U.S. MAGNET DEVELOPMENT PROGRAM

Current Status of BELFEM

Christian Messe

Tue Jul 2nd, 2024

BERKELEY LAB

Outline

- Motivation and Fundamentals
- Current Status and Lessons Learned
- Homologies and Cohomoligies
- Tasks to Make the Code 3D-Ready
- Next Steps

Motivation and Fundamentals

BELFEM: Motivation and Project Goals

want ability to model:

- **quasi-magnetodynamic modeling**
  - understand electromagnetic behavior of cables
- **coupled thermal modeling**
  - thermal behavior and physical coupling with EM
  - quench behavior
- **other phenomena**
  - current sharing
  - …

[Source: 10.1088/0953-2048/28/6/065007]

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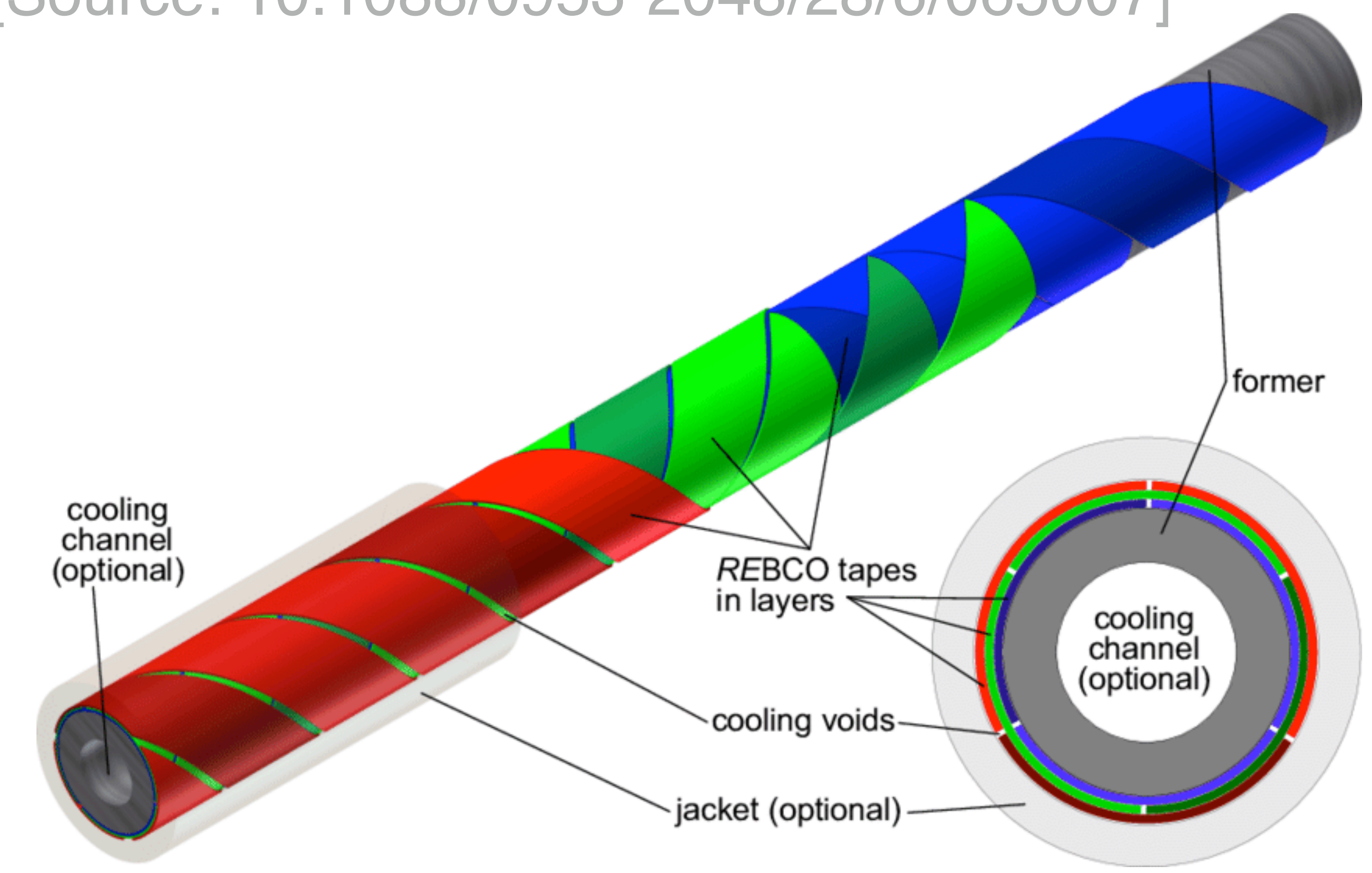

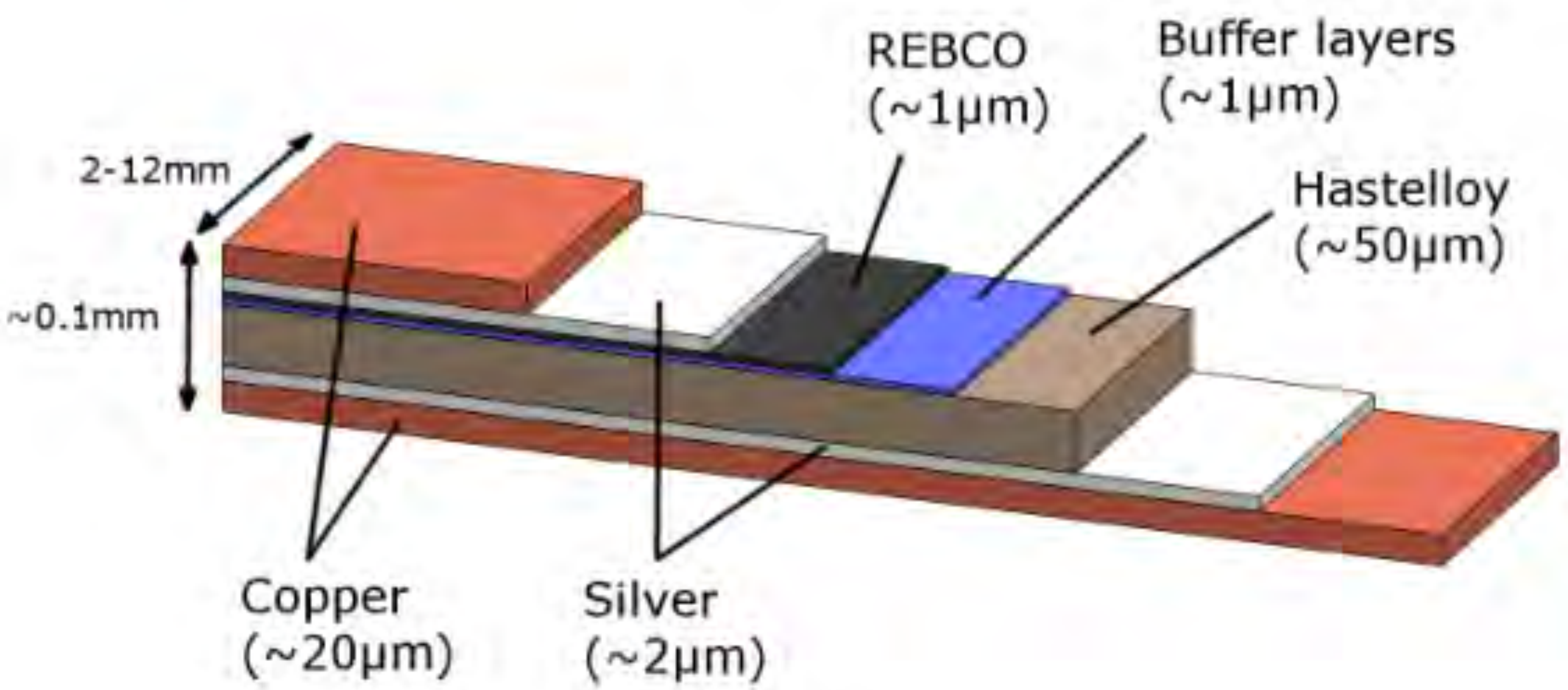

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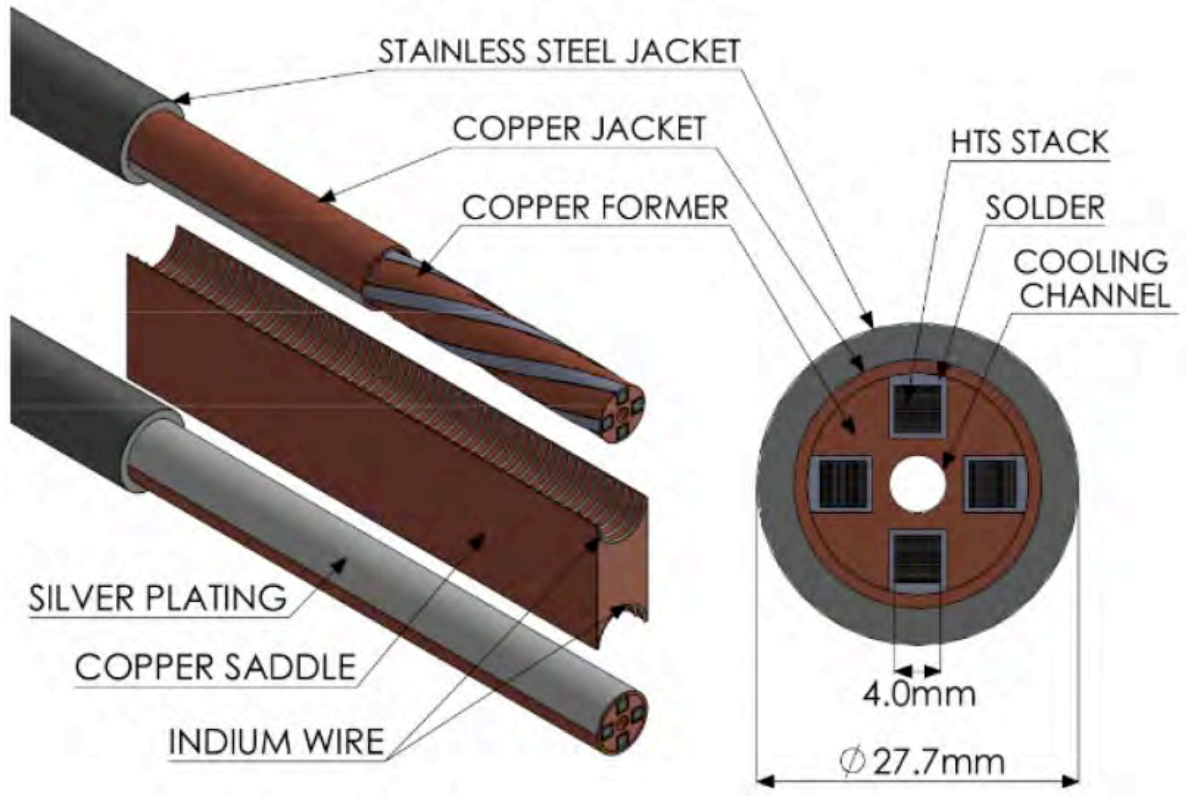


[Source: 10.1088/1361-6668/abb8c0]

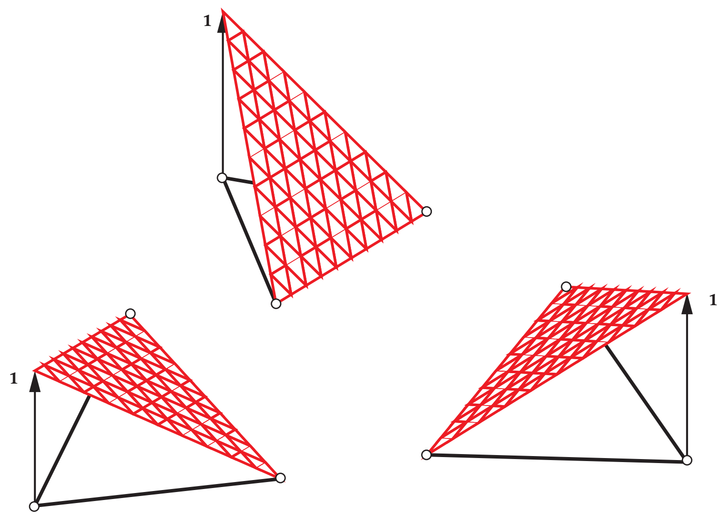
Recap: Maxwell in FEM

FEM Weak forms are based on different laws

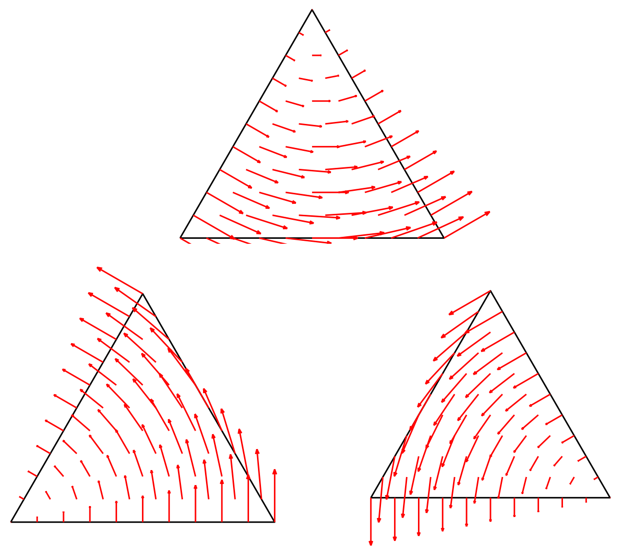
- h-formulation is based on Faraday's law
- a-formulation is based on Ampéré's law
- ϕ-formulation can be based on either Faraday or Gauß

