TMD collaboration meeting, LBNL, Berkeley, September 16, 2019

Gluon TMDs from Quarkonium Production in proton collisions including TMD evolution

Marc Schlegel
Department of Physics
New Mexico State University

in collaboration with J.-P. Lansberg, C. Pisano, F. Scarpa
based on **PLB 784, 217 (2018)** ; **NPB 920, 192 (2017)**

recent update: Scarpa, Boer, Echevarria, Lansberg, Pisano, M.S., arXiv:1909.05769

Gluon TMDs = Transverse Momentum Dependent Parton Distributions of gluons in nucleon

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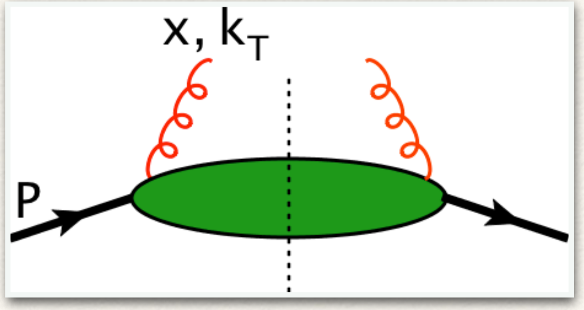

TMD gluonic matrix element

$$\Gamma^{\alpha\beta;\,[\mathcal{W},\mathcal{W}']}(x,k_T) = \int \frac{d\lambda d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{ix\lambda + ik_T\cdot z_T}\, \langle P,S|\, F^{n\alpha}(0)\, \mathcal{W}\, F^{n\beta}(\lambda n + z_T)\, \mathcal{W}'\, |P,S\rangle$$

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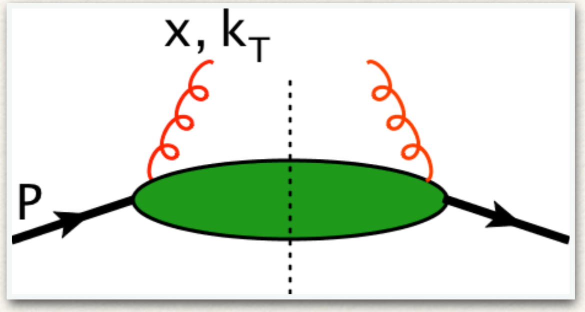

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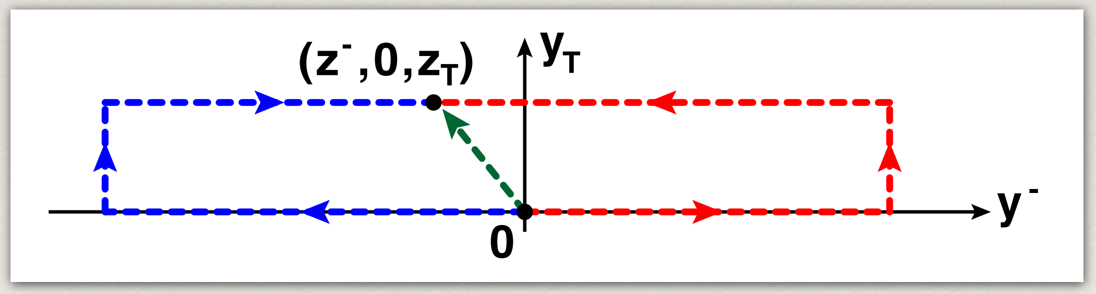
Wilson line in fundamental representation

Color Gauge invariant definition of TMDs ➙ Wilson line

$$\mathcal{W}[a,b] = \mathcal{P}e^{-ig\int_a^b ds\cdot A^a(s)\, t^a}$$

➙ Wilson line for TMD:
nontrivial, process dependent:



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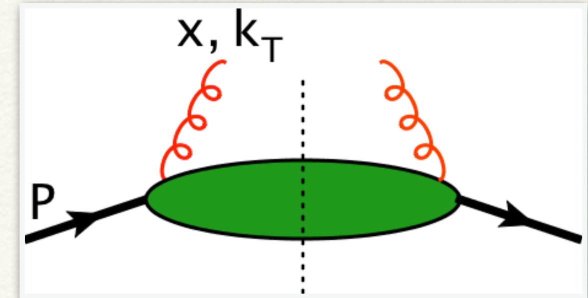

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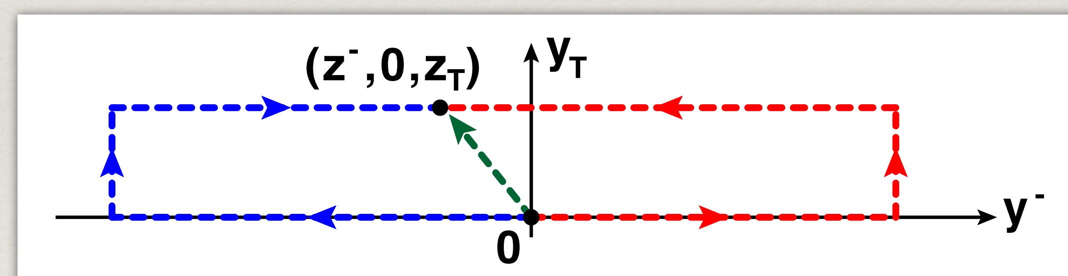

past-pointing WL →
Initial State Interactions:
pp → color singlet + X (DY-type)

$$\Gamma^{\alpha\beta;[-,-]}$$

future-pointing WLs →
Final State Interactions:
ep → jets + X (SIDIS-type)

$$\Gamma^{\alpha\beta;[+,+]}$$

Proper Definition & Soft function

[Collins; Sun, Xiao, Yuan; Aybat, Rogers; see also: Echevarria, Kasemats, Mulders, Pisano, JHEP 07, 158 (2015)]

Inclusion of Soft Function $\Longrightarrow$

$$S(b_T) = \frac{1}{N_c}\mathrm{Tr}_c\langle 0|\,\mathcal{W}_n^\dagger(-b_T/2)\,\mathcal{W}_{\bar{n}}(-b_T/2)\,\mathcal{W}_{\bar{n}}^\dagger(b_T/2)\,\mathcal{W}_n(b_T/2)\,|0\rangle$$

$$\tilde{\Gamma}^{\alpha\beta}(x, b_T) \to \frac{\tilde{\Gamma}^{\alpha\beta}(x, b_T)}{\sqrt{S(b_T)}}$$

Modification needed in order to have:

1) Renormalizable Matrix Element
2) Finite matching coefficients at small b_T

→ TMDs subject to RG and CS equations → scale dependence

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Parameterization

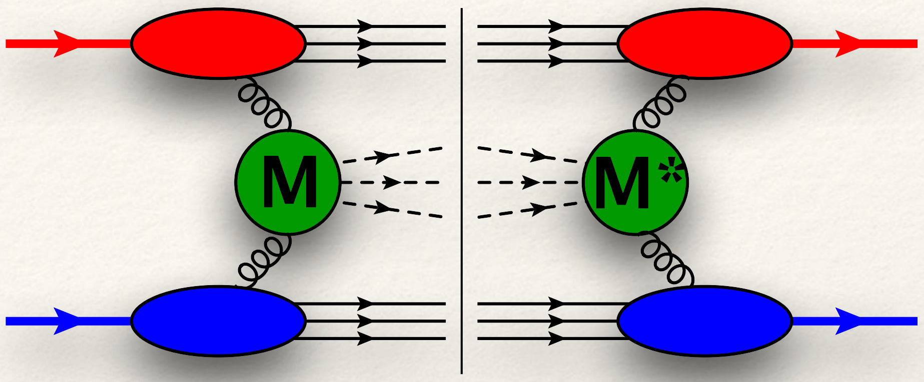
unpolarized & linearly polarized gluons :
helicity flip TMDs ➙ azimuthal modulations

$$\Gamma^{\alpha\beta}(x, k_T) = \frac{1}{2x}\left[-g_T^{\alpha\beta}\; f_1^g(x, k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2}k_T^2 g_T^{\alpha\beta}}{M^2}\; h_1^{\perp g}(x, k_T^2)\right]$$

Gluon TMDs from pp - collisions

[J.-P. Lansberg, C. Pisano, M.S., NPB 920, 192 (2017)]

General TMD expression for gluon fusion:



$$\frac{d\sigma}{d^4q...}(q_T \ll Q) \propto \mathcal{C}[\Gamma^{\alpha\alpha'}(x_a, k_{aT})\,\Gamma^{\beta\beta'}(x_b, k_{bT})]\,\left(\mathcal{M}_{\alpha\beta}\,(\mathcal{M}_{\alpha'\beta'})^*\right)$$

$$\mathcal{C}[w\,f\,g] = \int d^2k_{aT} \int d^2k_{bT}\,\delta^{(2)}(k_{aT} + k_{bT} - q_T)\,w(k_{aT}, k_{bT})\,f(x_a, k_{aT})\,g(x_b, k_{bT})$$

