Finite-size scaling analysis of proton cumulants

Agnieszka Sorensen

w/ Paul Sorensen

arxiv:2405.10278

INSTITUTE for NUCLEAR THEORY

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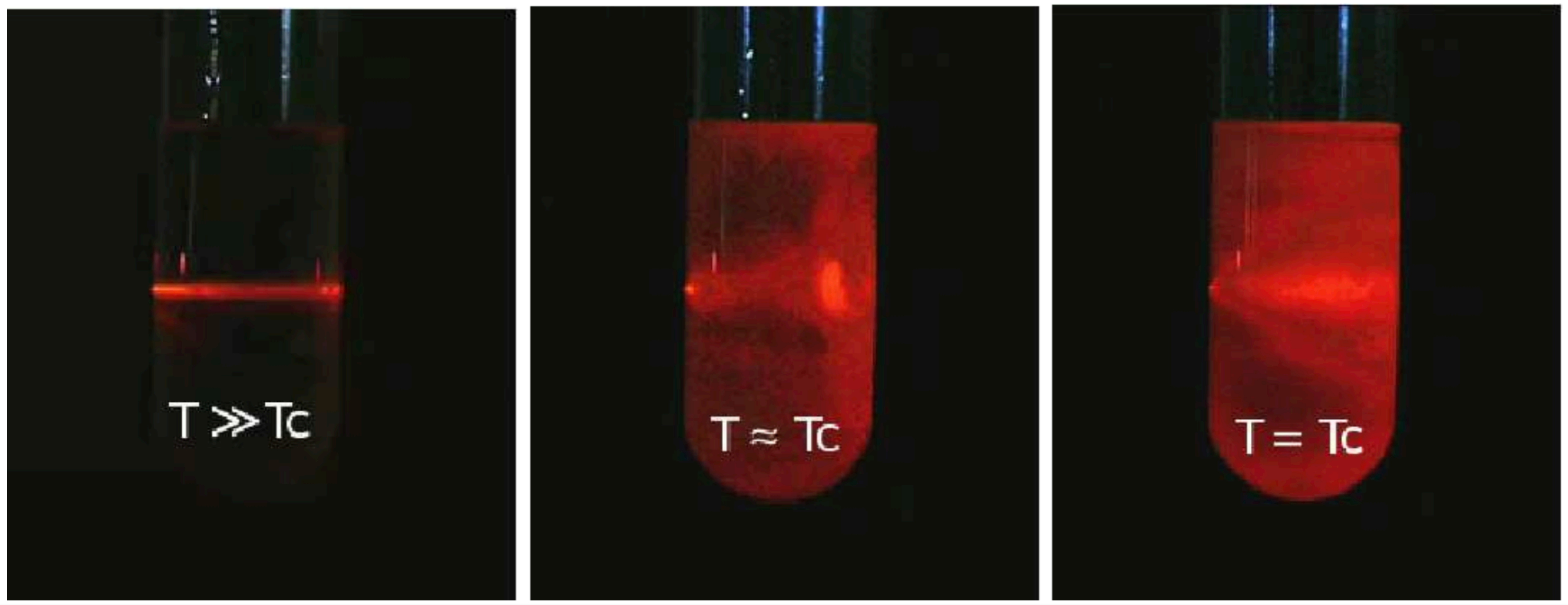
UNIVERSITY of WASHINGTON

Behavior near a critical point

- Critical point (CP):
  a single point in the phase diagram where change from an ordered to disordered phase occurs

- The endpoint of a 1st order phase transition

  As systems approach the CP, latent heat decreases
  ⇒ it costs little energy for components of one phase to form a local “bubble” of the other phase
  ⇒ as CP is approached, correlation length ξ increases = large fluctuations (large bubbles)
  ⇒ critical opalescence phenomenon:
  → “bubbles” grow to sizes comparable with visible light wavelengths ($\xi \approx \lambda$)
  → light can be scattered and a translucent system becomes cloudy (like fog)
  ⇒ at CP, correlation length formally diverges;
  system experiences correlations of all sizes
  (proof: critical opalescence in methanol+cyclohexane persists at CP where ξ ~ 1 cm)



Universal behavior

Near CP:

$$t \equiv \frac{T - T_c}{T_c}$$

$$m \equiv \frac{\mu - \mu_c}{\mu_c}$$

$$c_\infty(t, 0) \sim |t|^{-\alpha}$$

$$\tilde{n}_\infty(t, 0) \sim (-t)^{\beta}$$

$$\tilde{n}_\infty(0, m) \sim m^{\frac{1}{\delta}}$$

$$\chi_\infty(t, 0) \sim |t|^{-\gamma}$$

$$\xi_\infty(t, 0) \sim |t|^{-\nu}$$

$$\xi_\infty(0, m) \sim |m|^{-\nu_c}$$

CP: infinite volume concept

In real world ξ does not go to infinity = thermodynamic functions do not exhibit singularities

ξ is bound by the size of the system L

It can be shown that

$$X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}} \quad \Rightarrow \quad X_L(t_L) \sim L^{\frac{\sigma}{\nu}} \quad \Rightarrow \quad X_L(t_L) = L^{\frac{\sigma}{\nu}} \phi(t, L) = L^{\frac{\sigma}{\nu}} \phi\left(t L^{\frac{1}{\nu}}\right)$$

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one can find CP by plotting

$$X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}} \quad \Rightarrow \quad X_L(t_L) \sim L^{\frac{\sigma}{\nu}} \quad \Rightarrow \quad X_L(t_L) = L^{\frac{\sigma}{\nu}} \phi(t, L) = L^{\frac{\sigma}{\nu}} \phi\left(t L^{\frac{1}{\nu}}\right)$$

Finite size vs. window size

$$X_L(t_L) = L^{\frac{\sigma}{\nu}} \phi\left(tL^{\frac{1}{\nu}}\right)$$

Finite-size scaling (original): change the size of the system, calculate $X_L(t_L)$, repeat

Changing SIZE is not always possible or doesn't really probe the same system (bird flocks, heavy-ions)

Solution: study the dependence of X on the size of the *subsystem* that is considered

D. Martin, T. Ribeiro, S. Cannas, *et al.*, Box scaling as a proxy of finite size correlations, Sci Rep 11, 15937 (2021)

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$$\chi_\infty(t,0) \sim |t|^{-\gamma}$$

$$\frac{\chi_2}{\chi_1} = \frac{C_2}{C_1} \quad \Rightarrow \quad \chi_2 = \frac{C_2}{C_1}\chi_1$$

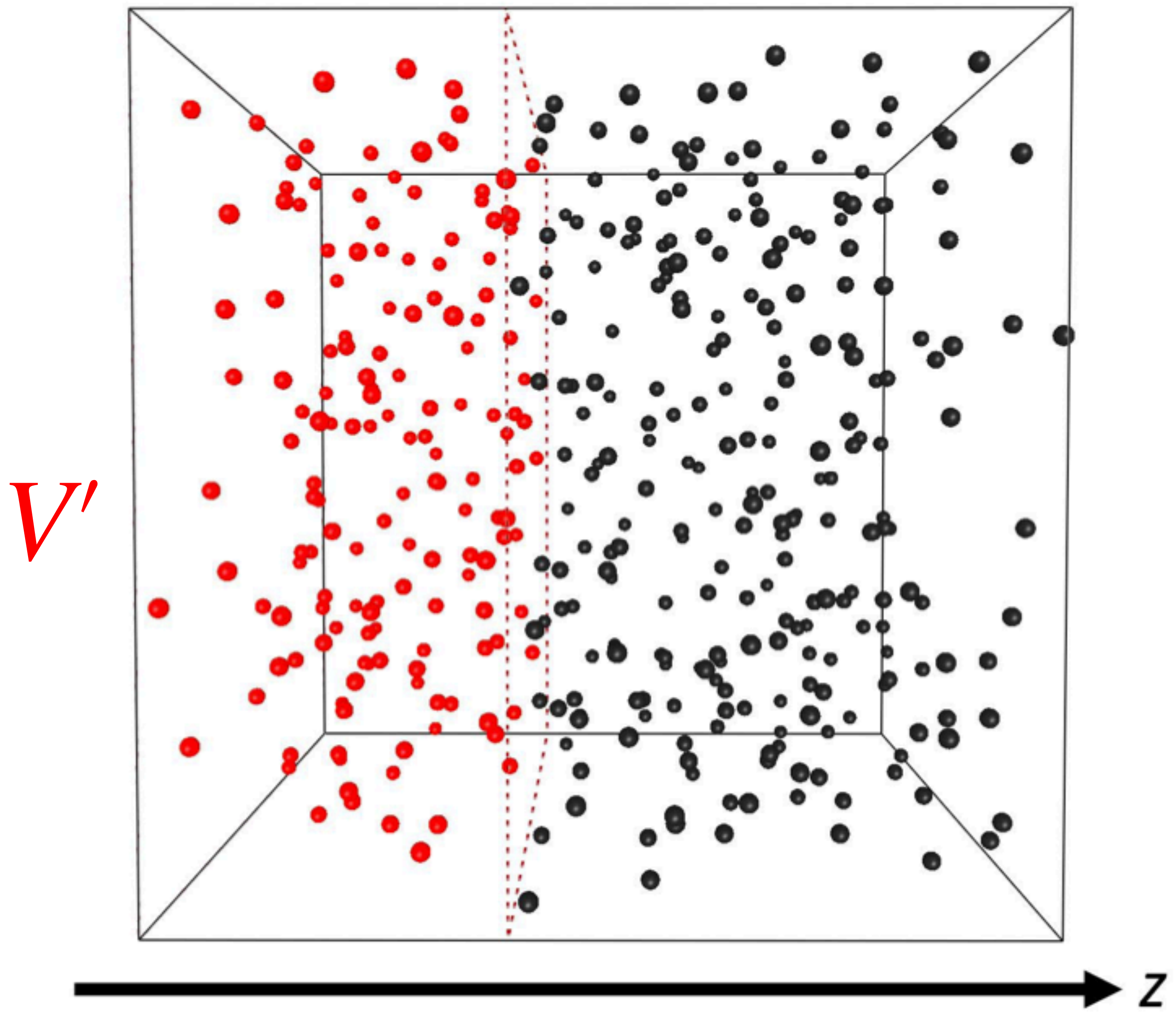
$$\chi_1 = \frac{C_1}{VT^3} = \frac{n_B}{T^3}$$

