Quark Gluon Interactions and Partonic Angular Momentum

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Collaborators

Orbital Angular Momentum

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Equation of State Neutron stars

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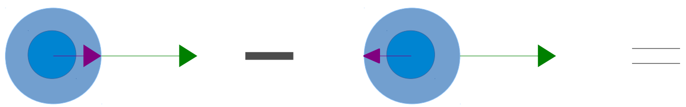
Outline

- Partonic Orbital Angular Momentum
- Wandzura Wilczek and genuine twist three contributions to twist three GPDs
- Role of gauge link
- Extending to chiral odd sector
- Results on the equation of state of neutron stars at short distances

Proton Spin Crisis



$g_1^P(x)$ Quark Spin Contribution

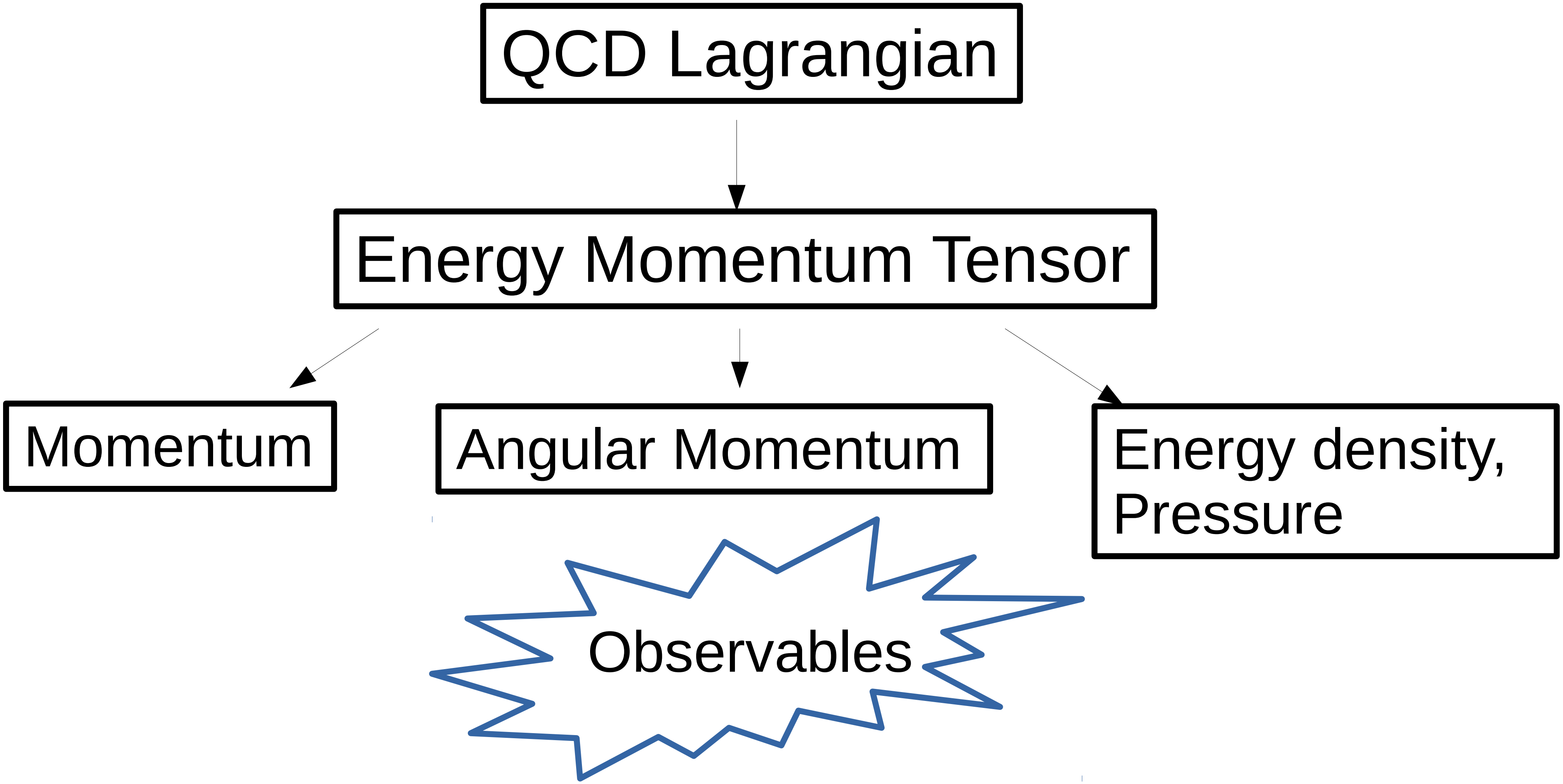
$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in 1980s to be small, present values about 30% of total !!

What are other sources ?

