Nuclear Matter at High Orders from Chiral Effective Field Theory

Christian Drischler

Science talk at NSD Staff Meeting

Berkeley
UNIVERSITY OF CALIFORNIA

nuclear matter

nuclear physics
structure & reactions of nuclei

astrophysics
neutron stars, supernovae, …

chiral effective field theory

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Nuclear Matter at High Orders from Chiral Effective Field Theory

Direct detection of gravitational waves

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ligo.caltech.edu

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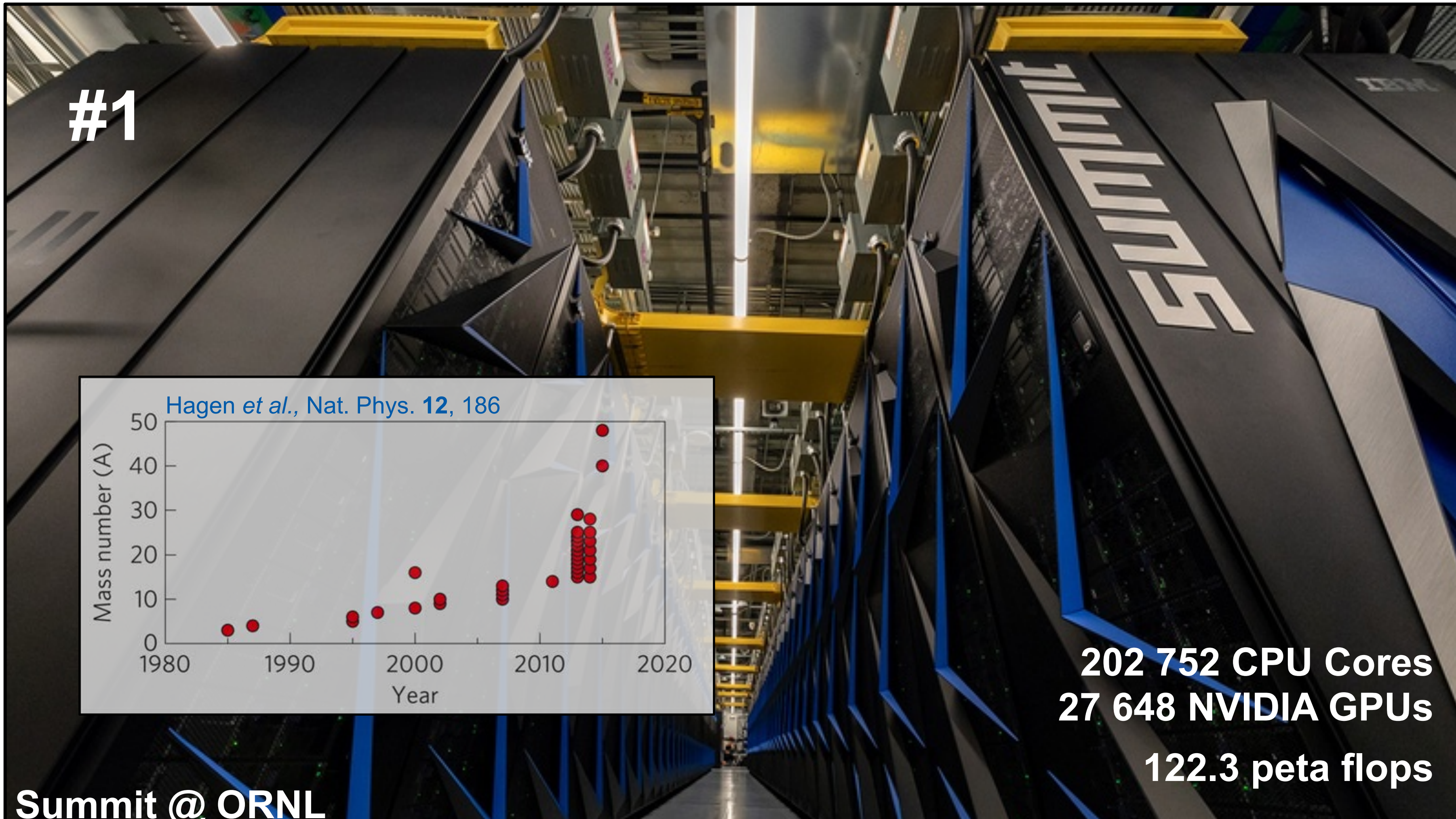
Next-generation supercomputers



Nuclear Matter at High Orders from Chiral Effective Field Theory

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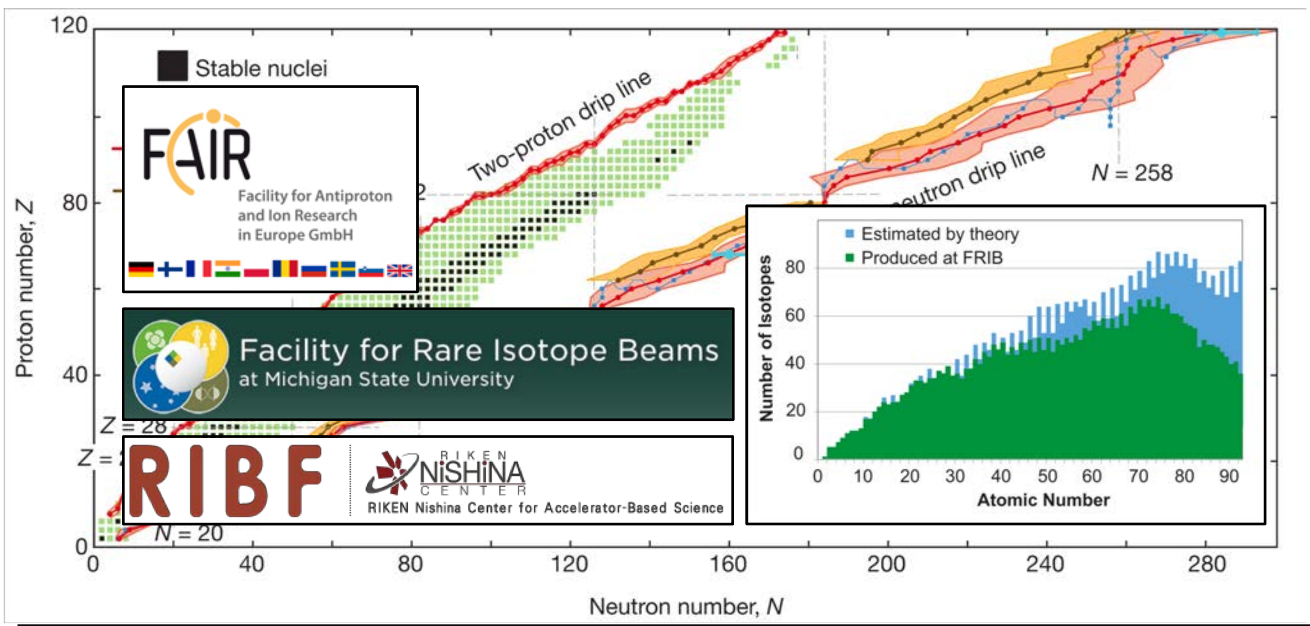
Erler *et al.*, Nature **486**, 509–512

Where do heavy elements come from?

How does the nuclear chart emerge from QCD?

How to predict properties of nuclei?



- observables
- many-body framework
- effective field theory
- quantum chromodynamics

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see also Hebeler *et al.*, ARNPS **65**, 457

equation of state
neutron-star matter | finite temperatures

many-body perturbation theory
computational efficient
many-body uncertainty estimates

chiral effective field theory
uncertainty estimates
many-body potentials (at full N^3LO)

CalLat ...
see McElvain, Haxton, arXiv:1902.03543

observables

many-body framework

effective field theory

quantum chromodynamics

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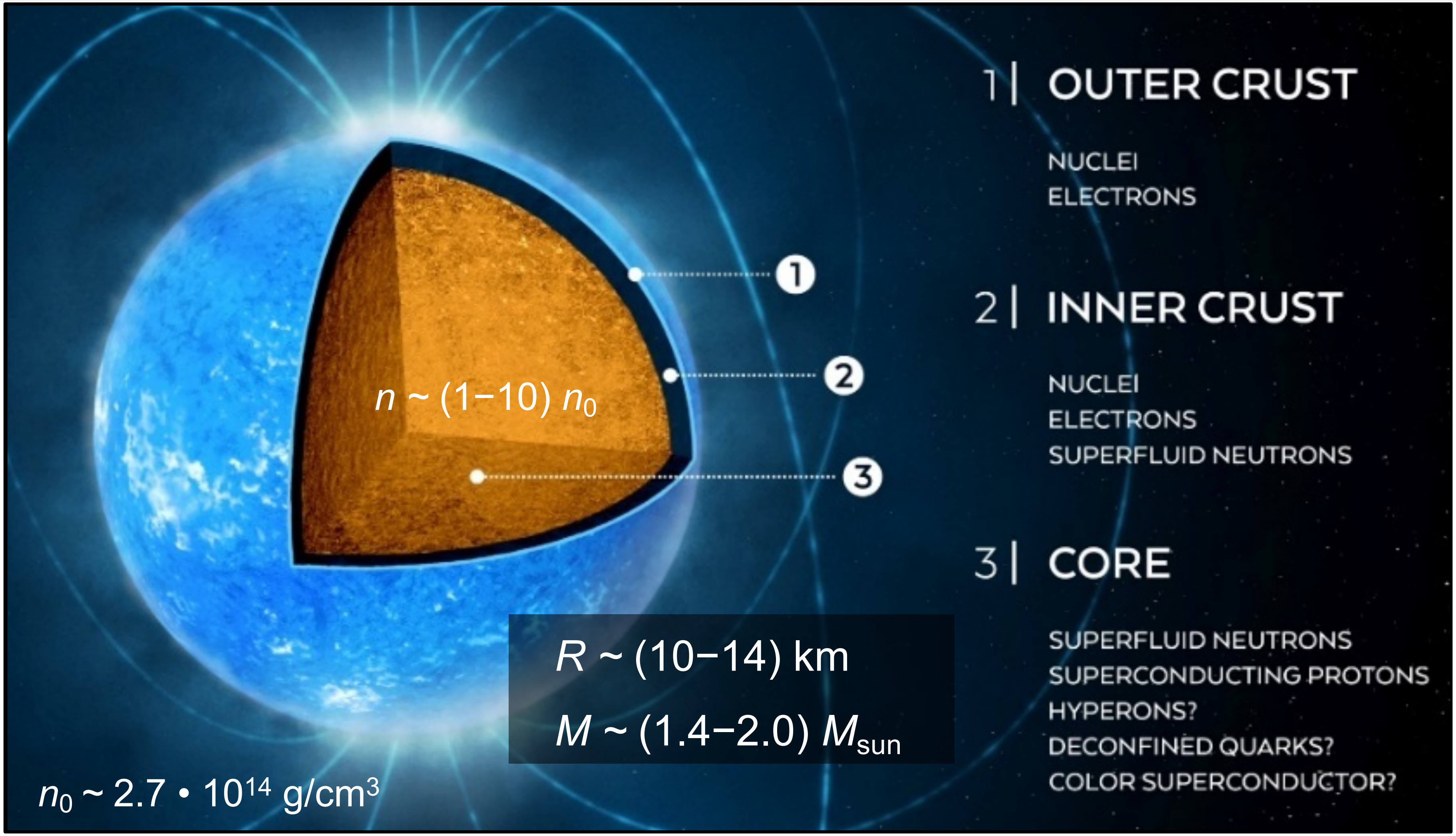
Neutron stars

