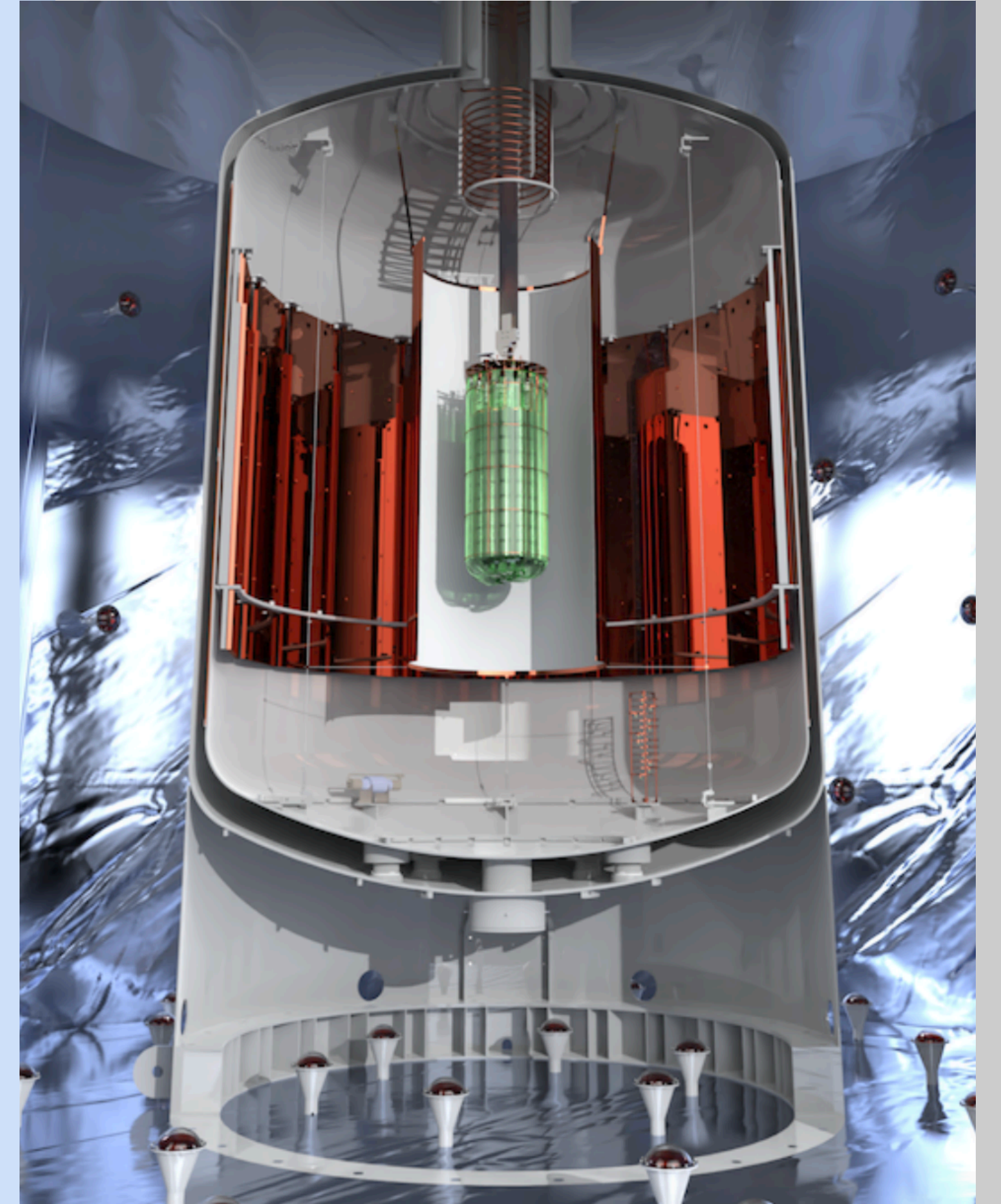


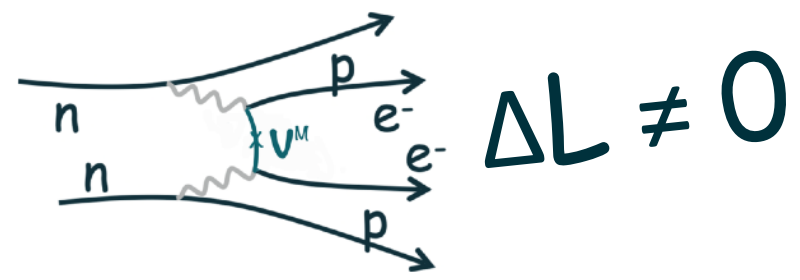
# Bayesian Uncertainty Quantification for Neutrinoless Double-Beta Decay

LEGEND

Large Enriched  
Germanium Experiment  
for Neutrinoless  $\beta\beta$  Decay

**Ann-Kathrin Schuetz, Alan Poon**  
Lawrence Berkeley National Laboratory





# Mass constraints

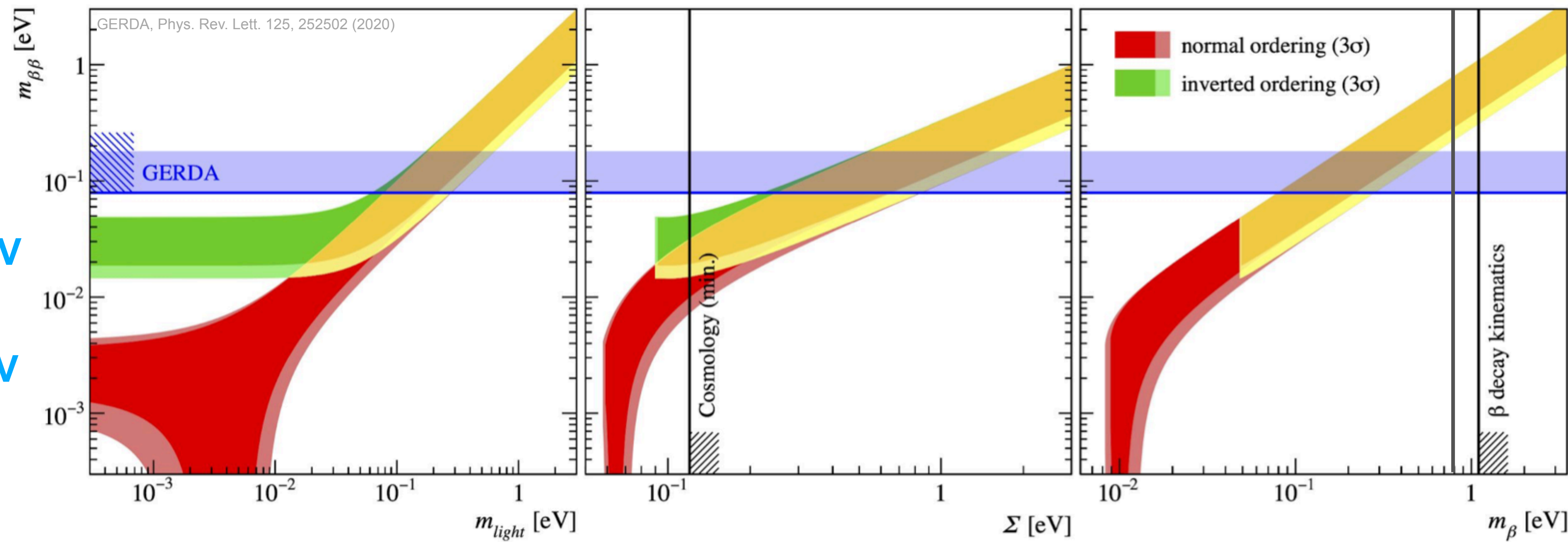
Most obvious decay mechanism: exchange of **light Majorana neutrinos**

$$1/T_{1/2}^{0\nu} = G \cdot \text{NME}^2 \cdot m_{\beta\beta}^2$$

effective neutrino mass
phase space
nuclear matrix element

**LEGEND-200:**  
 $m_{\beta\beta} < [0.33, 0.78] \text{ eV}$

**LEGEND-1000:**  
 $m_{\beta\beta} < [0.09, 0.21] \text{ eV}$

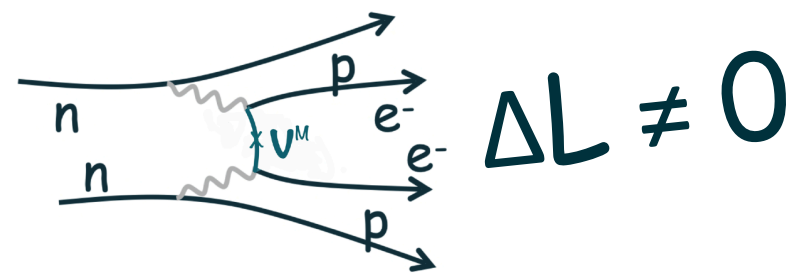


Planck+BAO:  $\Sigma < 0.12 \text{ eV}$   
 [Aghanim et al., arXiv:1807.06209]

KATRIN:  $m_{\beta} < 0.8 \text{ eV}$   
 [Nat. Phys. **18**, 160–166 (2022)]

interplay with cosmology / direct mass measurements

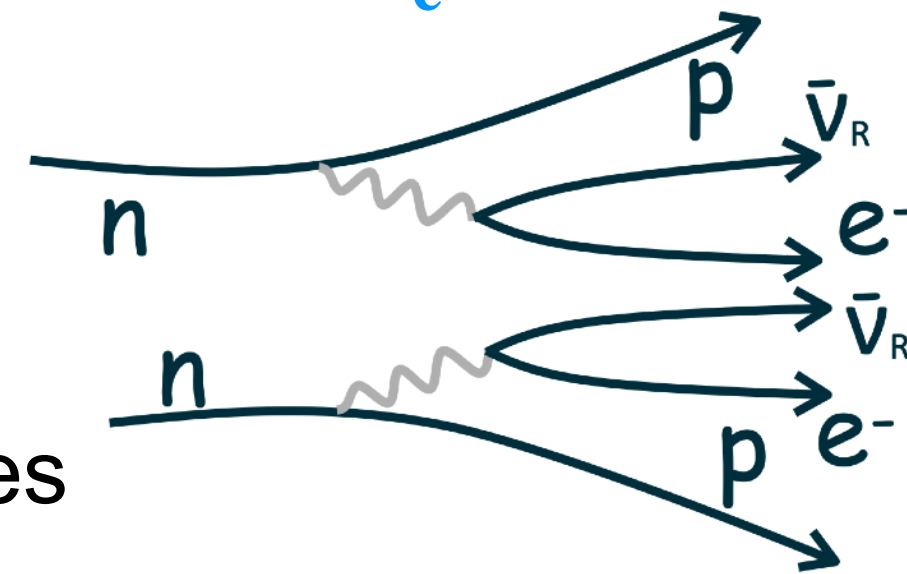




# Double-beta decay and lepton number violation

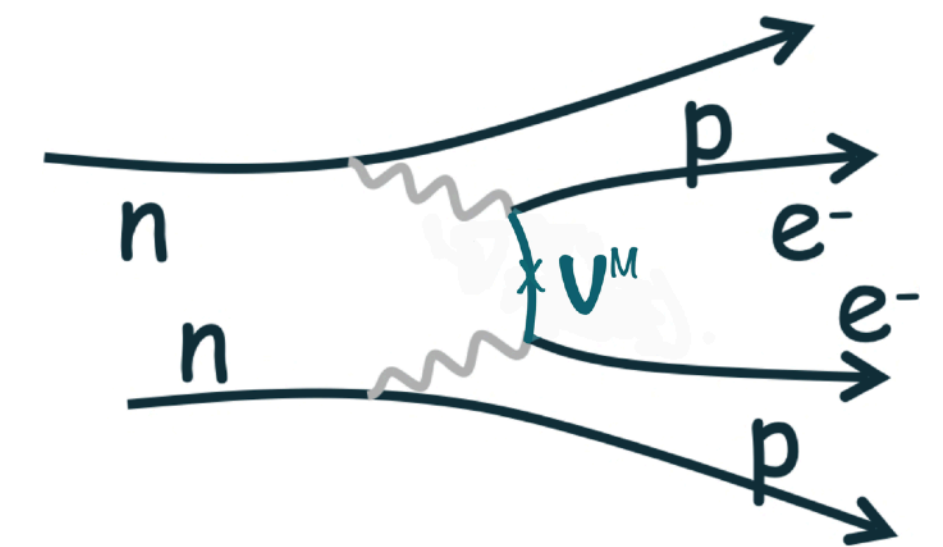
$2\nu\beta\beta : (A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$

- $\Delta L=0$
- SM allowed
- observed in many isotopes
- $T_{1/2} \sim (10^{18} - 10^{24}) \text{ yr}$

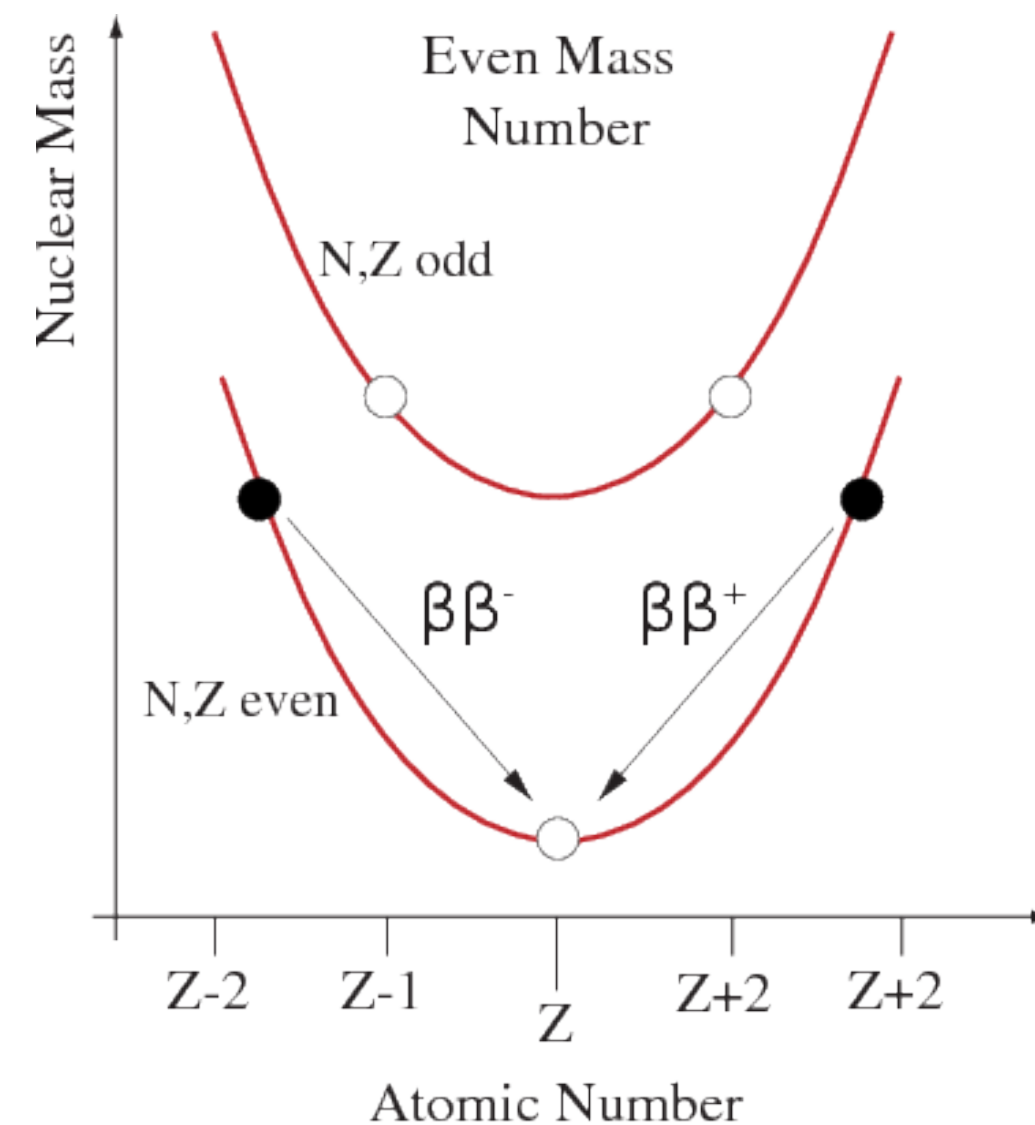


$0\nu\beta\beta : (A, Z) \rightarrow (A, Z + 2) + 2e^-$

- $\Delta L=2$
- beyond SM
- Majorana mass component
- via light Majorana  $\nu$  exchange or other L-violating processes

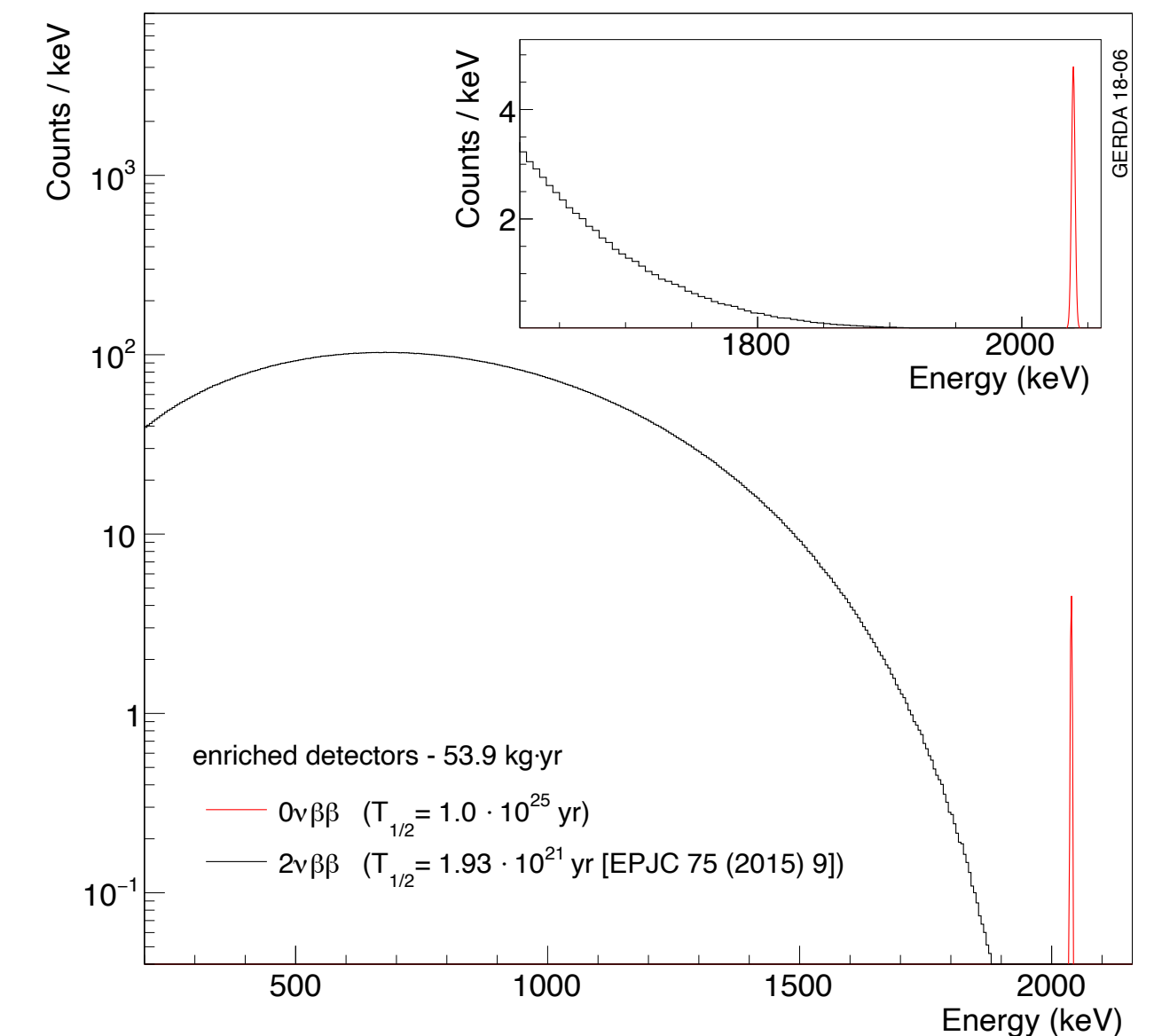


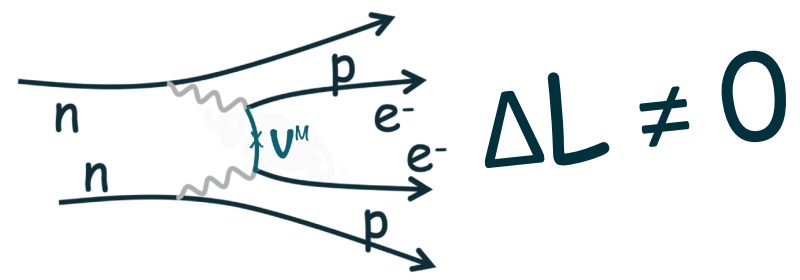
double-beta decay is possible when energetically favored



- measure sum energy spectrum of electrons
- $2\nu\beta\beta \rightarrow$  continuum
  - $0\nu\beta\beta \rightarrow$  mono-energetic peak @  $Q_{\beta\beta}$

$Q_{\beta\beta} = E_{e1} + E_{e2} - 2m_e$



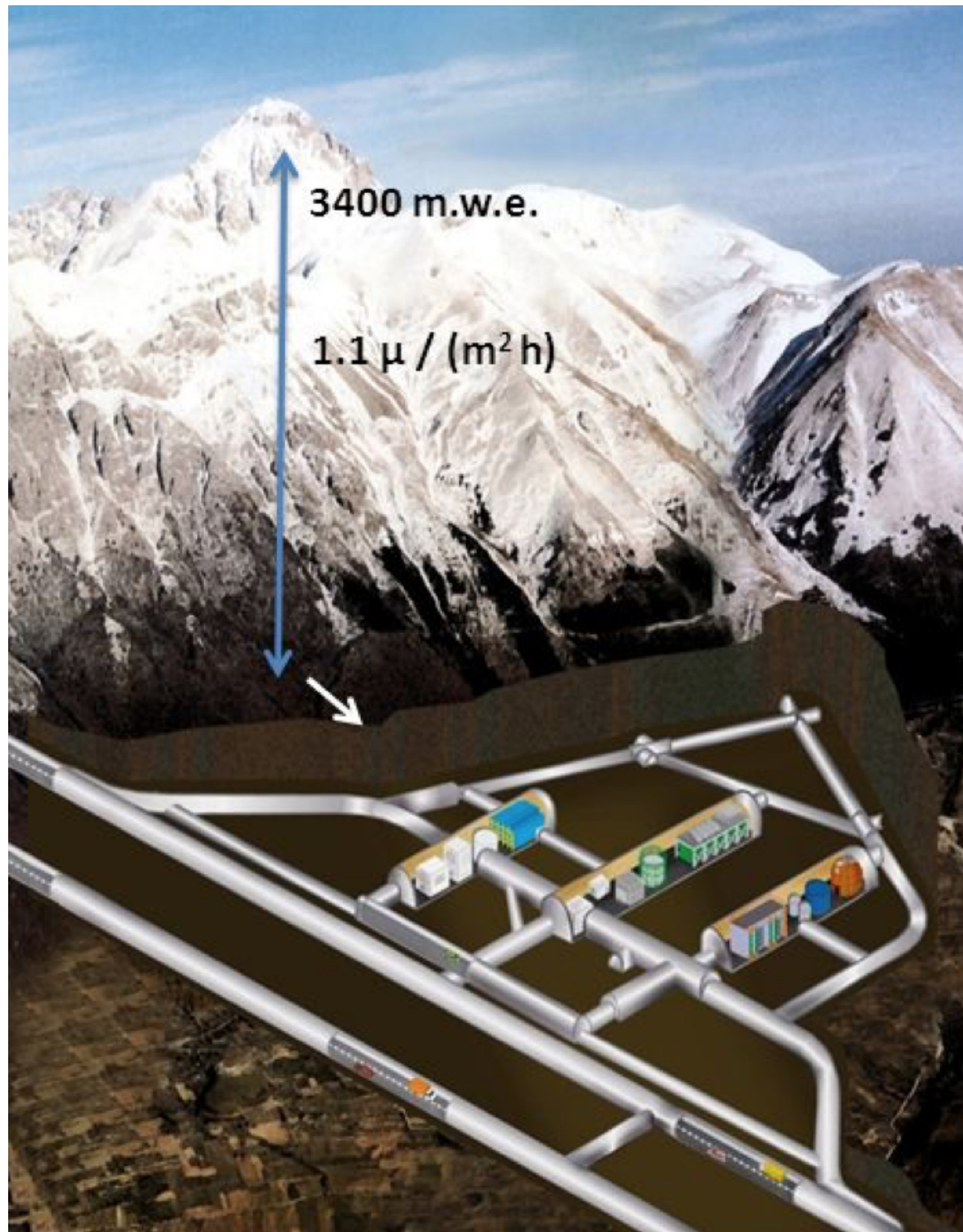
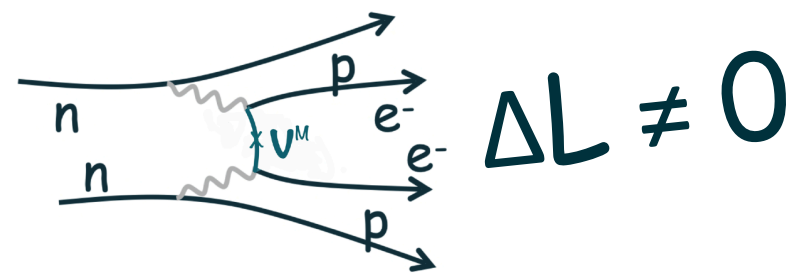


How rare?

