Current Status of BELFEM

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Tue Oct 8th, 2024

BERKELEY LAB

Outline

- Recap: Thin Shell Approach
- a-formulation vs ϕ-formulation
- Current BCs using Cohomology cuts
- Implementing jump conditions

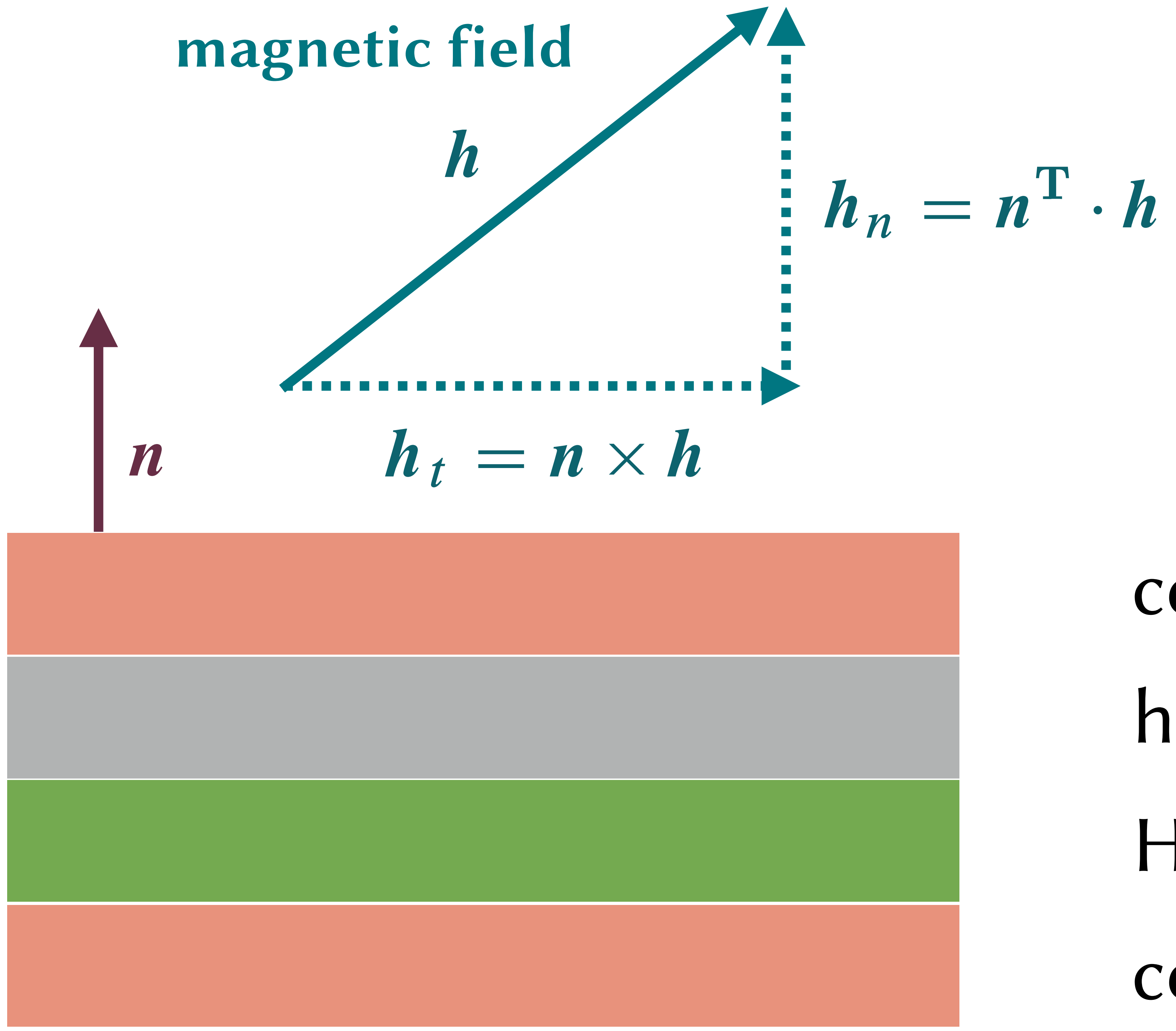
Recap: Thin Shells using H-ϕ

Sirois' Thin Shell Approach: How it Works

Typical HTS stack

copper	20 µm
hastelloy	80 µm
HTS	2 µm
copper	20 µm

Sirois' Thin Shell Approach: How it Works



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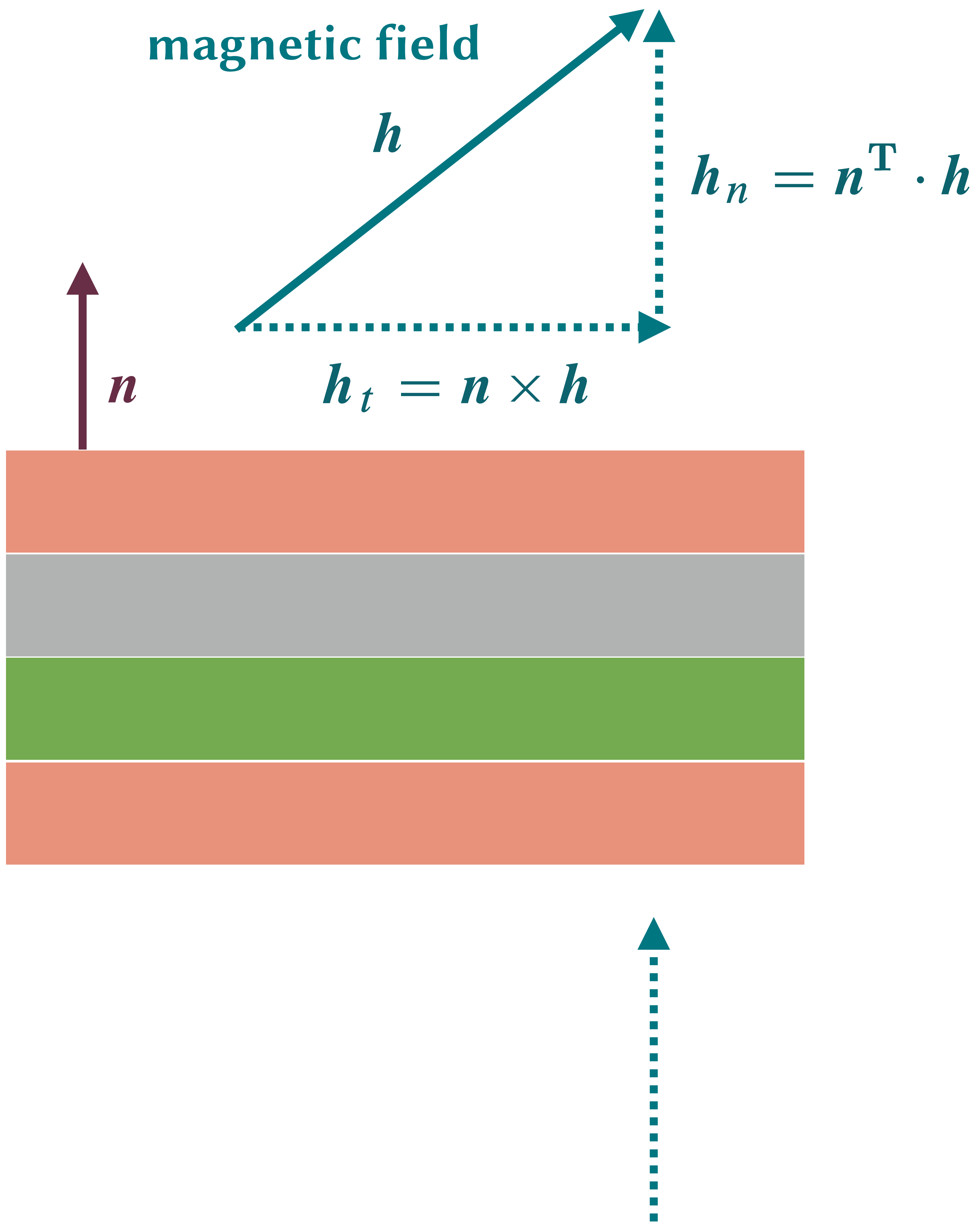

- normal component of **h** is assumed to be constant along thickness
- only in-plane component of **h** is resolved along layers
- Nédélec interpolation of **ht** in plane
- Lagrange interpolation of **ht** within layers along thickness
- coupled to connected air elements using static condensation

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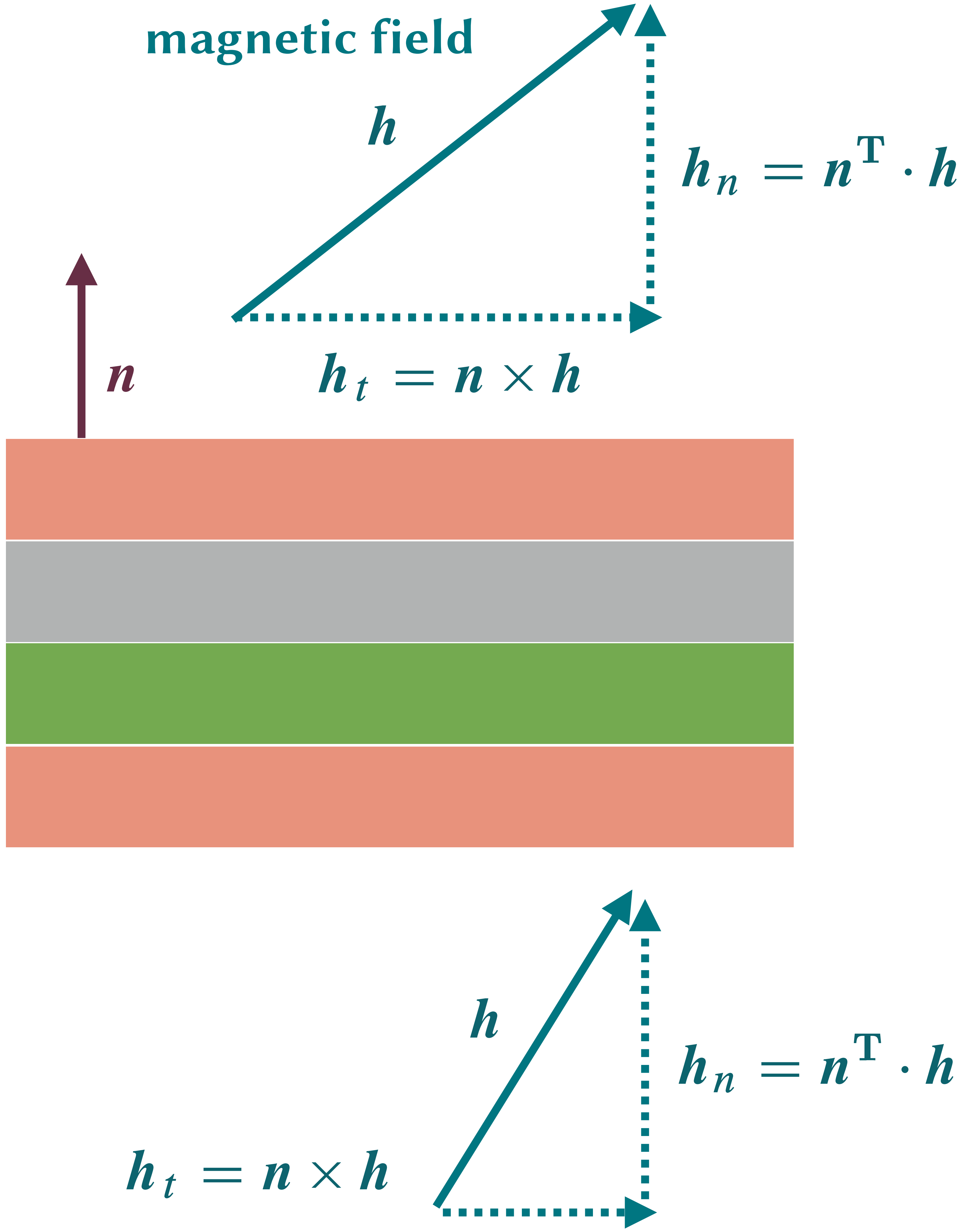