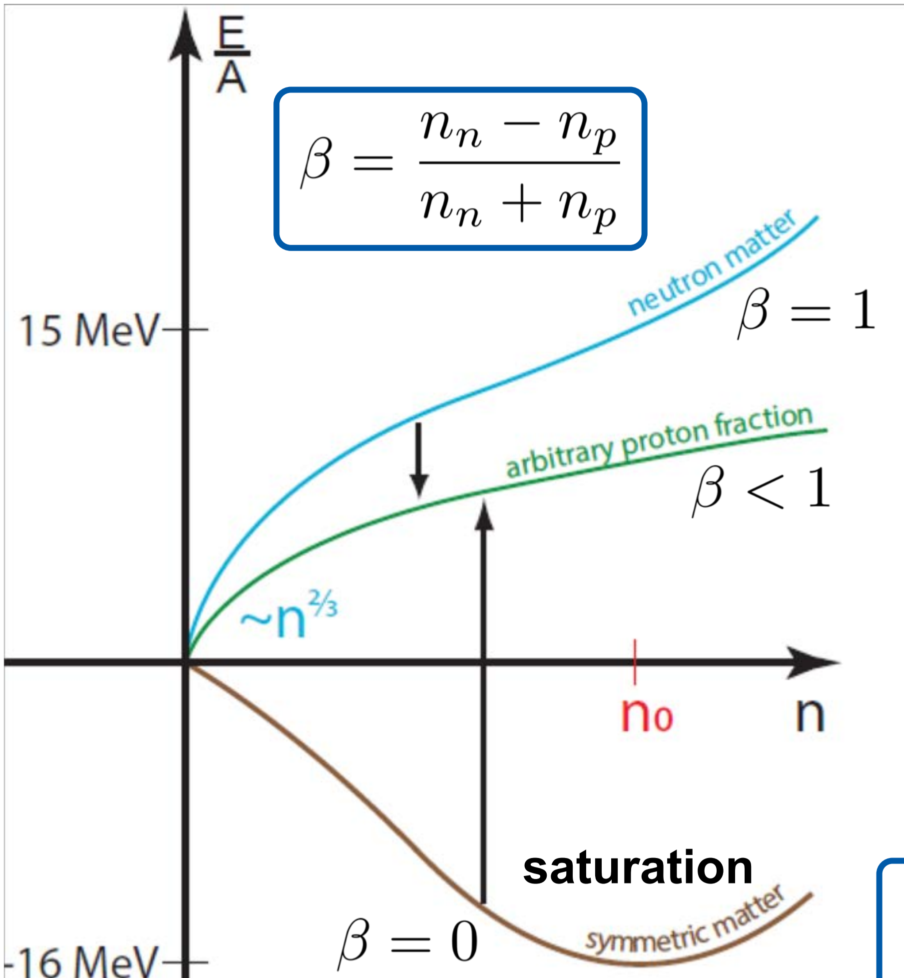
e.g., Watts *et al.*, Rev. Mod. Phys. **88,** 021001



Nuclear Matter at High Orders from Chiral Effective Field Theory

Homogeneous nuclear matter

see also Baym, arXiv:1902.0127



theoretical **testbed** for benchmarking nuclear forces

- **saturation point:** ($n_0 \sim 0.16$ fm^{-3}, $a_v \sim 16$ MeV)
- **incompressibility:** ($K \sim 240$ MeV)
- **symmetry energy** ($S_v \sim 32$ MeV) and its **slope** ($L \sim 55$ MeV) at n_0

Bethe–Weizsäcker formula

$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\mathrm{sym}}(n)$$

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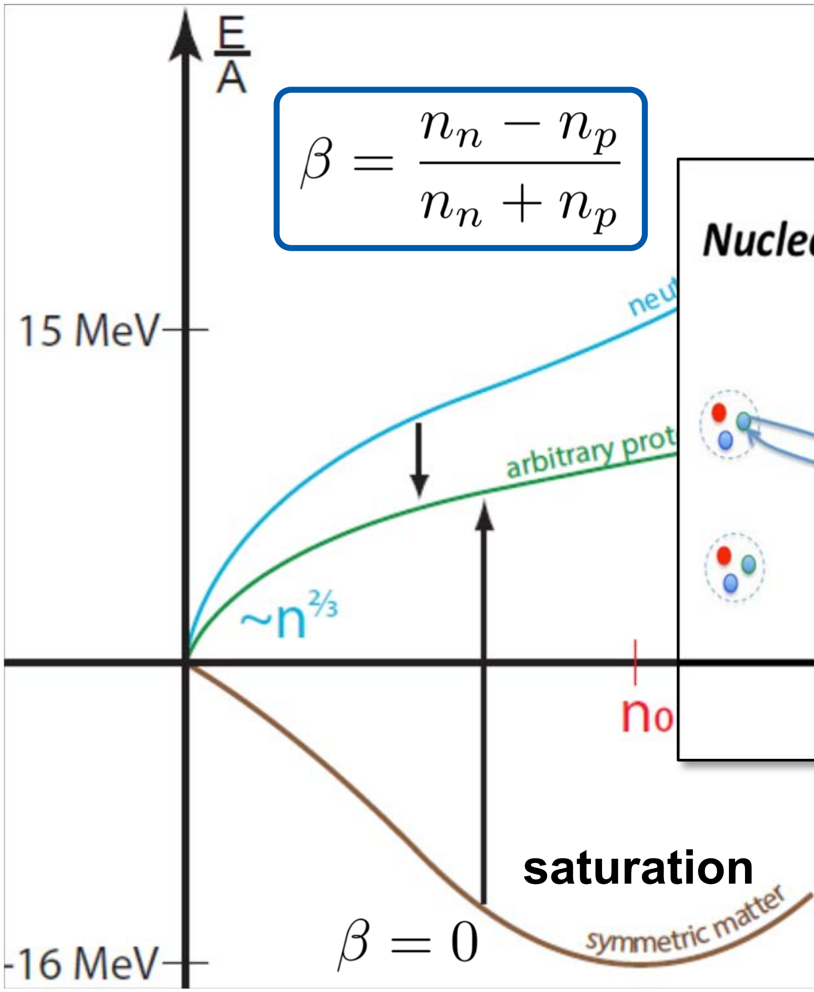
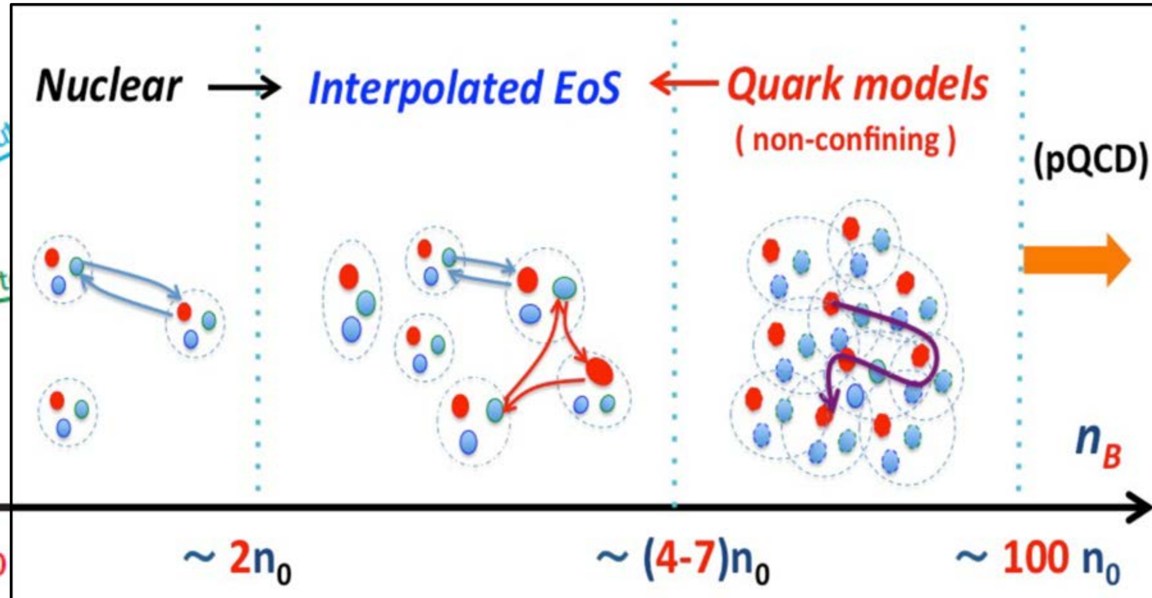
see also Baym, arXiv:1902.0127



