Nonperturabtive Renormalization of Quasi-TMD on the Lattice

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Sep 16, 2019

4th TMD Collaboration Meeting & POETIC 2019,
Lawrence Berkeley National Laboratory, CA 09/16-18, 2019

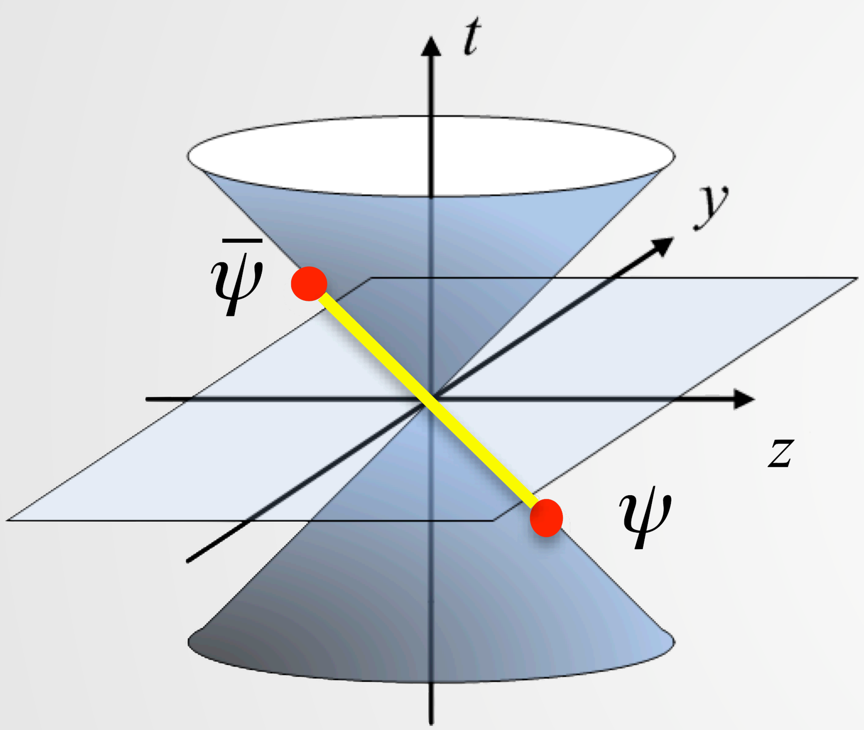
Outline

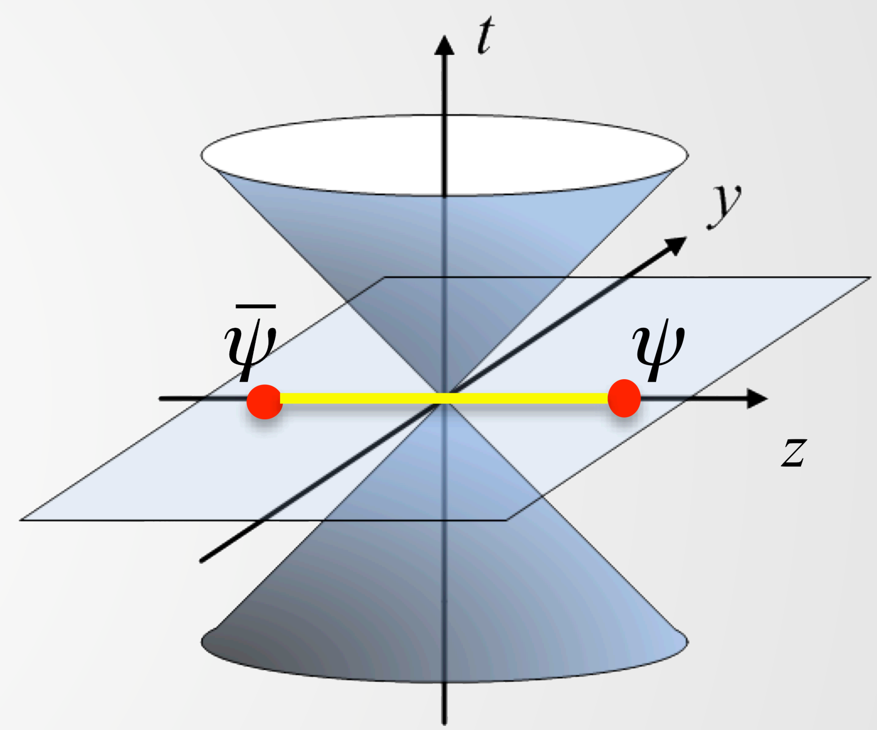
- Nonperturbative renormalization
  - Collins-Soper evolution kernel from lattice QCD
  - Renormalizability and operator mixing
  - Lattice renormalization in the RI’/MOM scheme
- Conversion to the MSbar scheme
  - One-loop conversion factors
  - Numerical results

Large-momentum effective theory

- Quasi-PDF:

- Ji, PRL110 (2013);
- Ji, SCPMA57 (2014).





PDF $q(x)$:
Cannot be calculated on the lattice

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- Xiong, Ji, Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

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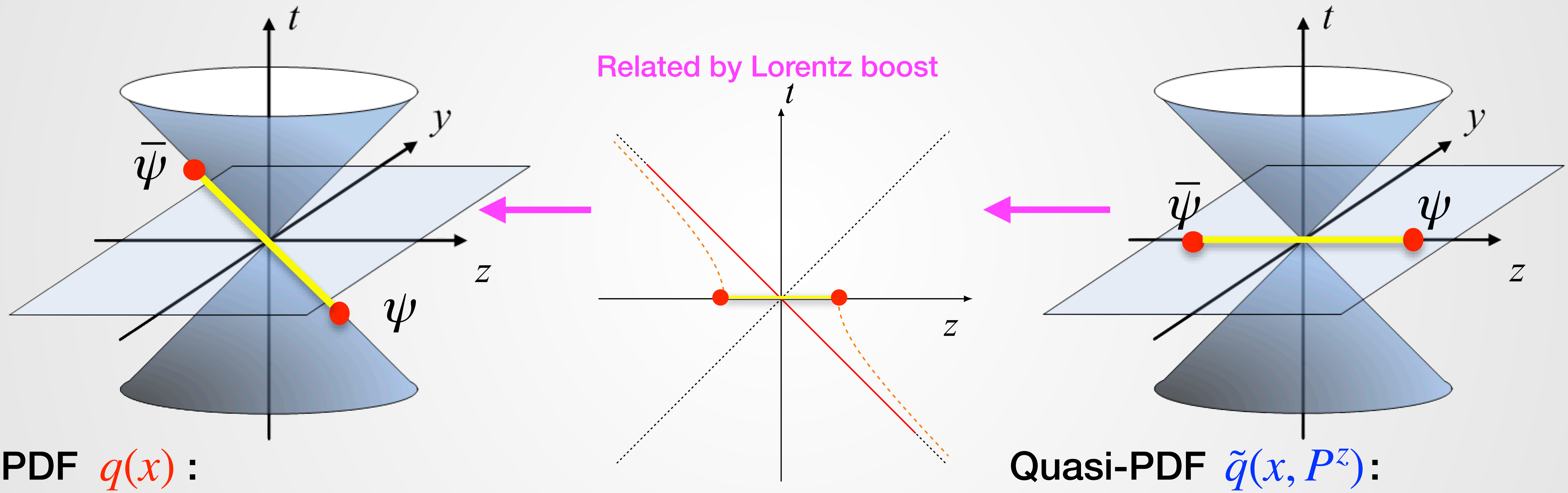

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Quasi-TMDPDF

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- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., JHEP09(2019)037.

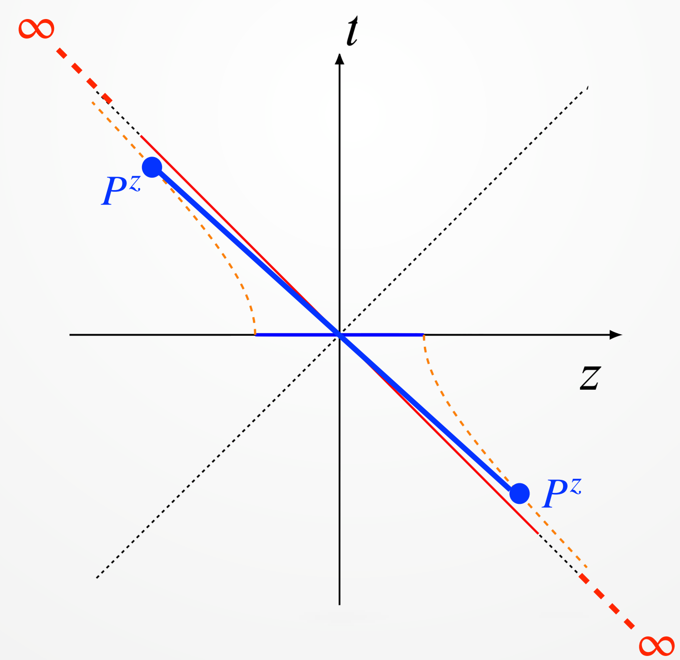
- **Definition:**

$$\tilde{f}_q^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\mathrm{UV}}(b^z, \mu, a) \frac{\tilde{B}_q(b^z, \overrightarrow{b}_T, a, L, P^z)}{\sqrt{\tilde{S}_q(b_T, a, L)}}$$

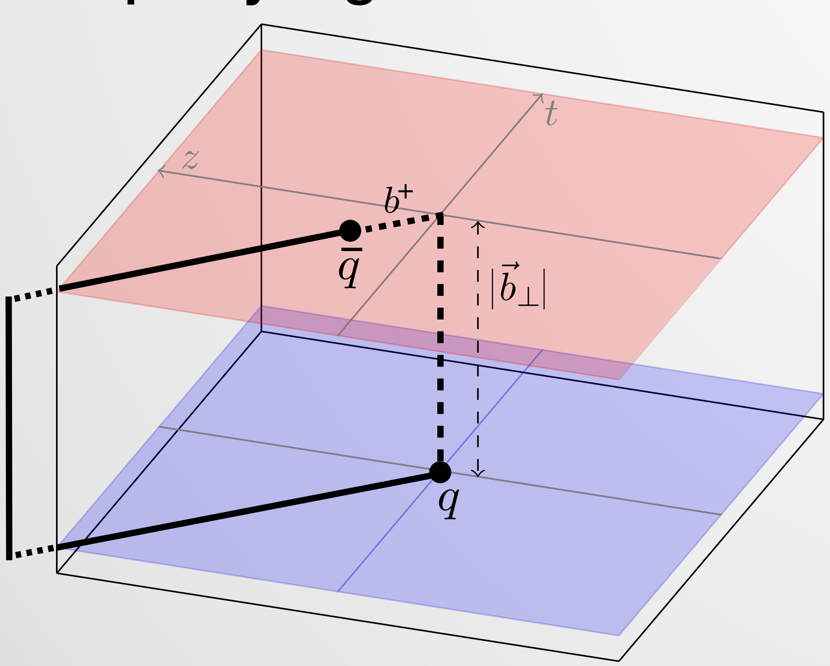
$B_q(x, \overrightarrow{b}_T, \epsilon, \tau, xP^-)$