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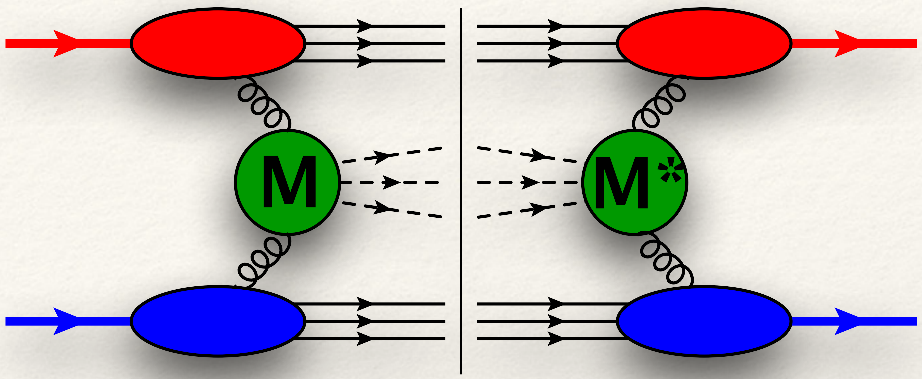

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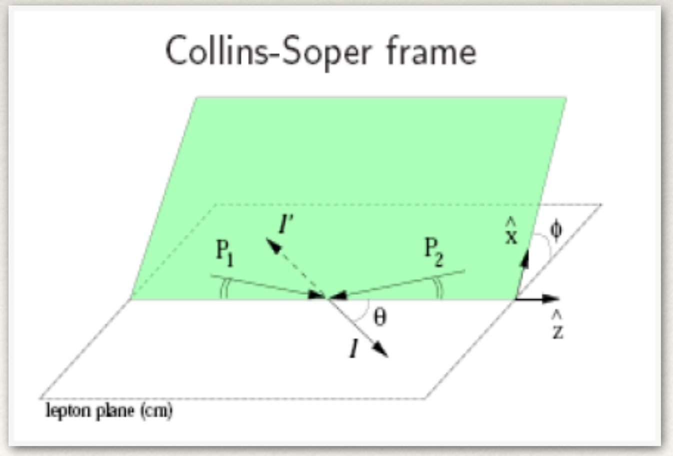
2-particle (pair) final states in pp - collisions

$$\frac{d\sigma^{gg}}{d^4q\;d\Omega}\Big|_{q_T\ll Q} = \hat{F}_1\;[f_1^g \otimes f_1^g] + \hat{F}_2\;[h_1^{\perp g}\otimes h_1^{\perp g}] + \cos(2\phi)\hat{F}_3\;[h_1^{\perp g}\otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\hat{F}_4\;[h_1^{\perp g}\otimes h_1^{\perp g}]$$



Evaluate cross section in
c.m. frame of the produced pair

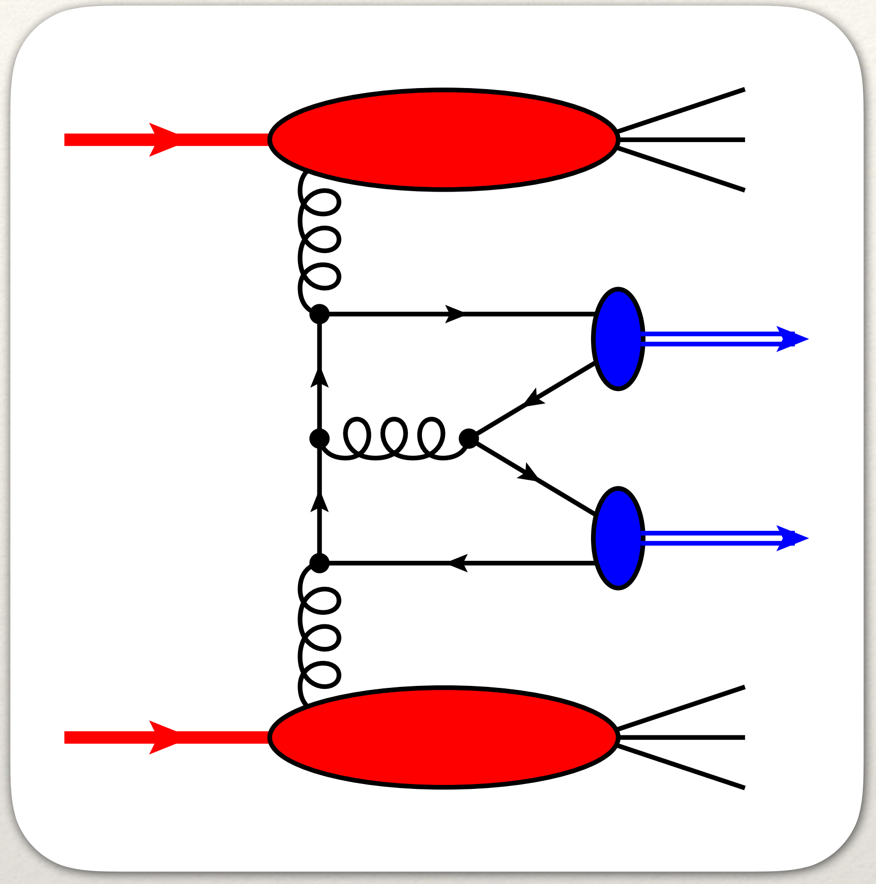
Collins - Soper angles θ , ϕ

Gluon TMDs do not appear in Drell-Yan

Double J/ψ(ϒ)- production

[Lansberg, Pisano, Scarpa, M.S., PLB 784, 217 (2018)]

TMD - formalism: J/ψ pair back-to-back; Color singlet



Here: Sample diagram (of about O(40))

Full analytical LO amplitude

$$g + g \to J/\psi + J/\psi$$

(J/ψ in color singlet configuration)

Presented in Appendix of

[Qiao, Sun, Sun; 0903.0954] (Kudos!)

Helicity formalism: contract with polarization vectors

→ perturbative factors F_1, F_2, F_3, F_4

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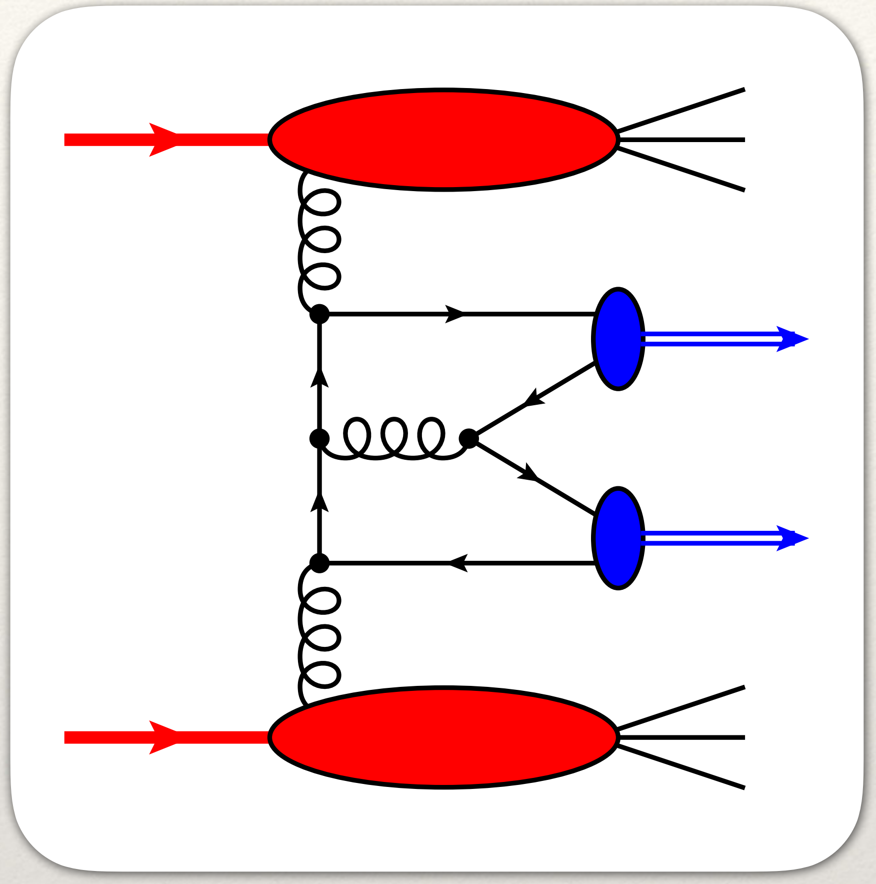