$$\beta = \frac{n_n - n_p}{n_n + n_p}$$

$$\beta = 0$$

theoretical **testbed** for benchmarking nuclear forces



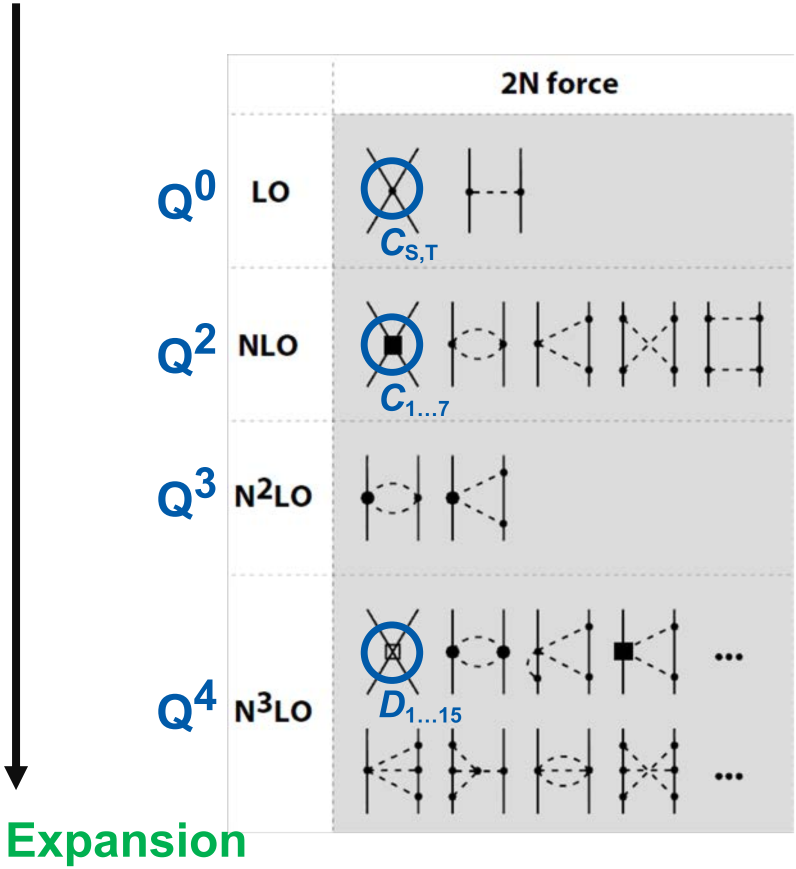
Bethe–Weizsäcker formula

$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\rm sym}(n)$$

Nuclear Matter at High Orders from Chiral Effective Field Theory

Chiral effective field theory

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



modern approach to nuclear forces:

- QCD is nonperturbative at the low-energy scales of nuclear physics
- use relevant instead of the fundamental degrees of freedom: *e.g.*, **nucleons** and **pions**
- **pion exchanges** and short-range **contact interactions** ($\propto$ LEC)
- systematic expansion enables improvable **uncertainty estimates**

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \sim \frac{1}{3}$$

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Chiral effective field theory

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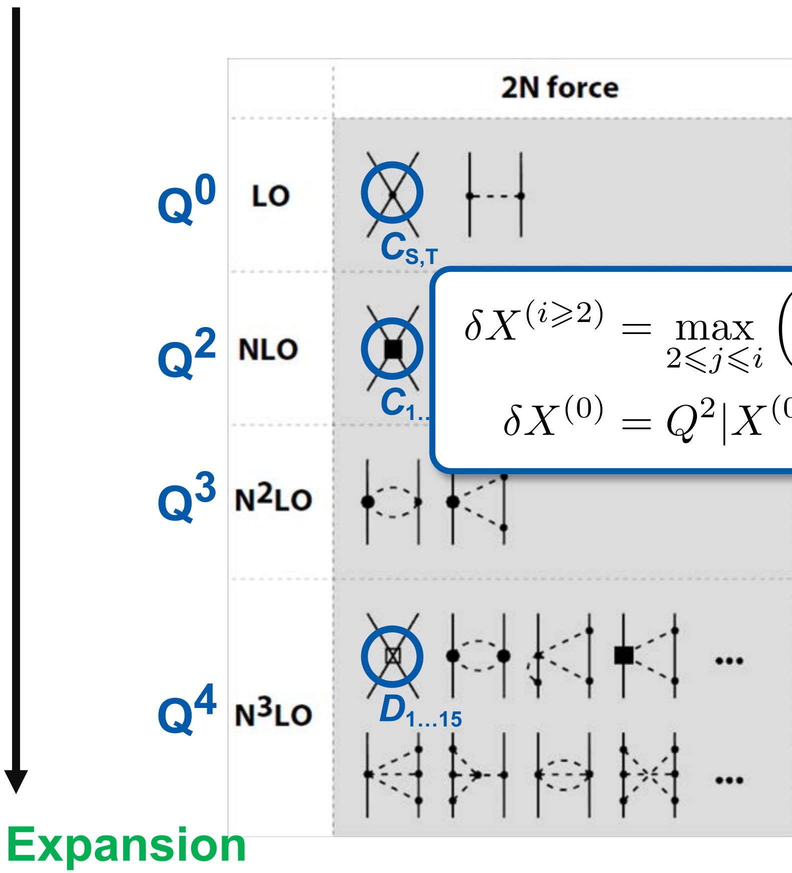

modern approach to nuclear forces:

- QCD is nonperturbative at the low-energy scales of nuclear physics
- use relevant instead of the ...ees of freedom: ...nd **pions**
- **pion exchanges** and short-range **contact interactions** (∝ LEC)
- systematic expansion enables improvable **uncertainty estimates**

$$\delta X^{(i\geqslant 2)} = \max_{2\leqslant j\leqslant i}\left(Q^{i+1}|X^{(0)}|,\, Q^{i+1-j}|\Delta X^{(j)}|\right)$$

$$\delta X^{(0)} = Q^2|X^{(0)}|$$

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \sim \frac{1}{3}$$

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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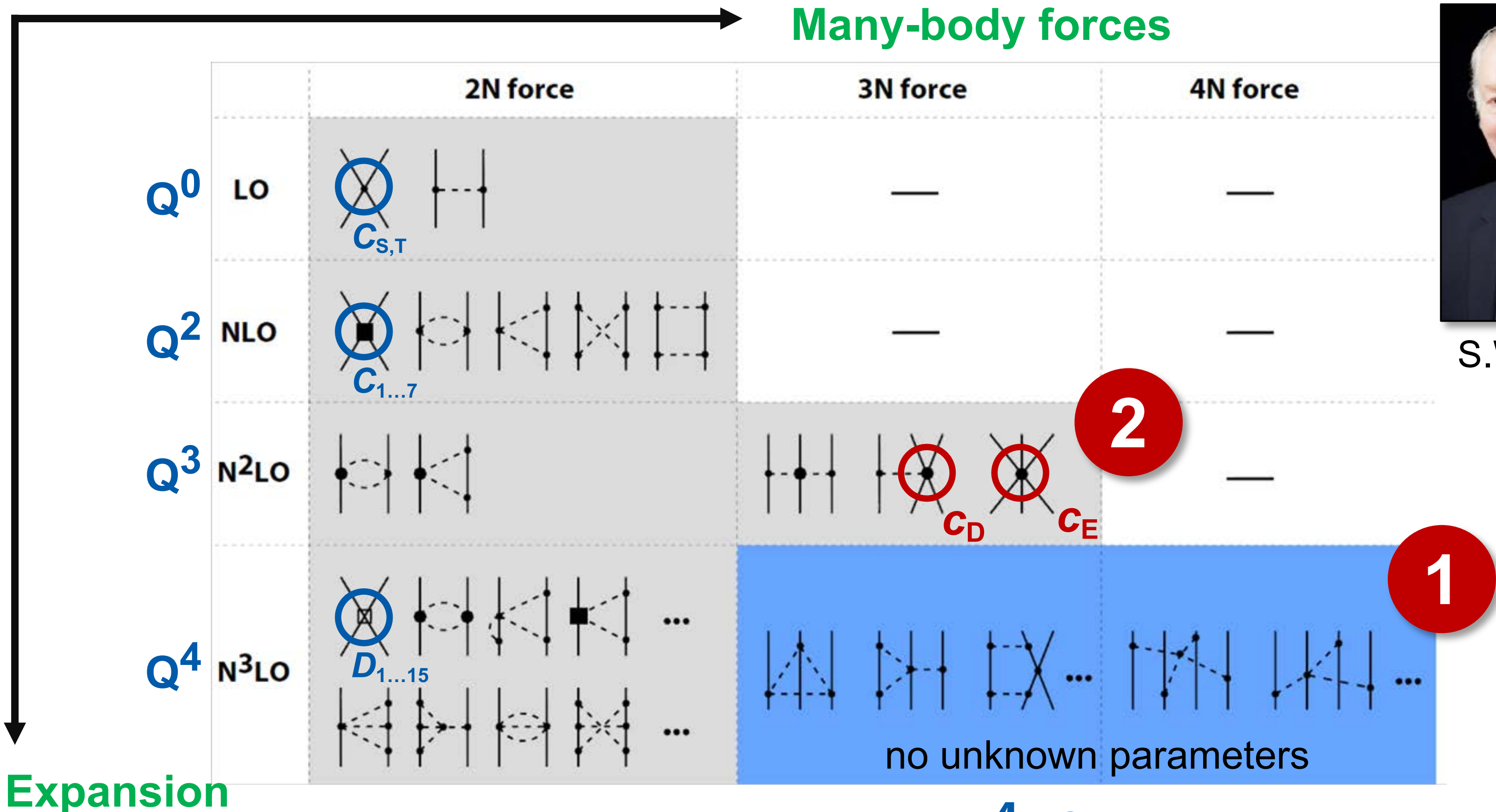
Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