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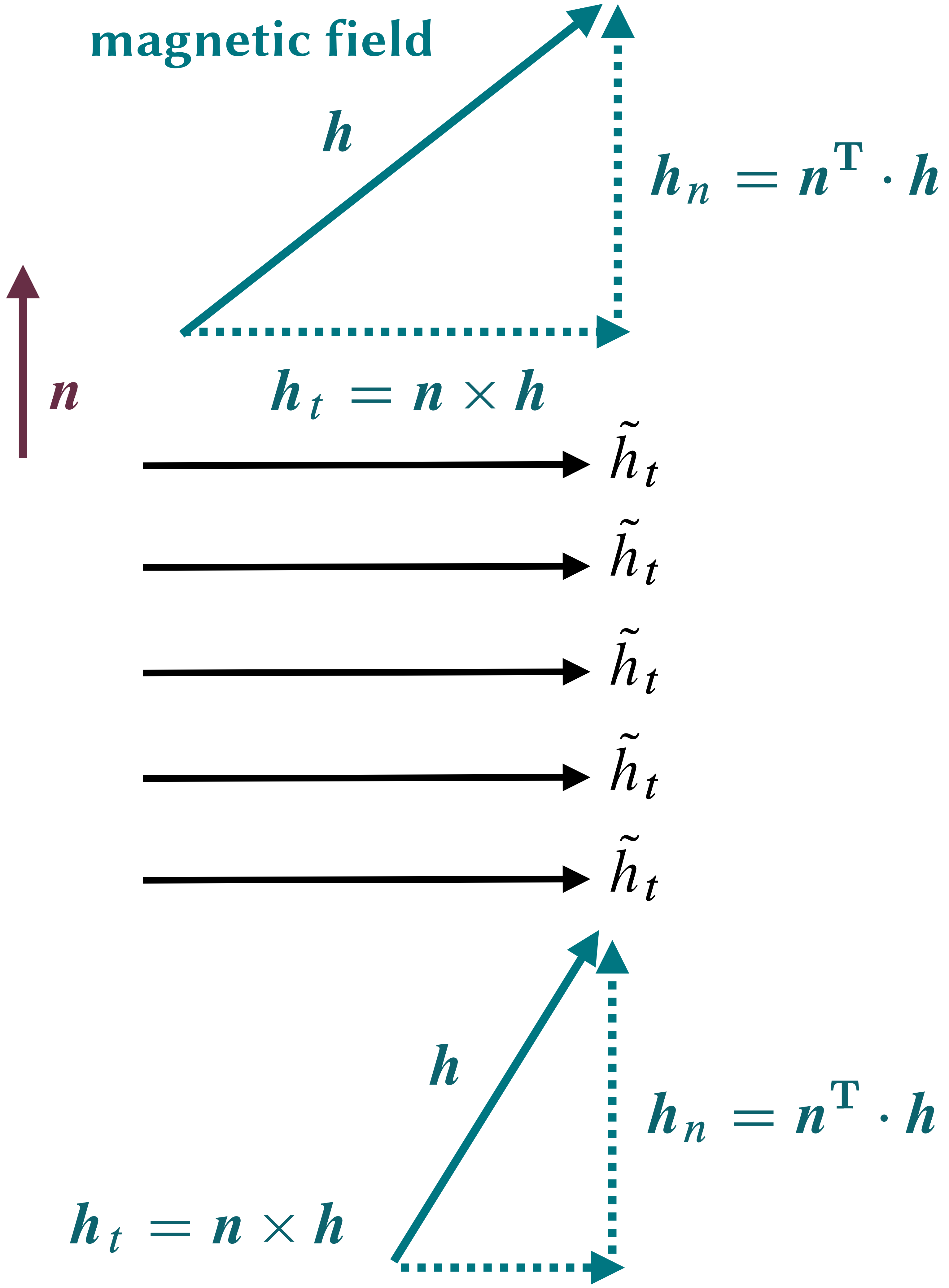

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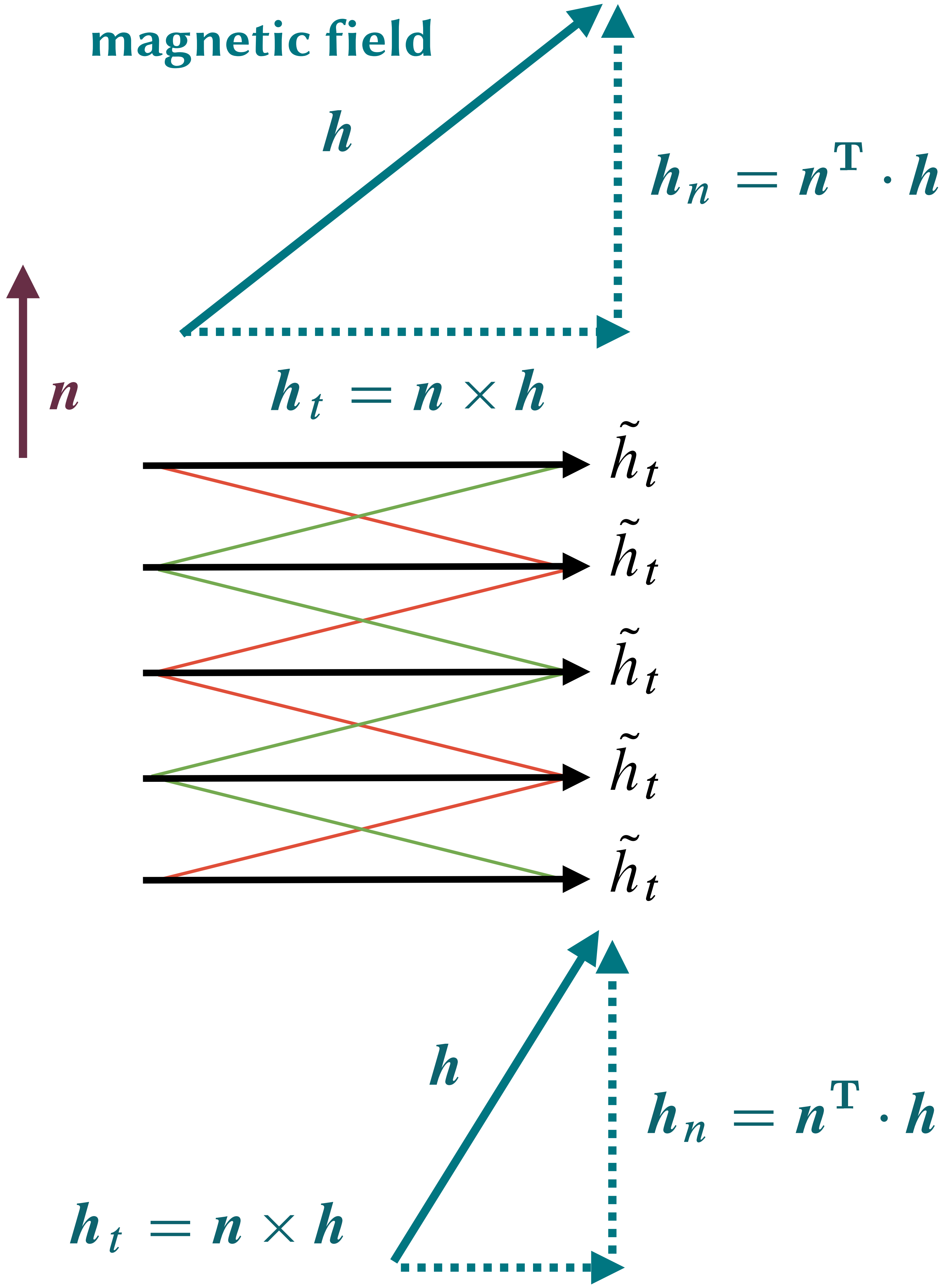

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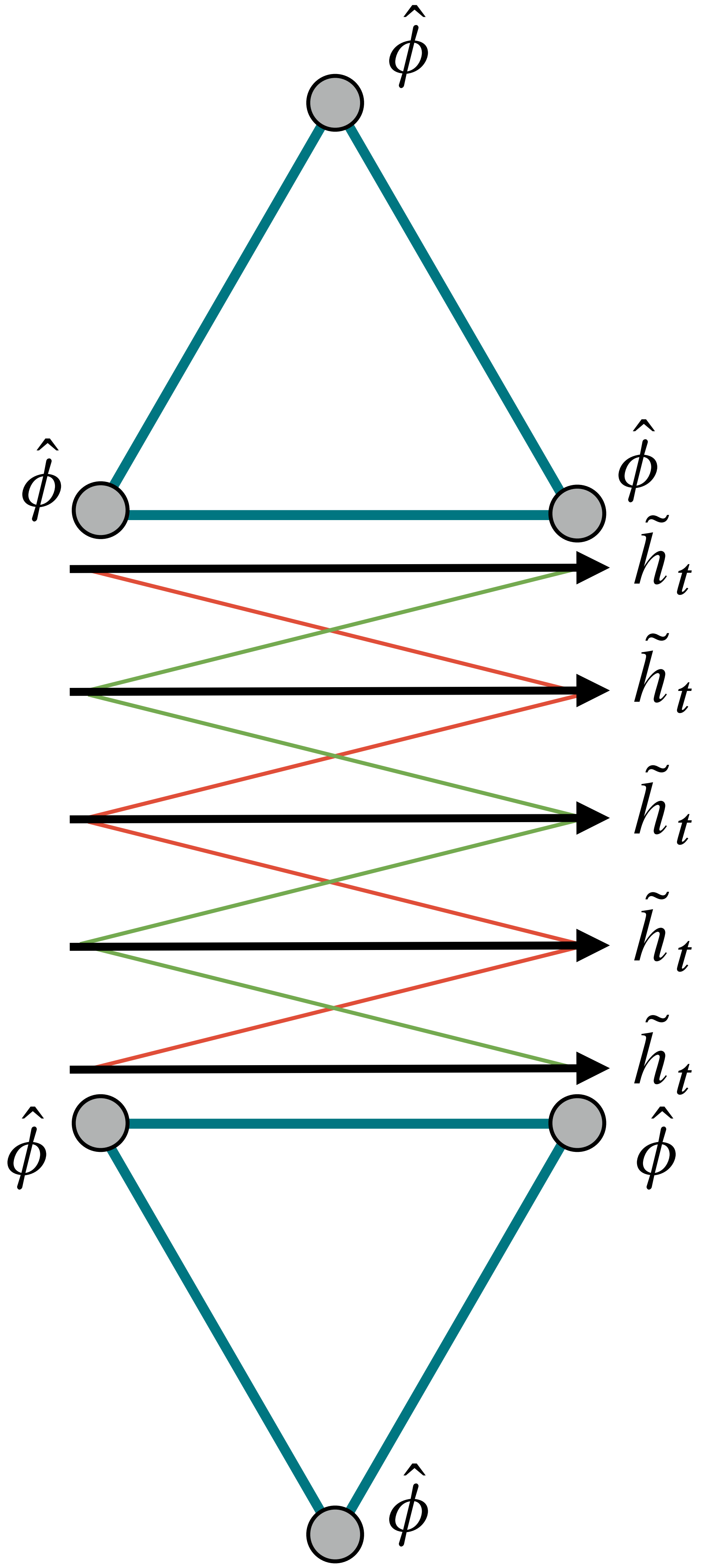

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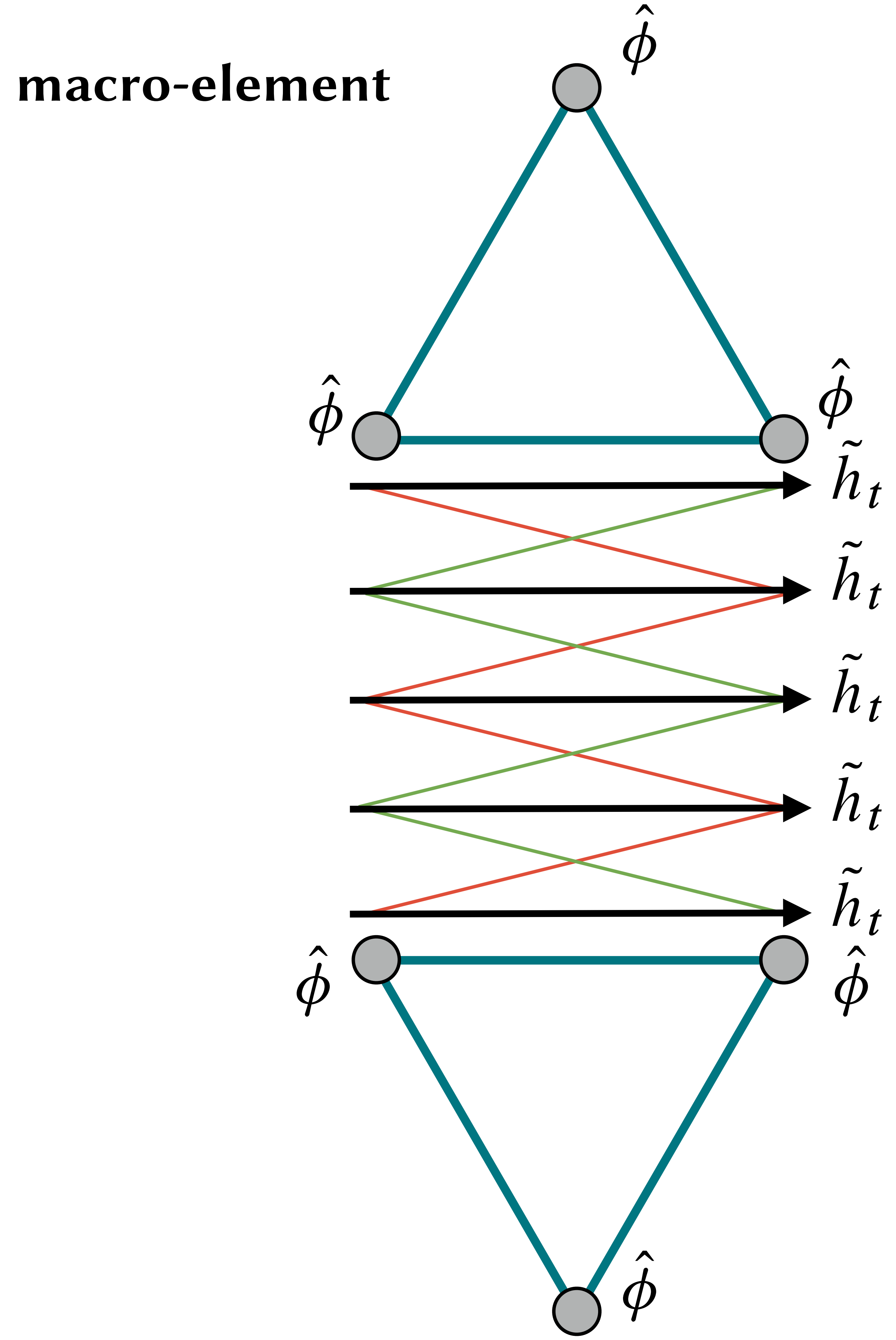