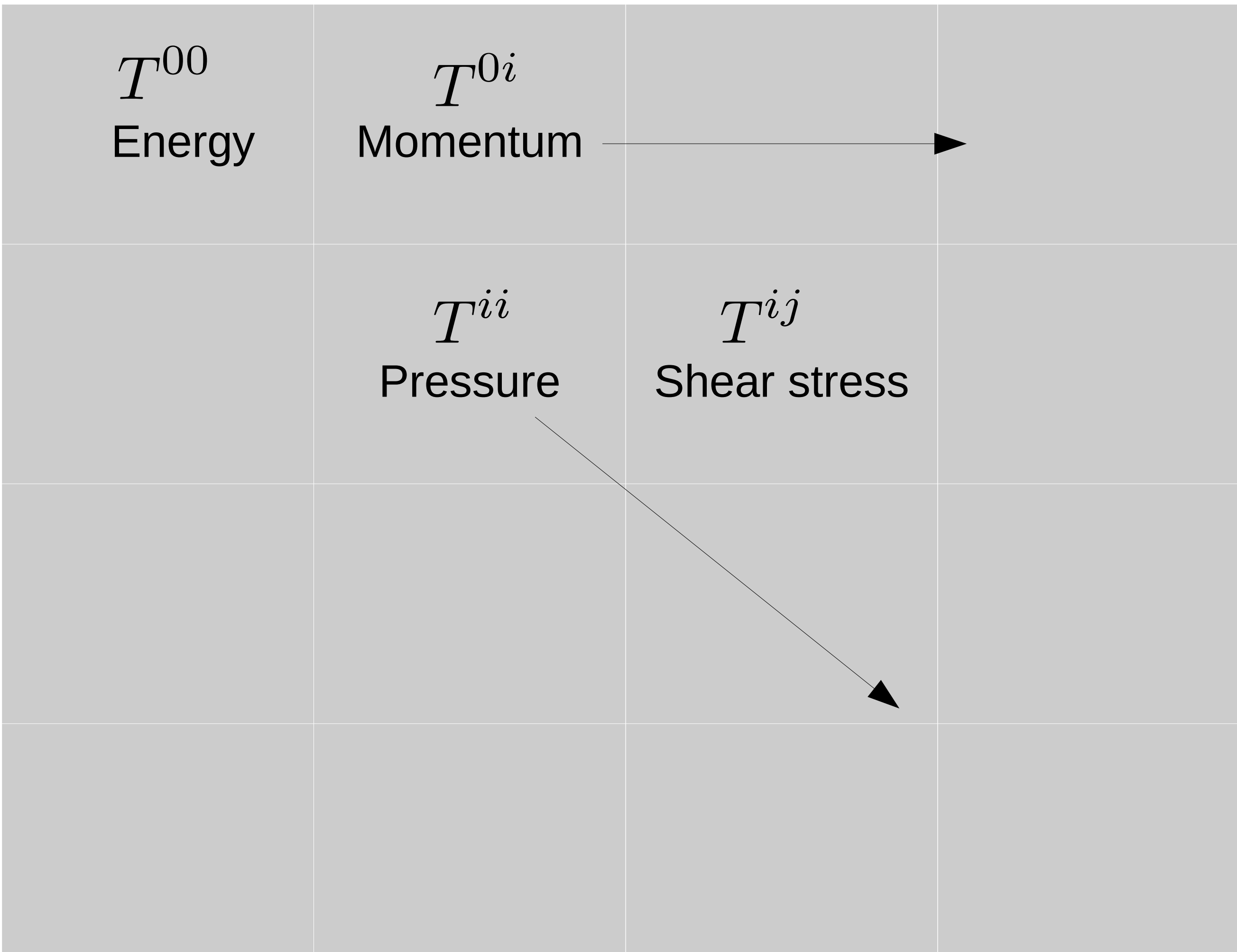
Partonic Orbital Angular Momentum

QCD Energy Momentum Tensor



Deeply Virtual Compton Scattering, moments of GPDs etc.

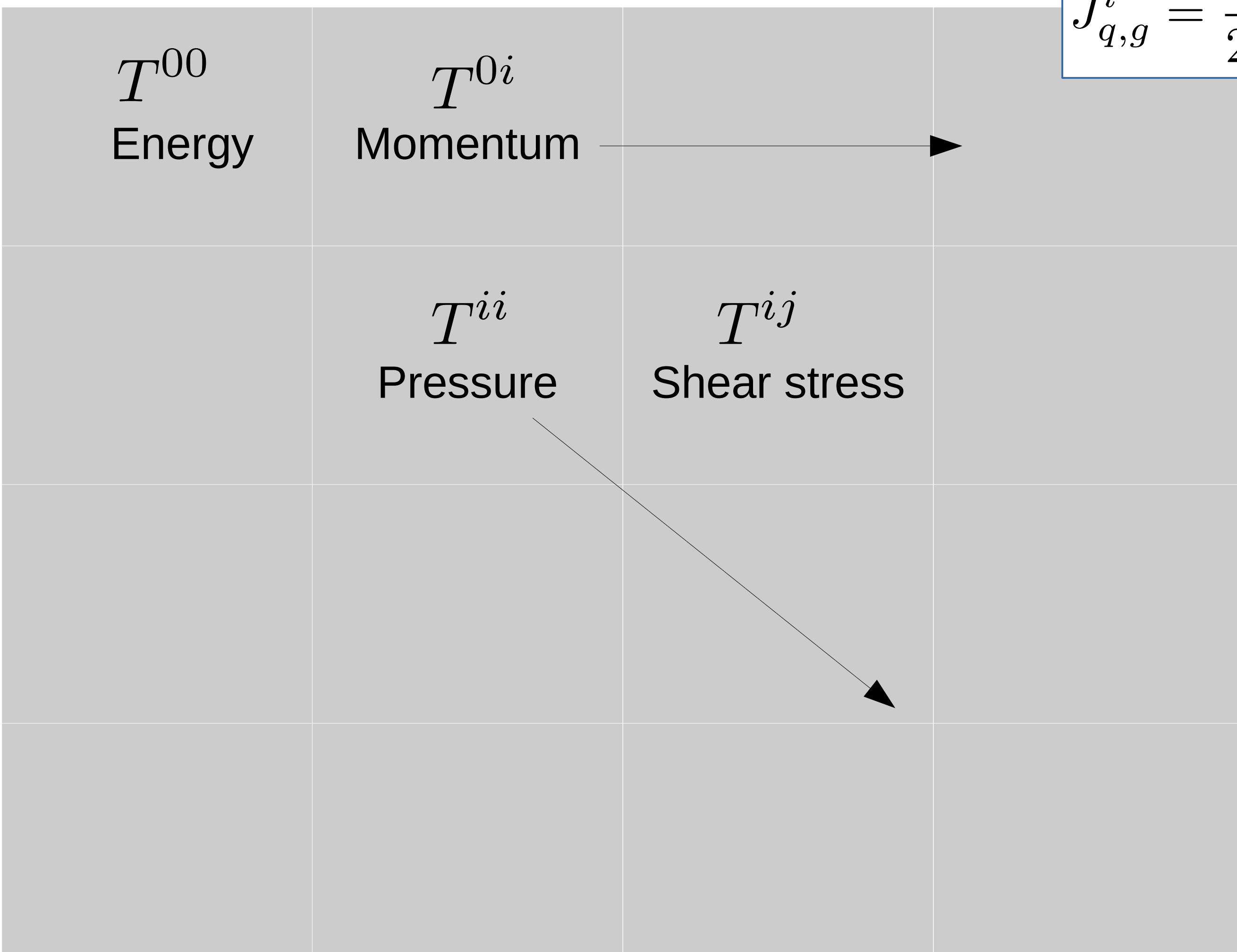
QCD Energy Momentum Tensor



QCD Energy Momentum Tensor

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x\,\left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

Angular Momentum



QCD Energy Momentum Tensor

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x\,\left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

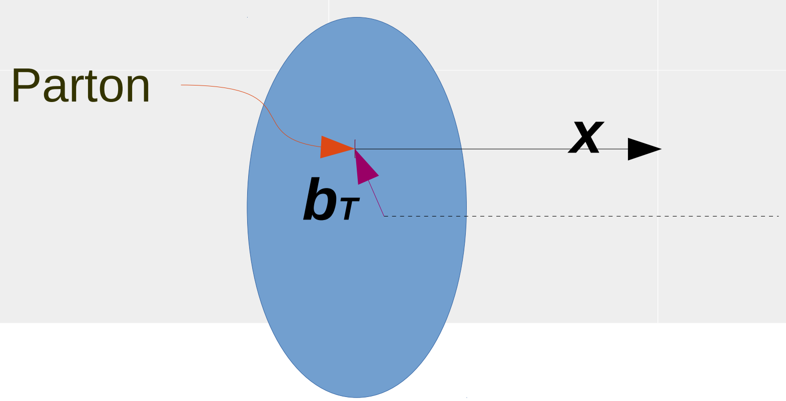
Angular Momentum

Energy Momentum

$$J_q = \frac{1}{2}\int dx x(H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

Angular Momentum described by
GPDs which are collinear



$$J = \mathcal{L} + \frac{1}{2}\Delta\Sigma$$

Total

OAM

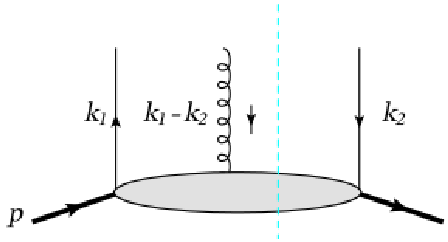
Spin

Do we have a direct description of orbital angular momentum ?

