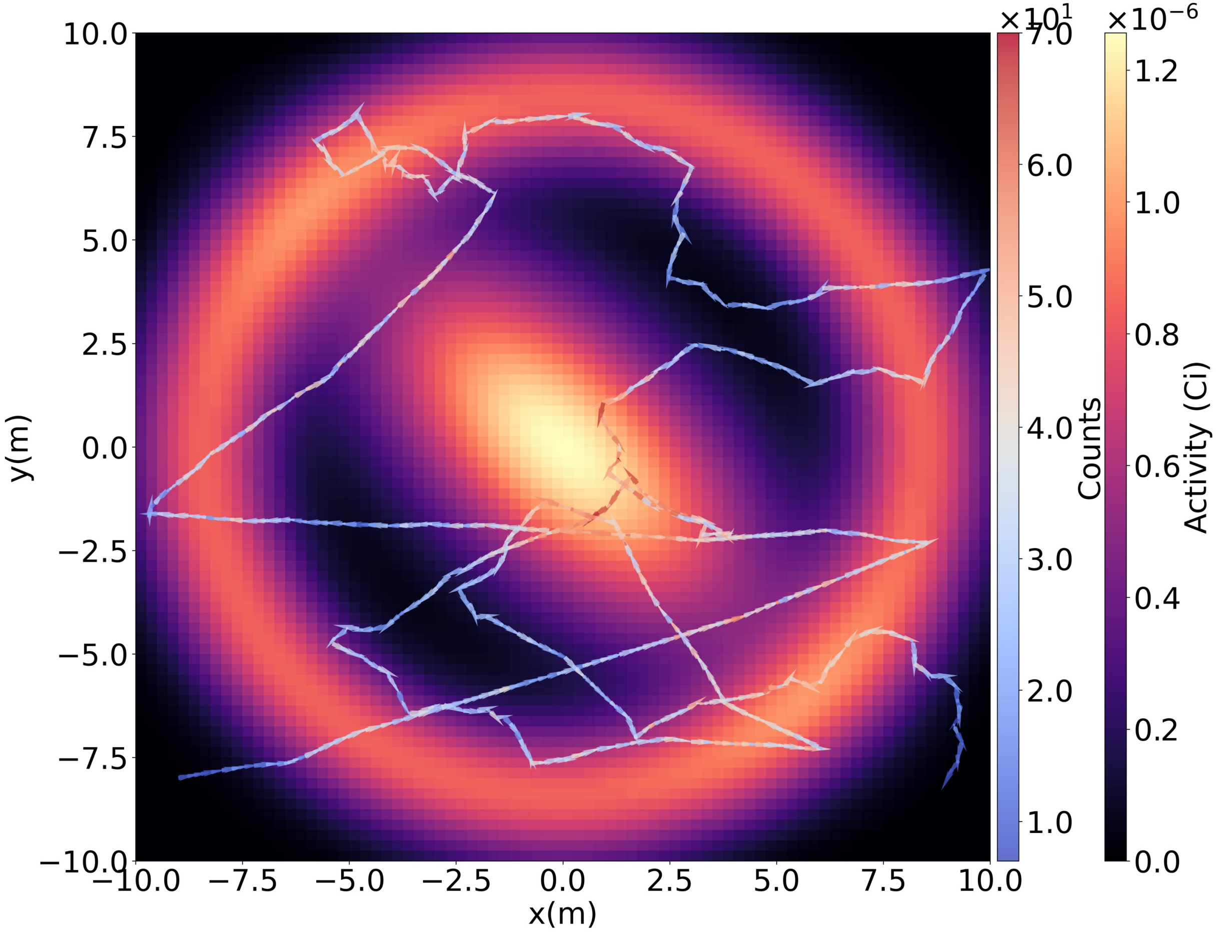
Red : True value above the interval
Green : True value within the interval
Blue : True value below the interval



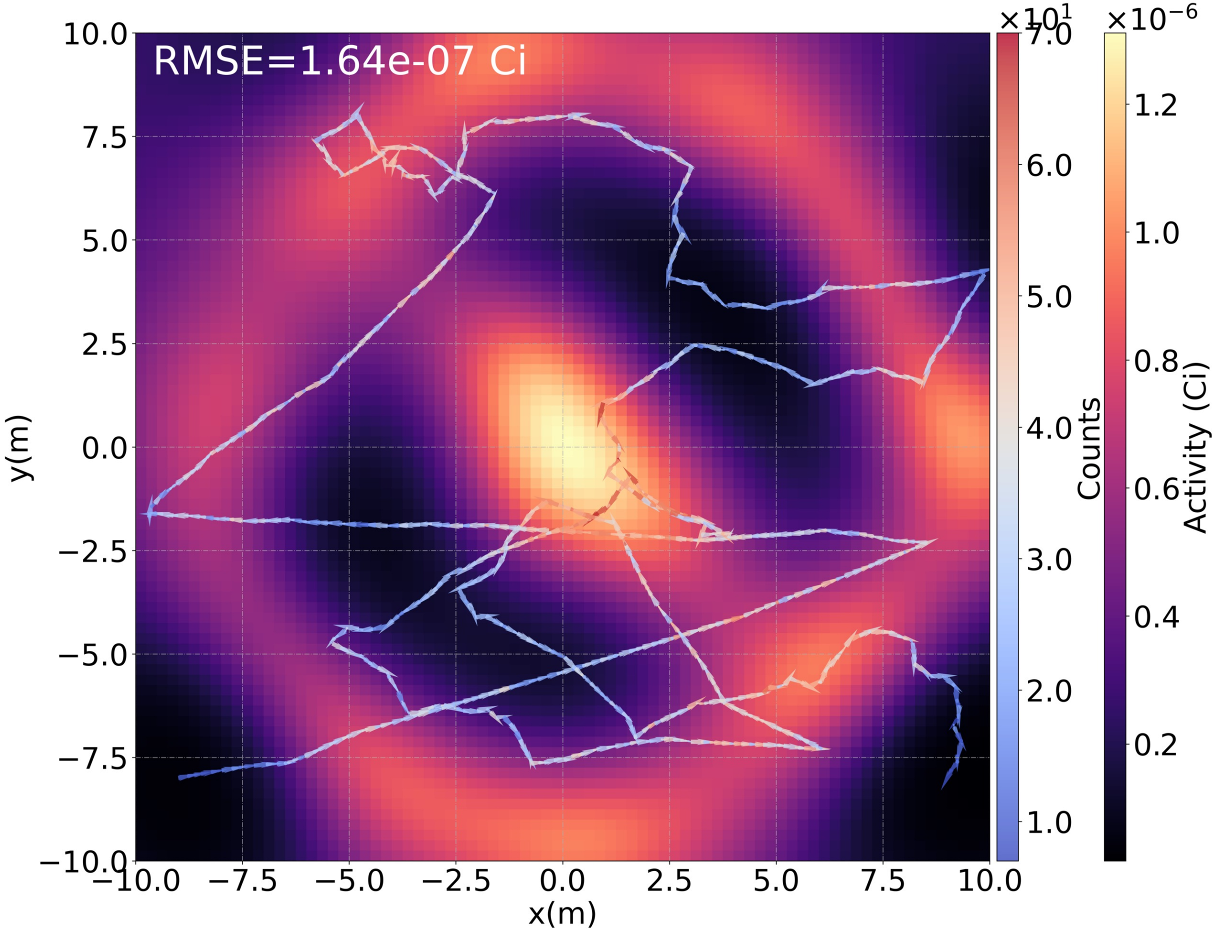
Boolean map (pCN MCMC ~45 min)

Red : True value above the interval
Green : True value within the interval
Blue : True value below the interval