Mixed formulations:

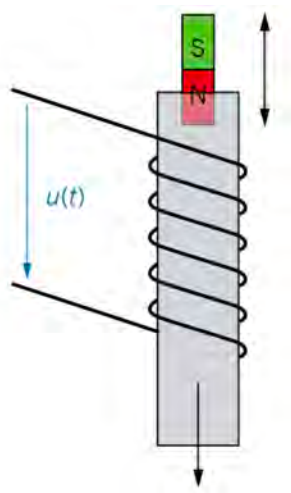
- there is no "universally best" formulation
- mixed formulations aim to combine benefits of individual formulations



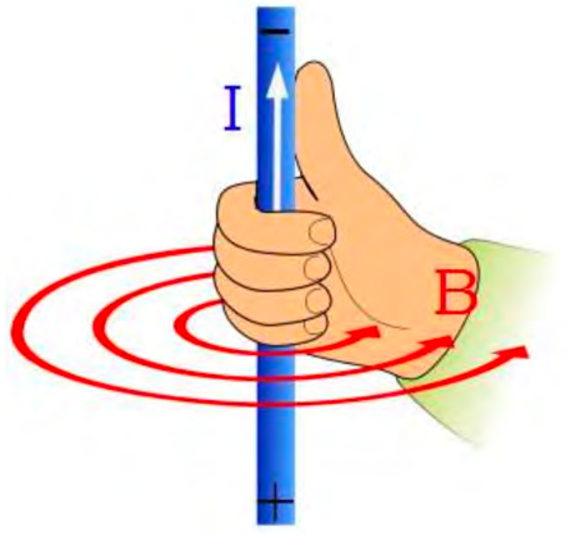
Lagrange Elements
(ϕ formulation)



Nédélec Elements
(h-formulation)



Faraday's law
$\nabla \times \boldsymbol{e} = -\dot{\boldsymbol{b}}$



Ampére's law
$\nabla \times \boldsymbol{h} = -\boldsymbol{j}$

	Air / Vacuum	Conductor	Ferromagnetic Alloy
Governing Equation	$\nabla \times \boldsymbol{h} = \boldsymbol{0}$ Ampére-Maxwell	$\nabla \times \boldsymbol{h} = \boldsymbol{j}$ Ampére-Maxwell	$\nabla \times \boldsymbol{e} = \dot{\boldsymbol{b}}$ Faraday's Law
Degree of Freedom	$\boldsymbol{h} = -\nabla\phi$ Magnetic Scalar Potential	$\boldsymbol{h}$ Magnetic Field	$\boldsymbol{b} = \nabla \times \boldsymbol{a}$ Magnetic Vector Potential
Transport Law	none	$\boldsymbol{e} = \rho \cdot \boldsymbol{j}$ Ohm's Law	$\boldsymbol{h} = \nu \cdot \boldsymbol{b}$ Magnetic Law
Comment	minimal number of dofs	need edge elements	simple material law implementation

Goals and Challenges

Modeling Goals: Make the code ready for 3D!

- fast implementation
  - require low/moderate conditioning of system matrix
- stable implementation
  - robust timestepping (including higher order)
  - handling numerical noise (⟶ e.g. "bleeding")

" no cutting corners!"

Modeling Challenges as learned last year:

- integral current boundary conditions (⟶ cohomologies)
  - "thin cuts vs thick cuts"
- h-ϕ domain interfaces
  - Lagrange Multipliers vs. Static Condensation
- ϕ-a domain interfaces
  - handling of strong nonlinearities (⟶ XFEM)

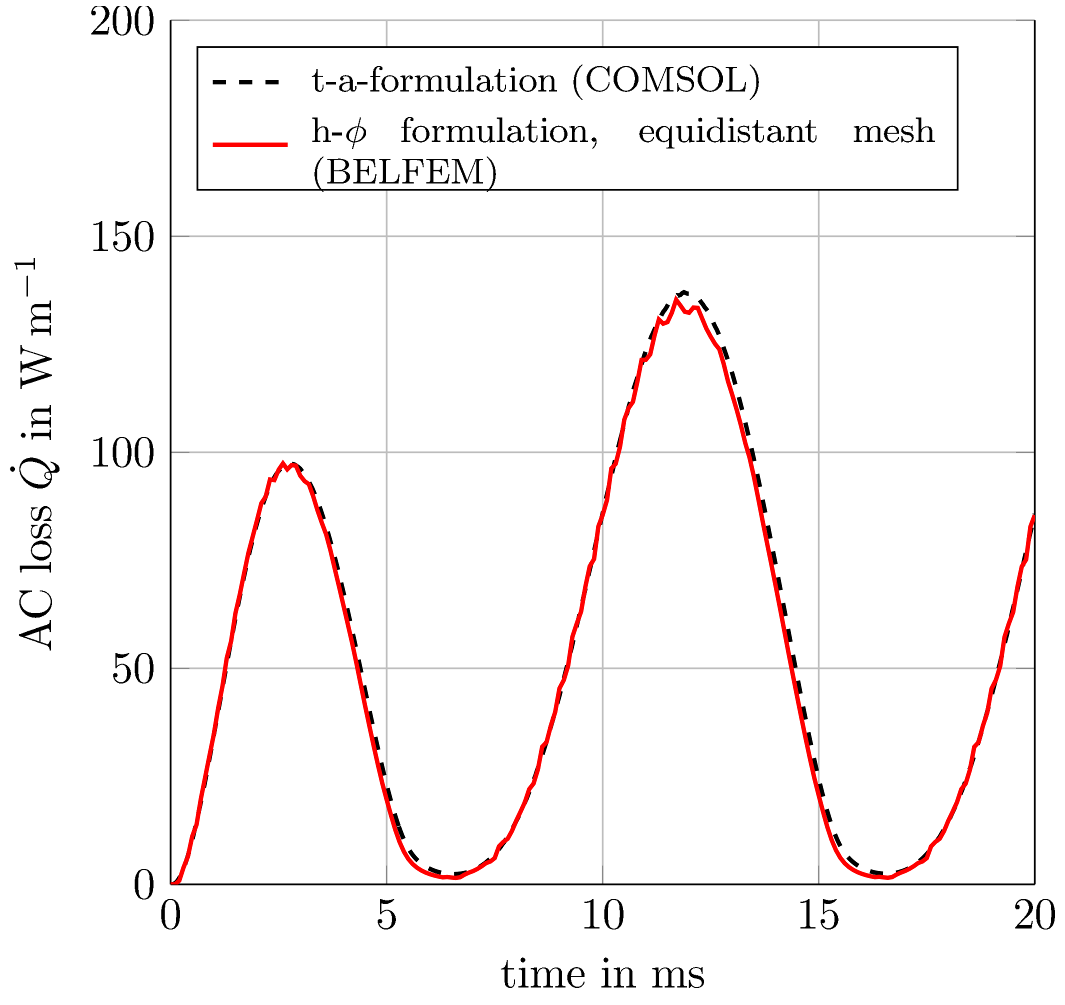
Current Status and Lessons Learned

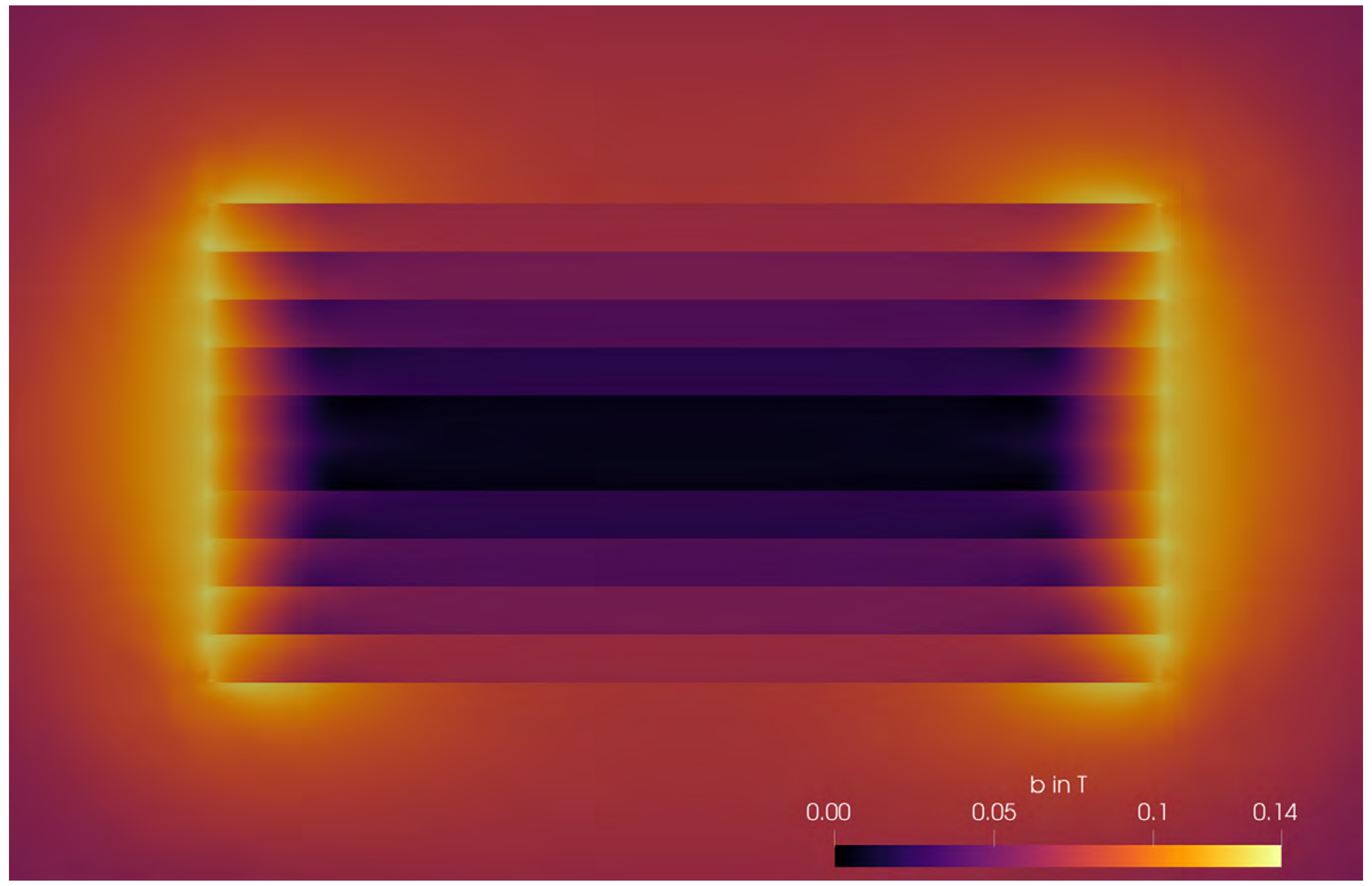
Thin Shell Formulation: 2D Results 2023

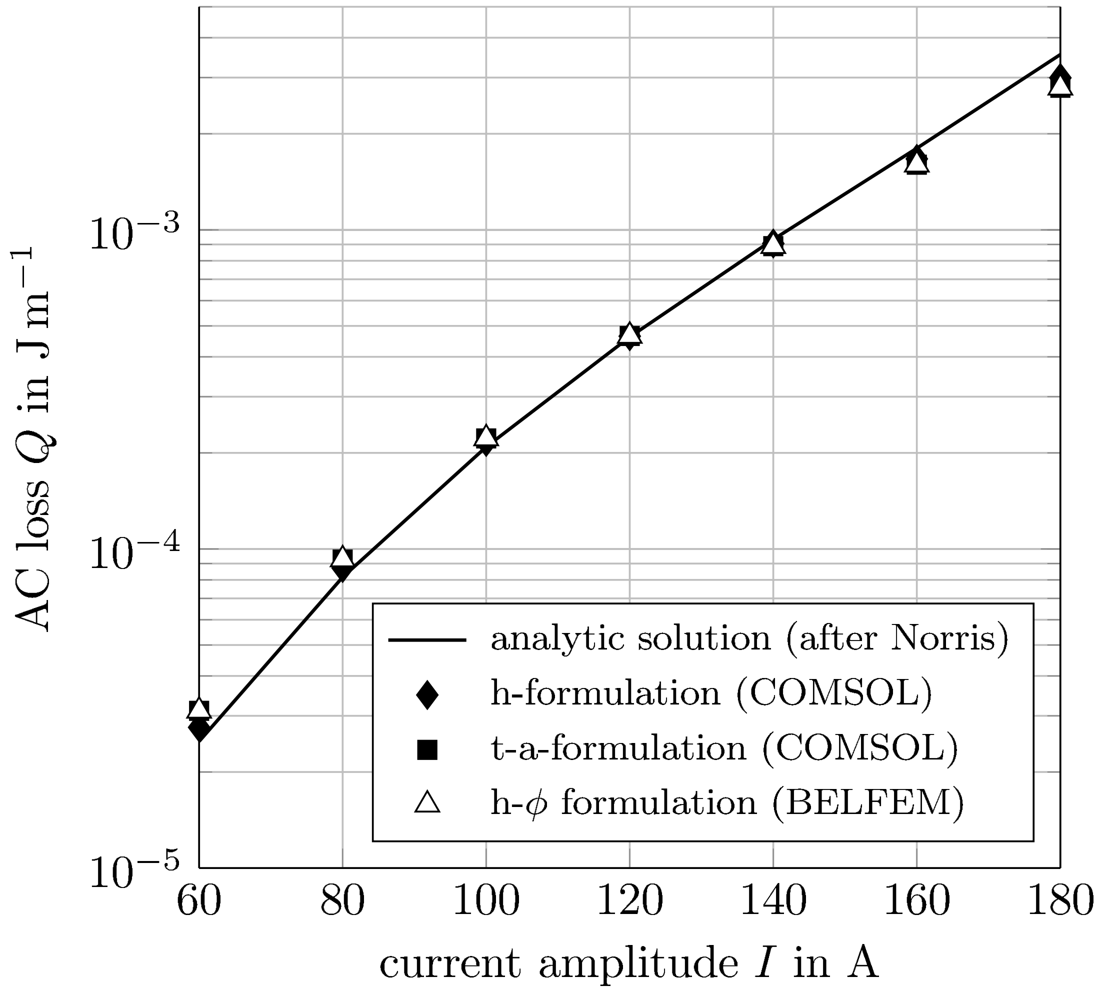
Published Paper in SuST 2023

- Christian Messe, Berkeley Lab
- Nico Riva, MIT
- Sofia Viarengo, Politechnico di Torino
- Gregory Giard & Frédéric Sirois, Polytechnique Montreal