ϵ : UV regulator

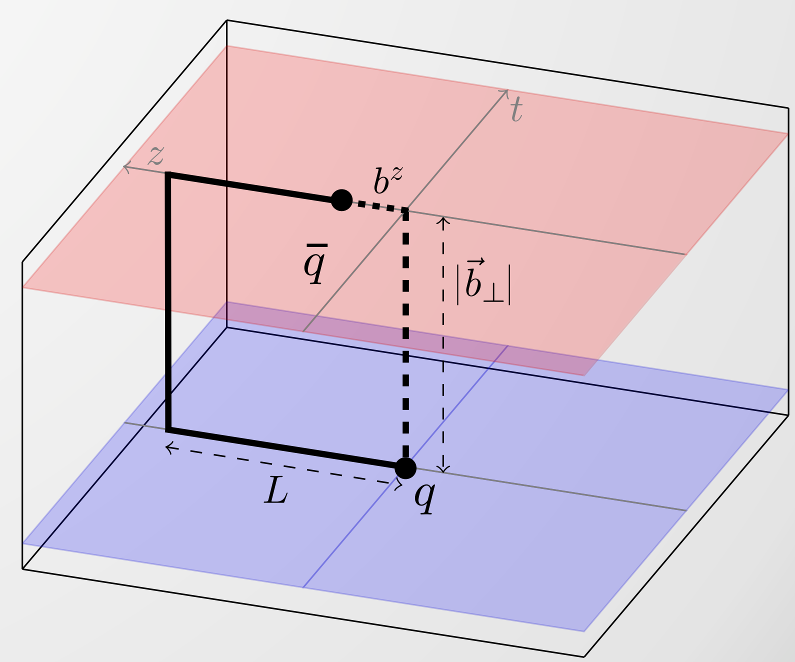
τ : rapidity regulator



$\tilde{B}_q(b^z, \overrightarrow{b}_T, a, L, P^z)$



Lorentz boost and $L \to \infty$



Quasi-TMDPDF

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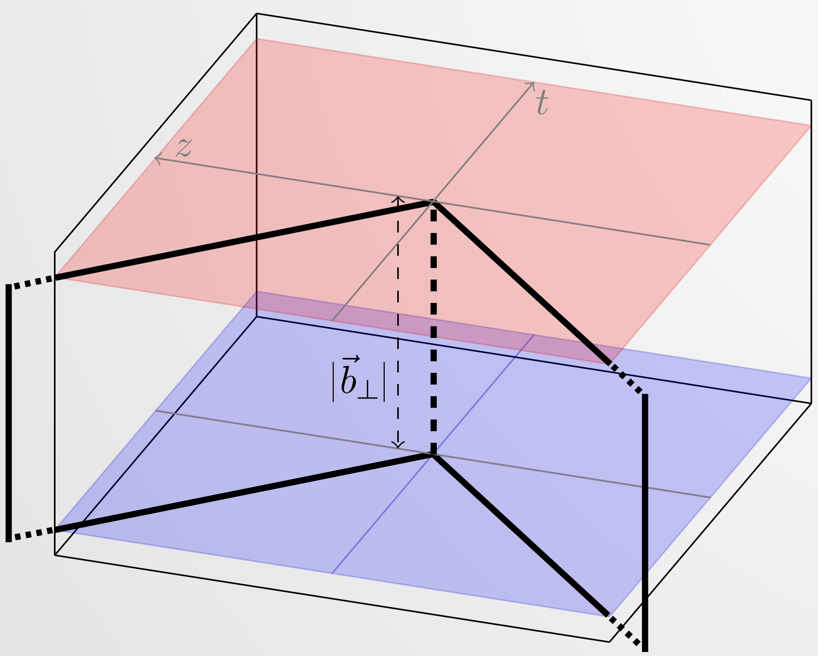
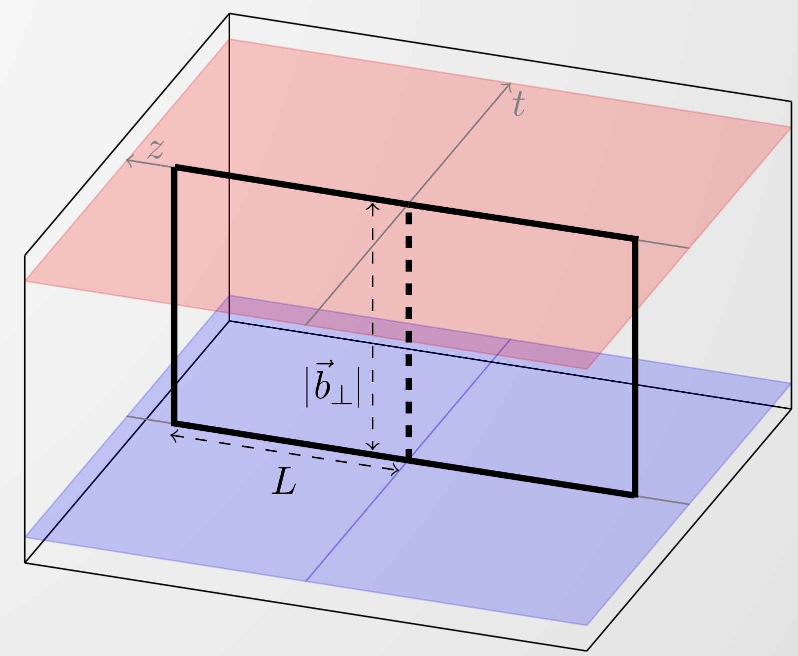
$S_q(b_T, \epsilon, \tau)$

Naive $\tilde{S}_q(b_T, a, L)$



Cannot be related by Lorentz boost



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- Relationship to the physical TMDPDF:

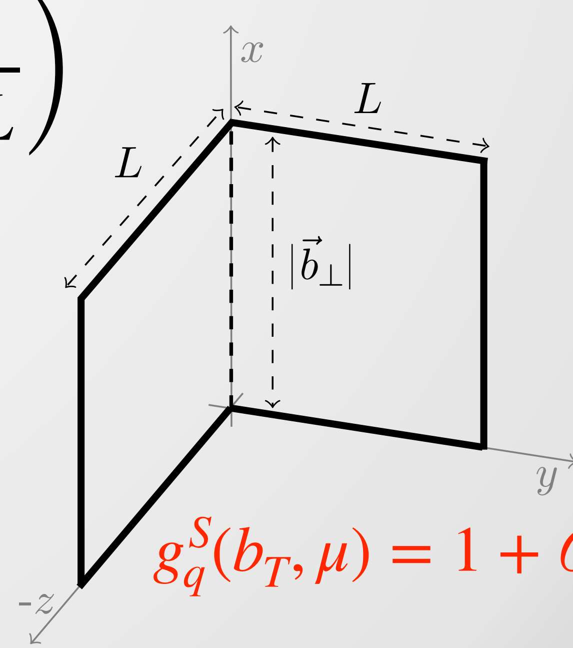
- M. Ebert, I. Stewart, Y.Z., JHEP09(2019)037.

$$\tilde{f}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right) g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln\frac{(2xP^z)^2}{\zeta}\right]$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll L \qquad \times f_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

$C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right)$ Perturbative matching coefficient

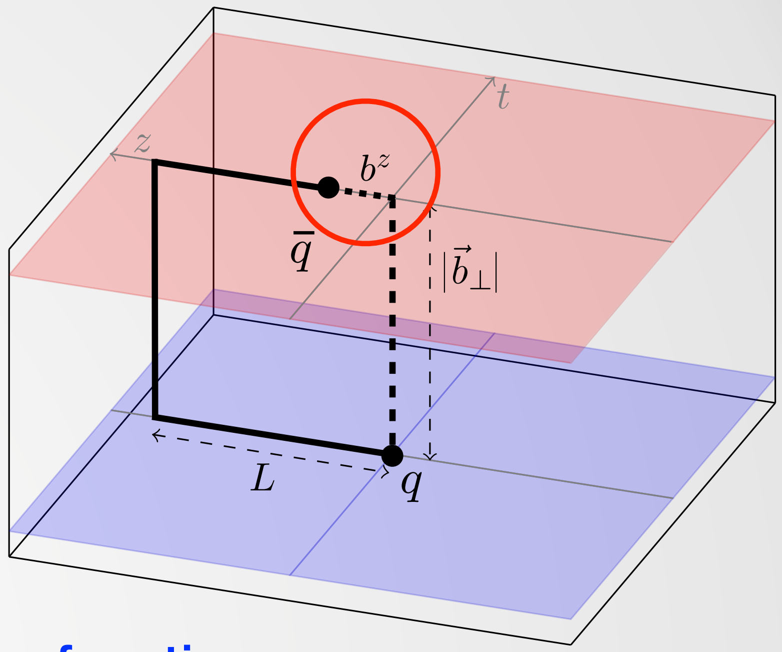
$g_q^S(b_T, \mu)$ Nonperturbative function for large b_T, depending on the choice of the quasi-soft factor



$$g_q^S(b_T, \mu) = 1 + \mathcal{O}(\alpha_s^2)$$

Collins-Soper kernel of TMDPDF from lattice QCD

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., JHEP09(2019)037;
- M. Ebert, I. Stewart, Y.Z., in progress.



$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

Quasi beam function
(or unsubtracted quasi-TMD)

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\int db^z\, e^{ib^z x P_1^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\overrightarrow{b}_T,a,L,P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\int db^z\, e^{ib^z x P_2^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\overrightarrow{b}_T,a,L,P_2^z)}$$

The idea of forming ratios to cancel the soft function has been used in the calculation of *x*-moments of TMDPDFs by
Hagler, Musch, Engelhardt, Yoon, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

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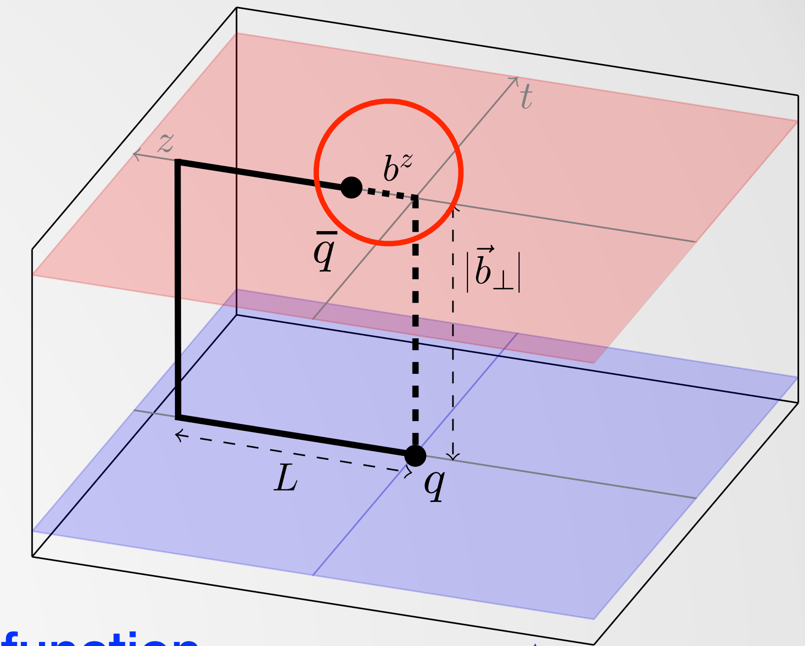

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(or unsubtracted quasi-TMD)

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

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Collins-Soper (CS) kernel does not depend on the external hadron state, which means that one can calculate it with a pion state including heavier than physical valence quarks.

work in progress with

Phiala Shanahan (MIT) and Michael Wagman (MIT).

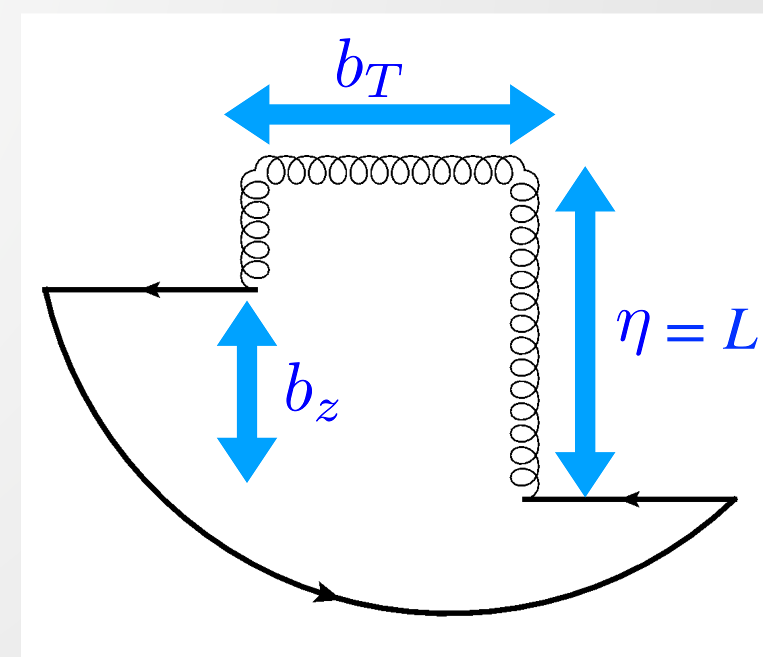
Procedure of lattice calculation

- 1. Lattice simulation of the bare quasi-beam function

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\int db^z\, e^{ib^z x P_1^z}\, \tilde Z'(b^z,\mu,\tilde\mu)\tilde Z_{\rm UV}(b^z,\tilde\mu,a)\tilde B_{\rm ns}(b^z, \vec{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\int db^z\, e^{ib^z x P_2^z}\, \tilde Z'(b^z,\mu,\tilde\mu)\tilde Z_{\rm UV}(b^z,\tilde\mu,a)\tilde B_{\rm ns}(b^z, \vec{b}_T, a, L, P_2^z)}$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll \eta < \frac{L_{\rm Lat}}{2}$$



Procedure of lattice calculation

- 2. Renormalization and conversion to the MSbar scheme

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\int db^z\, e^{ib^z x P_1^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\int db^z\, e^{ib^z x P_2^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_2^z)}$$

Nonperturbative Renormalization: $\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)$

Conversion to MSbar scheme: $\tilde{Z}'(b^z,\mu,\tilde{\mu})$

Procedure of lattice calculation

- 3. Fourier transform and calculate the ratio at different P^z

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z \; e^{ib^z x P_1^z} \; \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z \; e^{ib^z x P_2^z} \; \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_2^z)}$$

  - Independent of the choice of *x*!
  - Independent of P^z!
  - One may still seek alternatives to Fourier transforms that can be done directly in coordinate space.