Laplace Approximation Uncertainty Quantification



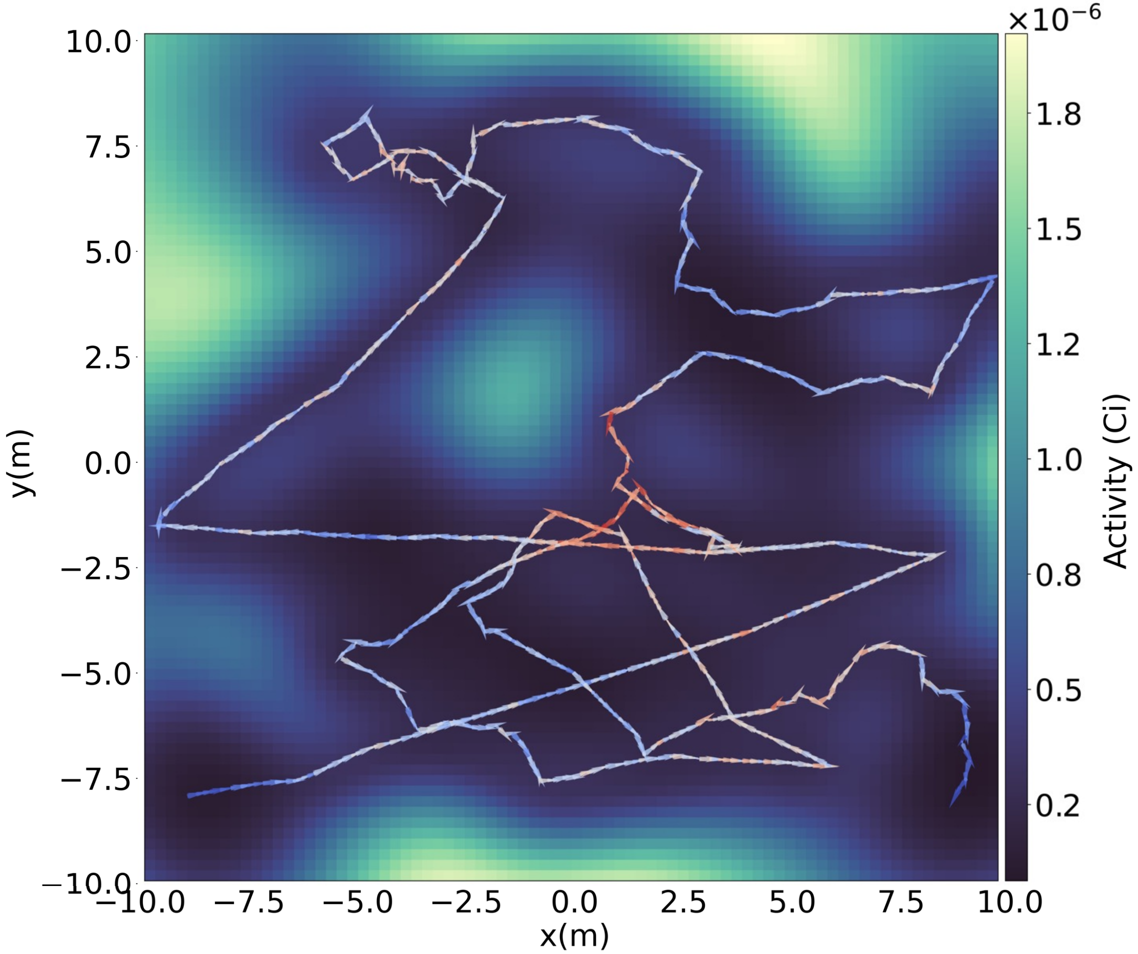
Ground truth



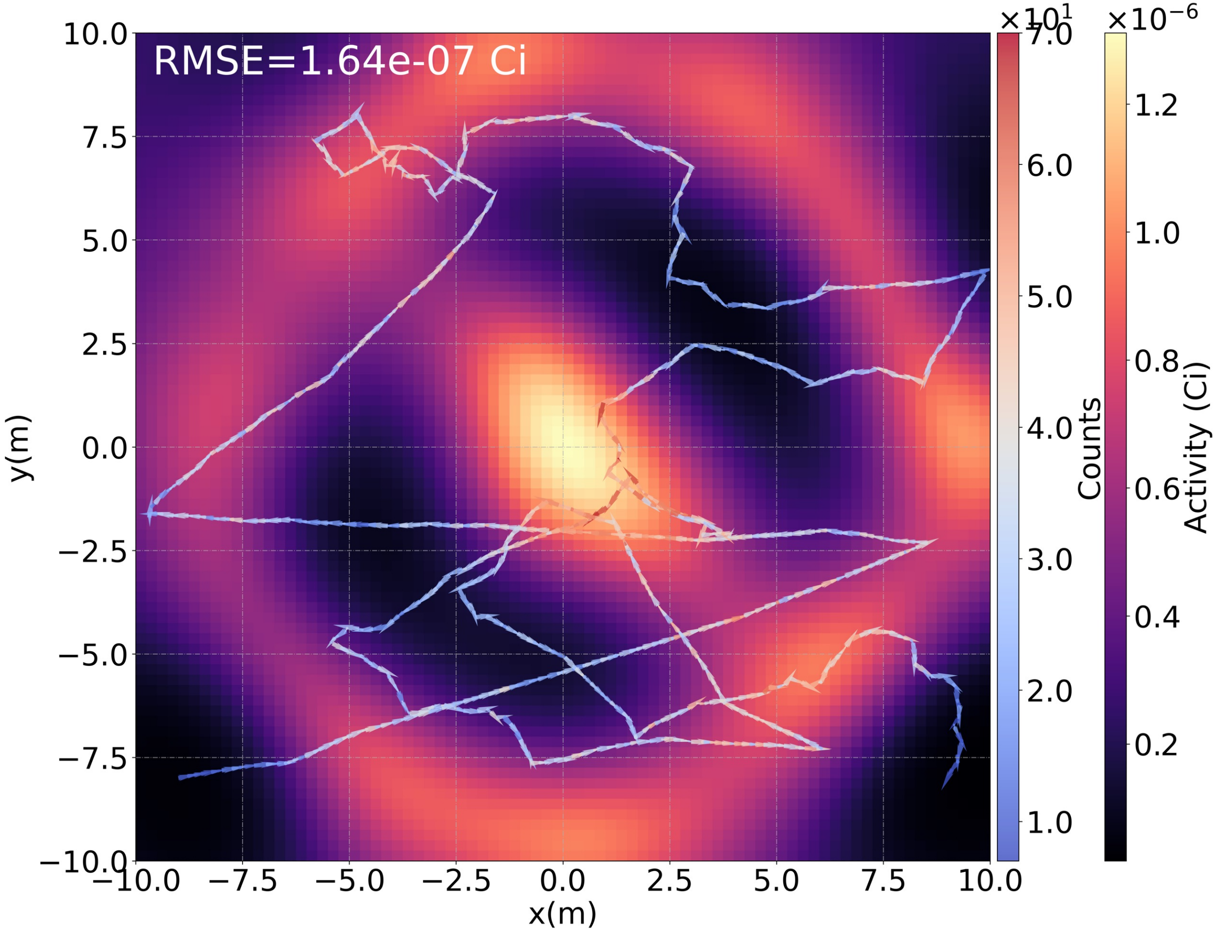
GPP reconstruction

$\sigma = 2.4\ m,\ \lambda = 6.0\times10^{-5} Bq^{-1}$

Laplace Approximation Uncertainty Quantification



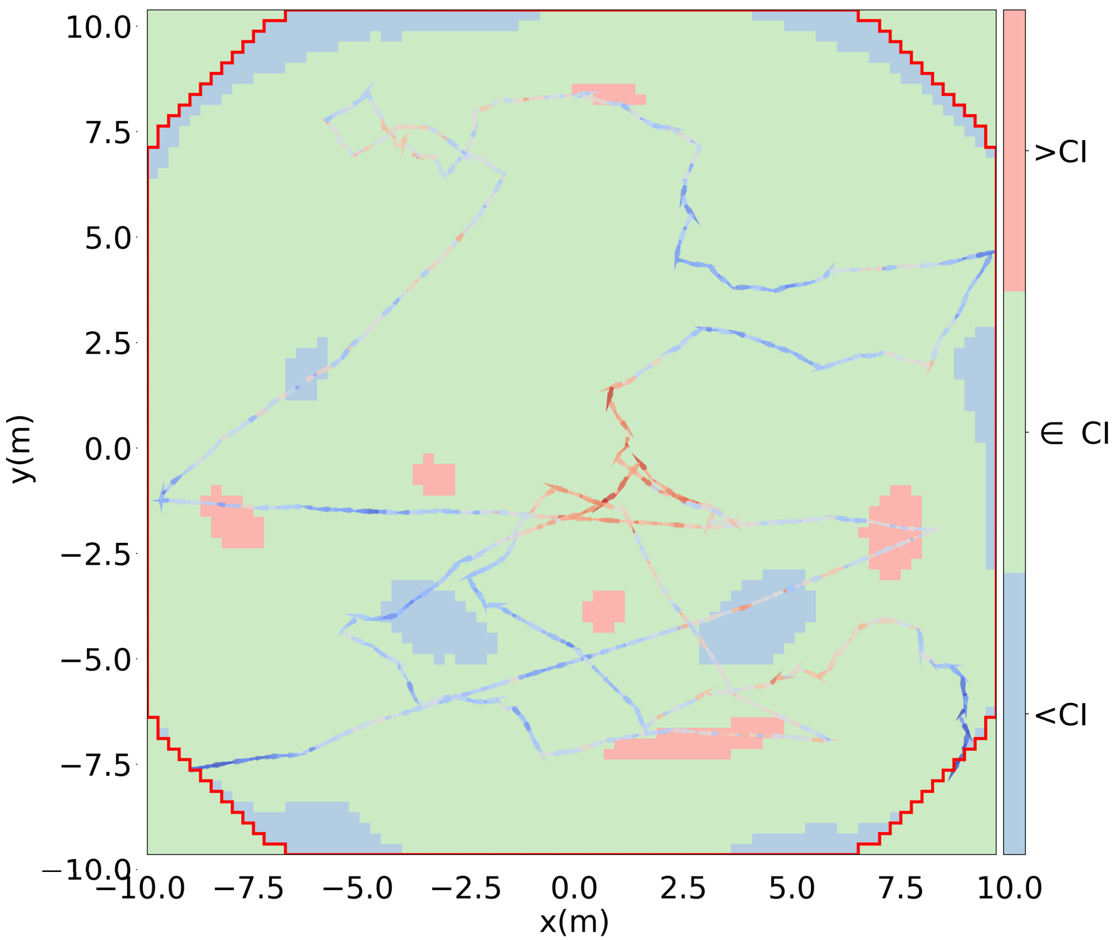
90% Bayesian credible interval (Laplace approx., ~0.5 s)



GPP reconstruction

$\sigma = 2.4\, m,\ \lambda = 6.0\times10^{-5} Bq^{-1}$

Laplace Approximation Uncertainty Quantification



Boolean map (Laplace approx. ~0.5 s)

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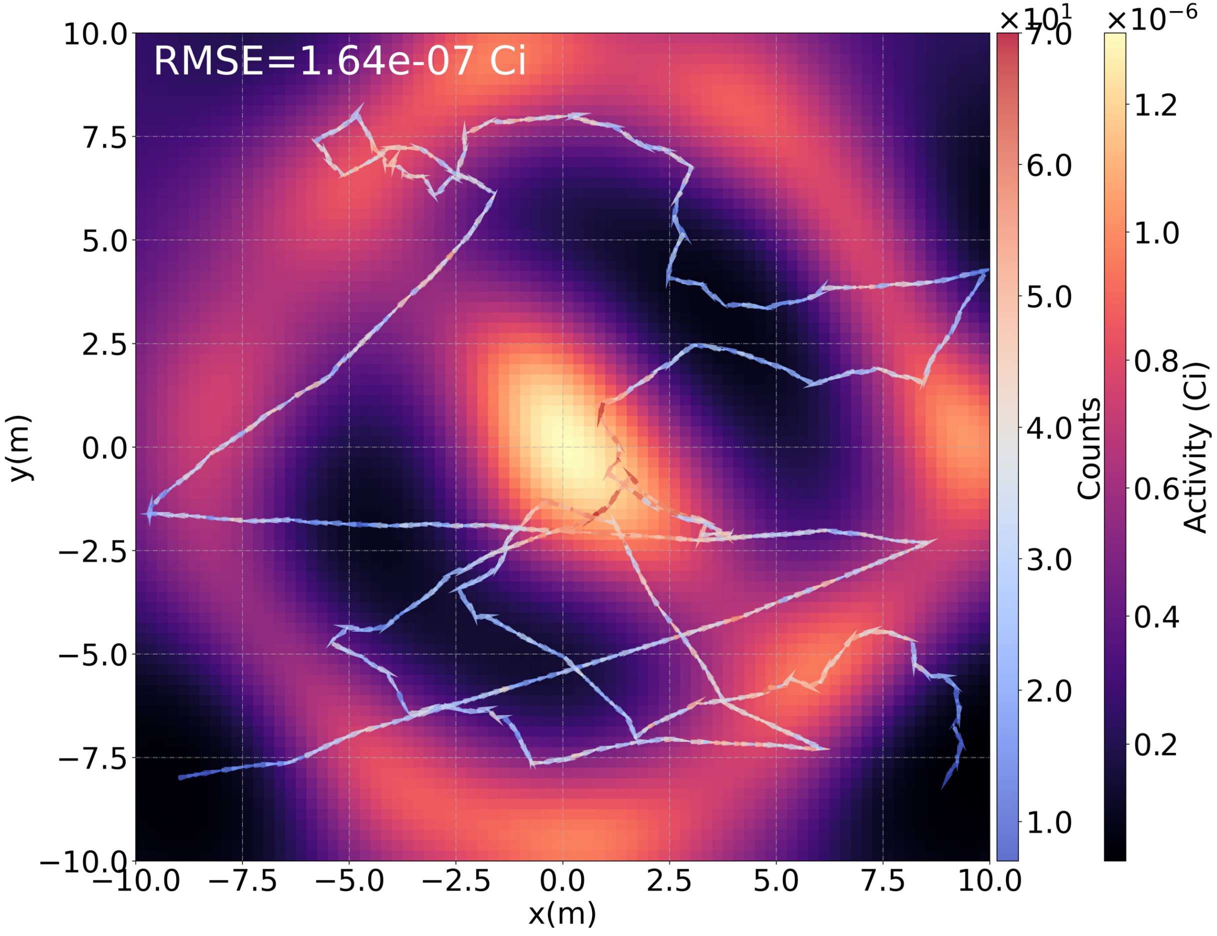

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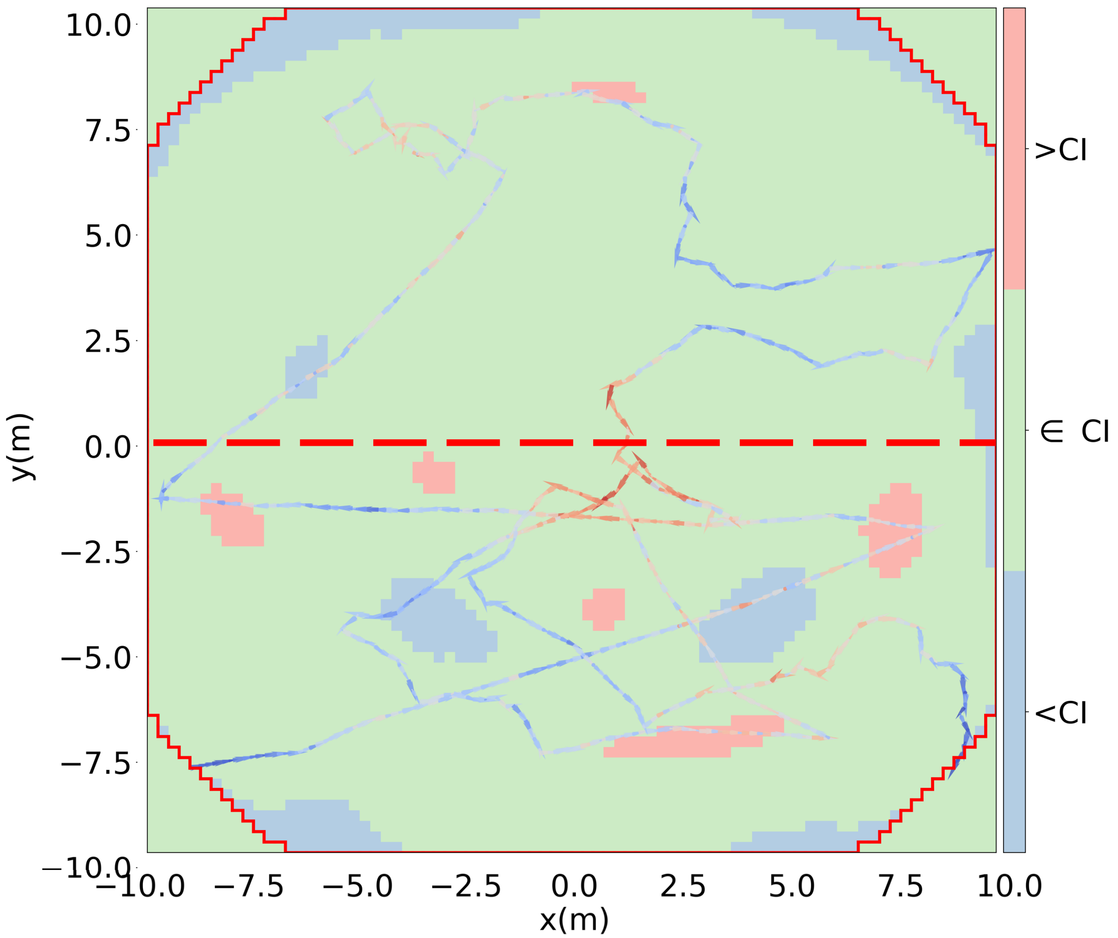
Laplace Approximation Uncertainty Quantification