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Degree of polynomial $N_i = 6,\ 4,\ 5,\ 6$

Q: invariant mass of Quarkonium pair

$$F_i(Q, \cos^2\theta) \propto \frac{\sum_{k=0}^{N_i} c_k(\alpha)\,(\cos^2\theta)^k}{(1-(1-\alpha^2)\cos^2\theta)^4}$$

$$\alpha = \frac{2M_{J/\psi}}{Q}$$

$\alpha = 1$ threshold limit

$\alpha = 0$ high-energy limit

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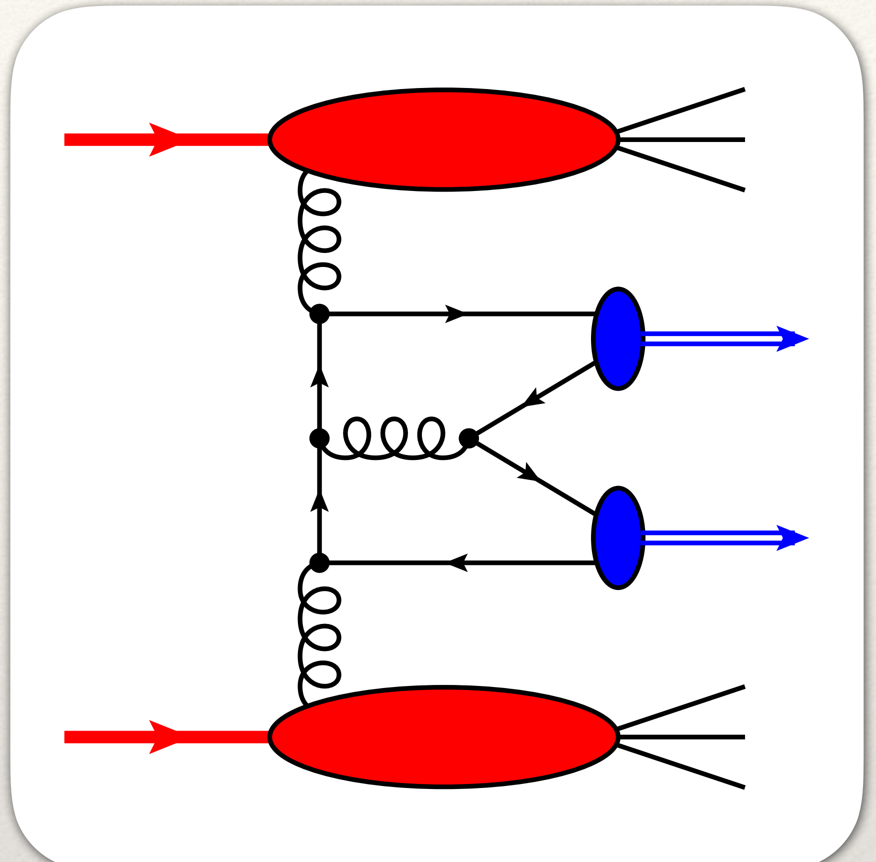

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Numerically: $F_2(Q, \cos^2\theta) \ll F_1(Q, \cos^2\theta)$ $F_4(\theta = \frac{\pi}{2}, Q \gg 2M_{J/\psi}) \to F_1$ Large!

q_T - spectrum: Data on J/ψ - pairs available from LHC

Measurements from CMS and ATLAS at 7,8 TeV, but not suitable for TMD exploration

LHCb data (2017) at 13 TeV slightly above threshold, Q = 8 GeV

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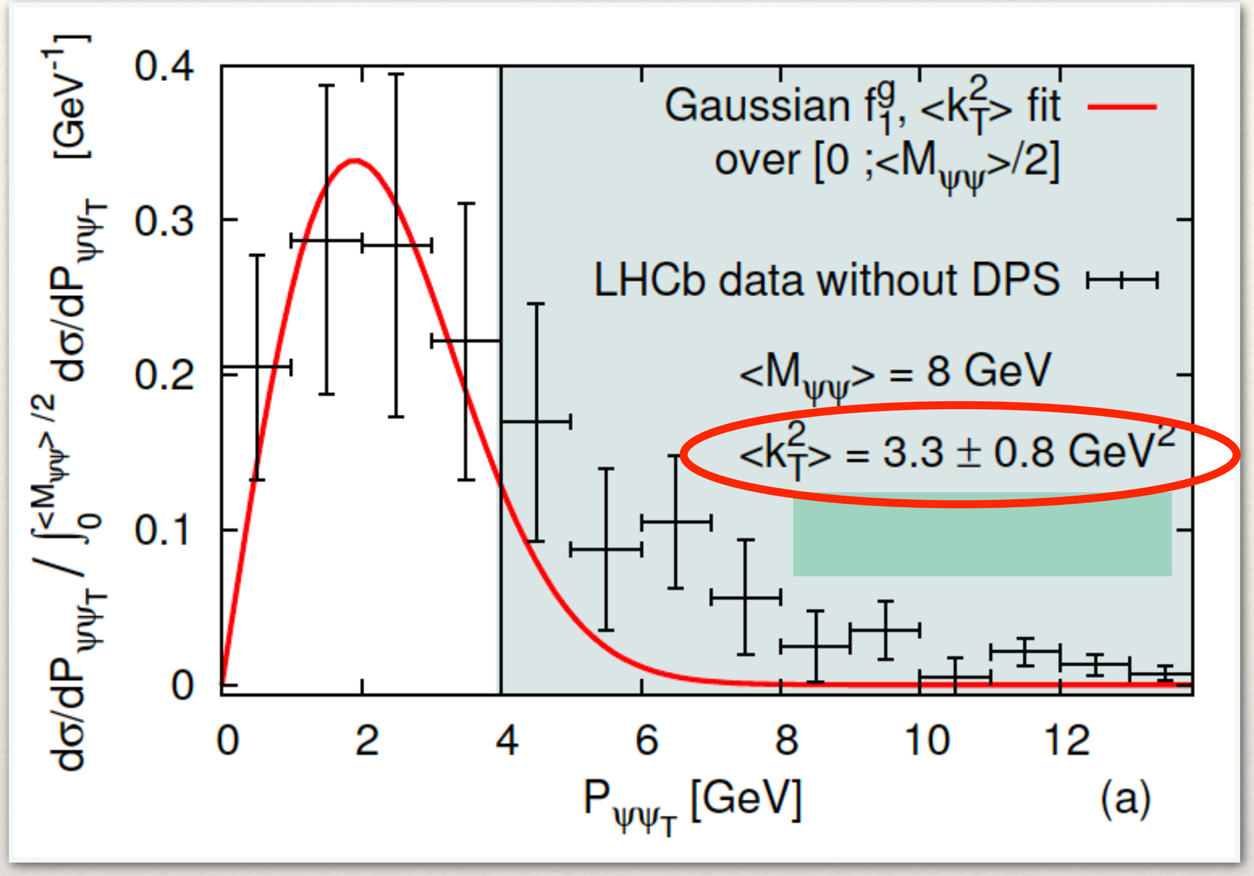

Fit observable

$$\frac{\left[\frac{d\sigma}{dq_T}\right]_{\text{bins}}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T}\right]_{\text{bins}}} \propto \frac{\mathcal{C}[f_1^g\, f_1^g]}{\int_0^{Q/2} dq_T\, \mathcal{C}[f_1^g\, f_1^g]}$$

First glimpse on unpol. gluon TMD:

No fits for f_1 so far, Gaussian ansatz

$$f_1^g(x, k_T^2; Q = 8\,\text{GeV}) = \frac{g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-k_T^2 / \langle k_T^2 \rangle\right)$$