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{\int db^z \, \bar{C}_{\rm ns}(\mu, y - b^z P_1^z, P_1^z)\tilde{f}_{\rm ns}(b^z, \overrightarrow{b}_T, \mu, P_1^z)}{\int db^z \, \bar{C}_{\rm ns}(\mu, y - b^z P_2^z, P_2^z)\tilde{f}_{\rm ns}(b^z, \overrightarrow{b}_T, \mu, P_2^z)}$$

- M. Ebert, I. Stewart, Y.Z., in progress. $\bar{C}_{\rm ns}(\mu, b^z P^z, P^z) = \int dx \, e^{-ix(b^z P^z)} \left[C_{\rm ns}(\mu, xP^z)\right]^{-1}$

Renormalizability

- Renormalization in the continuum theory:

$$\begin{aligned}\mathcal{O}_0^\Gamma(b^\mu, L) &\equiv \bar{q}_0(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger q_0(0) \\ &= Z_{q,W} e^{\delta m(L+|L-b^z|+b_T)} \left(\bar{q}(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger q(0) \right)_R \\ &\equiv \tilde{Z}_{\mathcal{O}} \left(\bar{q}(b^\mu) W_{\hat{z}} \frac{\Gamma}{2} W_T W_{\hat{z}}^\dagger q(0) \right)_R\end{aligned}$$

Can be proved using the auxiliary field formalism.

$$\begin{aligned}\mathcal{O}_0^\Gamma(b^\mu, L) \equiv \langle & q_0(b^\mu) \Gamma Q_1(b^\mu) \bar{Q}_1(\infty, b_T) Q_2(\infty, b_T) \\ & \times \bar{Q}_2(\infty, 0_T) Q_3(\infty, 0_T) \bar{Q}_3(0) q_0(0) \rangle_{Q_1, Q_2, Q_3}\end{aligned}$$

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);

Operator mixing

- Renormalization on the lattice (with broken chiral symmetry):
  - For the quasi-PDF, which corresponds to a straight Wilson line,
    - γ^z , mixing with 1 at $O(a^0)$;
    - γ^t , no mixing at $O(a^0)$.

  - Constantinou and Panagopoulos, PRD96 (2017);
  - J. Green et al., PRL121 (2018);
  - Chen et al, arXiv:1710.01089.

  - For the quasi beam function, according to one-loop lattice perturbation theory, for b^z=0,

  - M. Constantinou et al., PRD99 (2019)

    - γ^z , no mixing at $O(a^0)$;
    - γ^t , mixing with σ^{tz} at $O(a^0)$.
    - Nevertheless, based on the symmetry of lattice and the staple-shaped operator, one still cannot rule out mixings such as γ^z and 1, γ^t and σ^{tx}, etc;
    - At finite b^z, the mixing pattern is more complicated.

RI'/MOM Renormalization

Green's function:

$$G(b,p) = \sum_x \left\langle \gamma_5 S^\dagger(p, b+x)\gamma_5 U(b+x, x)\frac{\Gamma}{2}S(p,x)\right\rangle$$

Amputated Green's function (or vertex function):

$$\Lambda(b,p) = \left(\gamma_5 \left[S^{-1}(p)\right]^\dagger\right) G(b,p) S^{-1}(p)$$

RI'/MOM scheme:

$$Z_{\mathcal{O}}^{-1}(b, p_R^\mu) Z_q(p_R^2)\ G(b,p)\Big|_{p=p_R} = G^{\text{tree}}(b, p_R)\,,$$

- I. Stewart and Y.Z., PRD97 (2018);
- Constantinou and Panagopoulos, PRD96 (2017);
- M. Constantinou et al., PRD99 (2019).

$$Z_q(p_R^2) = \frac{1}{12}\text{Tr}\left[S^{-1}(p)S^{\text{tree}}(p)\right]\Big|_{p=p_R}$$

Parametrization of the amputated Green's function:

$$\Lambda_{\gamma^t}(z,p) = \tilde{F}_t\gamma^t \ + \ \underline{\tilde{F}_z\gamma^z \ + \ \tilde{F}_T\frac{b\!\!\!/_T}{b_T} \ + \ \tilde{F}_{p\!\!\!/}\frac{p^t p\!\!\!/}{p^2}}$$

Chiral symmetry preserving

$$\underline{+\ \tilde{F}_{\sigma_{tz}}\sigma^{tz} \ + \ \tilde{F}_{\sigma_{tT}}\sigma^{tT} + \tilde{F}_A\epsilon^{tzT\rho}\gamma_5\gamma_\rho + \ldots}$$

Chiral symmetry breaking

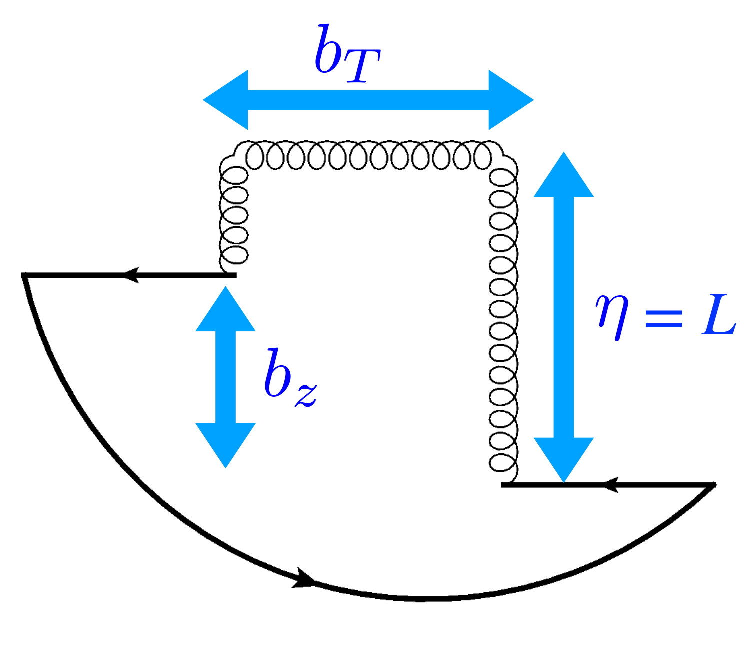
Lattice renormalization

- Lattice setup:

generated by Michael Endres

  - Quenched Wilson gauge configurations;

β	a [fm]	$L^3 \times T$	η
6.1005	0.08	$24^3 \times 48$	7,9,11
6.3017	0.06	$32^3 \times 64$	10,12,14
6.5977	0.04	$48^3 \times 96$	15,18,21



  - Probe valence pion with $m_\pi \sim 1.2\ \mathrm{GeV}$

$$0 \le b^z, b_T \le \eta$$

  - N_{cfg}=30.

Lattice renormalization

- Choice of lattice momenta:

n_μ	$\sqrt{p^2}$ [GeV]	p_z [GeV]	$p^{[4]}/(p^2)^2$
(2,2,2,2)	2.4	1.3	0.27
(2,2,2,4)	2.7	1.3	0.25
(2,2,2,6)	3.1	1.3	0.31
(3,3,3,2)	3.5	1.9	0.30
(3,3,3,4)	3.7	1.9	0.26
(3,3,3,6)	4.0	1.9	0.25
(3,3,3,8)	4.3	1.9	0.28
(4,4,4,4)	4.7	2.6	0.28
(4,4,4,6)	4.9	2.6	0.26
(4,4,4,8)	5.2	2.6	0.25

$$p^{[4]} = \sum_{\mu=1}^{4} p_\mu^4$$

Lattice renormalization

- For simplicity, we parameterize the amputated Green's function as:

$$\Lambda_\Gamma(z,p) = \sum_{\Gamma'=1}^{16} C_{\Gamma\Gamma'}\Lambda_{\Gamma'}(z,p)$$

  - And obtain the 16 by 16 mixing matrix.

- Considering that the mixings are all UV finite, we diagonalize the mixing matrix and renormalize the operator as $\mathcal{O}_\Gamma^{\overline{\mathrm{RI}'}}(p_R) = \left(Z_{\mathcal{O}}^{\mathrm{RI}'}(p_R)^{-1}\right)_{\Gamma\Gamma'}\mathcal{O}_{\Gamma'}^{\mathrm{latt}}$.

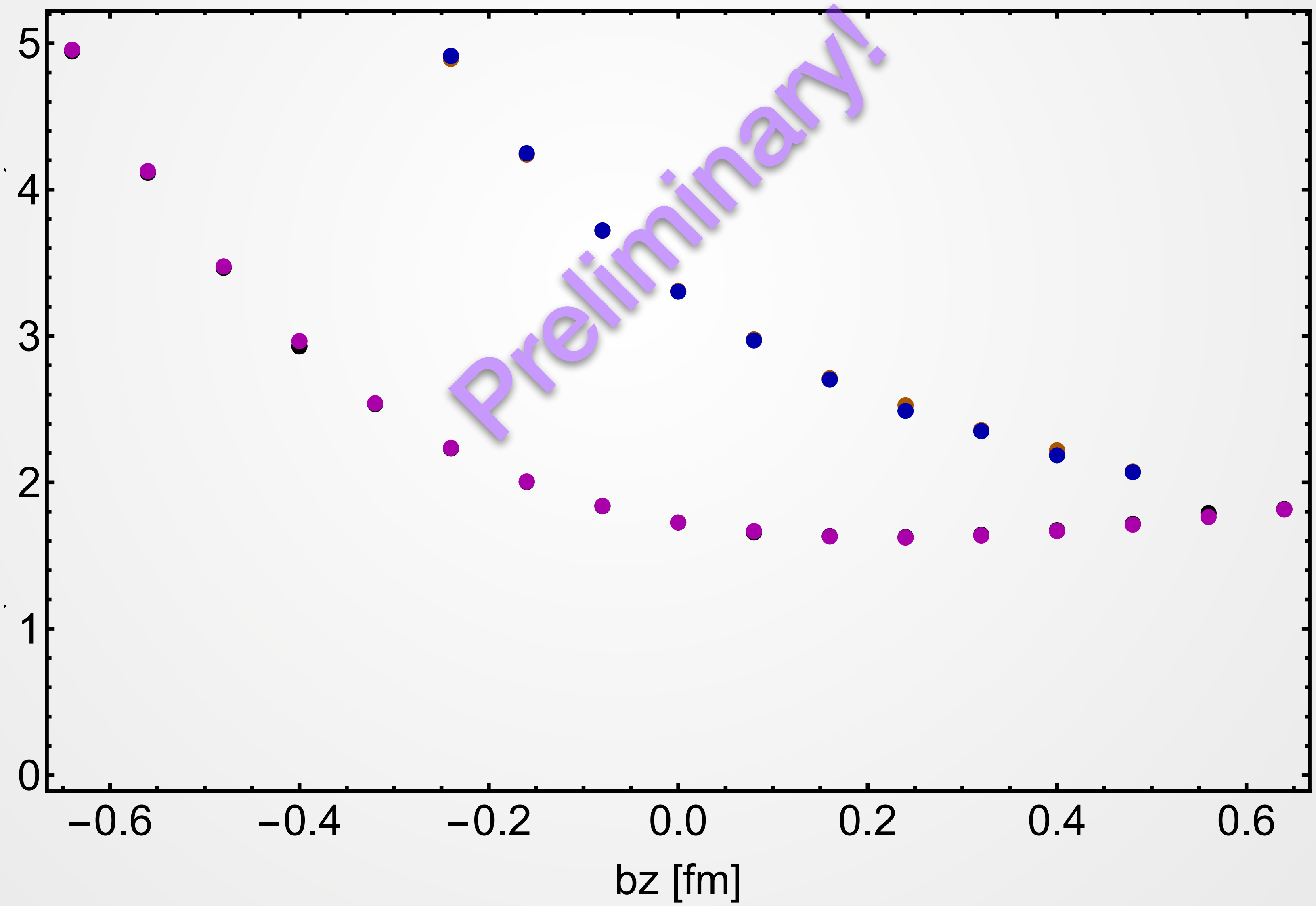
Numerical results

- RI'/MOM renormalization factors:

$\rho = \lambda = 0$

$a = 0.08\ \mathrm{fm}$

$b_T = 2a$



● $\eta = 7a$

● $\eta = 9a$

Each color with two b_T orientations.