Boolean map (Laplace approx. ~0.5 s)

Red : True value above the interval
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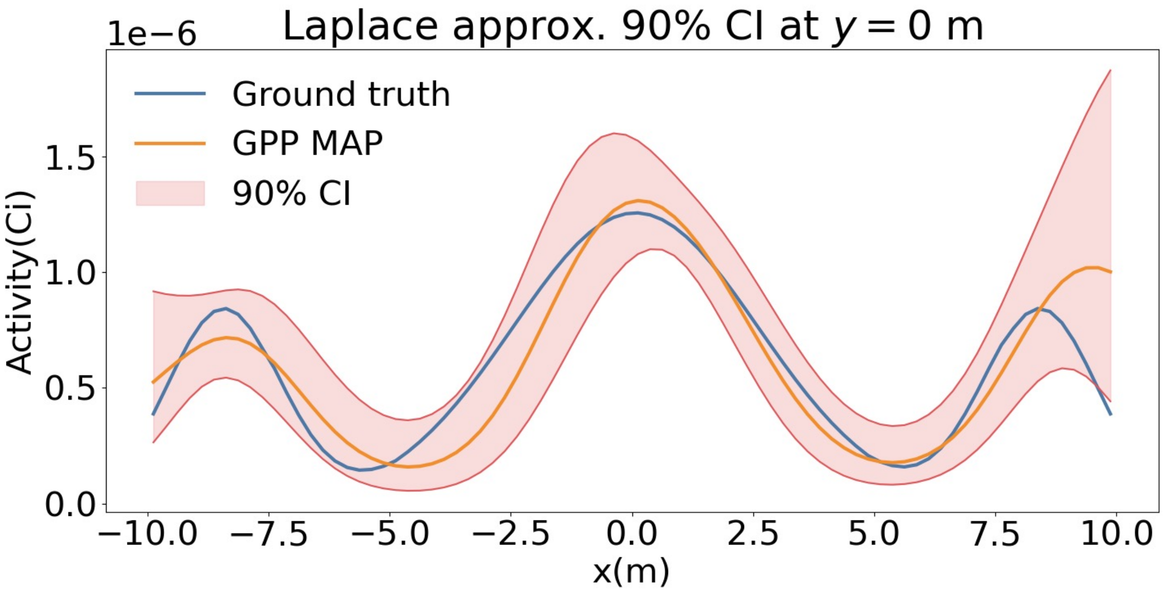
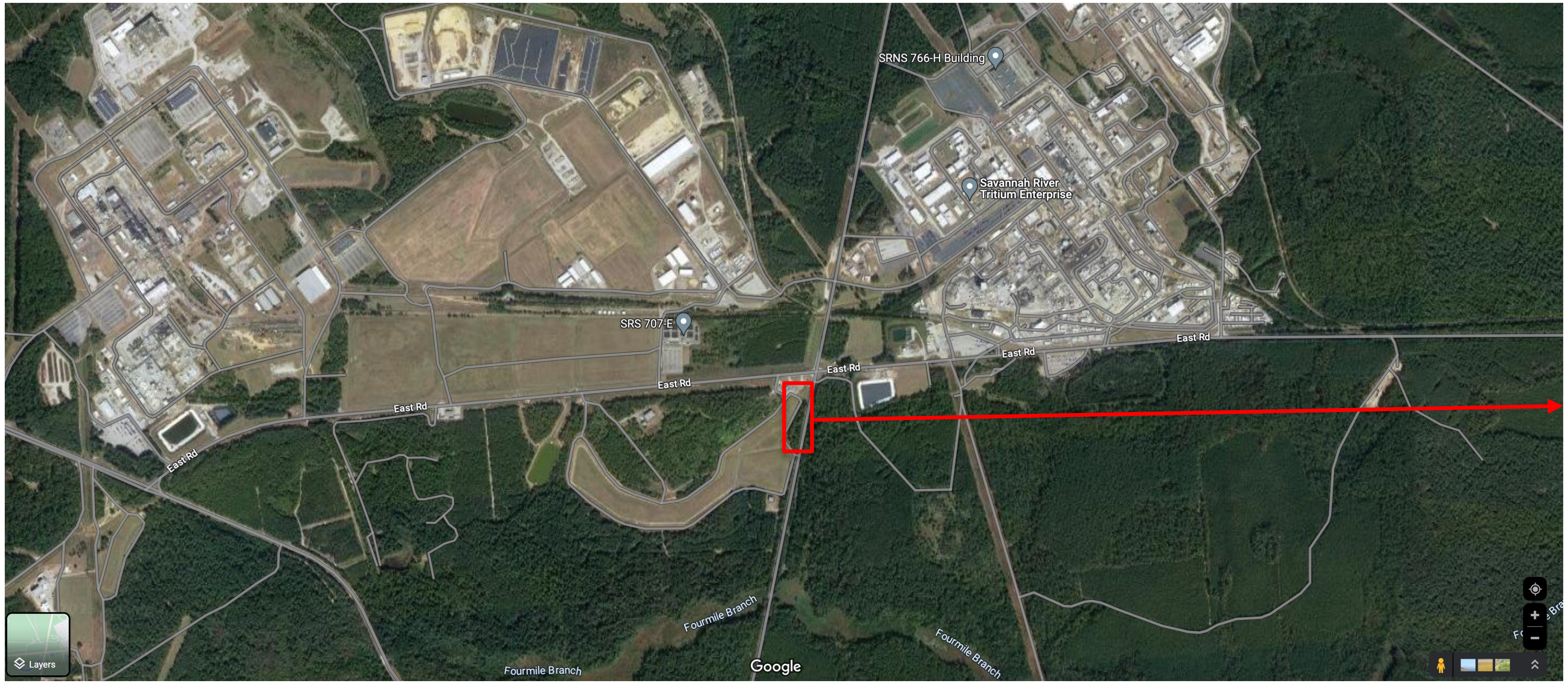


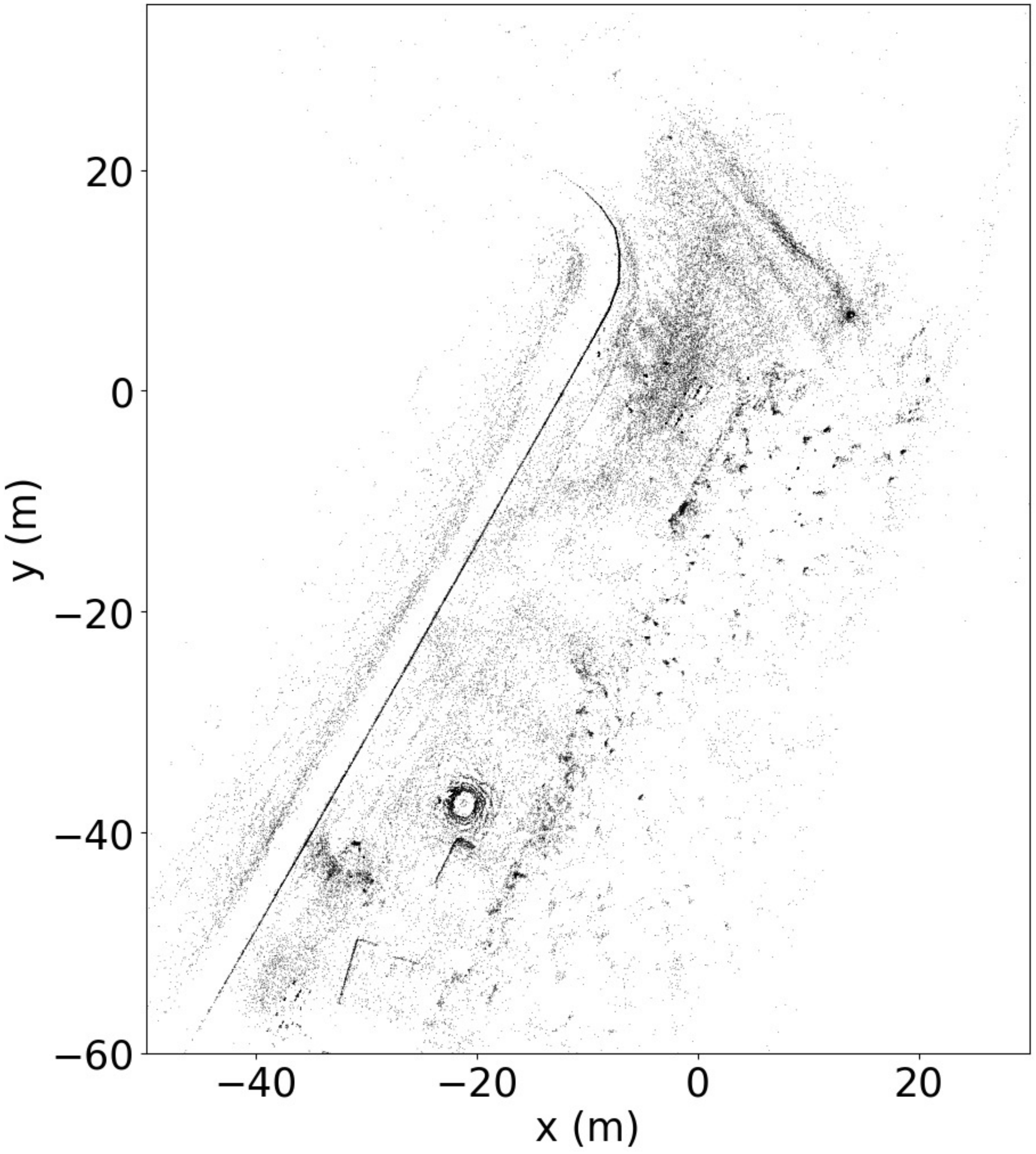
Simultaneous Spectral Decomposition and Full Spectral Imaging

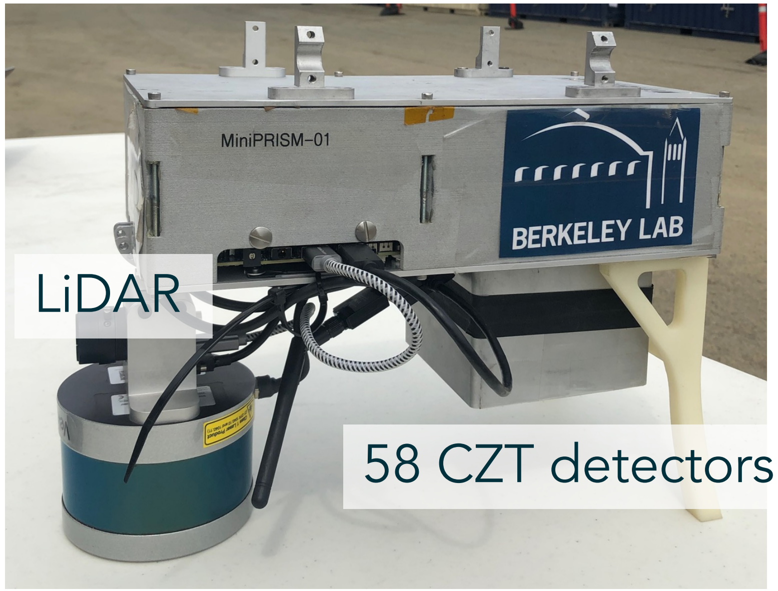
SRNL H Area Contamination Measurements



Savannah River National Lab (SRNL)