Partonic Orbital Angular Momentum

$$\int dx x G_2 = \int dx x (H + E) - \int dx \tilde{H}$$

$$G_2 \equiv \tilde{E}_{2T} + H + E$$



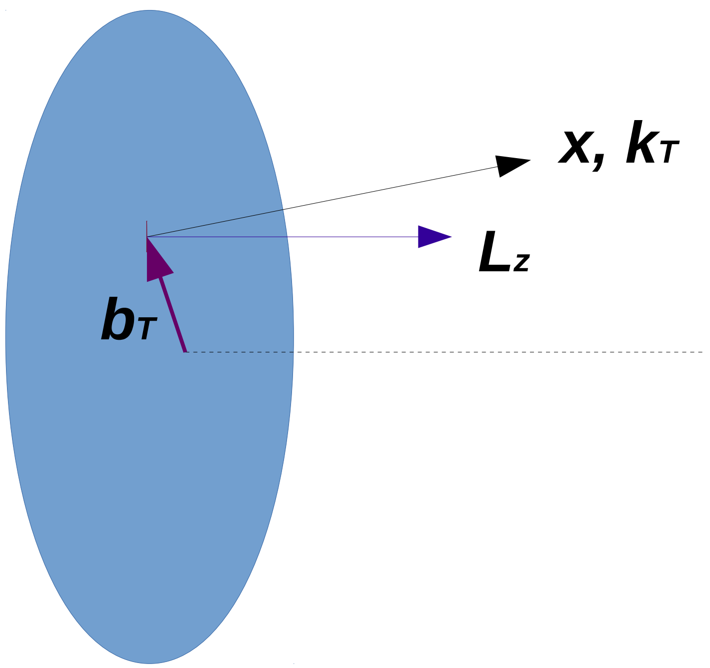
Twist 3

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G_2 shown to be OAM
- Twist three GPD, implicitly includes quark gluon interactions

Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z} = \mathbf{b_T} \times \mathbf{k_T}$



$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel,
JHEP 0908 (2009)

The Two Definitions

$F_{14}^{(1)}$

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q)$$

$$\frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

Distribution of OAM in x

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

Twist two Twist three

AR, Engelhardt and Liuti PRD 98 (2018)

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- Parametrization of the quark quark correlator at different levels
- LIRs occur because the number of GPCFs is less than the number of GTMDs.

$$\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} = \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F + i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)$$

Integrate over k^-

Explicit k_T coefficient

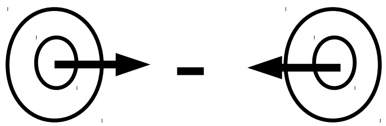
$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Integrate over k_T

$$F^{[\gamma^i]}_{\Lambda,\Lambda'} = \frac{1}{2(P^+)^2}\bar{U}\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta_T^i}{2M}E_{2T} + \frac{P^+\Delta_T^i}{M^2}\tilde{H}_{2T} - \frac{P^+\gamma^i}{M}\tilde{E}_{2T}\right]U$$

$$\int \frac{d^4z}{2\pi}e^{ik.z}\langle p',\Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p,\Lambda\rangle$$

Generalized Lorentz Invariance Relations



Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

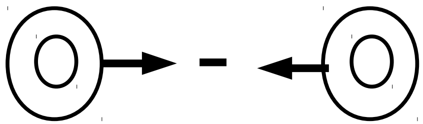
$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

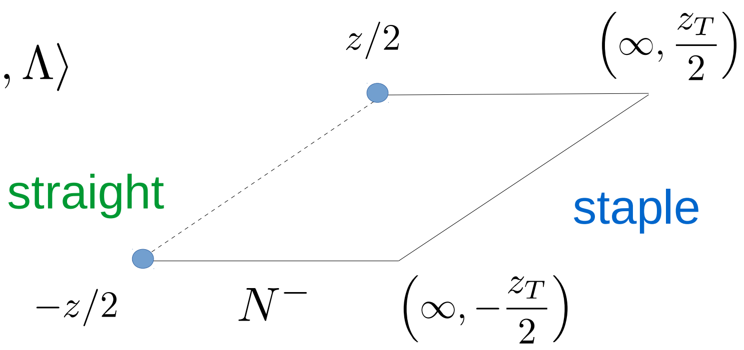
Generalized Lorentz Invariance Relations

- As the quark quark correlator is non-local, the parameterization depends on the choice of gauge link.
- At the completely unintegrated level, we have no knowledge of the light-cone direction if we consider a straight gauge link.
- This is at variance with the staple gauge link case – even at the completely unintegrated level introduction of the *N*⁻ vector along which the staple lies introduces more functions that parameterize the correlator.
- As a result we need to introduce the LIR violating term.

Gauge Link

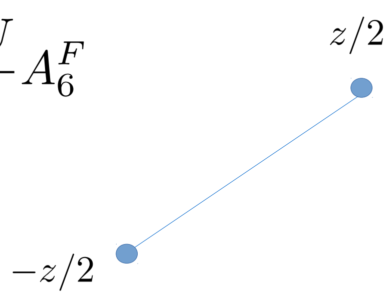
$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\mathcal{U}\psi(z/2) \mid p, \Lambda\rangle$$

Completely unintegrated correlator



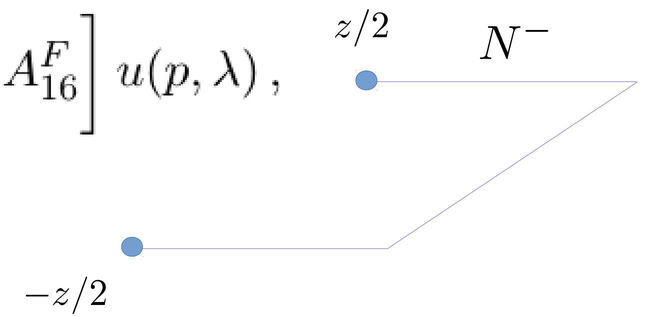
Generalized Lorentz Invariance Relations

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma \mathcal{U} \psi(z/2) \mid p, \Lambda \rangle$$