- P. Shanahan, M. Wagman, Y.Z., in progress.

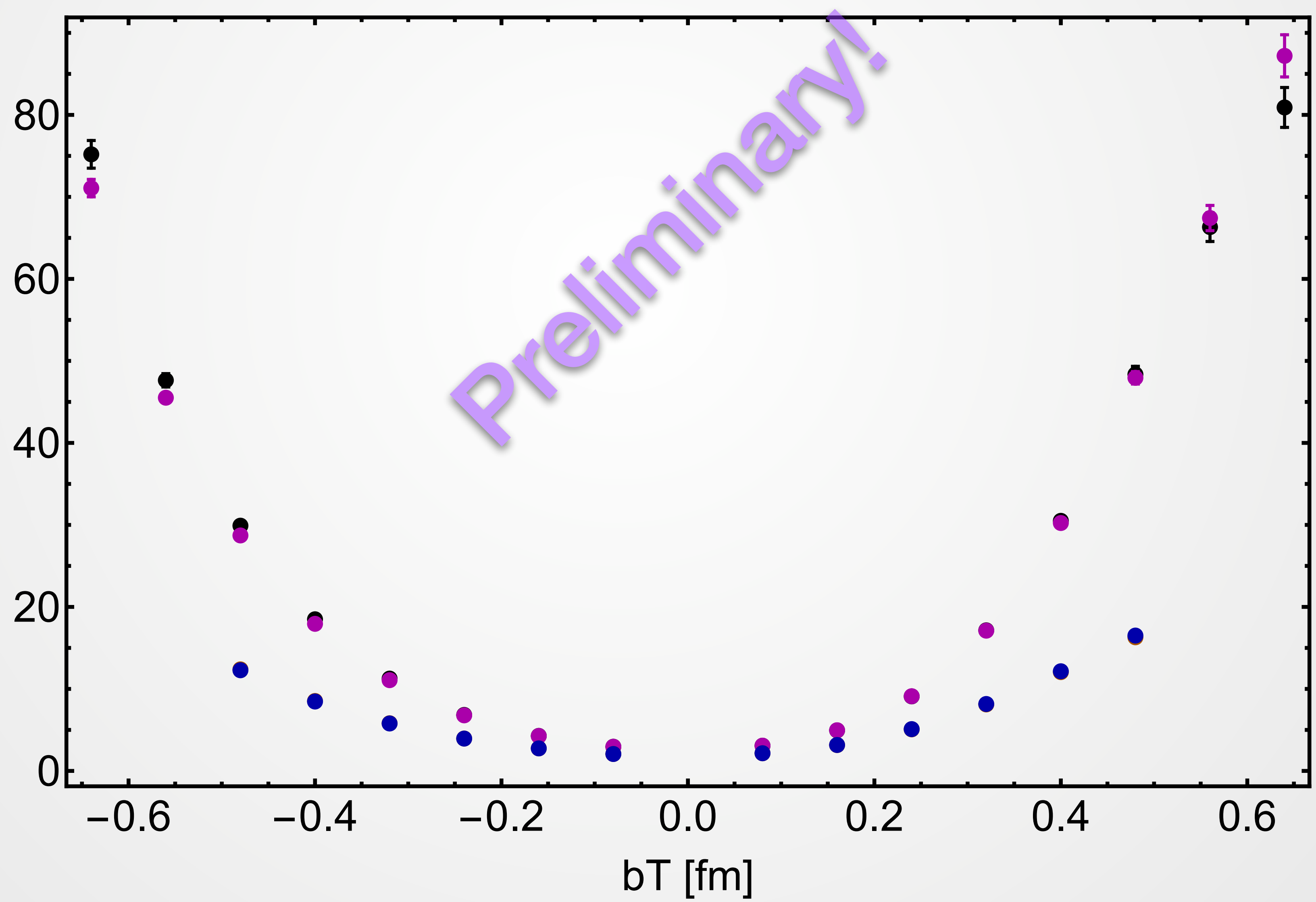
Numerical results

- RI'/MOM renormalization factors:

$\rho = \lambda = 0$

$a = 0.08\ \text{fm}$

$b^z = -8a$



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Each color with two b_T orientations.

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Lattice results

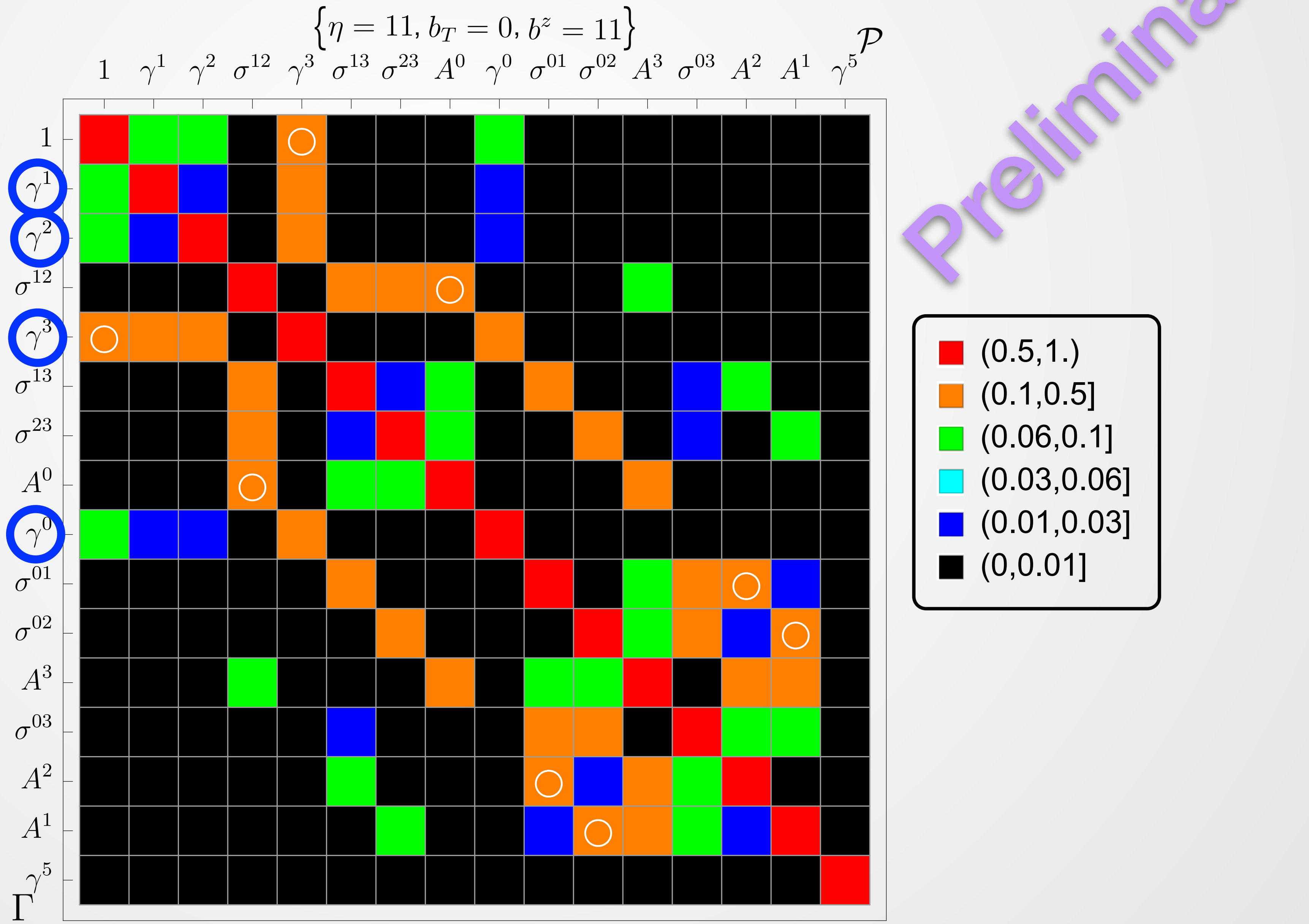
$a = 0.08\ \text{fm}\,, L_{\text{lat}} = 24\,a\,.$

- **Quasi-PDF:**

$$\frac{\text{Max}\{Z_{\Gamma\Gamma'}\}}{\text{Tr}[Z]/16}$$

@ all momentum choices.

"○" stands for predicted mixing.



Lattice results

$a = 0.08\ \mathrm{fm}\,, L_{\mathrm{lat}} = 24\,a\,.$

- Quasi beam function (b^z=0):

Preliminary!

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@ all momentum choices.

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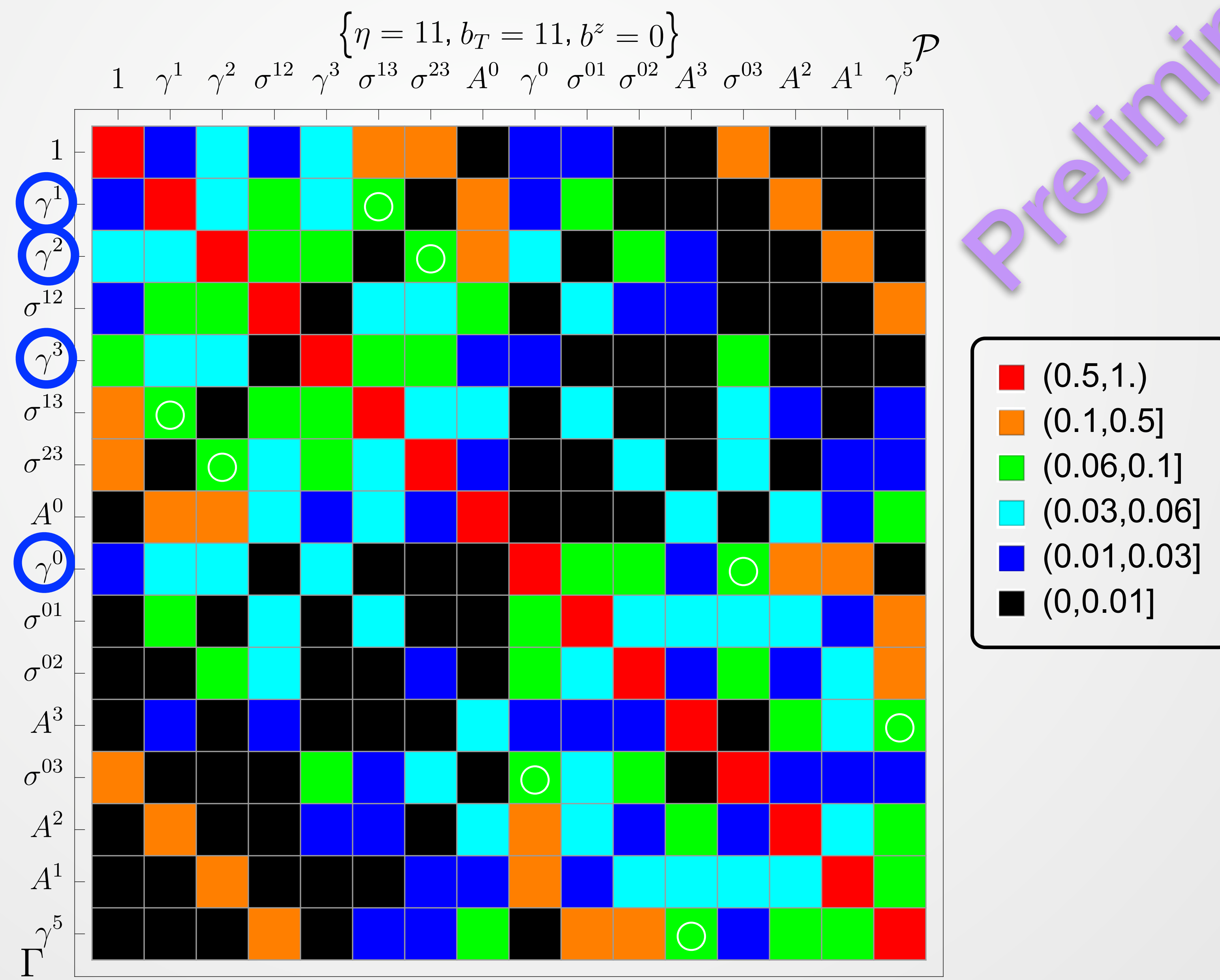