S.Weinberg

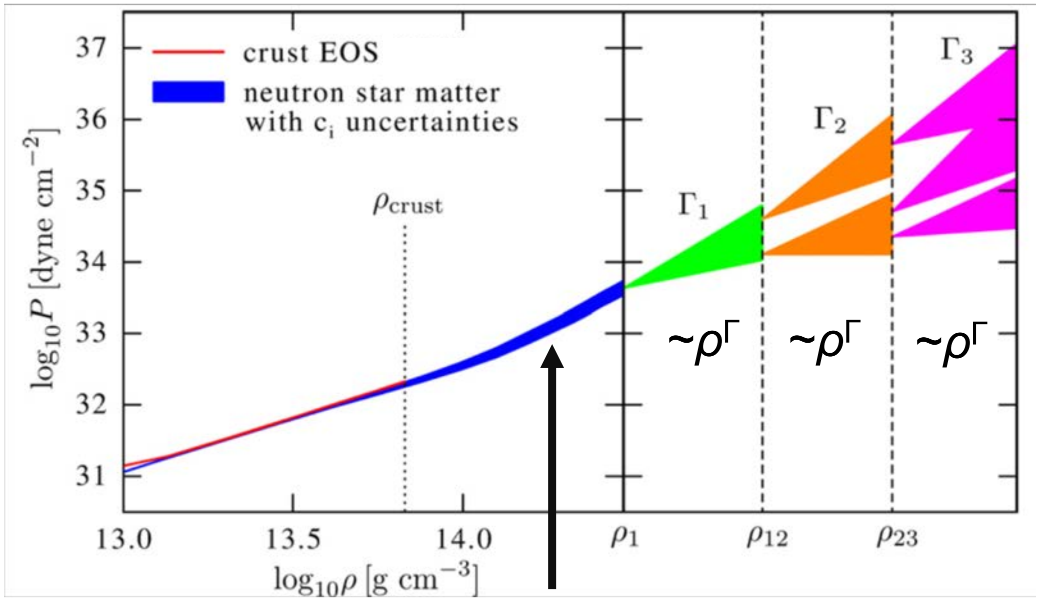
… and ongoing work at N⁴LO and even N⁵LO…

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

Nuclear Matter at High Orders from Chiral Effective Field Theory

Mass–radius relation

Read *et al.*, PRD **79**, 124032; Greif *et al.*, arXiv:1812.08188



Neutron matter extrapolated to β equilibrium

Nuclear Matter at High Orders from Chiral Effective Field Theory

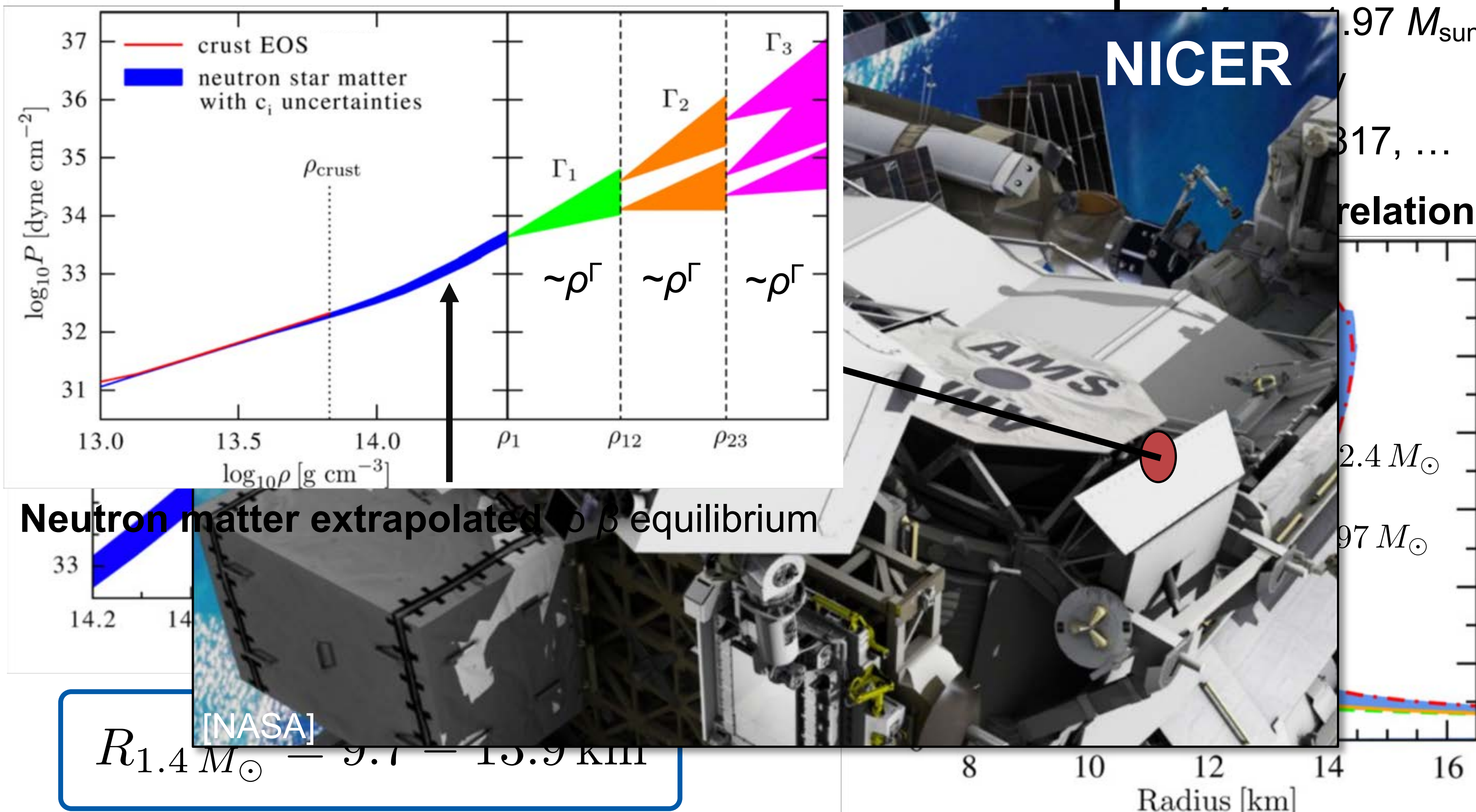
Mass–radius relation

Read *et al.*, PRD **79**, 124032; Greif *et al.*, arXiv:1812.08188

.97 M_{sun}

317, …

relation



Neutron matter extrapolated to β equilibrium

$$R_{1.4\,M_\odot} = 9.7 - 13.9 \text{ km}$$

Nuclear Matter at High Orders from Chiral Effective Field Theory

Mass–radius relation

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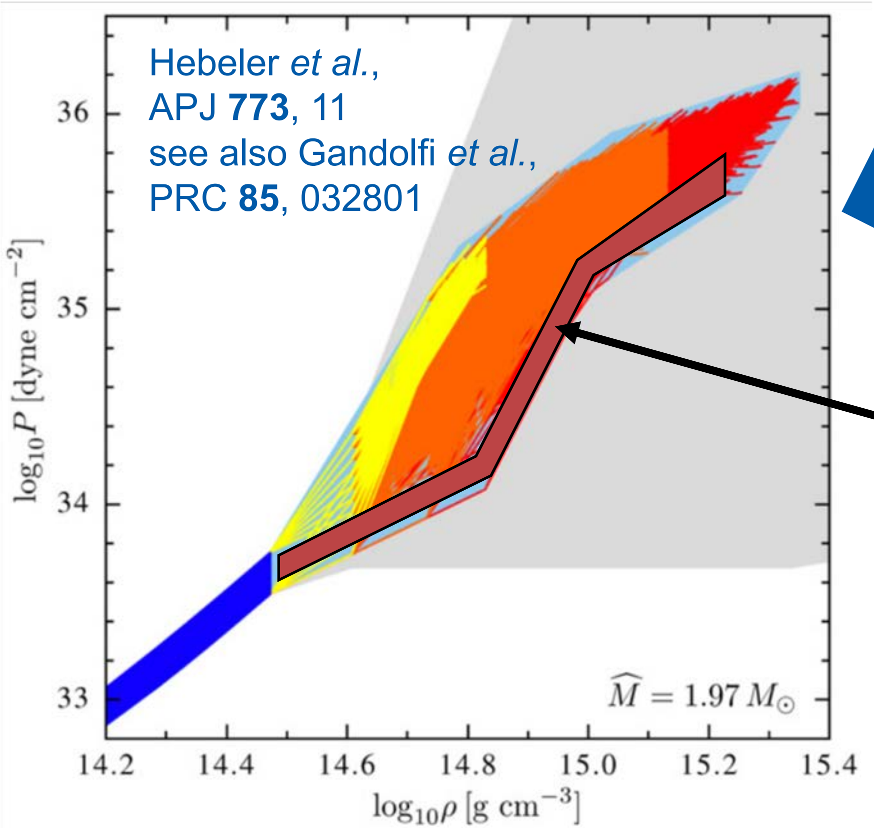
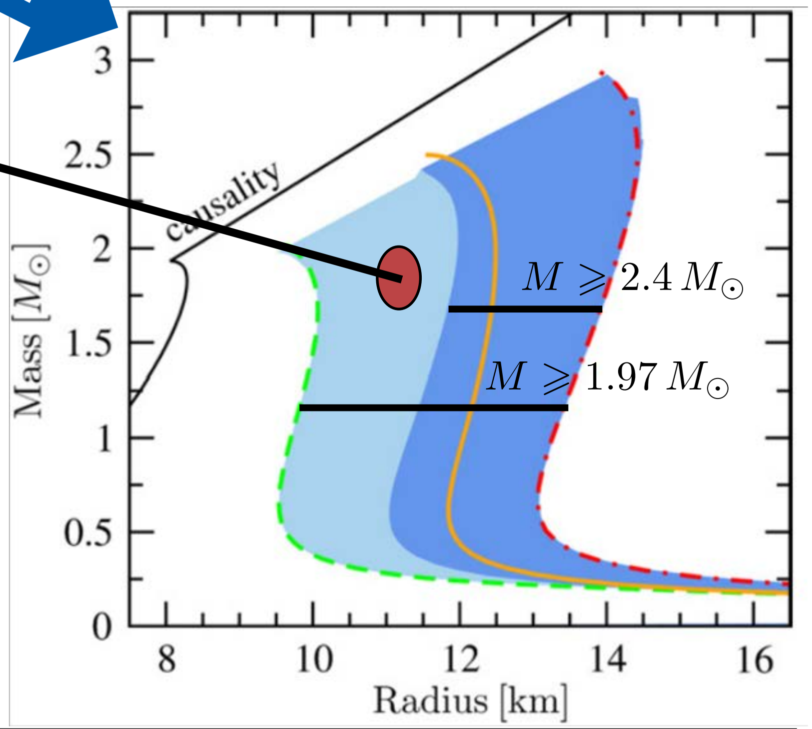


$$R_{1.4\,M_\odot} = 9.7 - 13.9\,\mathrm{km}$$

constraints from
- + $M_{max} \geq 1.97\,M_{sun}$
- + causality
- + GW170817, …

TOV eqs.