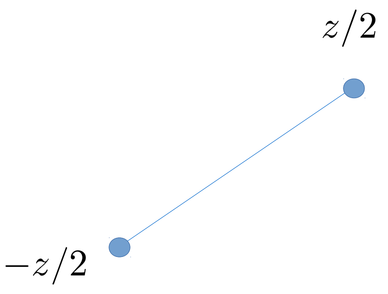
$$\begin{aligned} \mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} &= \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M} A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M} A_6^F \\ &+ i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F) \end{aligned}$$

$$\begin{aligned} & W^{[\gamma^\mu]}_{\lambda\lambda'}(P, k, \Delta, N; \eta) \\ &= \bar{u}(p', \lambda')\left[\frac{P^\mu}{M} A_1^F + \frac{k^\mu}{M} A_2^F + \frac{\Delta^\mu}{M} A_3^F + \frac{N^\mu}{M} A_4^F + \frac{i\sigma^{\mu k}}{M} A_5^F + \frac{i\sigma^{\mu\Delta}}{M} A_6^F + \frac{i\sigma^{\mu N}}{M} A_7^F \right. \\ &+ \frac{P^\mu\, i\sigma^{k\Delta}}{M^3} A_8^F + \frac{k^\mu\, i\sigma^{k\Delta}}{M^3} A_9^F + \frac{N^\mu\, i\sigma^{k\Delta}}{M^3} A_{10}^F + \frac{P^\mu\, i\sigma^{kN}}{M^3} A_{11}^F + \frac{k^\mu\, i\sigma^{kN}}{M^3} A_{12}^F \\ &\left. + \frac{N^\mu\, i\sigma^{kN}}{M^3} A_{13}^F + \frac{P^\mu\, i\sigma^{\Delta N}}{M^3} A_{14}^F + \frac{\Delta^\mu\, i\sigma^{\Delta N}}{M^3} A_{15}^F + \frac{N^\mu\, i\sigma^{\Delta N}}{M^3} A_{16}^F \right] u(p, \lambda), \end{aligned}$$



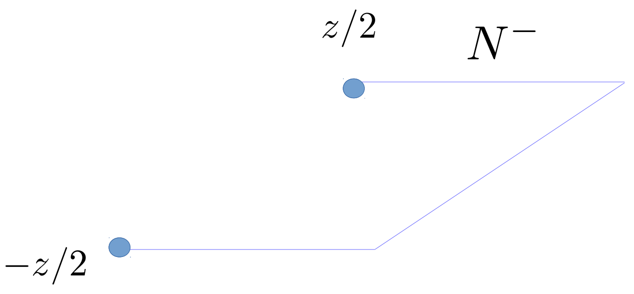
LIR violating term

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$



LIR violating term

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

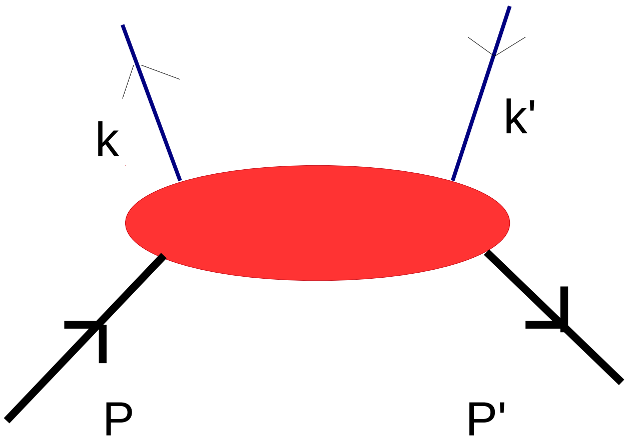


$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2} \int d^2k_T \int dk^- \left[\frac{k_T \cdot \Delta_T}{\Delta_T^2} (A_{11}^F + x A_{12}^F) + A_{14}^F + \frac{k_T^2 \Delta_T^2 - (k_T \cdot \Delta_T)^2}{\Delta_T^2} \left(\frac{\partial A_8^F}{\partial (k \cdot v)} + x \frac{\partial A_9^F}{\partial (k \cdot v)} \right) \right]$$

$$= \frac{dF_{14}^{(1)}}{dx} \quad - \quad \frac{dF_{14}^{(1)}}{dx}$$

staple **straight**

Intrinsic Momentum vs Momentum Transfer Δ

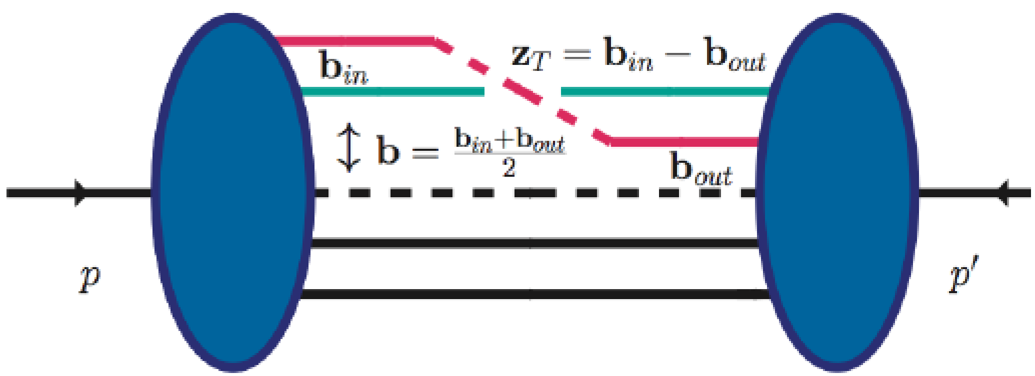


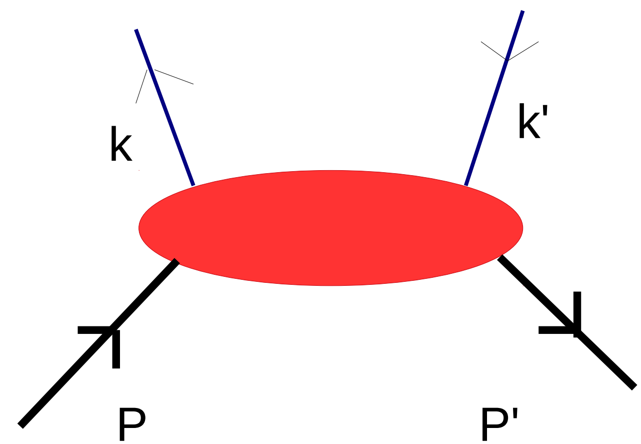
Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

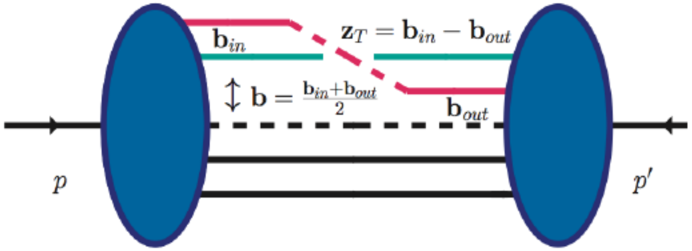
$$\begin{aligned}\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,\\ \bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0\end{aligned}$$

Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$
$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m)i\sigma^{i+}\gamma_5\mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m)i\sigma^{i+}\gamma_5\mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta} \int dz^- d^2z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

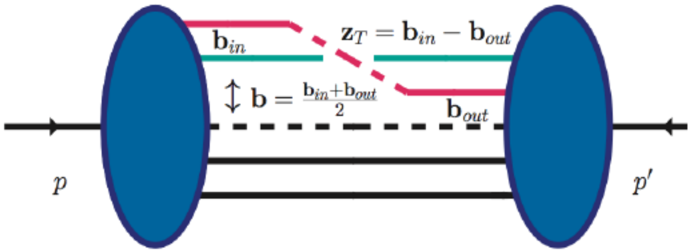
Equations of Motion R

Crucial for understanding qgq contribution to GPDs!!