Lattice results

$$a = 0.08\ \text{fm}\,, L_{\text{lat}} = 24\,a\,.$$

- Quasi beam function ($b^z \neq 0$):

$$\frac{\text{Max}\{Z_{\Gamma\Gamma'}\}}{\text{Tr}[Z]/16}$$

@ all momentum choices.

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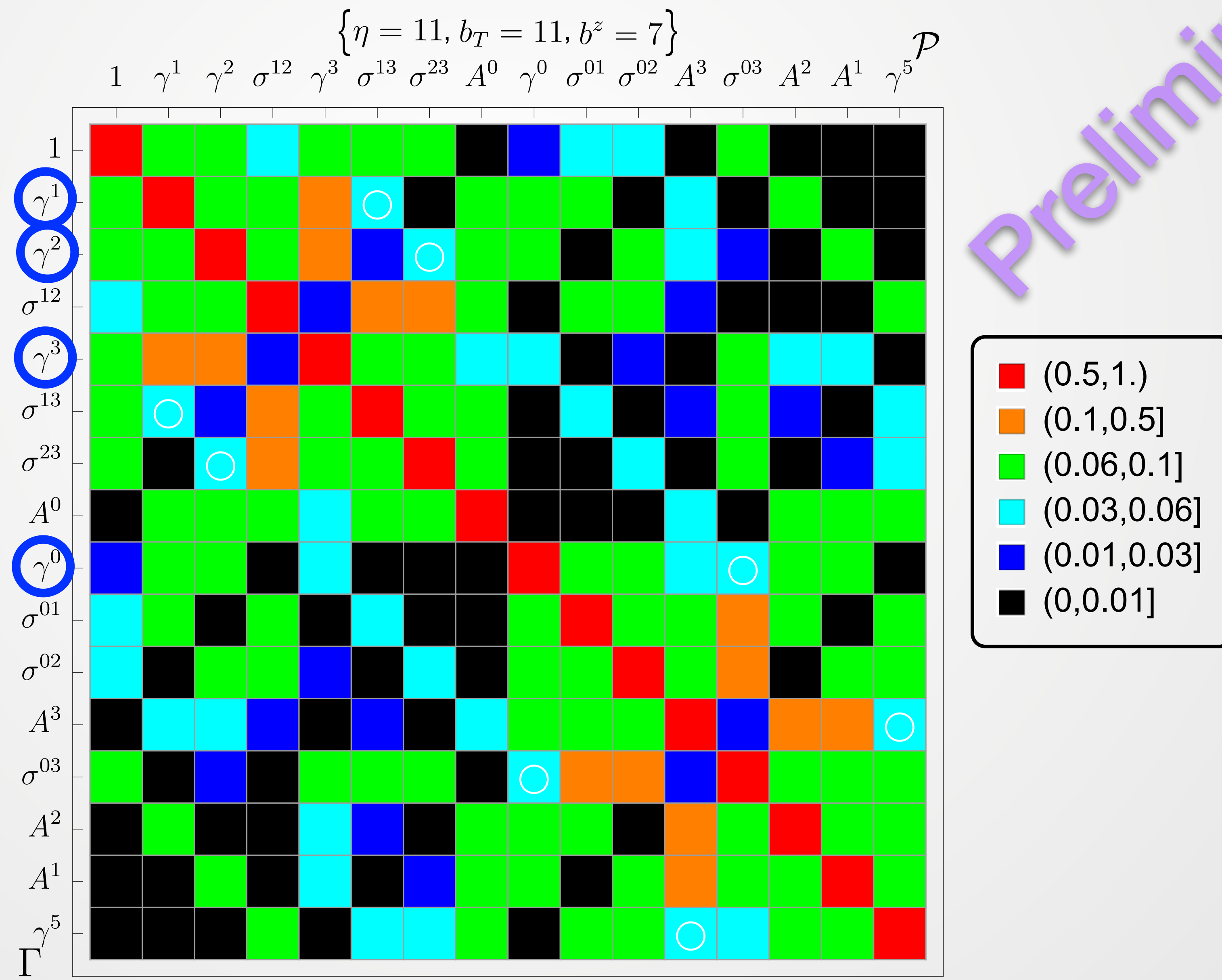

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Conversion to the MSbar scheme

$$\mathcal{O}_{\Gamma}^{\overline{\mathrm{MS}}}(\mu) = \mathcal{R}_{\Gamma\Gamma''}(\mu, p_R)\left(Z_{\mathcal{O}}^{\mathrm{RI}'}(p_R)^{-1}\right)_{\Gamma''\Gamma'} \mathcal{O}_{\Gamma'}^{\mathrm{latt}}\,.$$

$$\mathcal{R}_{\Gamma\Gamma''}^{\overline{\mathrm{MS}}}(b, L, \mu, p_R) = \left(Z^{\overline{\mathrm{MS}}}(\epsilon, \mu)\right)^{-1}\left[Z_{\mathcal{O}}^{\mathrm{RI}'}(b, L, p_R, \epsilon)\right]_{\Gamma\Gamma''}$$

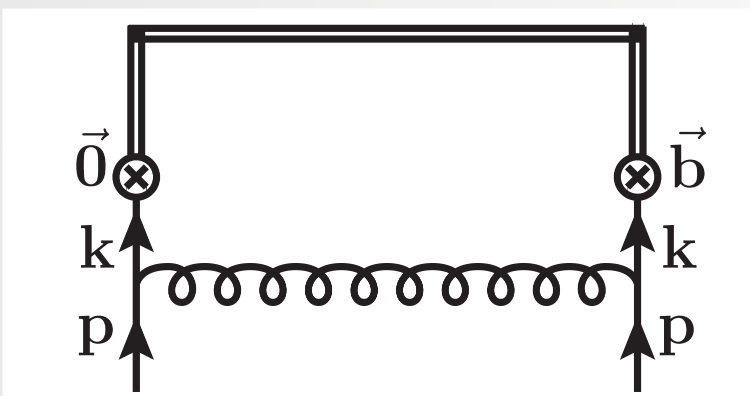
At one-loop order in continuum perturbation theory, the conversion factor has been calculated for:

- $\Gamma = \Gamma'' = \{1, \gamma_5, \gamma^\lambda, \gamma_5\gamma^\lambda, \sigma^{\lambda\rho}\}$ at b^z=0;
    - M. Constantinou et al., PRD99 (2019)
- $\Gamma = \gamma^\lambda,\ \Gamma'' = \{1, \gamma_5, \gamma^\rho, \gamma_5\gamma^\rho, \sigma^{\rho\sigma}\}$ for all b^z.
    - M. Ebert, I. Stewart, Y.Z., in progress.

Conversion to the MSbar scheme

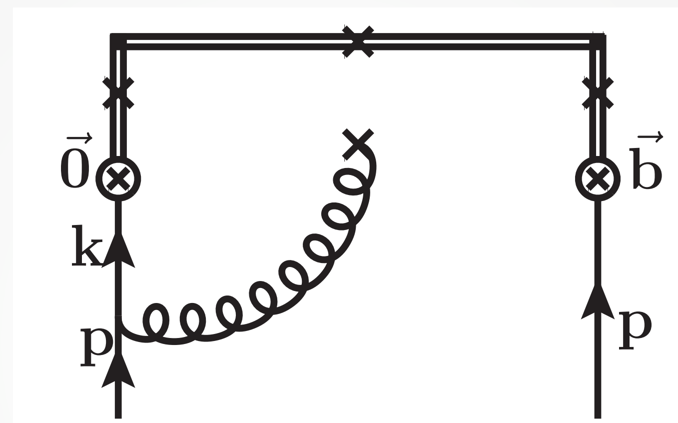
- One-loop calculation:

- M. Ebert, I. Stewart, Y.Z., in progress.



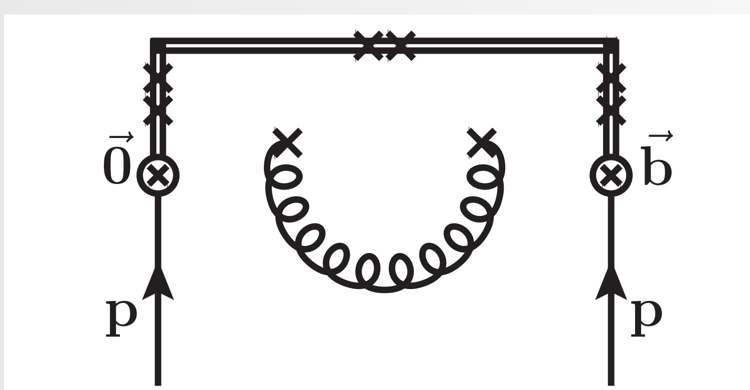
(a) Vertex diagram

+



(b) Sail topology

In the limit of $L \gg b_T$, at most logarithmically dependent on the linear combinations of $L,\ b_T,\ b^z$



(c) Wilson line self energy

$$\ni -2\frac{b^z}{b_T}\arctan\frac{b^z}{b_T} + 2\frac{L}{b_T}\arctan\frac{L}{b_T} + 2\frac{L-b^z}{b_T}\arctan\frac{L-b^z}{b_T}$$

In the limit of $L \gg b_T$, linearly dependent on ***L/b*$_T$**.
This will lead to a significant one-loop correction.