Most measured half-lives for  $2\nu\beta\beta$  are  $O(10^{21})$  years

- Compare to lifetime of the universe:  $10^{10}$  years
- Compare to Avogadro's number:  $6 \text{ Å} \sim 10^{23}$
- A mole of the isotope will produce  $\sim 1$  decay/day
- If it exists, the half-lives of  $0\nu\beta\beta$  would be much longer
- $^{76}\text{Ge}$   $0\nu\beta\beta$  limit is  $> 10^{26}$  years,  $^{130}\text{Te}$   $0\nu\beta\beta$  limit is  $> 10^{24}$  years
- A mole of  $^{76}\text{Ge}$  produces  $< 1$  neutrinoless decay/year

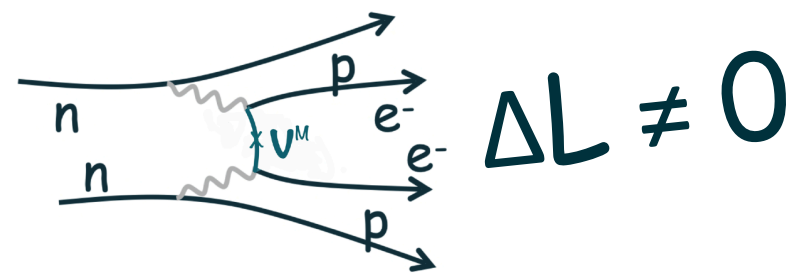




LEGEND mission: “The collaboration aims to develop a phased,  $^{76}\text{Ge}$  based double-beta decay experimental program with discovery potential at a half-life beyond  $10^{28}$  years, using existing resources as appropriate to expedite physics results.”



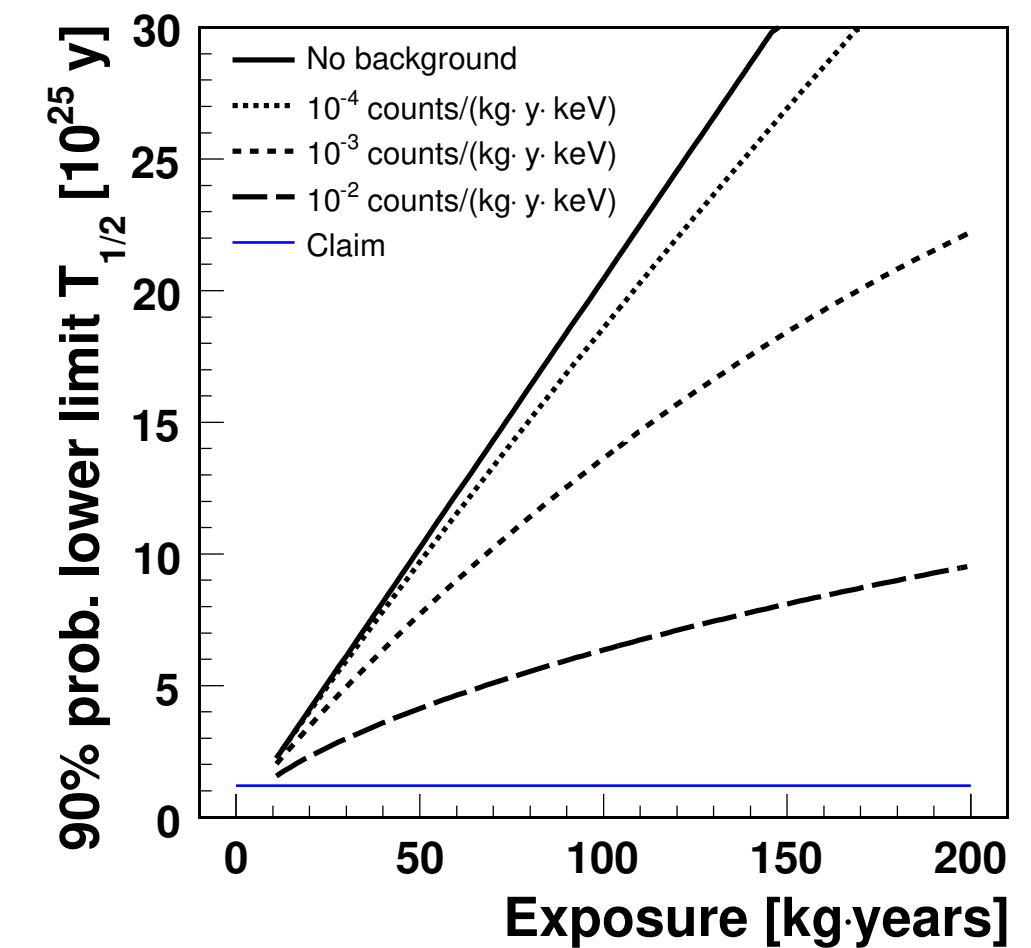




## Experimental sensitivity

background (BI) < 1  $T_{1/2}^{0\nu} \propto \varepsilon \cdot a \cdot M \cdot t$

background (BI) > 1  $T_{1/2}^{0\nu} \propto \varepsilon \cdot a \cdot \sqrt{\frac{M \cdot t}{BI \cdot \Delta E}}$



**$0\nu\beta\beta$  search in  $^{76}\text{Ge}$ :  $^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-$**

✓ source = detector

⇒ **high  $\varepsilon$**

✓ high purity Ge (HPGe) detectors

⇒ **low intr. BI**

✓  $\Delta E$  @  $Q_{\beta\beta} \sim 0.2\%$

⇒ **excellent  $\Delta E$**

✓ high density

⇒  **$0\nu\beta\beta$  peak-like events**

- low  $Q_{\beta\beta}$  value ( $Q_{\beta\beta} = 2039$  keV)

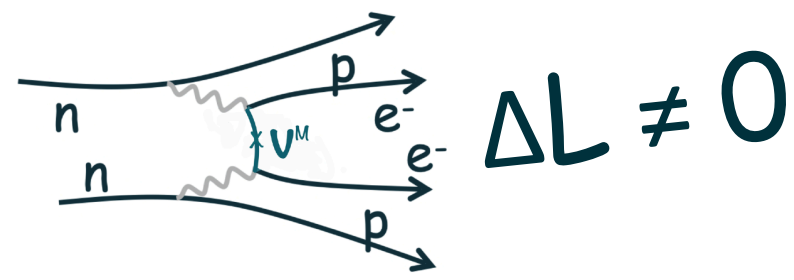
⇒ **possible external BI (e.g.  $^{208}\text{Tl}$ )**

-  $a=7.8\%$  for  $^{76}\text{Ge}$

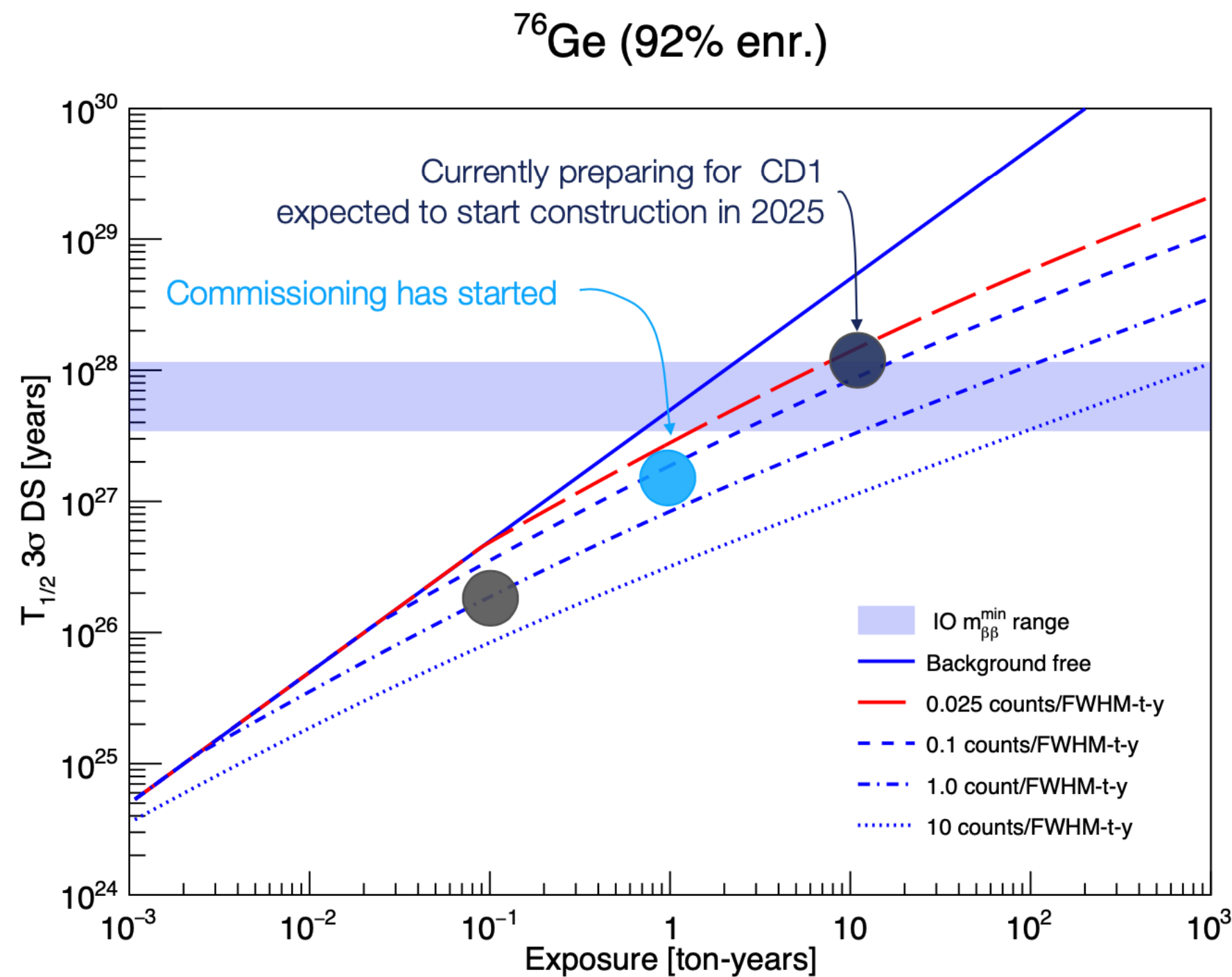
⇒ **enrichment necessary**

**→ increase sensitivity by background reduction (BI) at  $Q_{\beta\beta}$  and simultaneous increase of mass (M) and improvement of the energy resolution ( $\Delta E$ )**





# Building on past strengths

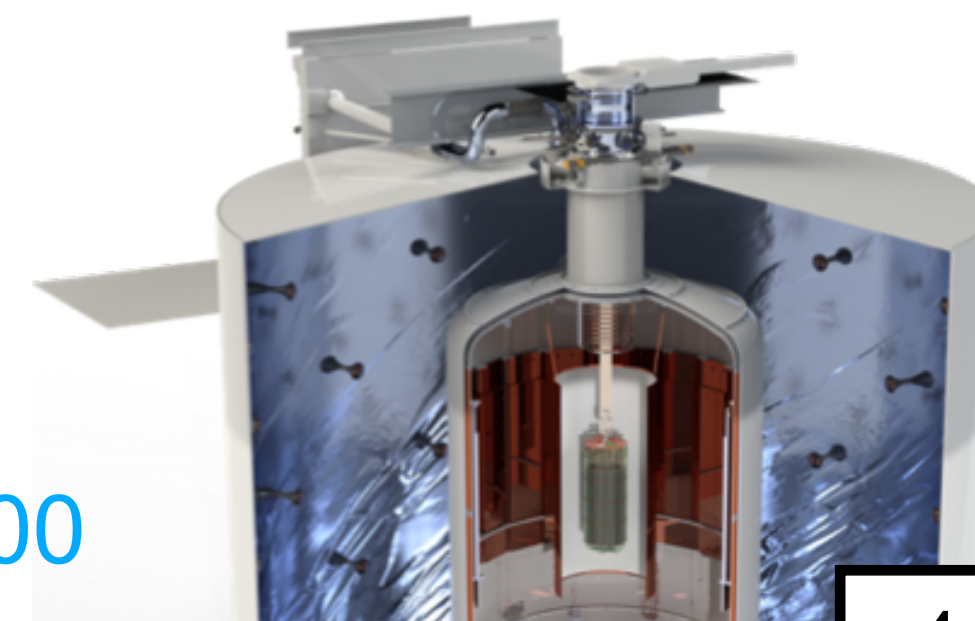


GERDA - Germanium Detector Array

- operation of bare Ge crystals immersed in LAr, LAr scintillation for active veto
- LNGS (Italy) until Nov 2019
- $\Delta E$ : 2.6 keV (BEGe FWHM at  $Q_{\beta\beta}$ )
- **BI:  $5.2 \times 10^{-4}$  cts / (kg·keV·yr)**

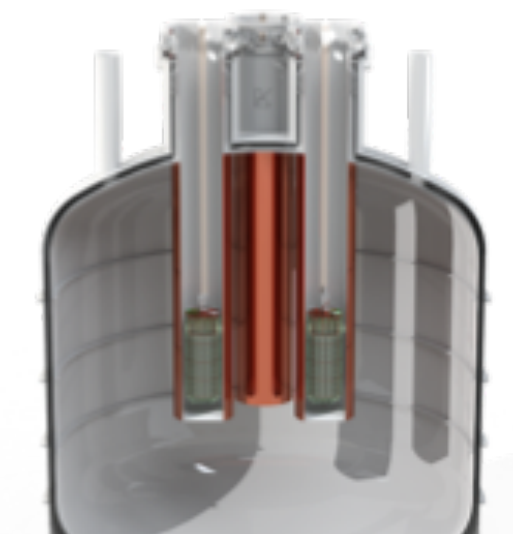
MJD - MAJORANA DEMONSTRATOR

- operation of Ge crystals in vacuum cryostat, ultra-clean underground electro-formed copper
- SURF (South Dakota) until 2020
- **$\Delta E$ : 2.53 keV (FWHM at  $Q_{\beta\beta}$ )**
- BI:  $4.7 \times 10^{-3}$  cts / (kg·keV·yr)



LEGEND-200

LEGEND-1000

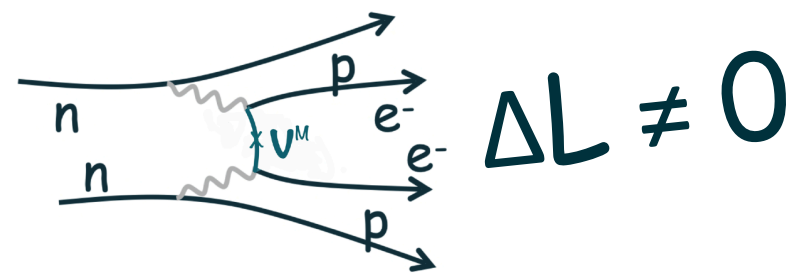


- currently commissioning; will start data-taking in 2022
- requires only  $\times 2-3$  background improvement w.r.t. GERDA
- goal (5 yr runtime):
- discovery sensitivity  $T_{1/2} > 10^{27}$  yr (90% C.L.)  $\rightarrow m_{\beta\beta} < 33-71$  meV

- 4x L-200 design  $\sim 400$  detectors
- large reentrant tube filled with UGLAr/ all LAr vol. with veto system
- baseline site is SNOLAB but LNGS possible  $\rightarrow$  rich R&D program
- goal (10 yr runtime):
- discovery sensitivity  $T_{1/2} > 10^{28}$  yr (90% C.L.)  $\rightarrow m_{\beta\beta} < 9-19$  meV

LEGEND-1000	Detector mass: 1000 kg Exposure: 10 ton-yr	$T_{1/2}^{0\nu} > 10^{28}$ yr
LEGEND-200	Detector mass: 200 kg Exposure: 1 ton-yr	$T_{1/2}^{0\nu} > 10^{27}$ yr
GERDA / MAJORANA DEMONSTRATOR	Detector mass: 44/ 44 kg Exposure: 127/ 65 kg-yr	$T_{1/2}^{0\nu} > 10^{26}$ yr / $10^{25}$ yr

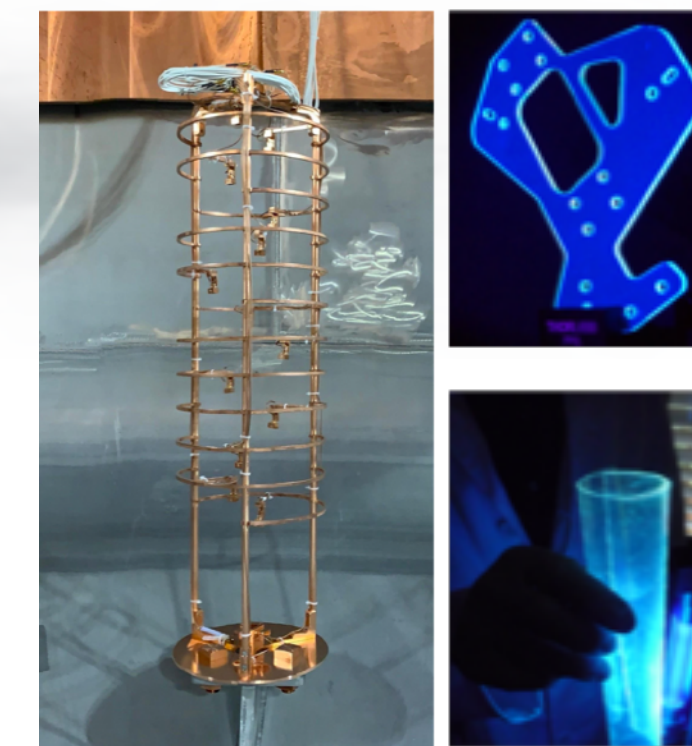
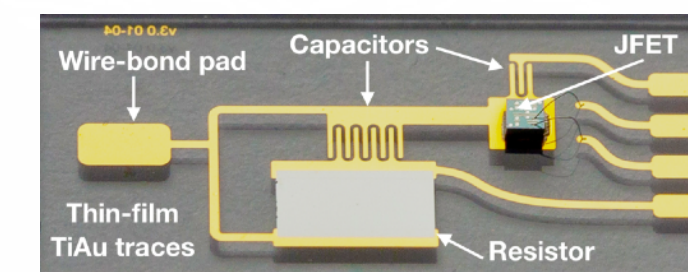
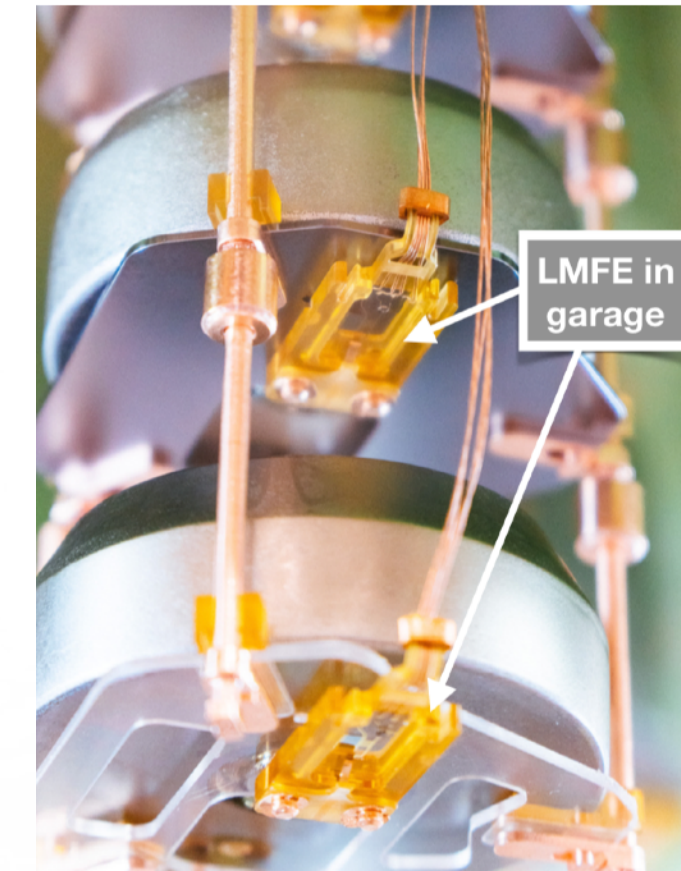
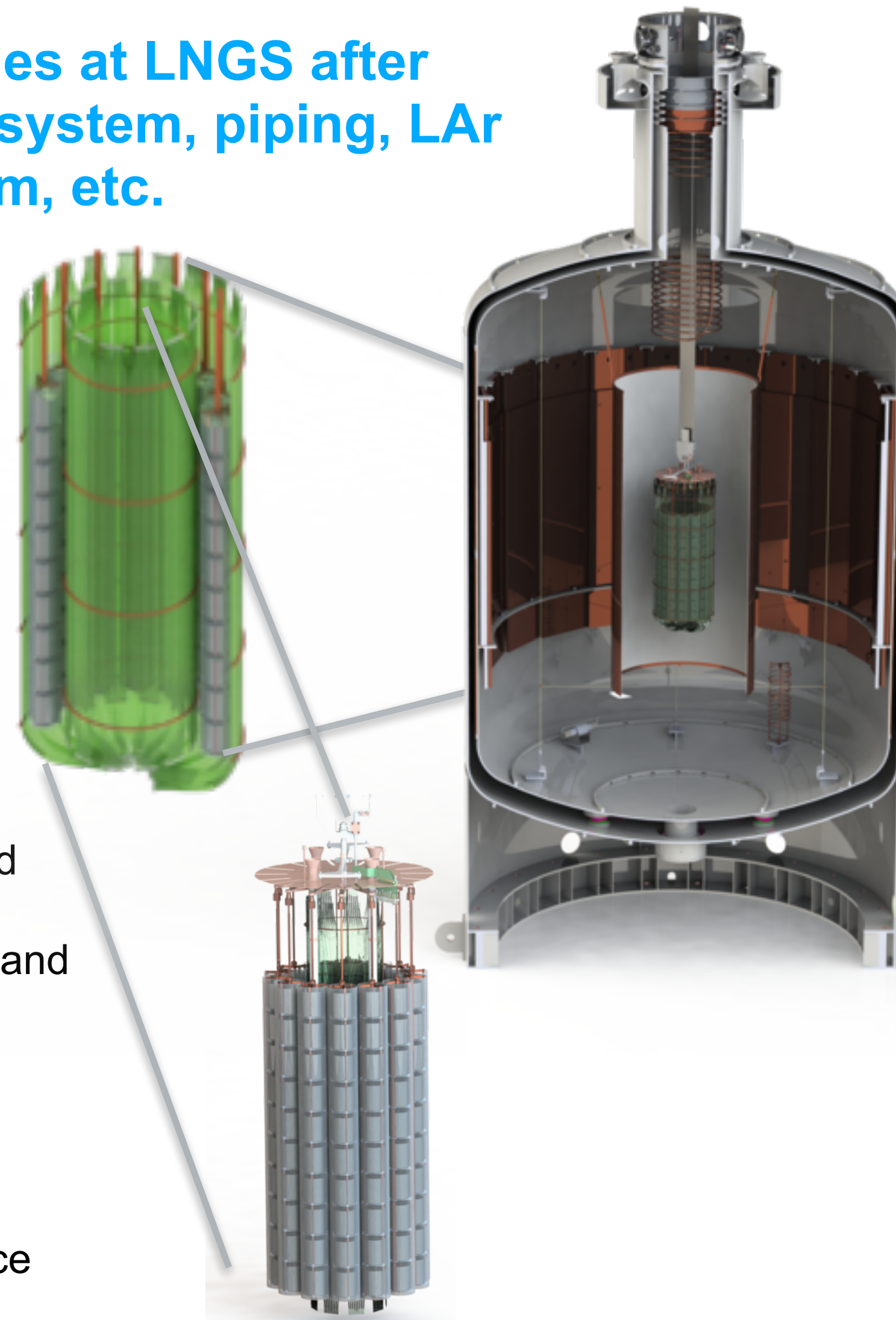




## First stage: LEGEND-200

Reuse of GERDA facilities at LNGS after upgrades on e.g. lock system, piping, LAr veto, calibration system, etc.