SRNL H Area Contamination Measurements

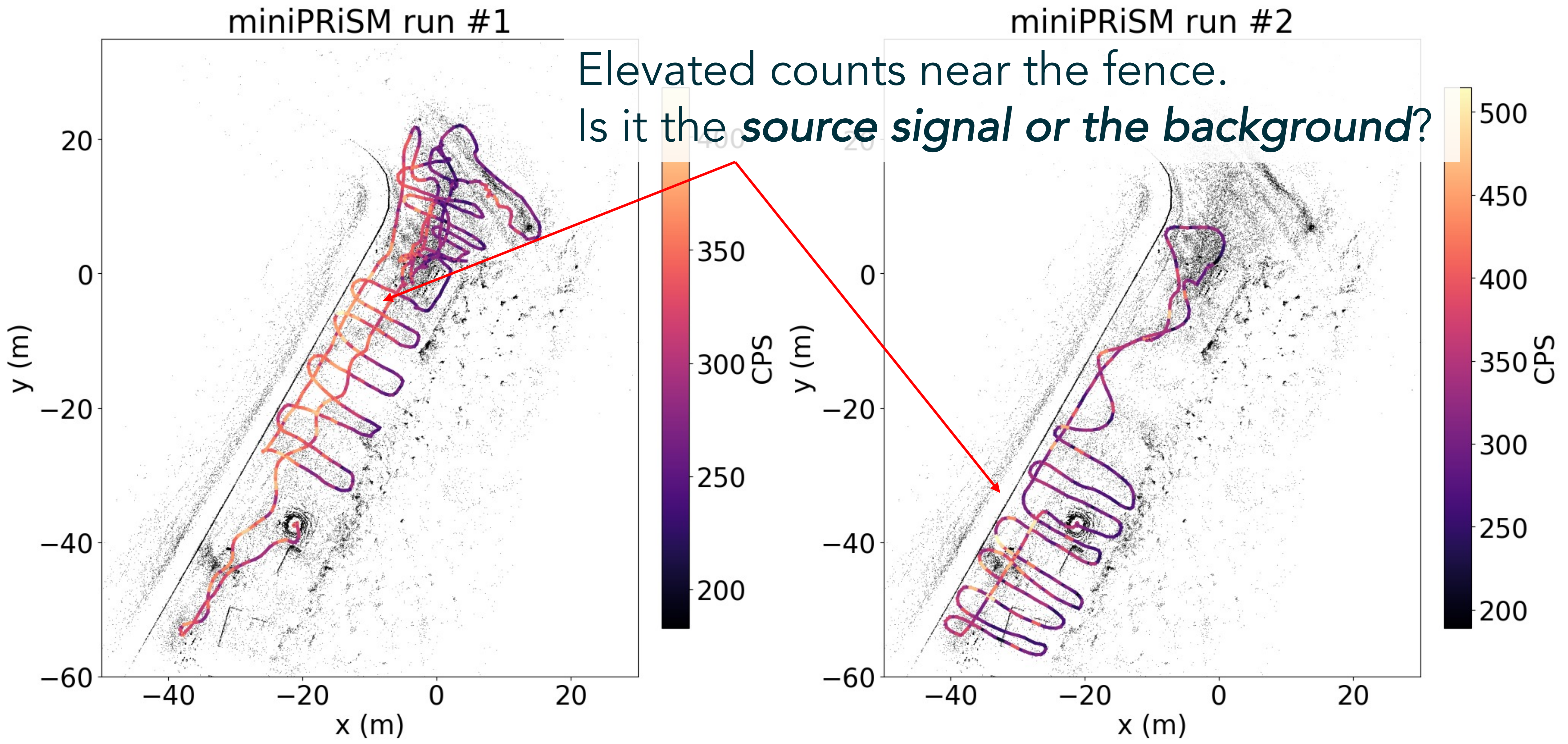




MiniPRiSM imaging system

H-area LiDAR point cloud map

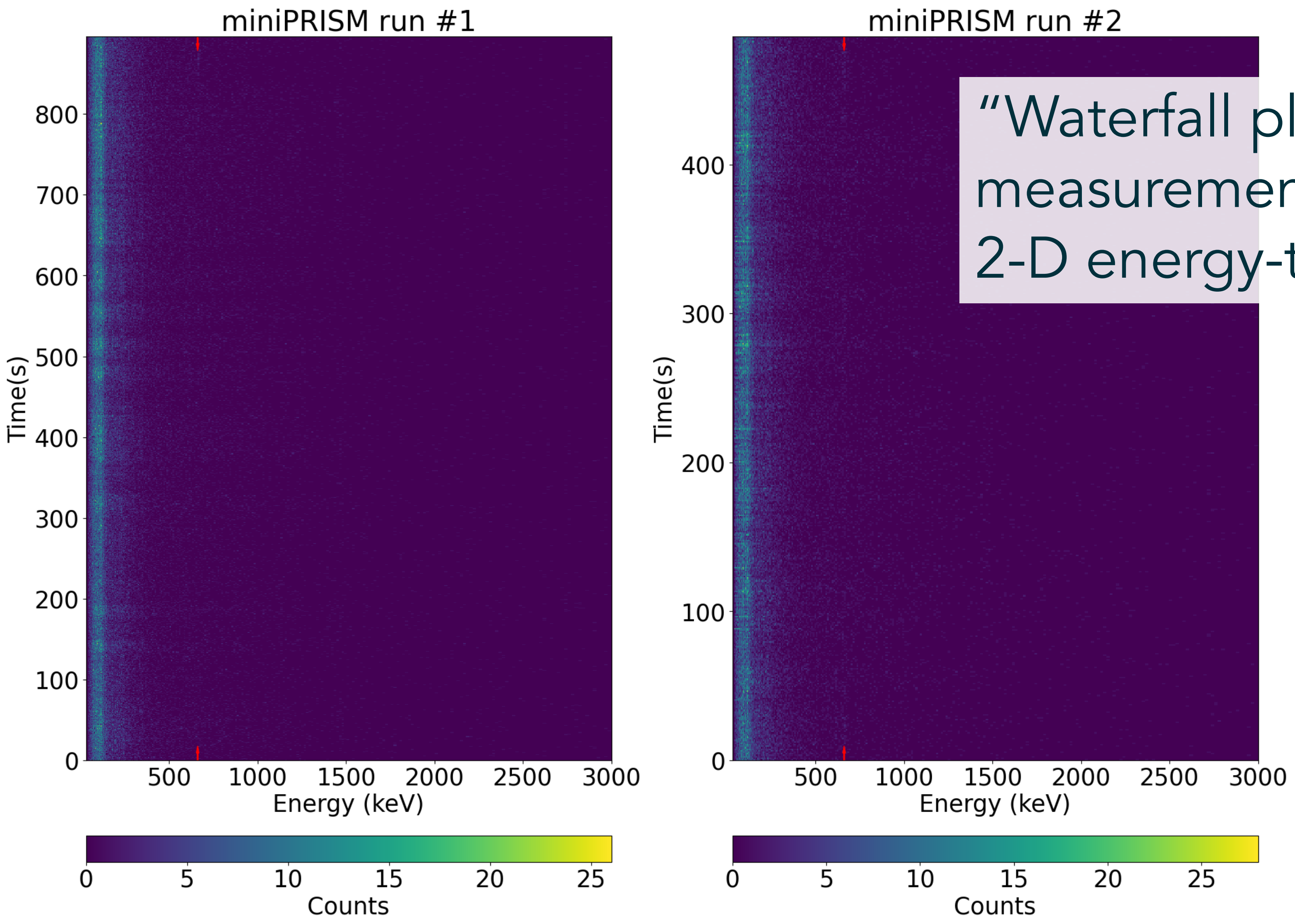
SRNL H Area Contamination Measurements



Elevated counts near the fence.

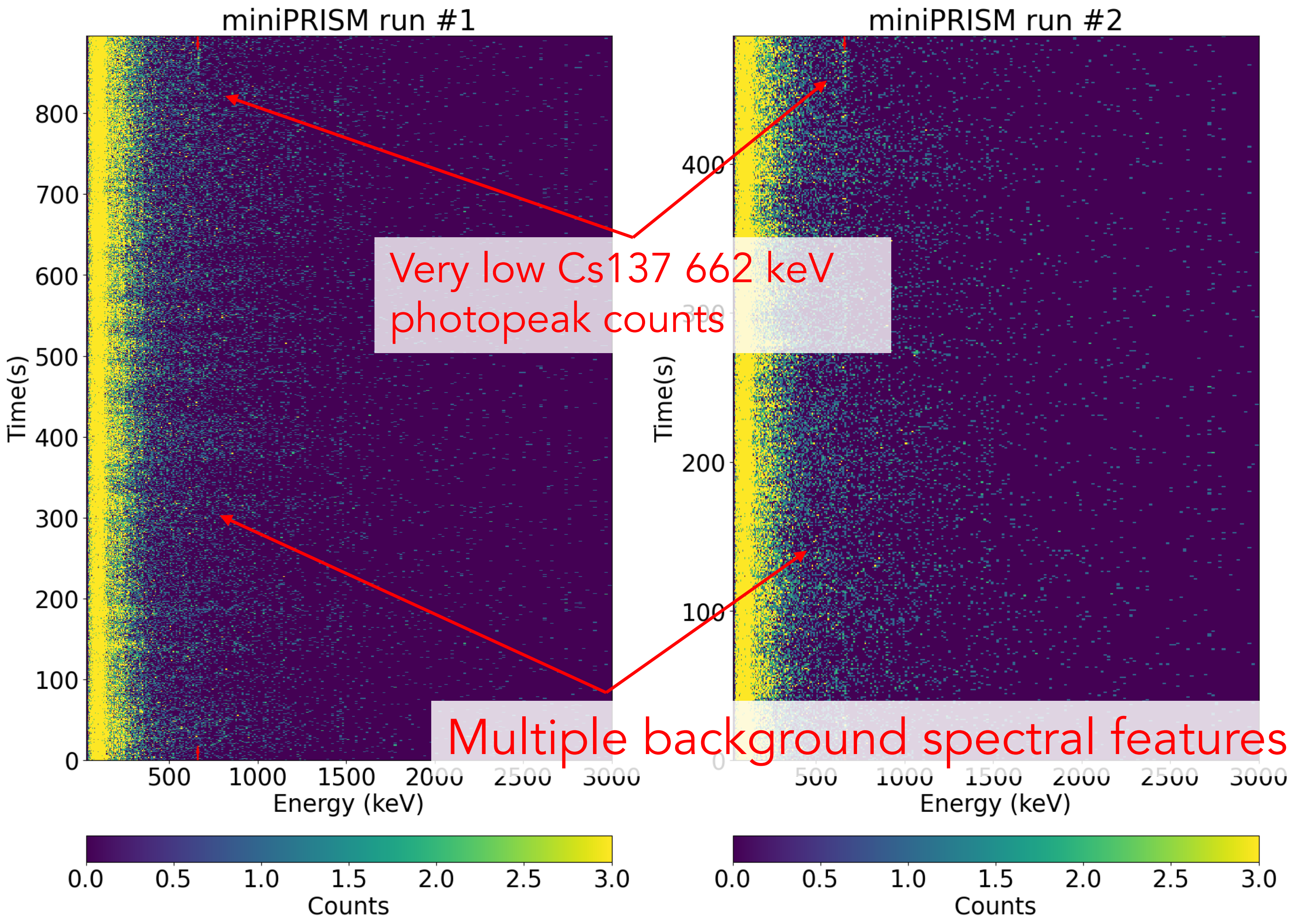
Is it the ***source signal or the background***?