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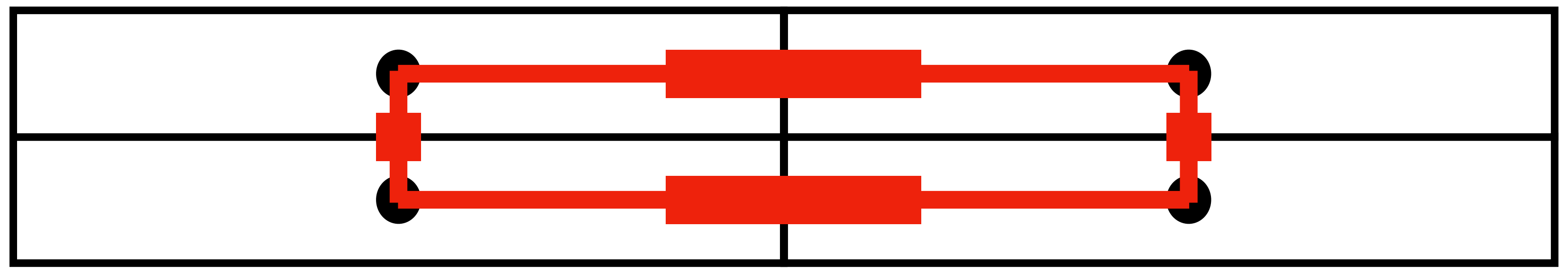
Sirois' TS approach : Differences to T-Formulation



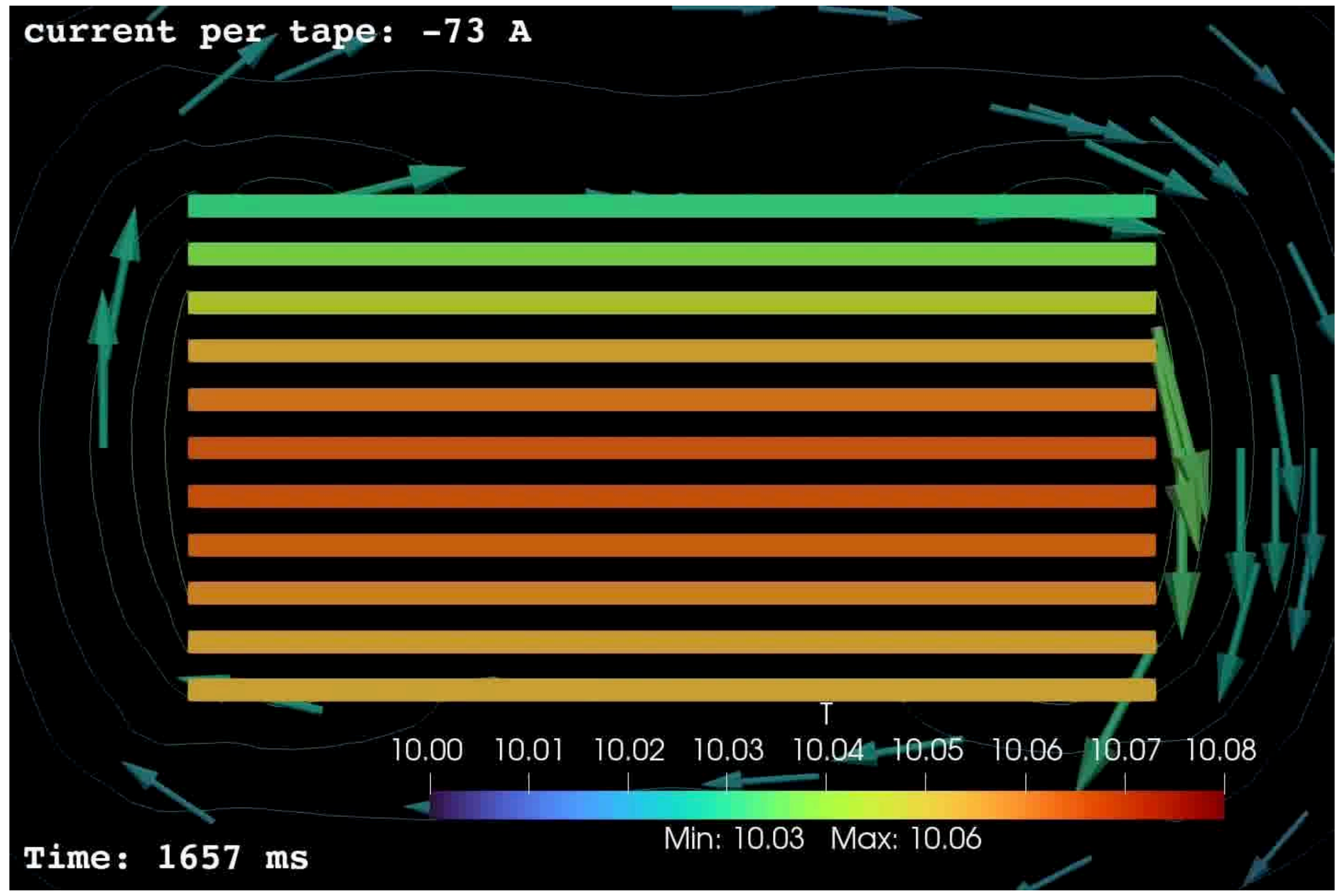
H-Formulation in tapes

- Directly uses the magnetic field H, no gauging needed!
  ➡ Important for field-dependent material properties.

- Can resolve individual layers of the tape
  ➡ No need for homogenization methods.

- In 3D: "Out-of-the-box" support for inter-layer current sharing

- Very simple thermal coupling using "thermal resistor mesh"
  ➡ "Out-of-the-box" support for quenching. (Unpublished!)



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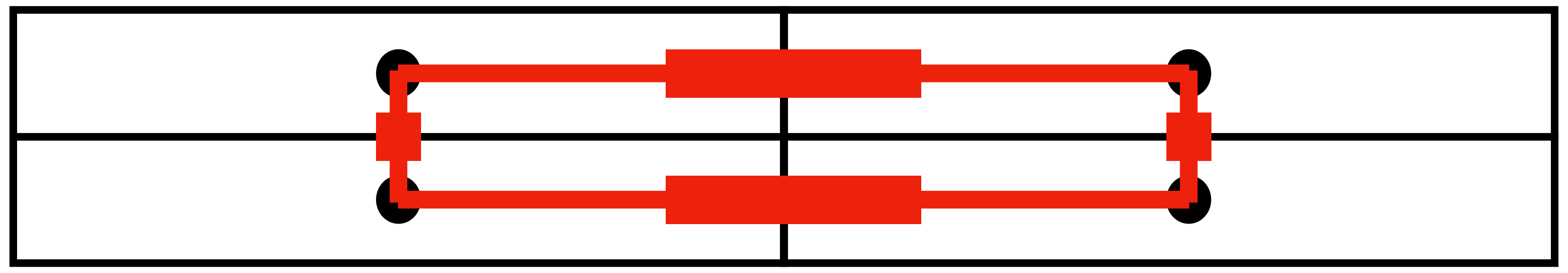
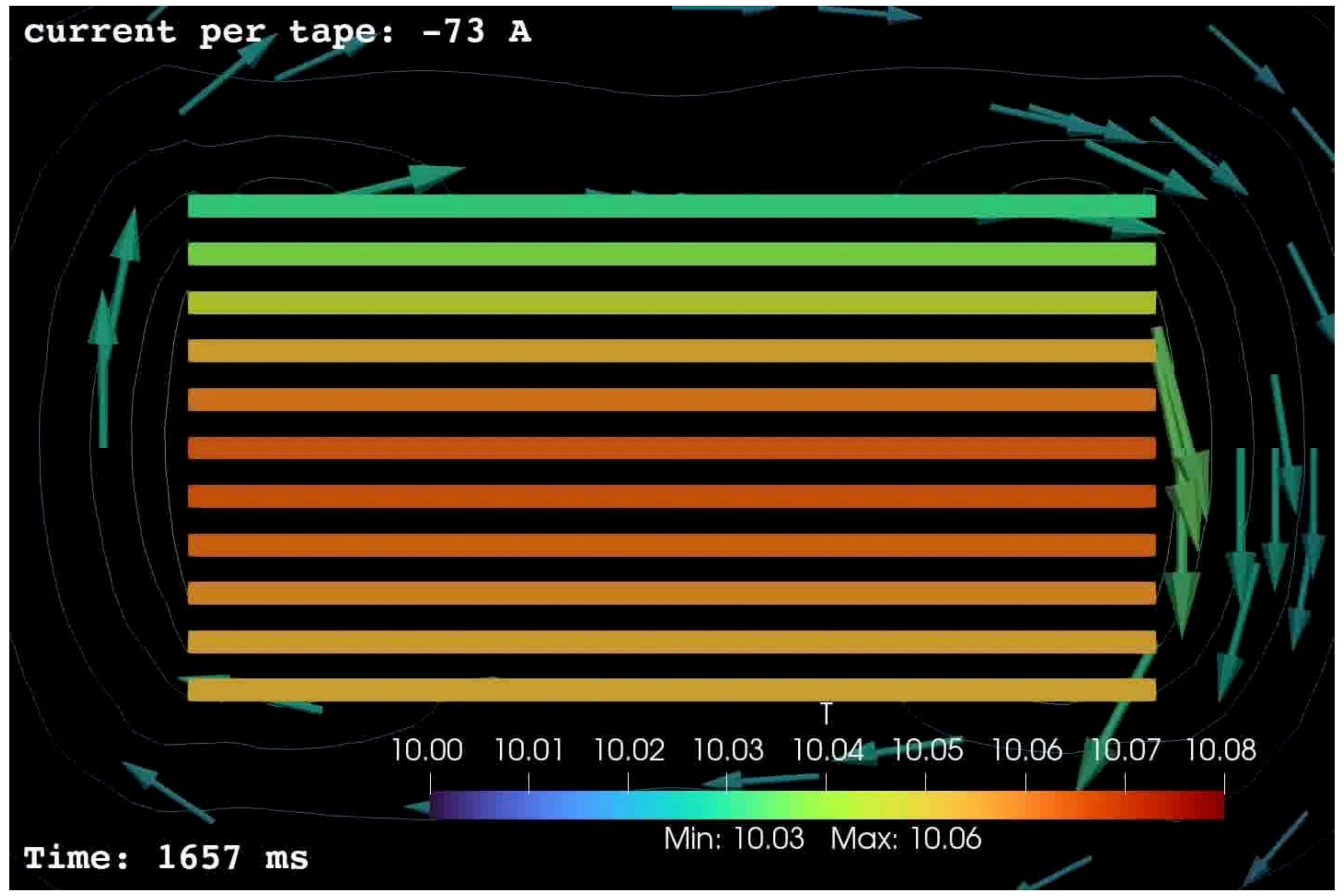




Non-Conducting domain A-Formulation vs. ϕ-Formulation

A vs ϕ Formulation for non-conducting Domains

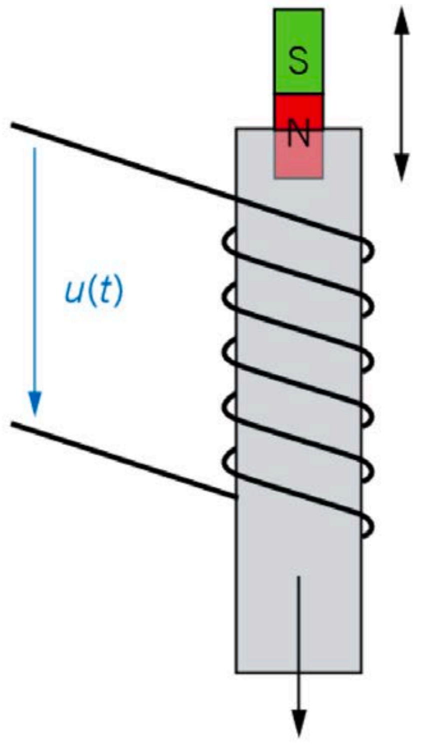
A-Formulation

- Uses magnetic vector potential

$$\boldsymbol{b} = \nabla \times \boldsymbol{a}$$

- Based on Faraday's law

$$\nabla \times \boldsymbol{e} = -\dot{\boldsymbol{b}}$$



- Dofs associated with edges (in 3D)
- Pros:
  - Well established
  - Straight forward current BCs
- Cons:
  - More DoFs than H-Formulation

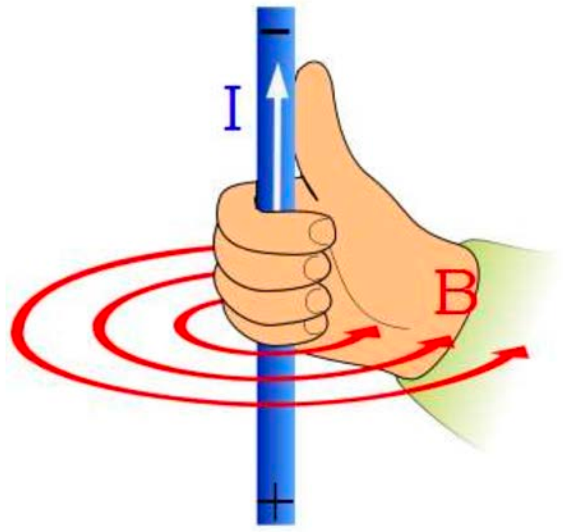
H-Formulation

- Uses magnetic scalar potential

$$\boldsymbol{h} = -\nabla \phi$$

- Based on Ampere's law

$$\nabla \times \boldsymbol{h} = -\boldsymbol{j}$$



- Dofs associated with nodes
- Pros:
  - Significantly less DoFs
- Cons:
  - Complicated Current BCs

Recap: Cohomologies

H-ϕ formulation: Homologies

Boundary Conditions

current is applied over Ampere's circuital law:

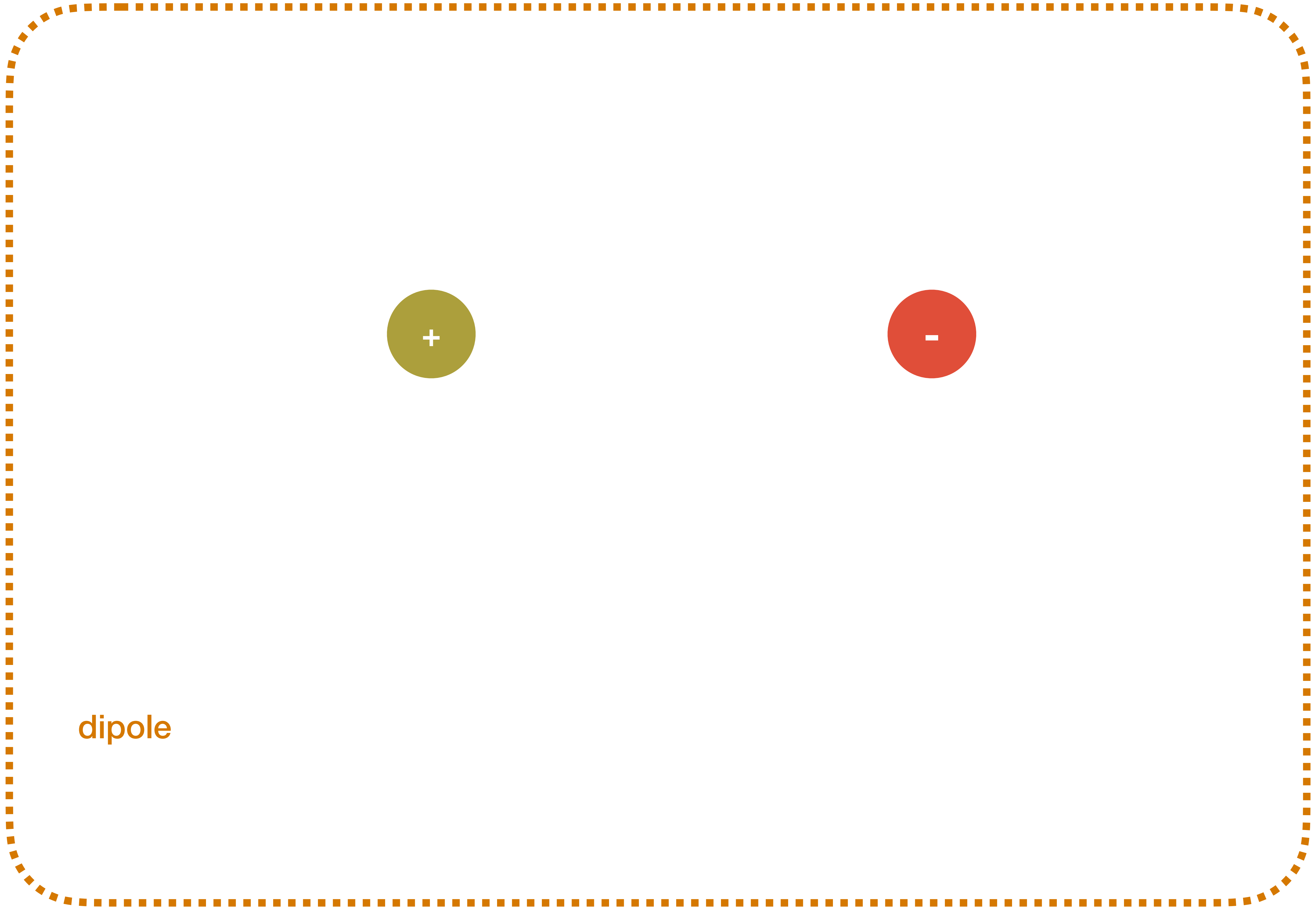
Ampéré's circuital law $$\oint \boldsymbol{h}\, \mathrm{d}l = I$$

- homologies represent the loops that can be drawn around the conducting regions that fulfill Ampéré's law

- only integral current "**I [A]**" needs to be known

- cohomologies are cuts in the domain over which jumps in the magnetic potential ϕ are imposed so that Δϕ=I.

➡ <u>very elegant mathematics!</u>
➡ <u>homology definition not user friendly</u>
➡ <u>difficult to implement in commercial codes</u>



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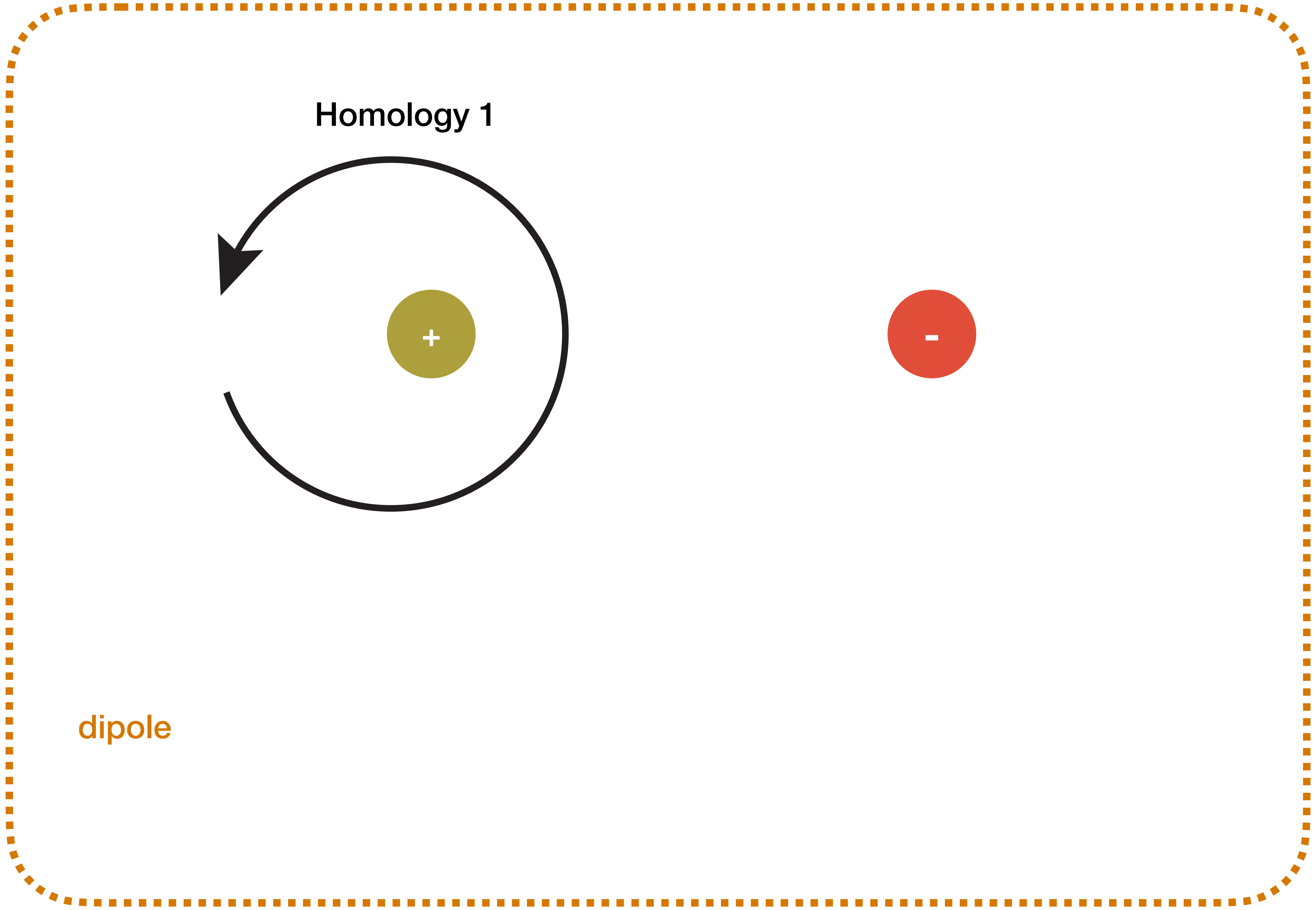

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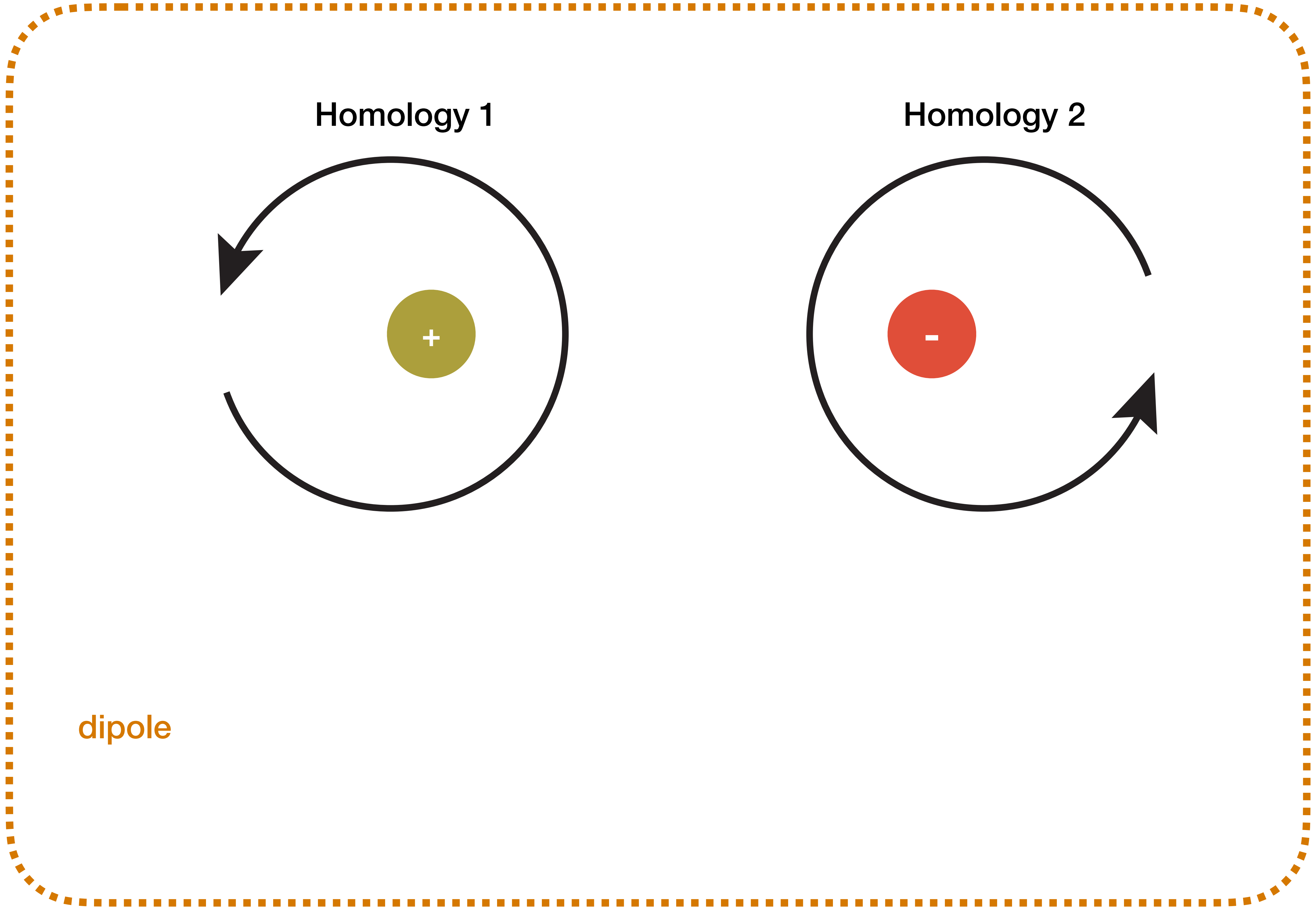

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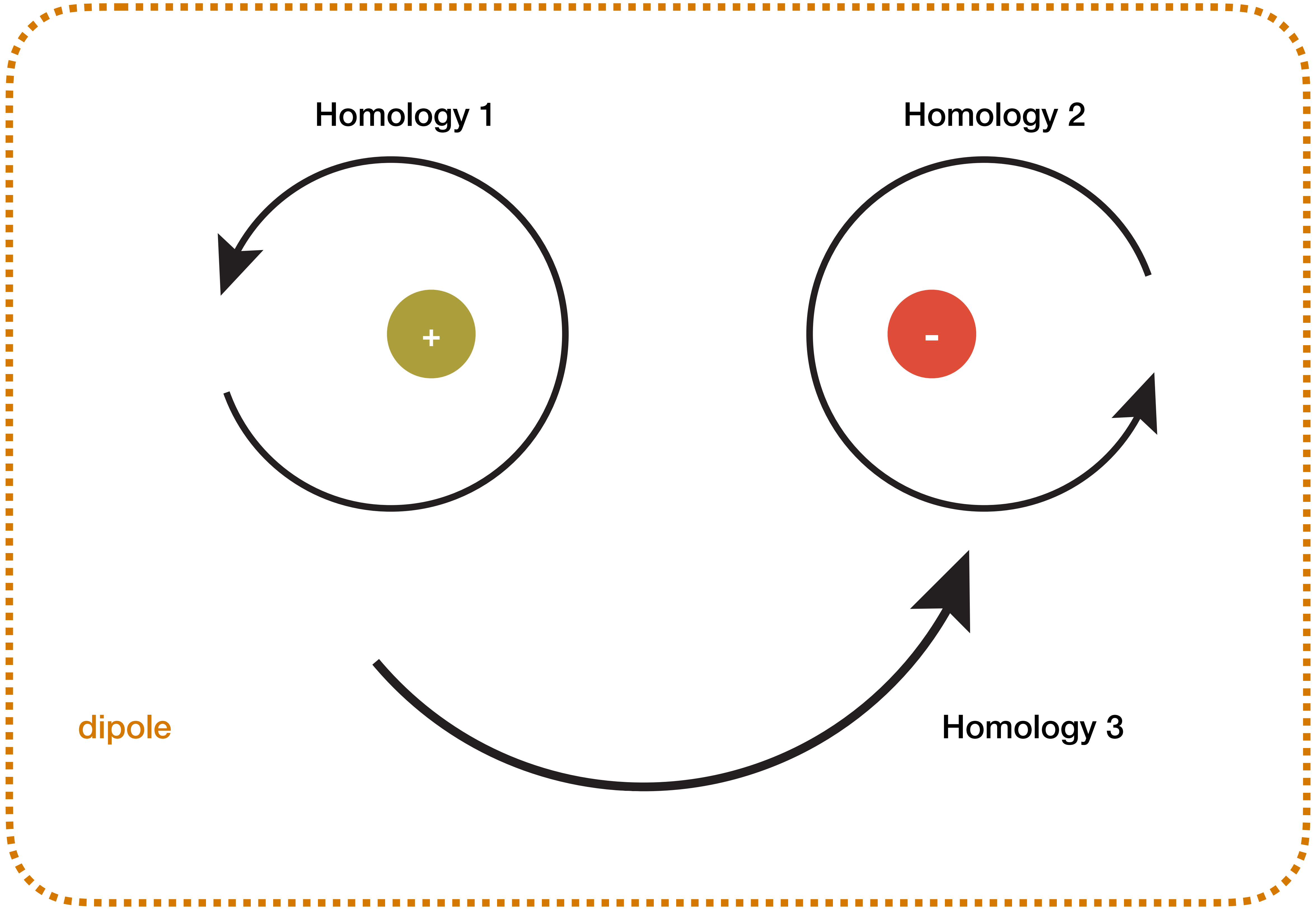