$$\Rightarrow \quad \chi_2 = \frac{C_2}{C_1}\frac{n_B}{T^3}$$

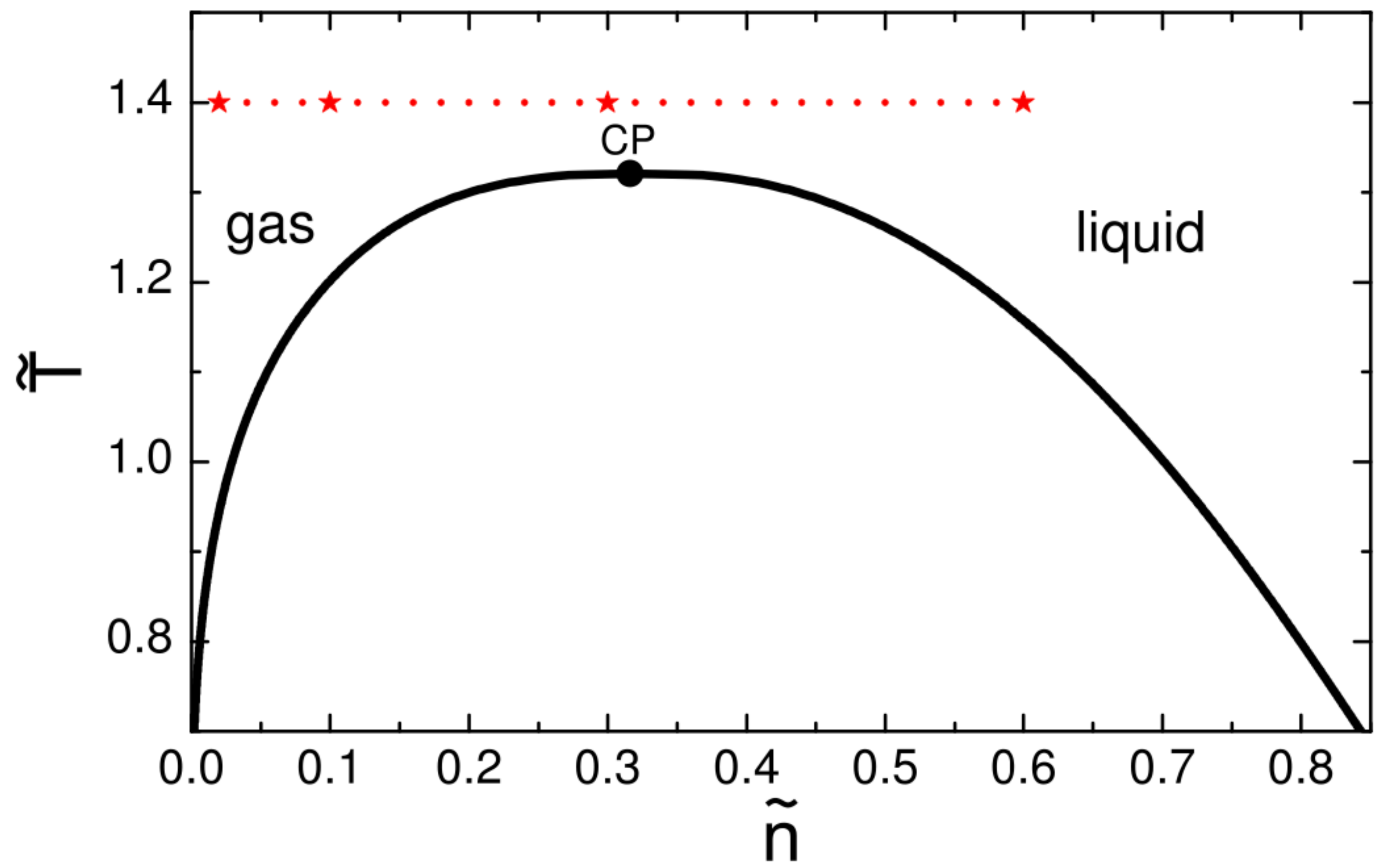
Does it work??

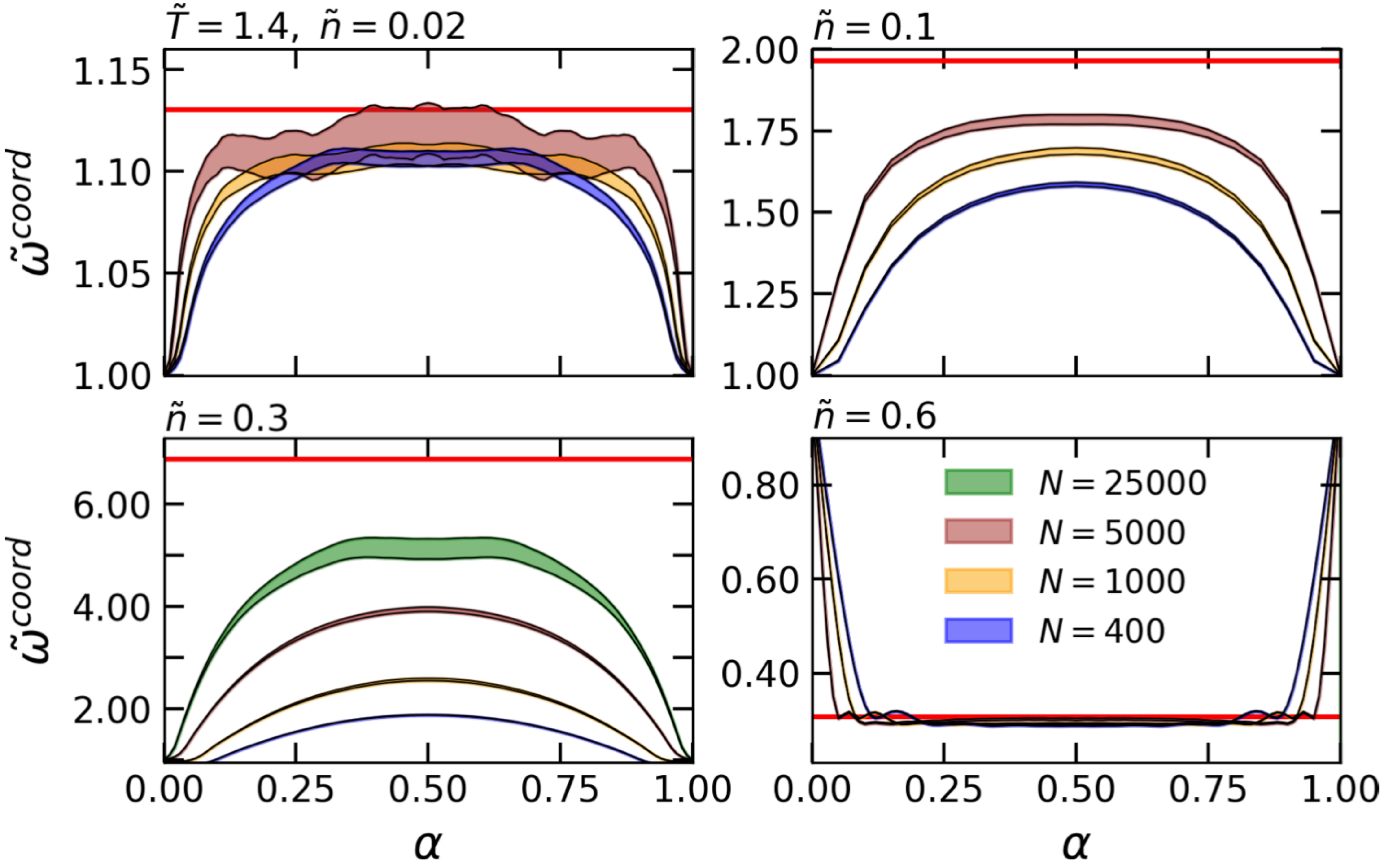
Motivation for VDF studies: cumulants in molecular dynamics

We will use some insights from V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, V. Vovchenko, Phys. Rev. C **105** no.4, 044903 (2022), arXiv:2201.08486



$$\omega = \frac{\kappa_2}{\kappa_1} \qquad \alpha = V'/V \qquad \tilde{\omega}^{coord} = \frac{\omega^{coord}}{1-\alpha} = \omega^{gce}$$