SRNL H Area Contamination Measurements

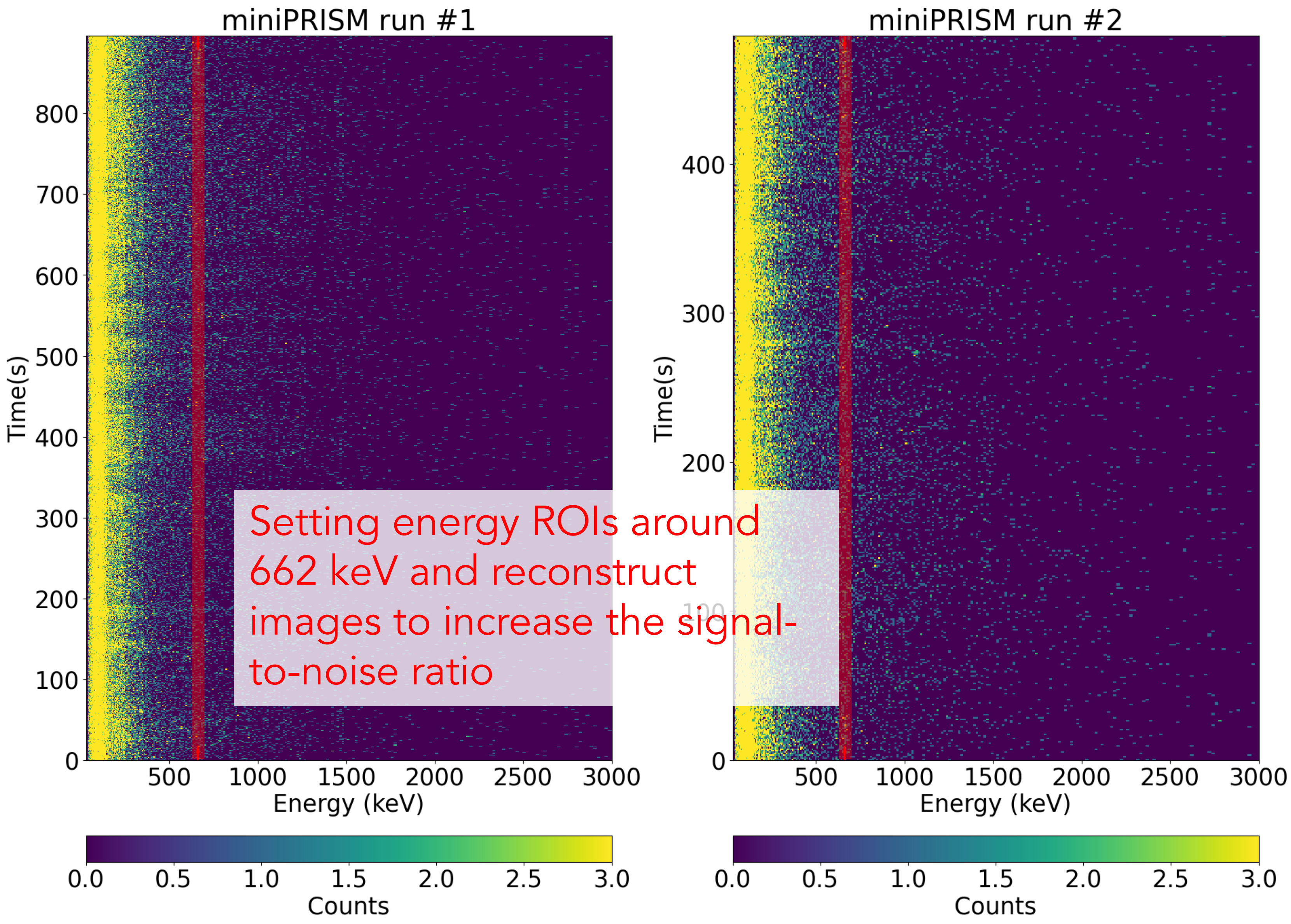


"Waterfall plots" from the measurements
2-D energy-time count histograms

SRNL H Area Contamination Measurements

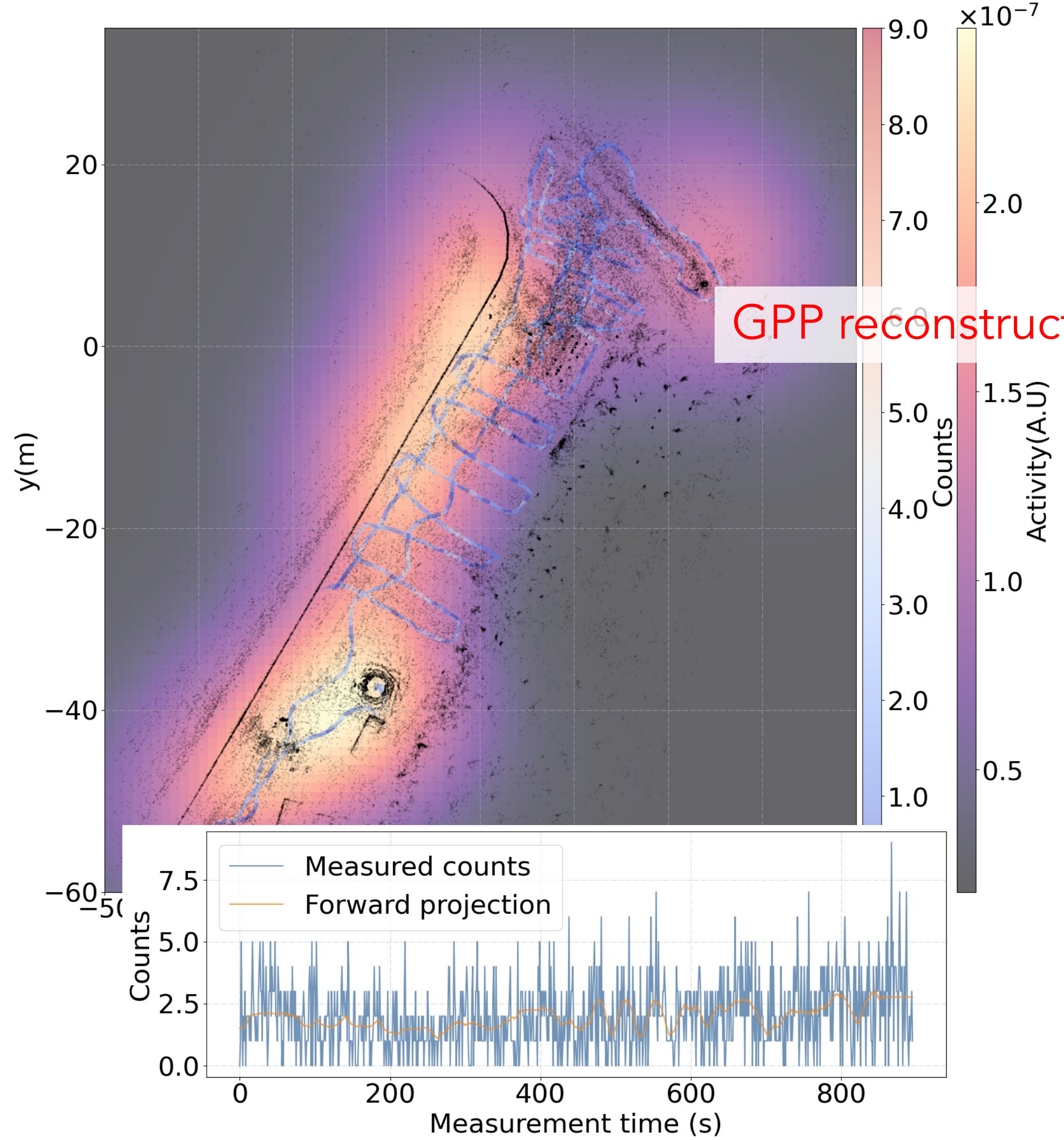


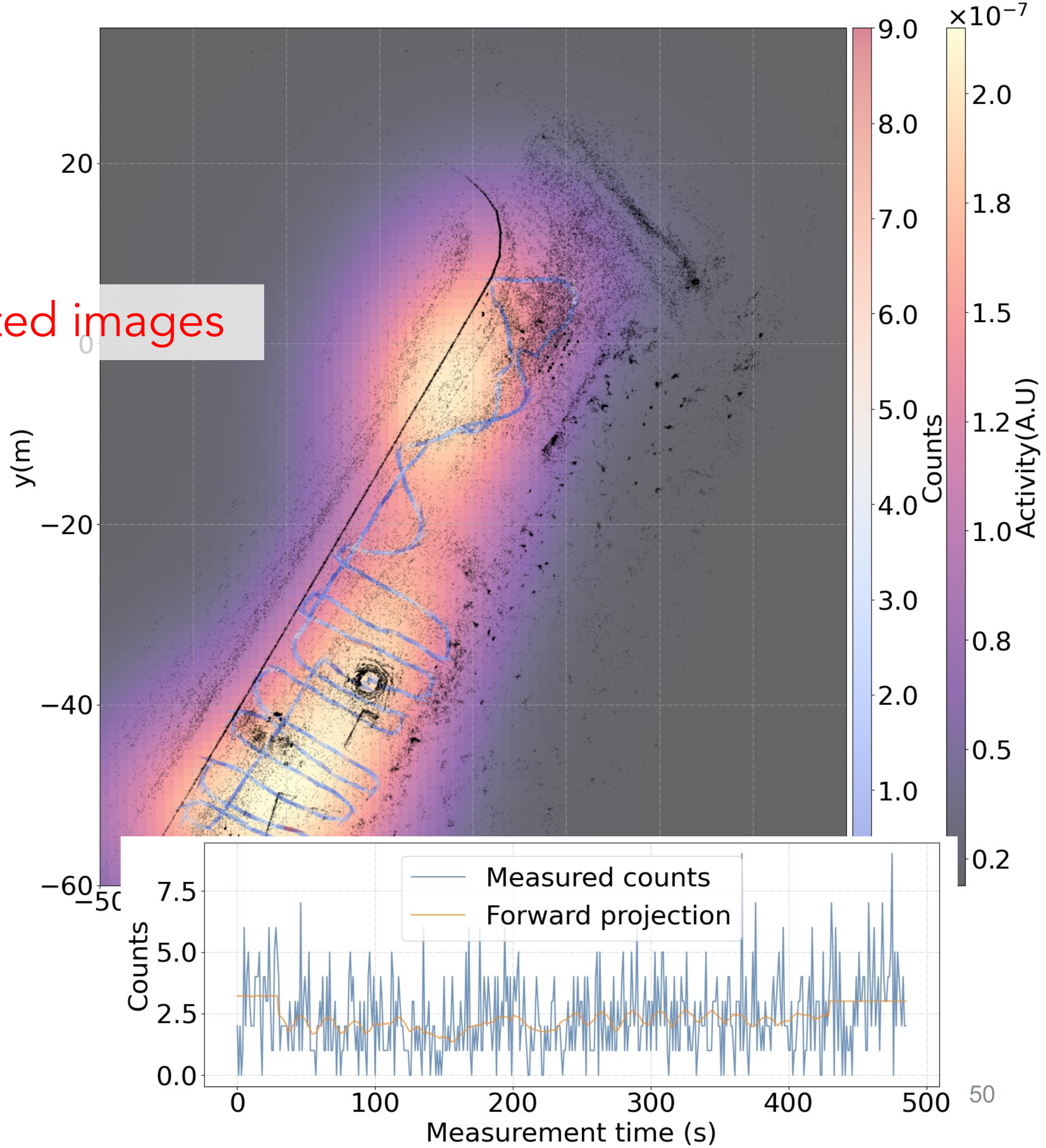
SRNL H Area Contamination Measurements



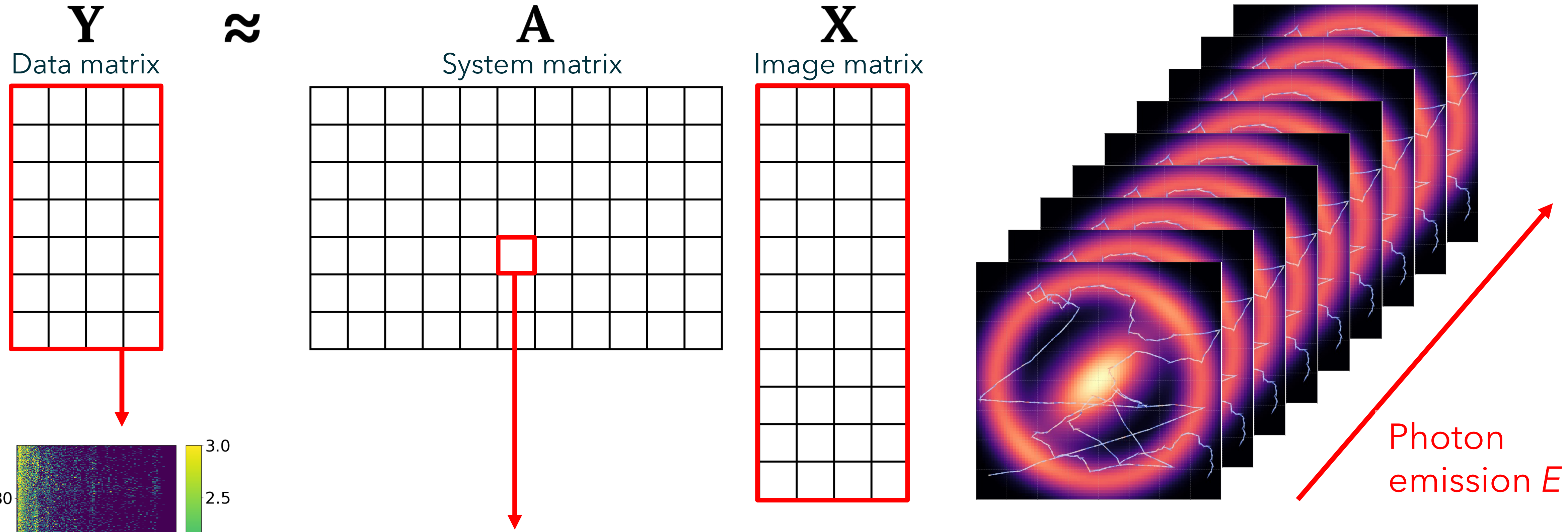
Setting energy ROIs around 662 keV and reconstruct images to increase the signal-to-noise ratio