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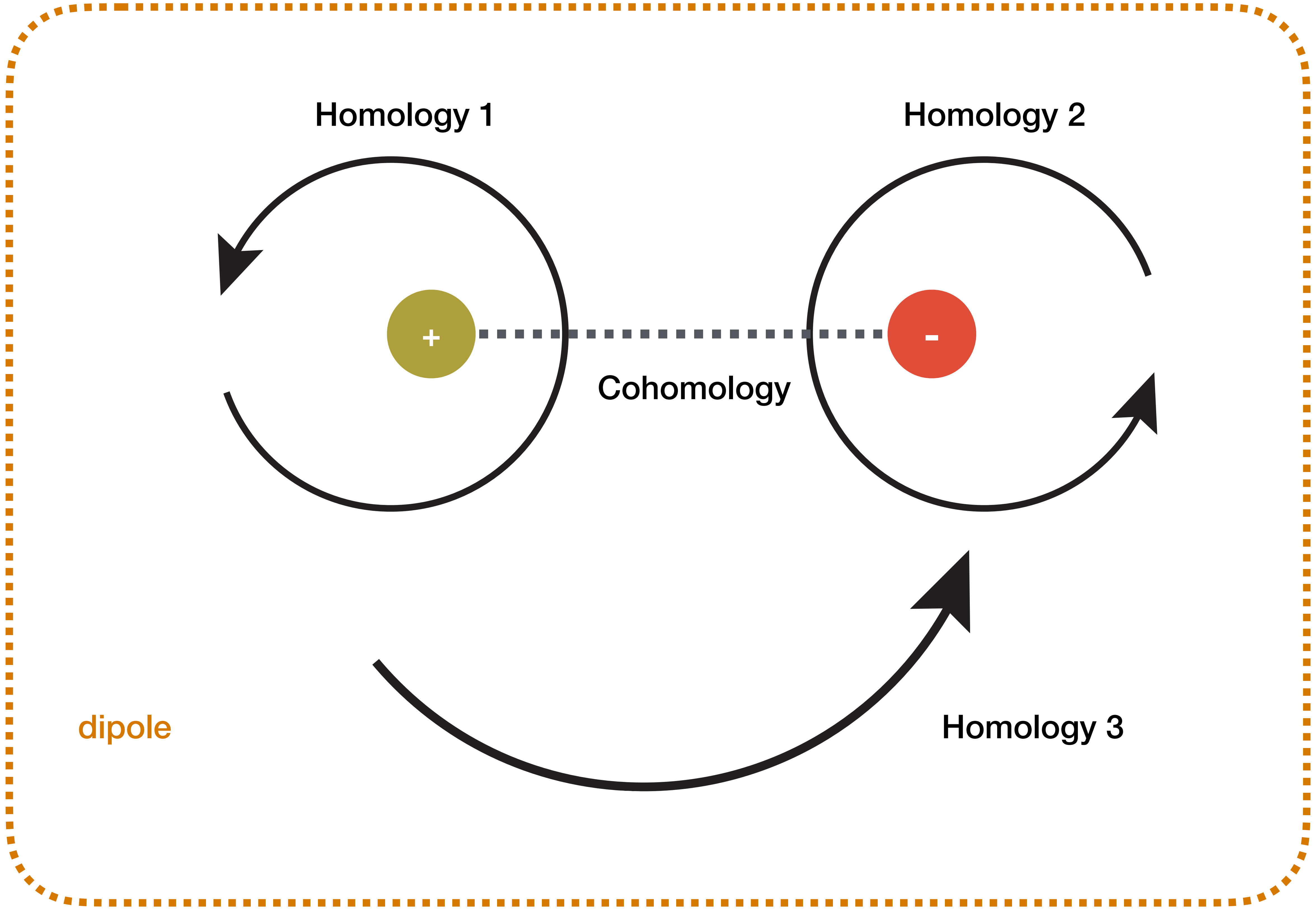
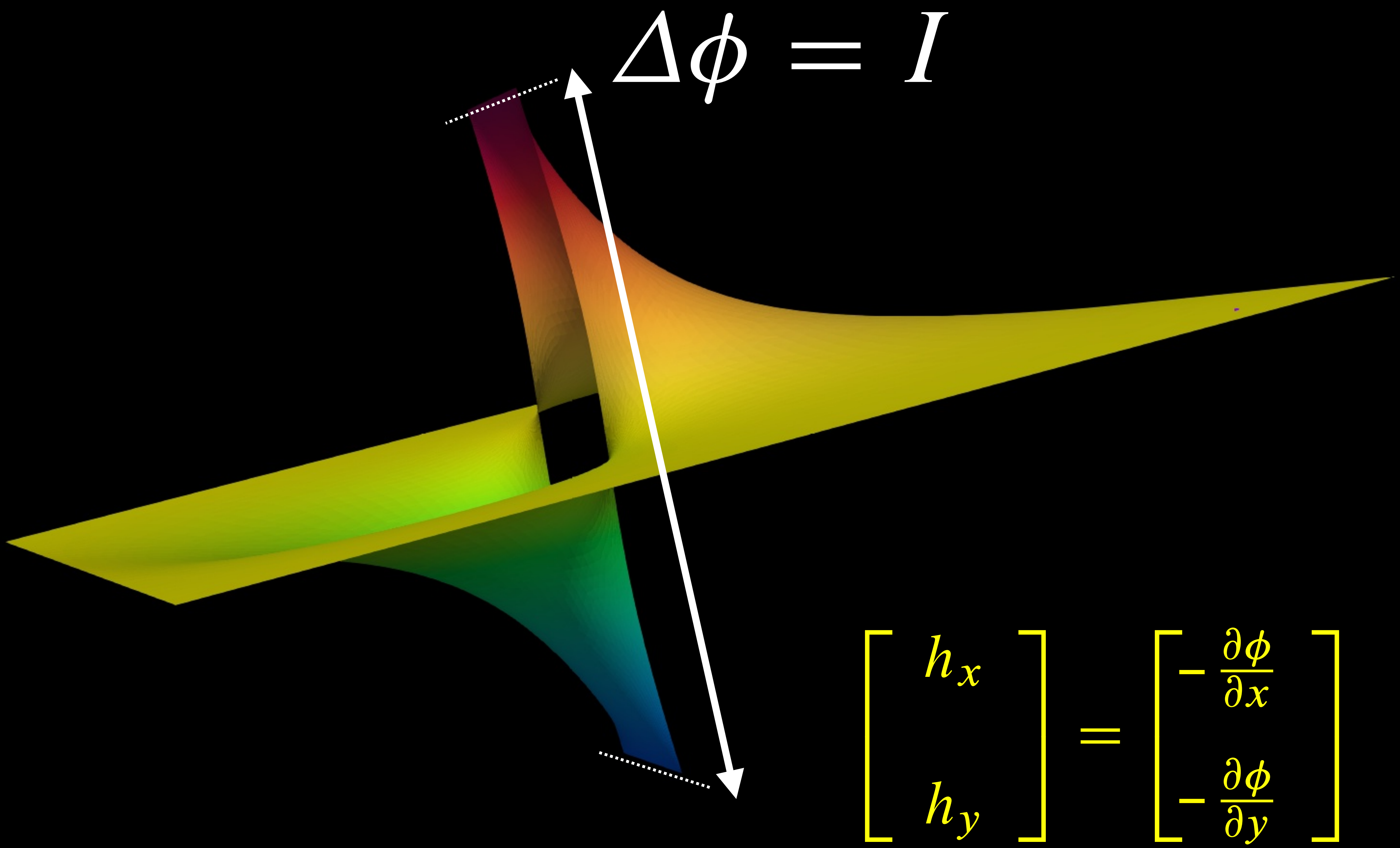
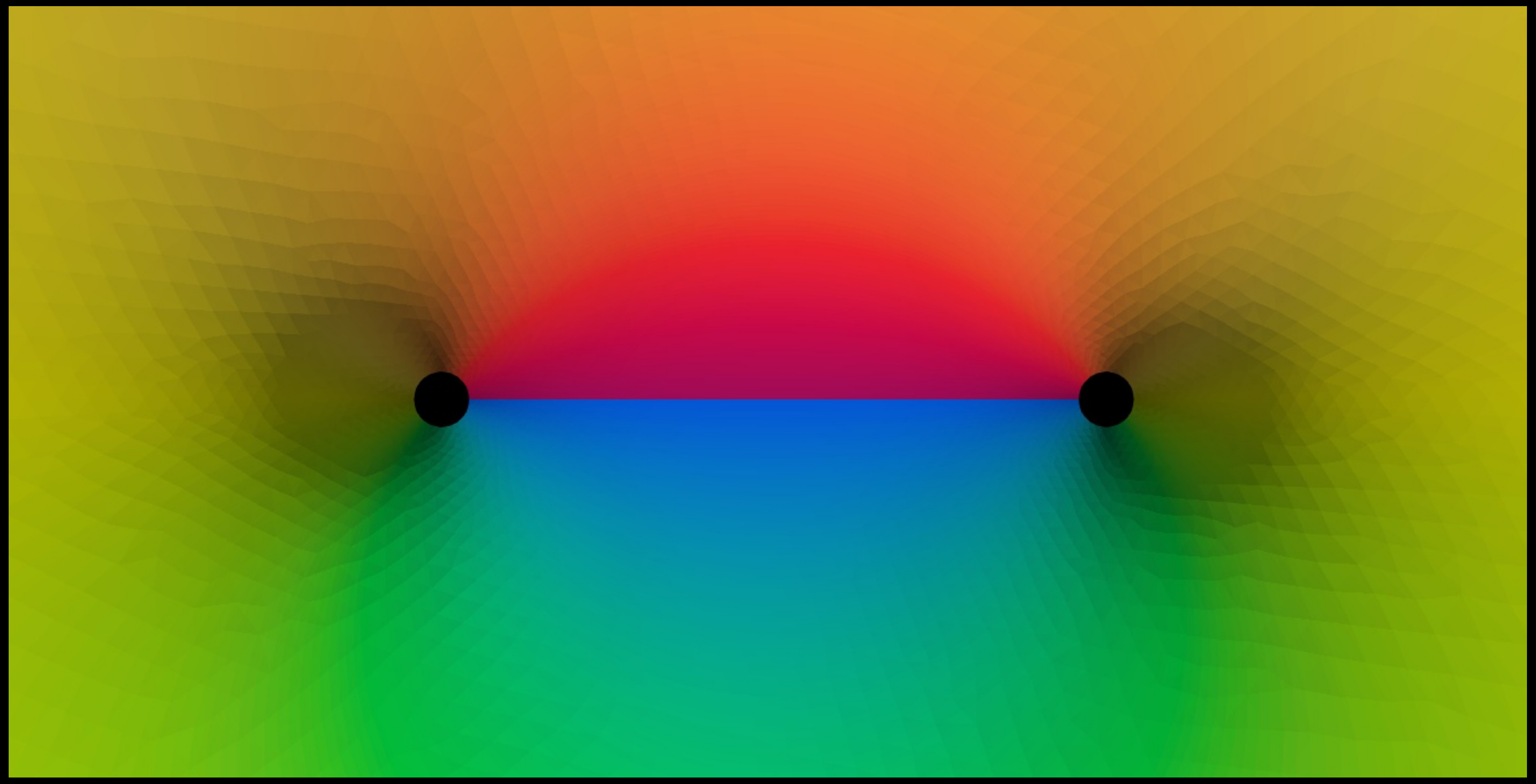