Conversion to the MSbar scheme

- Inclusion of the quasi soft factor:

$$\mathscr{R}^{\overline{\rm MS}}(b, L, \mu, p_R) = \mathscr{R}^{\overline{\rm MS}}(b, L, \mu, p_R)\tilde{\Delta}_s(b_T, \mu, L) = \frac{\mathscr{R}^{\overline{\rm MS}}(b, L, \mu, p_R)}{\sqrt{\tilde{S}^q(b_T, \mu, L)}}$$

  - The quasi soft factor does not depend on p_R or b^z, thus not affecting the ratio;
  - The quasi soft factor cancels the linearly divergent L/b_T terms in the limit of $L \gg b_T$;
  - The implementation of the one-loop quasi soft factor is not unique:

$$\left(1+\frac{\alpha_s C_F}{4\pi}\mathscr{R}^{\overline{\rm MS}\,(1)}\right)\times\left(1+\frac{\alpha_s C_F}{4\pi}\tilde{S}^{q\,(1)}\right)^{-1/2} + \mathscr{O}(\alpha_s^2)$$

$$\left(1+\frac{\alpha_s C_F}{4\pi}\mathscr{R}^{\overline{\rm MS}\,(1)}\right)\times\left(1+\frac{1}{2}\frac{\alpha_s C_F}{4\pi}\tilde{S}^{q\,(1)}\right)^{-1} + \mathscr{O}(\alpha_s^2)$$

$$\left(1+\frac{\alpha_s C_F}{4\pi}\mathscr{R}^{\overline{\rm MS}\,(1)}\right)\times\left(1-\frac{1}{2}\frac{\alpha_s C_F}{4\pi}\tilde{S}^{q\,(1)}\right) + \mathscr{O}(\alpha_s^2)$$

Numerical results

- **Parameters:**

$$a = 0.06\ \text{fm}\,, L_{\text{lat}} = 32\,a\,, \mu = 3.0\text{GeV}\,, \alpha_s(\mu) = 0.2492\,.$$

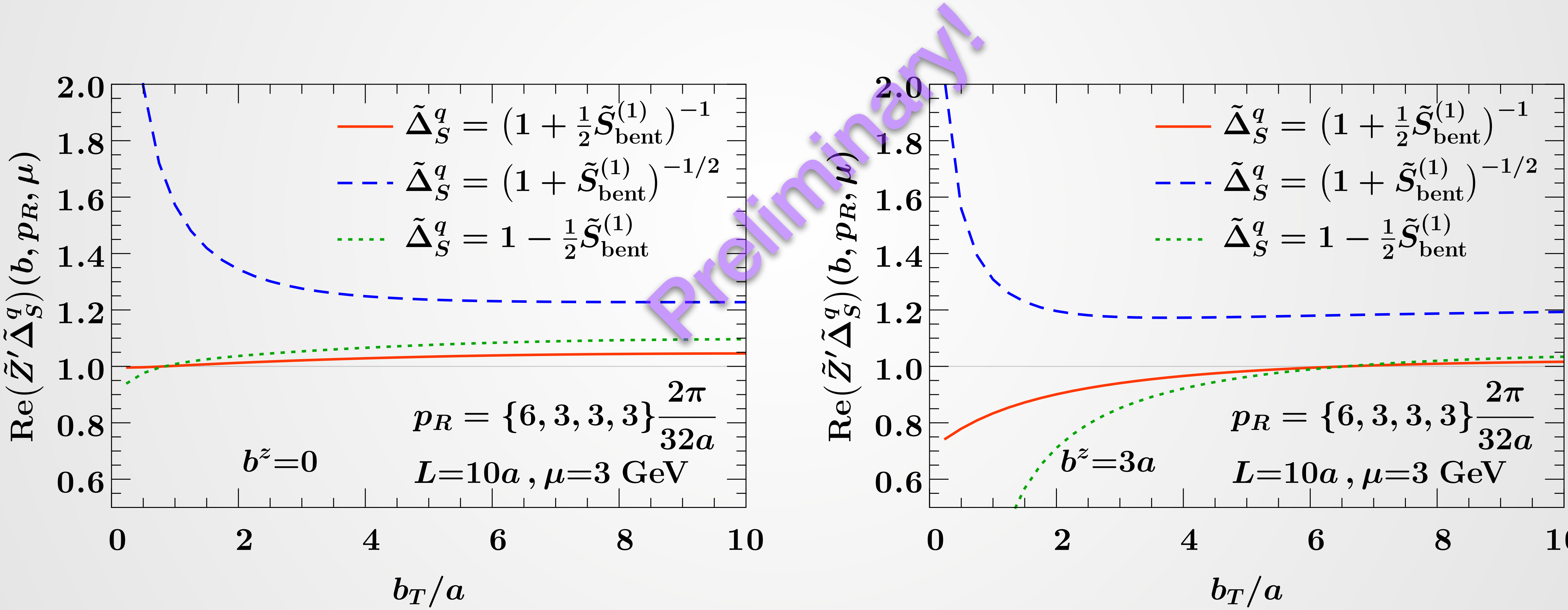
$$p_R = (3,3,3,6) = (p^1, p^2, p^3, p^4) = \frac{2\pi}{L_{\text{lat}}} \approx (1.9,1.9,1.9,3.9)\ \text{GeV}$$

$$p_R^2 = 63\left(\frac{2\pi}{L_{\text{lat}}}\right)^2 \approx (5.1\ \text{GeV})^2$$

Numerical results

- $\mathscr{R}^{\overline{\mathrm{MS}}}$ **v.s.** b_T

$\rho = \lambda = 0,\ 3 \quad \mathscr{R}^{\overline{\mathrm{MS}}} = \tilde{Z}'^{\rho\lambda}$

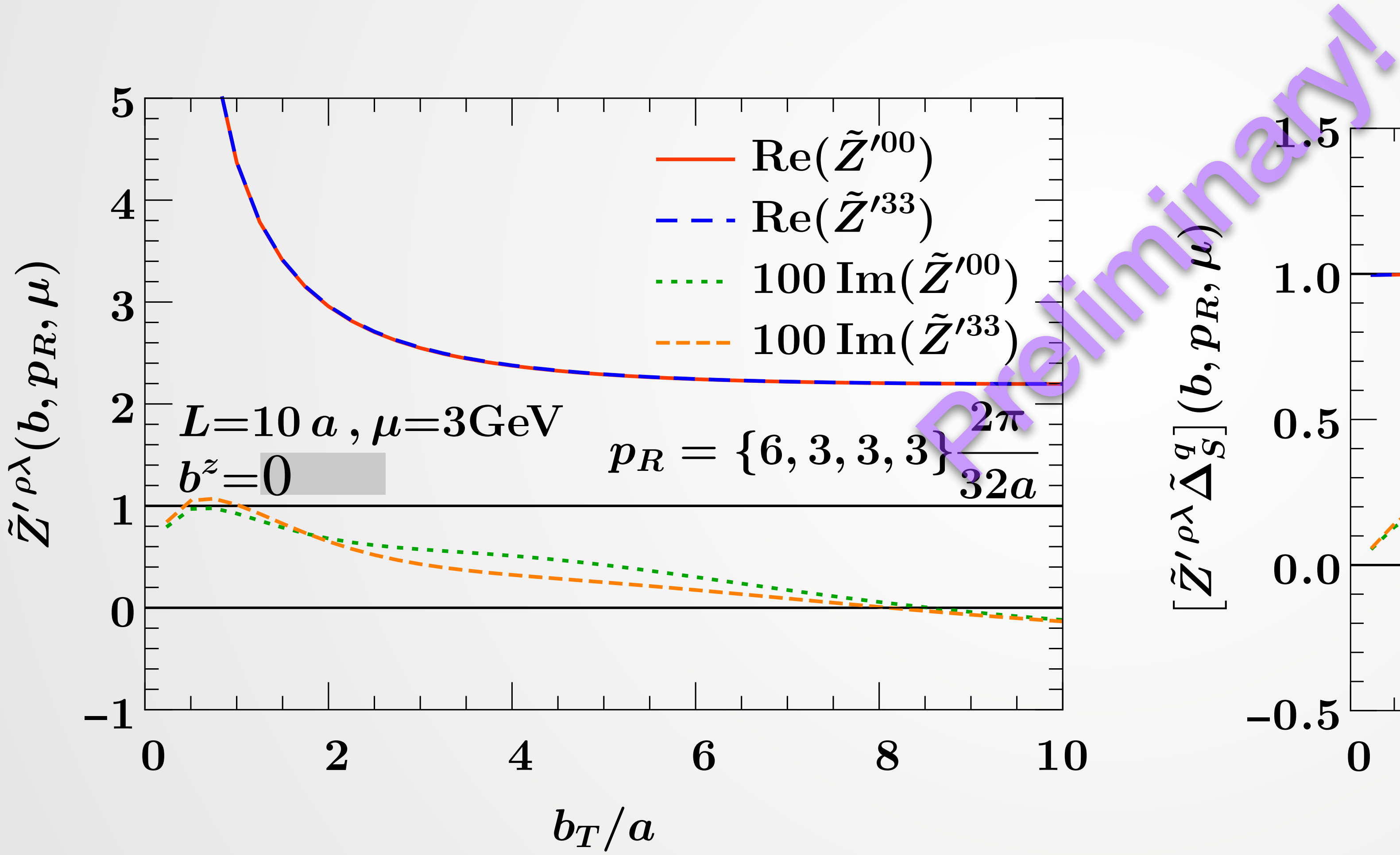


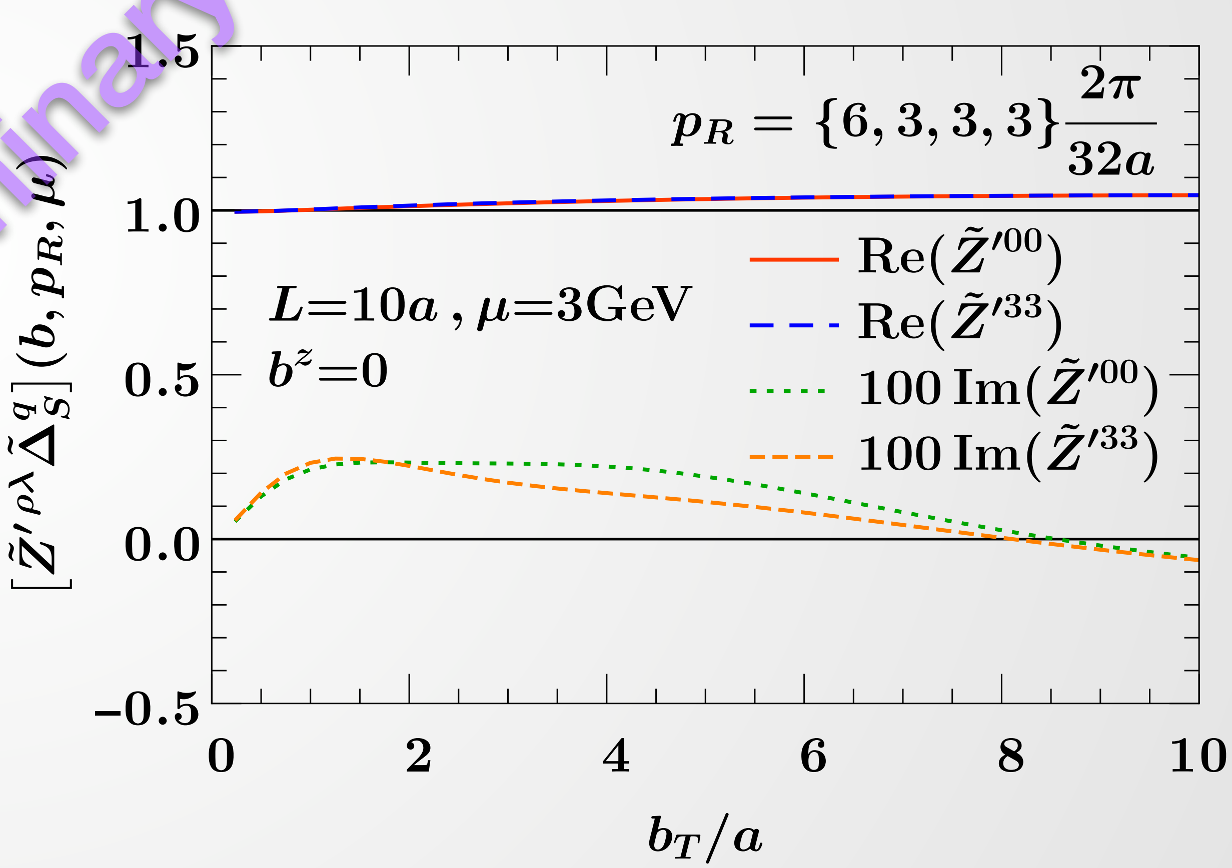
- M. Ebert, I. Stewart, Y.Z., in progress.