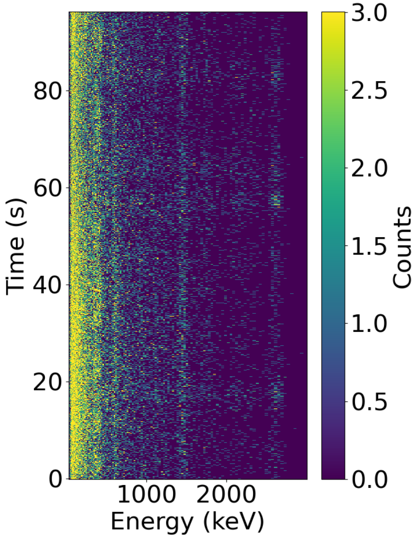
SRNL H Area Contamination Measurements





Full Spectral Imaging and Spectral Decomposition



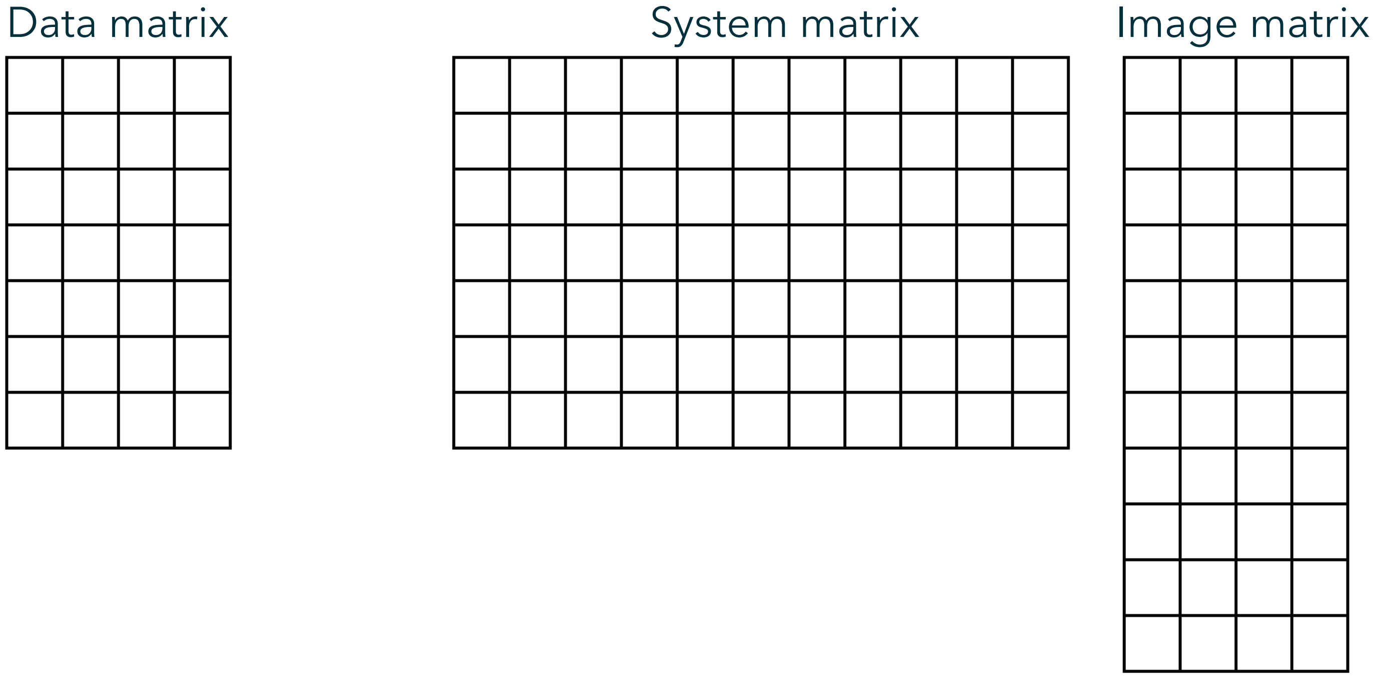


A_{ijkl} : Probability of a photon emitted from voxel x_j with the emission energy E_k detected in the measurement time y_i and records energy E_l

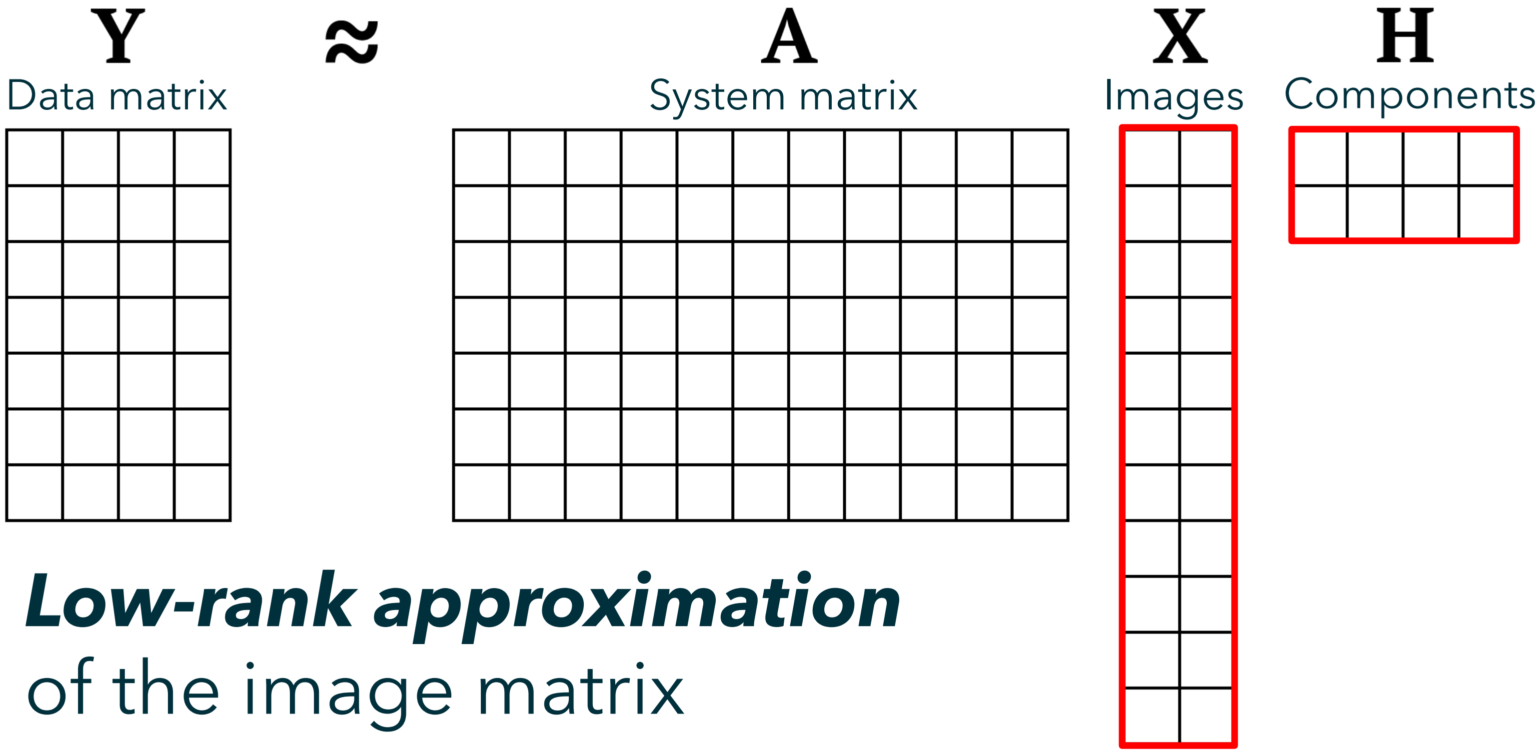
Problem – Too large image matrix and the system matrix

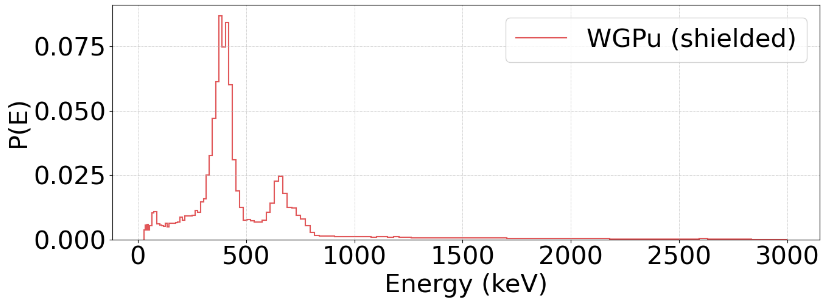
Full Spectral Imaging and Spectral Decomposition

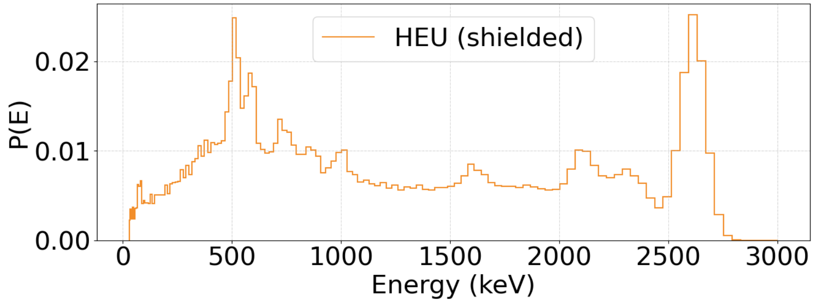
$$\mathbf{Y} \approx \mathbf{A} \quad \mathbf{X}$$



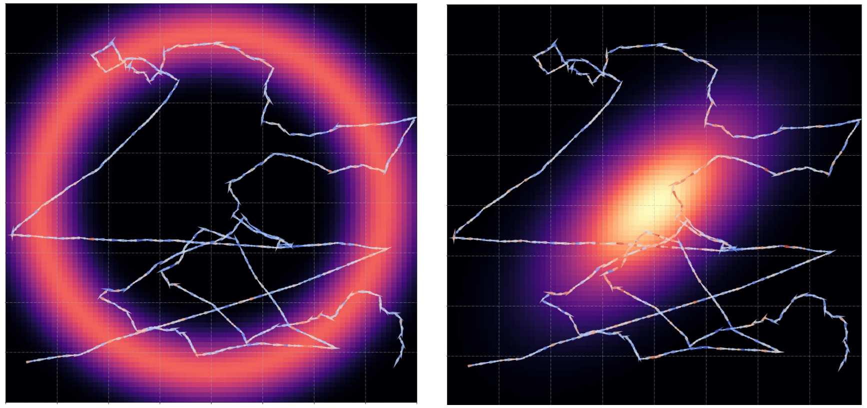
Full Spectral Imaging and Spectral Decomposition