- detect LAr scintillation light
- fiber shroud: fibers were coated with tetraphenyl butadiene (TPB) for wavelength shifting from VUV to blue regime
- ~ 200 kg of detectors distributed over 12 strings
- reuse of detectors from GERDA and MJD + 140 kg of additional inverted-coaxial point-contact (ICPC) detectors
- ICPC detectors:
  - Active mass > 3 kg
  - Excellent PSD performance



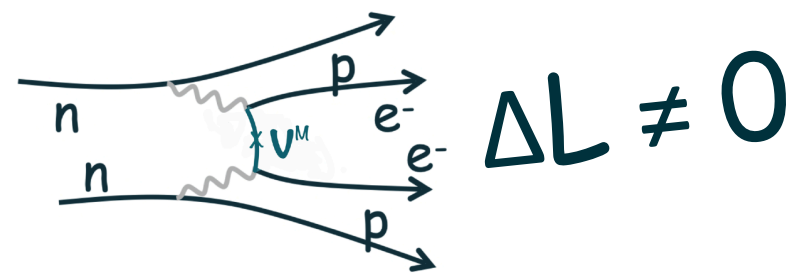
**Read-out:** *New in LEGEND-200*

- Low-mass front end (LMFE) close to detector (MJD-style)
- differential signal output (reducing noise)
- new digitizer: Flashcam
- new custom made head electronics, HV filter boards, slow control

**LAr veto:** *New in LEGEND-200*

- independent trigger (veto  $^{77(m)}\text{Ge}$ , neutrons)
- in-situ LAr purity monitoring (LLAMA)
- measuring light yield and triplet lifetime
- Scintillating/ transparent detector holders (PEN)
- LAr purification system (fill and continuous)
- cryostat refilled with new LAr Sep.2021

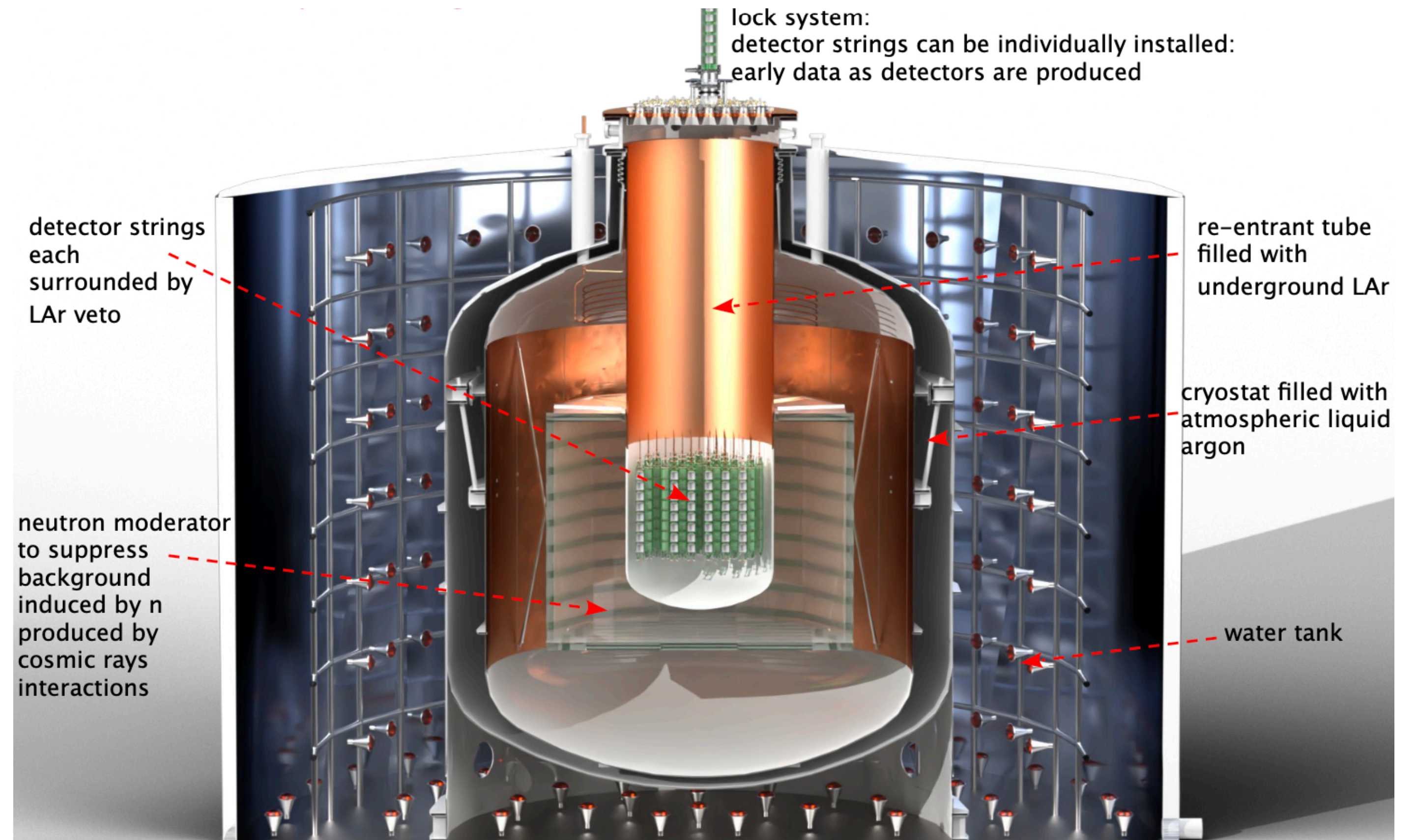




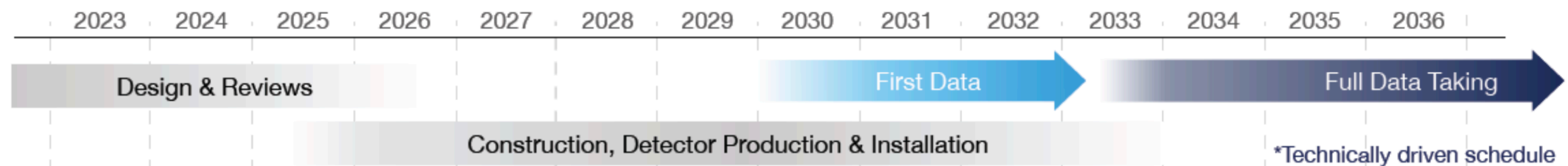
## Second stage: LEGEND-1000

### Performance parameters

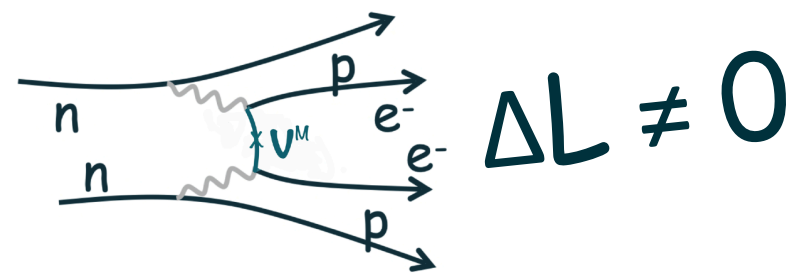
$0\nu\beta\beta$ isotope	$^{76}\text{Ge}$
$Q_{\beta\beta}$	2039 keV
Total mass	1000 kg
$\Delta E$ at $Q_{\beta\beta}$	2.5 keV FWHM
Signal acceptance	0.69
Total exposure	10 t·yr
Background goal	$< 10^{-5}$ cts/(keV·kg·yr)
$T_{1/2}^{0\nu}$	1.3·10 <sup>28</sup> yr (90% C.L. discovery) 1.8·10 <sup>28</sup> yr (90% C.L. sensitivity)
$m_{\beta\beta}$	9.4-21.4 meV (99.7% C.L. discovery) 8.5-19.4 meV (90% C.L. sensitivity)



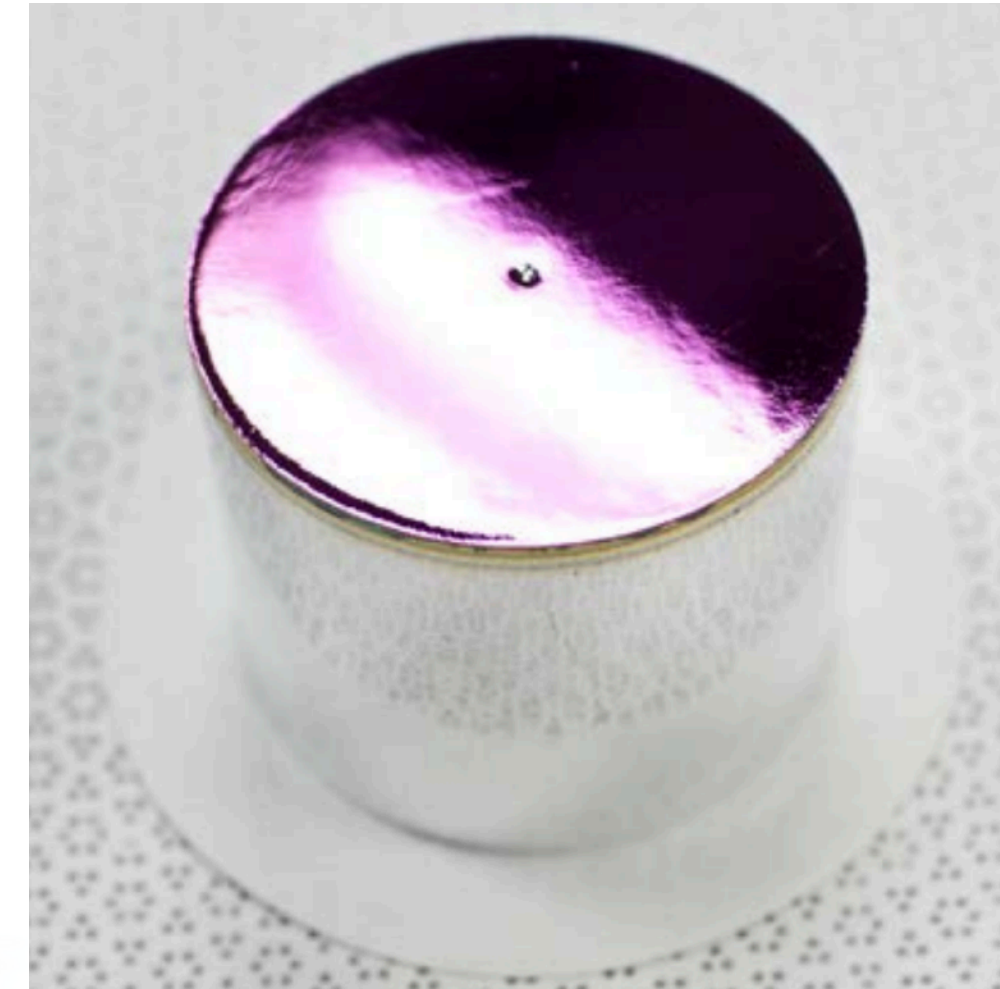
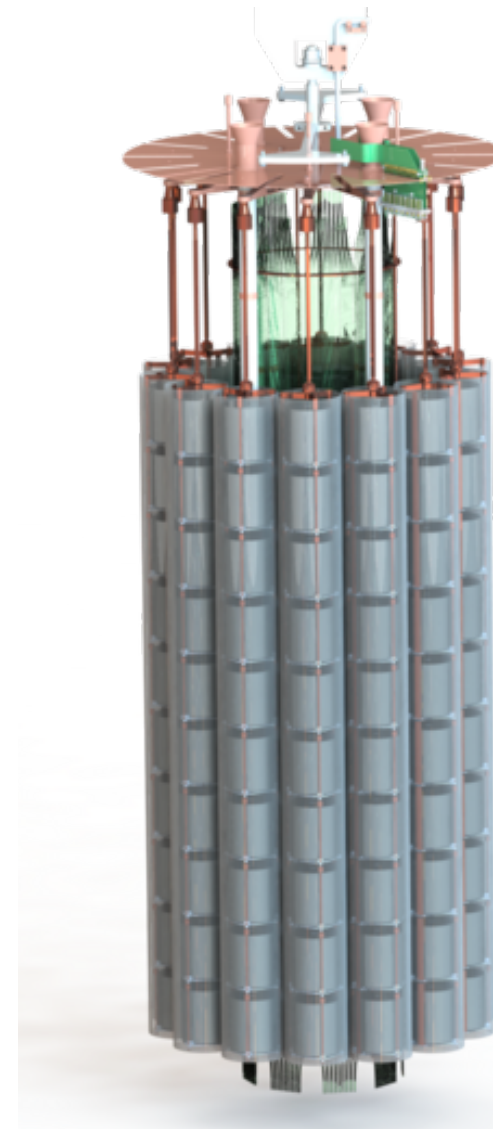
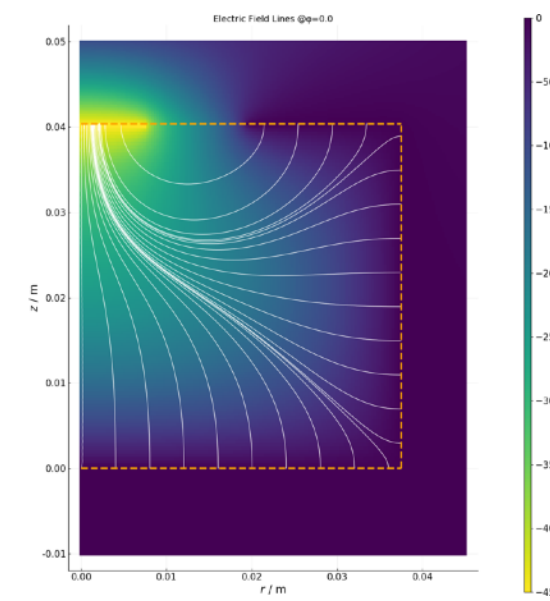
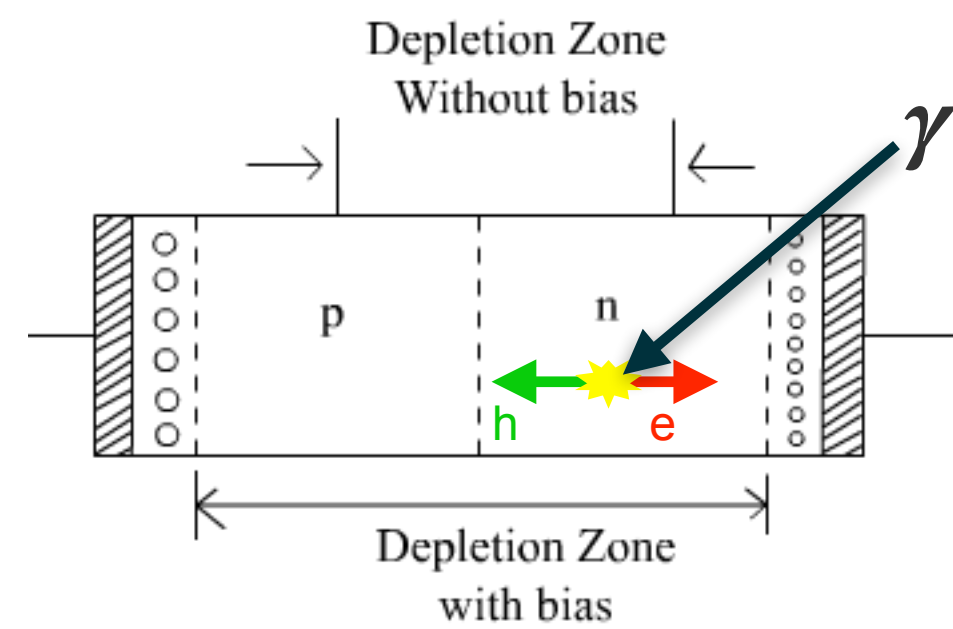
### Timeline