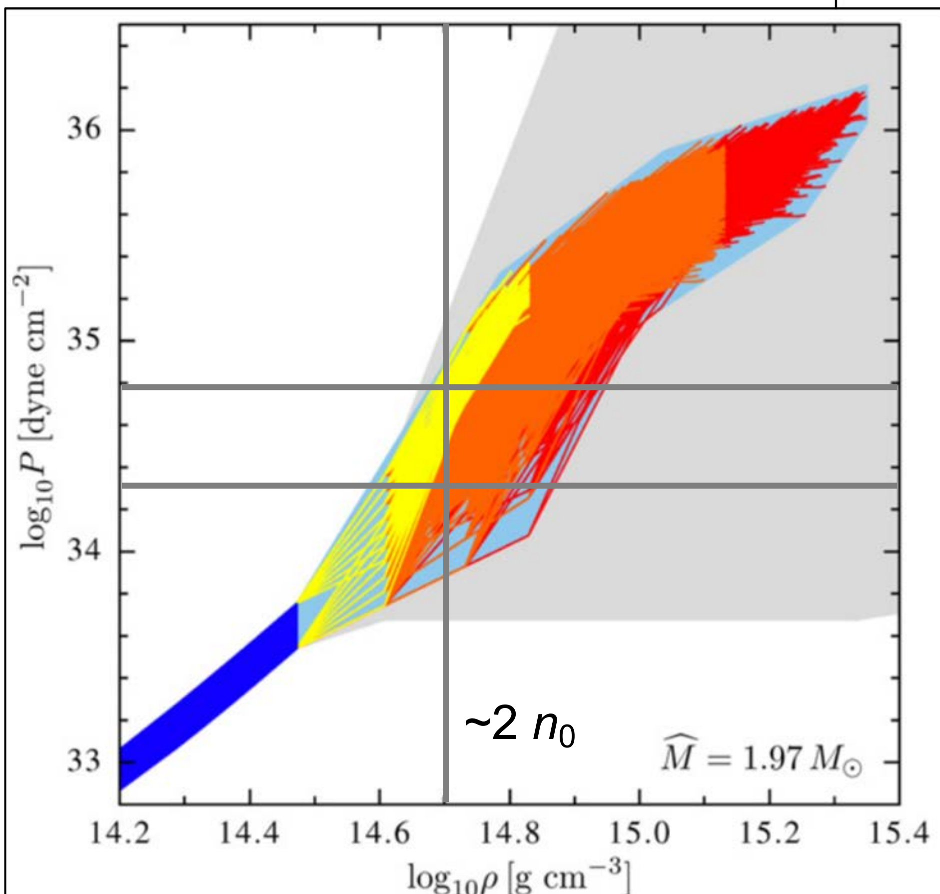
mass–radius relation



Nuclear Matter at High Orders from Chiral Effective Field Theory

Constraining the EOS

Abbott *et al.* (LIGO/Virgo), PRL **121**, 161101



GW170817: Measurements of Neutron Star Radii and Equation of State

B. P. Abbott *et al.*[*]
(The LIGO Scientific Collaboration and the Virgo Collaboration)

eived 5 June 2018; revised manuscript received 25 July 2018; published 15 October 2018)

ugust 2017, the LIGO and Virgo observatories made the first direct detection of gravitational
the coalescence of a neutron star binary system. The detection of this gravitational-wave signal,
, offers a novel opportunity to directly probe the properties of matter at the extreme conditions
e interior of these stars. The initial, minimal-assumption analysis of the LIGO and Virgo data
traints on the tidal effects of the coalescing bodies, which were then translated to constraints on
radii. Here, we expand upon previous analyses by working under the hypothesis that both bodies
n stars that are described by the same equation of state and have spins within the range observed in
nary neutron stars. Our analysis employs two methods: the use of equation-of-state-insensitive
etween various macroscopic properties of the neutron stars and the use of an efficient
tion of the defining function $p(\rho)$ of the equation of state itself. From the LIGO and Virgo
and the first method, we measure the two neutron star radii as $R_1 = 10.8^{+2.0}_{-1.7}$ km for the heavier
$= 10.7^{+2.1}_{-1.5}$ km for the lighter star at the 90% credible level. If we additionally require that the
state supports neutron stars with masses larger than 1.97 $M_\odot$ as required from electromagnetic
s and employ the equation-of-state parametrization, we further constrain $R_1 = 11.9^{+1.4}_{-1.4}$ km and
$^{+1.4}_{-1.4}$ km at the 90% credible level. Finally, we obtain constraints on $p(\rho)$ at supranuclear densities,
re at twice nuclear saturation density measured at $3.5^{+2.7}_{-1.7} \times 10^{34}$ dyn cm^{-2} at the 90% level.

Annala, Gorda *et al.*, PRL **120**, 172703
Margalit, Metzger, APJ **860**, 2
Most, Weih *et al.*, PRL **120**, 261103
Lim, Holt, PRL **121**, 062701
Tews, Margueron *et al.*, PRC **98**, 045804
Coughlin, Dietrich *et al.*, arXiv:1812.04803
Tews, Margueron *et al.*, arXiv:1901.09874

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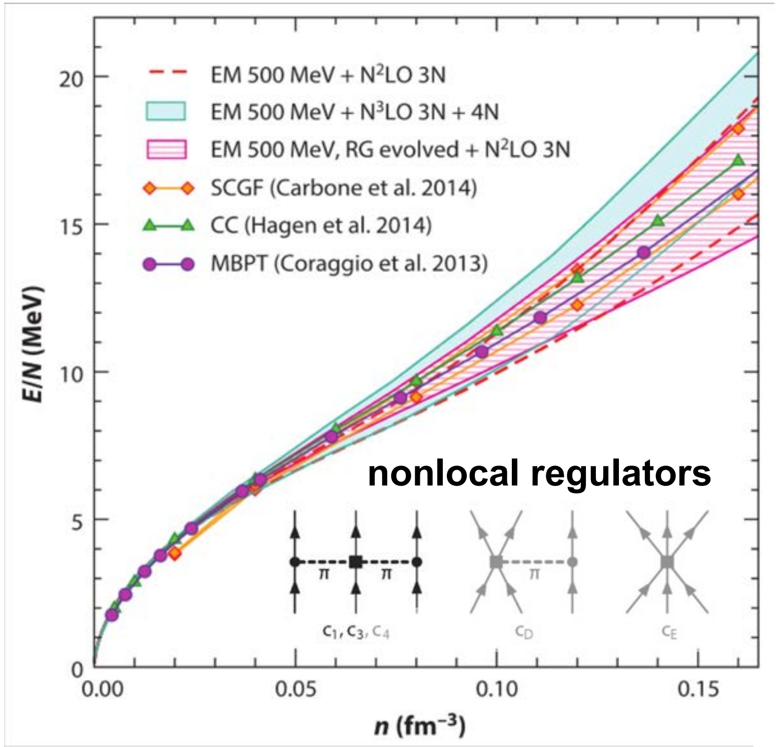
Neutron-matter EOS

Hebeler *et al.*, ARNPS **65**, 457



for QMC, see Carlson, Gandolfi, Tews, …

Remarkable agreement between **many-body frameworks** and **different Hamiltonians**

not so obvious in **symmetric matter**

Explore, quantify, reduce uncertainties from

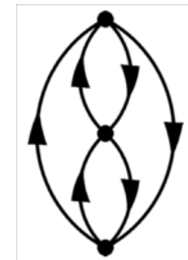
1. chiral convergence
2. many-body convergence

» **push state-of-the-art MBPT calculations to higher orders**

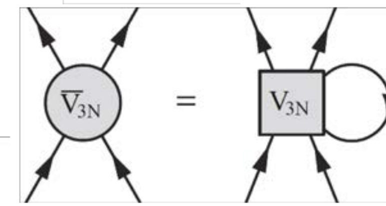
Nuclear Matter at High Orders from Chiral Effective Field Theory

Significant challenges!

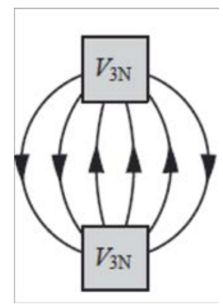
CD, Hebeler, Schwenk, PRL **122**, 042501



✗ **Higher orders: particle-hole contributions**
Coraggio *et al.*, PRC **89**, 044321; Holt, Kaiser, PRC **95**, 034326

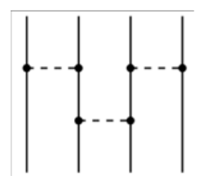


✗ **Approximated normal-ordering**
Holt *et al.*, PRC **81**, 024002; Hebeler, Schwenk, PRC **82**, 014314



✗ **Neglected residual 3N diagrams**
Hagen *et al.*, PRC **89**, 014319; Kaiser, EPJ A **48**, 58

✗ **Higher many-body forces**
Hebeler *et al.*, PRC **91**, 044001



» **development of a novel Monte Carlo framework**

Stevenson, Int. J. Mod. Phys. C **14**, 1135

The number of diagrams increases rapidly!

1, 3, 39, 840, 27 300, 1 232 280, …

n =	2	3	4	5	6	7

Integer sequence A064732:
Number of labeled Hugenholtz diagrams with n nodes.

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Number of diagrams in MBPT

Stevenson, Int. J. Mod. Phys. C **14**, 1135

The number of diagrams increases rapidly!

	1,	**3,**	**39,**	**840,**	**27 300,**	**1 232 280,**	**…**
n =	2	3	4	5	6	7	

Integer sequence A064732:

Number of labeled Hugenholtz diagrams with n nodes.