- validated against analytical methods + COMSOL / GetDP
- first research promises faster and more detailed results than other established methods such as t-a







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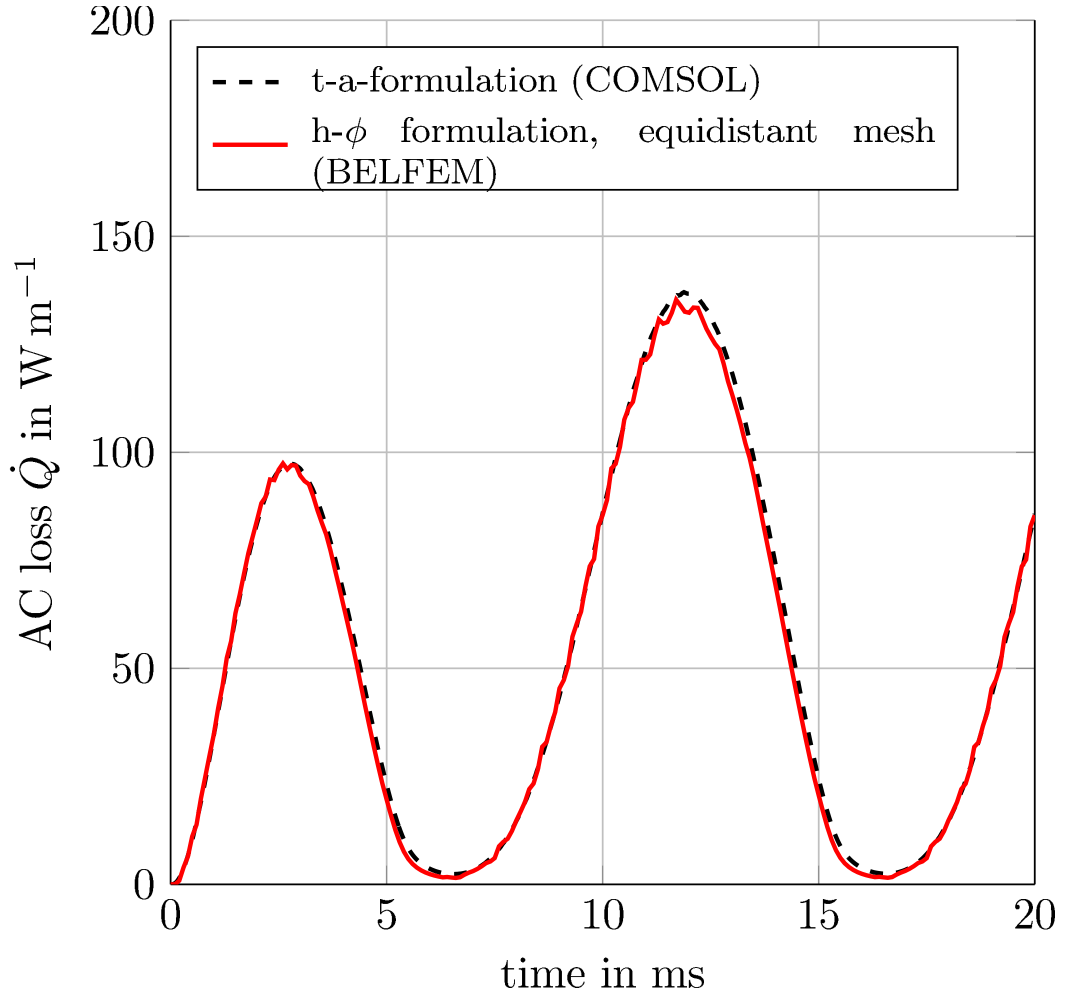

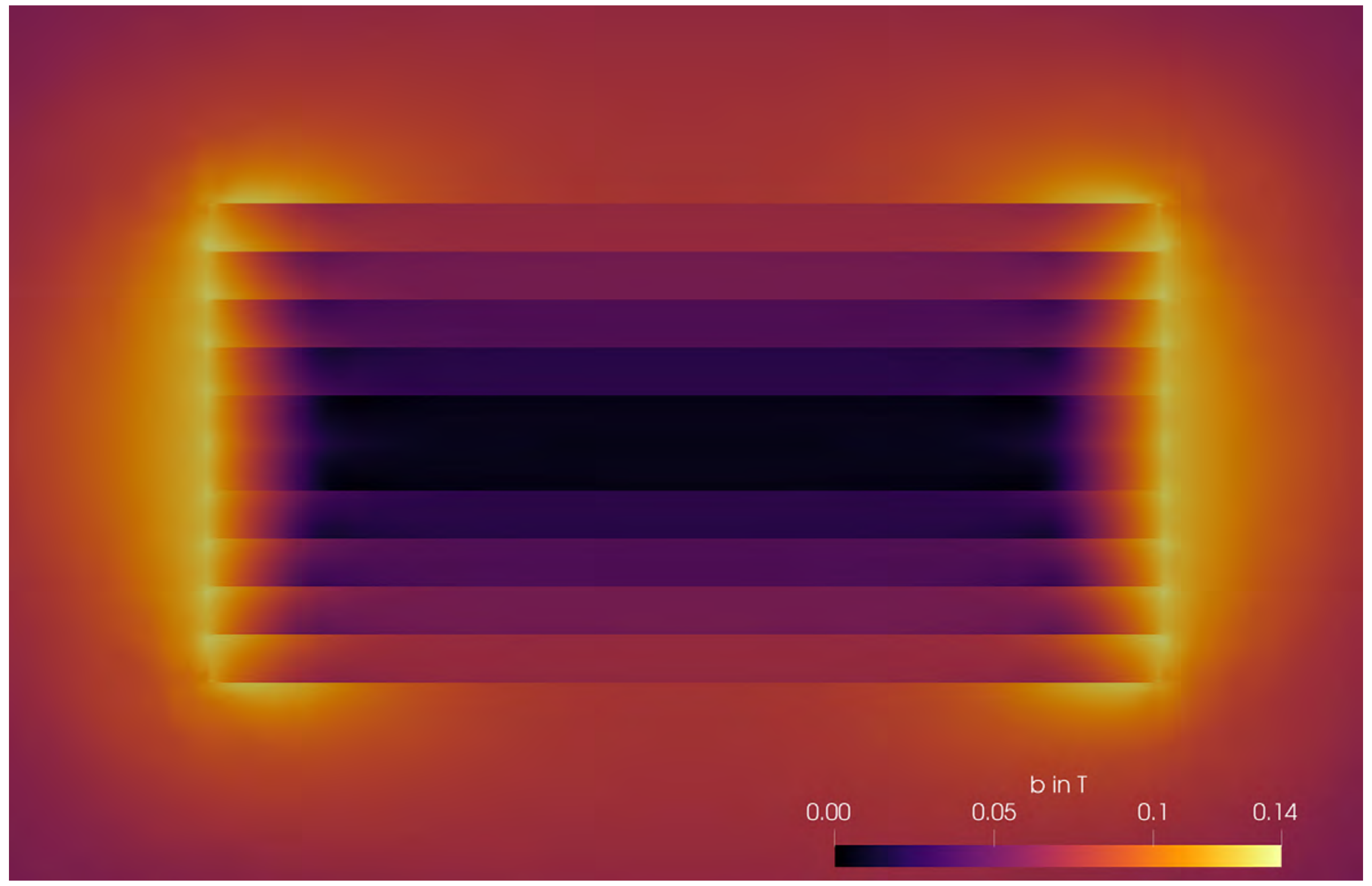

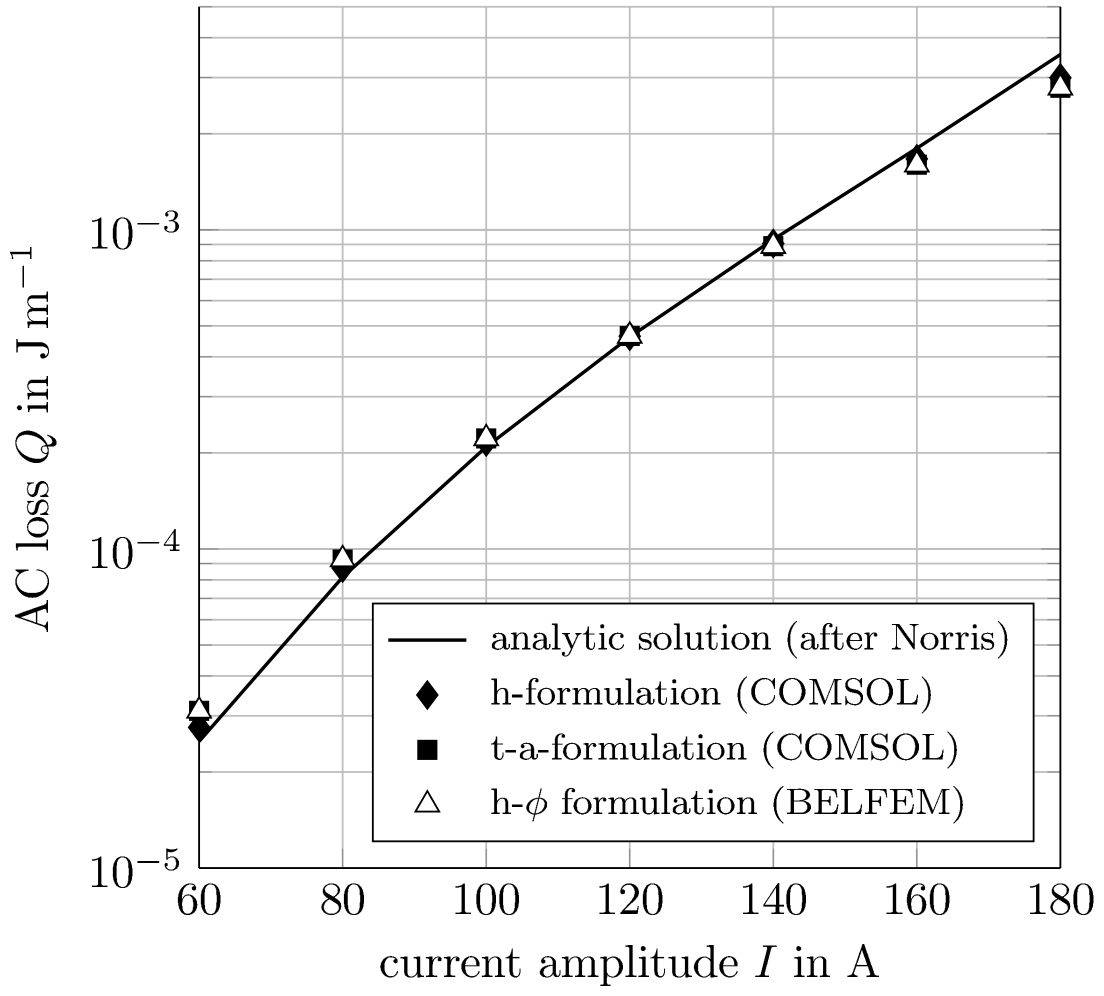

Thin Shell Formulation: 2D Results 2023

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BELFEM: a special purpose finite element code for the magnetodynamic modeling of high-temperature superconducting tapes

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Abstract
Predicting the performance and reliability of high-temperature superconducting (HTS) cables and magnets is a critical component of their research and development process. Novel mixed finite element formulations, particularly the $\boldsymbol{h}$-ϕ-formulation with thin-shell simplification, present promising opportunities for more efficient simulations of larger geometries. To make these new methods accessible in a flexible tool, we are developing the Berkeley Lab Finite Element Framework (BELFEM). This paper provides an overview of the relevant formulations, discusses the current state of the art, and discusses the main aspects of the BELFEM code structure. We validate a first 2D thin-shell implementation in BELFEM against selected benchmarks computed in COMSOL Multiphysics and compare the performance of our code with a comparable formulation in GetDP. We also outline the next steps in the development process, paving the way for more advanced and robust modeling capabilities.