Finite-size scaling analysis of cumulants in a periodic box

VDF potentials in SMASH hadronic transport

AS and V. Koch, Phys. Rev. C **104**, 3, 034904 (2021)
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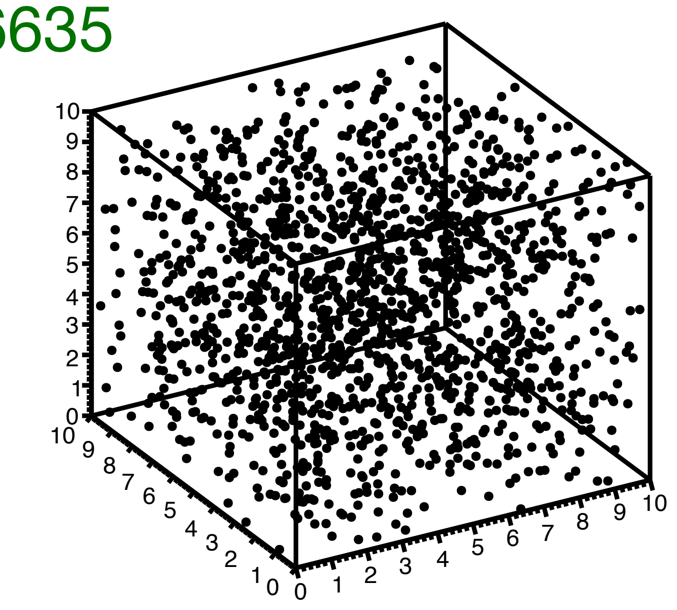
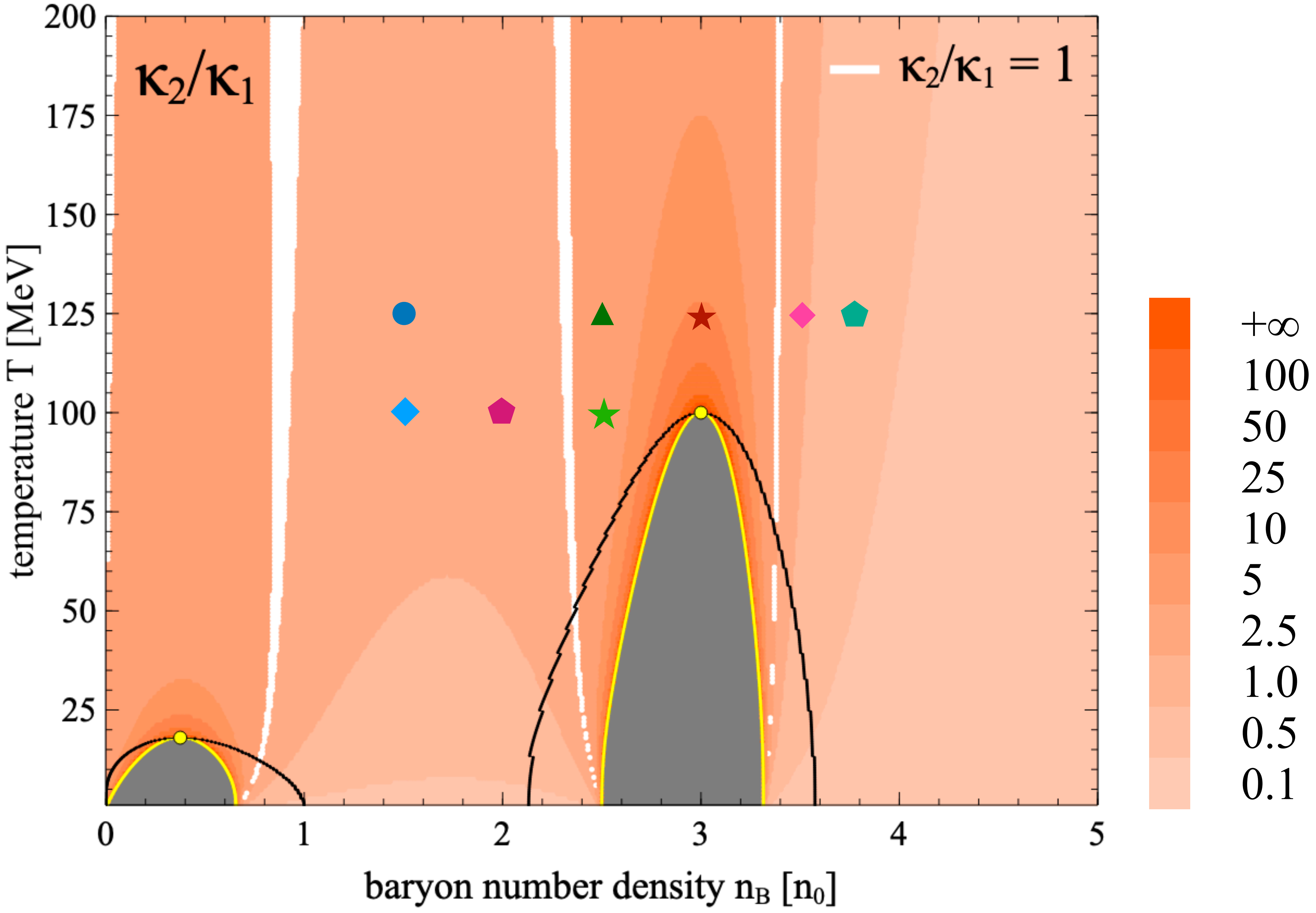


cubic subvolumes
(good definition of probed L)



	◆	⬟	★	●	▲	★	◆	⬟
T [MeV]	100	100	100	125	125	125	125	125
nB [n0]	1.5	2.0	2.5	1.5	2.5	3.0	3.5	3.75
k2/k1 (VDF)	0.67	0.70	1.46	0.72	1.36	5.44	0.59	0.85

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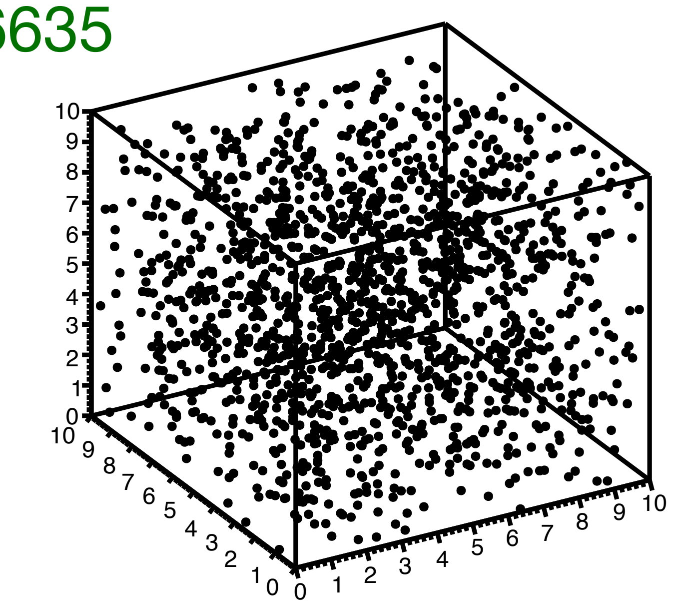

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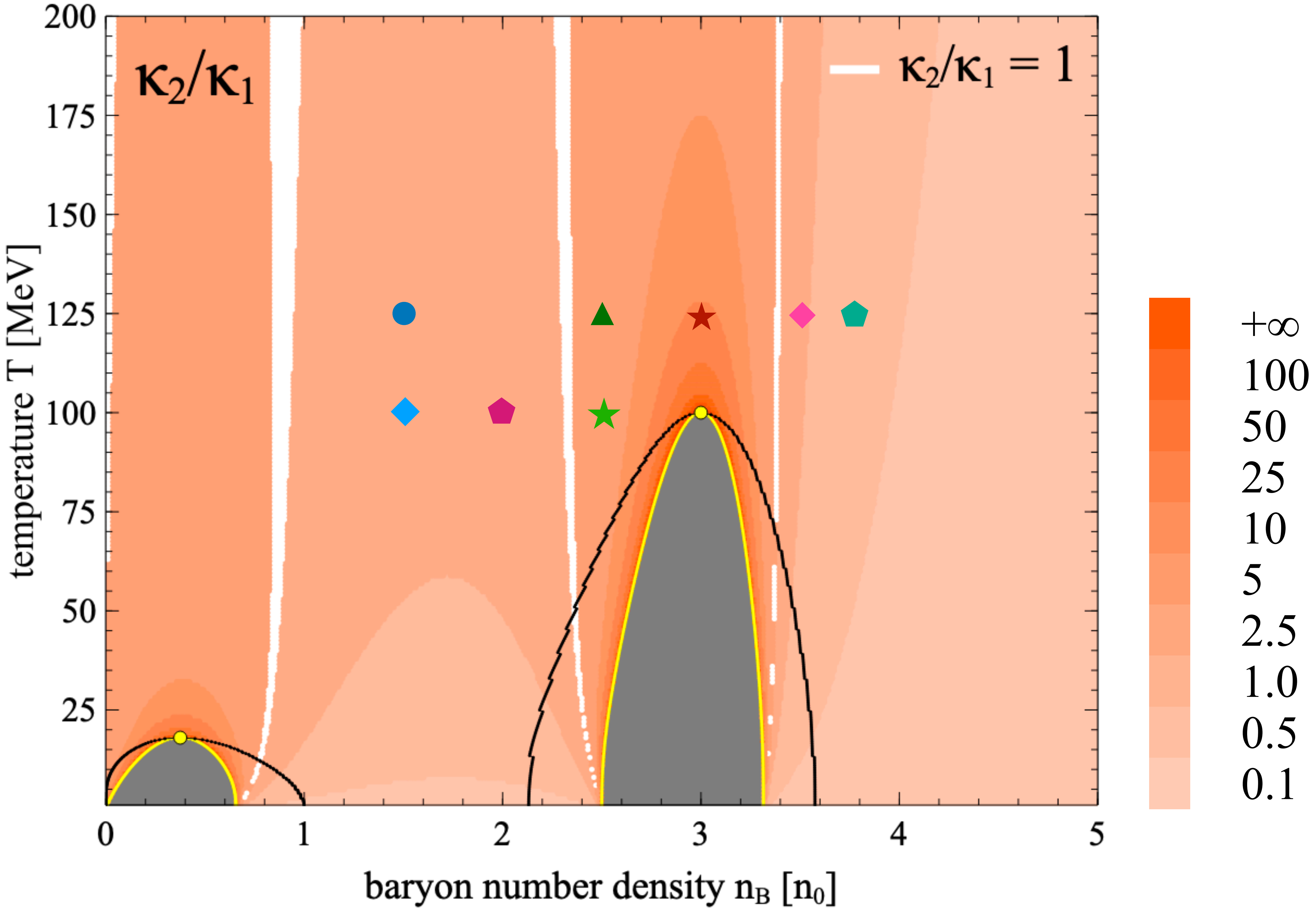