# Point contact HPGe detectors

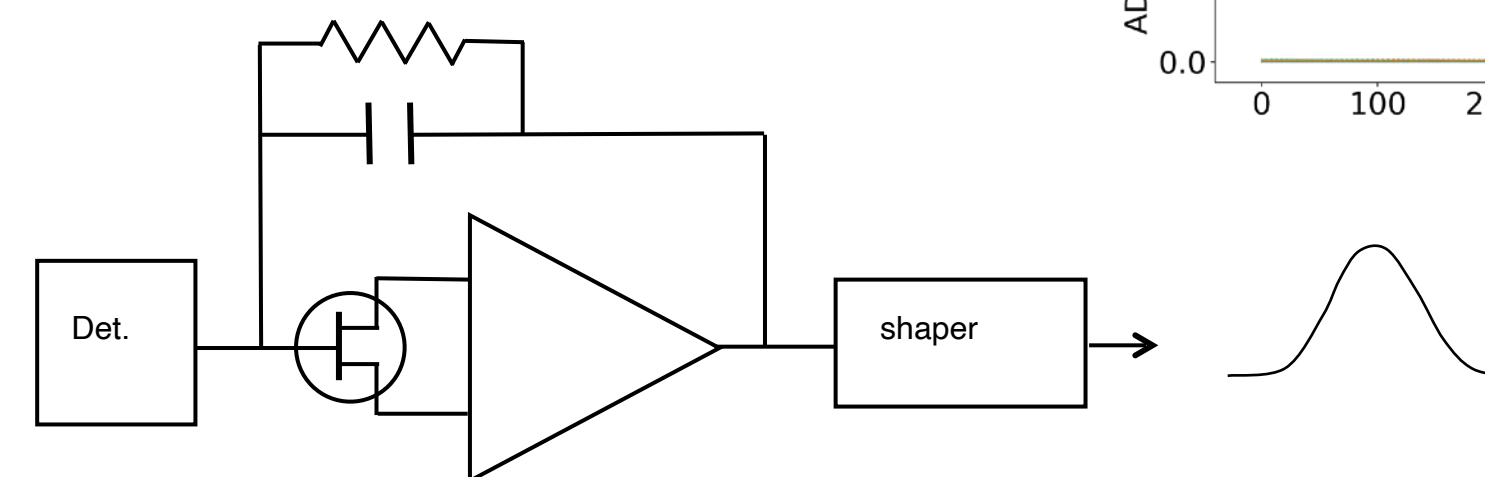
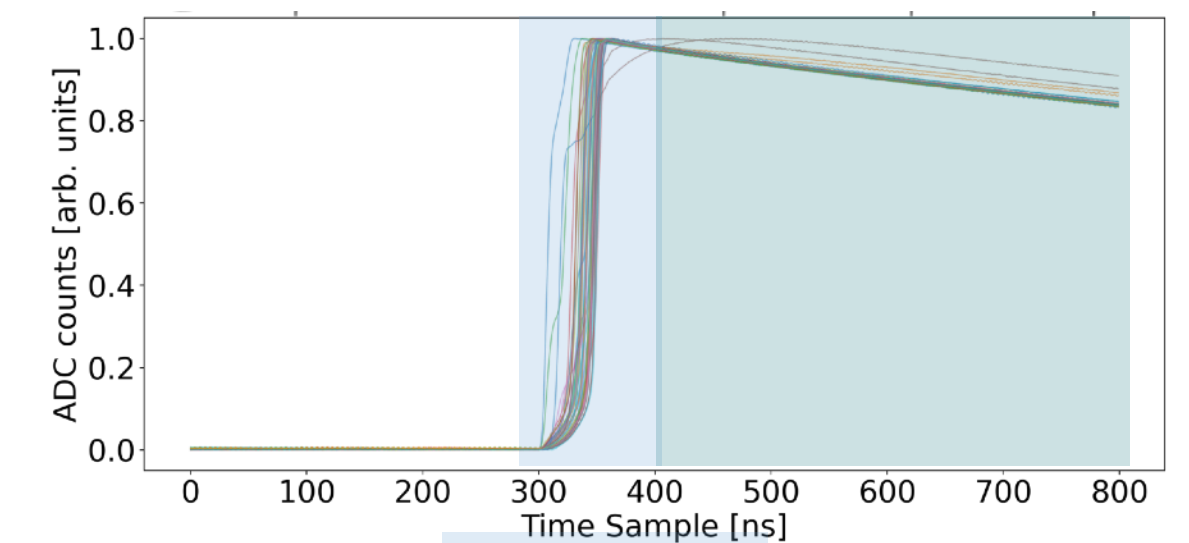
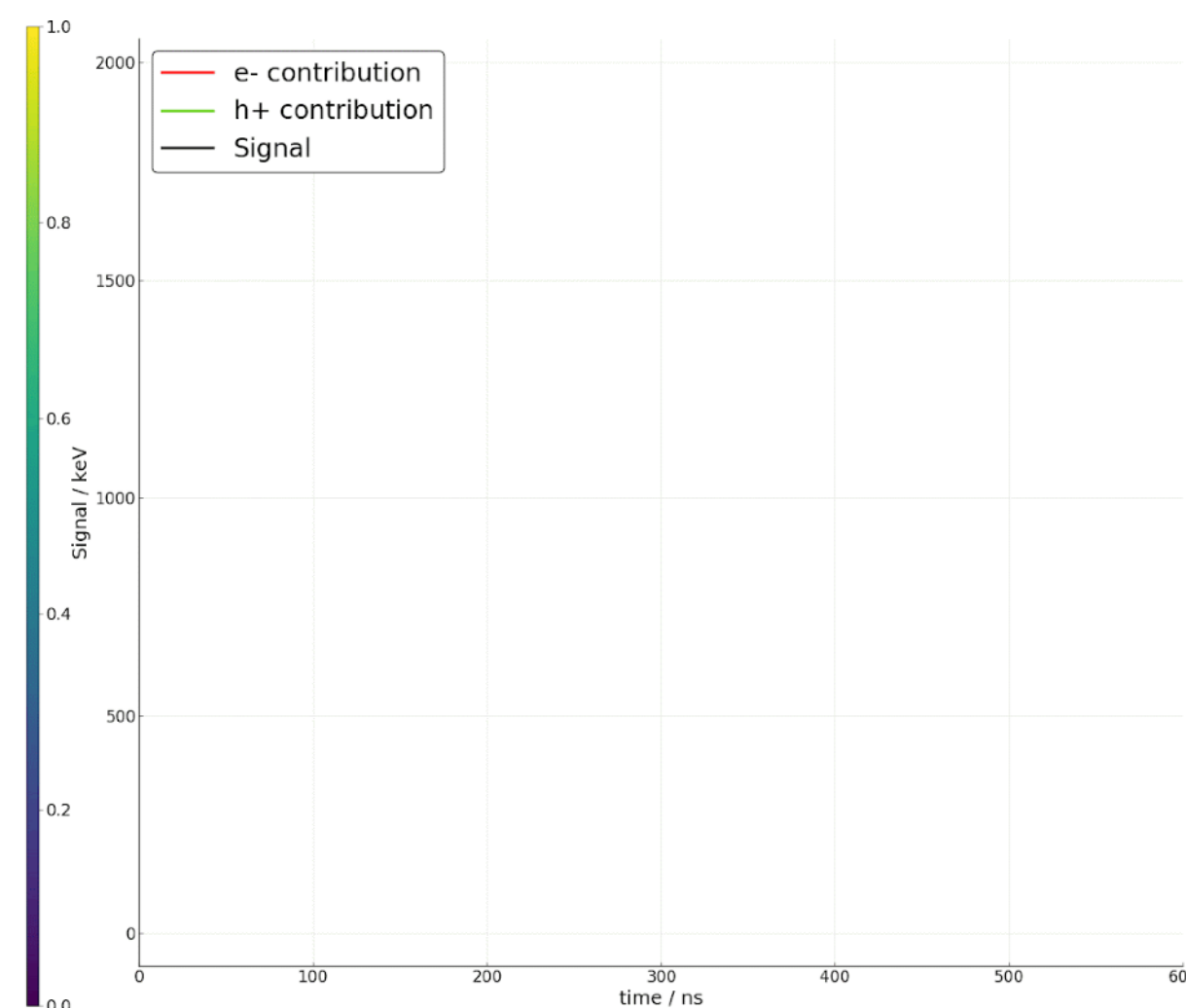
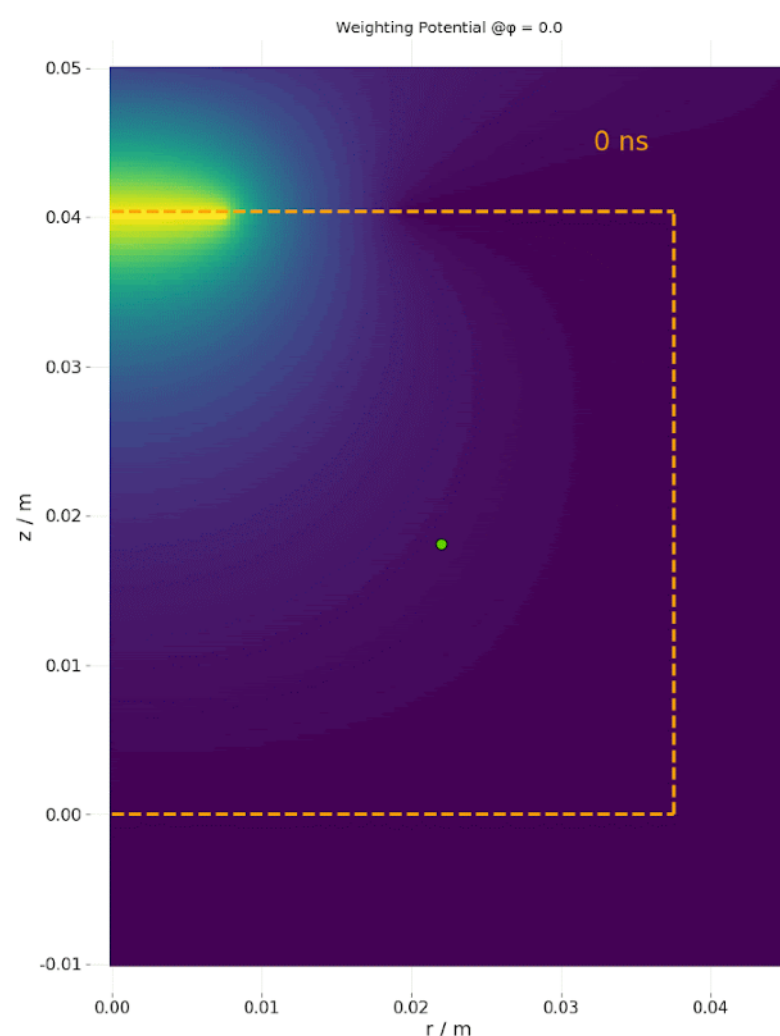


## P-type Point Contact Detector

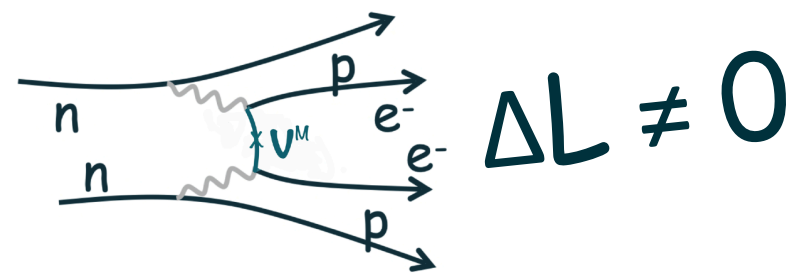
- Signal cable
  - ➔ Less background
- Small p-contact
  - ➔ Low capacitance
  - ➔ Less noise
- Thick dead layer
  - ➔ Less alpha and beta events
- Lower bias voltage
- Bigger detectors
- Smaller surface to volume ratio

- Incident ionizing radiation creates electron - hole pairs  $\propto E$
- $e^-$  and  $h^+$  created in the depletion region move to the respective electrodes  $\rightarrow$  **detectable current**

- Charge drift & signal formation by using Shockley-Ramo Theorem: Charge induced on given electrode by motion of charge carriers
- Resulting signal convoluted with the electronic response function (detector-channels-wise)







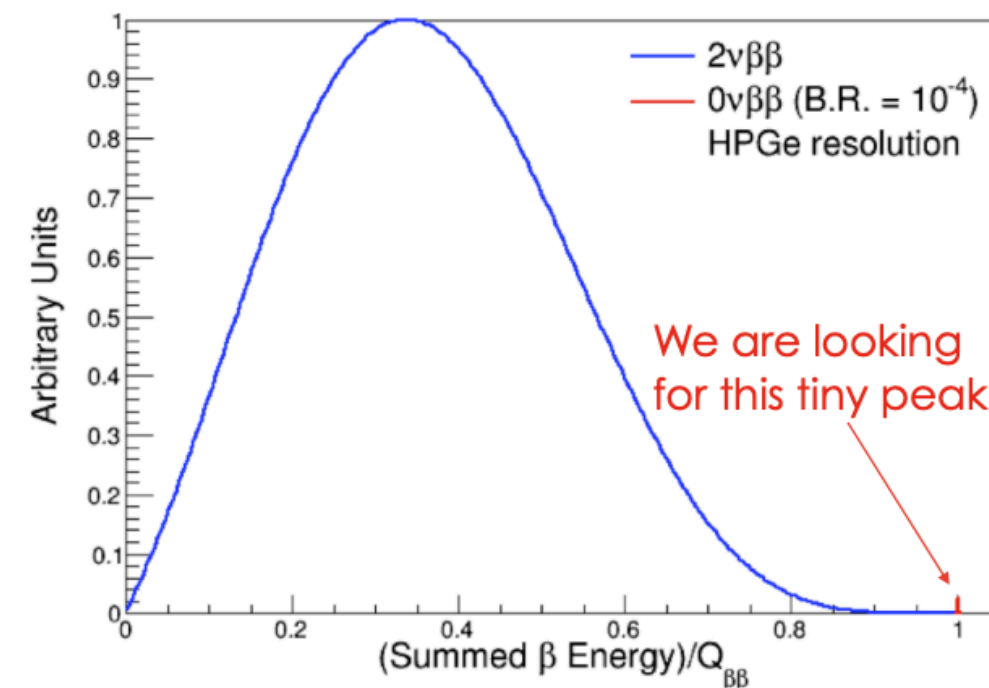
# Bayesian Inference and UQ for $0\nu\beta\beta$ decay

Experimental goal is to measure mono-energetic peak at  $Q_{\beta\beta}$

$$0\nu\beta\beta : (A, Z) \rightarrow (A, Z + 2) + 2e^-$$

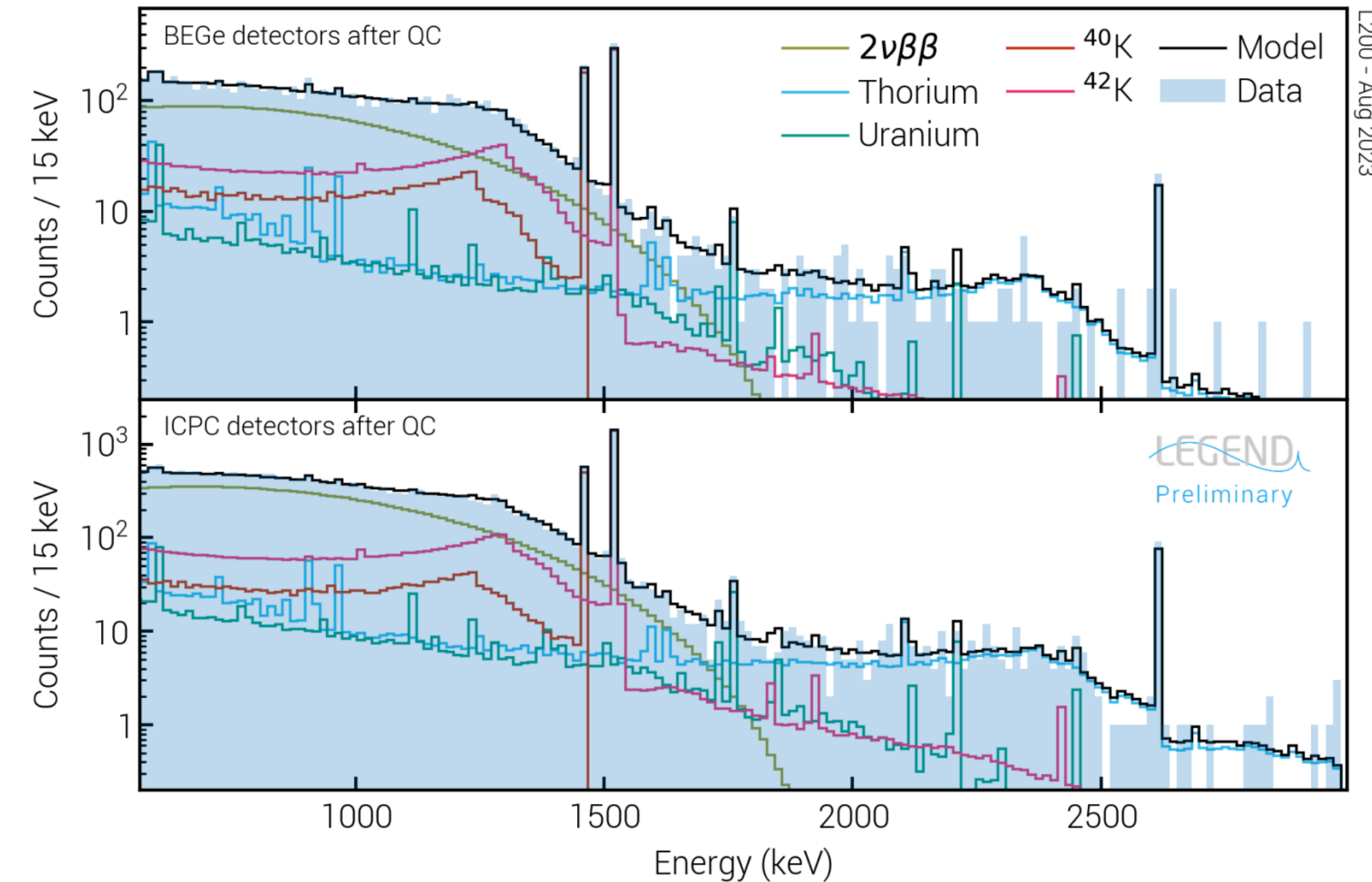
Experimental sensitivity:

$$\text{background (BI)} > 1 \quad T_{1/2}^{0\nu} \propto \varepsilon \cdot a \cdot \sqrt{\frac{M \cdot t}{BI \cdot \Delta E}}$$

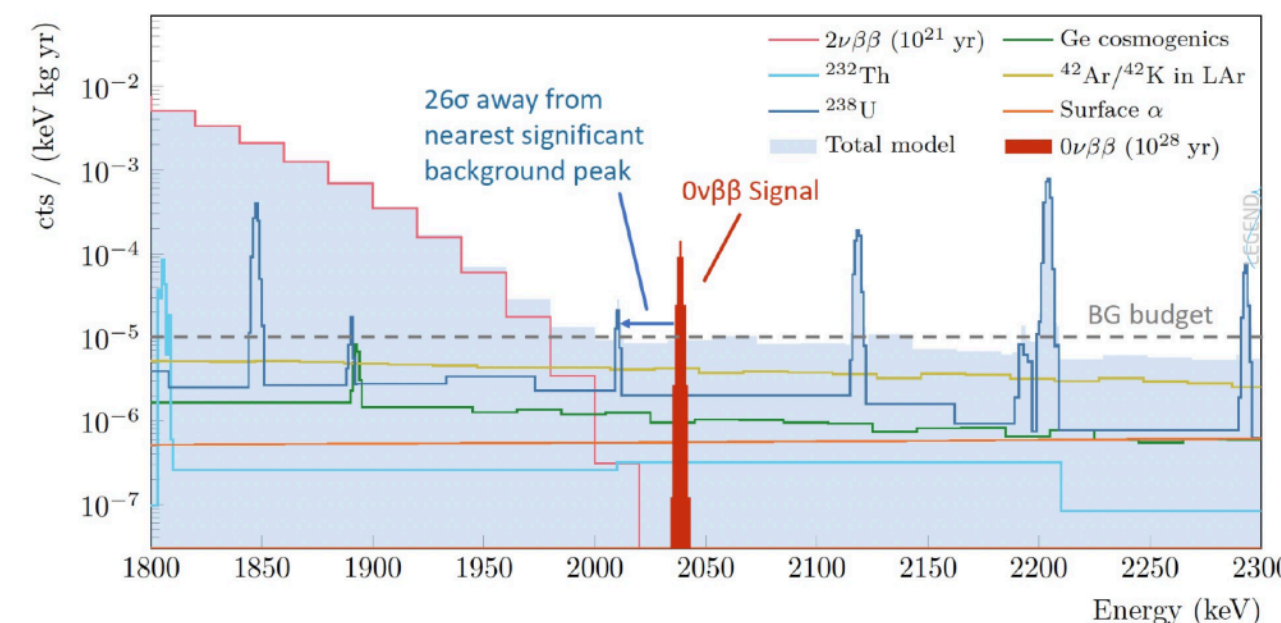


measure sum energy spectrum of electrons

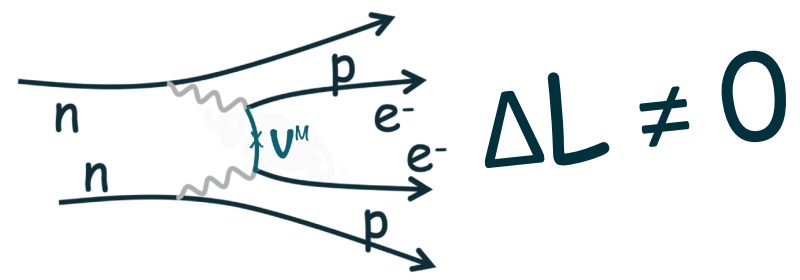
- $2\nu\beta\beta \rightarrow$  continuum
- $0\nu\beta\beta \rightarrow$  mono-energetic peak @  $Q_{\beta\beta}$



But this signal is buried under other backgrounds...



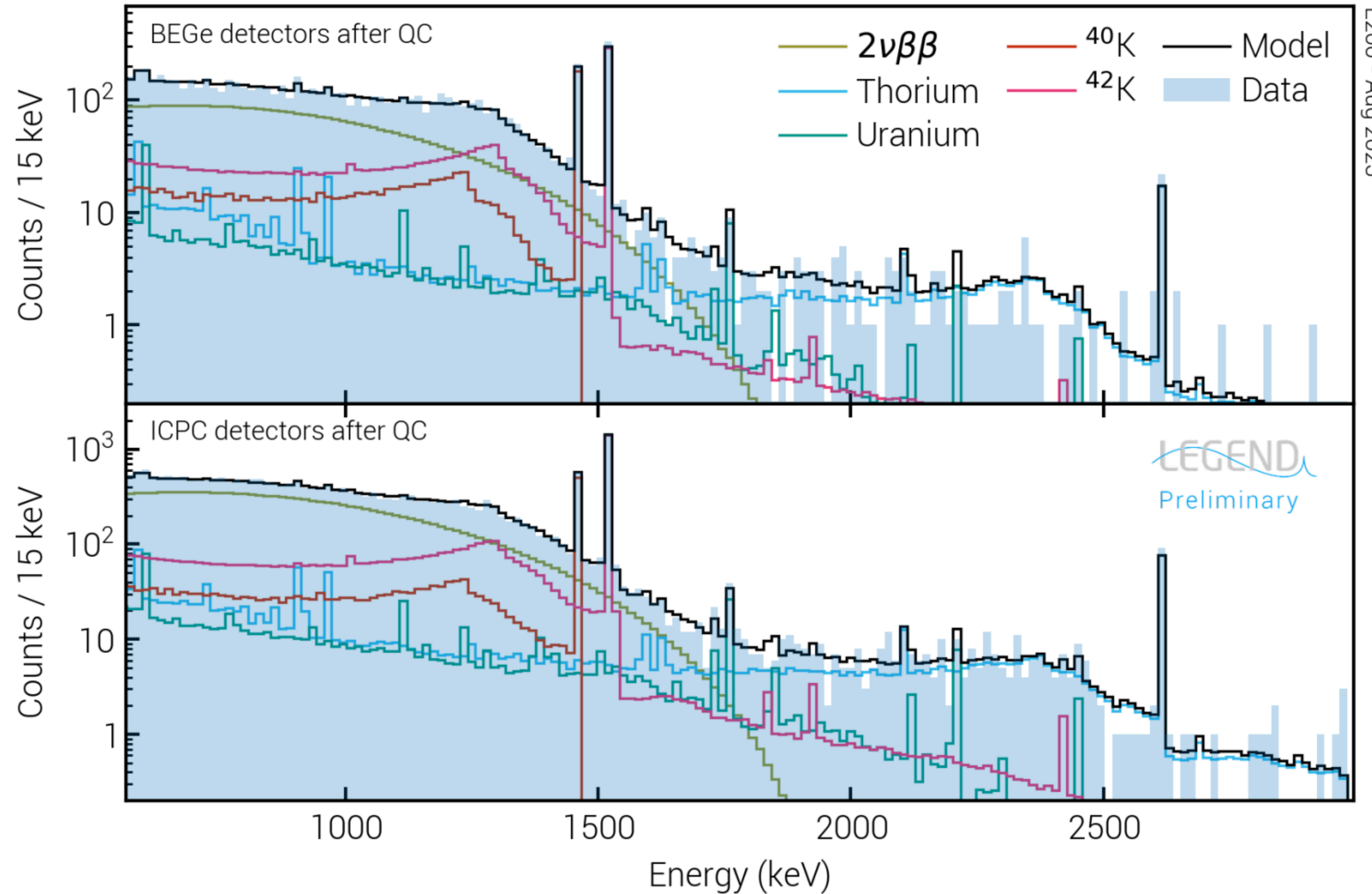
→ increase sensitivity by **background reduction (BI)** at  $Q_{\beta\beta}$  and simultaneous increase of mass ( $M$ ) and improvement of the energy resolution ( $\Delta E$ )