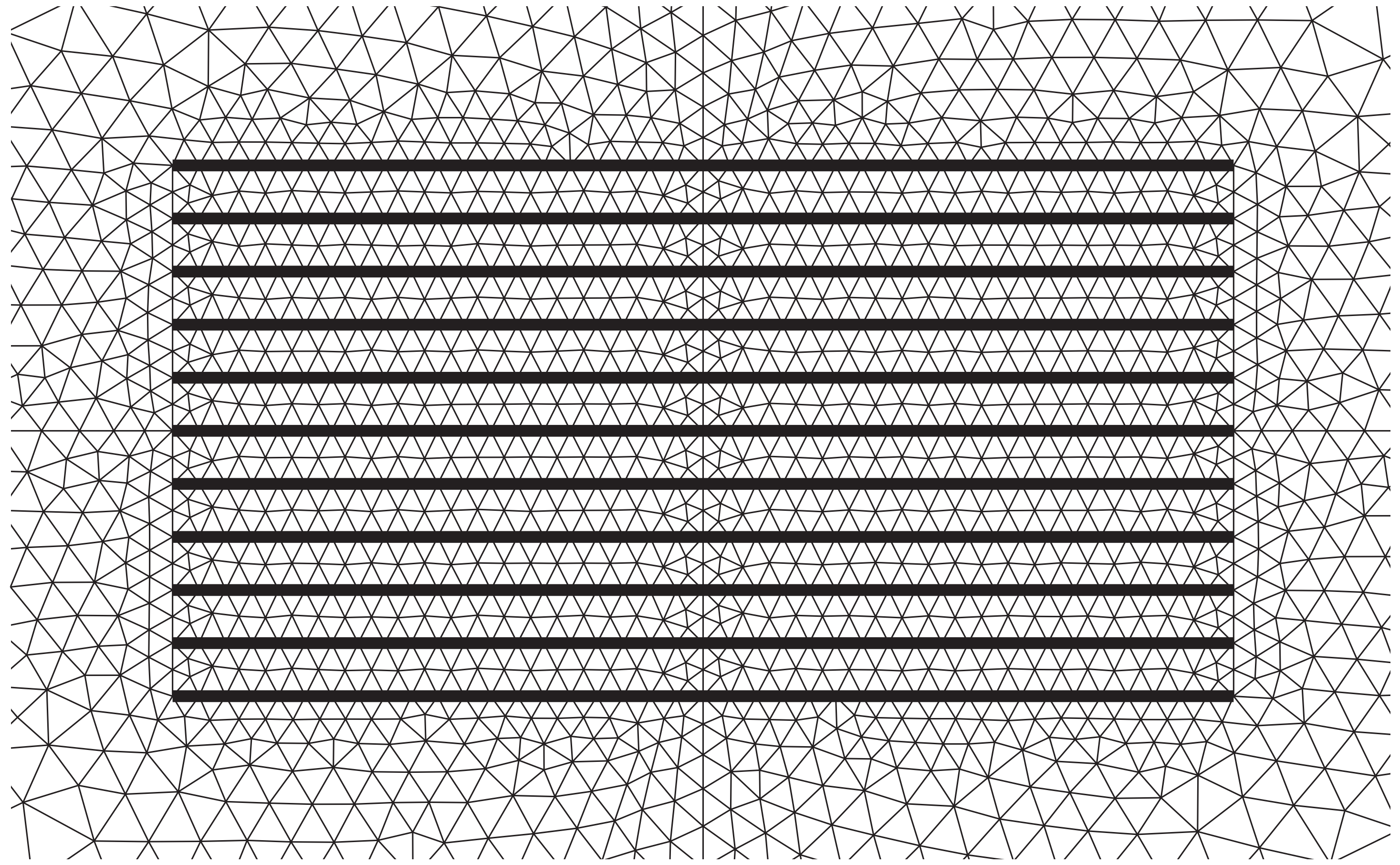
Keywords: HTS, FEM, modeling, h-ϕ-formulation, code development

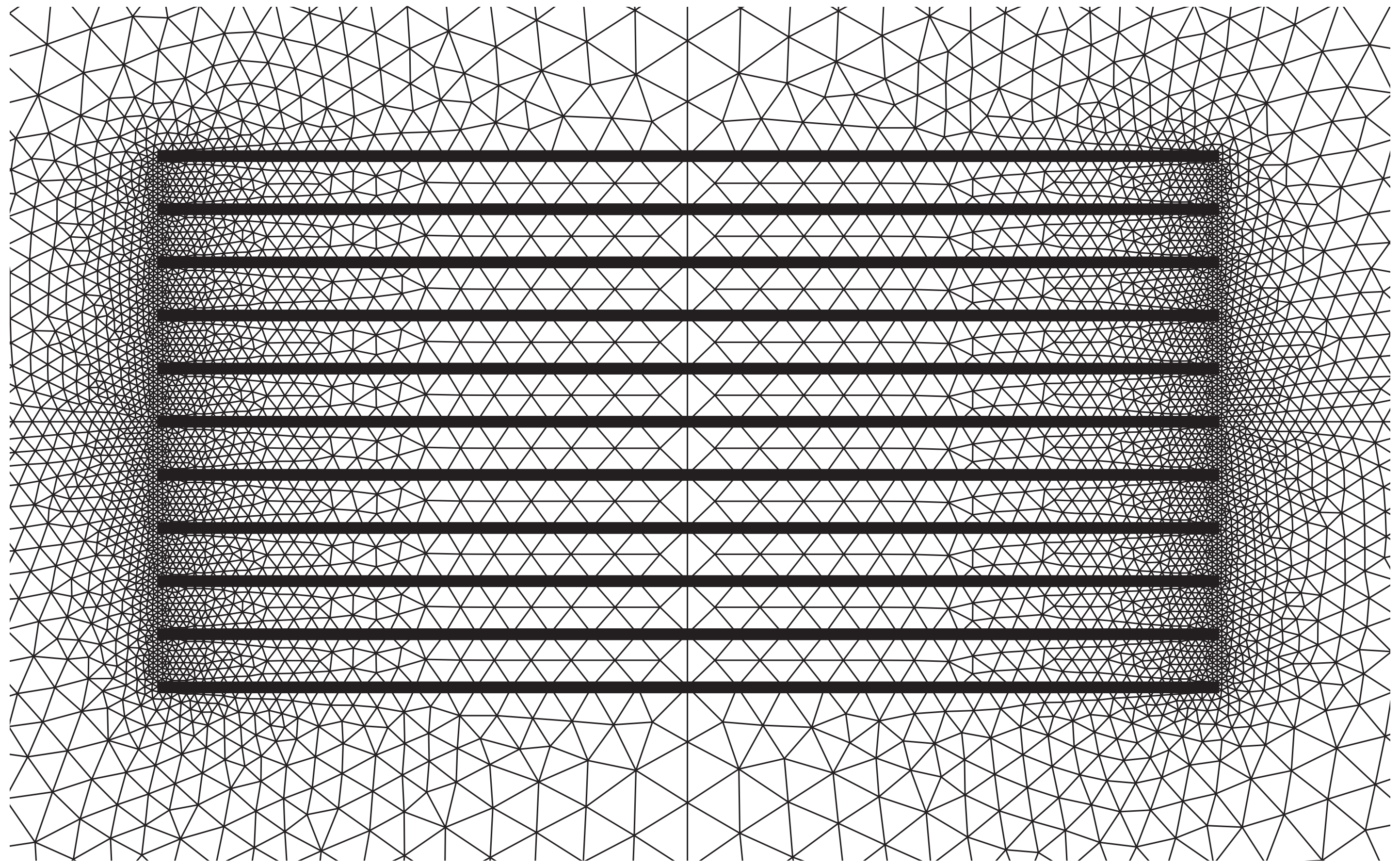
(Some figures may appear in colour only in the online journal)

1. Introduction

High-temperature superconducting (HTS) cables and magnets play a crucial role in nuclear fusion applications [1], particle accelerators [2], medical devices [3], and even spacecraft engines [4]. To ensure their safe operation, it is necessary to analyze and understand their electrodynamic and thermal performance. Here, advanced finite element methods are essential and very powerful tools: not only do they solve the Maxwell's equations in their magnetodynamic form and the heat transfer equation that describe the physical behavior of these devices, they are also able to realistically model the highly nonlinear behavior of the used materials. The computational cost of these methods, however, remains an important challenge. In recent years, significant progress has been made in the development of so called *mixed formulations*, which promise to be more efficient than traditional finite element

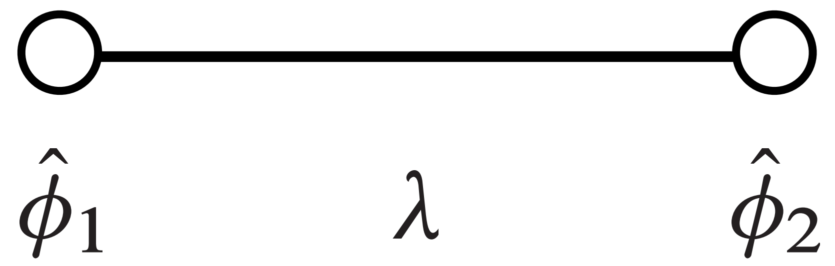
* Author to whom any correspondence should be addressed.





Code and Library	Constant time step		Adaptive time step	
	Coarse Mesh	Fine Mesh	Coarse Mesh	Fine Mesh
GetDP (MUMPS)	5:13	10:57	2:36	7:58
BELFEM (MUMPS)	1:56	7:15	0:23	1:09
BELFEM (STRUMPACK)	0:55	2:54	0:11	0:25

Lagrange Multipliers: Basic Concept



Lagrangian Multipliers:

- well known from “contact problems”
- used for h-ϕ by Arsenault 2020 [10.1109/tasc.2020.3033998]
- “pointwise constraint” in COMSOL

Pro:

- easy implementation

Con:

- poor conditioning
- zeros on main diagonal

$\Longrightarrow$ **approach not feasible for large 3D problems!**

Solution:

- use static condensation instead!

potential

$$\Pi = \lambda \left(\hat{\phi}_1 - \hat{\phi}_2 - I \right) \stackrel{!}{=} \text{min.}$$

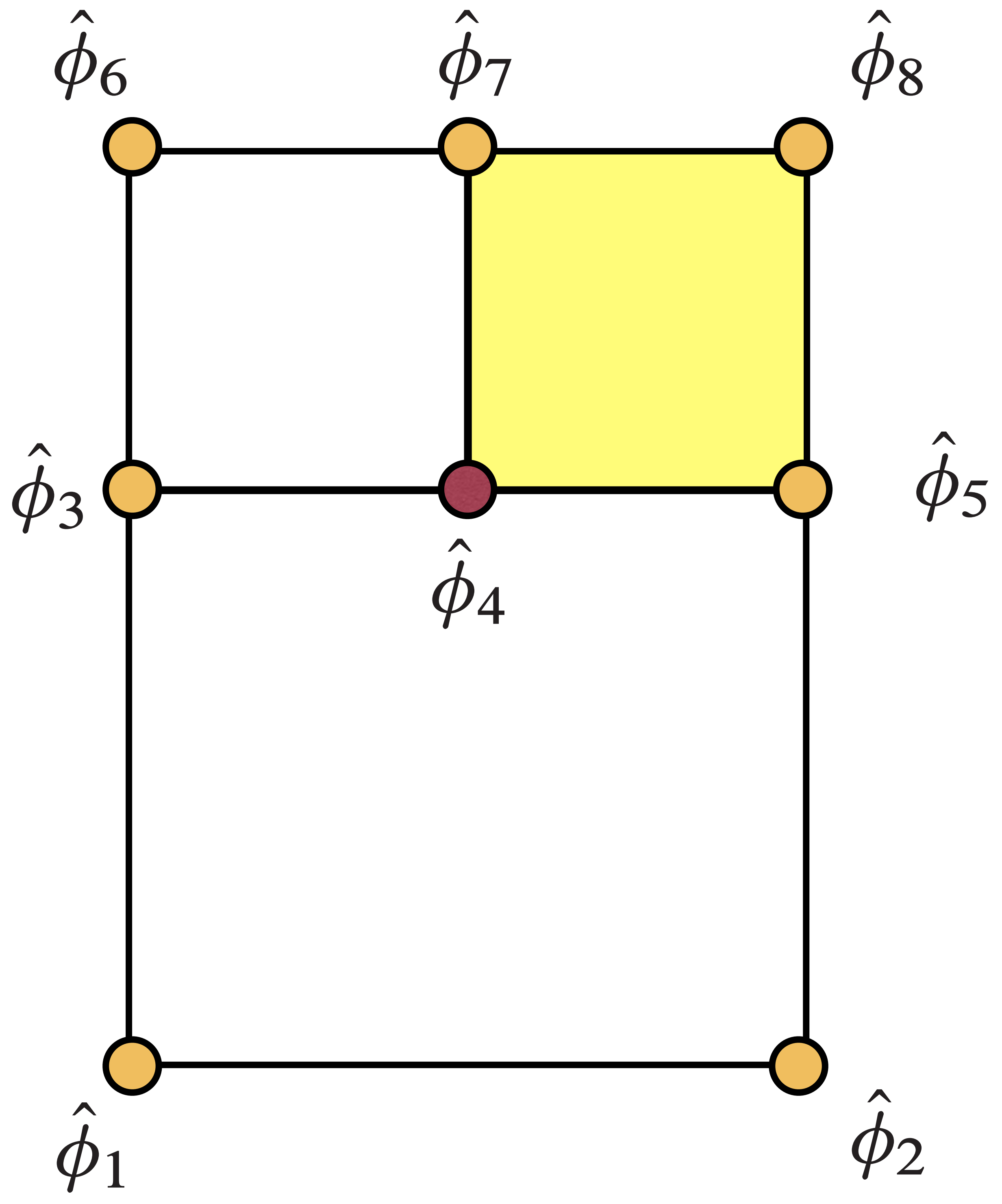
first variation

$$\delta\Pi = \delta\lambda \left(\hat{\phi}_1 - \hat{\phi}_2 - I \right) + \lambda \left(\delta\hat{\phi}_1 - \delta\hat{\phi}_2 \right) = 0$$

rewrite in matrix form

$$\begin{bmatrix} \delta\hat{\phi}_1 & \delta\hat{\phi}_2 & \delta\lambda \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} \delta\hat{\phi}_1 & \delta\hat{\phi}_2 & \delta\lambda \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