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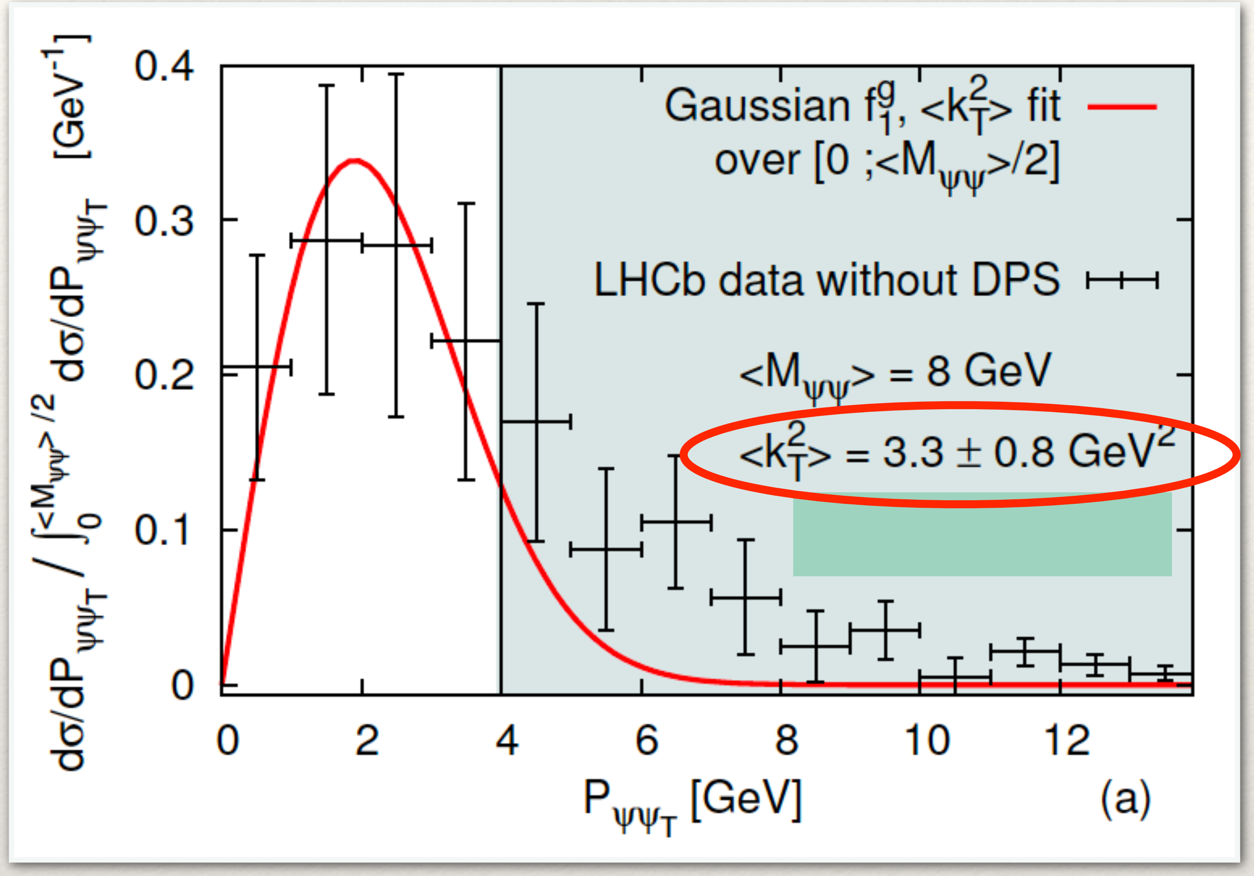

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- One-parameter fit for $<k_T^2>$, χ^2 = 1.08: *effective*, but not *intrinsic* width
- LHCb data corrected for double-parton scattering
- Expectation: Color Singlet Mode dominant

[Lansberg, Shao, PLB 2015; Ko, Yu, Lee, JHEP 2011; Li, Xu, Liu, Zhang, JHEP 1023]

Azimuthal modulation: no data available → predictions

Suggested observables: weighted cross section ratios

$$\frac{\int d\phi \cos(2\phi) \left[\frac{d\sigma\, d\phi}{dq_T}\right]_{\text{bins}}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T}\right]_{\text{bins}}} \propto \frac{\mathcal{C}[w_3\, h_1^{\perp g}\, f_1^g] + \mathcal{C}[w_3\, f_1^g\, h_1^{\perp g}]}{\int_0^{Q/2} dq_T\, \mathcal{C}[f_1^g\, f_1^g]}$$

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Input for linearly pol. gluon TMD: models

Model 1

$$h_1^{\perp g}(x, k_T^2) = \frac{M^2}{\langle k_T^2 \rangle} \frac{g(x)}{\pi \langle k_T^2 \rangle} \exp\left(1 - \frac{3\, k_T^2}{2 \langle k_T^2 \rangle}\right)$$

Model 2: Saturation of positivity bound

$$h_1^{\perp g}(x, k_T^2) = \frac{2M^2}{k_T^2}\, f_1^g(x, k_T^2)$$

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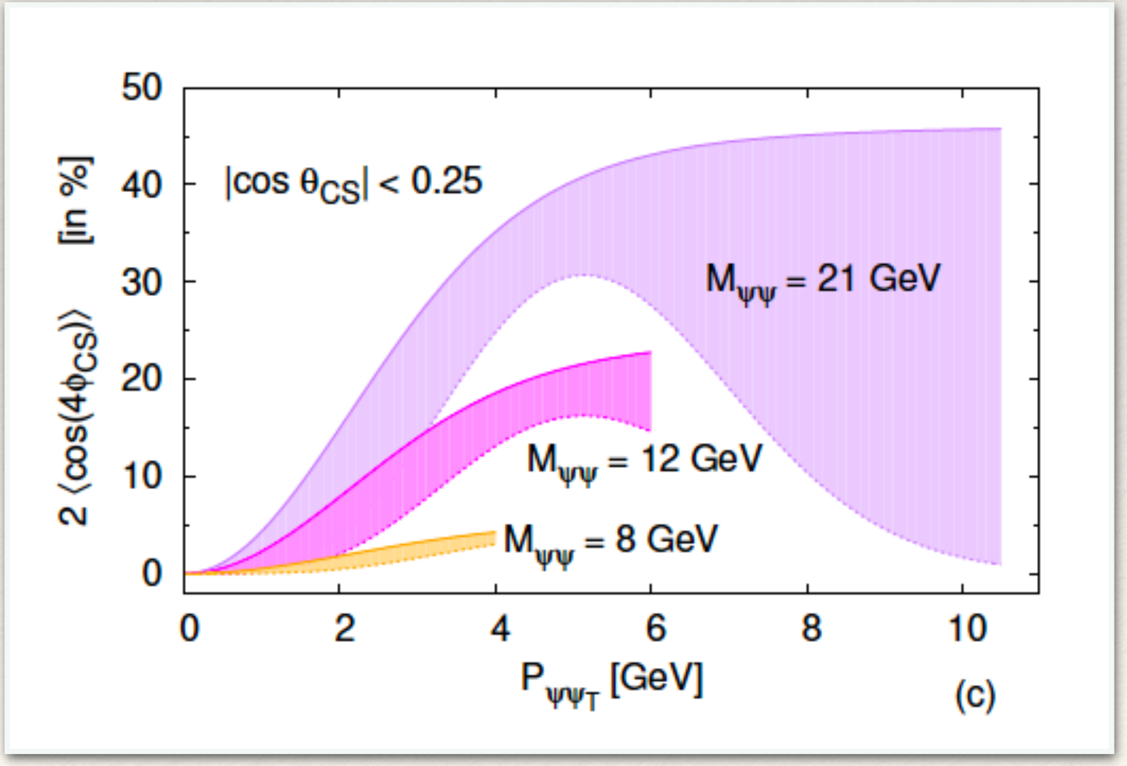