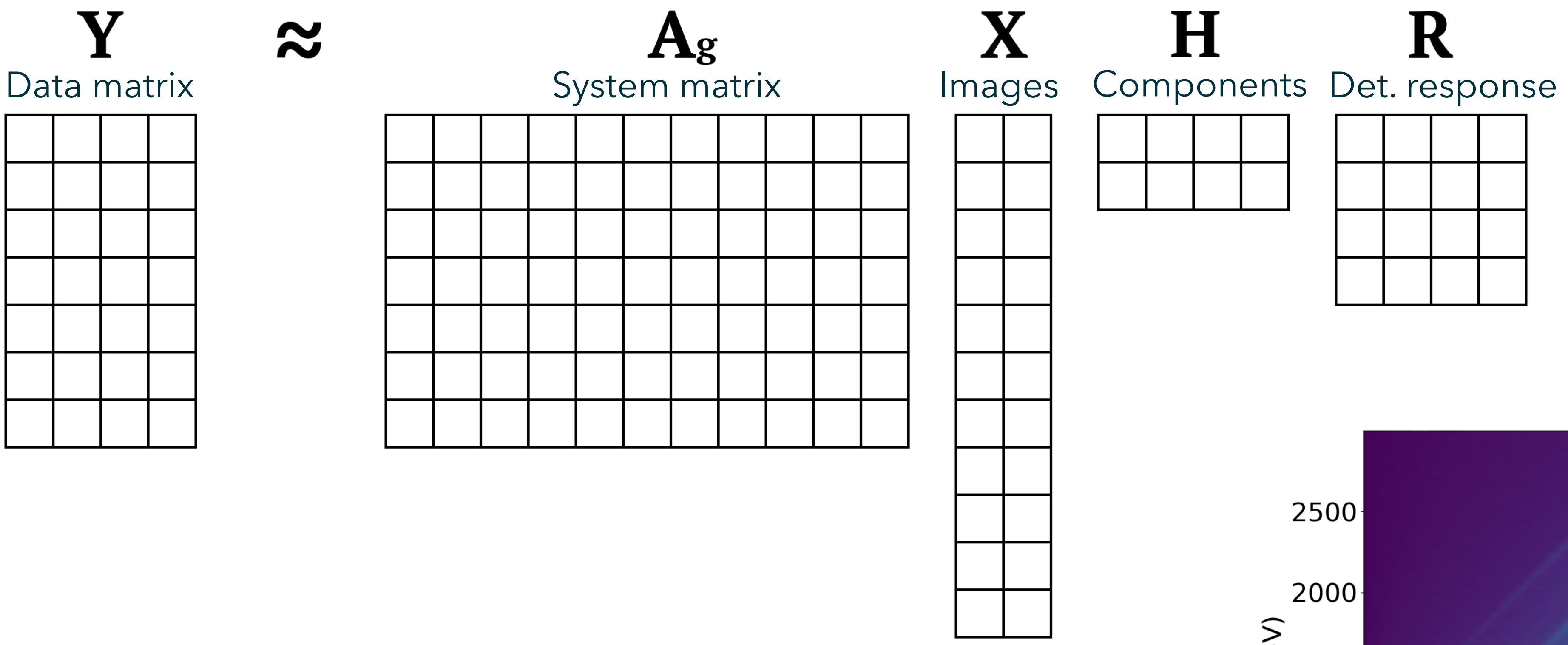


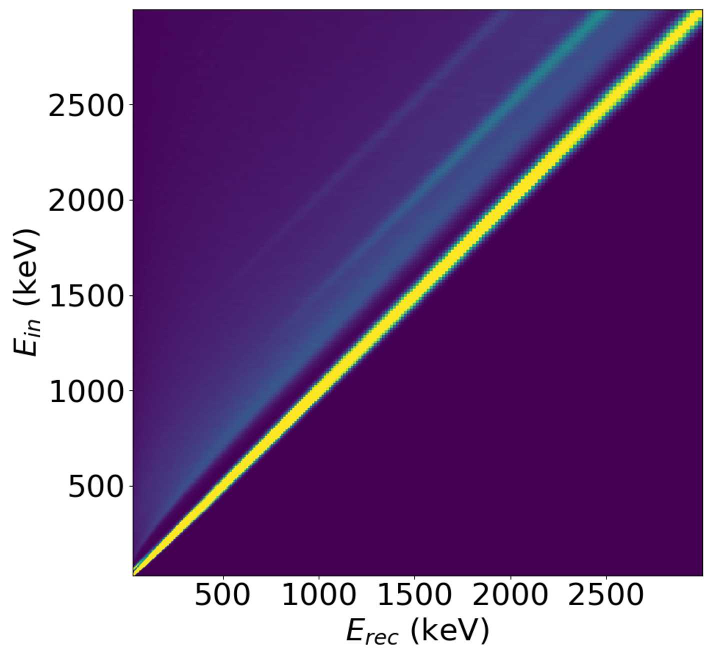


Low-rank approximation
of the image matrix



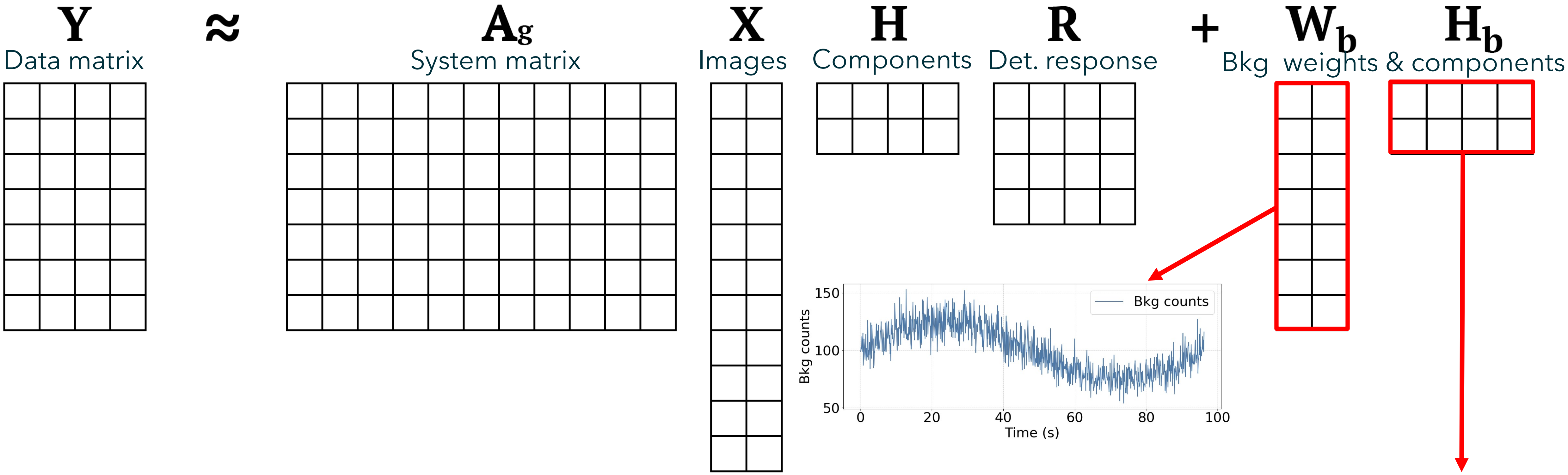
Full Spectral Imaging and Spectral Decomposition

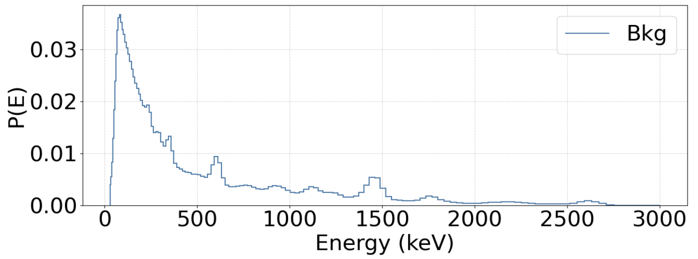




Decompose the system matrix A into the ***geometric efficiency matrix*** $\boldsymbol{A_g}$ and the ***detector response matrix R***

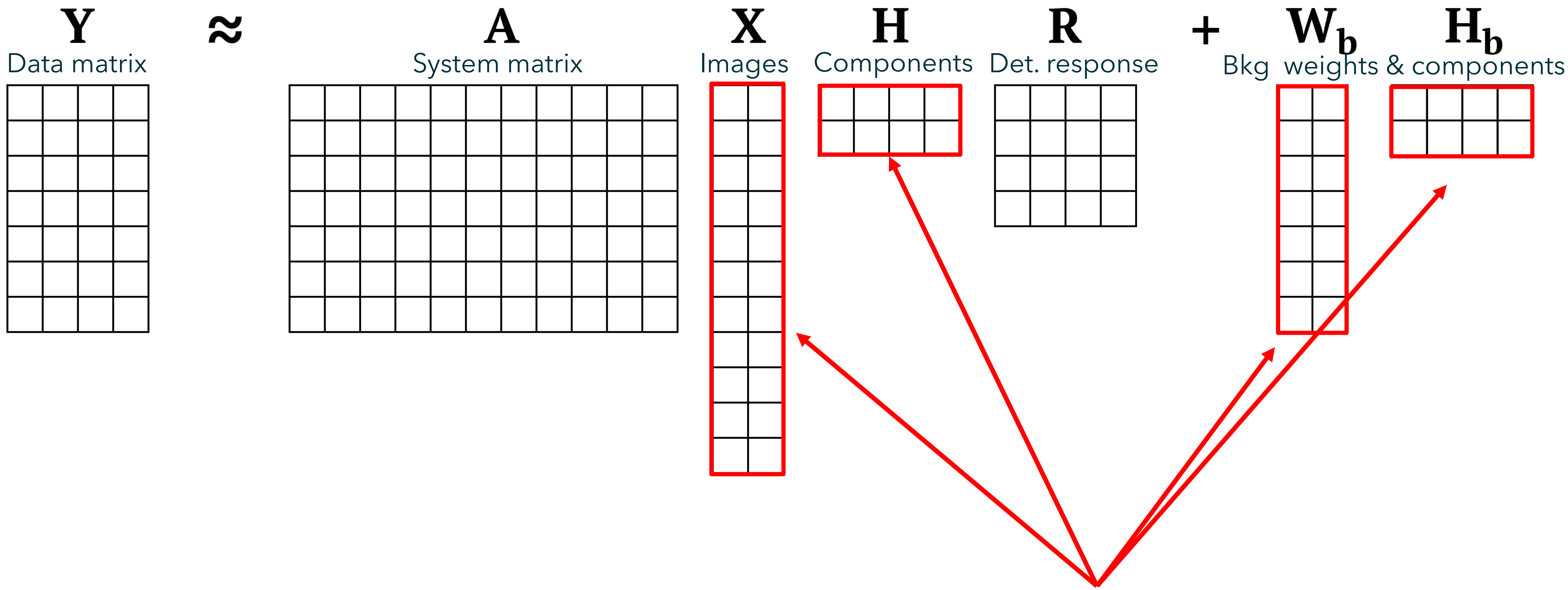
Full Spectral Imaging and Spectral Decomposition





The background can also be decomposed into ***different background components*** and their corresponding ***count contribution.***

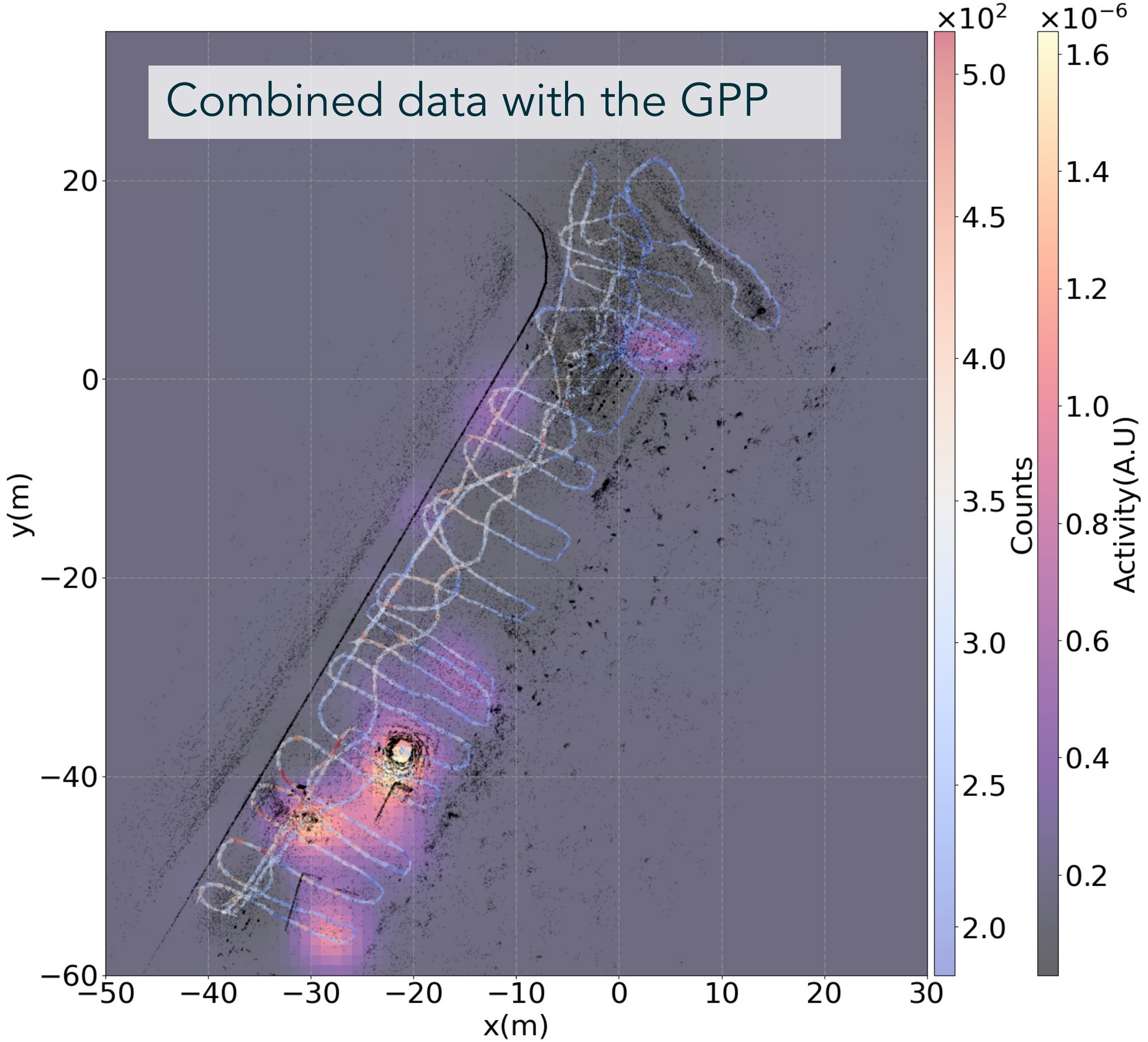
Full Spectral Imaging and Spectral Decomposition