Static Condensation: Basic Concept



- Node 4 is "hanging" $\longrightarrow$ dof 4 is expressed as linear combination of dof 3 and 5

$$\begin{bmatrix} \delta\hat{\phi}_4 & \delta\hat{\phi}_5 & \delta\hat{\phi}_8 & \delta\hat{\phi}_7 \end{bmatrix} \mathbf{K} \begin{bmatrix} \hat{\phi}_4 \\ \hat{\phi}_5 \\ \hat{\phi}_8 \\ \hat{\phi}_7 \end{bmatrix} = \begin{bmatrix} \delta\hat{\phi}_3 & \delta\hat{\phi}_5 & \delta\hat{\phi}_8 & \delta\hat{\phi}_7 \end{bmatrix} \mathbf{T}^{\mathbf{T}} \mathbf{K}\, \mathbf{T} \begin{bmatrix} \hat{\phi}_3 \\ \hat{\phi}_5 \\ \hat{\phi}_8 \\ \hat{\phi}_7 \end{bmatrix}$$

- Change of basis us performed using T-Matrix

$$\mathbf{T} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Method can be used for both domain interfaces and domain cuts

Lessons Learned

“Bleeding”

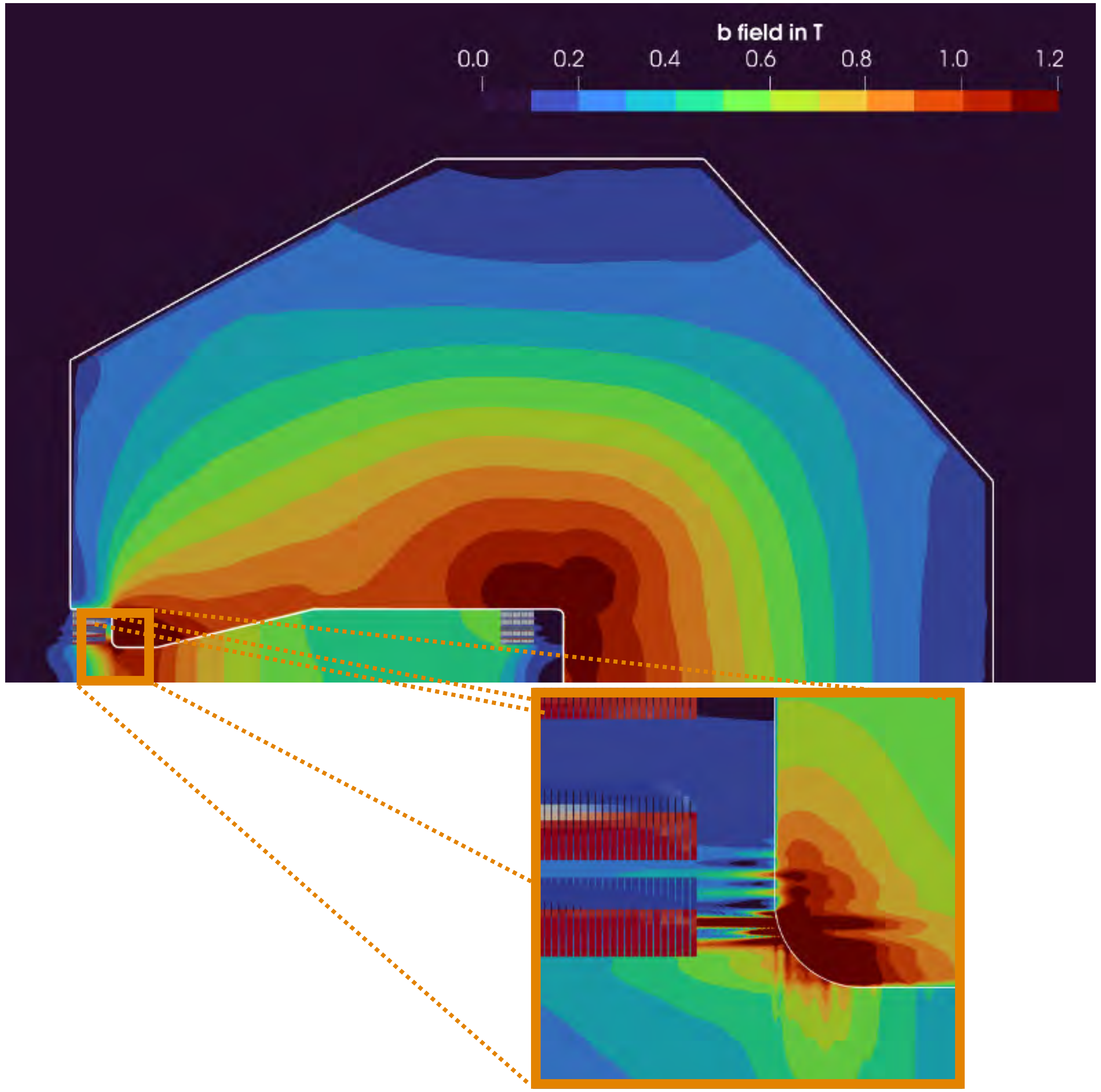
- caused by abrupt change of μ along ϕ-a interface
- documented in Dular 2021 [10.1109/TASC.2021.3098724]

Solution:

- stabilize interpolation on ϕ-side using enriched shape functions

$$\phi \approx N_i\,\hat{\phi}_i + E_{ij} N_j\ \hat{\psi}_{ij}$$

- this XFEM approach is well known in other physical domains



Homologies and Cohomologies

H-ϕ formulation: Homologies and Cohomologies

Boundary Conditions

current is applied over Ampere's circuital law:

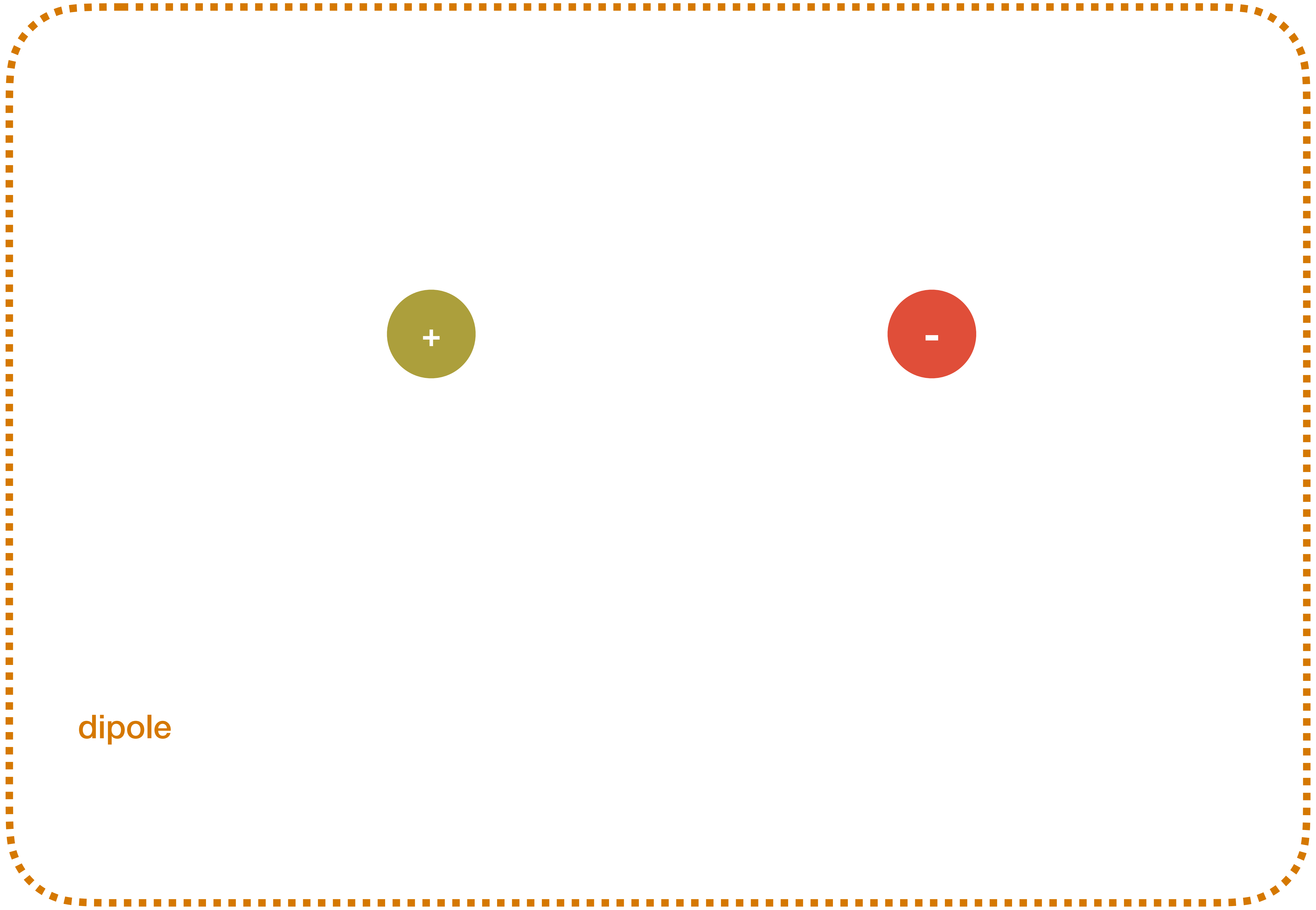
Ampére's circuital law $\oint \boldsymbol{h}\,\mathrm{d}l = I$

- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampere's law

- only integral current I needs to be known

- cohomologies are cuts in the domain over which jumps in the magnetic potential ϕ are imposed so that Δϕ=I.

➡ <u>very elegant mathematics!</u>
➡ <u>homology definition not user friendly</u>
➡ <u>difficult to implement in commercial codes</u>



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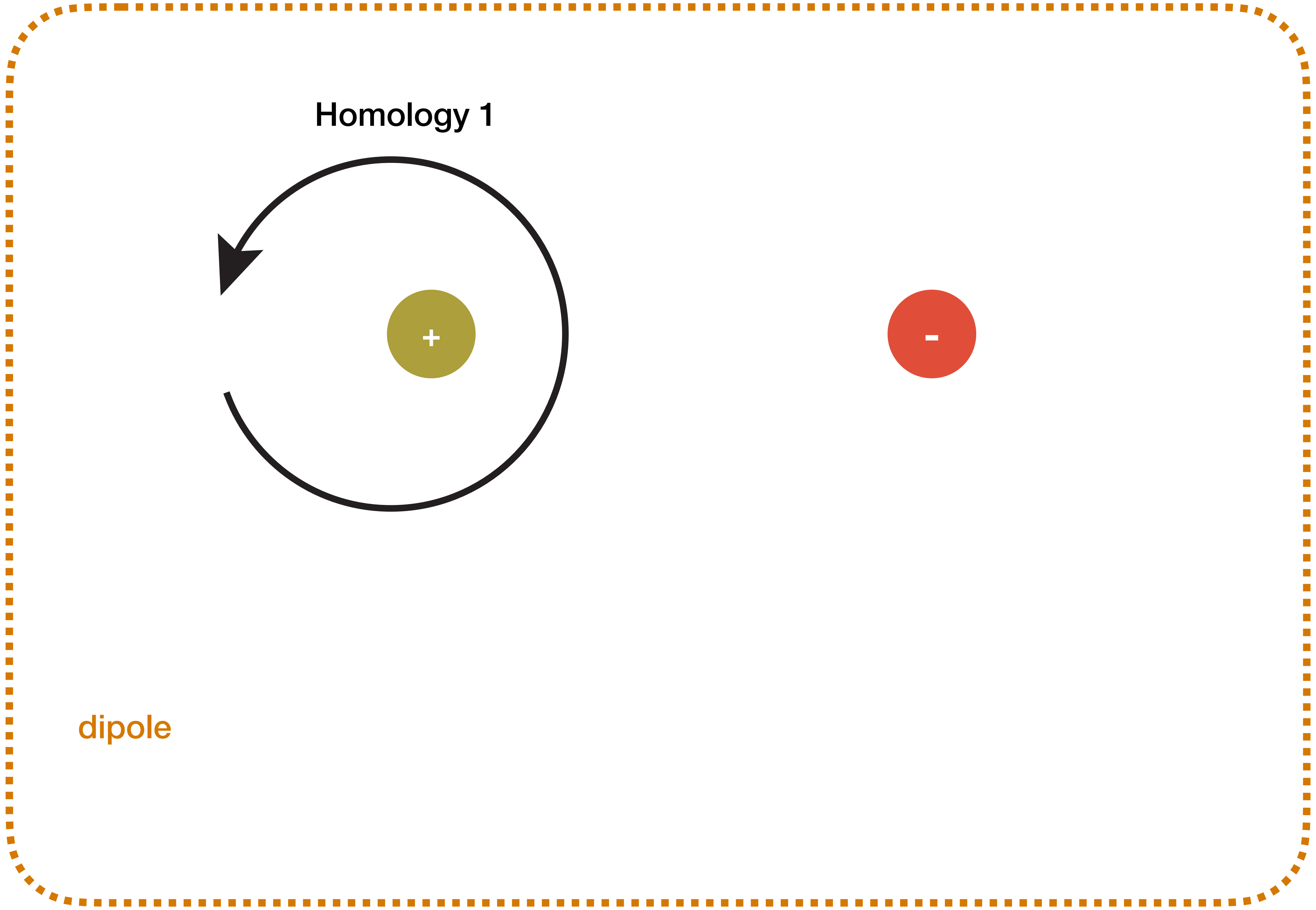