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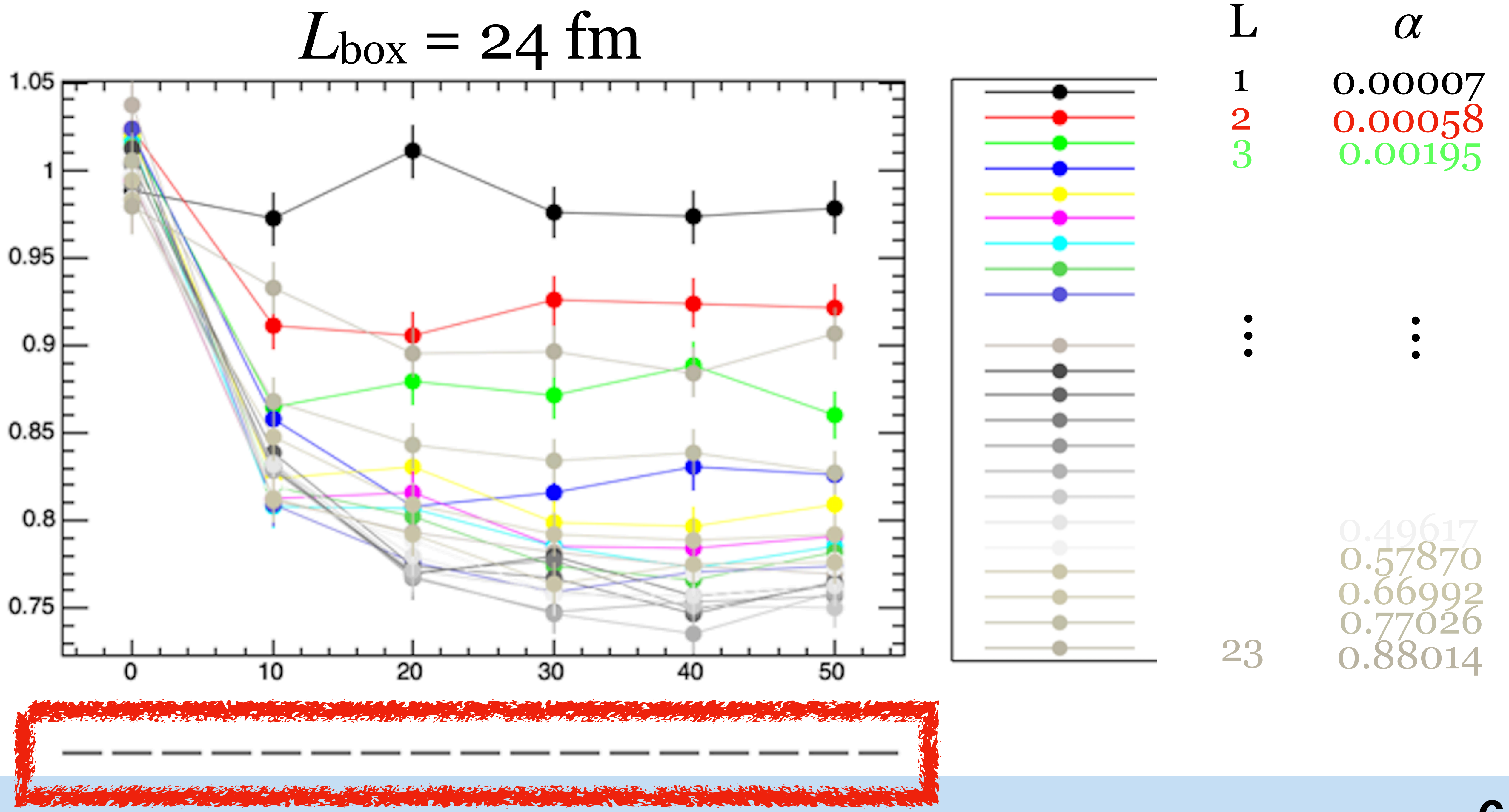
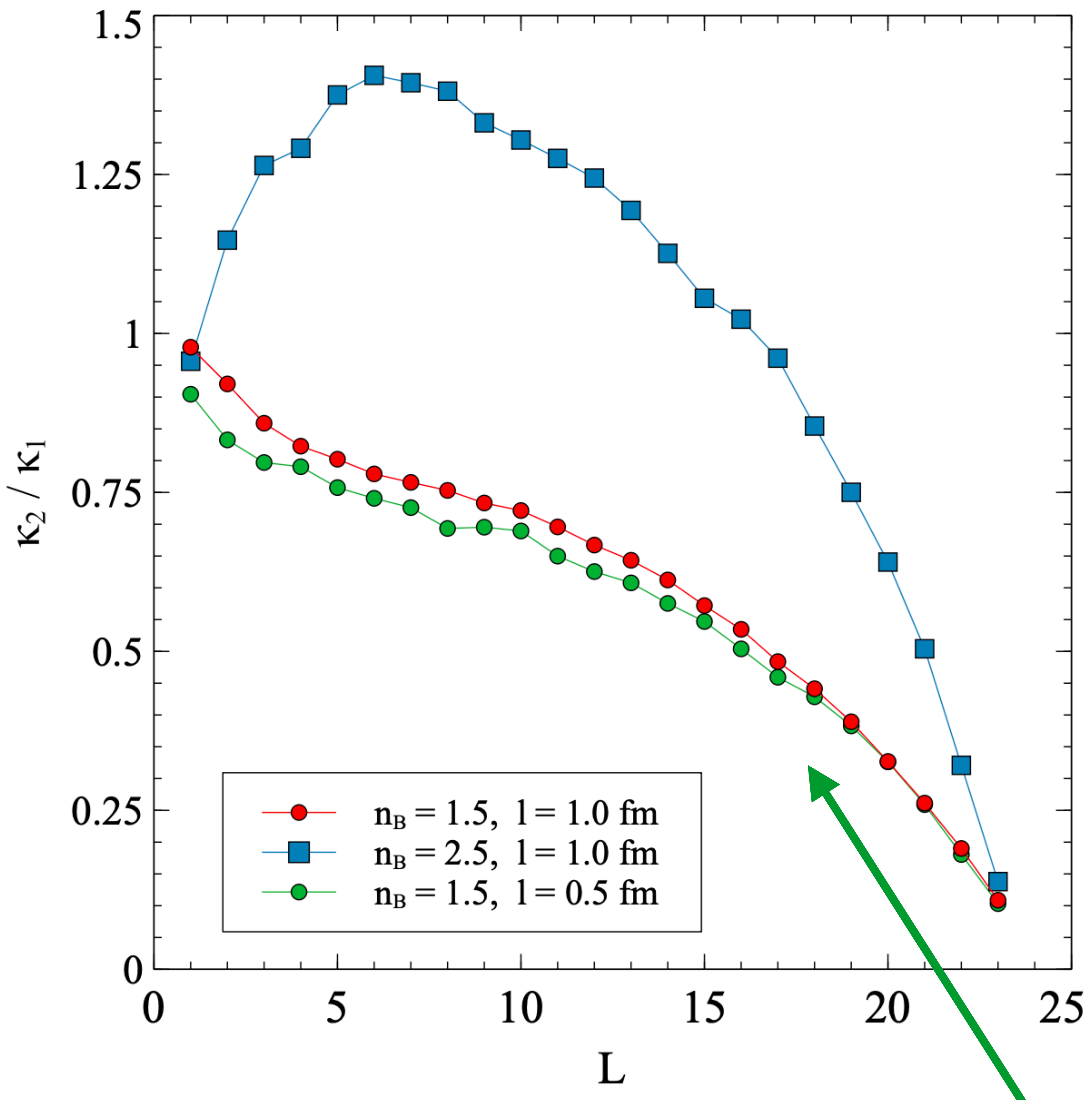
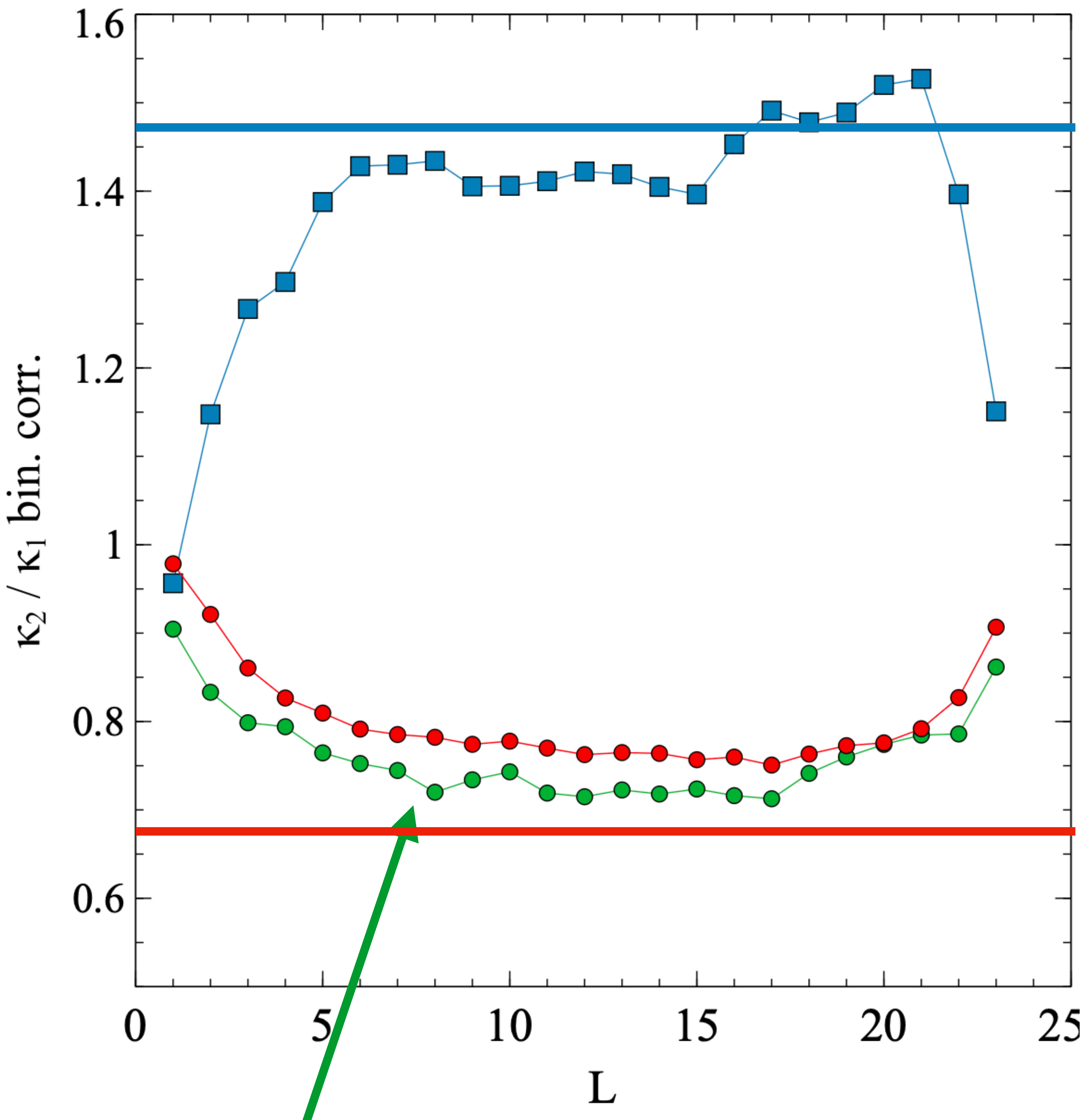