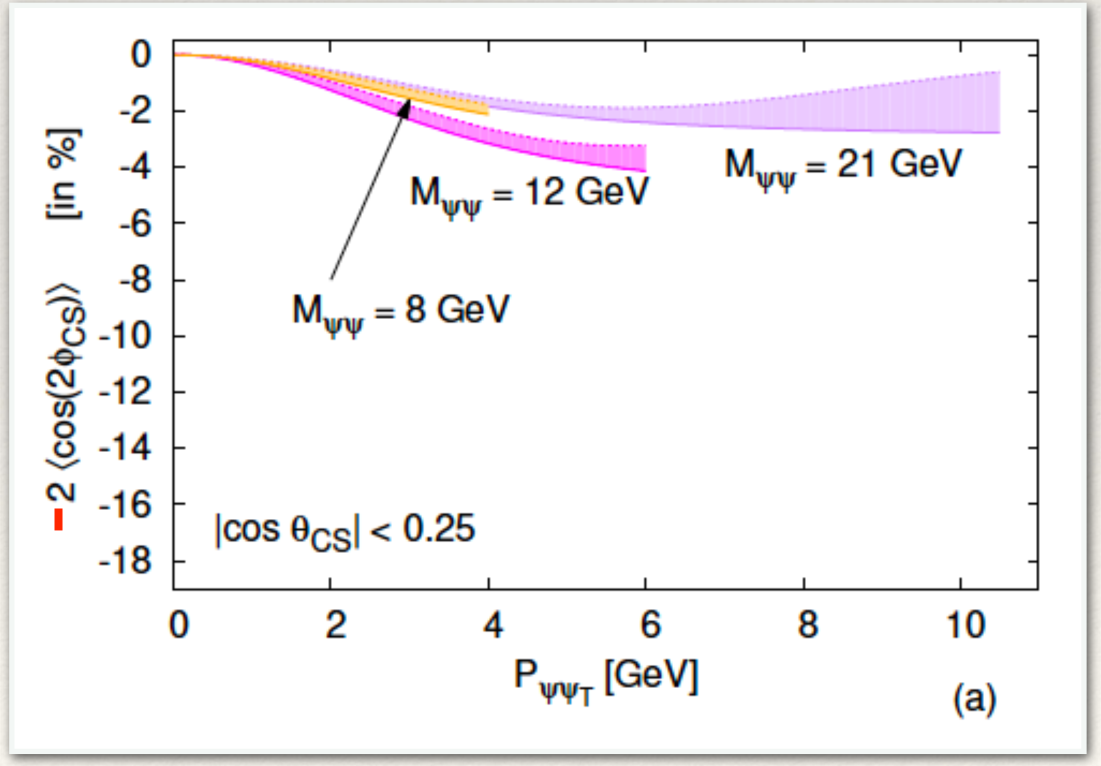

- Large effects for cos(4ϕ) for Q larger than threshold: 10% - 20%, cos(2ϕ) few percent

Azimuthal modulations including TMD evolution

[Scarpa, Boer, Echevarria, Lansberg, Pisano, M.S., arXiv:1909.05769]

Solution of CS/RG equations: Evolved unpolarized gluon TMDs (in b_T space)

$$f_1^g(x, b^2; Q) \propto \left[\sum_{q,g} \int dz\, C_1^{q,g}(z)\, f_1^{q,g}(z) \right]_{\mu \propto \frac{1}{b_*}} \quad e^{-\frac{1}{2} S_{\rm pert}(b_*, Q)}\, e^{-\frac{1}{2} S_{\rm non-pert}(b_c))}$$

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TMD at “large k_T”, matching coeff known to NNLO

here: LO $f_1^g(x) + \mathcal{O}(\alpha_s)$

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separation of pert. and non-pert. regimes: b*-prescription

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T\max}}\right)^2}}$$

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{M_{QQ}}\right)^2}$$

$$b_{T,\max} = 1.5\,\text{GeV}^{-1}$$

$$1/b_{T,\max} < \mu_b < Q$$

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here: LO $f_1^g(x) + \mathcal{O}(\alpha_s)$

here: NLL resummation

$$S_A(b_T;\zeta,\mu) = 2\frac{C_A}{\pi}\int_{\mu_b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\ln\left(\frac{\zeta}{\bar{\mu}^2}\right) \times \left[\alpha_s(\bar{\mu}^2) + \left(\left(\frac{67}{9} - \frac{\pi^2}{3}\right) - \frac{20 T_f n_f}{9}\right)\frac{\alpha_s^2(\bar{\mu}^2)}{4\pi}\right] + 2\frac{C_A}{\pi}\int_{\mu_b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\alpha_s(\bar{\mu}^2)\left[-\frac{11 - 2n_f/C_A}{6}\right],$$

separation of pert. and non-pert. regimes: b*-prescription

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T\rm max}}\right)^2}}$$

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{M_{QQ}}\right)^2}$$

$$b_{T,\rm max} = 1.5\,\mathrm{GeV}^{-1}$$
$$1/b_{T,\rm max} < \mu_b < Q$$

Linearly polarized gluon TMDs

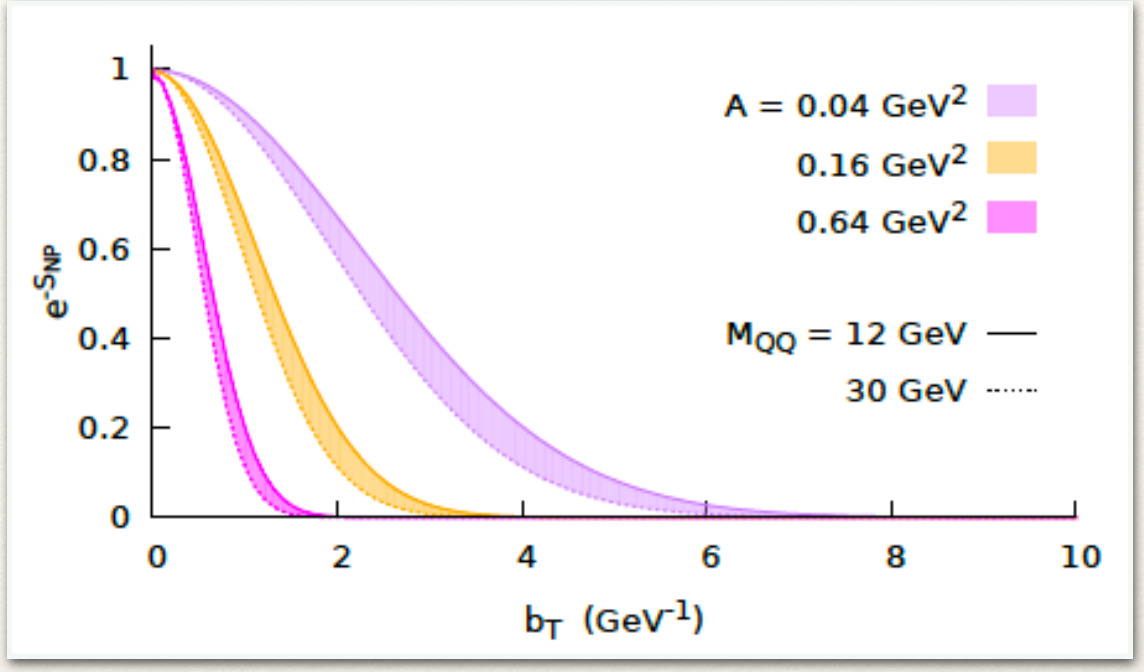
$$h_1^{\perp,g}(x, b_T; Q) \propto \left[\sum_{q,g} \int dz\, C_1^{\perp;q,g}(z)\, f_1^{q,g}(z)\right]_{\mu \propto \frac{1}{b_*}} e^{-\frac{1}{2} S_{\rm pert}(b_*, Q)}\, e^{-\frac{1}{2} S_{\rm non-pert}(b_c))}$$

LO: $\frac{\alpha_s}{\pi}\int_x^1 d\xi\, C_{A,F}\left(\frac{\xi - x}{\xi\, x}\right) f_1^{g,q}(\xi) + \mathcal{O}(\alpha_s^2)$ α_s suppression!

Non-perturbative Sudakov factor: data fits, or here: Model

$$S_{\text{non-pert}} = A \ln(Q)\, b_T^2$$

A (GeV^2)	$b_{T\,\text{lim}}$ (GeV^{-1})	r (fm $\sim 1/(0.2$ GeV$)$)
0.64	2	0.2
0.16	4	0.4
0.04	8	0.8