# Background Decomposition

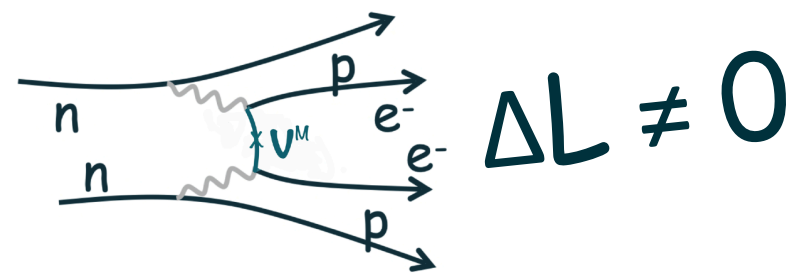
Decomposition before analysis cuts

- Well described by expected contributions with current statistics

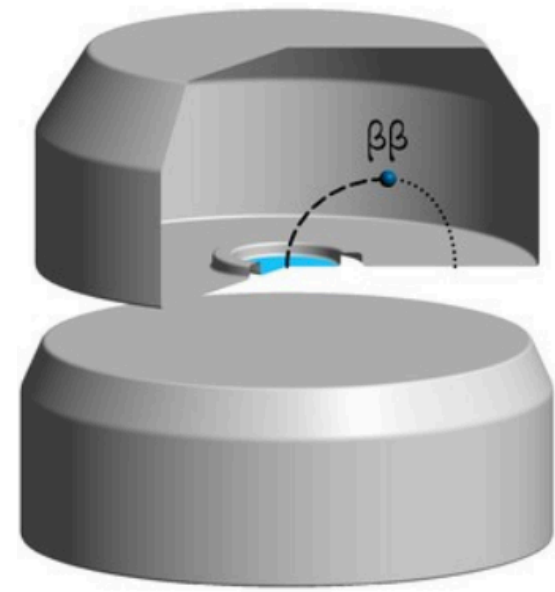


L200 - Aug 2023



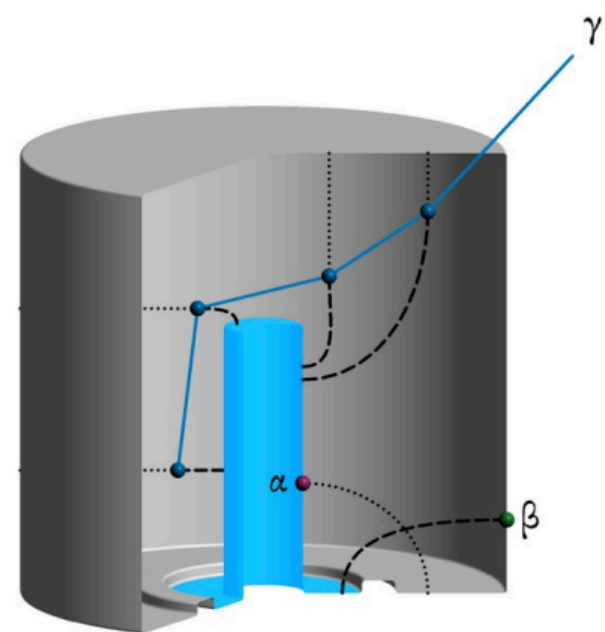
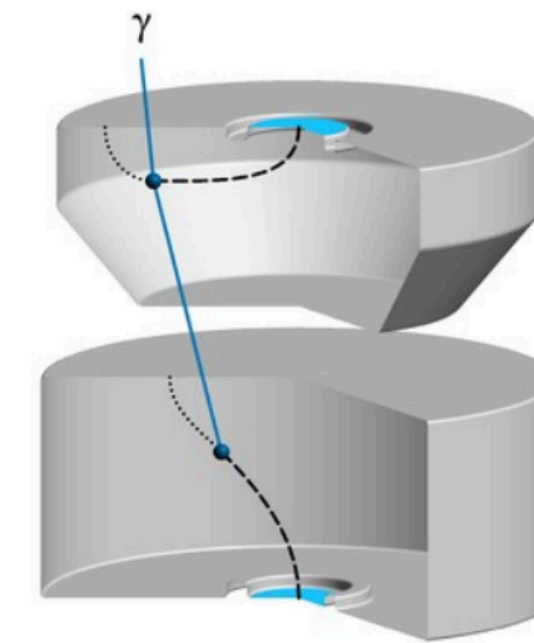


# Background reduction



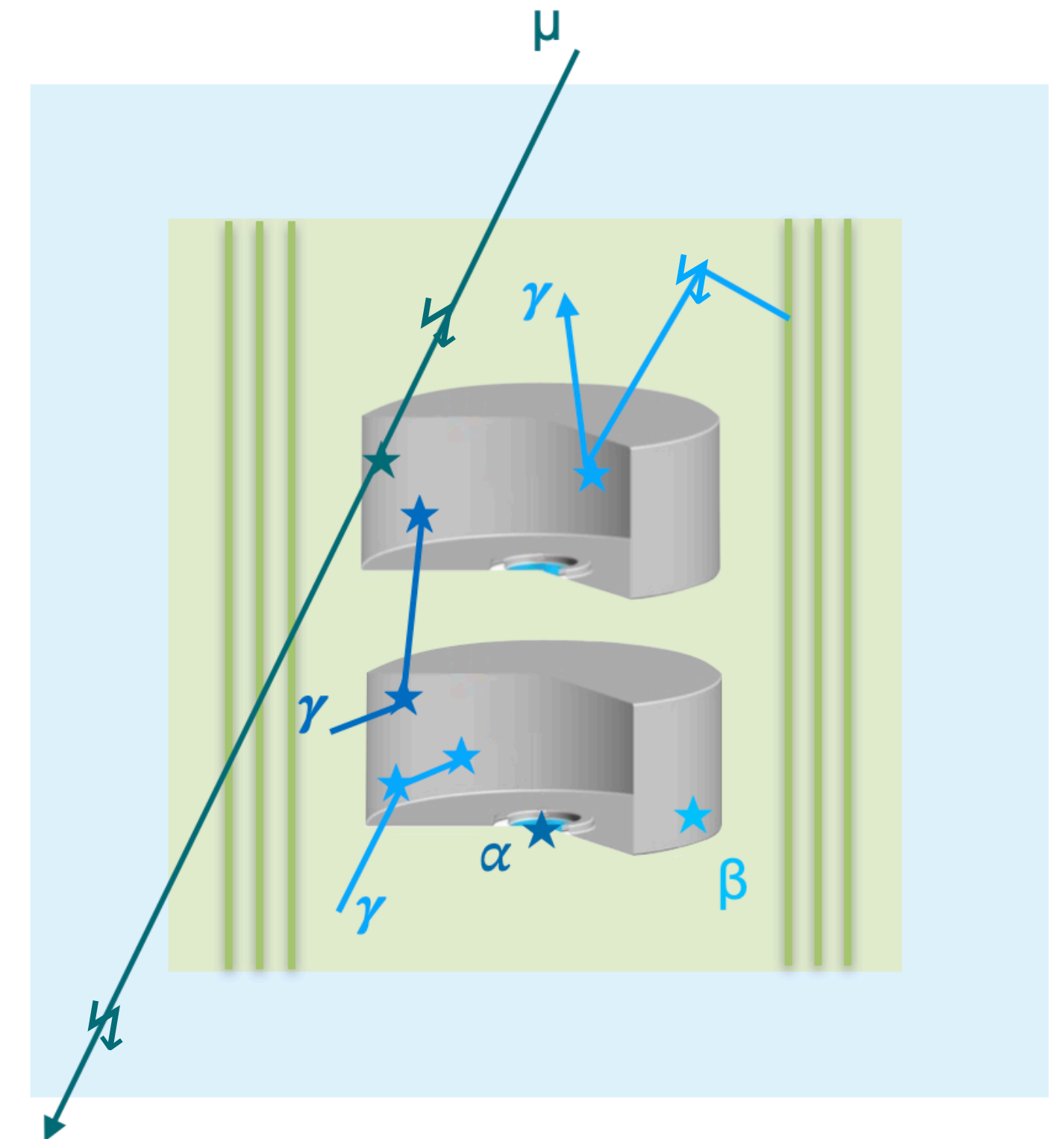
**Signal-like events ( $0\nu\beta\beta$ / $2\nu\beta\beta$  events)**  
local energy deposit in single detector

**(1) Background events ( $\gamma$  events)**  
coincident energy deposition in more than one detector  
→ detector anti-coincidence

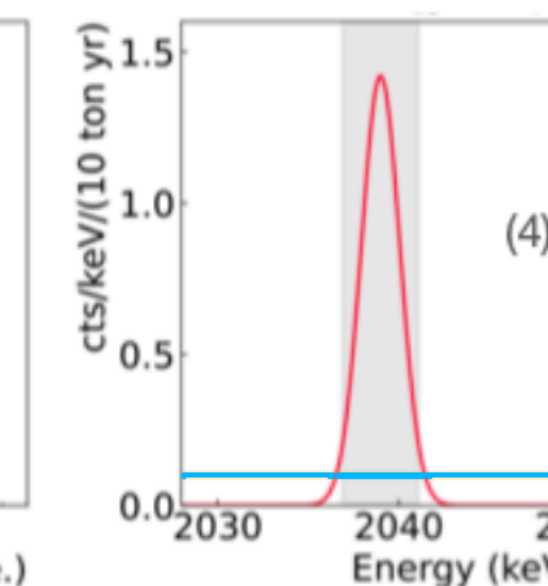
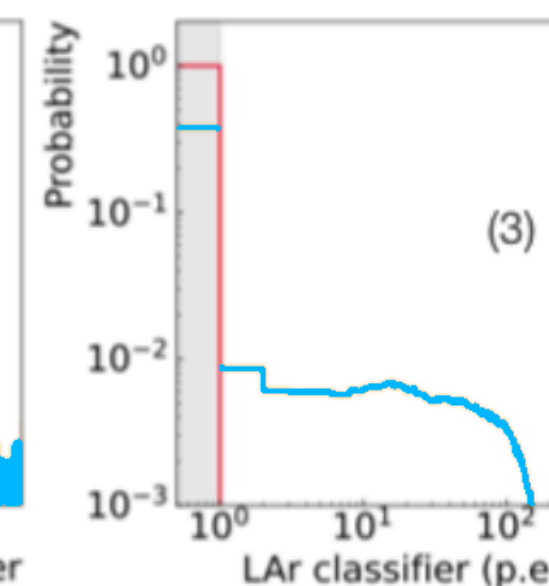
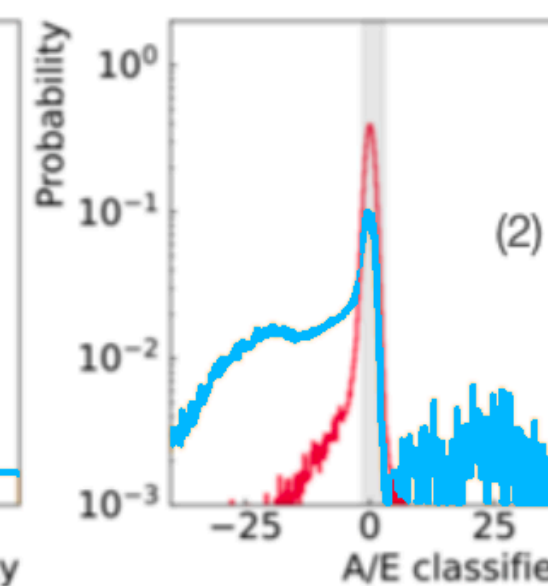
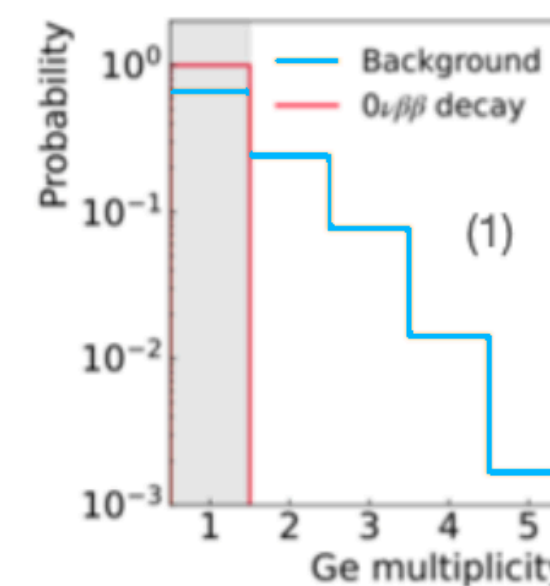
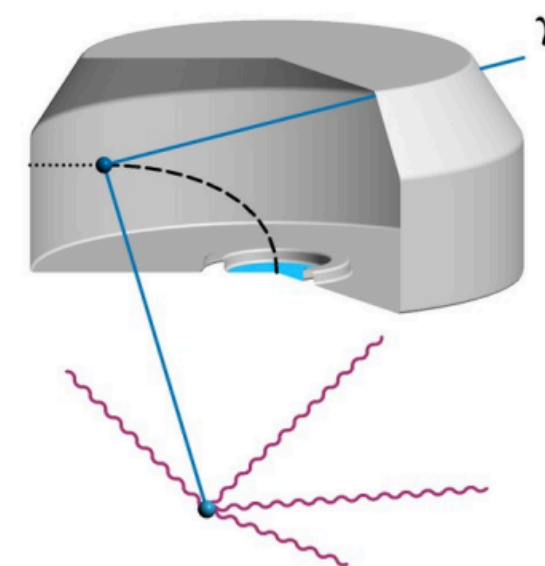


**(2) Background events ( $\gamma$  events)**  
deposition in multiple locations (MSE)  
→ PSD (analysis of time profile of current signal)

**Surface events ( $\alpha/\beta$  events)**  
energy deposited on or close by the detector contacts  
→ PSD (short (p+) or long (n+) current pulse)

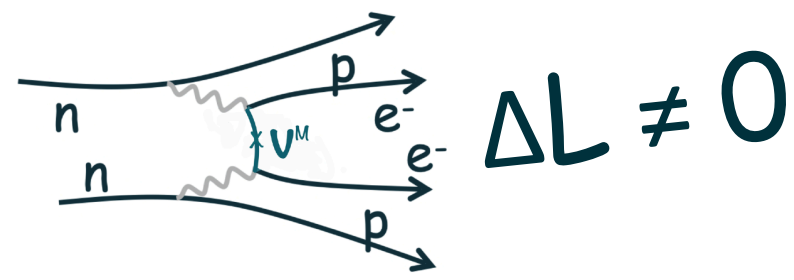


**(3) Background events ( $\gamma$  events)**  
additional energy deposition in LAr  
→ LAr veto

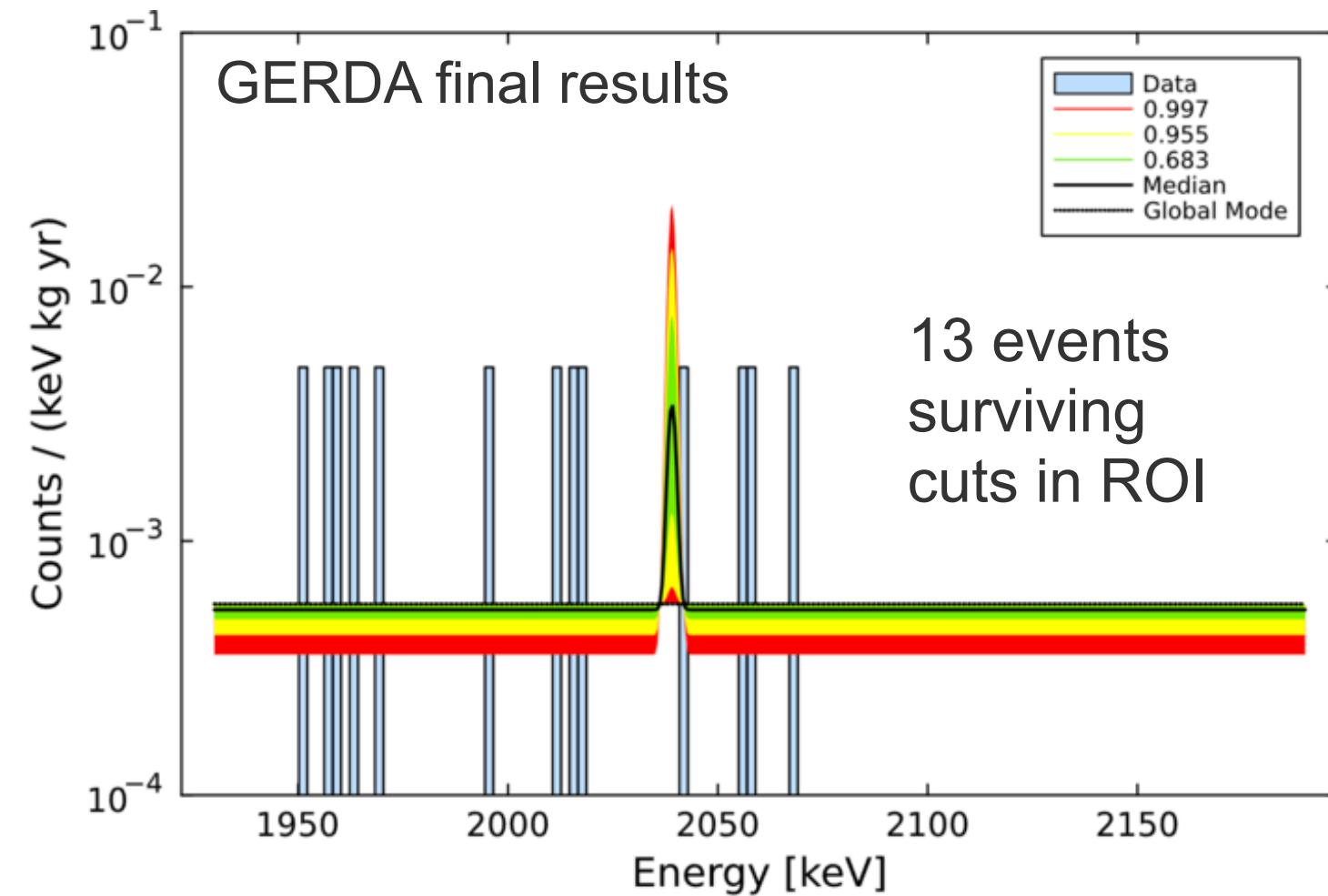


Energy resolution:  
all backgrounds with continuous energy





# Bayesian Inference and UQ for $0\nu\beta\beta$ decay



Example GERDA/ L200 commissioning:  
 < 10 detector datasets (< 20 cts in all datasets)  
 $Q_{\beta\beta}$ ,  $\sigma_i$ ,  $\mu_i^B$ ,  $\mu_i^S$   
 → < 50 nuisance parameters

LEGEND-1000: 10x detector channels  
 ~ 100 measurement campaigns  
 ~ 150 detector datasets  
 $Q_{\beta\beta}$ ,  $\sigma_i$ ,  $\mu_i^B$ ,  $\mu_i^S$   
 →  $10^5$ - $10^6$  nuisance parameters

The total likelihood is constructed as the product of all  $\mathcal{L}_i$  weighted with the Poisson term:

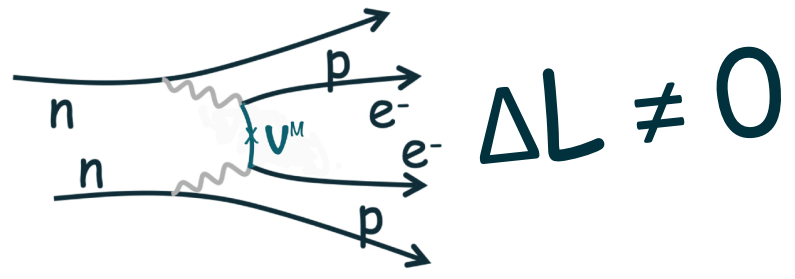
$$\mathcal{L} = \prod_k \left[ \frac{(\mu_{s,k} + \mu_{b,k})^{N_k} e^{-(\mu_{s,k} + \mu_{b,k})}}{N_k!} \right] \times \prod_{i=1}^{N_k} \frac{1}{\mu_{s,k} + \mu_{b,k}} \times \left( \frac{\mu_{b,k}}{\Delta E} + \frac{\mu_{s,k}}{\sqrt{2\pi}\sigma_k} e^{-\frac{(E_i - Q_{\beta\beta})^2}{2\sigma_k^2}} \right)$$

Annotations in the diagram:  
 - Poisson weight: points to the first term in the product.  
 - Flat background: points to the  $\frac{1}{\mu_{s,k} + \mu_{b,k}}$  term.  
 - Gaussian signal: points to the term in parentheses.  
 - Expected counts: points to the  $\mu_{s,k} + \mu_{b,k}$  term in the denominator of the second product.

where:

- $N_k$  total number of events observed in the  $i$ th partition
- $E_i$  individual event energies in the  $i$ th partition
- $\sigma_k = FWHM_i / (2\sqrt{2\ln 2})$ : energy resolution in ROI
  - the average FWHM across partitions is 3.29 keV

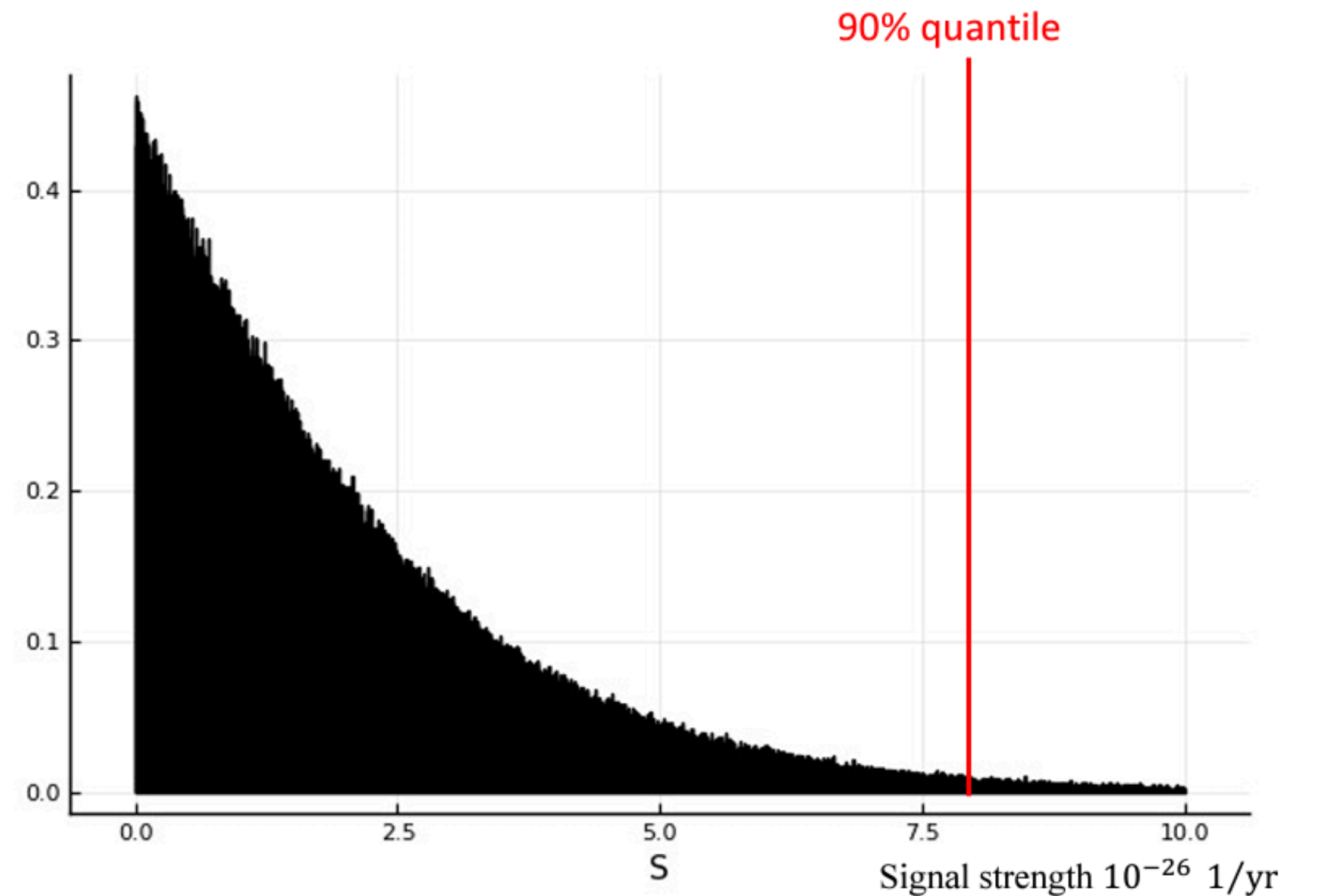




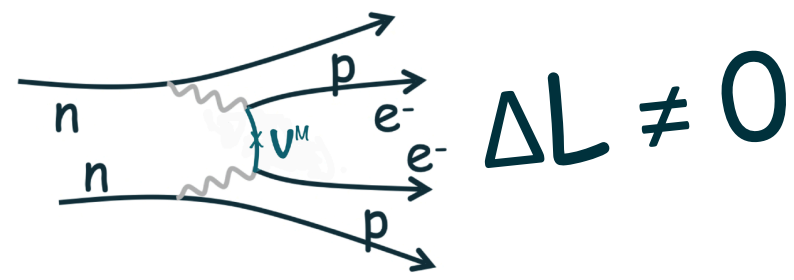
# Models for the Signal strength

There are 2 different priors on the signal strength  $S = \frac{1}{T_{1/2}^{0\nu}}$  (which ranges from 0 to  $10^{-24}$  1/yr):