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D}-m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$
$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma_5\mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A}+m)i\sigma^{i+}\gamma_5\mathcal{U} = 0$$

$$b = \frac{z_{in}+z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta} \int dz^- d^2z_T e^{-ik\cdot z} \langle p',\Lambda'|\bar{\psi}\left[(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}}-m)\right]\psi|p,\Lambda\rangle = 0$$

Use LIRs and Equation of Motion Relations to derive Wandzura Wilczek Relations

- The equations of motion connect k_T dependent quantities with collinear objects.
- These k_T dependent quantities are also connected to collinear objects by LIRs. This is independent of equation of motion relations.
- Use the LIR to eliminate the k_T dependent quantities in equation of motion relations. This results in the Wandzura Wilczek relations for twist 3 GPDs.

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3 **Twist 2** **Genuine Twist 3 (explicit gluon)**

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\partial\!\!\!/} - igA\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + igA\!\!\!/)\Big|_{z/2}\right] \psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0) | p, \Lambda\rangle$$

Wandzura Wilczek Relations

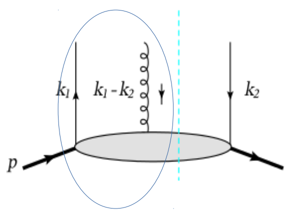
$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three vector GPD

Twist two

Axial vector GPD contributes to a vector GPD

Genuine Tw 3



$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

Twist three PDF

Twist two

Genuine Tw 3

Role of Gauge Link

Equation of Motion

$$0 = x\tilde{E}_{2T} + \tilde{H} - F_{14}^{(1)} + \int d^2k_T \frac{\Delta^i}{\Delta_T^2}\left(\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--}\right)$$

GPDs are not affected by staple

Eliminated using LIR

Quark gluon quark correlator - affected by choice of gauge link

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p',\Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\partial\!\!\!/} - ig A\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + ig A\!\!\!/)\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p,\Lambda\rangle_{z^+=0}$$

Role of Gauge Link

Equation of Motion

$$0 = \boxed{x\tilde{E}_{2T} + \tilde{H}} - F_{14}^{(1)} + \int d^2k_T \frac{\Delta^i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} - \mathcal{M}_{--}^{i,S}\right)$$

GPDs are not affected by staple

Eliminated using LIR

Quark gluon quark correlator - affected by choice of gauge link

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

$$\mathcal{A}_{F_{14}}(x) = \frac{d}{dx}(\mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0})$$

$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}^F + xA_{12}^F) + A_{14}^F + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8^F}{\partial(k\cdot v)} + x\frac{\partial A_9^F}{\partial(k\cdot v)}\right)\right]$$

Role of Gauge Link – calculations in the diquark model

$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2} \int d^2k_T \int dk^- \left[\frac{k_T \cdot \Delta_T}{\Delta_T^2} (A_{11}^F + x A_{12}^F) + A_{14}^F \right.$$
$$\left. + \frac{k_T^2 \Delta_T^2 - (k_T \cdot \Delta_T)^2}{\Delta_T^2} \left(\frac{\partial A_8^F}{\partial (k \cdot v)} + x \frac{\partial A_9^F}{\partial (k \cdot v)} \right) \right]$$

With Brandon Kriesten and Simonetta Liuti

Back to Wandzura Wilczek Relations

staple

$$\widetilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\frac{\widetilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\widetilde{H}\right] - \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

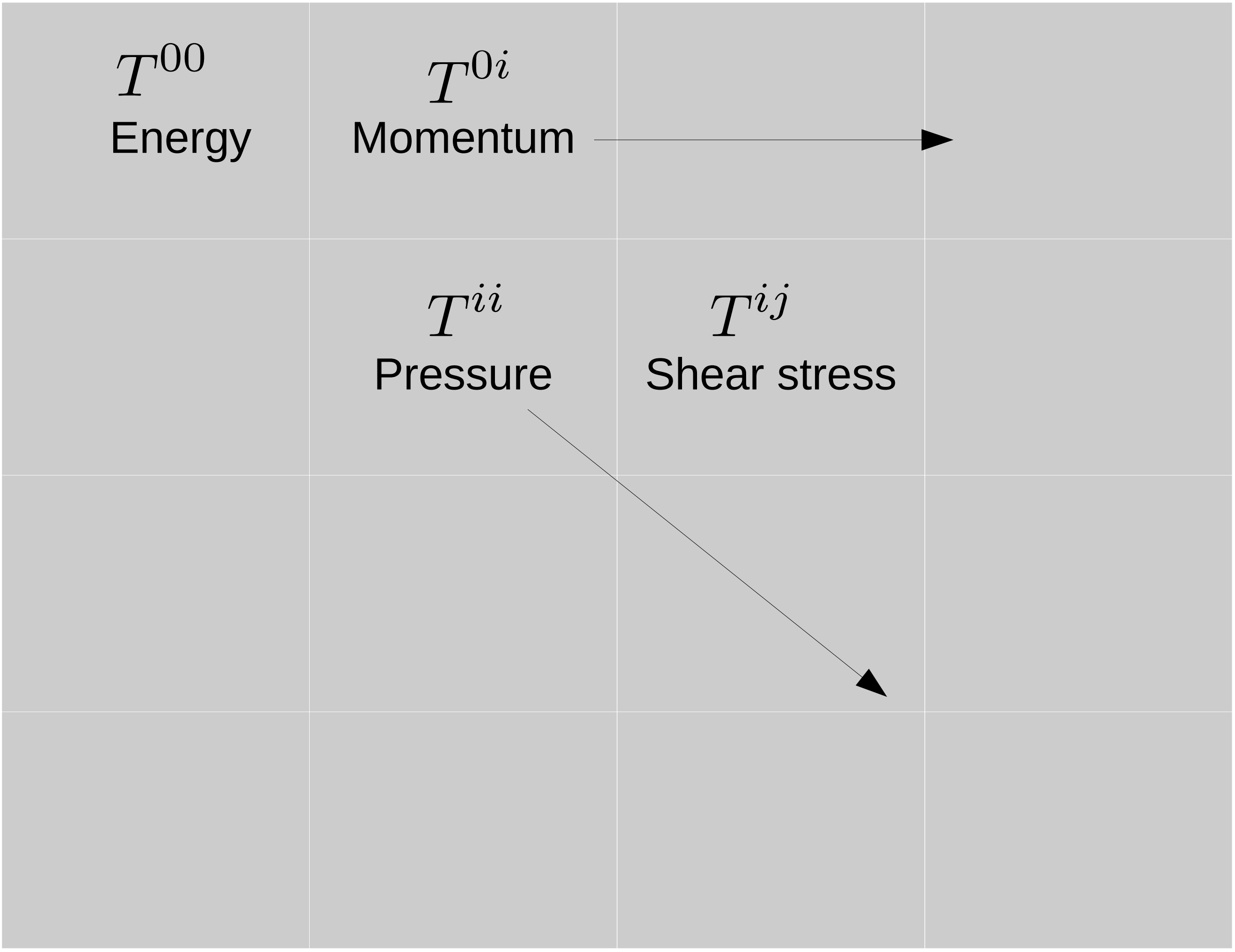
straight

$$\widetilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\frac{\widetilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\widetilde{H}\right] - \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

GPDs are collinear. They carry no memory of the staple gauge link.