Finite-size scaling analysis of cumulants in a periodic box

L_{box} = 24 fm





	◆	⬟	★
T [MeV]	100	100	100
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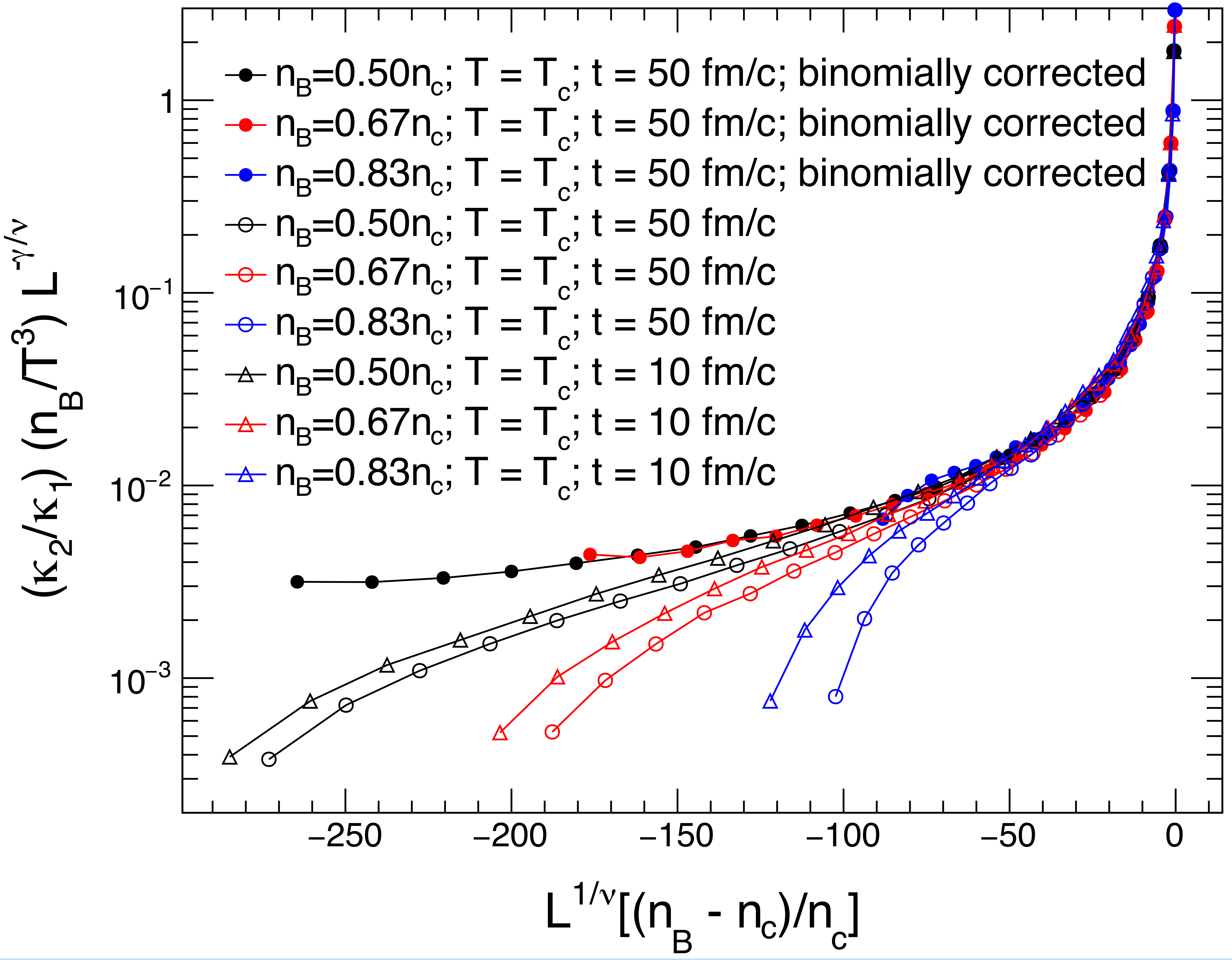
small dependence on the microscopic scale

Finite-size scaling analysis of cumulants in a periodic box

$L_{\text{box}} = 24$ fm



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T [MeV]	100	100	100
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Thermal model

$$\chi_2(W,\mu_{\rm fo}) = \frac{C_2(W,\mu_{B,{\rm fo}})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$$

- We used published thermal model fits for $T_{\rm fo}$ and $\mu_{B,{\rm fo}}$
- We parameterize $dV_{\rm fo}/dy$ from several publications. At lower energies, HBT and some fits give a much smaller $dV_{\rm fo}/dy$. We use the larger volume, lower density solution. For 2.4 GeV, $T_{\rm fo}^3 V$ is highly uncertain, ranging from 0.5 to 5
- Experiments can improve results by publishing $dV_{\rm fo}/dy$, $T_{\rm fo}$ and $\mu_{B,{\rm fo}}$ from thermal model fits for specific W

$\sqrt{s_{\rm NN}}$ (GeV)	$y_{\rm beam}$	$\mu_{\rm fo}$ (GeV)	$T_{\rm fo}$ (GeV)	$dV_{\rm fo}/dy$ (fm^3)
2.4	0.73	0.776	0.050	17157
3.0	1.05	0.720	0.080	4850
7.7	2.09	0.398	0.144	1044
11.5	2.50	0.287	0.149	1047
14.5	2.73	0.264	0.152	1080
19.6	3.04	0.188	0.154	1137
27	3.36	0.144	0.155	1218
39	3.73	0.103	0.156	1341
54.4	4.06	0.083	0.160	1487

J. Adamczewski-Musch et al. (HADES), Phys. Rev. C 102, 024914 (2020), arXiv:2002.08701 [nucl-ex].

M. Abdallah et al. (STAR), Phys. Rev. C 104, 024902 (2021), arXiv:2101.12413 [nucl-ex].

M. Abdallah et al. (STAR), Phys. Rev. C 107, 024908 (2023), arXiv:2209.11940 [nucl-ex].

A. Andronic, P. Braun-Munzinger and J. Stachel, Acta Phys. Polon. **B** 40, 1005-1012 (2009), [arXiv:0901.2909 [nucl-th].

A. Motornenko, J. Steinheimer, V. Vovchenko, R. Stock, and H. Stoecker, Phys. Lett. B 822, 136703 (2021), arXiv:2104.06036 [hep-ph].