- $p(S) \sim \text{Uniform}$ 
  - equiprobable signal strengths
- $p(S) \sim \frac{1}{\sqrt{S}}$ 
  - equiprobable Majorana neutrino masses  $m_{\beta\beta}$
  - $S \propto m_{\beta\beta}^2$



Get limit at 90% C. I. from posterior distribution of  $S$

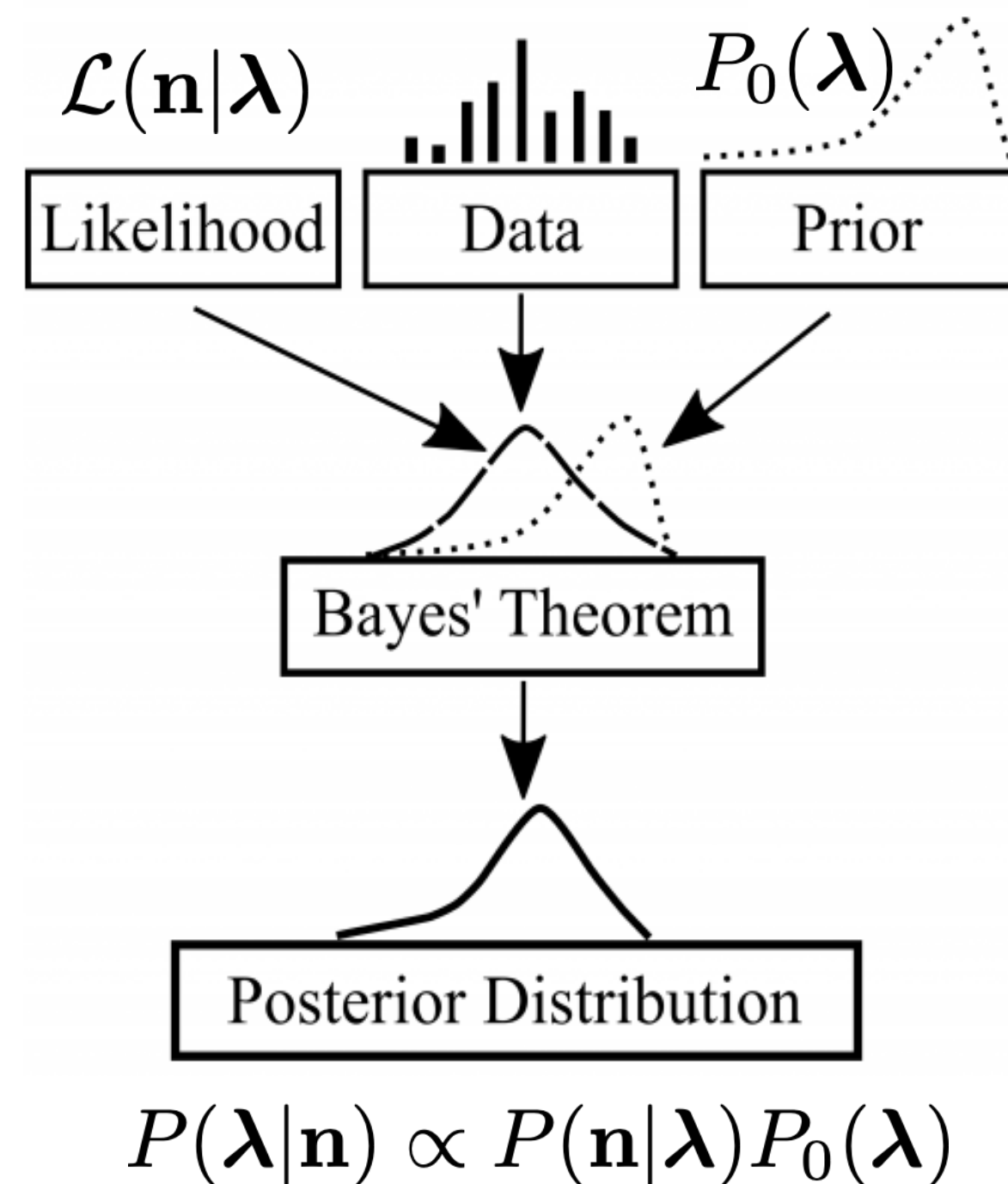


# Limitation MCMC sampling

## Limitation: High dimensional parameter space:

- Metropolis-Hastings sampling with random walk → expensive
- gradient-based sampling like Hamiltonian Monte Carlo → slow for high dimensional correlations

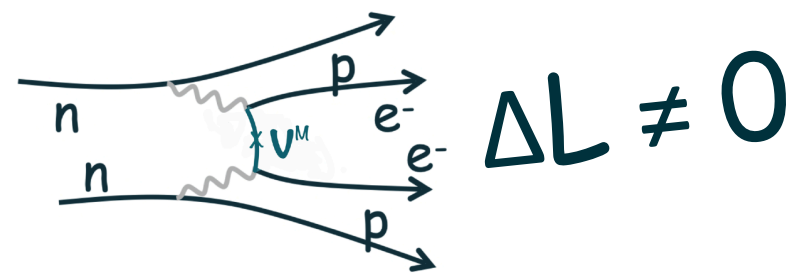
## How can we address MCMC sampling limitations?



### Ideas:

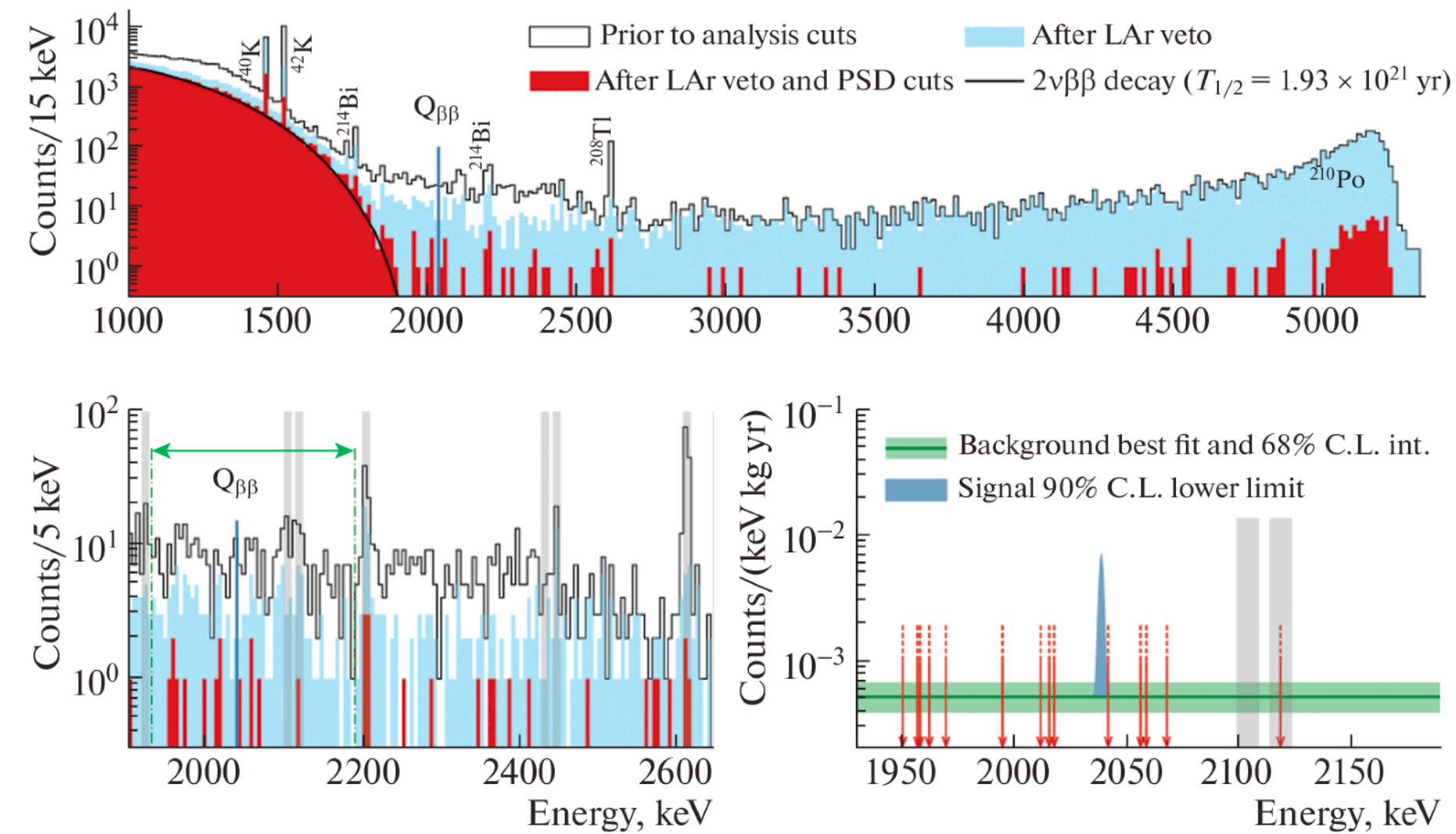
- GP surrogates for Bayesian inference [arXiv:1809.10784] → physics posteriors densities mostly non-gaussian with tails
- Variational Bayesian Monte Carlo with Noisy Likelihoods
- Hamiltonian Monte Carlo
- Langevin Monte Carlo with modifications
  - LMC (w/o MH adjusted)
  - Random coordinate descent LMC → Is the gradient less expensive to calculate?
  - Deterministic LMC with Normalizing Flow [arXiv:2205.14240] (proposed by Uroš Seljak)
  - LMC underdamped
- NN which learns the conditional probability (posterior) [arXiv:2006.02369] → potential large uncertainty?
- Sampling with support points [arXiv:1609.01811]
- Dimension reduction by embedded (sub-)structures/ correlations



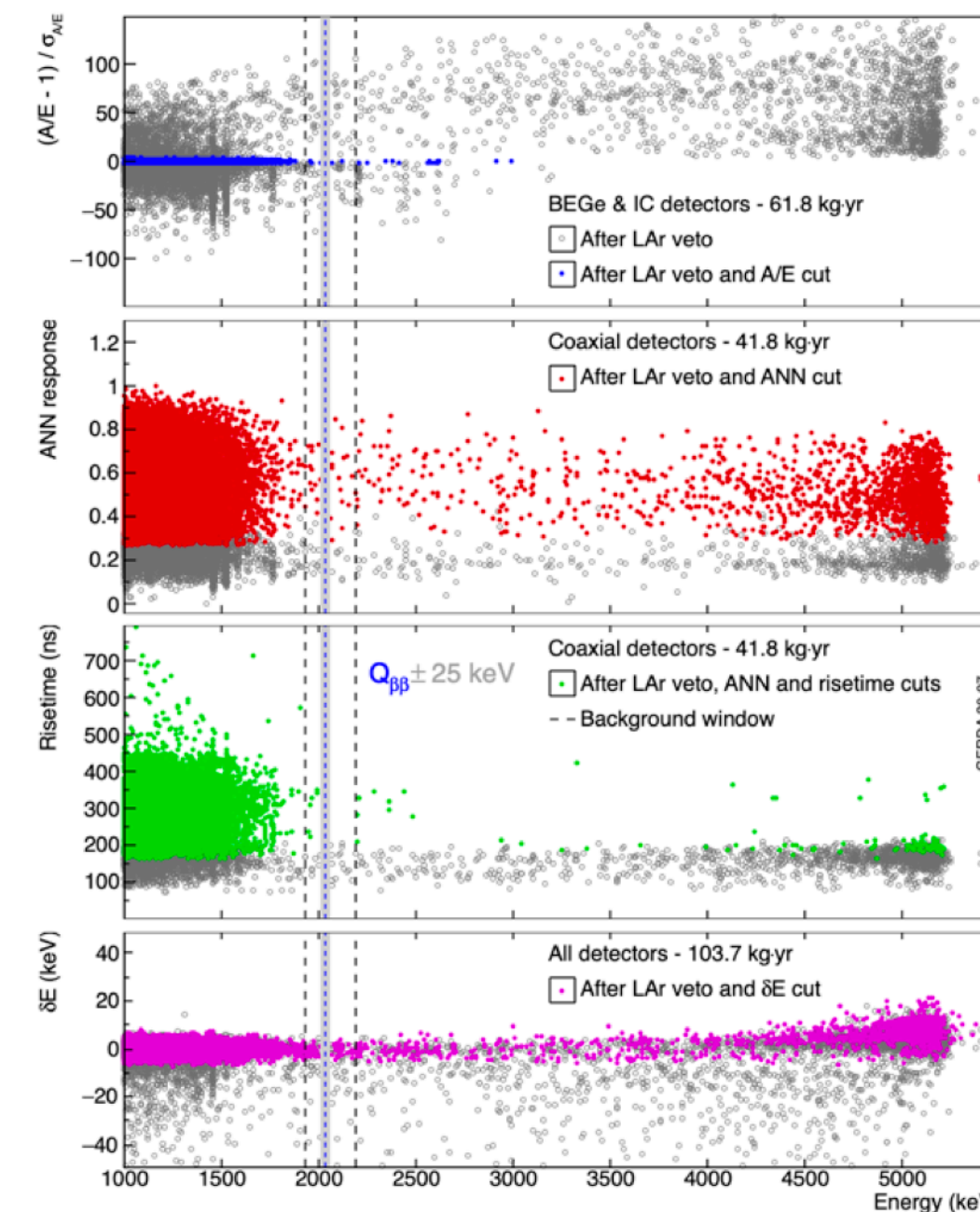


# Extended Bayesian Inference and UQ for $0\nu\beta\beta$

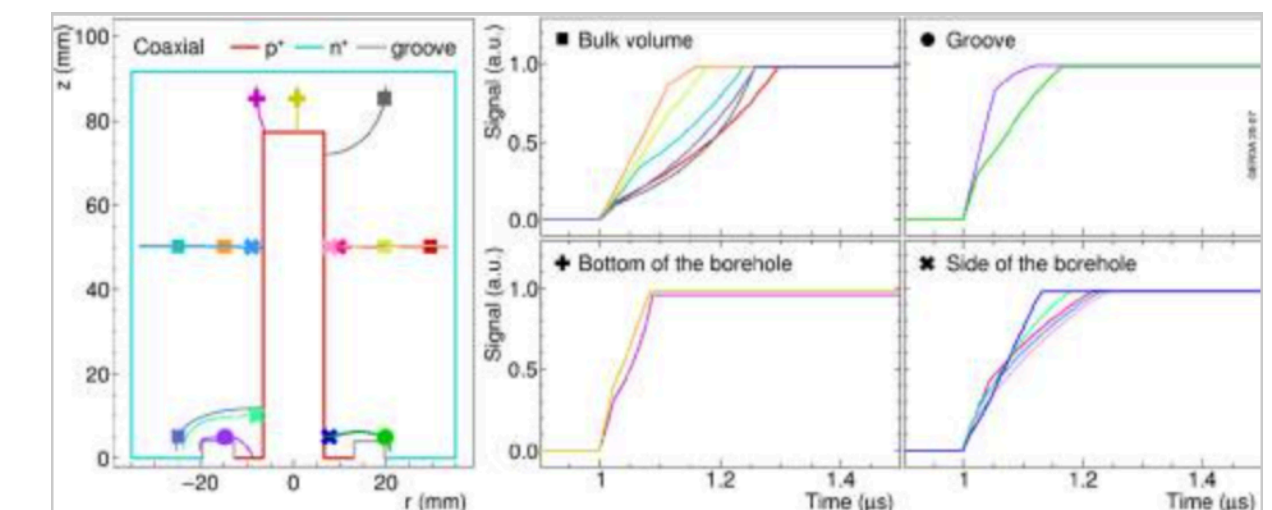
So far, background rejection based on single pulse shape/ veto parameters prior to spectral fit



## Correlations are everywhere!



**Pulse shape analysis:** analysis of the time profile of individual pulses used to reject different backgrounds by a single parameter

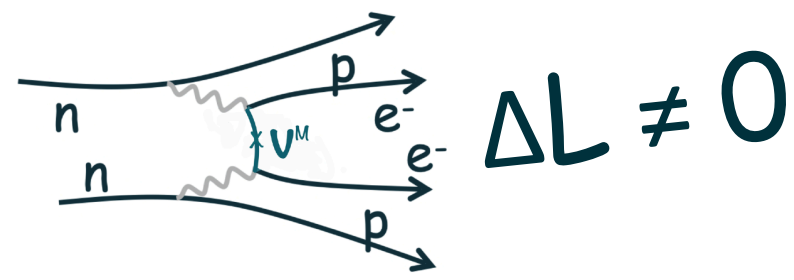


**Goal:** model which incorporates additional informations into the likelihood (e.j. PSD, veto signals,...) indicating the background probability of an event

### Questions:

- How to integrate PSD simultaneously for all type of pulse shapes?
- How would the mathematical formulation of the fit look like?