Equation of state of Neutron Stars at Short Distances

QCD Energy Momentum Tensor



Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F_{\alpha}^{\nu}$$

- The quark and gluon contributions to the energy momentum tensor are parameterized by the gravitational form factors.

$$\langle P' | (T^{\mu\nu})_R | P \rangle =$$

$$\bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

Off forward

Quark and gluon contributions not conserved separately

GPD moments and the EMT

- Mellin moments of GPDs give the gravitational form factors that parameterize the energy momentum tensor.

$$\int xH(x,\xi,t) = A_{20} + 4\xi^2 C_{20}$$

$$\int xE(x,\xi,t) = B_{20} - 4\xi^2 C_{20}$$

$$\langle P'|(T^{\mu\nu})_R|P\rangle =$$

$$\bar{u}(P')\left[\underline{A_{q,g}}\gamma^{(\mu}\bar{P}^{\nu)} + \underline{B_{q,g}}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + \underline{C_{q,g}}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

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$$\int xH(x,\xi,t) = A_{20} + 4\xi^2 C_{20} \qquad \int xE(x,\xi,t) = B_{20} - 4\xi^2 C_{20}$$

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GPD moments and the EMT

Moments

$$\int xH(x,\xi,t) = A + 4\xi^2 C \qquad \int xE(x,\xi,t) = B - 4\xi^2 C$$

Physical Interpretation

$$\frac{1}{2}(A+B) = J$$ Angular Momentum $$J^i = \int d^3 r \epsilon^{ijk} r_j T_{0k}$$

$$C = \int d^3 r (r^i r^j - \delta^{ij} r^2) T_{ij}$$ Internal Forces

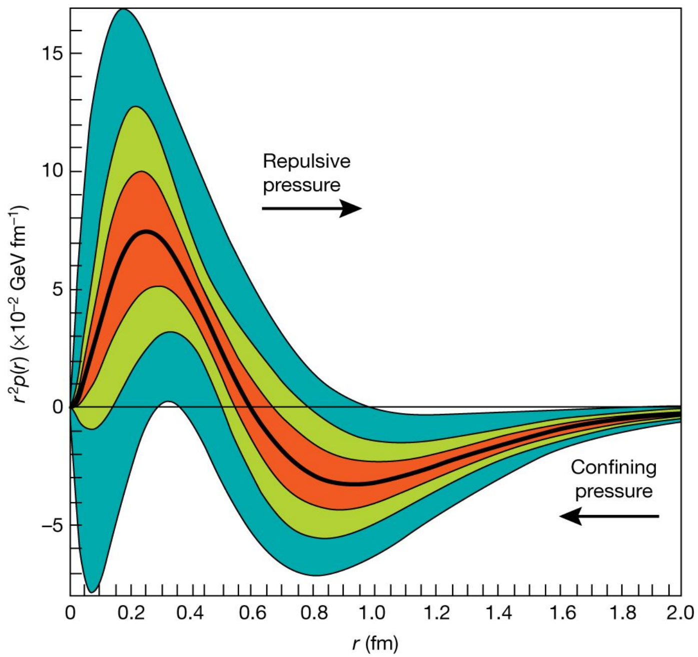
D-term and pressure

$$T^{ij} = \left(\frac{r^i r^j}{r^2} - \frac{1}{2}\delta_{ij}\right) s(r) + \delta_{ij} p(r)$$

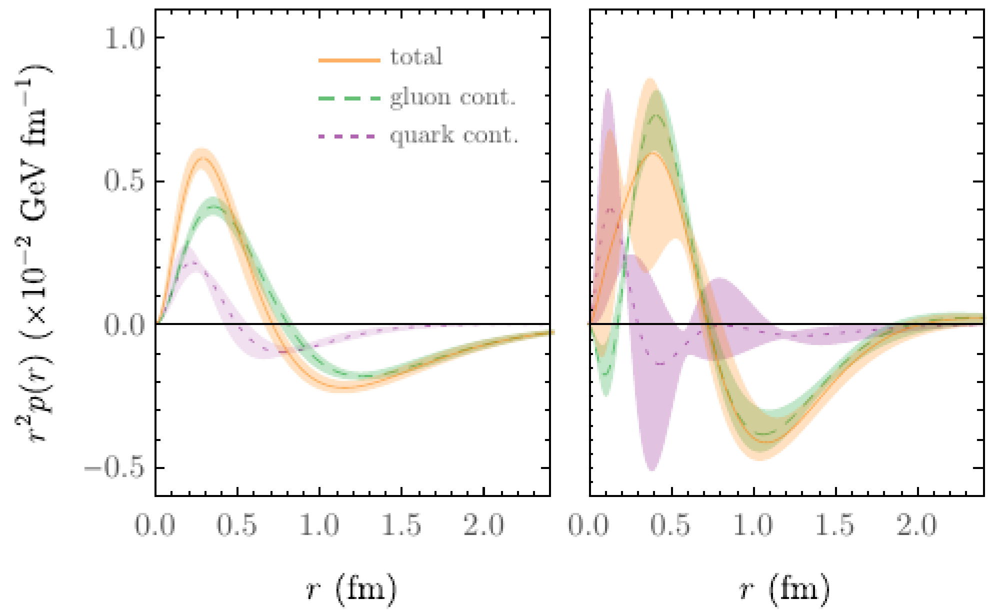
shear pressure

Landau&Lifshitz, Vol.7
M. Polyakov, hep-ph/0210165
M. Polyakov, P. Schweitzer, arXiv:1805.06596