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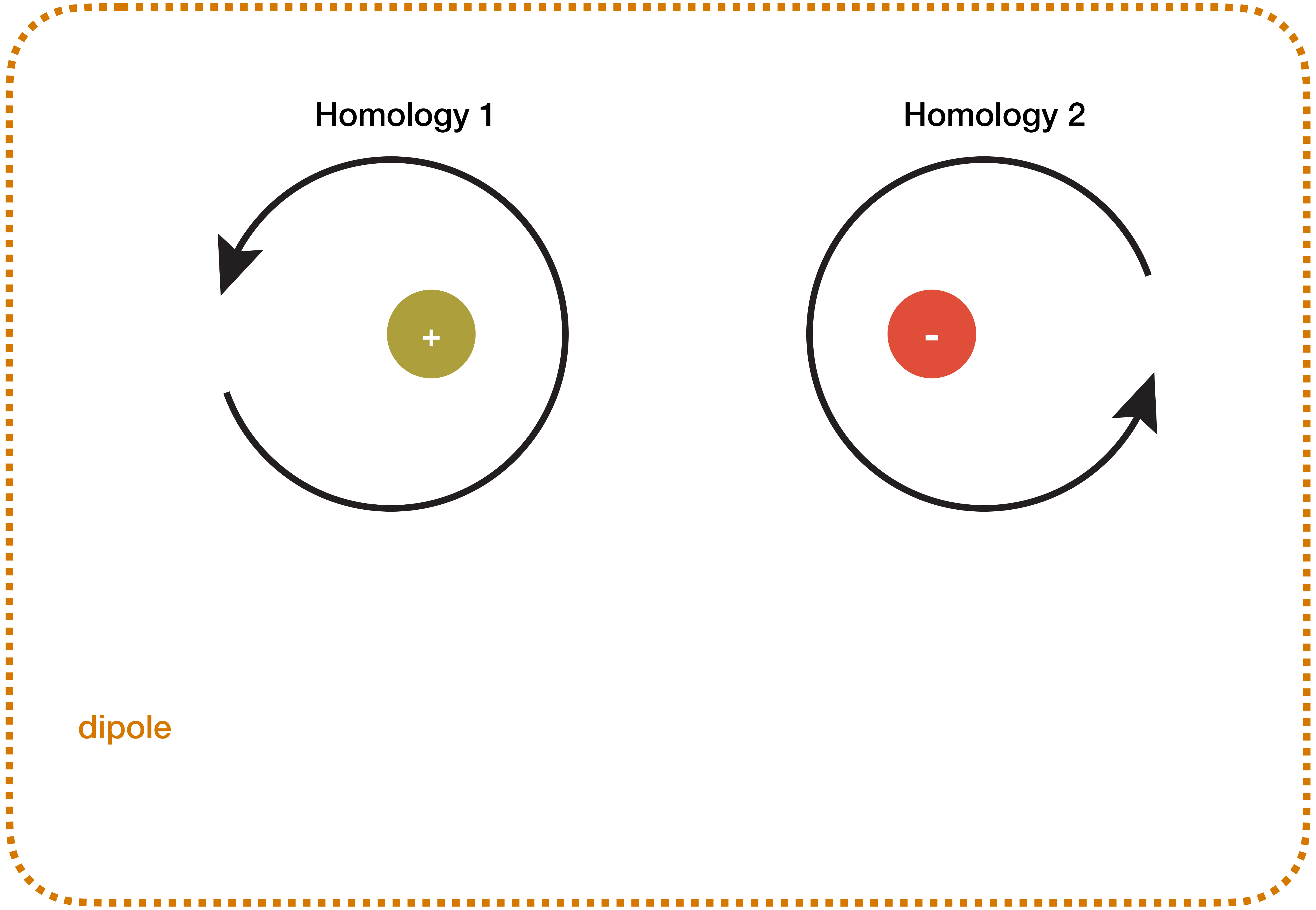

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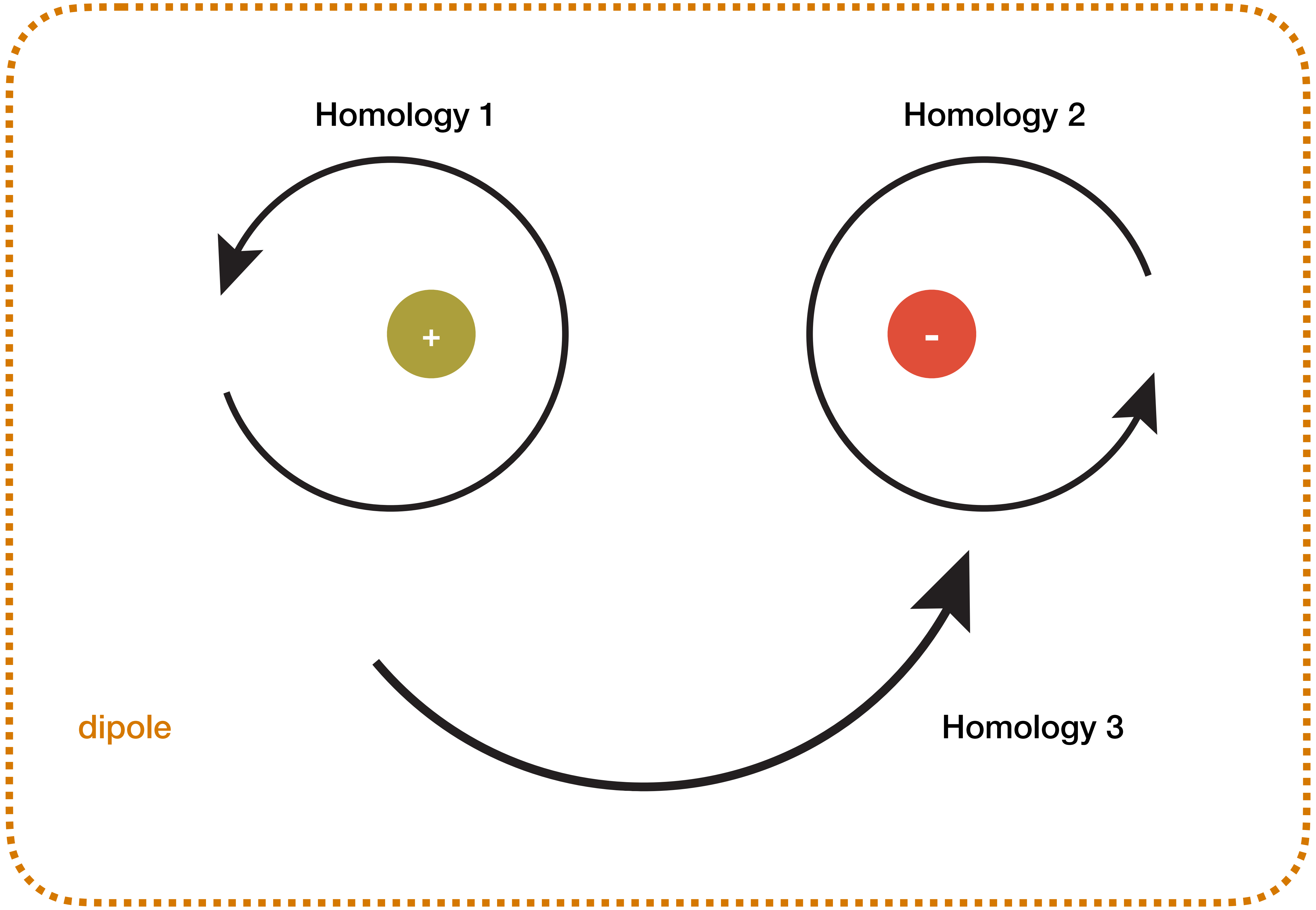