Nuclear Matter at High Orders from Chiral Effective Field Theory

Efficient Monte Carlo framework

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CD, Hebeler, Schwenk, PRL **122**, 042501

efficient evaluation of **diagrams** in **MBPT** (single-particle basis)

- using analytic expressions: **NN**, **3N**, **4N** forces (nonlocal) up to N^3LO
- **implementing diagrams** has become **straightforward** (all terms)
- multi-dimensional momentum integrals: Lepage's Vegas algorithm
- **computational beast:** automated computation of many diagrams

EOS at high orders

automatic code generation

analytic form of forces / diagrams

Nuclear Matter at High Orders from Chiral Effective Field Theory

Nuclear saturation

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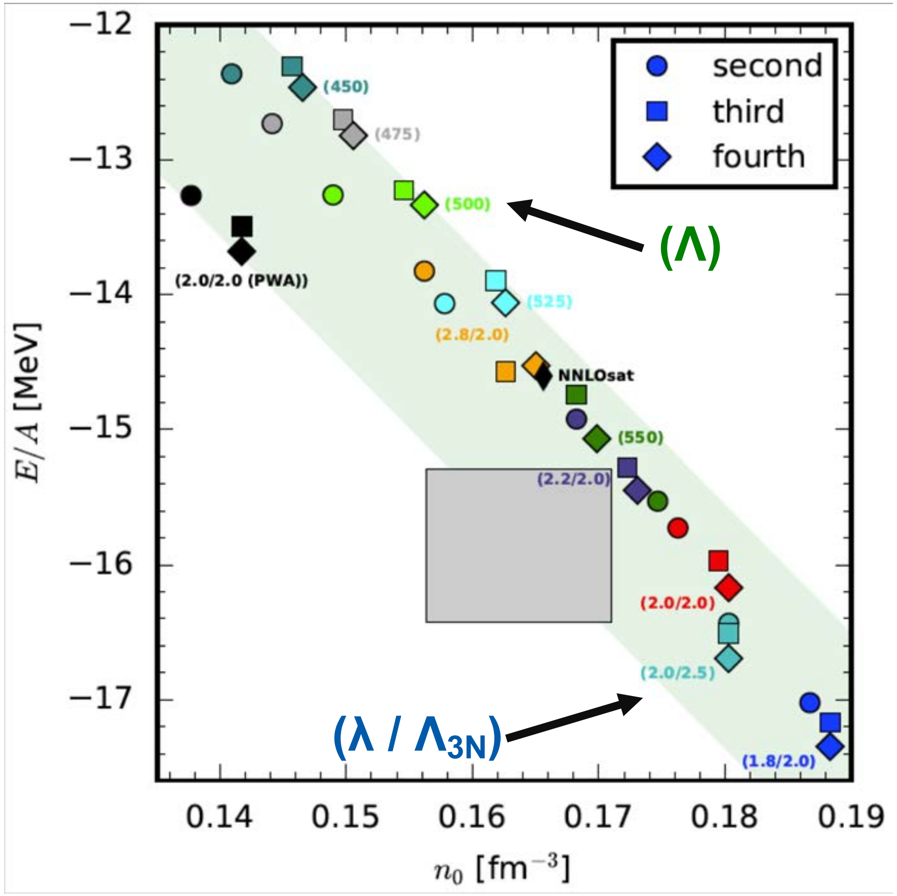
potentials: Hebeler *et al.*, PRC **83**, 031301



CD, Hebeler, Schwenk, PRL **122**, 042501

contributions from

- **NN (4th)**, NN plus 3N (3rd),
- **residual 3N–3N term (2nd)**

Hebeler *et al.*, PRC **83**, 031301
Carlsson *et al.*, PRX **6**, 011019

good many-body convergence

» **interactions are perturbative** for these densities

Coester-like linear correlation

Coester *et al.*, PRC **1**, 769

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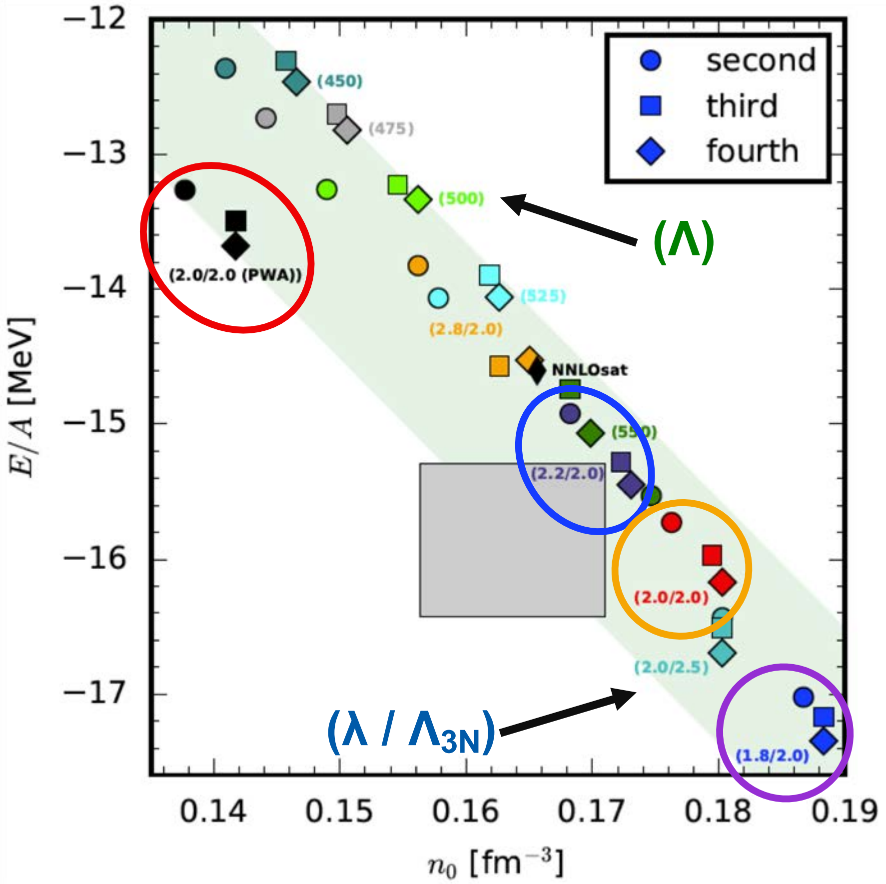
Nuclear saturation