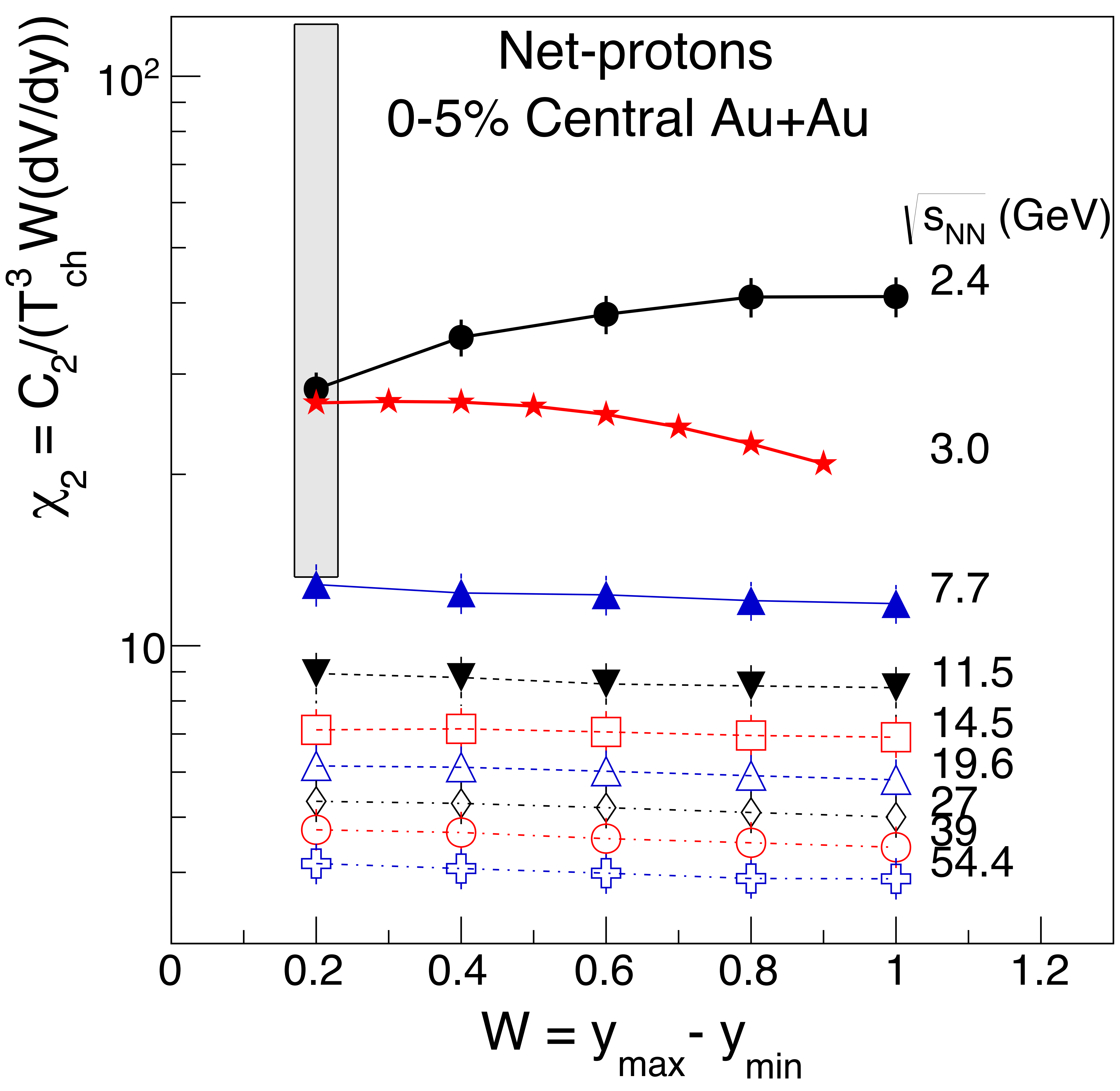
S. Chatterjee, S. Das, L. Kumar, D. Mishra, B. Mohanty, R. Sahoo, and N. Sharma, Adv. High Energy Phys. 2015, 349013 (2015).

Susceptibility

$$\chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T^3_{\rm fo} W dV_{\rm fo}/dy}$$

- Grey band shows uncertainty from freezeout ambiguities for the 2.4 GeV data.
  Uncertainty precludes any conclusion about observing a maximum in χ_2

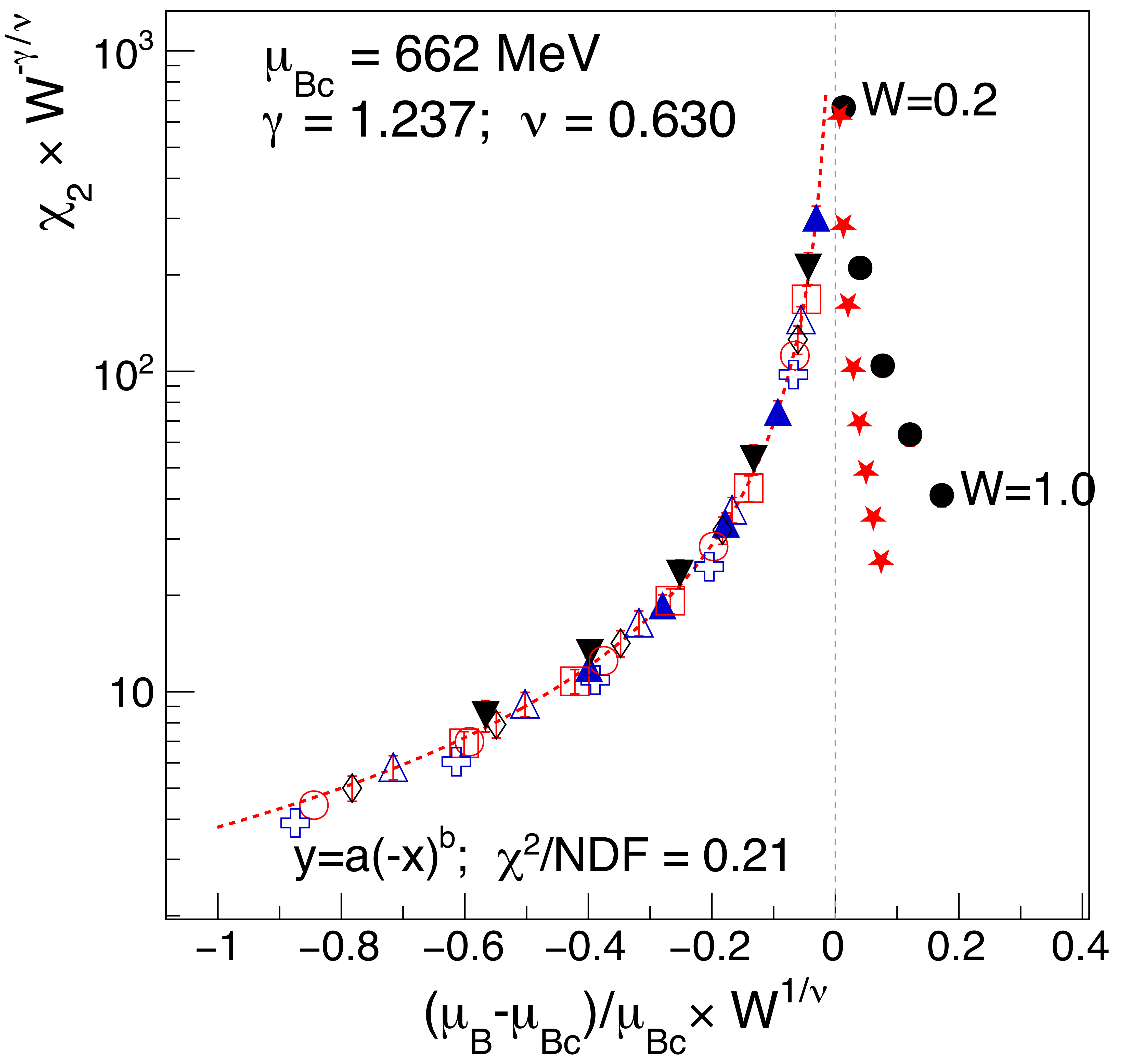
- Data do indicate a change in slope at higher μ_B and at small W:
  χ_2 decreases with increasing W for 7.7-54.4 GeV but changes slope at 2.4 GeV
  (3.0 GeV is ~flat)



Scaled susceptibility

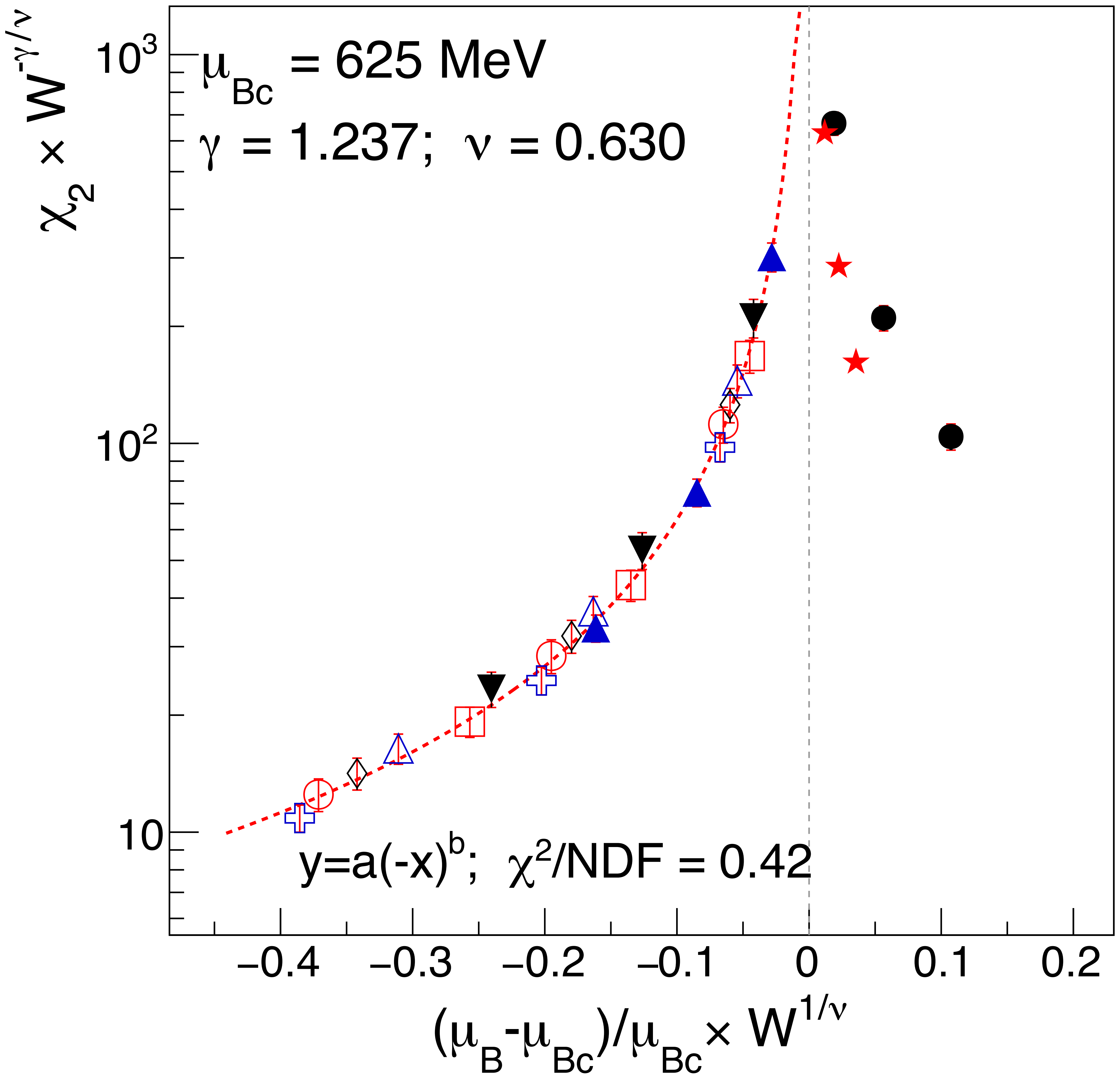
$$\chi_2(W, m) = W^{\gamma/\nu}\Phi(mW^{1/\nu})$$
$$m = (\mu_B - \mu_{B,c})/\mu_{B,c}$$