H-ϕ formulation: Jump Visualization



$$\Delta\phi = I$$

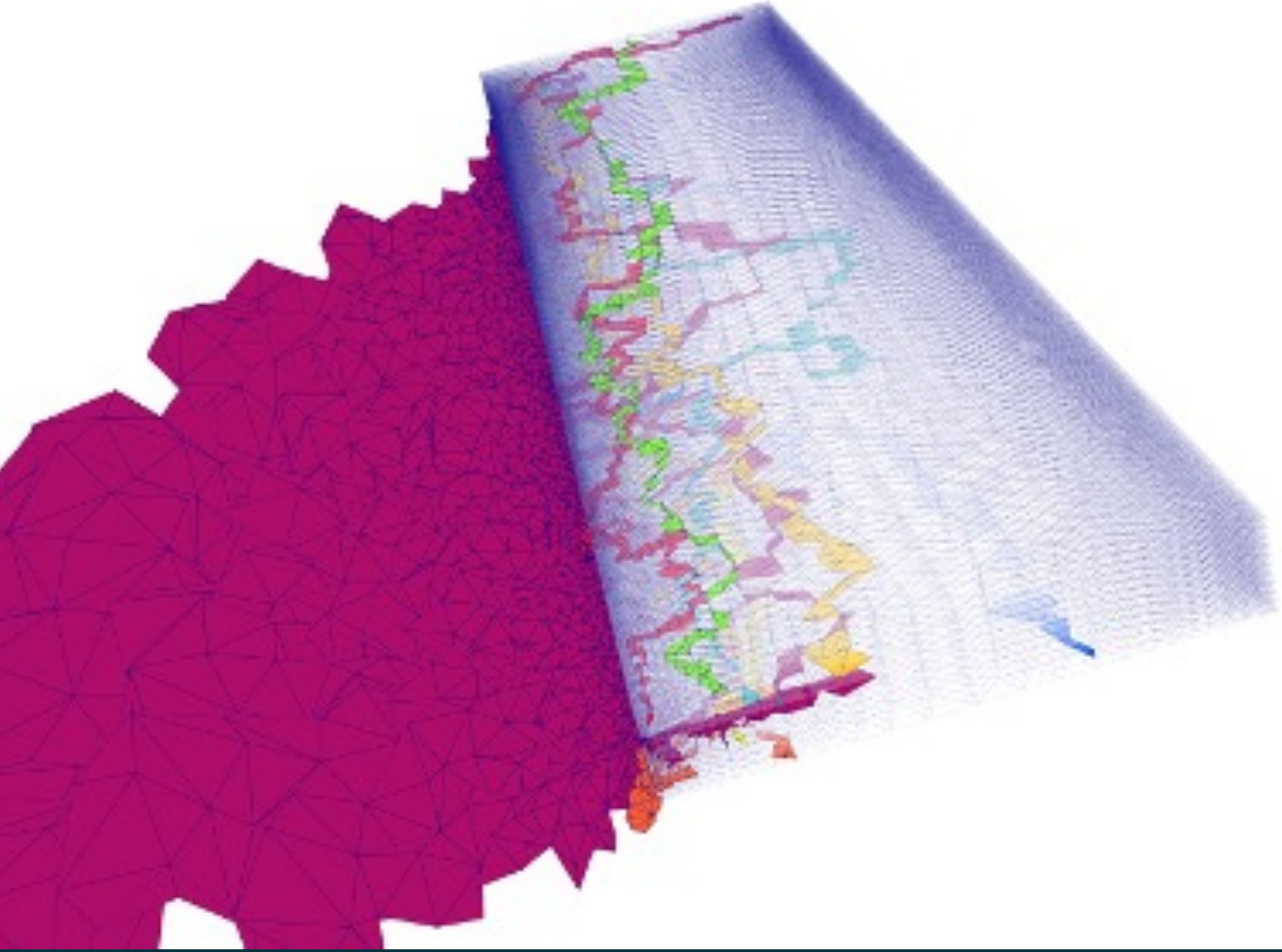
$$\begin{bmatrix} h_x \\ h_y \end{bmatrix} = \begin{bmatrix} -\frac{\partial\phi}{\partial x} \\ -\frac{\partial\phi}{\partial y} \end{bmatrix}$$



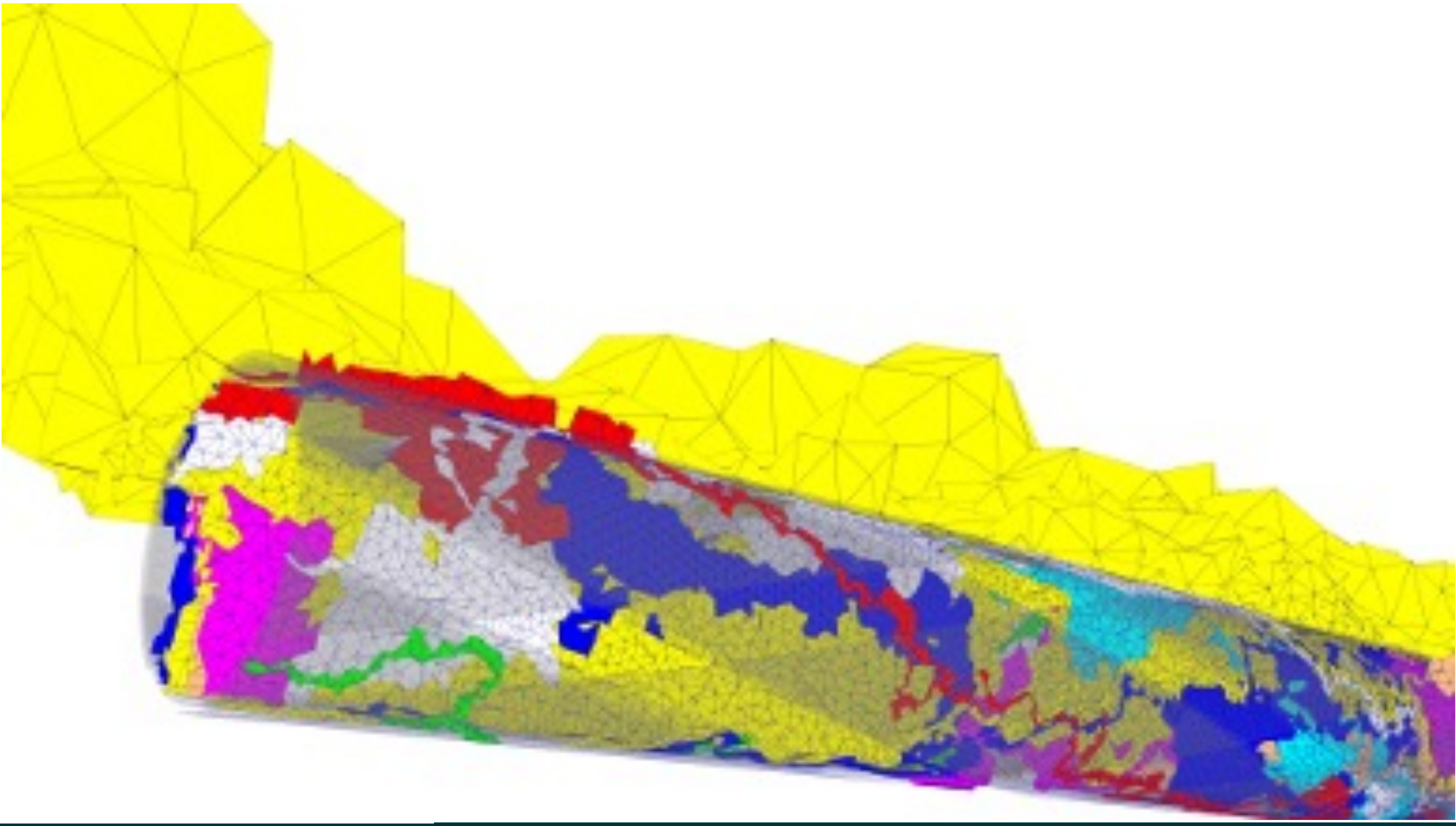
Discontinuity Conditions

H-ϕ formulation: Homology Examples

3D Tapestack



CORC geometry

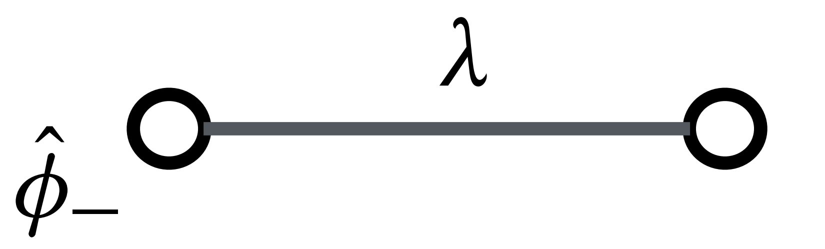


Jump Conditions

Lagrange Multipliers

- Well known from contact conditions in structural dynamics
- In COMSOL: “Pointwise Constraint"
- Pros:
  - Very easy to implement
- Cons:
  - Element matrix non-positive definite
  ➡ More work for solver

element topology:



$$\hat{\phi}_+ \overset{!}{=} \hat{\phi}_- + I$$

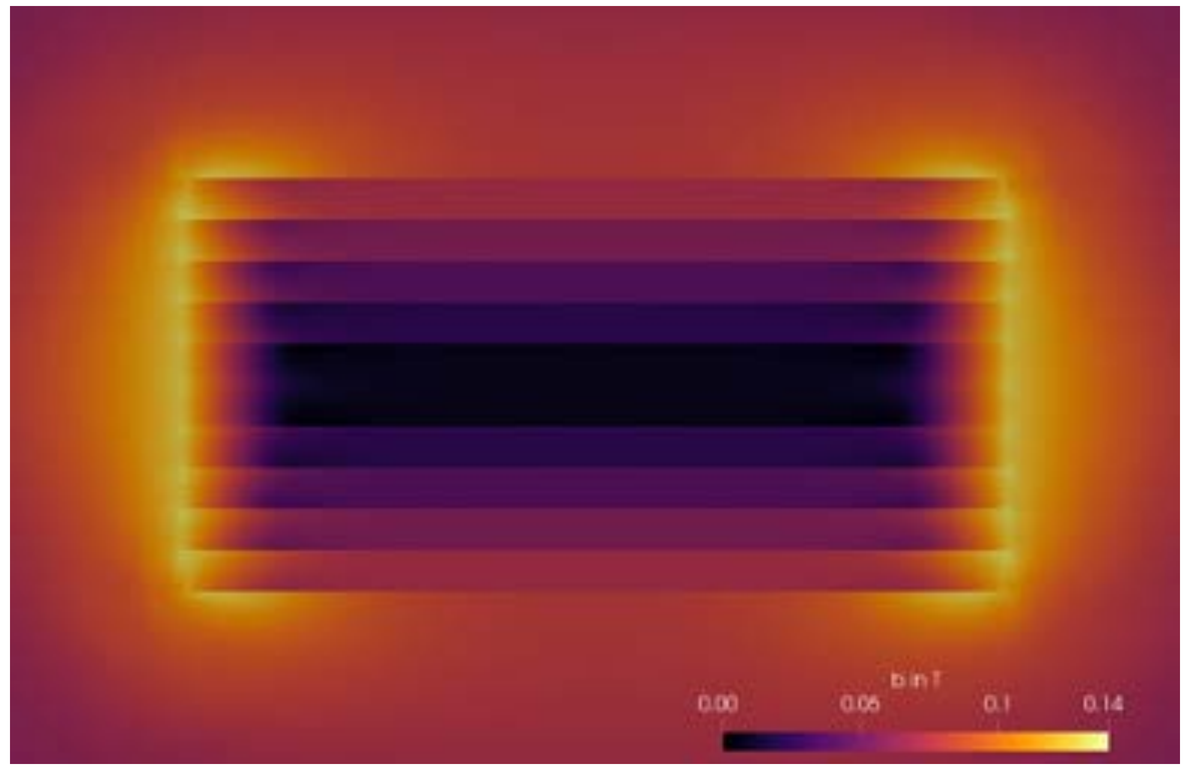
virtual work equation:

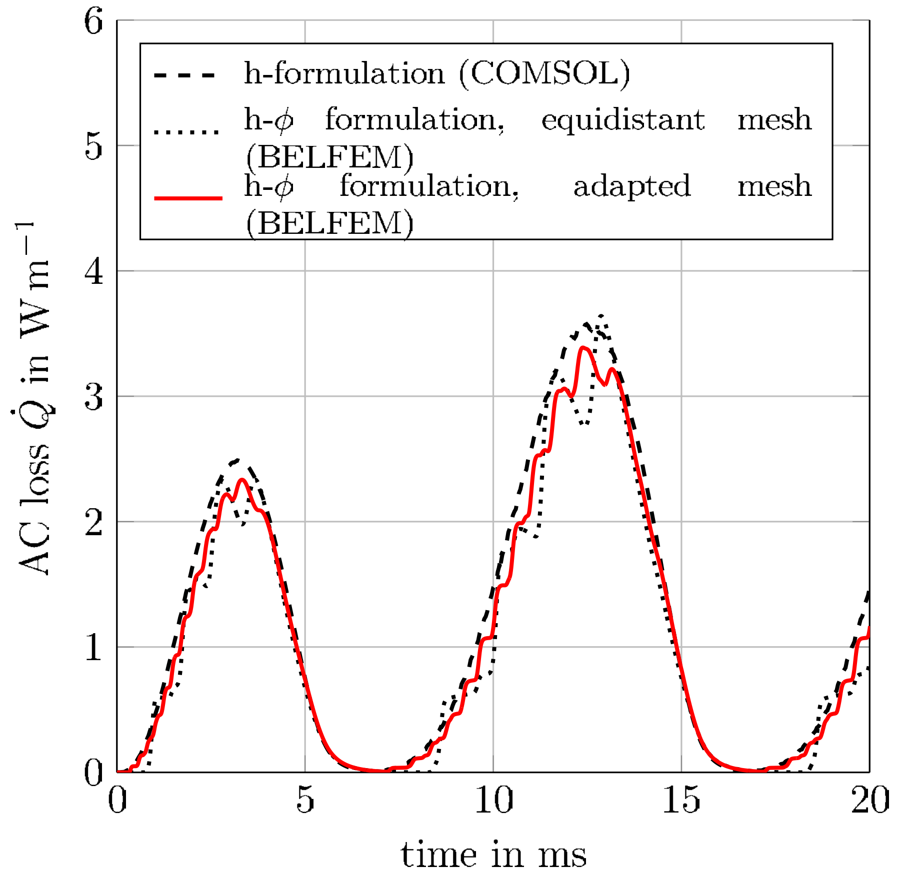
$$\Pi = \lambda \left(\hat{\phi}_+ - \hat{\phi}_- - I \right) = 0$$