Pressure Distribution inside the Proton

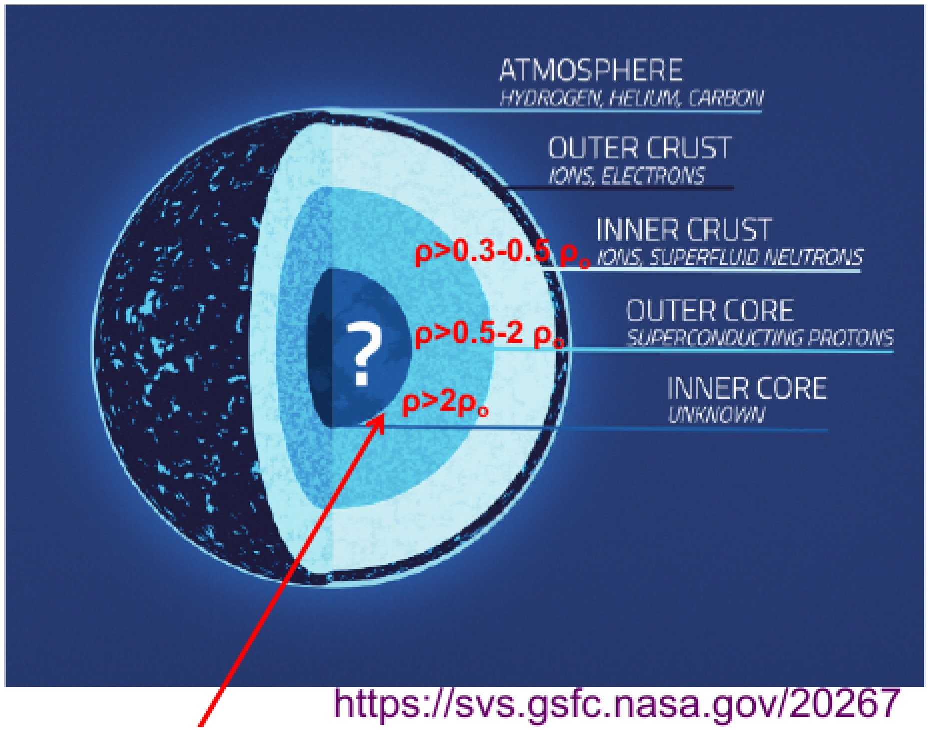


Burkert, Elouadrhiri, Girod (Nature, 2018)



Detmold and Shanahan (PRL, 2019)

Neutron Stars



https://svs.gsfc.nasa.gov/20267

Gravitational collapse is countered by pressure generated by nuclear forces

TOV Equations

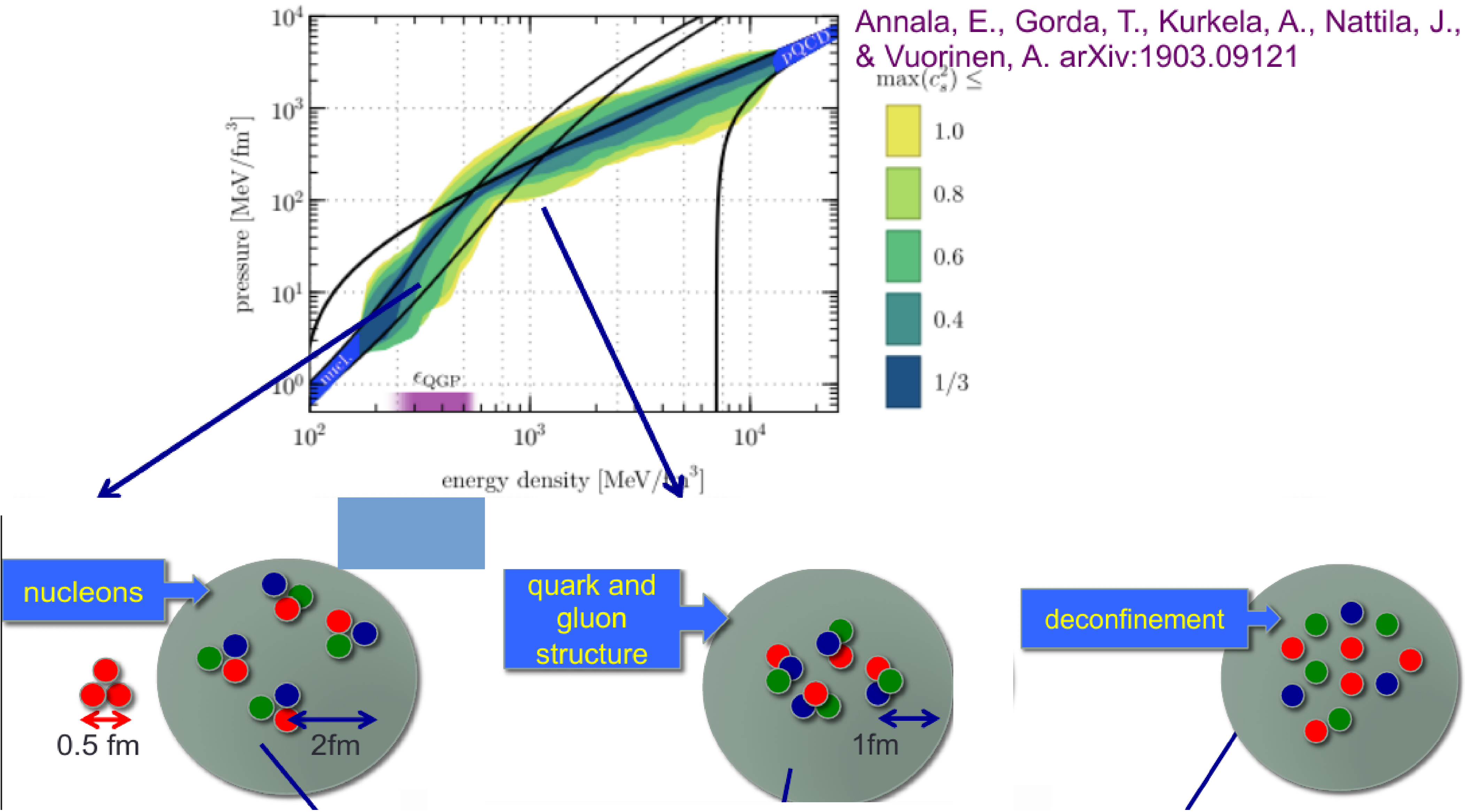
energy density

pressure

matter distribution

$$\frac{dp(r)}{dr} = -\frac{G}{r}\left(\varepsilon(r) + p(r)\right)\left(m(r) + 4\pi r^3 p(r)\right)\left(r - 2\,G\,m(r)\right)^{-1}$$