Numerical results

- $\mathscr{R}^{\overline{\mathrm{MS}}}$ v.s. b_T

$\rho = \lambda = 0,\, 3 \quad \mathscr{R}^{\overline{\mathrm{MS}}} = \tilde{Z}'^{\rho\lambda}$



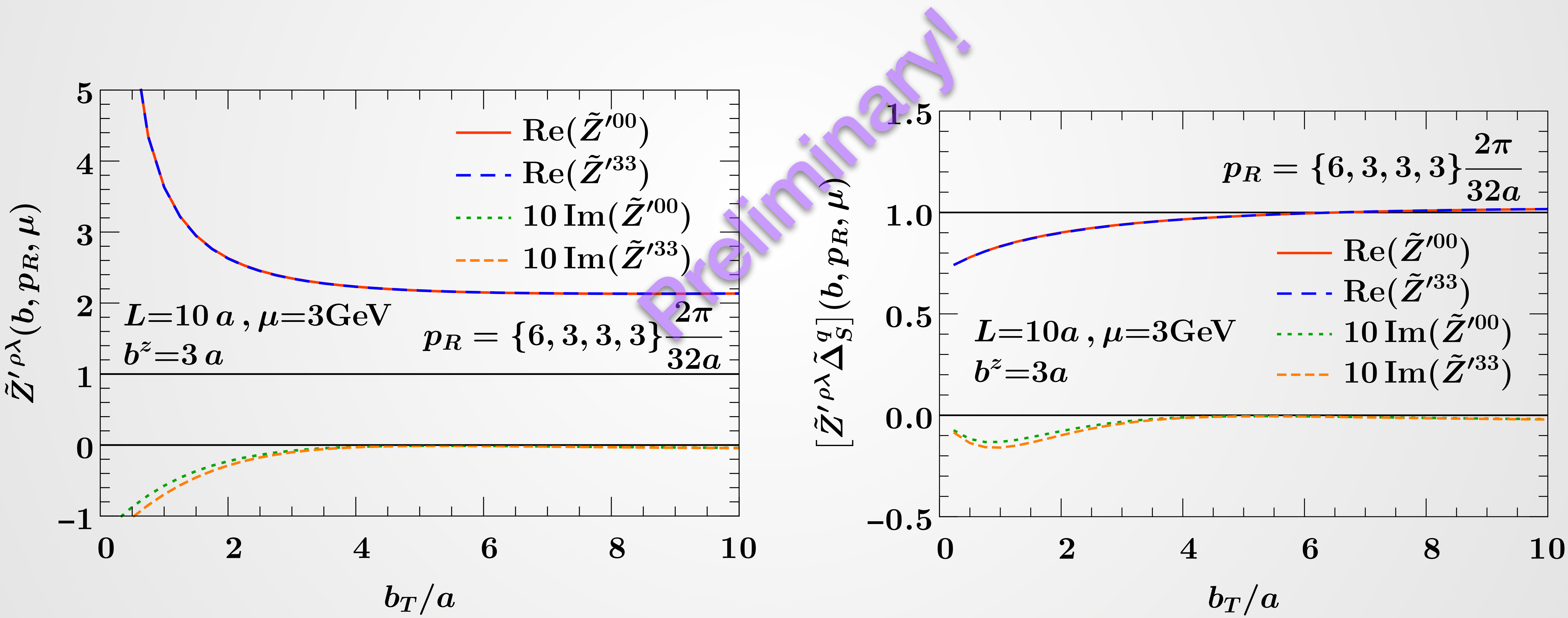


- M. Ebert, I. Stewart, Y.Z., in progress.

Numerical results

- $\mathscr{R}^{\overline{\mathrm{MS}}}$ **v.s.** b_T

$\rho = \lambda = 0,\ 3 \quad \mathscr{R}^{\overline{\mathrm{MS}}} = \tilde{Z}'^{\rho\lambda}$

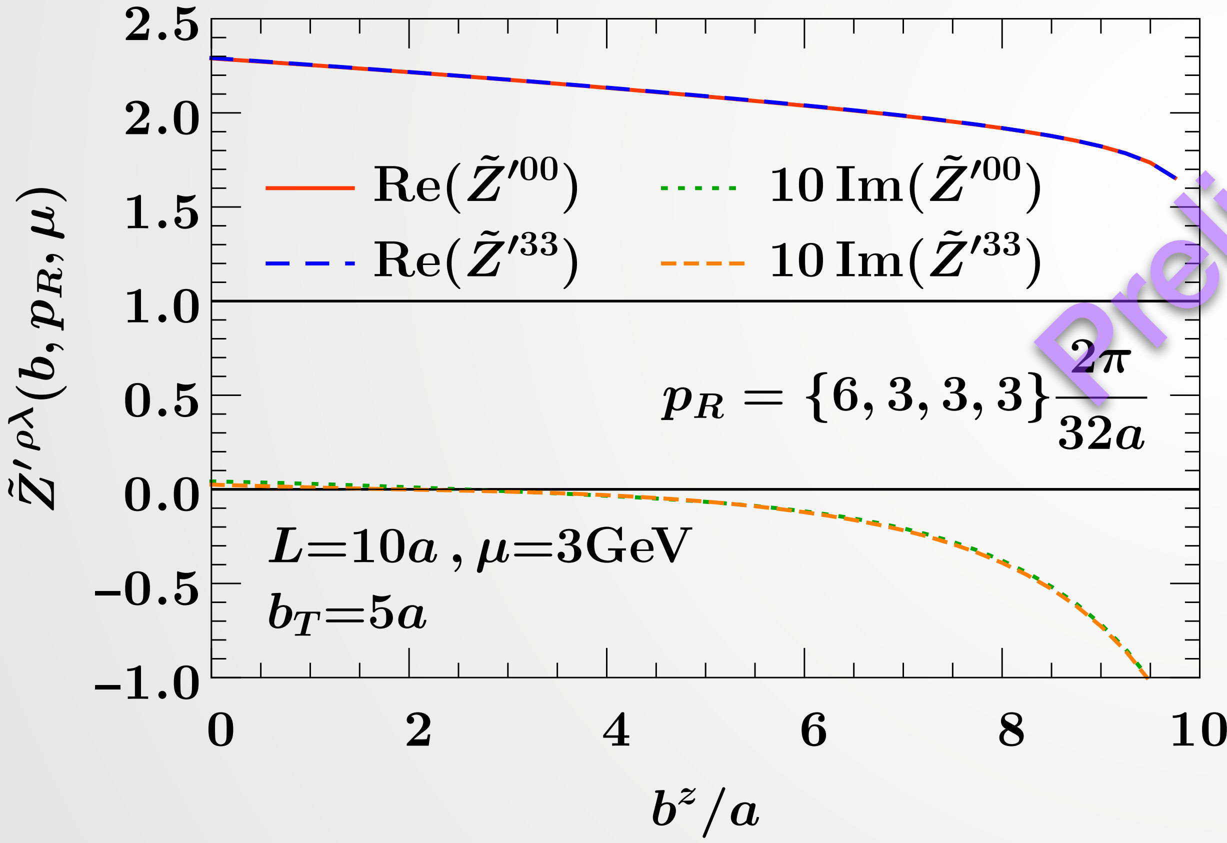


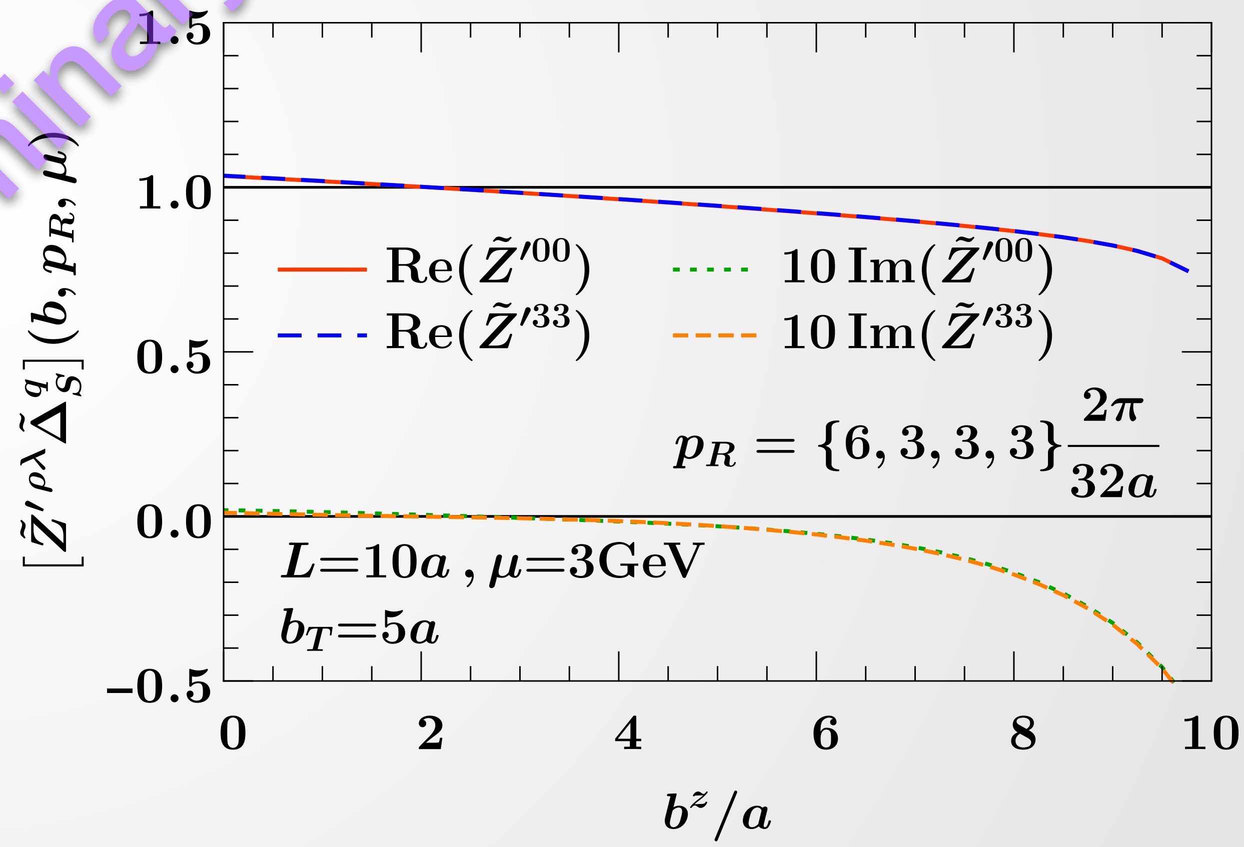
- M. Ebert, I. Stewart, Y.Z., in progress.

Numerical results

- $\mathscr{R}^{\overline{\mathrm{MS}}}$ v.s. b^z

$\rho = \lambda = 0,\ 3 \quad \mathscr{R}^{\overline{\mathrm{MS}}} = \tilde{Z}'^{\rho\lambda}$