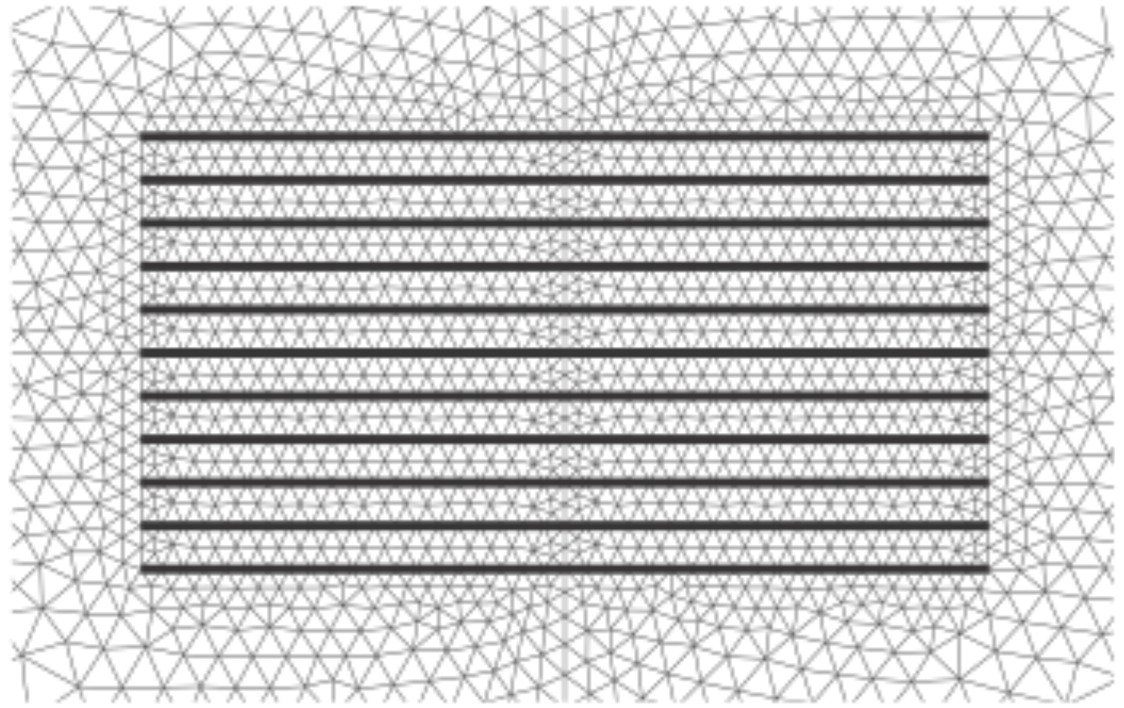
element matrix:

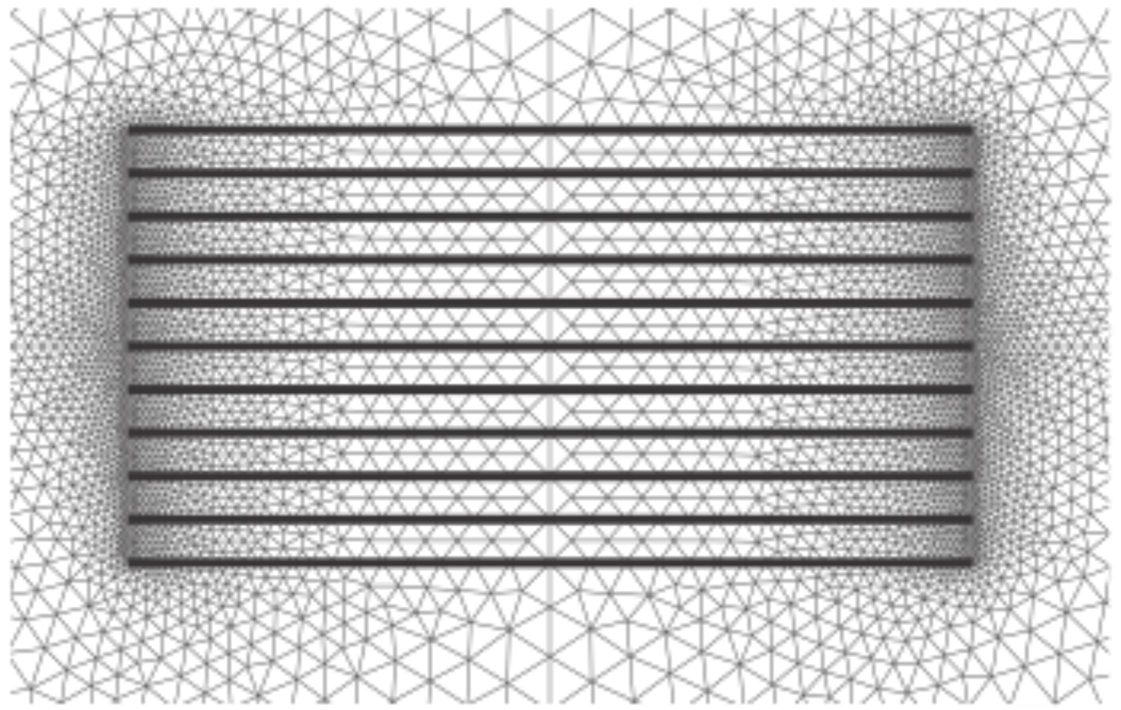
$$\begin{bmatrix} \delta\hat{\phi}_- & \delta\hat{\phi}_+ & \delta\lambda \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\phi}_- \\ \hat{\phi}_+ \\ \lambda \end{bmatrix} = \begin{bmatrix} \delta\hat{\phi}_- & \delta\hat{\phi}_+ & \delta\lambda \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

2D Benchmark from 2023







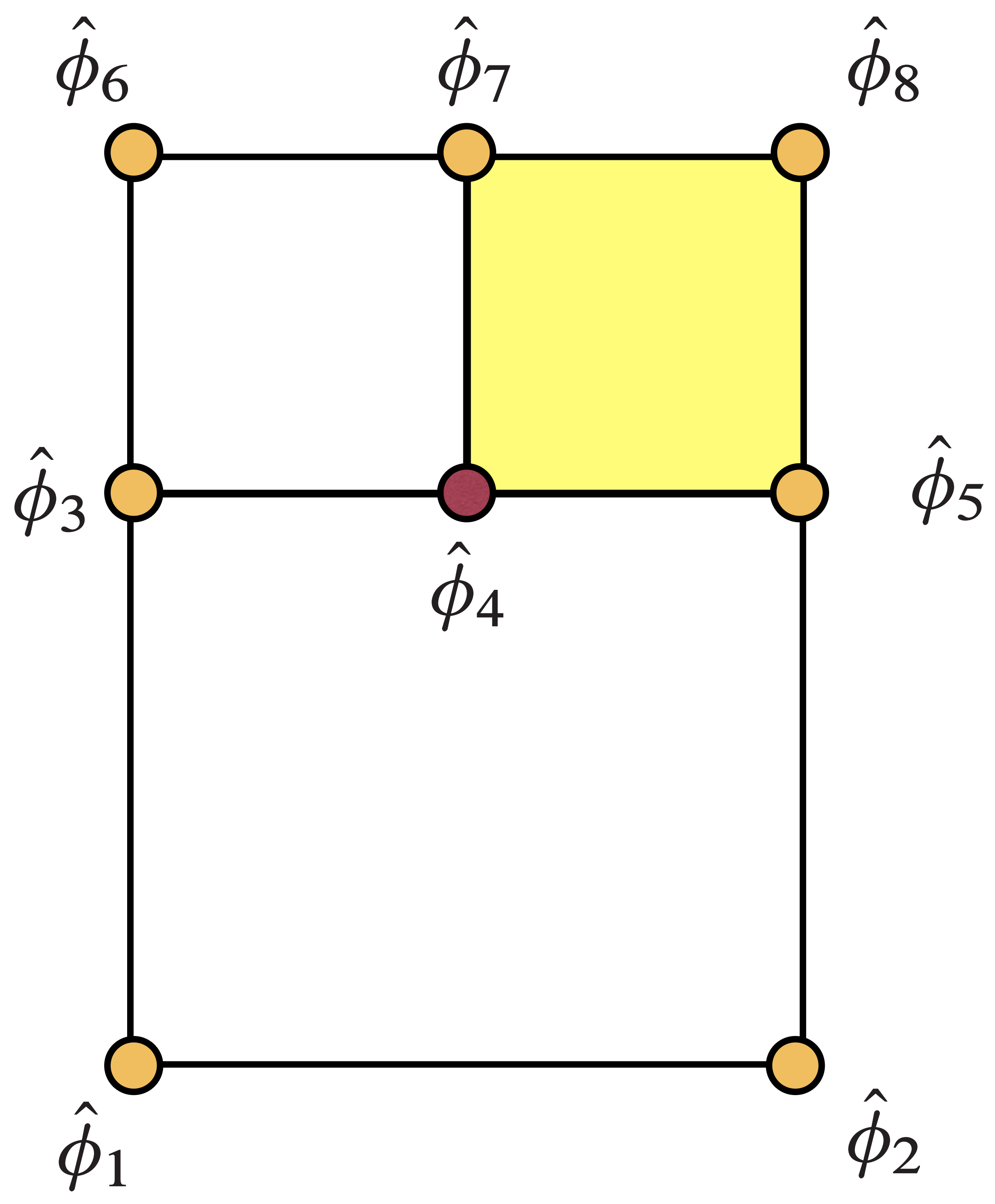


- Very promising results, however, the Lagrange multipliers are a reason to be concerted about very large models since the non-positive definite matrices will be problematic.

code and library		constant time step		adaptive time step	
		coarse mesh	fine mesh	coarse mesh	fine mesh
GetDP	MUMPS	5:13	10:57	2:36	7:58
BELFEM	MUMPS	1:56	7:15	0:23	1:09
BELFEM	STRUMPACK	0:55	2:54	0:11	0:25

The concept of “Hanging Dofs” for non-conformal meshes

Better Approach: Eliminate Implicit Dof:



- Node 4 is “hanging” ⟶ dof 4 is expressed as linear combination of dof 3 and 5

$$\begin{bmatrix} \delta\hat{\phi}_4 & \delta\hat{\phi}_5 & \delta\hat{\phi}_8 & \delta\hat{\phi}_7 \end{bmatrix} \mathbf{K} \begin{bmatrix} \hat{\phi}_4 \\ \hat{\phi}_5 \\ \hat{\phi}_8 \\ \hat{\phi}_7 \end{bmatrix} = \begin{bmatrix} \delta\hat{\phi}_3 & \delta\hat{\phi}_5 & \delta\hat{\phi}_8 & \delta\hat{\phi}_7 \end{bmatrix} \mathbf{T}^{\mathbf{T}} \mathbf{K}\, \mathbf{T} \begin{bmatrix} \hat{\phi}_3 \\ \hat{\phi}_5 \\ \hat{\phi}_8 \\ \hat{\phi}_7 \end{bmatrix}$$

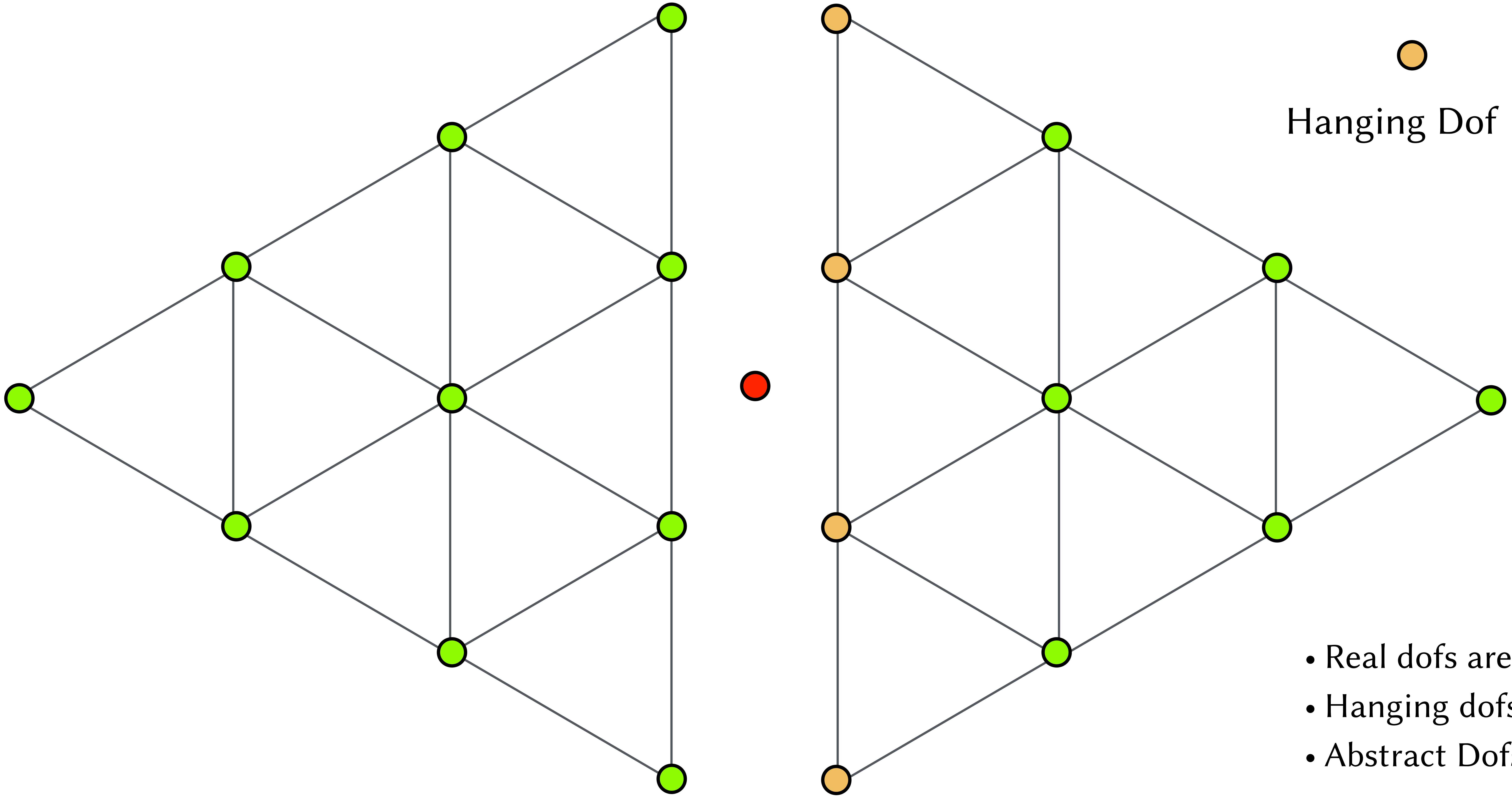
- Change of basis us performed using T-Matrix

$$\mathbf{T} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Method can be used for both domain interfaces and domain cuts

- But: Not a standard method supported in FE solvers!