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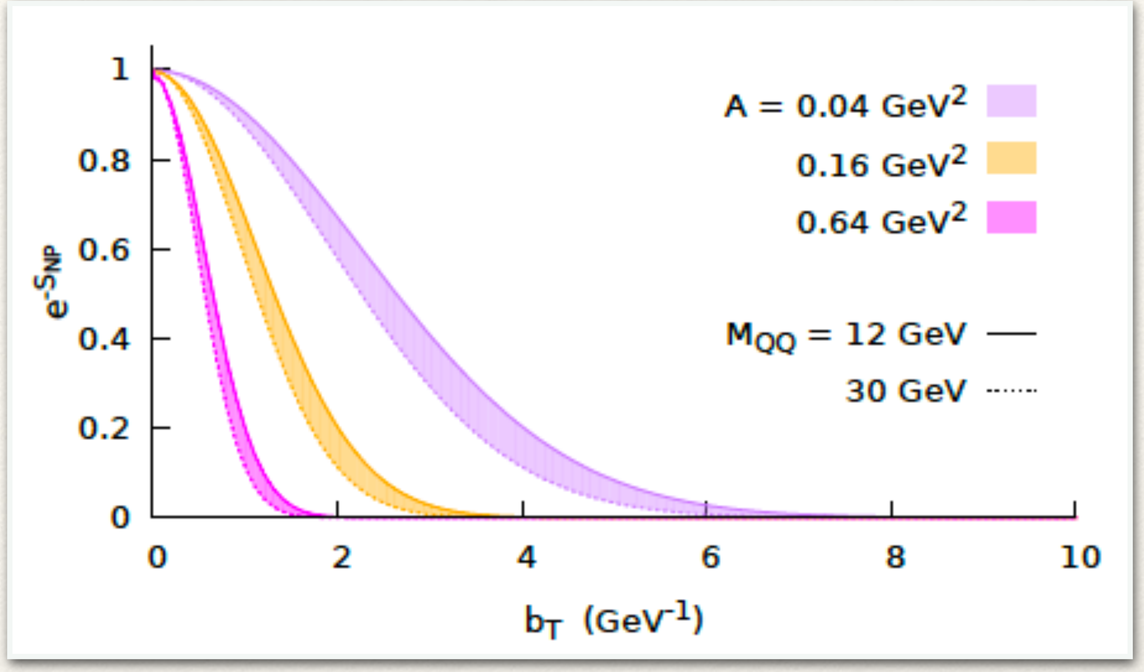
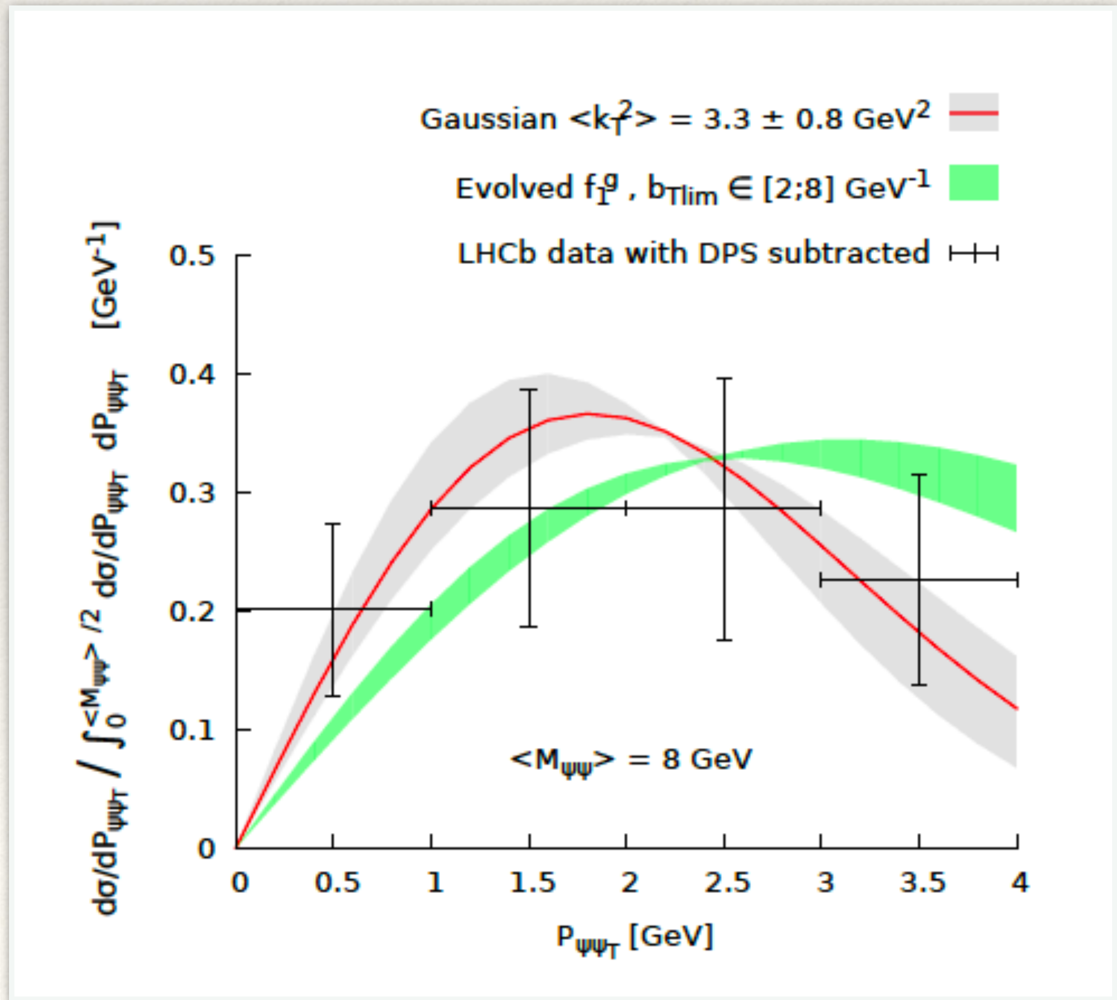
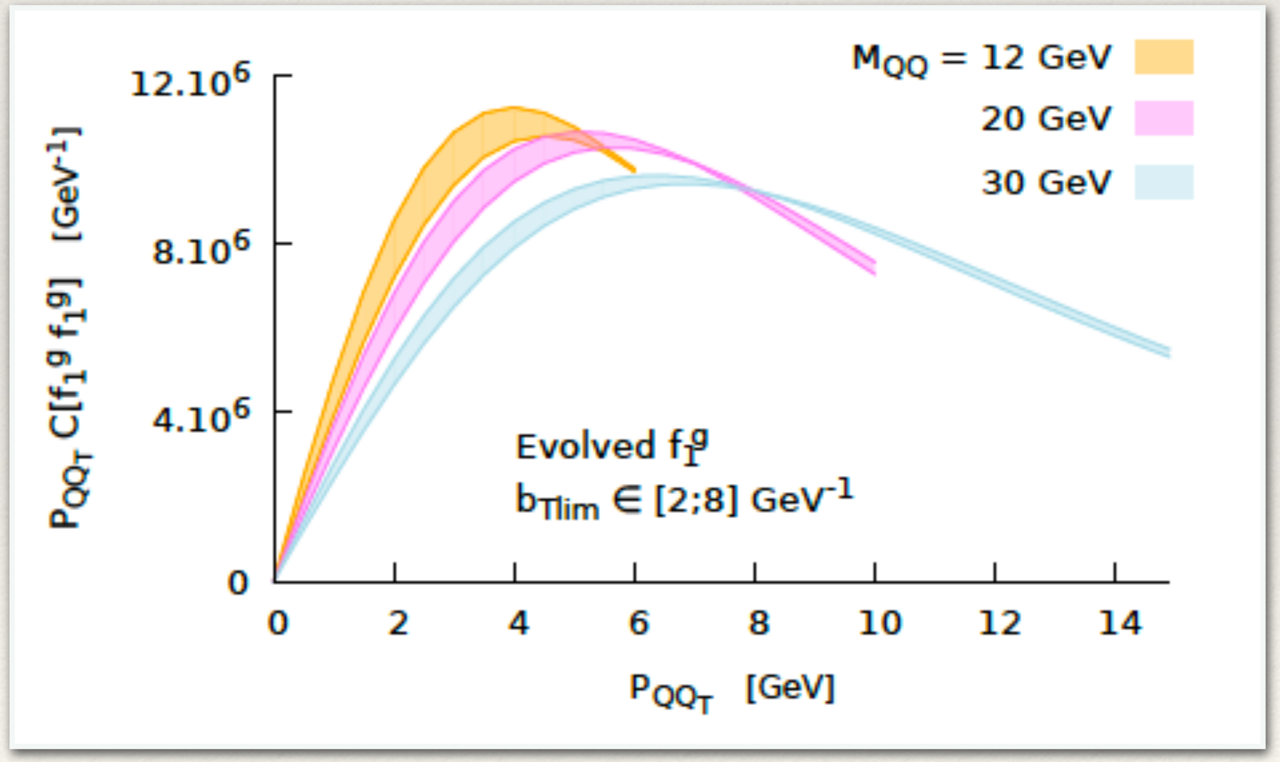