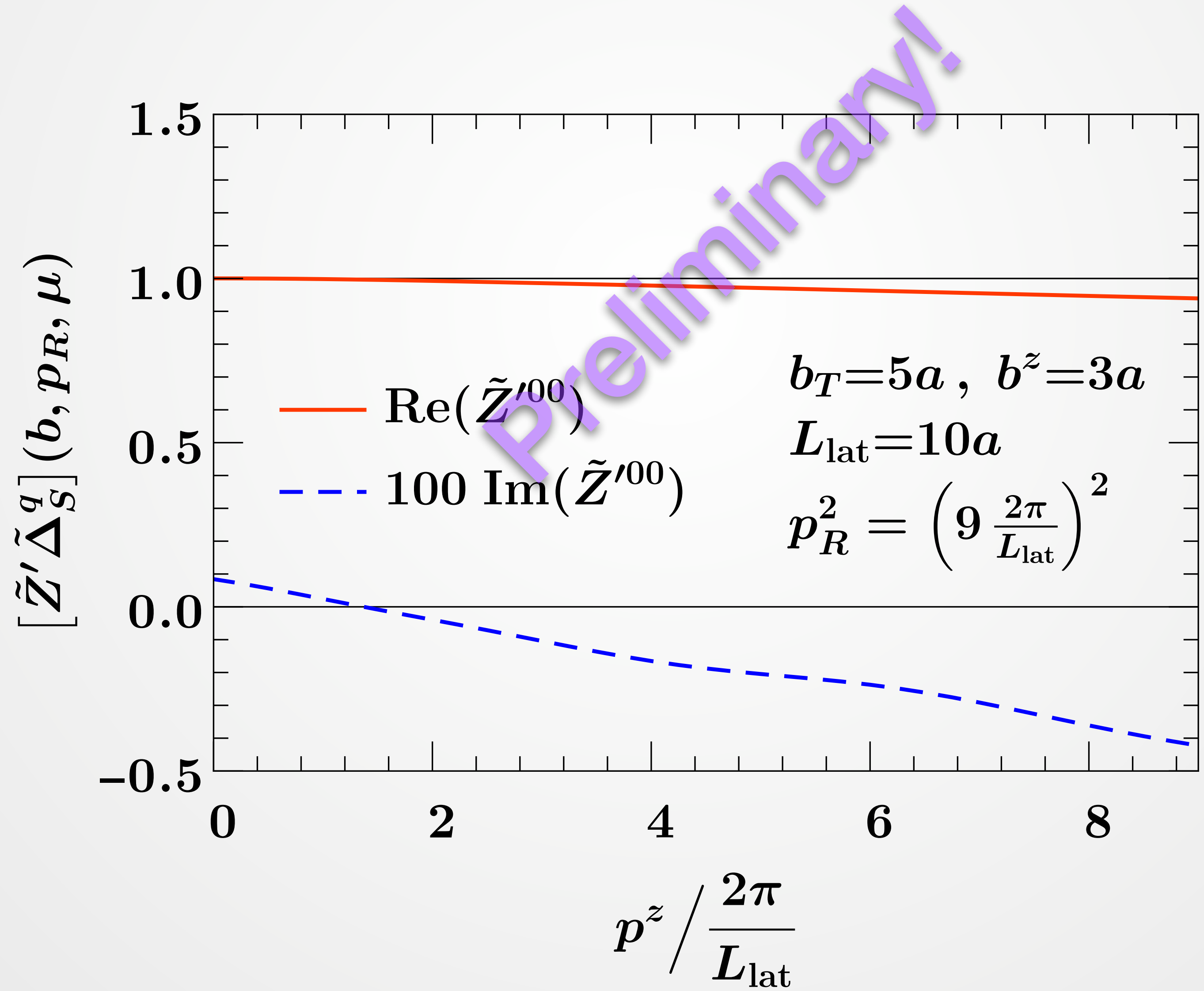
Preliminary!

- M. Ebert, I. Stewart, Y.Z., in progress.

Numerical results

- $\mathscr{R}^{\overline{\mathrm{MS}}}$ **v.s.** p_R^z

$\rho = \lambda = 0, 3 \quad \mathscr{R}^{\overline{\mathrm{MS}}} = \tilde{Z}'^{\rho\lambda}$



- M. Ebert, I. Stewart, Y.Z., in progress.

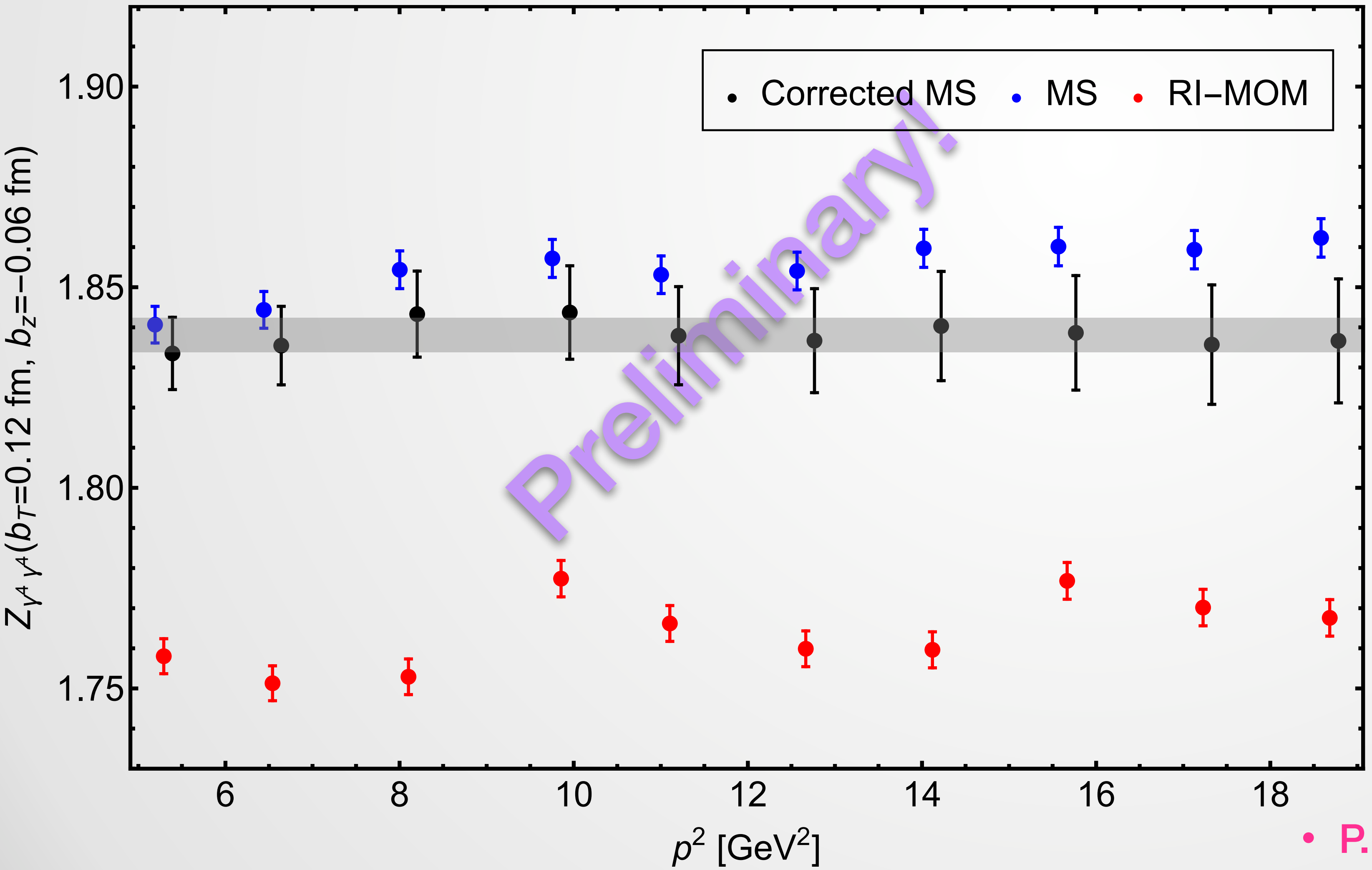
Numerical results

- Fitting the lattice artifacts:

$$\mathcal{Z}^{\overline{\rm MS}}(\mu, p_R) = \mathcal{R}(\mu, p_R)\left(Z_{\mathcal{O}}^{\rm RI'}(p_R)^{-1}\right)$$
$$= \mathcal{Z}^{\overline{\rm MS}}(\mu) + c_1\tilde{p}_z + c_2\tilde{p}^2 + c_3\tilde{p}_4^2 + c_4\frac{\tilde{p}^{[4]}}{\tilde{p}^2} + c_5\tilde{p}^2\ln(\tilde{p}^2) + \frac{d_1}{\tilde{p}^2} + \ldots$$

$$\tilde{p}_\mu \equiv \sin(ap_\mu)$$

$$\rho = \lambda = 0$$



- $Z^{\overline{\rm RI'}}(p_R)$
- $\mathcal{Z}^{\overline{\rm MS}}(\mu, p_R)$
- $\mathcal{Z}^{\overline{\rm MS}}(\mu, p_R)$ − lattice artifacts
- $\mathcal{Z}^{\overline{\rm MS}}(\mu)$

- P. Shanahan, M. Wagman, Y.Z., in progress.

A first look at the CS kernel

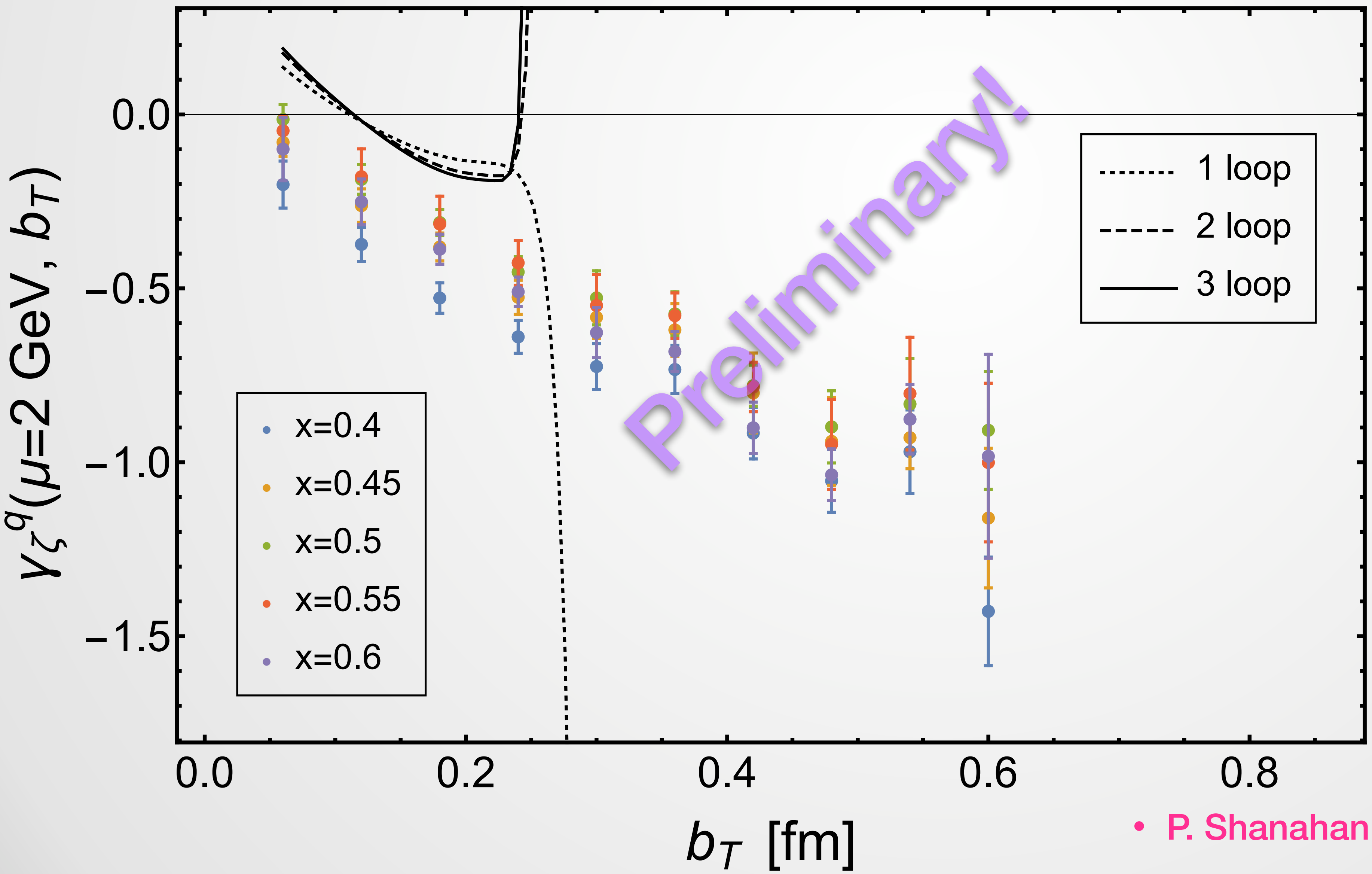
Caveat: low stats and mixing not considered.

$$\gamma_\zeta^i(\mu, b_T) = -2\int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'}\Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(1/b_T)]$$

α_s running with Nf=0.

$a = 0.06$ fm

$P_1^z = 1.3$ GeV, $P_2^z = 1.9$ GeV



• P. Shanahan, M. Wagman, Y.Z., in progress.

Conclusion

- The RI'/MOM renormalization of the quasi beam function has been implemented nonperturbatively on the lattice;
- In addition to the operator mixings predicted by one-loop lattice perturbation theory for the Wilson fermions, we have also observed other sizable mixings;
- The one-loop MSbar conversion factor has been calculated for the quasi beam function, and the inclusion of quasi soft factor significantly reduces the perturbative correction;
- The final MSbar renormalization factors can be well fitted with lattice artifacts;
- Future work will improve statistics and mixing analysis to determine the CS kernel from the lattice.