- Good scaling for negative m
- Low energy points do not scale well
- Scaling function Φ is well described by a power law; consistent with expectation for $m \to -\infty$.
- This scaling neglects variation of $t = (T - T_c)/T_c$; not a bad approximation for 7.7 GeV and above, but worse for 2.4 and 3.0 GeV.



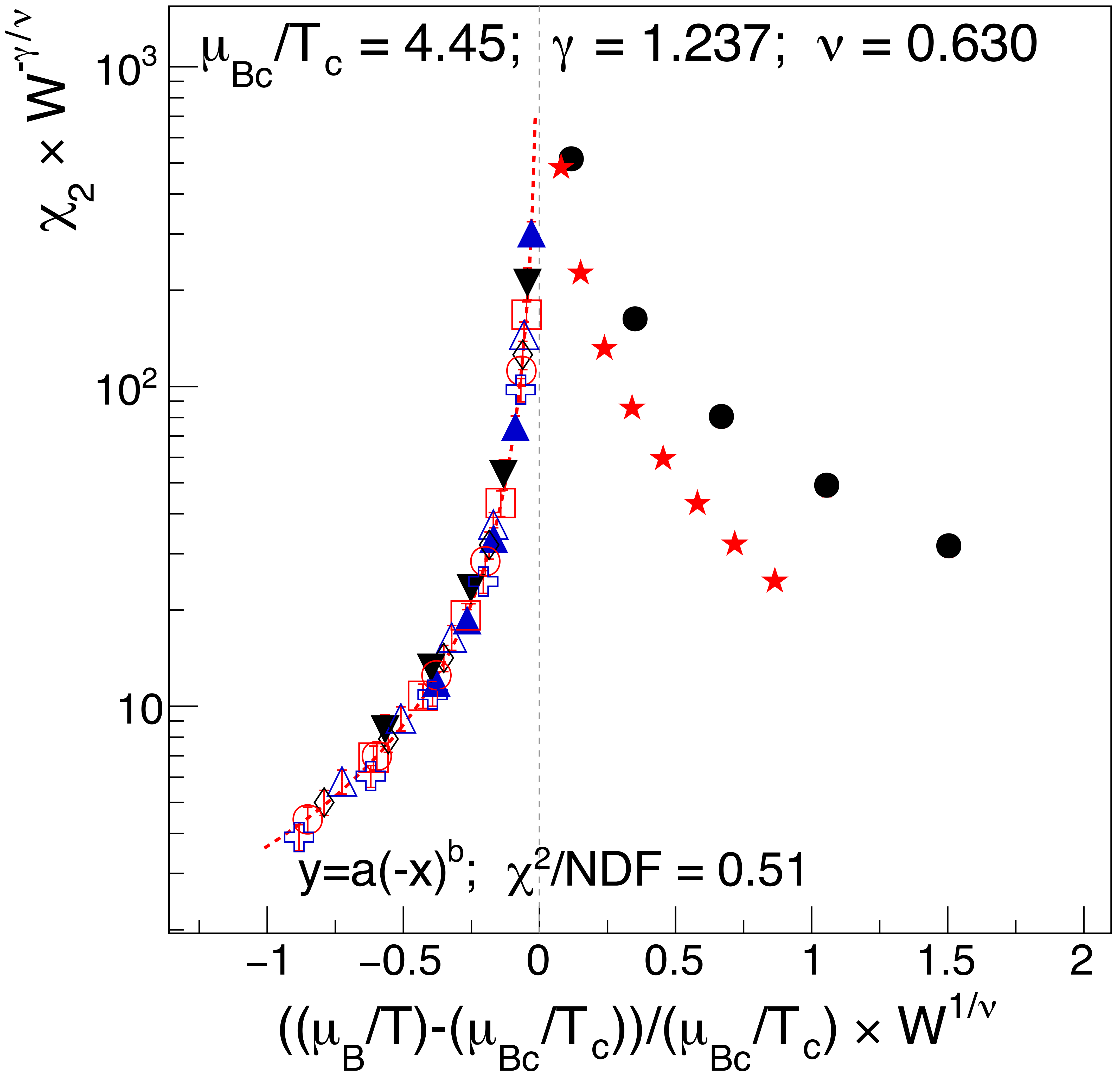
Scaled susceptibility: excluding widest bins

- Our simulations showed that baryon number conservation may spoil the scaling for larger values of *W*
- Excluding W=0.8 and 1.0 reduces the $\mu_{B,c}$ (as was expected from simulations)
- The fraction of measured baryons to total baryons is likely well below 25% for all these points except the 2.4 and 3.0 GeV data (not in the fit)



Scaled susceptibility: μ_B/T fit

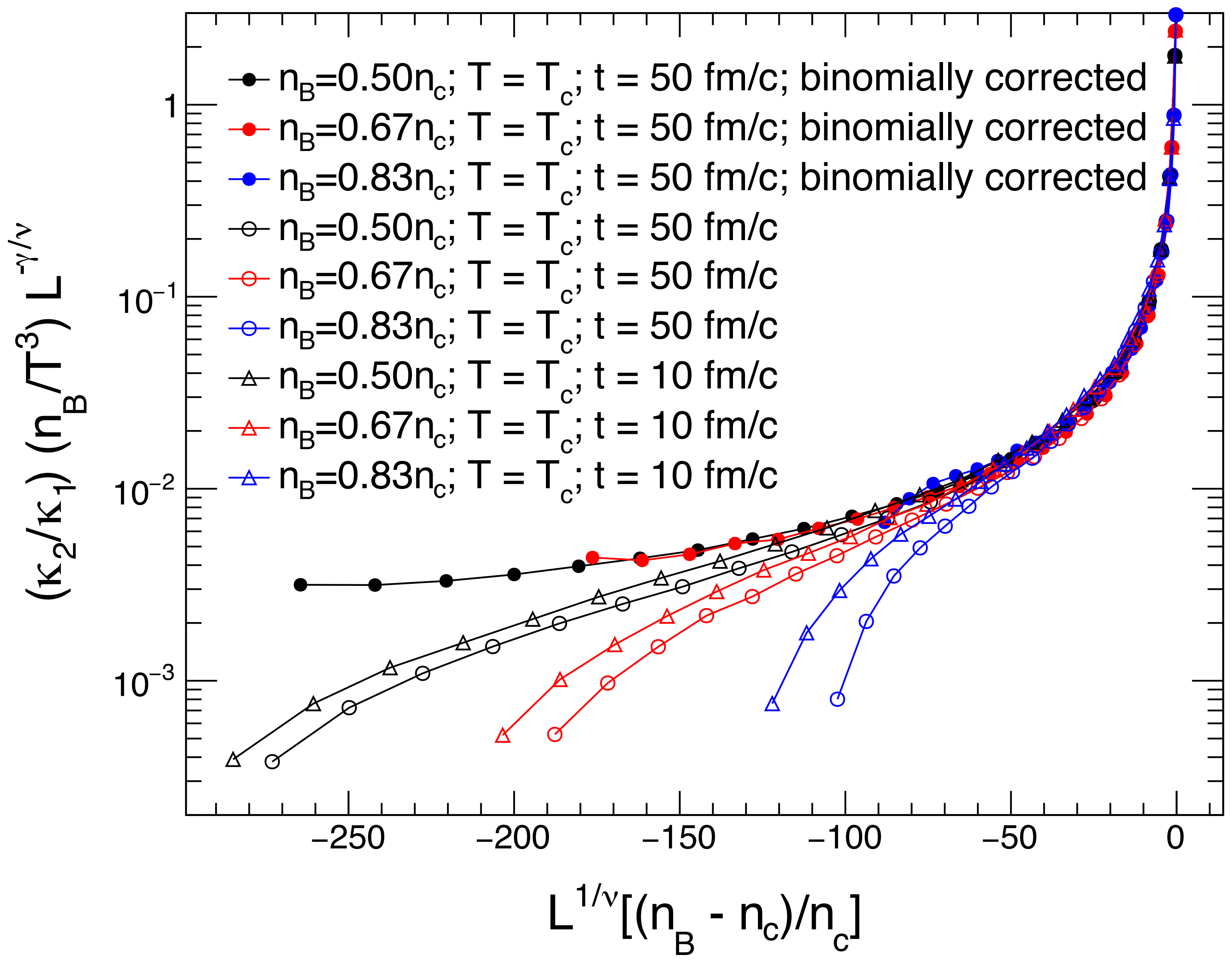
- To explore factoring in the temperature dependence used $(r - r_c)/r_c$ where $r = \mu/T$
- From that, we extract $T_c = 140 \pm 13$ MeV