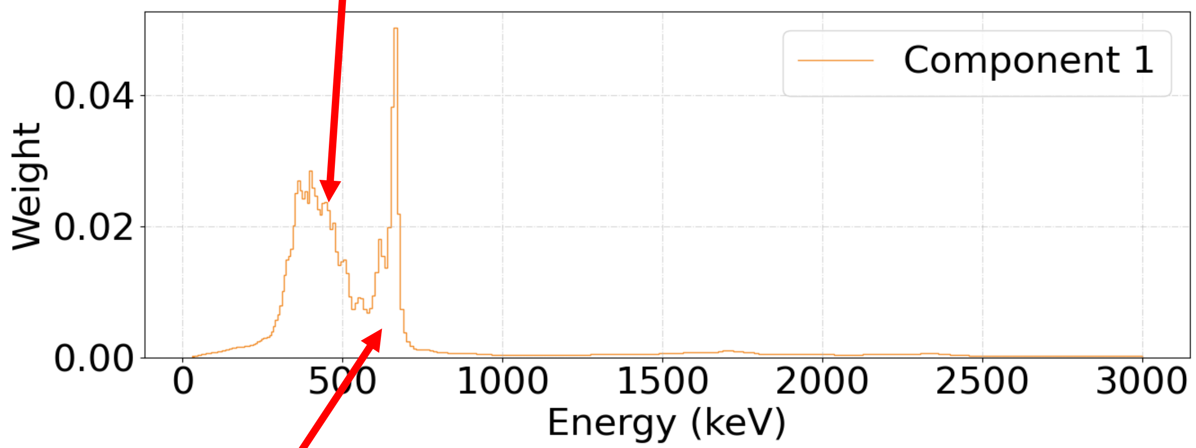


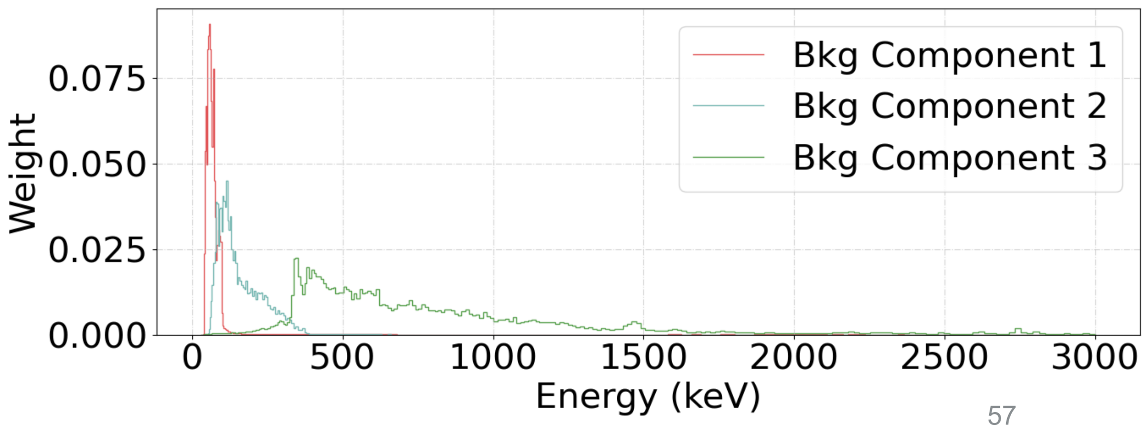
Apply the GPP to the images, components, and the background weights

Estimating 4 matrices altogether!

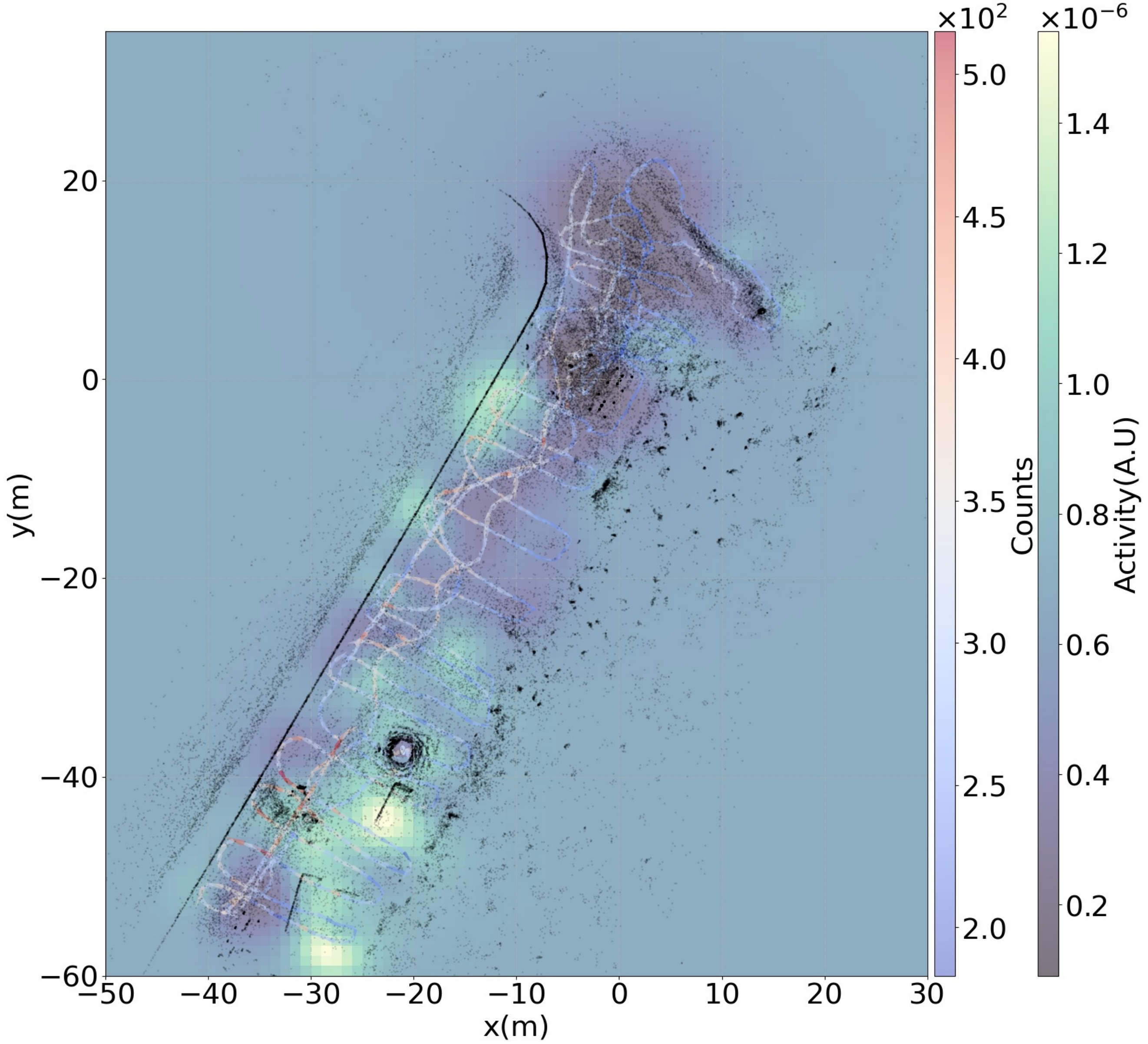
SRNL H Area Contamination Measurements



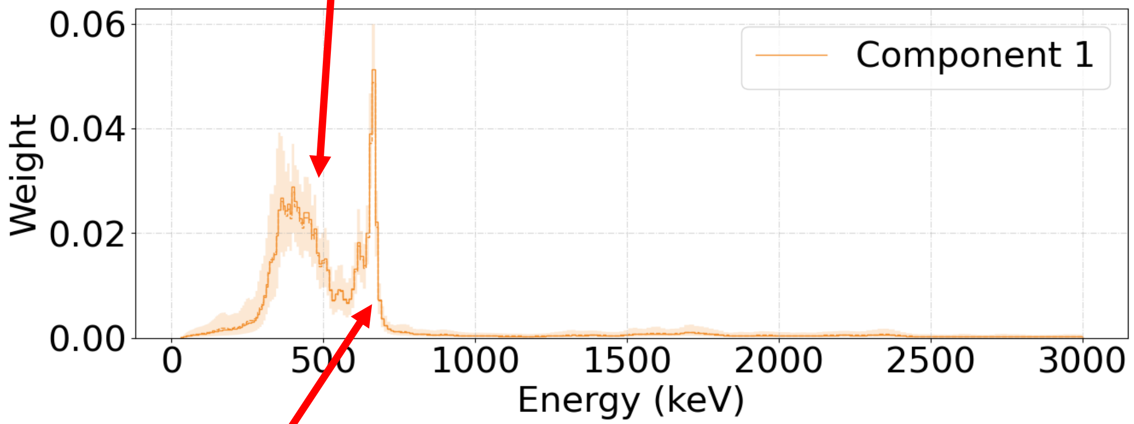




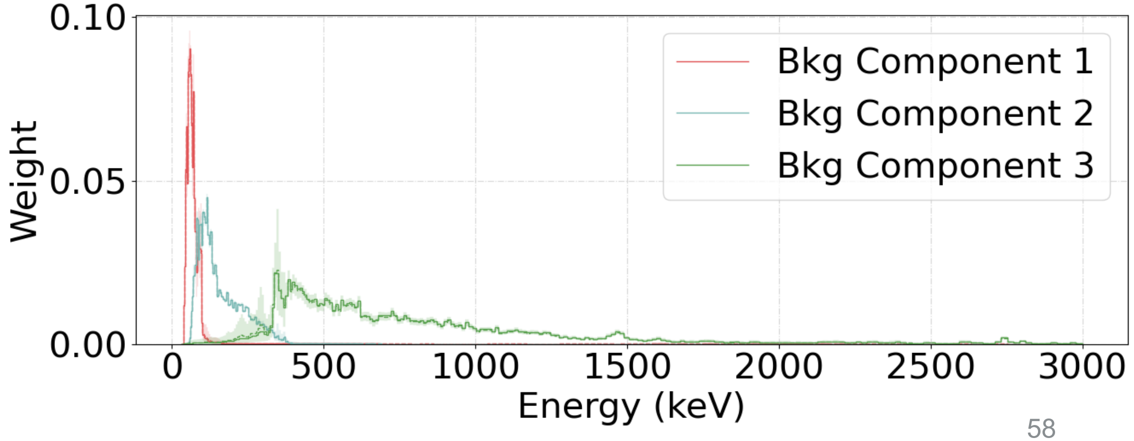
SRNL H Area Contamination Measurements



Compton edge at 477 keV



Component containing the 662keV peak



Conclusions / Future works

- ***Scene Data Fusion*** *(SDF) enables* ***imaging with a single detector,*** *which has* ***a significant potential impact on radiation safety and security applications****.*

- The proposed **GPP algorithm can significantly improve image reconstruction quality** over the conventional reconstruction methods while providing the **uncertainty quantification capabilities.**

- **The full spectral GPP** allows for radiation mapping in scenarios with **very low Signal-to-Noise Ratio (SNR) measurements and dynamically changing backgrounds.**

- Currently, we are working on **applying the framework to various related applications**, such as **medical image reconstruction** and **dose rate mapping within the Chernobyl NPP** shelter object.

Questions?