Transverse pair momentum (q_T) spectrum

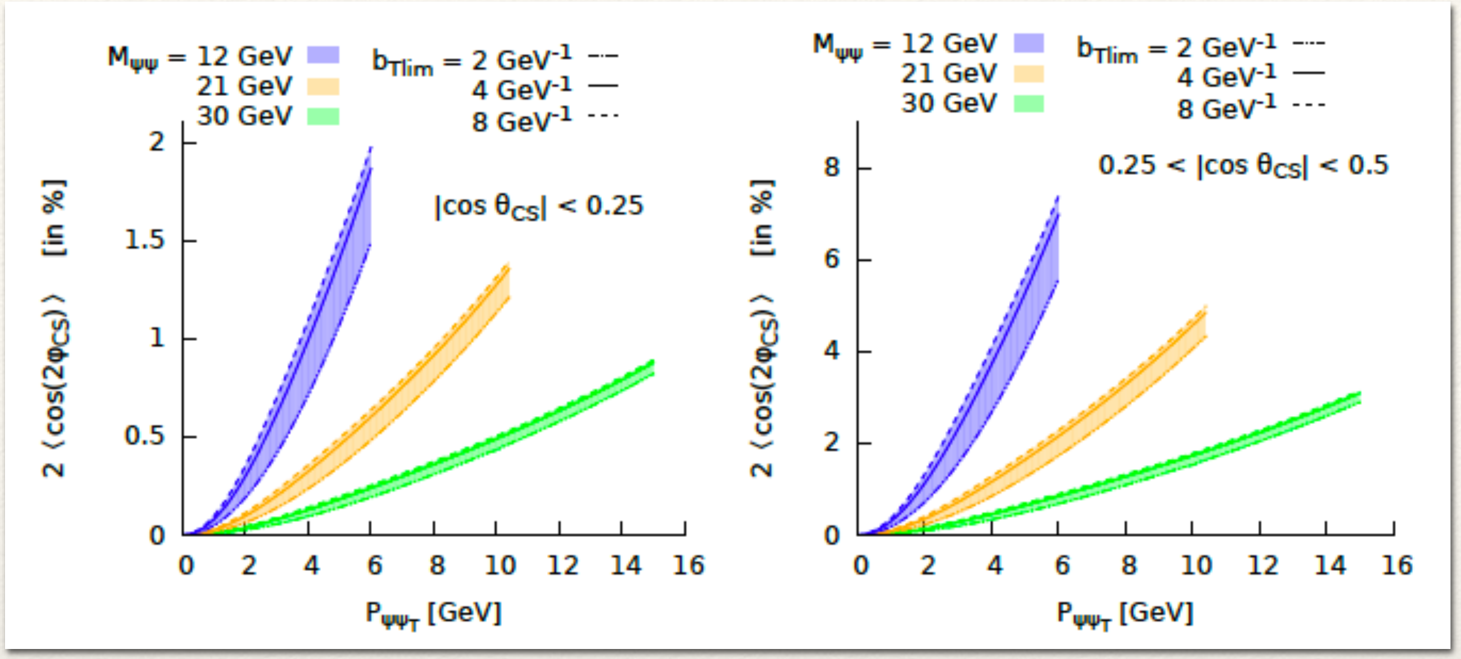




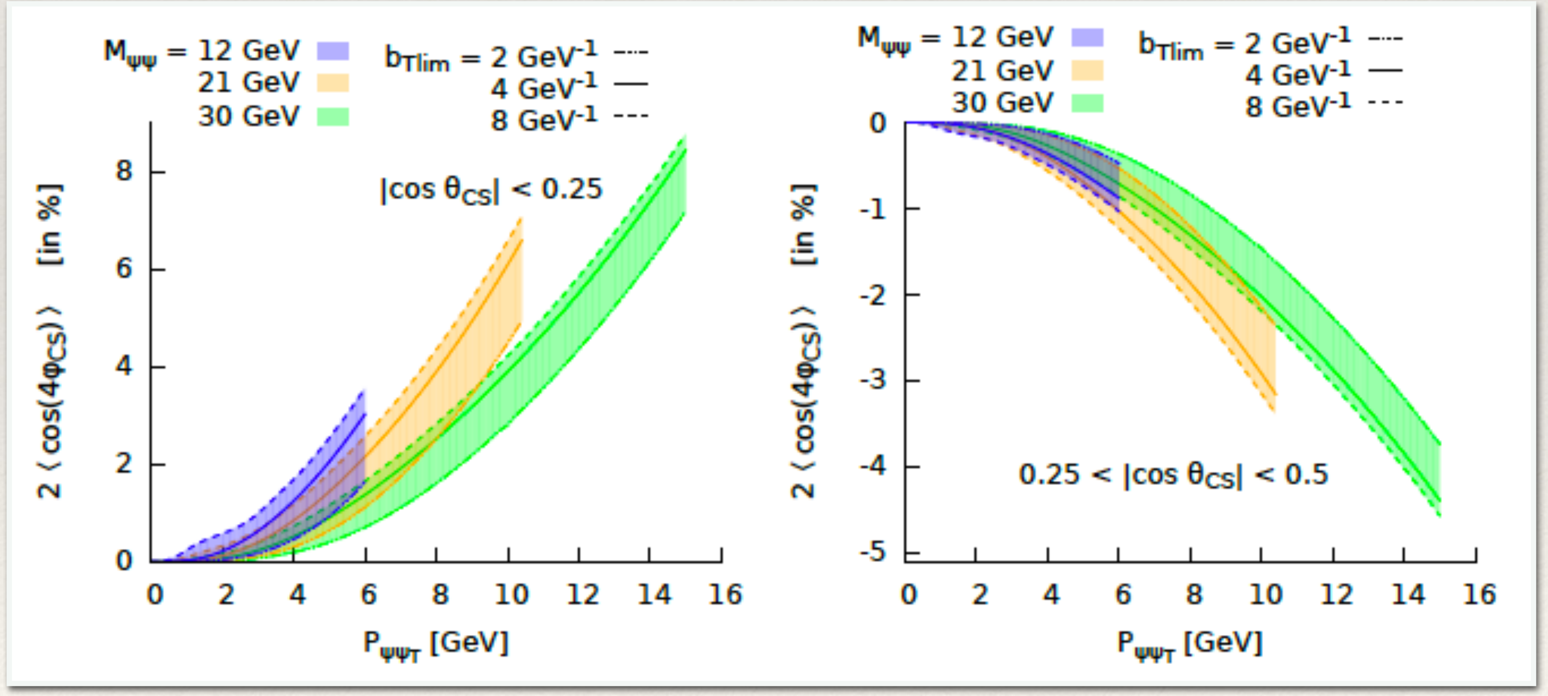
TMD evolution broadens the spectrum

TMD evolution effects azimuthal asymmetries

cos(2ϕ) modulation



cos(4ϕ) modulation



- TMD evolution diminished effects for cos(4ϕ) for Q larger than threshold: 4% - 8%
- May still be feasible at LHC → high-luminosity upgrade at LHC (ϒ - production)

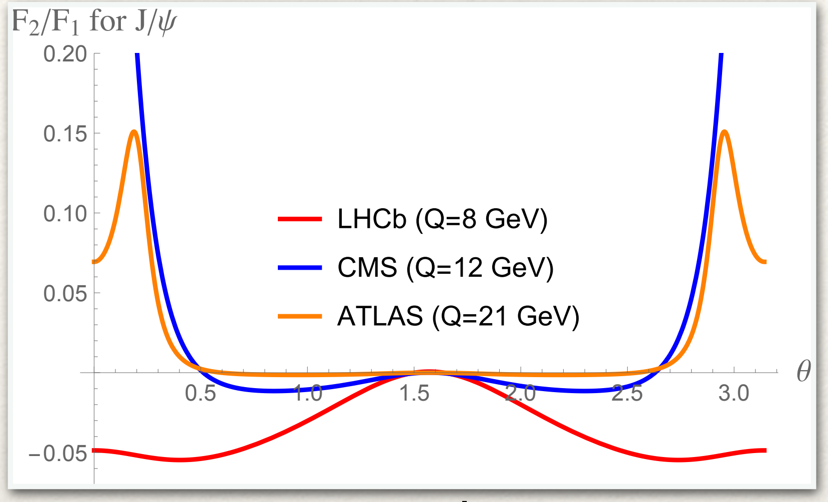
Summary

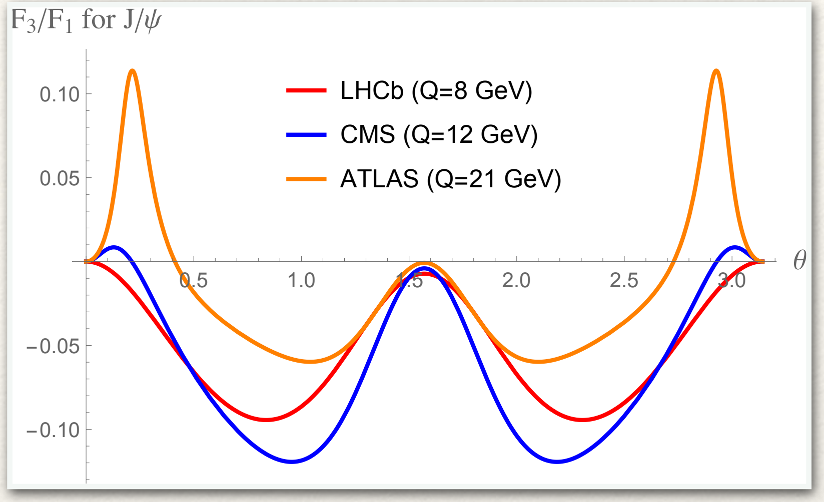
- Gluon TMDs ➝ new aspects on the 3D gluonic structure of the nucleon ➝ linear gluon polarization
- Promising final state in pp: J/ψ pairs at the LHC, particularly large: $\cos(4\phi)$ azimuthal modulation
- Data from LHCb: first extraction of unpolarized gluon TMD
- Evolution shrinks azimuthal modulation by a factor of about 2.
- Important for EIC: An idea what to expect for gluon TMDs