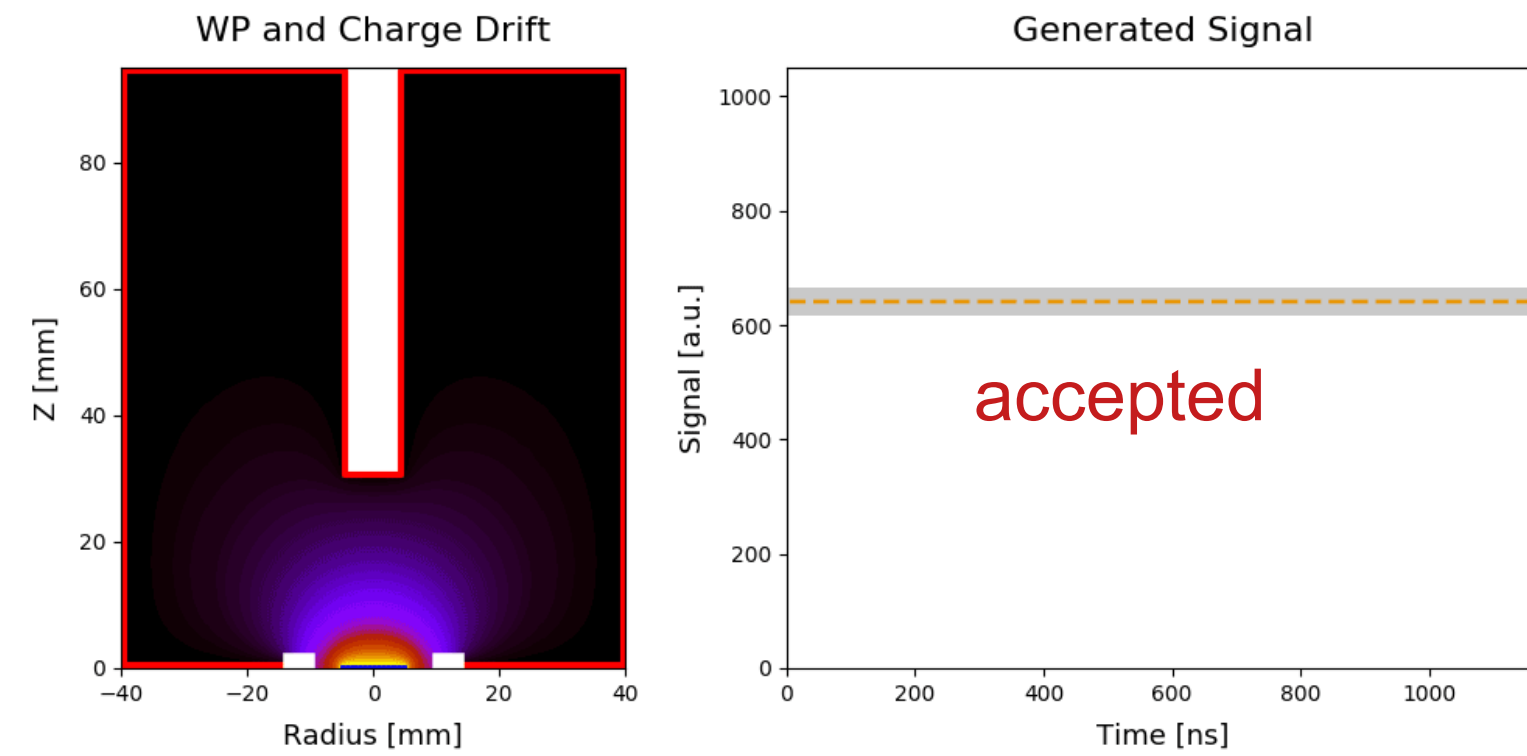




# Background rejection in point contact HPGe

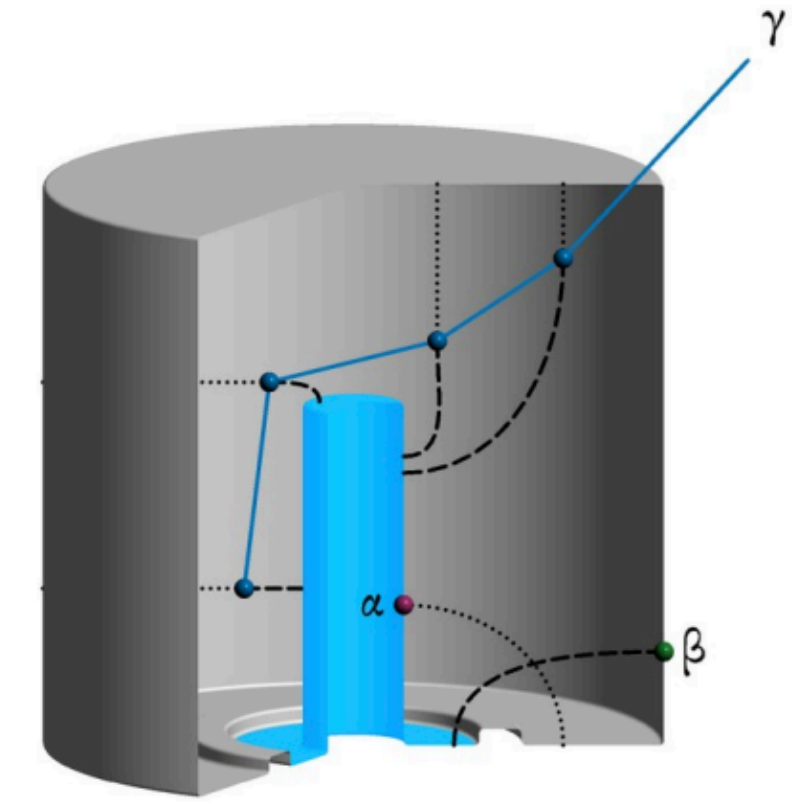
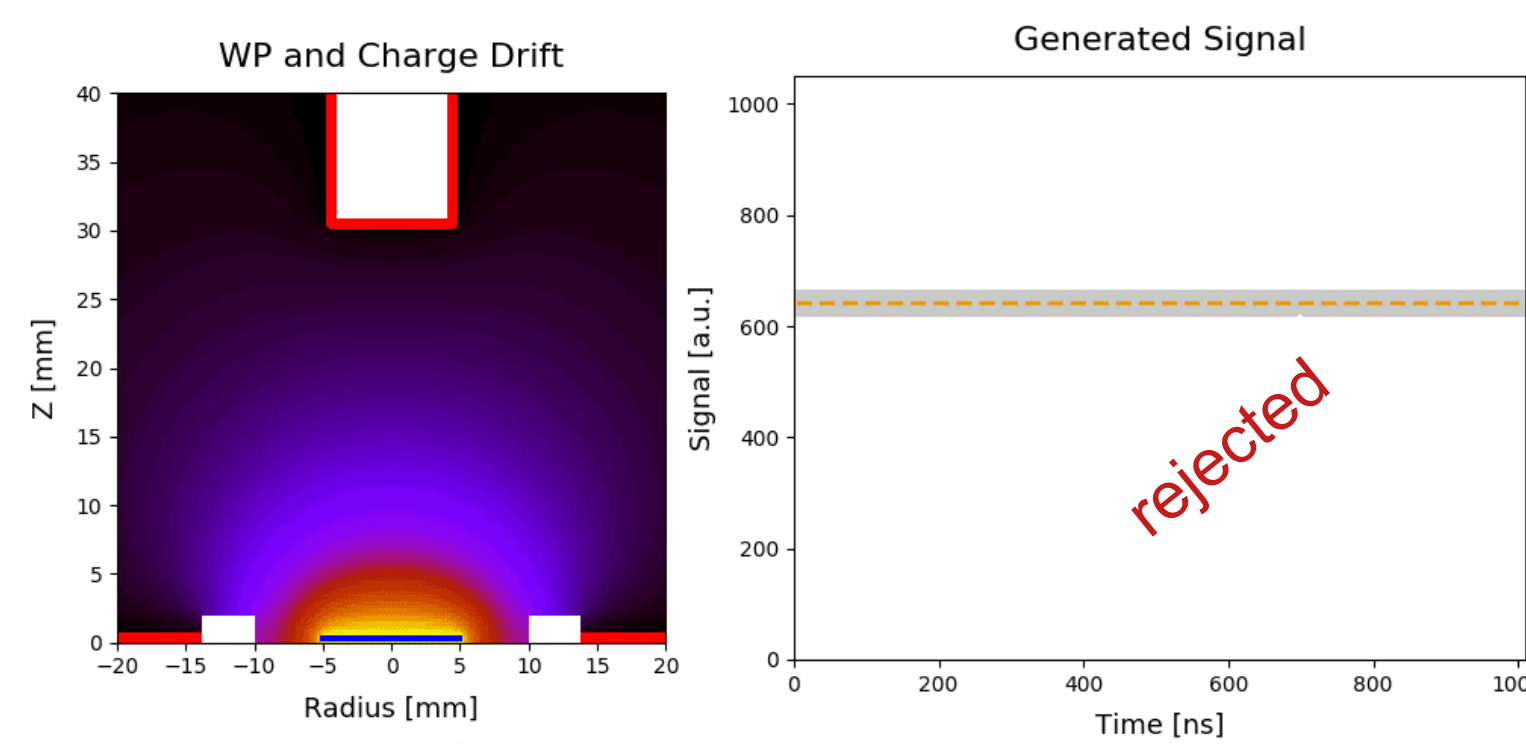
Weighting Potential and Charge Drift

$0\nu\beta\beta$  signal candidate (single-site)



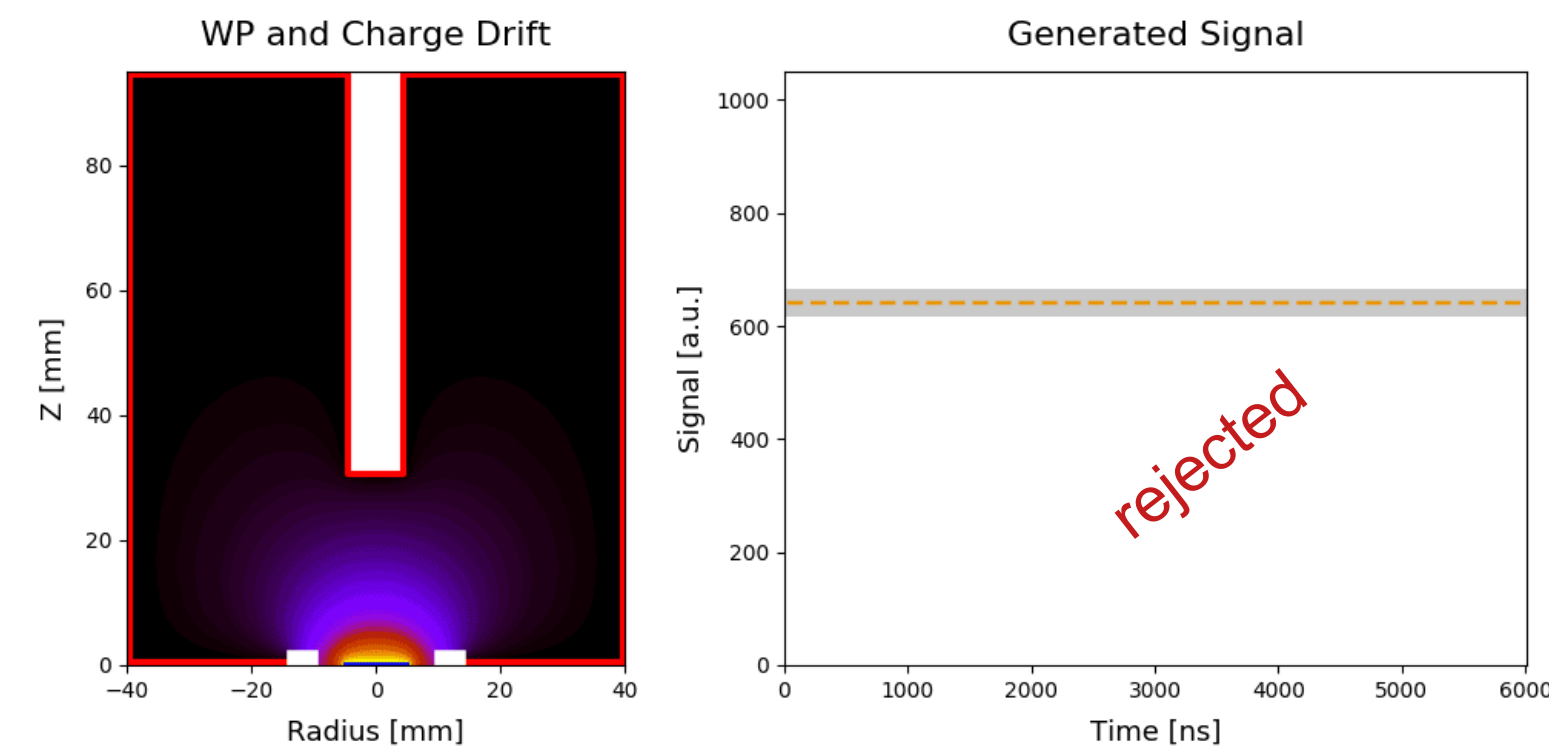
- Charge signal    - Current signal

$\alpha$ -background on p+ contact

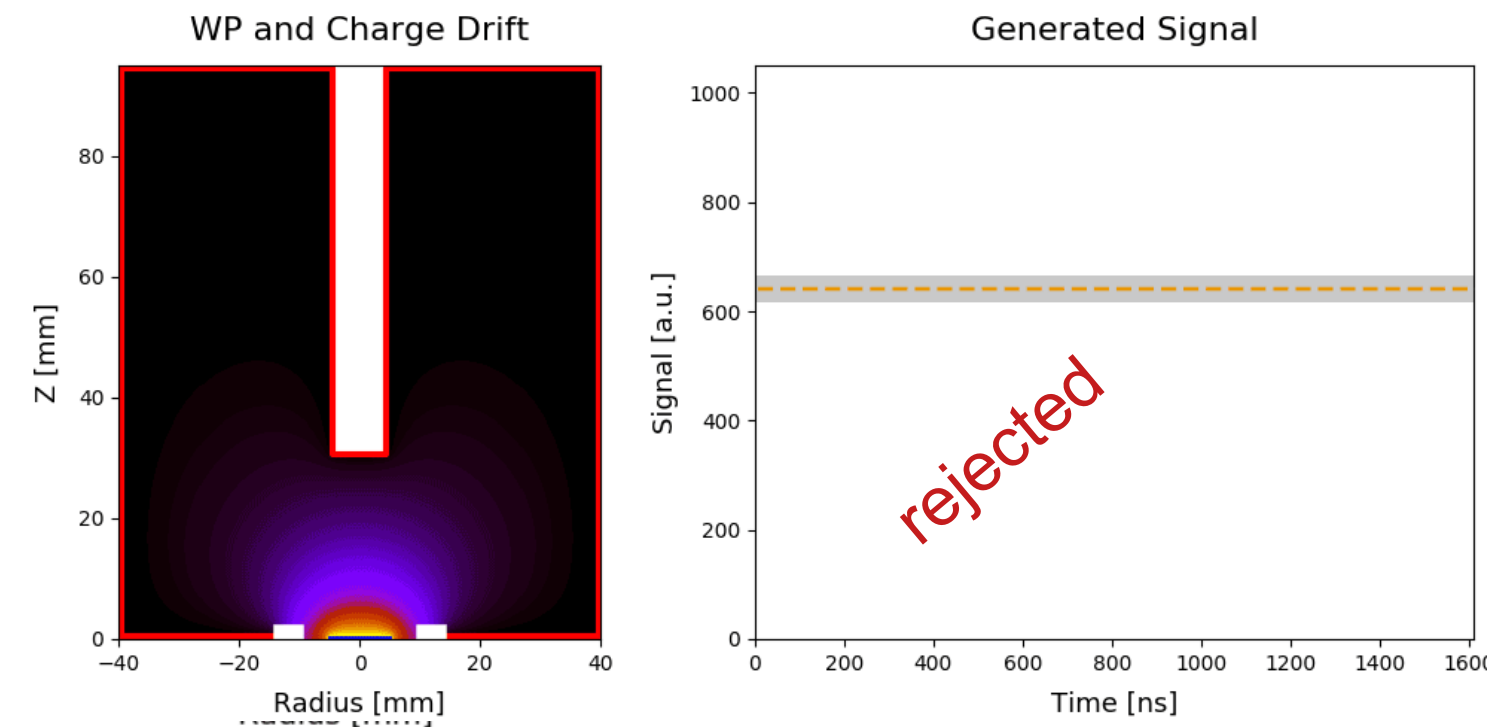


- amplitude of current pulse is suppressed for a **multi-site event** compared to a **single-site event** of the same event Energy

Surface- $\beta$ -background  $^{42}\text{K}$  ( $^{42}\text{Ar}$ ) on n+ contact

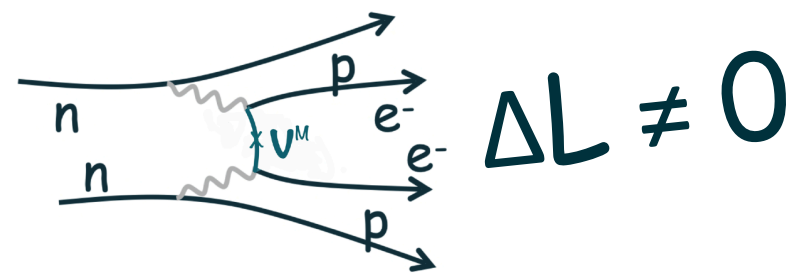


$\gamma$ -background (multi-site)



- comparing **A against E** effectively rejects multi-site backgrounds
- various powerful PSA event topology tools can be used to reject different backgrounds
- alternative machine learning algorithms are available





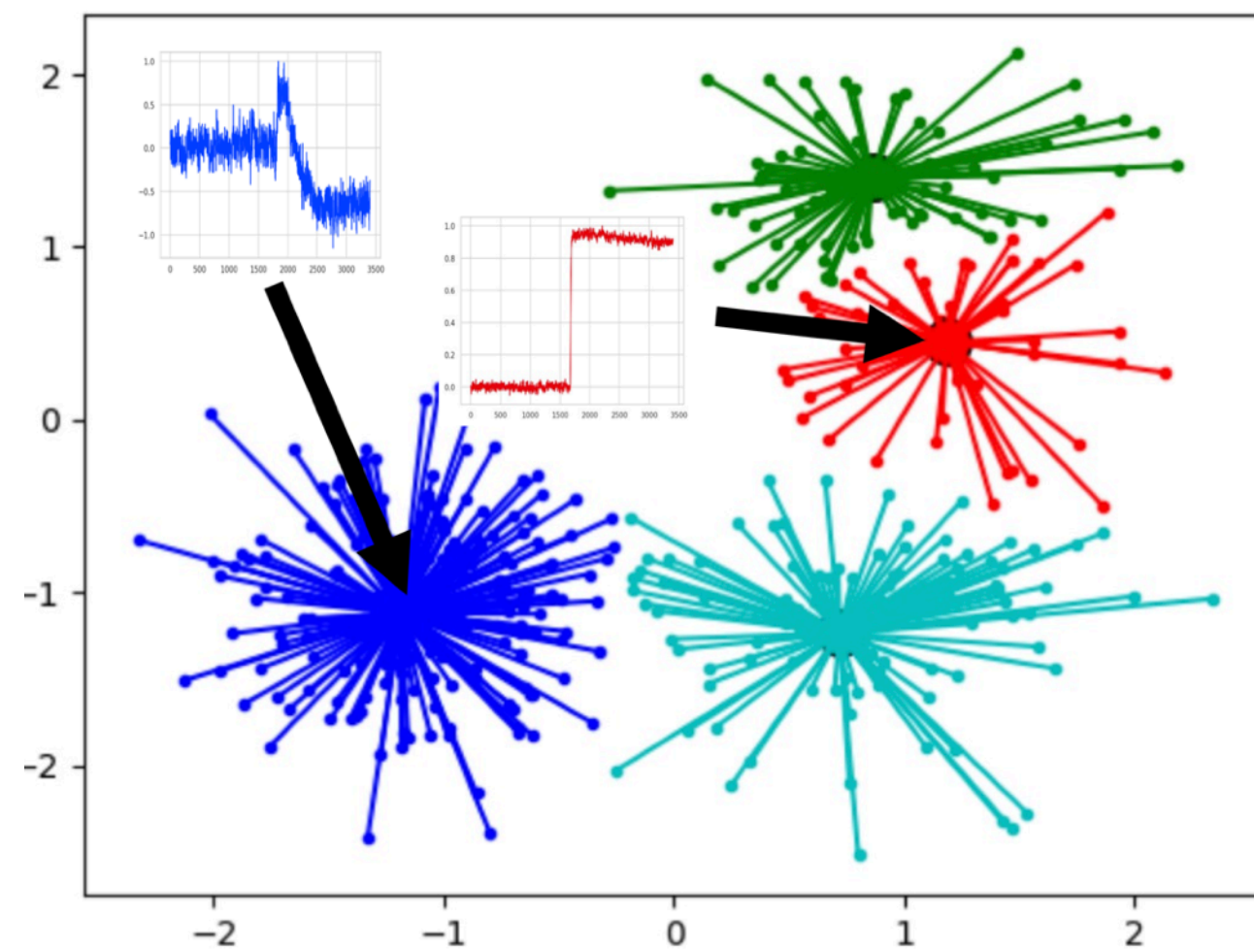
# PSD Extended Bayesian Inference $0\nu\beta\beta$ decay

## Can we integrate PSD and veto signals?

Find a tool analogue to LEGEND ML Data Cleaning using actual physics waveforms with  
 → probability of waveform classification to the categories shown before  
 + uncertainty estimation



## waveform of a physical event

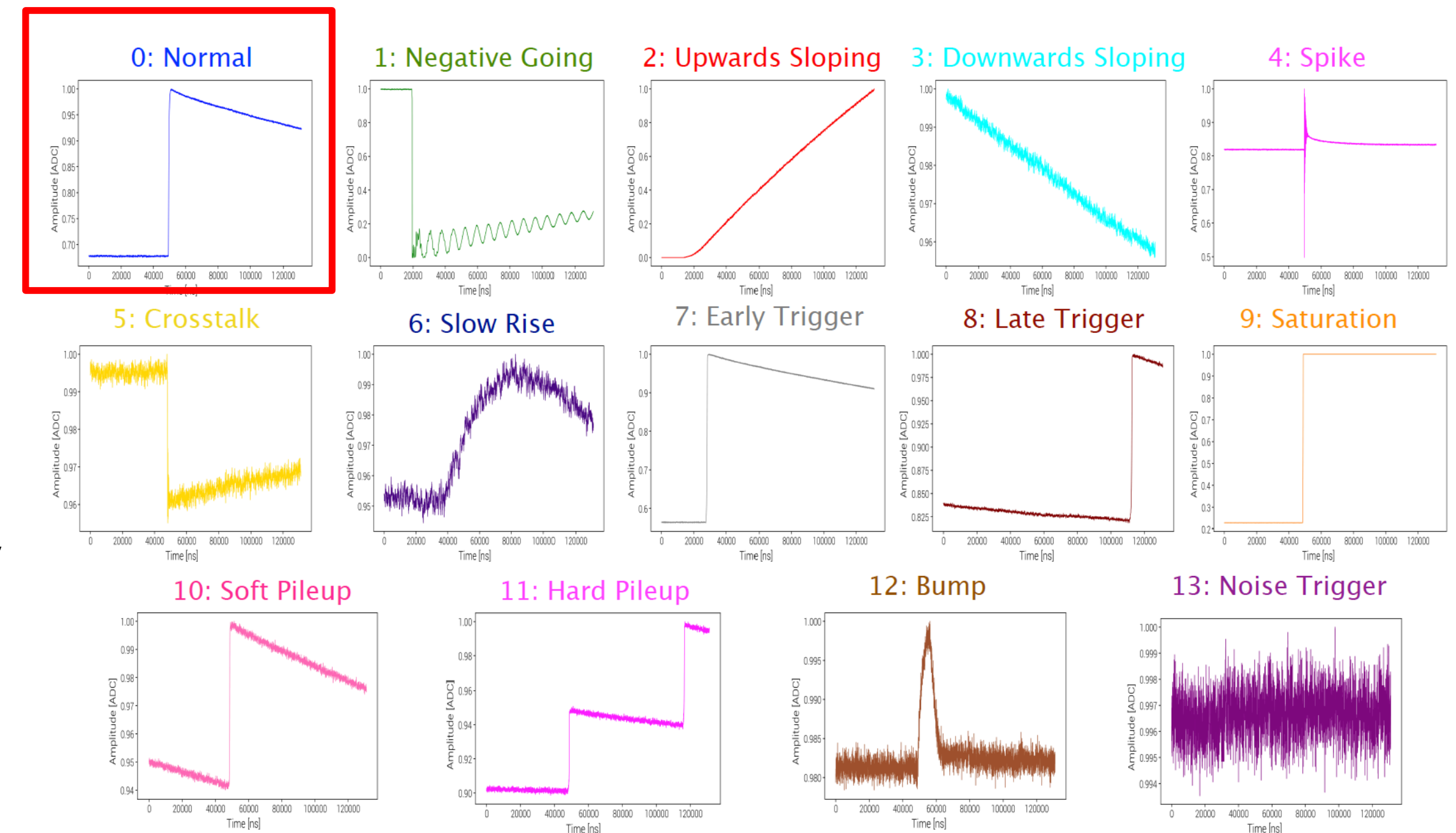


### Advantages of ML Data cleaning method over traditional analysis:

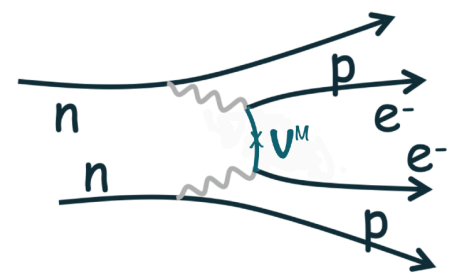
- Adapts to changing run conditions
- Allows ID of new populations during commissioning
- Could improve separability by using more waveform information

### Disadvantages

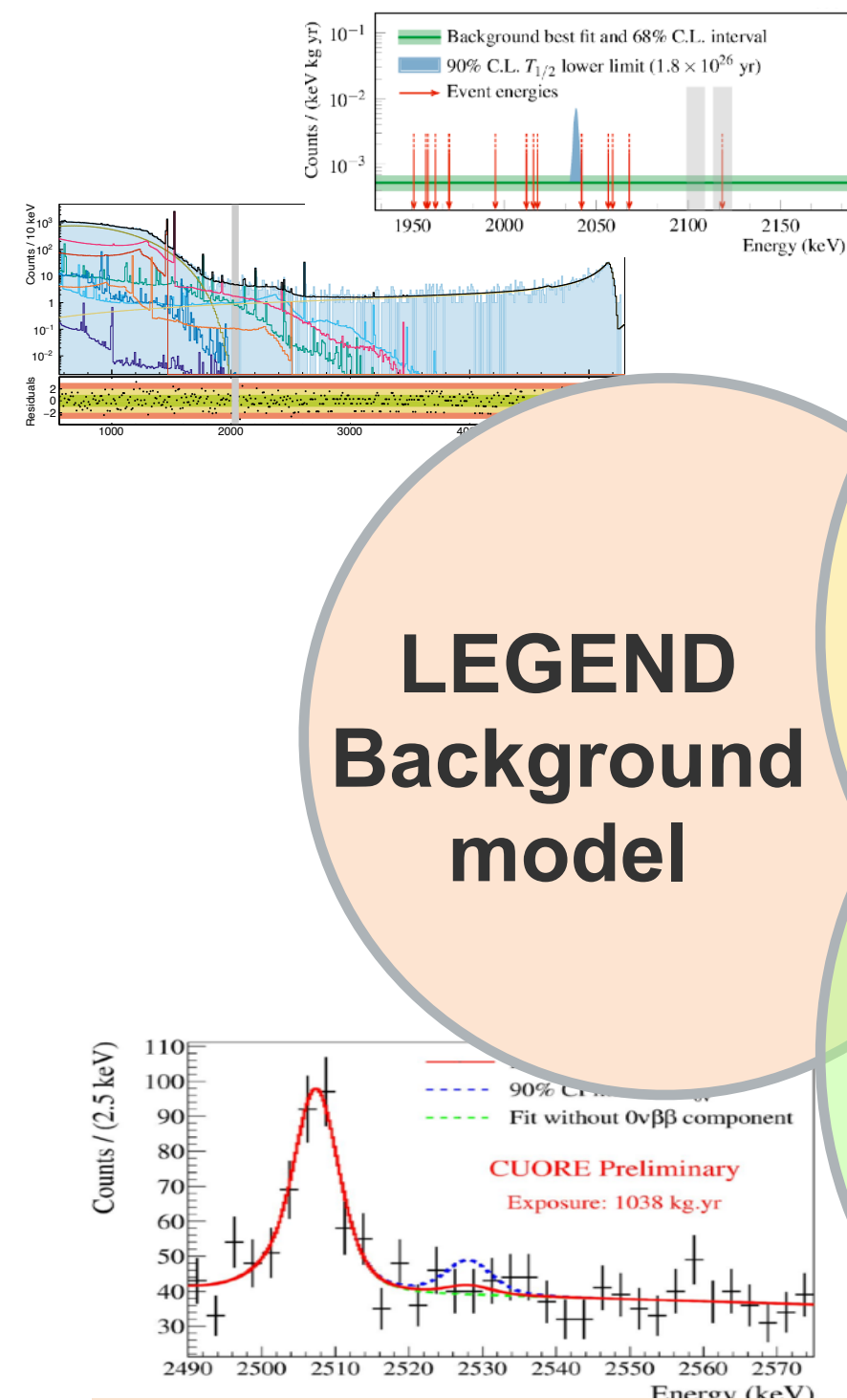
- no uncertainty estimation
- Classification 0 or 1 for class identity