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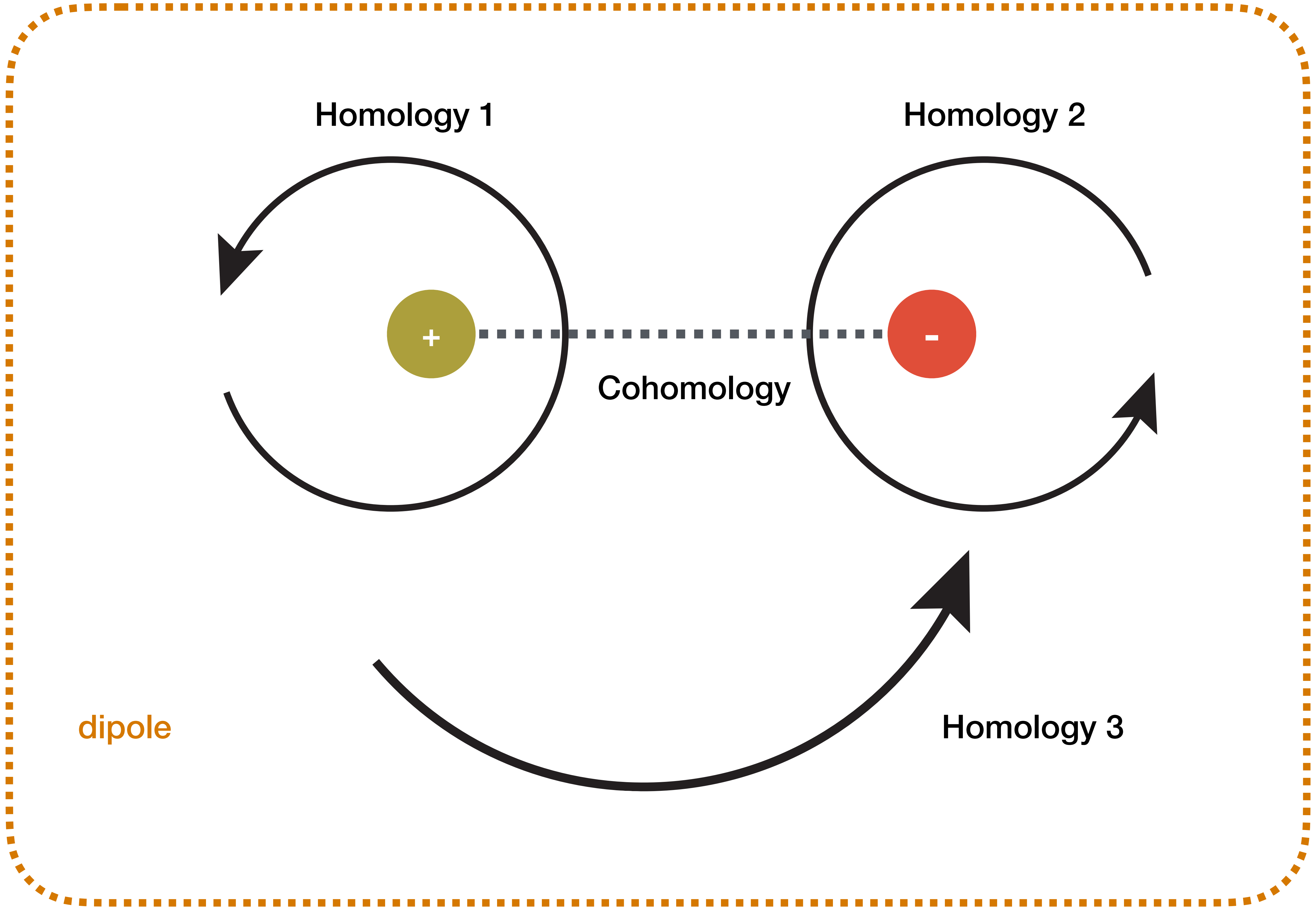
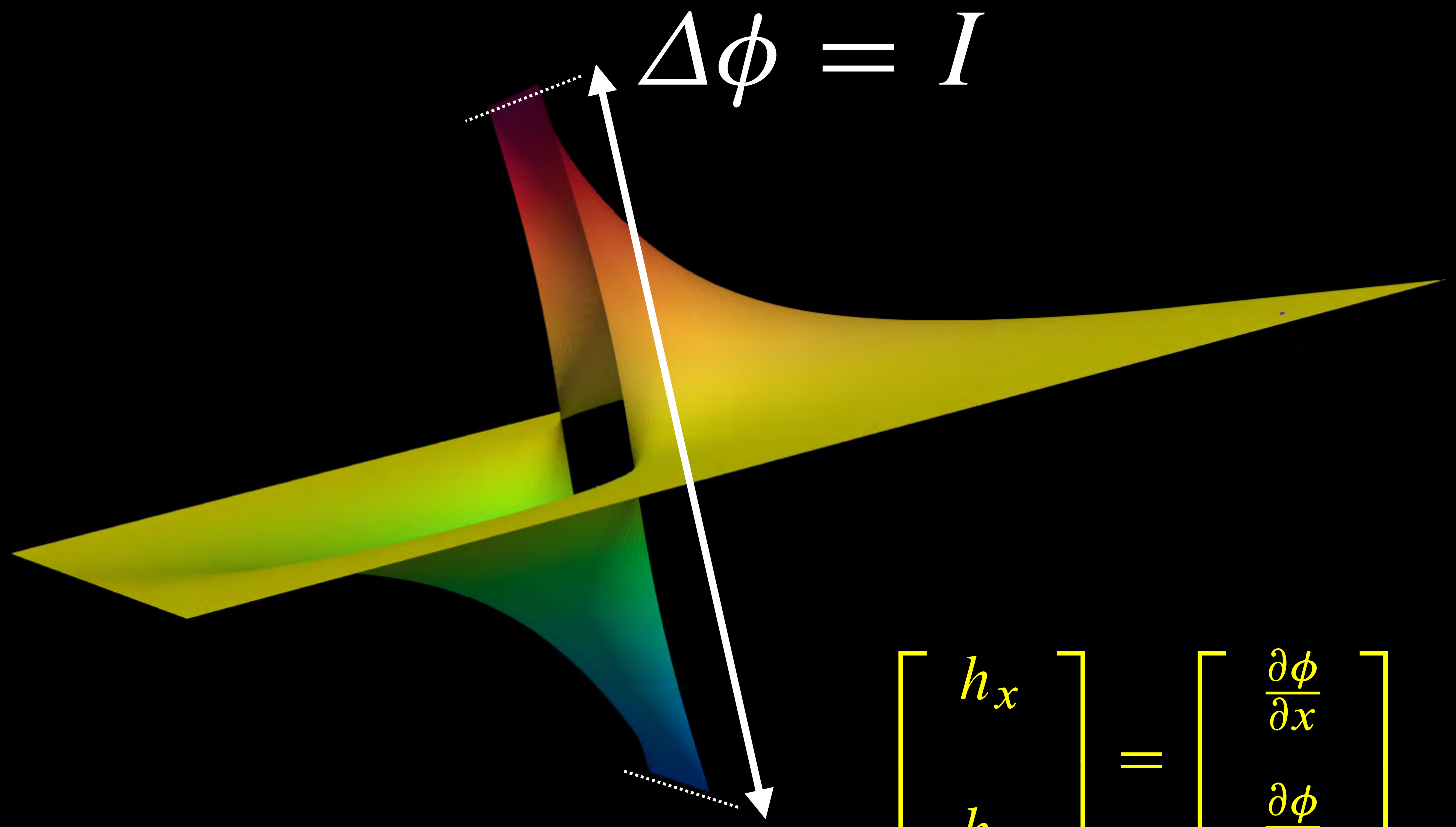
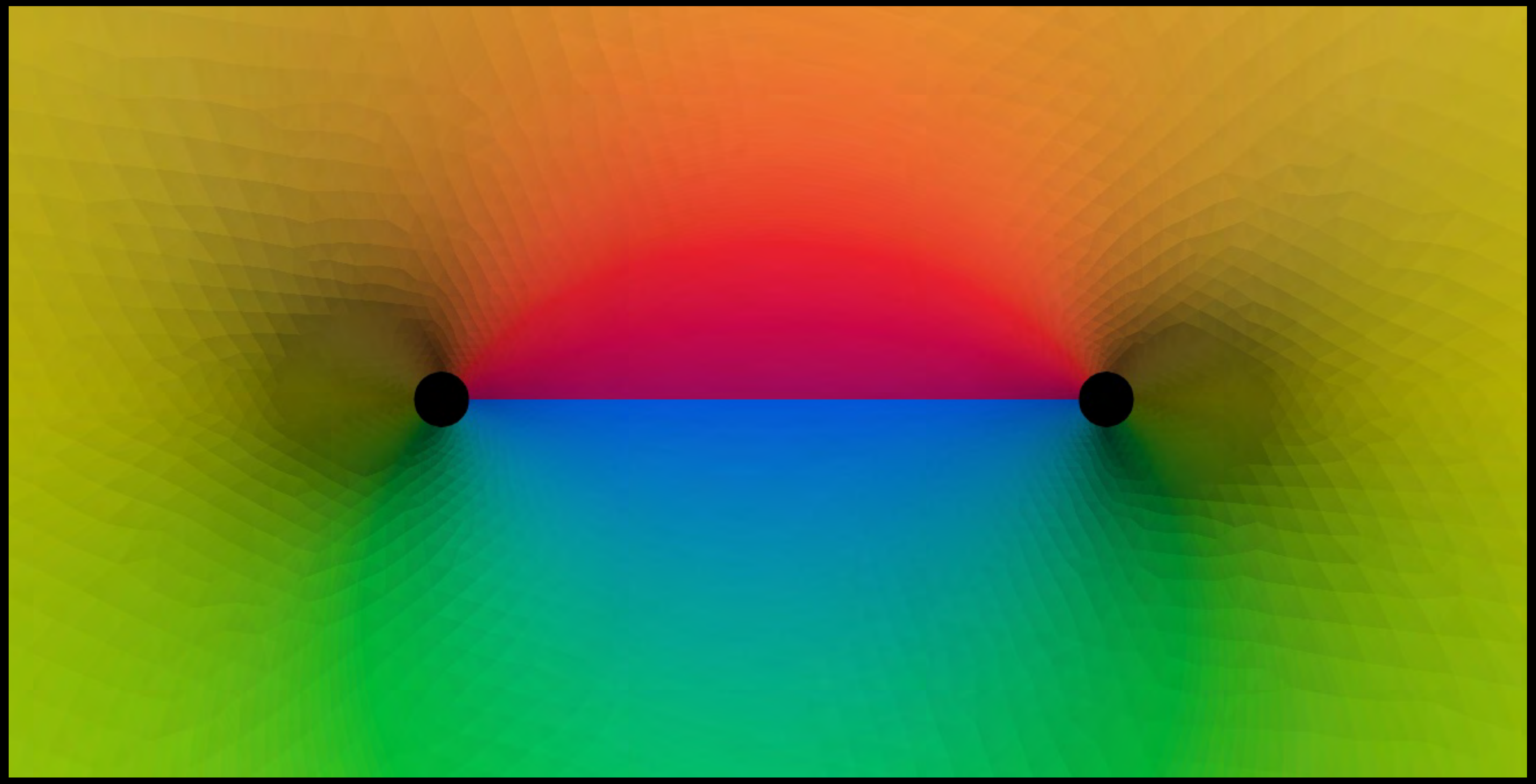


H-ϕ formulation: Homologies and Cohomologies



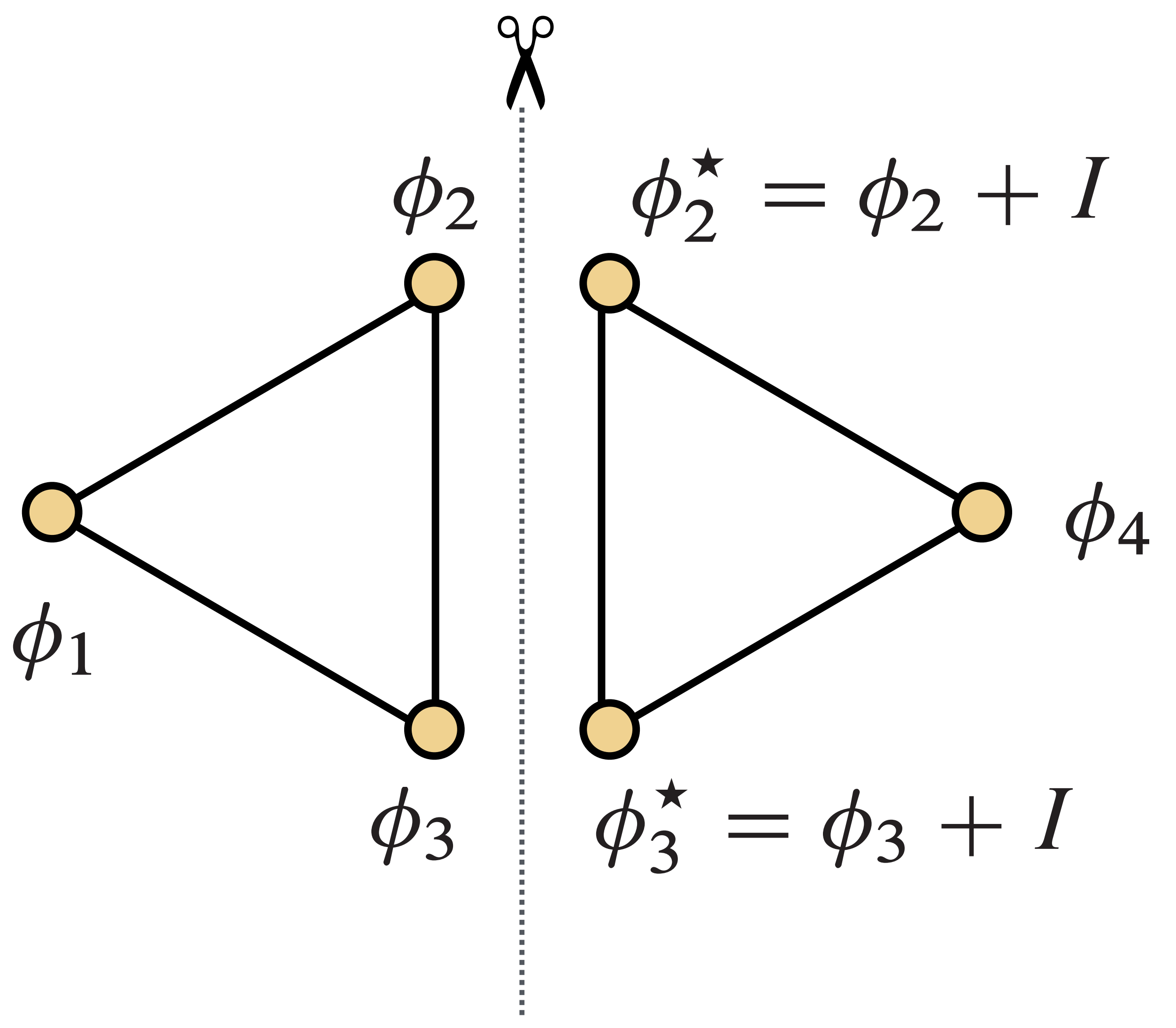
$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \frac{\partial\phi}{\partial y} \end{bmatrix}$$



H-ϕ formulation: Thin vs. Thick cuts

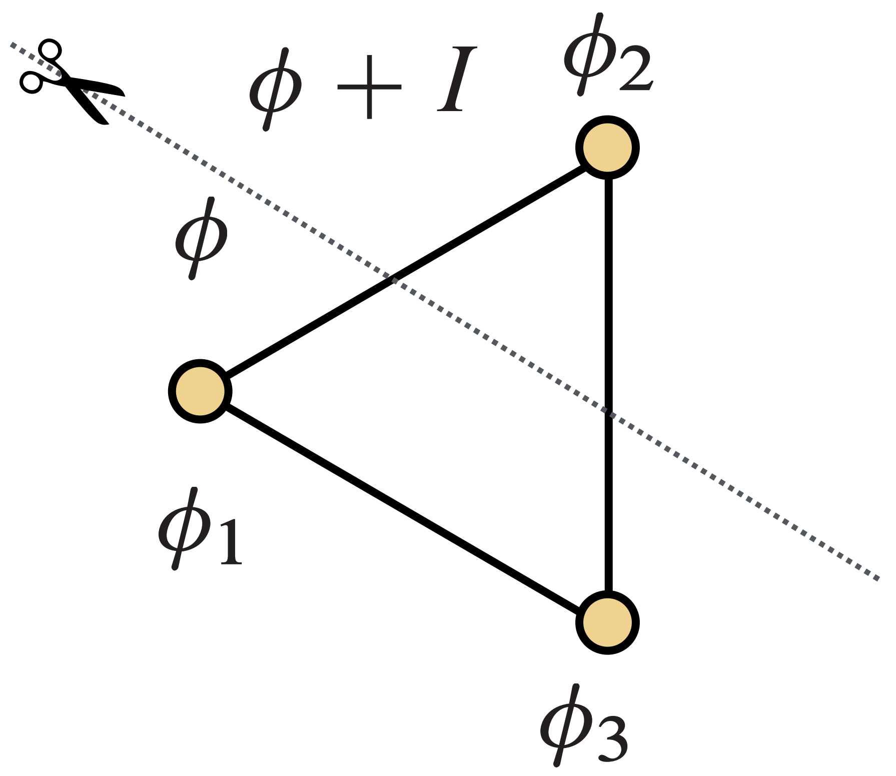
Thin Cut

- jump of potential is between elements
- nodes and degrees of freedom at cut are duplicated
- implemented using Lagrange Multipliers or Condensation



Thick Cut

- jump of potential is along element
- interpolation function needs to be enriched
- implemented using XFEM



- **Literature does not say which is faster or more stable!**
- **User can not be expected to define cuts themselves**
  - ➡ **need robust algorithm with minimal user interference**