Back-up slides

$$\frac{d\sigma^{gg}}{d^4q\; d\Omega}\Big|_{q_T \ll Q} = \hat{F}_1\; [f_1^g \otimes f_1^g] + \hat{F}_2\; [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi)\hat{F}_3\; [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\hat{F}_4\; [h_1^{\perp g} \otimes h_1^{\perp g}]$$

Ratios:





q_T - spectrum:
Modification from lin. pol. glue mostly negligible

cos(2ϕ):
about -10% at θ = π/4, 3π/4

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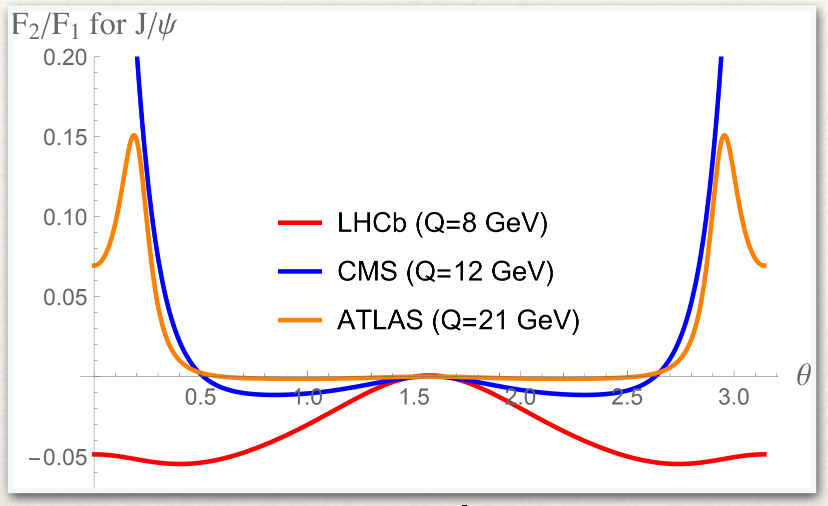
Ratios:



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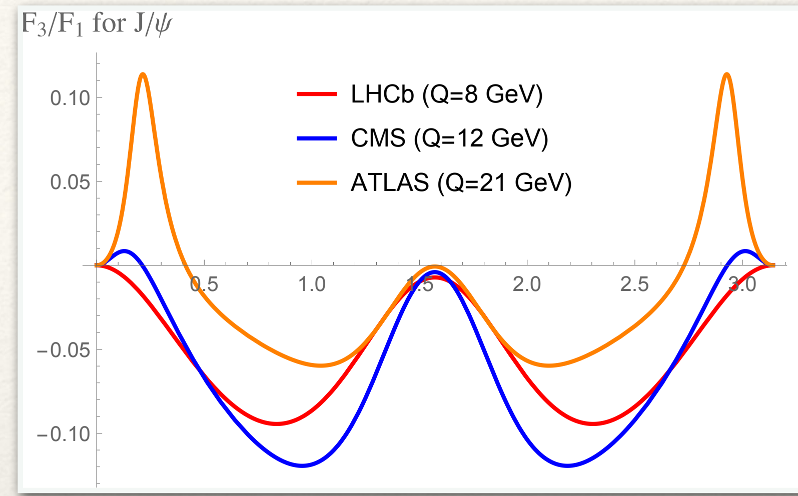
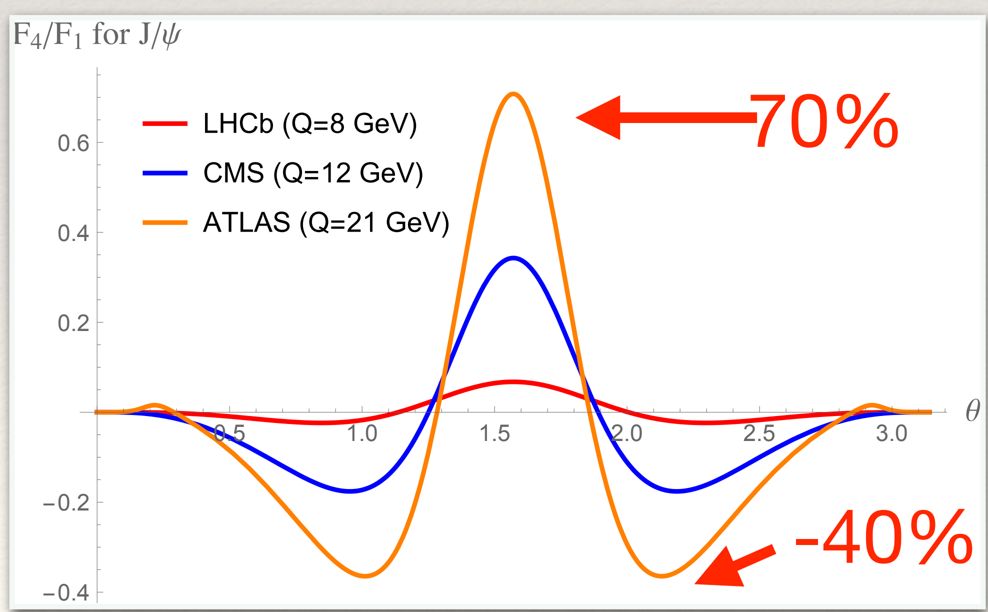
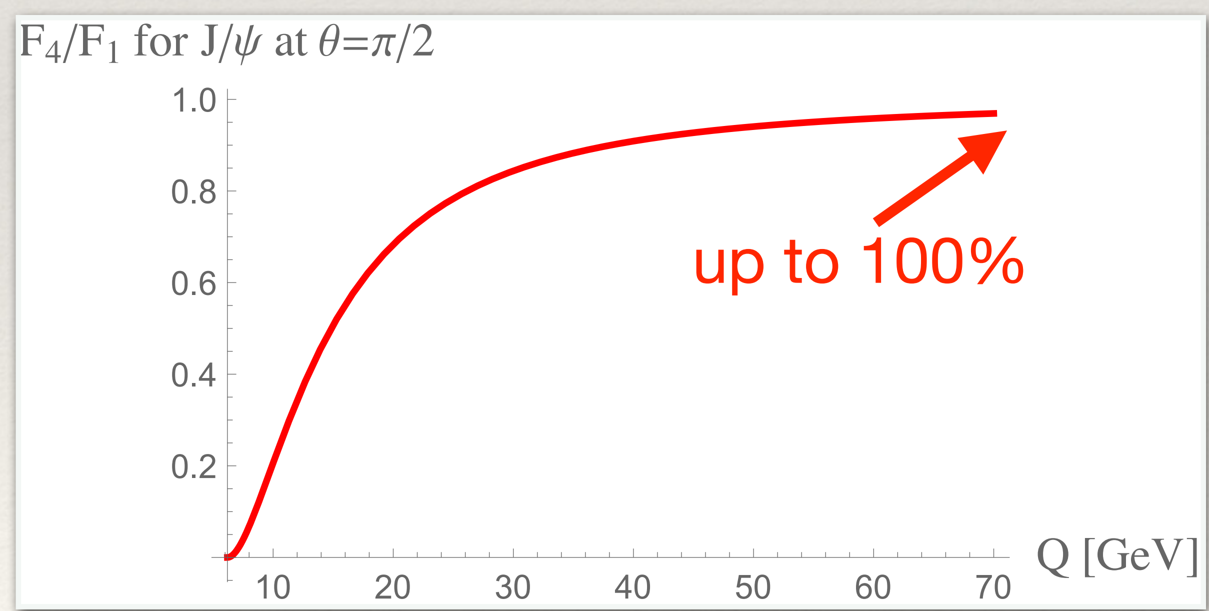


cos(2ϕ):
about -10% at θ = π/4, 3π/4

cos(4ϕ): Large at θ = π/2!





$$F_4(\theta = \tfrac{\pi}{2}, Q \gg 2M_{J/\psi}) \to F_1$$

unique to this process!