potentials: Hebeler *et al.*, PRC **83**, 031301

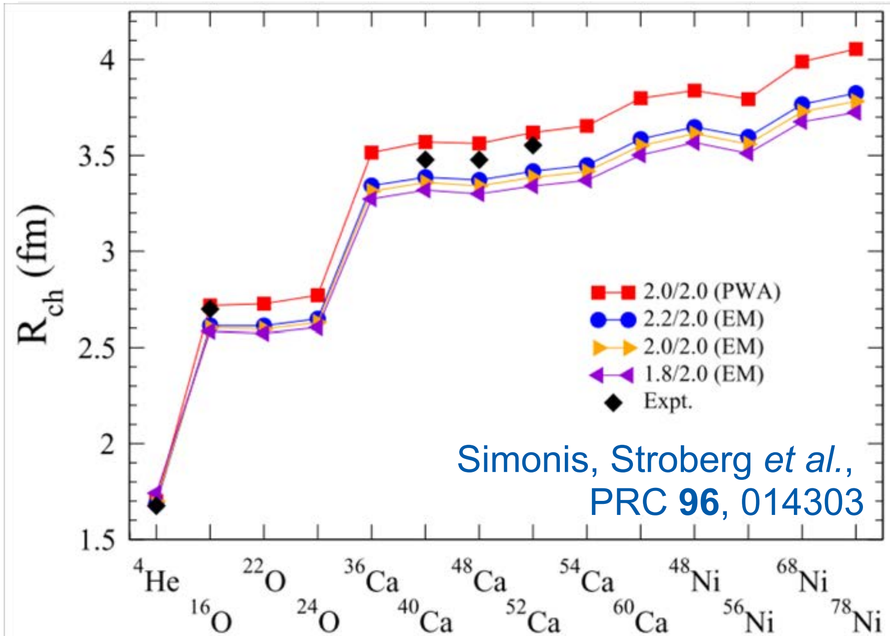
Homogeneous Matter



CD, Hebeler, Schwenk, PRL **122**, 042501

magic **1.8 / 2.0 (EM) agrees well with experimental data!**

Finite Nuclei

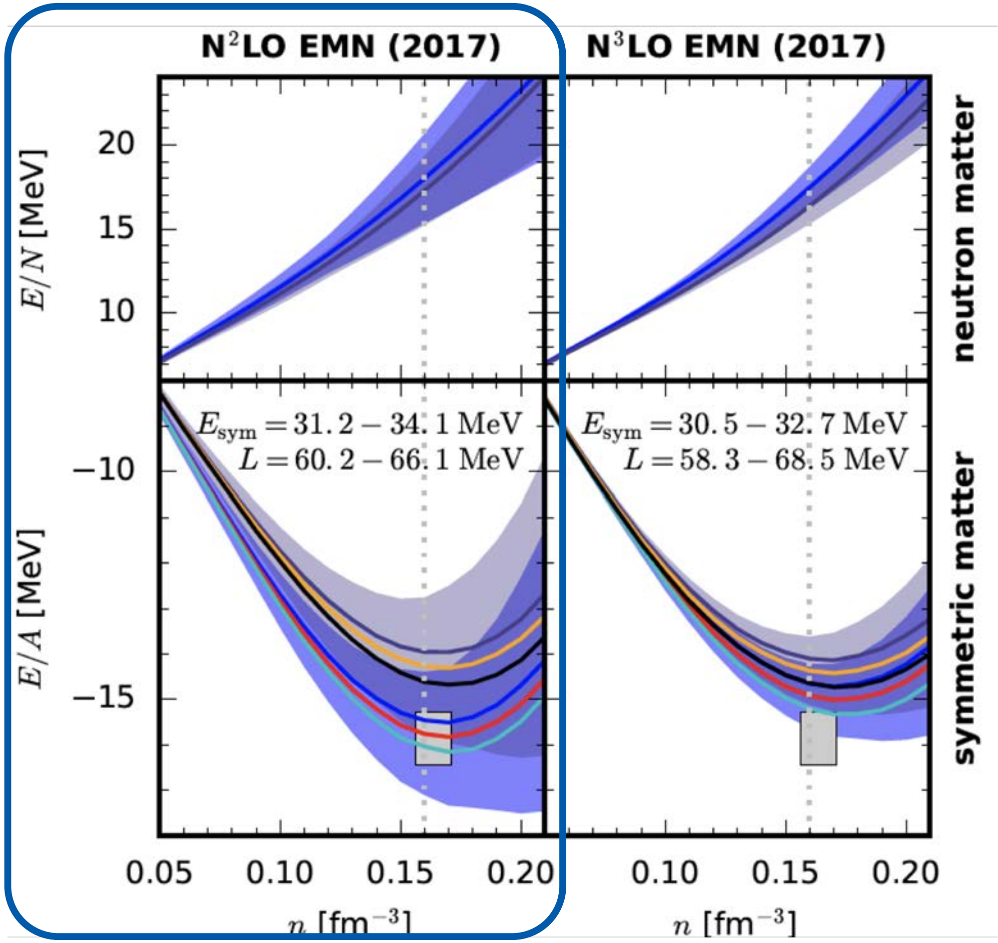


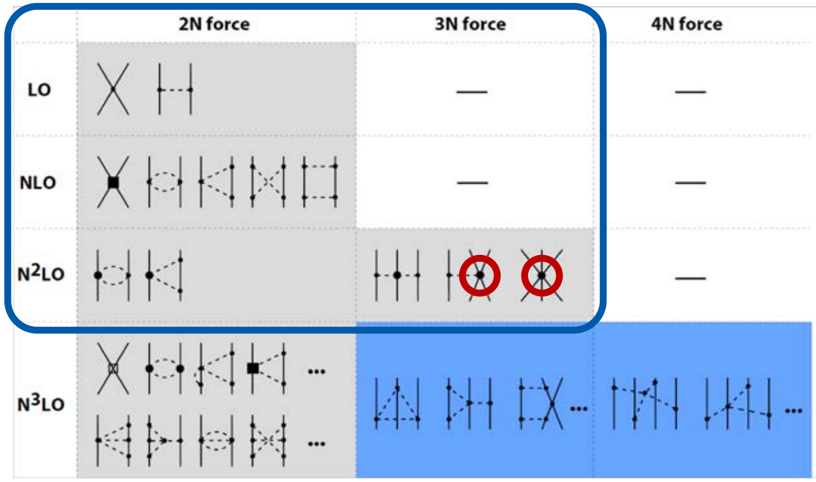
Simonis, Stroberg *et al.*, PRC **96**, 014303

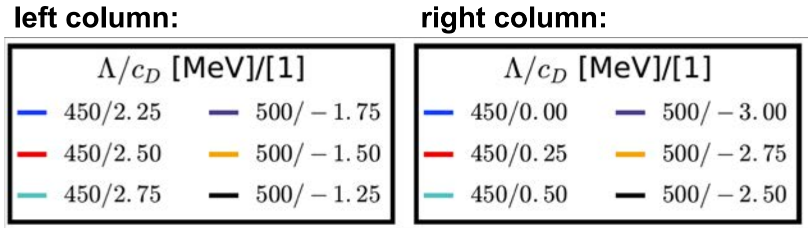
Nuclear Matter at High Orders from Chiral Effective Field Theory

Fits to empirical saturation point

CD, Hebeler, Schwenk, PRL **122**, 042501



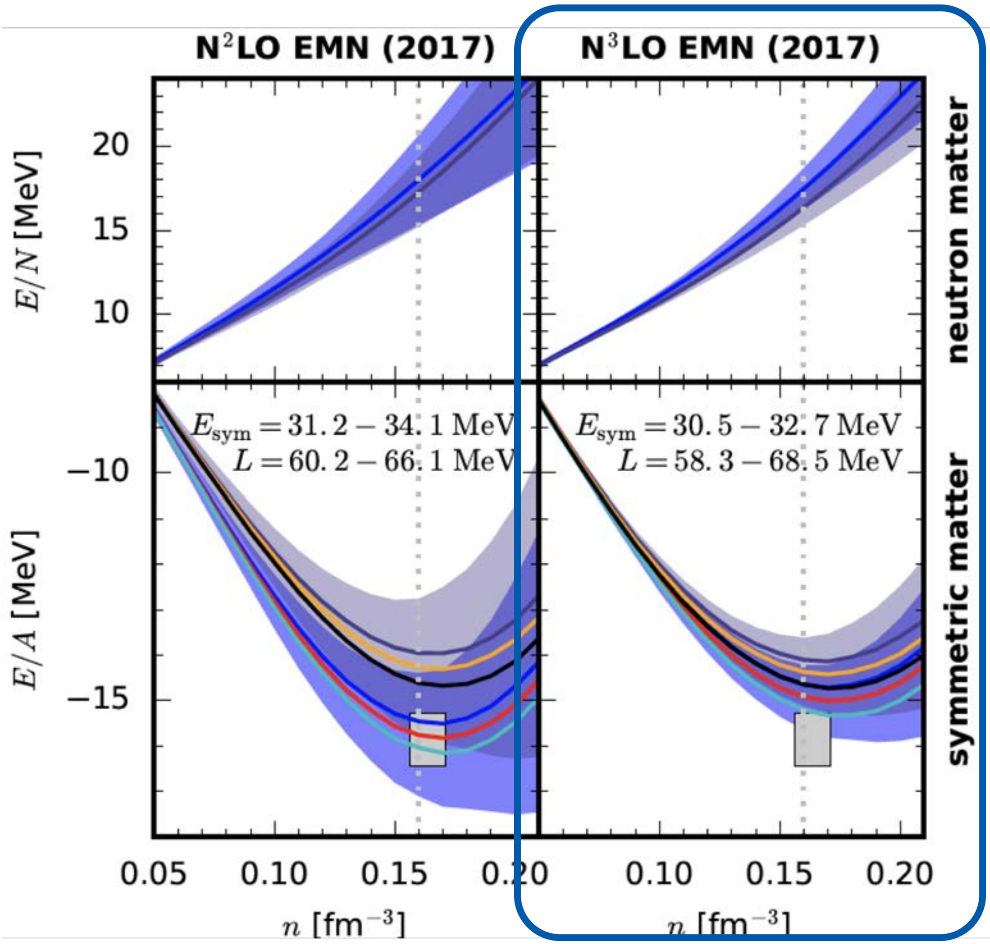


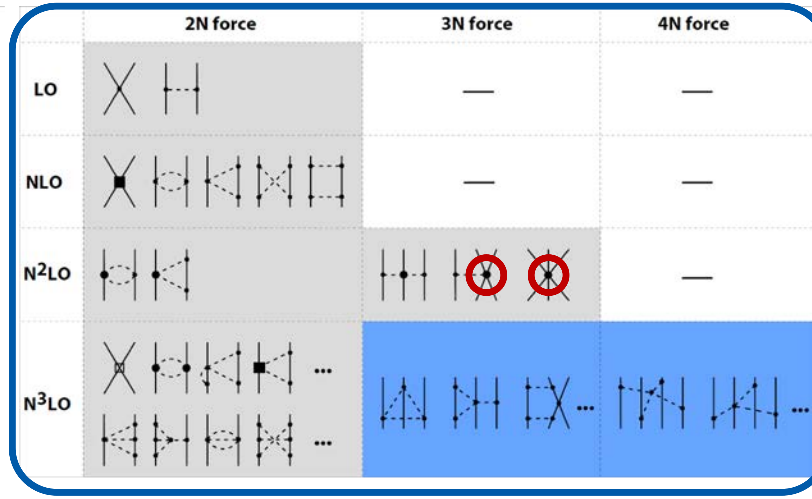


Nuclear Matter at High Orders from Chiral Effective Field Theory

Fits to empirical saturation point

CD, Hebeler, Schwenk, PRL **122**, 042501





left column:

Λ/c_D [MeV]/[1]	
450/2.25	500/−1.75
450/2.50	500/−1.50
450/2.75	500/−1.25

right column:

Λ/c_D [MeV]/[1]	
450/0.00	500/−3.00
450/0.25	500/−2.75
450/0.50	500/−2.50

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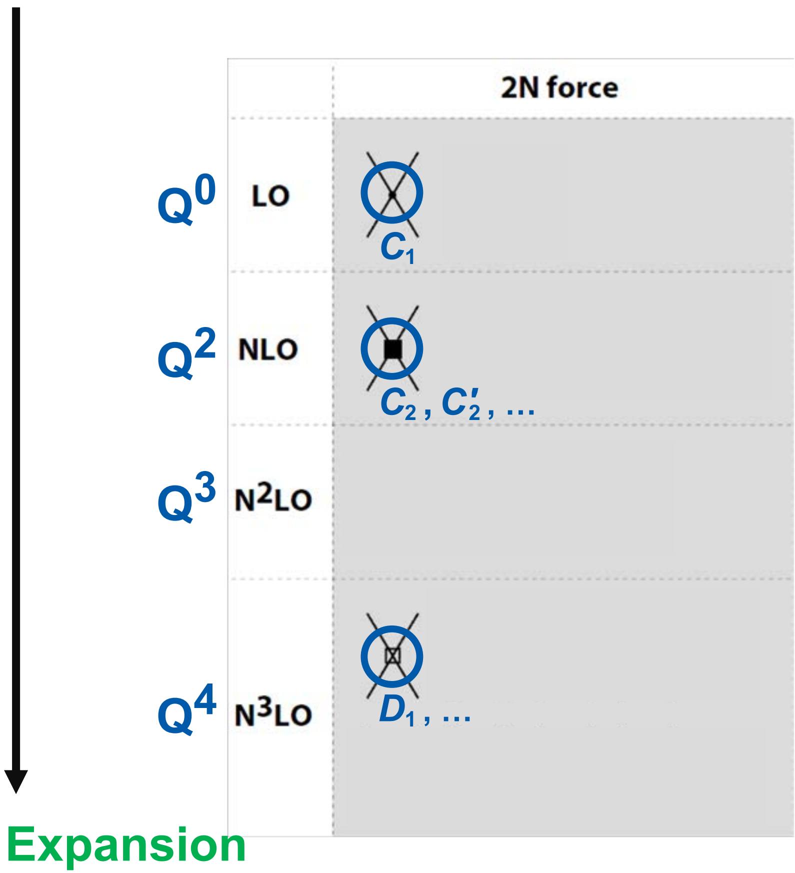
Pionless effective field theory