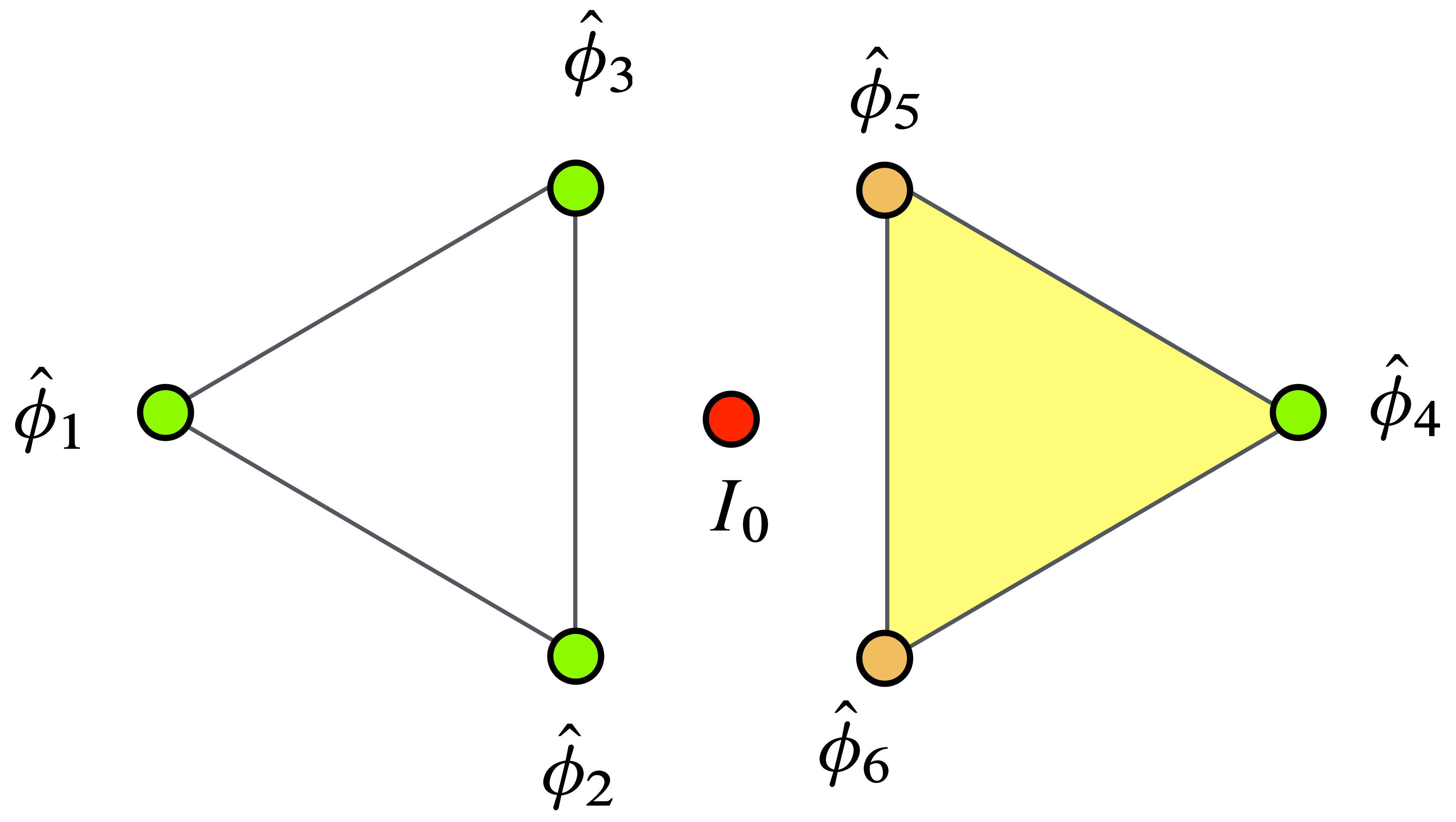
Hanging Dofs with Abstract Nodes (Unpublished!)



- Real dofs are associated with the mesh.
- Hanging dofs are linearly dependent on other dofs.
- Abstract Dofs are not represented in the mesh!

Hanging Dofs with Abstract Nodes

The concept of "Hanging Nodes" is well known in FE theory. Using this technique in for Jump conditions, however, is new, And according to my best knowledge unique to BELFEM!



- The "local dofs" of the yellow element are [ϕ4, ϕ5, ϕ6]
- We compute the element Jacobian A_l using these dofs.
- The DOF transformation matrix reads

$$\begin{bmatrix} \hat{\phi}_4 \\ \hat{\phi}_5 \\ \hat{\phi}_6 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \cdot \begin{bmatrix} \hat{\phi}_2 \\ \hat{\phi}_3 \\ \hat{\phi}_4 \\ I_0 \end{bmatrix}$$

- The Jacobian with respect to real dofs then reads

$$\mathbf{A} = \mathbf{T}^{\mathbf{T}}\, \mathbf{A}_l\, \mathbf{T}$$

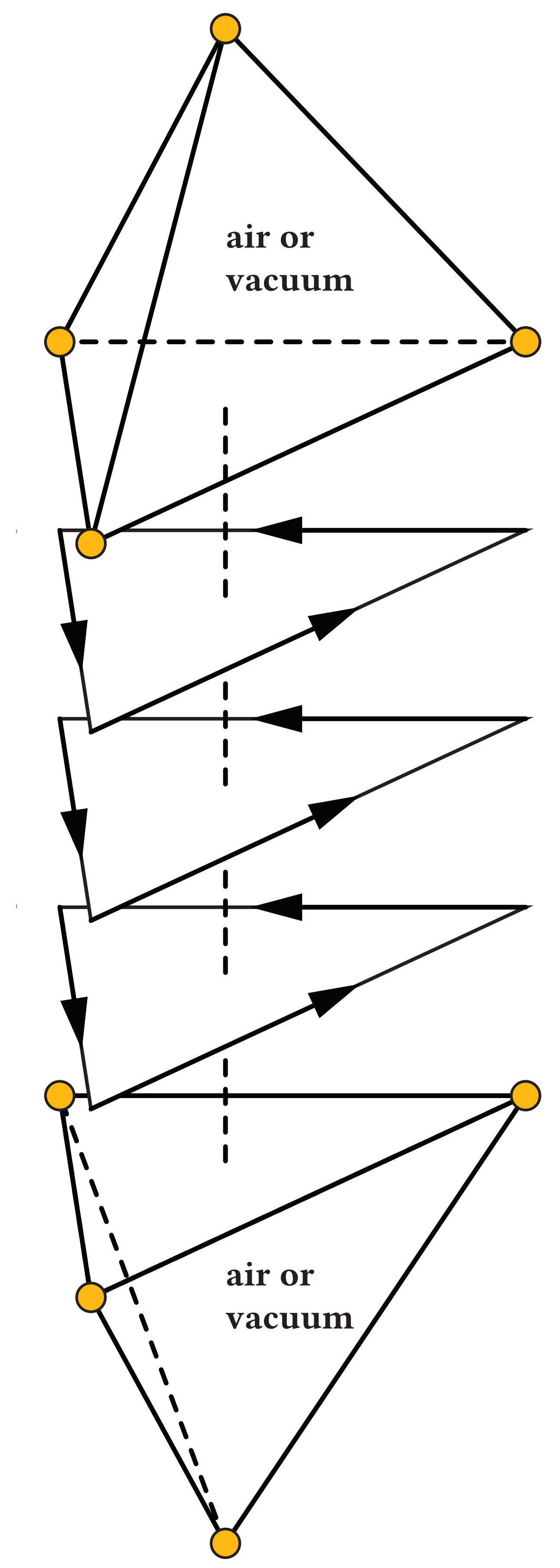
- Ultimately, I is imposed as Dirichlet condition

Summary and Timeline

Intermediate Summary:

- Thin Shell h-ϕ is still in development, but will probably outperform t-a in most aspects.
- In 3D, the ϕ-formulation needs significantly less degrees of freedom than the a-formulation.
- Current boundary conditions need to be applied over cohomologies
  - Cohomoligies for 3D problems not intuitive, need clever algorithms
  - Developed in cooperation with Polytechnique Montreal
- Current conditions:
  - Were implemented with Lagrange Multipliers in the past
  - Switched to new "Hanging DoF" concept to achieve better conditioning.

Current Efforts

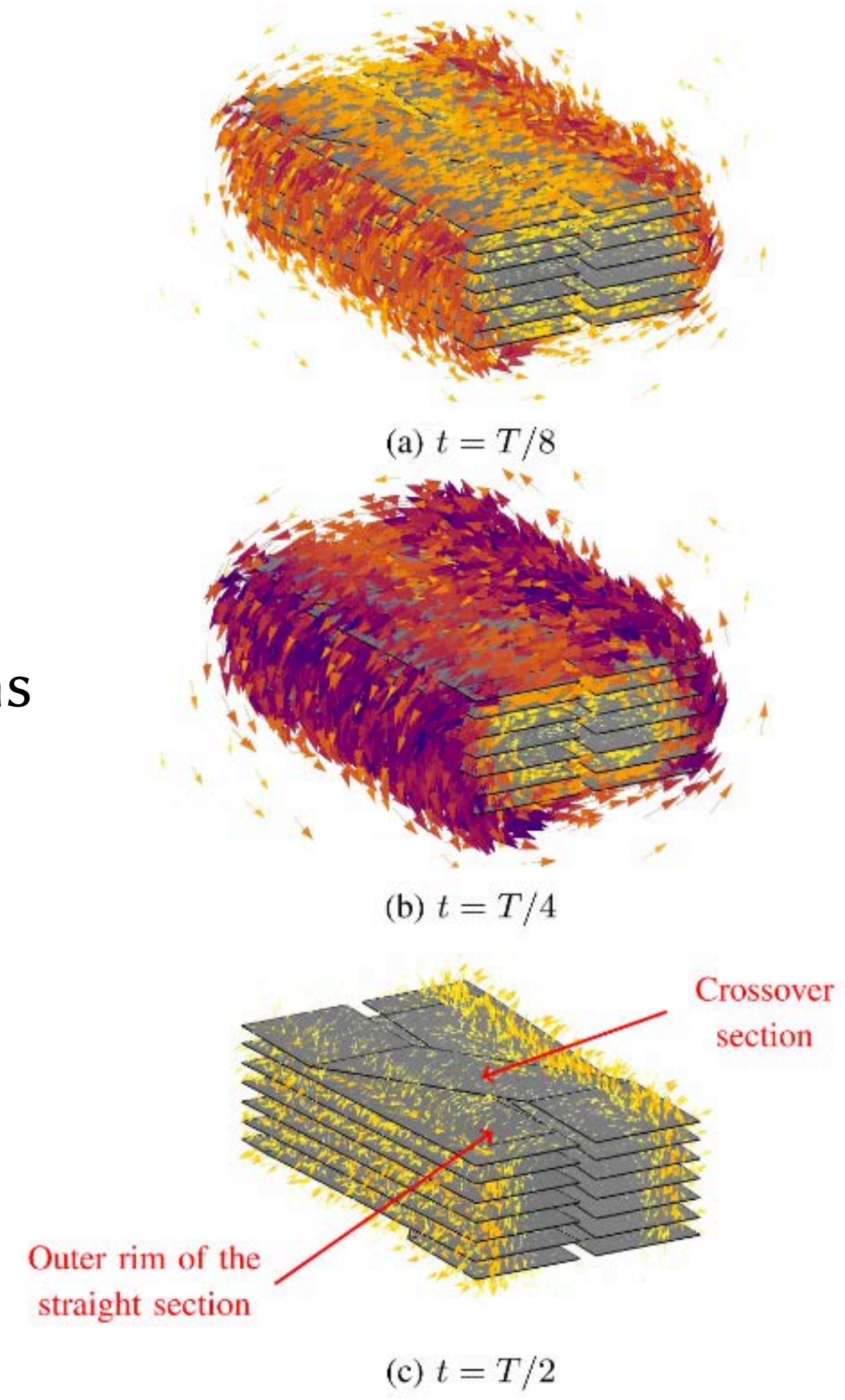


Goal:

- model a thin shell tapes tack in 3D after Alves et Al, 2022
- extend model to encompass solder and thermal model
- be able to do the coupled EM-Thermal quenching model by end of the year

Roadmap:

- overhaul data structure for simplified programming of weak governing equations
- improve degree of freedom management system (almost complete)
- first benchmark with 3D tapestack (Hopefully before Christmas!)
- implement solder and thermal model
- benchmark involving quench
- address contact sharing (Spring 2025)



[Alves et al, 10.1109/TASC.2022.3143076]