Cohomologies: Work in Progress

PhD Student Gregory Giard (Polytechnique Montreal):

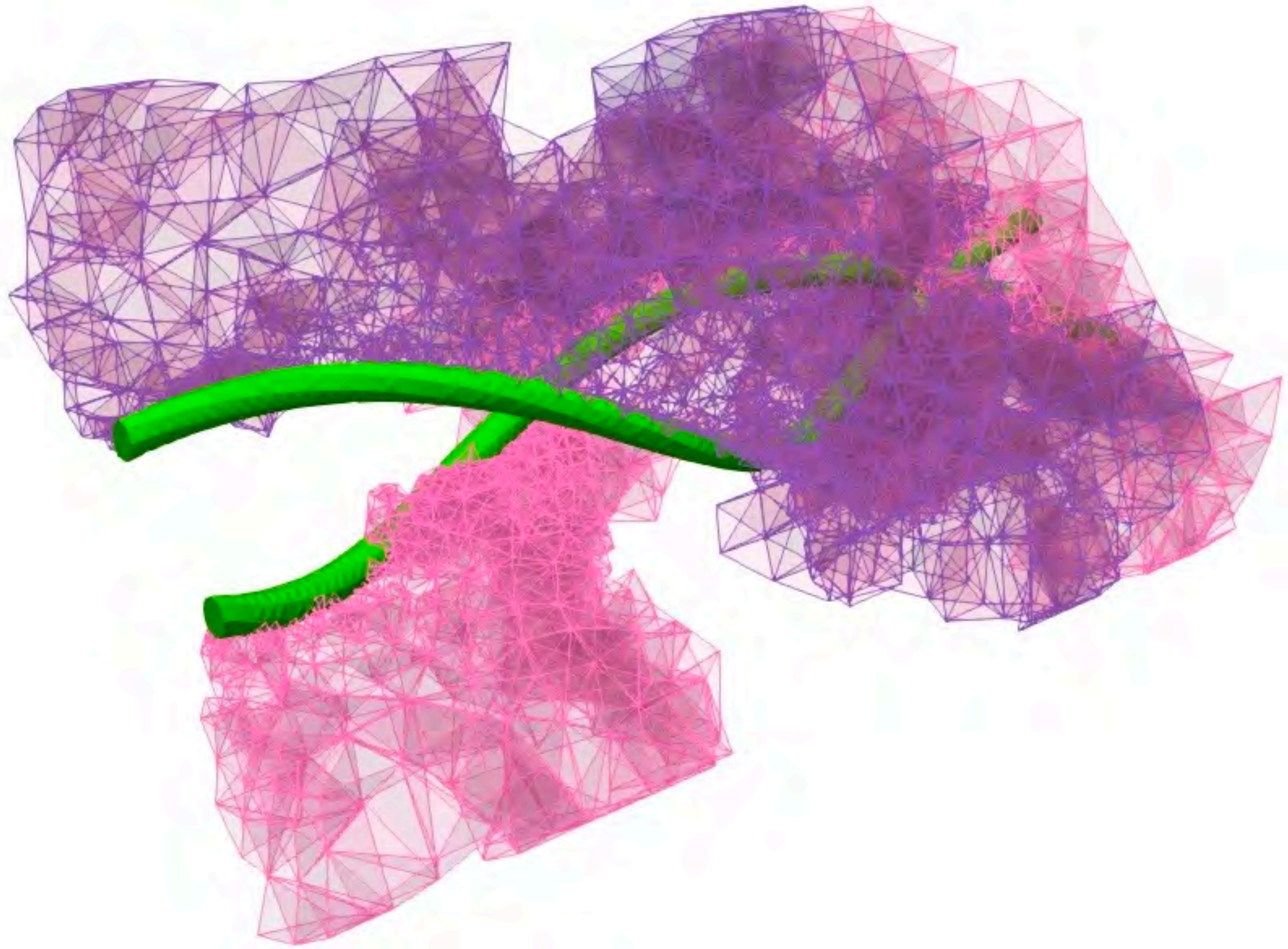
- visiting scholar at LBL from 01/23-06/23

- contribution to adaptive time stepping method
- development of 3D thermal conduction model

- implementing automated cohomology computation in 3D

- automated identification of cut orientations based on user provided currents (“the user shall not worry about cohomologies”)

- clean formulation of “thin” and “thick” cuts using “static condensation” for the former and XFEM for the latter.



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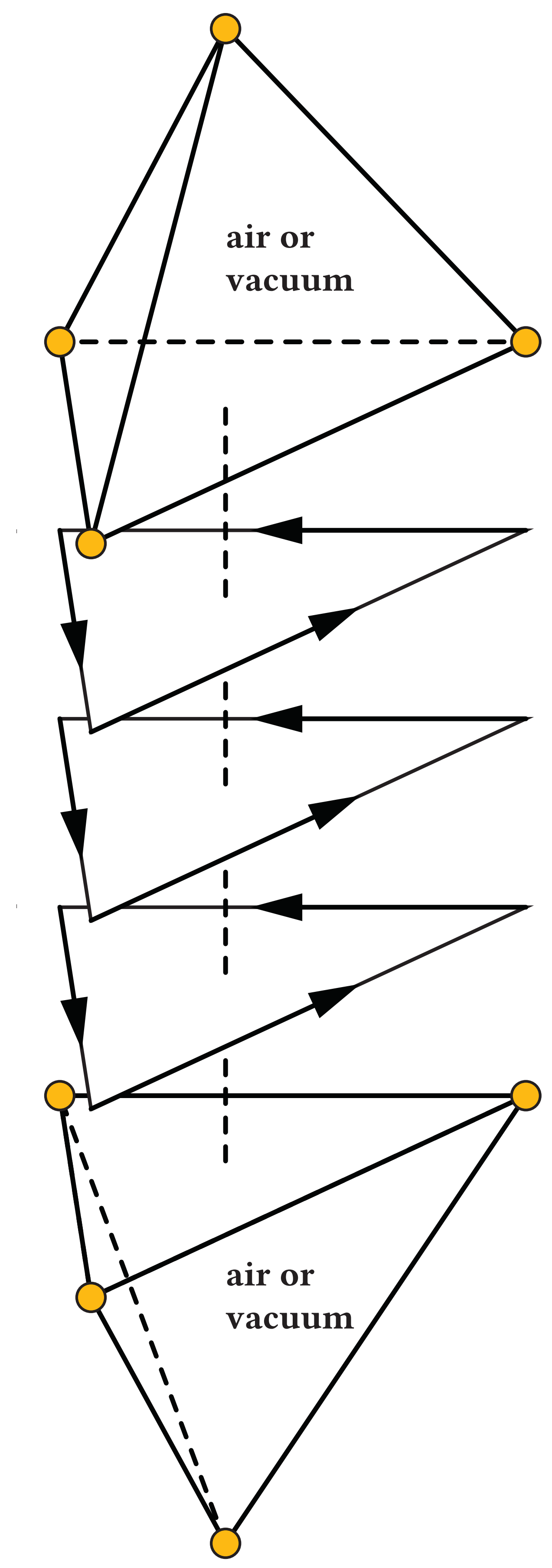
Next Steps

Current Efforts

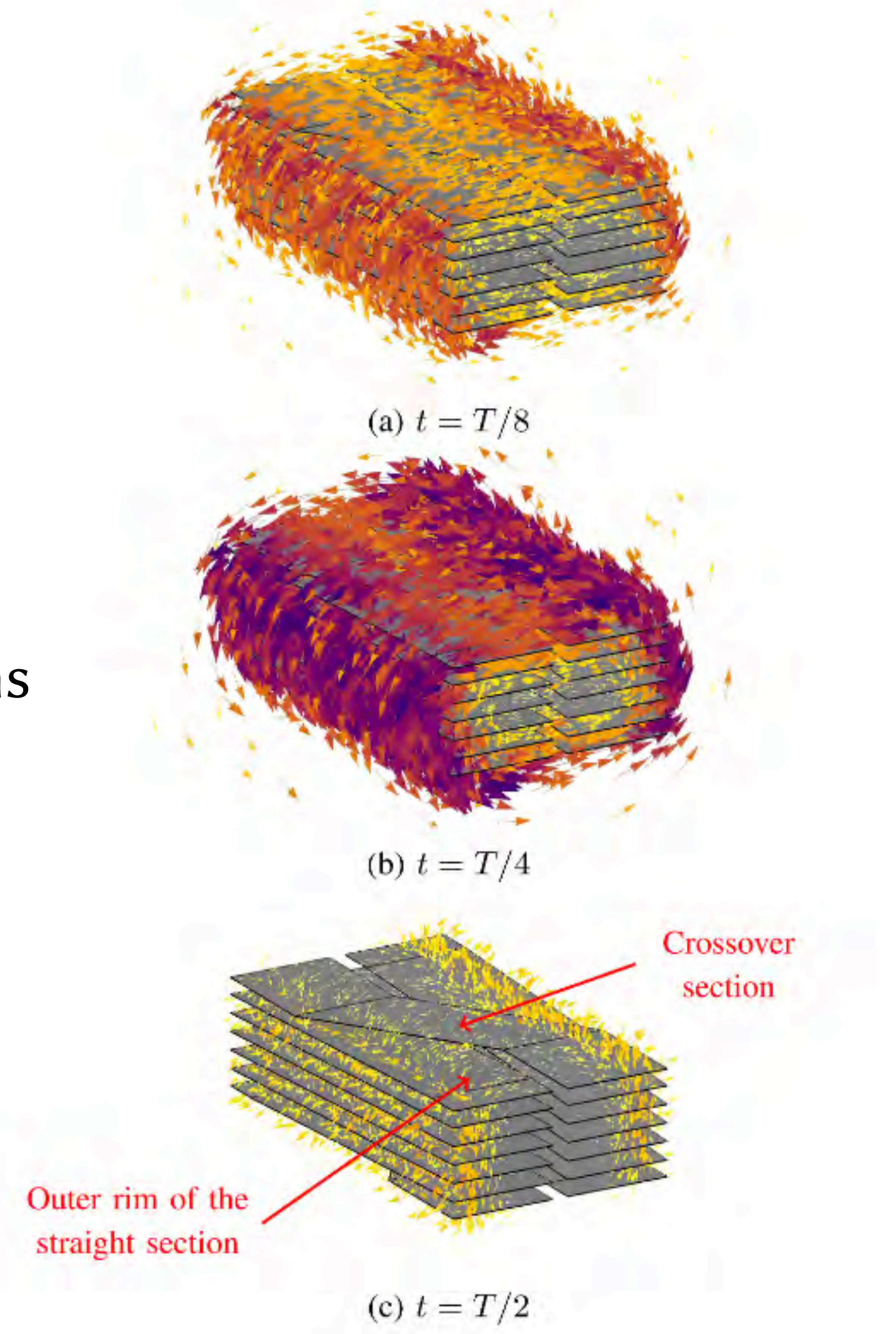


Goal:

- model a thin shell tapes tack in 3D after Alves et Al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

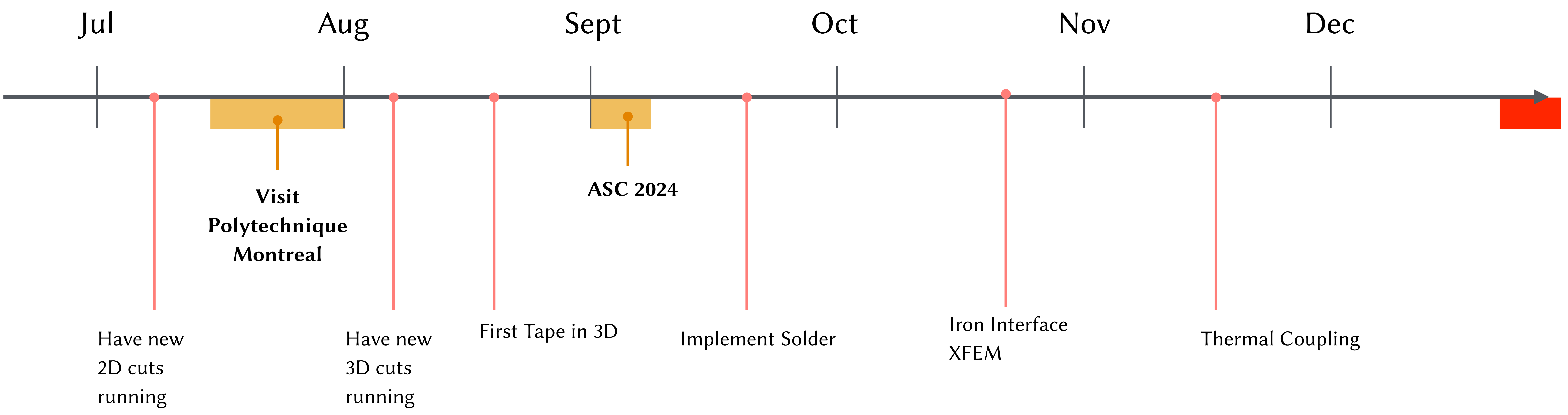
Roadmap:

- overhaul data structure for simplified programming of weak governing equations
- improve degree of freedom management system to support static condensation
- first benchmark with 3D tapestack
- implement solder and thermal model
- benchmark involving quench
- address contact sharing (Spring 2025)



[Alves et al, 10.1109/TASC.2022.3143076]

Tentative Timelime



Goals

- Demonstrate Quench Propagation in Winter
- Begin Intertape Current Sharing in Early 2025