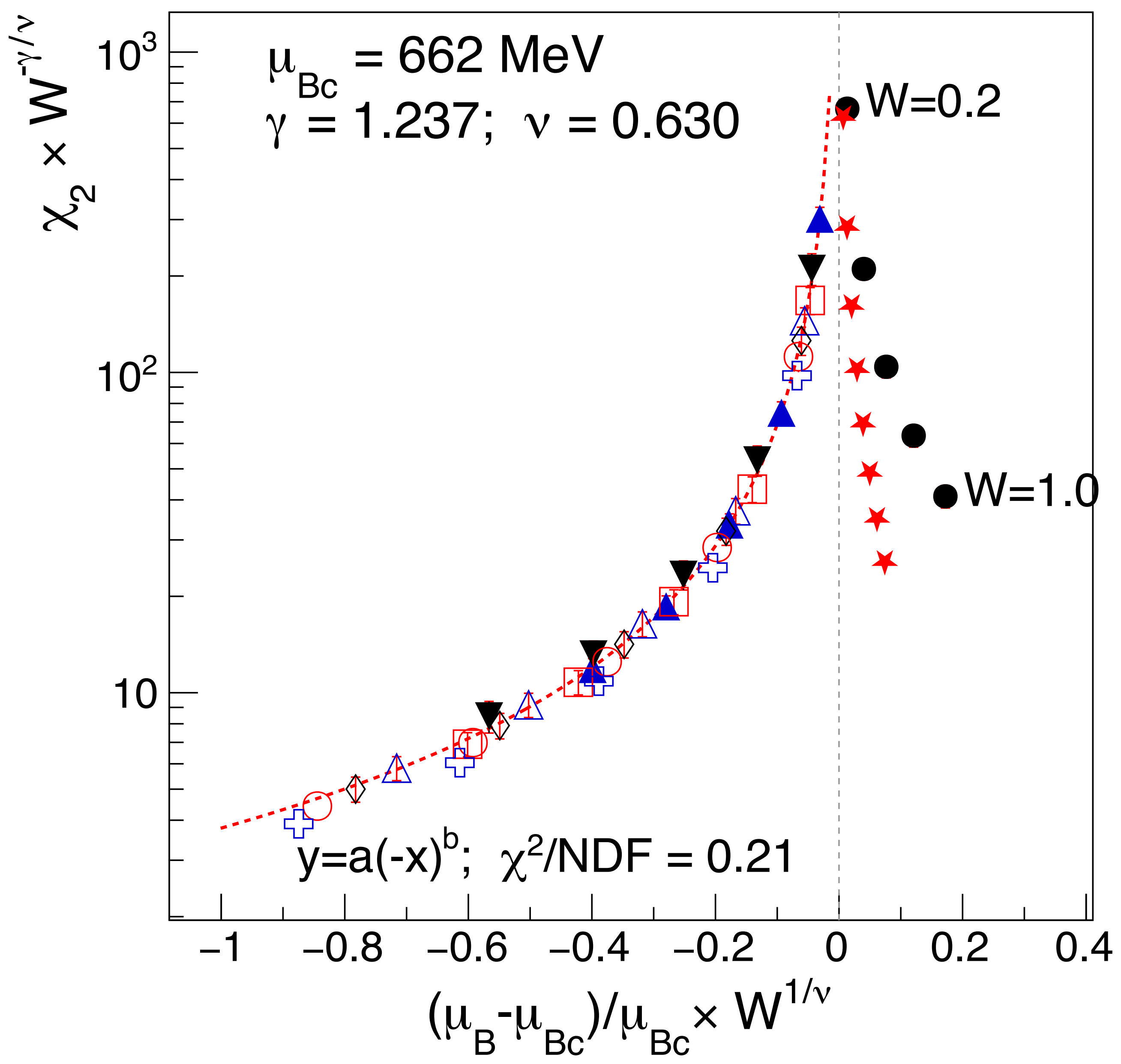


Summary

- Simulations show that window-size analysis works: effects due to finite time, baryon number conservation can be controlled by considering less than ~25% of the total volume
- We observe finite-size scaling for χ_2 extracted from 7.7-54.4 GeV data: we obtain $\mu_B \approx 625 \pm 60$ MeV and $T_c = 140 \pm 13$ MeV
- We explored a variety of fit ansaetze: μ_B , μ_B/T , (μ_B, T), different critical exponents…

Thank you
for your attention





Binder cumulants

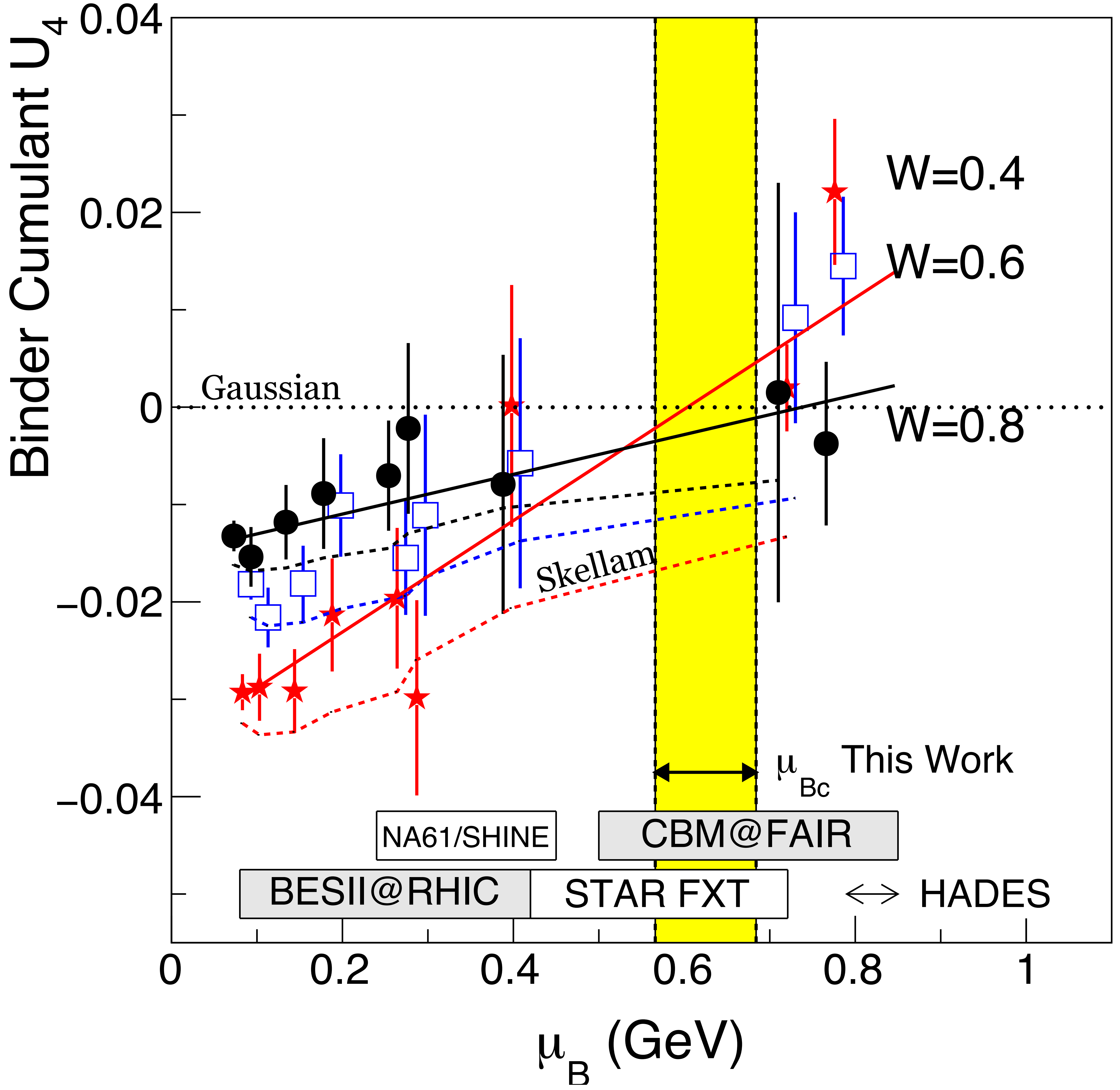
K. Binder, Z. Phys. B 43, 119 (1981).

$$U_4 = -C_4/(3C_2^2)$$

- Expectation: U_4 = 0 (Gaussian), 2/3 (bimodal), crosses at the critical point

$$U_4 \approx c_1 + c_2(\mu - \mu_c)W^{1/\nu}$$

- At low μ_b, U_4 follows Skellam with $U_4(W{=}0.8) > U_4(0.6) > U_4(0.4)$
- At μ_b>400, the ordering appears to reverse
- Data are consistent with a critical point between μ_b of 400 and 800 MeV



Different critical exponents

- We explored a broad range of critical exponents including mean-field (1.0, 0.5)
- For each selected critical exponent pair, we find the temperature that minimizes the Chi-square.
- Chi-square is shown in color and Tc as text.
- Most results are satisfactory Chi-square values so we do not interpret the Chi-square valley as necessarily providing the correct exponents