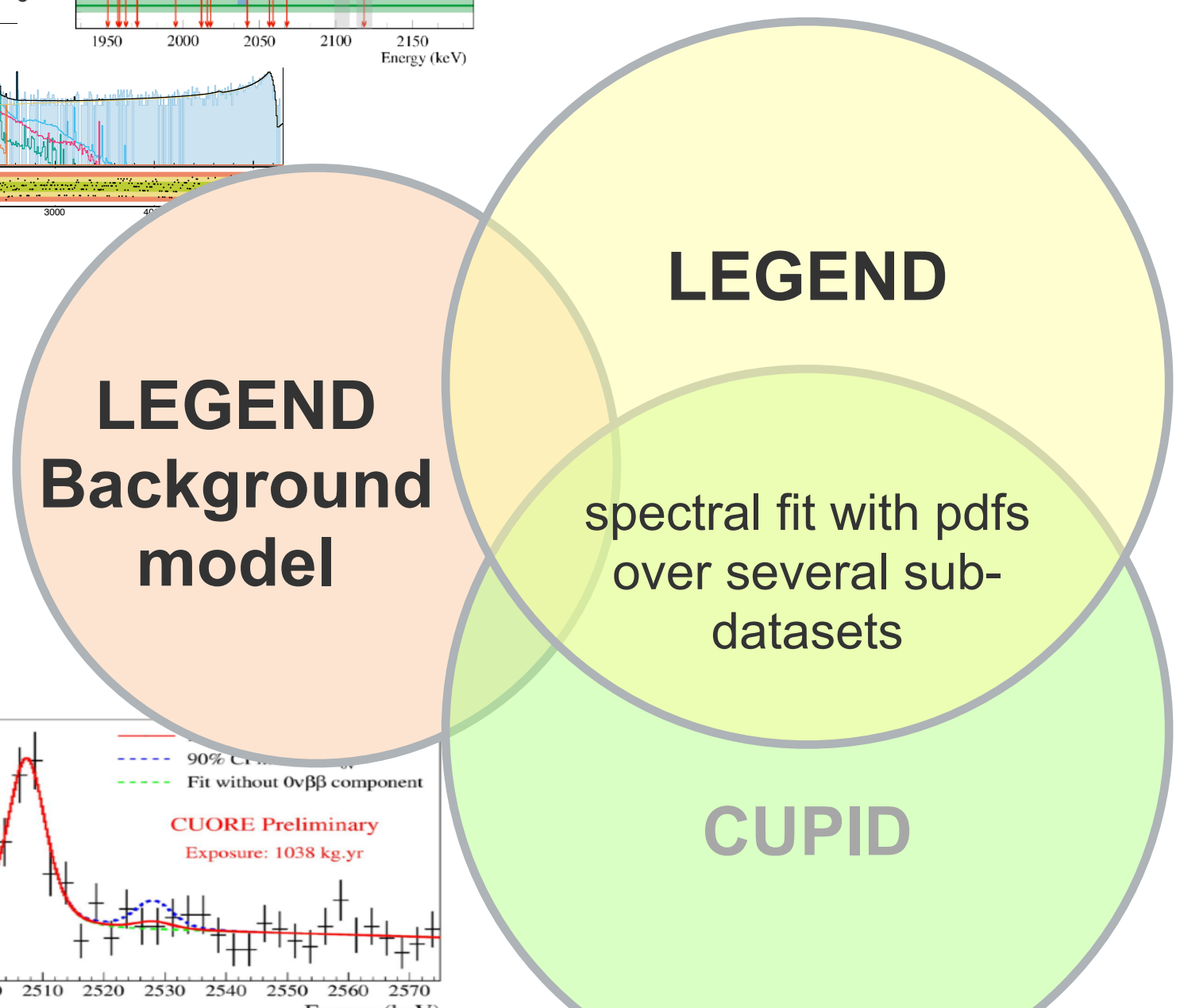




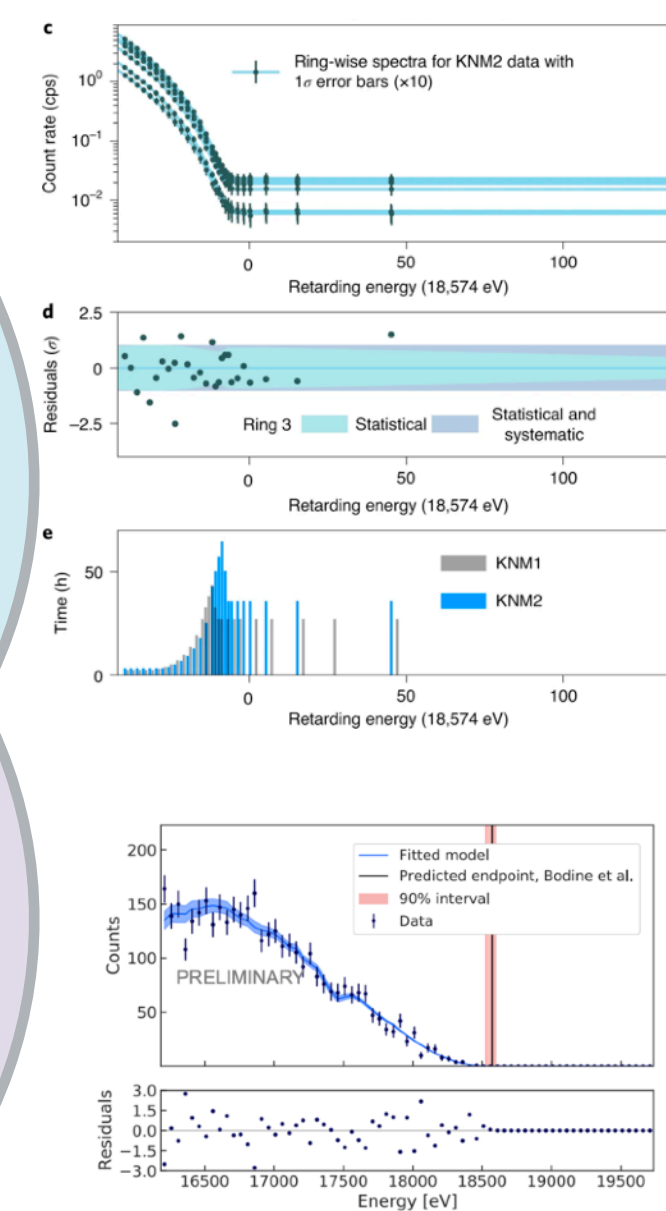
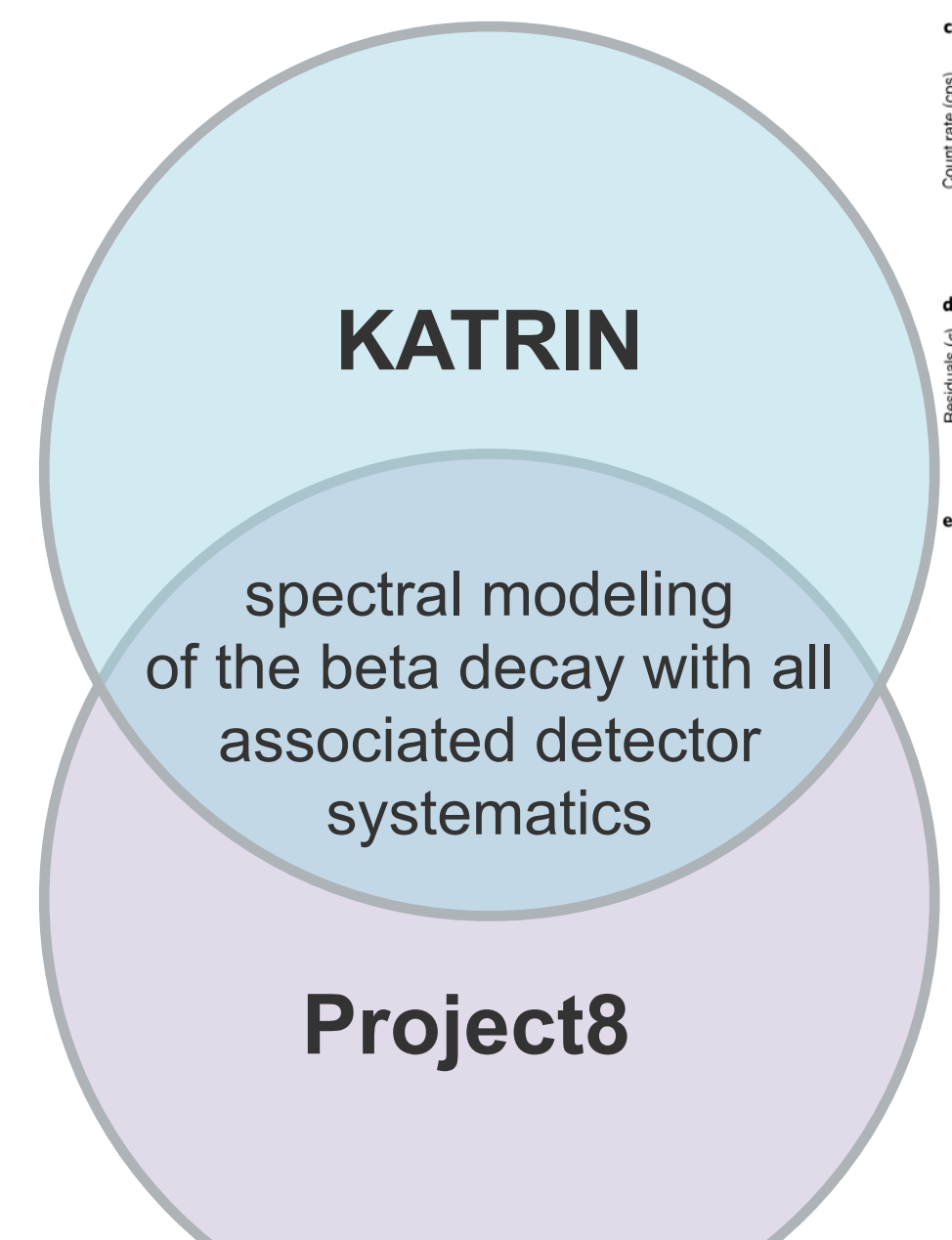
# $\Delta\mathcal{L} \neq 0$ Bayesian parameter estimation and uncertainty quantification in Nuclear Science with focus on (Double) Beta decay



0 $\nu\beta\beta$  decay



Beta decay - Neutrino mass



**Bottlenecks:**

- Increase of sub-datasets
- Marginalization over  $O(10^6)$  nuisance parameters describing detector systematics and bkg

**Need:**

- More performant parameter estimation tools and analysis strategies that will handle large datasets
- fit which incorporates additional informations into the likelihood (psd, veto signals,...)

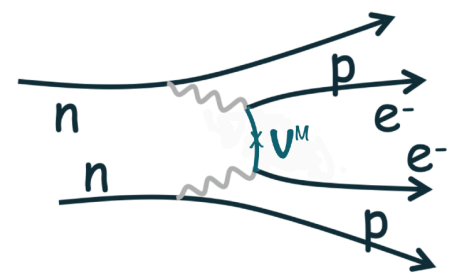
**Bottlenecks:**

- increasingly high dimensional data due to large number of detector systematics
- "Long" calculation time of detector response time

**Need:**

- differentiable model
- improved sampling tools e.j. gradient-based sampling
- multi-fidelity ML techniques

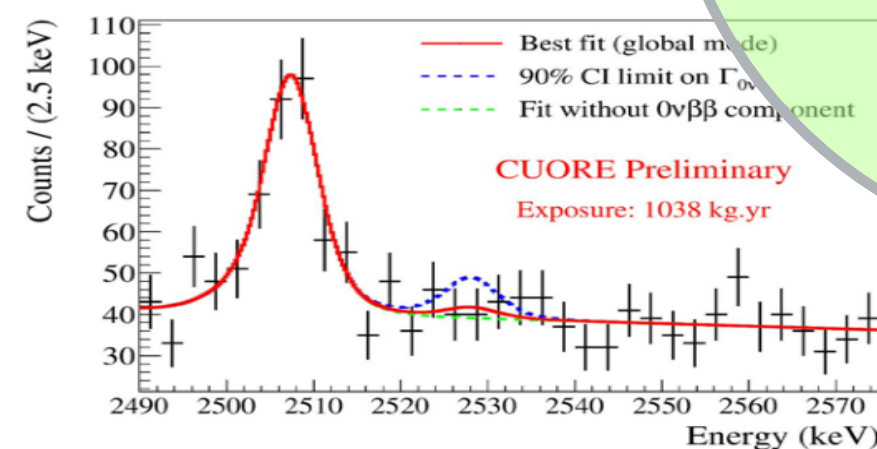
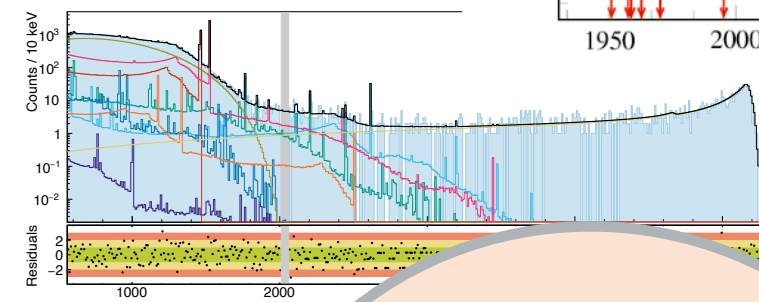
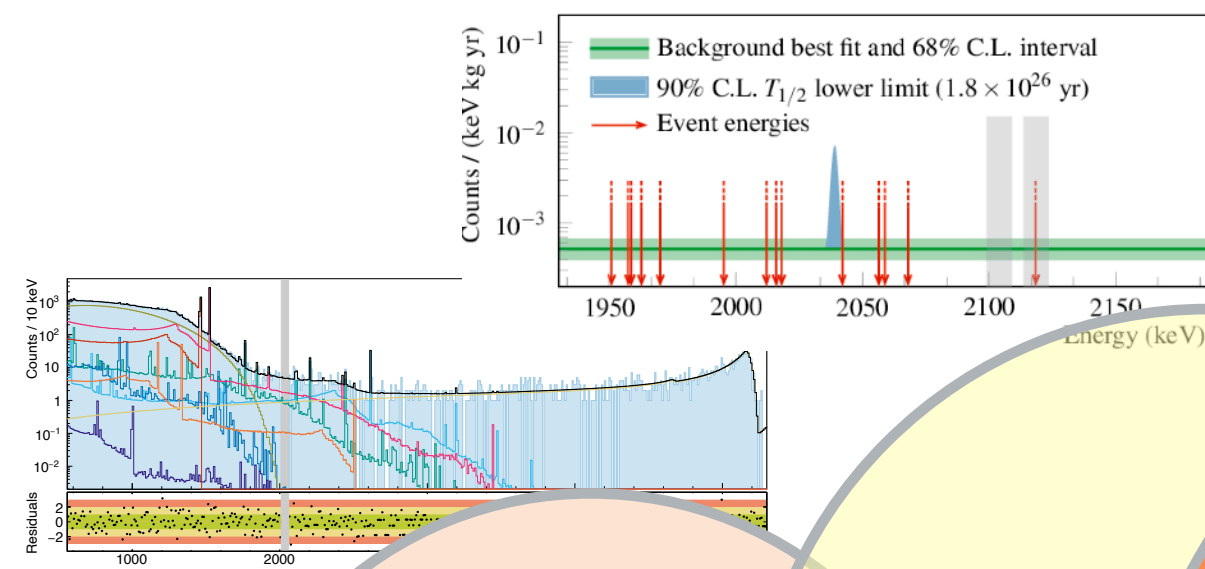




# $\Delta L \neq 0$ Bayesian parameter estimation and uncertainty quantification in Nuclear Science with focus on (Double) Beta decay

## Neutrinoless double beta decay

## Beta decay - Neutrino mass



**LEGEND**  
Background model

**LEGEND**  
spectra over several datasets

**CUORE**

**Bottlenecks:**

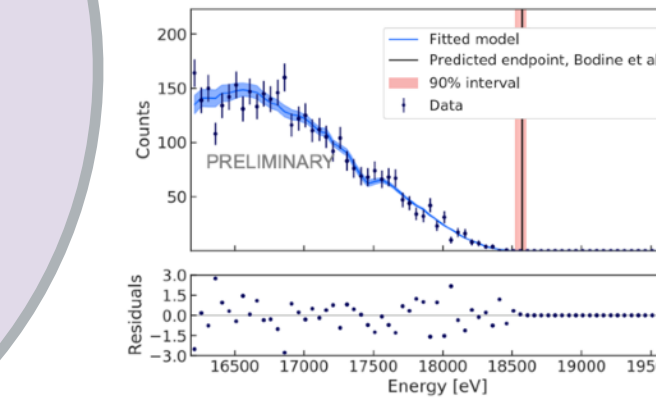
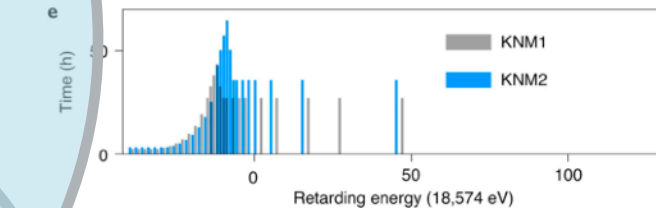
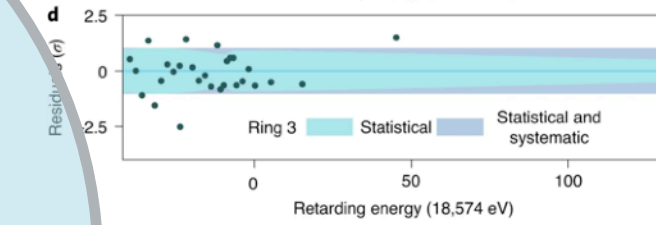
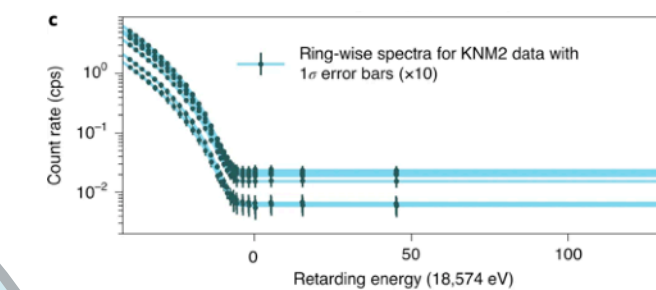
- Large number of datasets
- Large number of nuisance parameter

**Need:**

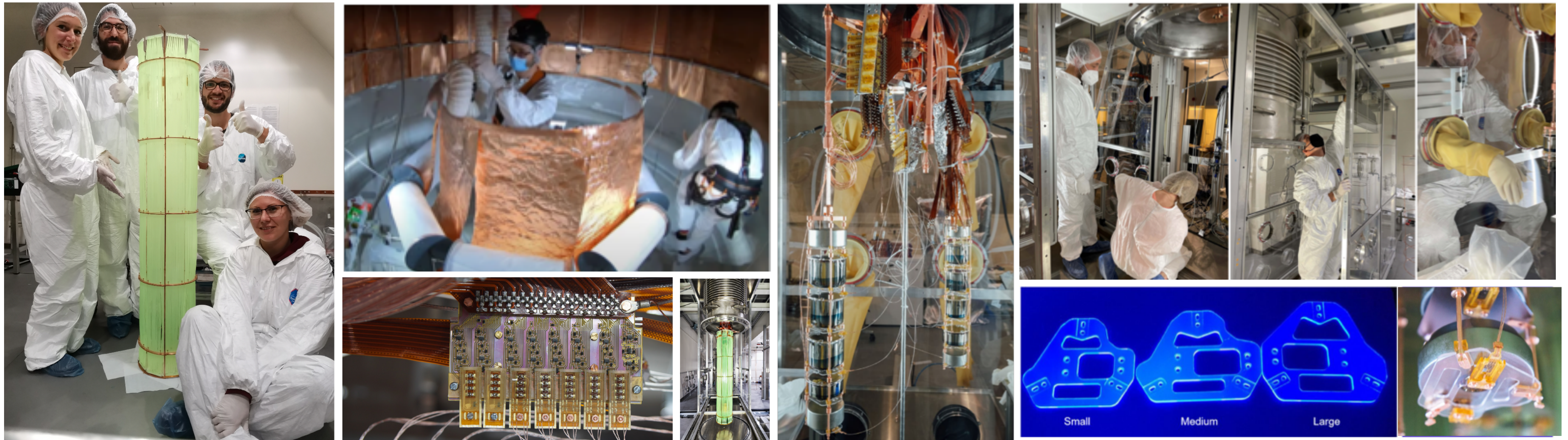
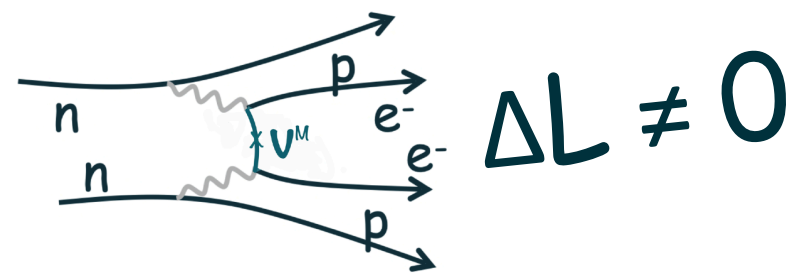
- More performant parameter estimation tools and analysis strategies
- Likelihoods which incorporates additional informations
- Potential ML-based dimensionality reduction
- “Advanced” likelihood sampling techniques
- Multi-dimensional sampling

**TRIN**  
modeling decay with all detector statistics

**Project8**







**Thank you for your attention!**  
**Question?**