Wellehofer, CD, Schwenk, arXiv:1812.08444



apply many-body perturbation theory

$$\langle \boldsymbol{p}' | V_{\mathrm{NN}} | \boldsymbol{p} \rangle = \left[C_0(\Lambda) + C_2(\Lambda) \frac{\boldsymbol{p}'^2 + \boldsymbol{p}^2}{2} + C_2'(\Lambda)\, \boldsymbol{p}' \cdot \boldsymbol{p} + \ldots \right] \times \theta(\Lambda - p)\, \theta(\Lambda - p')$$

$$\langle \boldsymbol{p}' \boldsymbol{q}' | V_{3\mathrm{N}} | \boldsymbol{p} \boldsymbol{q}' \rangle = \left[D_0(\Lambda) + \ldots \right] \times \theta(\Lambda - p)\, \theta(\Lambda - p') \theta(\Lambda - q)\, \theta(\Lambda - q')$$

match LECs to effective-range expansion

$$C_0 = \frac{4\pi a_s}{M} \qquad C_2 = C_0 \frac{a_s r_s}{2} \qquad C_2' = \frac{4\pi a_p^3}{M}$$

e.g.,

$$\mathcal{I}_{\mathrm{MBPT}} \sim \int_0^{\Lambda} dq\, \frac{q^2}{q^2 - p^2}$$

renormalize LECs perturbatively to cancel divergences: $\Lambda \rightarrow \infty$

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Fourth order

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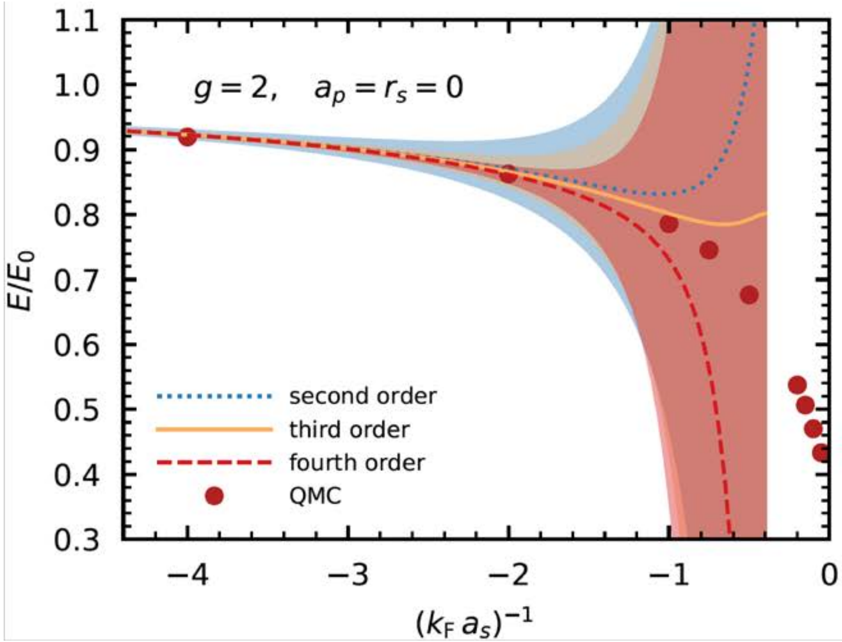
Wellehofer, CD, Schwenk, arXiv:1812.08444

due to Pauli-blocking: expansion is analytic in ($k_F\, a_s$) for $g = 2$

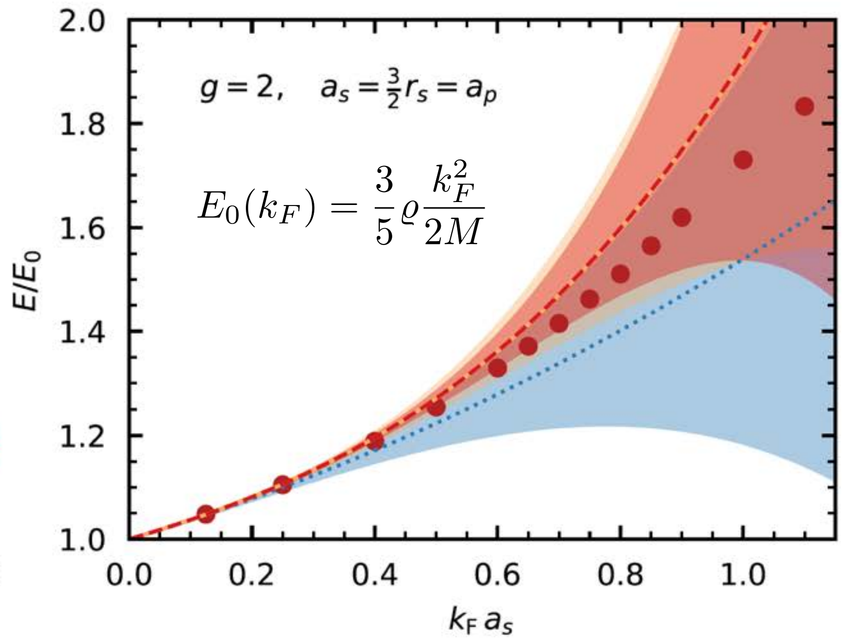
$$E(k_\mathrm{F}) = E_0 \left(1 + \sum_{\nu=1}^{N} X_\nu \,(k_\mathrm{F} a_s)^\nu \right)$$

uncertainty estimate at order *N*

$$X_{N+1} = \pm \max[X_{\nu \le N}]$$



QMC: Gandolfi *et al.*, ARNPS **65**, 303



QMC: Pilati *et al.*, PRL **105**, 030405

Nuclear Matter at High Orders from Chiral Effective Field Theory

Fourth order

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Wellehofer, CD, Schwenk, arXiv:1812.08444

due to Pauli-blocking: expansion is analytic in ($k_F\, a_s$) for $g = 2$

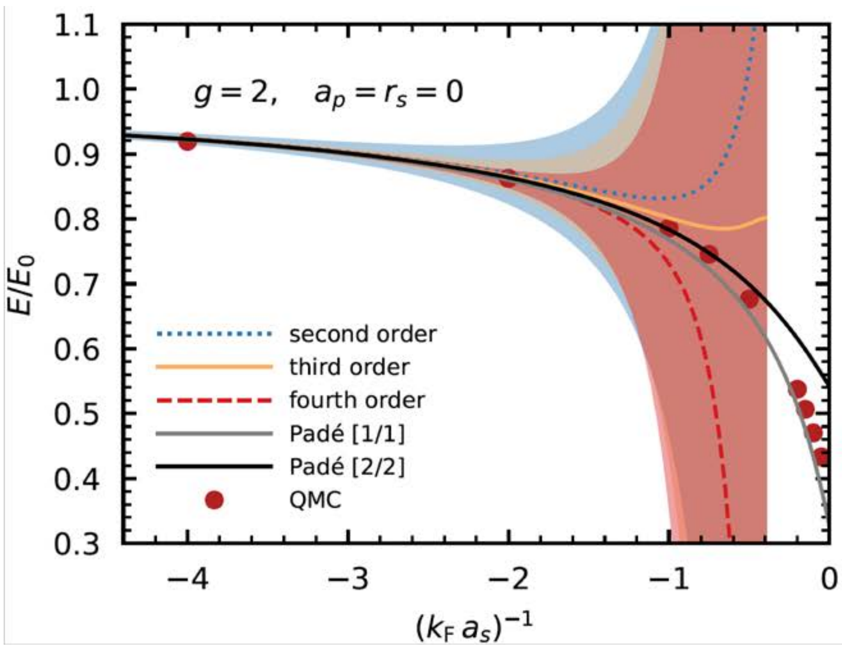
Bertsch parameter $\xi_n = 0.33\ldots0.54$

is consistent with cold atomic gases: $\xi_n = 0.45$

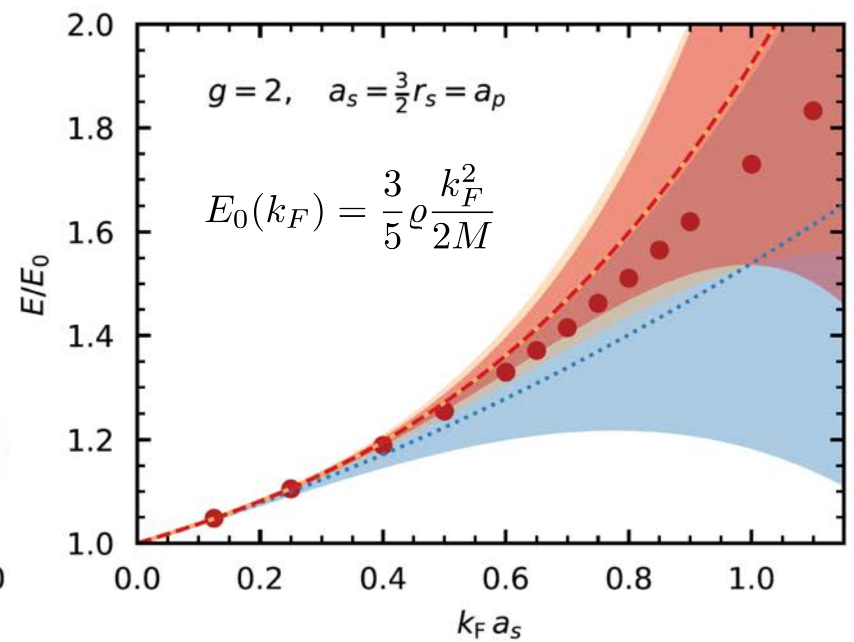
Ku *et al.*, Science **335**, 563

Padé resummation

$$[n,\, m] = \frac{\sum_{j=0}^{m} p_j x^j}{1 + \sum_{k=1}^{n} q_k x^k}$$



QMC: Gandolfi *et al.*, ARNPS **65**, 303



QMC: Pilati *et al.*, PRL **105**, 030405

Nuclear Matter at High Orders from Chiral Effective Field Theory

Summary and outlook

1. **Perform zero-T calculations (up to 6th order)**
   resummation, higher-order single-particle spectra, …
2. **Work out finite-T extension (to high orders)**
   finish developments, study thermodynamic properties, …
3. **Construct high-density/temperature EOS**
   observational constraints, interface to astrophysics, …
4. **Quantify theoretical uncertainties**
   Bayesian truncation errors: naturalness, breakdown scale, …

Thanks to:

K. Hebeler
K. McElvain
A. Schwenk
C. Wellenhofer

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