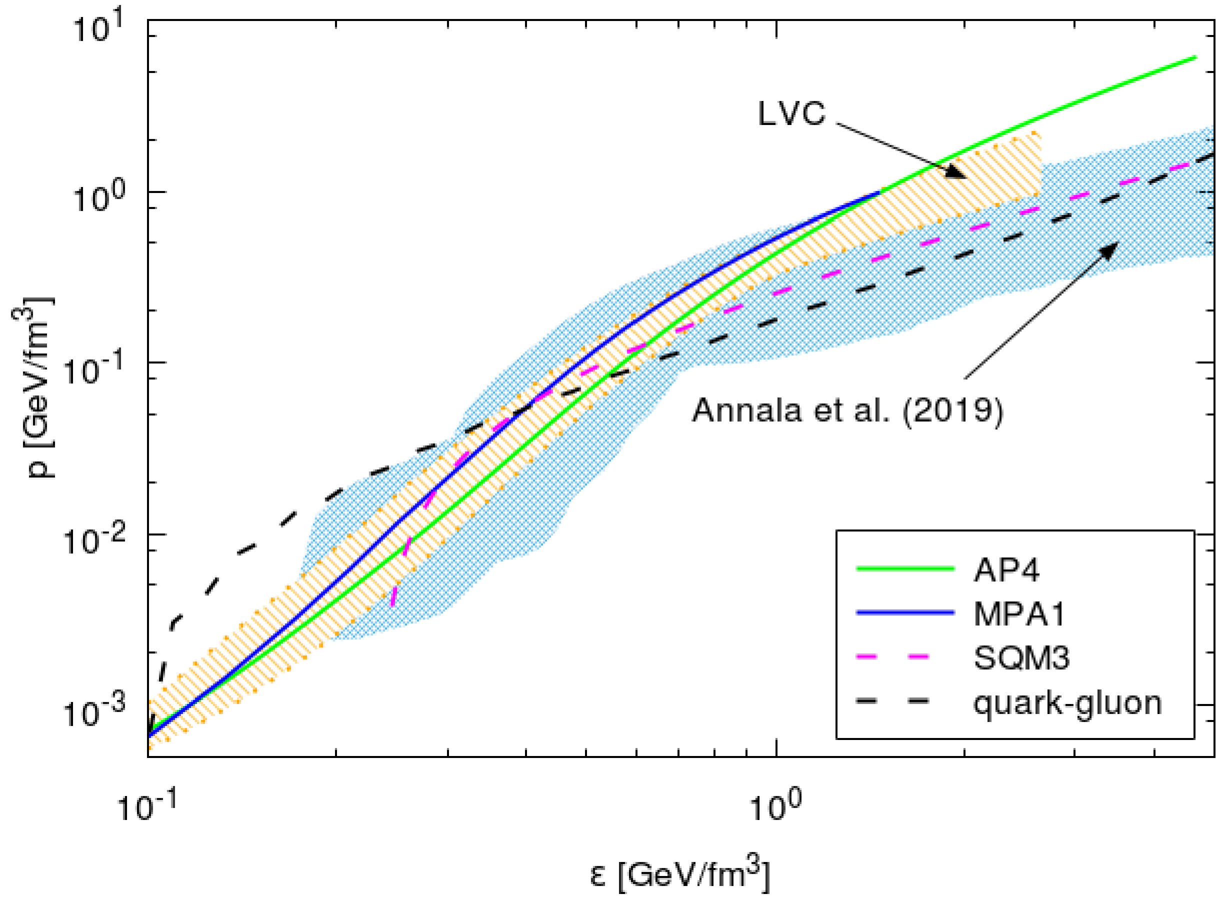
$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$



Annala, E., Gorda, T., Kurkela, A., Nattila, J., & Vuorinen, A. arXiv:1903.09121

G. Baym et al. arXiv:1707.04966

Equation of State of Neutron Stars



$$\epsilon_{q,g}(r) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_2^{q,g}(t),$$

$$p_{q,g}(r) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}}\, 2\,t\, C_2^{q,g}(t)$$

S Liuti, AR, K Yagi, T Gorda arxiv:1812.01479

$$\sum_{\Lambda,\lambda} \rho^q_{\Lambda\lambda}(\mathbf{b}) = H_q(\mathbf{b}^2) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_1^q(t).$$

Conclusions

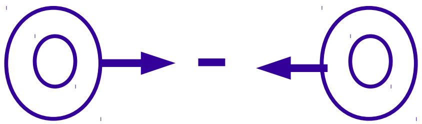
- Way of deriving the Wandzura Wilczek relations. Allows us to **write out precisely quark gluon contribution to twist 3. Study x dependence.**

- Gluons play a key role in the mass and pressure make up of the proton – even neutron stars!

- Quark gluon quark interactions are at the heart of unravelling the structure of the proton.

Thank you!

Moments of twist three GPDs -Quark gluon structure



$$\int dx \widetilde{E}_{2T} = -\int dx (H+E) \qquad \Rightarrow \int dx \left(\widetilde{E}_{2T} + H + E\right) = 0$$

$$\int dx\, \underline{x} \widetilde{E}_{2T} = -\frac{1}{2}\int dx\, x (H+E) - \frac{1}{2}\int dx \widetilde{H}$$

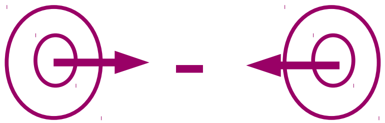
← **OAM**

$$\int dx\, \underline{x}^2 \widetilde{E}_{2T} = -\frac{1}{3}\int dx\, x^2 (H+E) - \frac{2}{3}\int dx\, x \widetilde{H} - \frac{2}{3}\int dx\, x\, \mathcal{M}_{F_{14}}\Big|_{v=0}$$

Genuine Twist Three

$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 F^{+i}(0) \psi(0) | p, \Lambda \rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) \;=\; -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx\underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) \;=\; -\frac{1}{2}\int dx x\widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

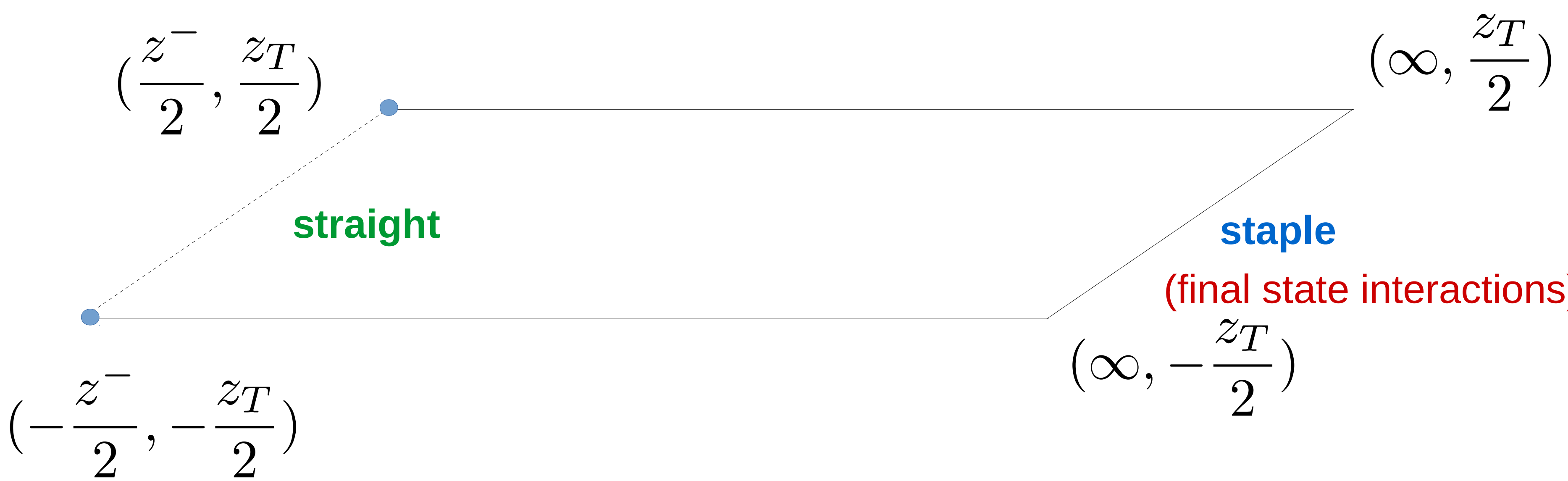
mass term

$$\int dx\, \underline{x}^2 \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) \;=\; -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)}$$
$$- \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} \;=\; \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p, \Lambda\rangle$$

Quark gluon quark contributions



$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} \quad = $$

$$i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} \quad = $$

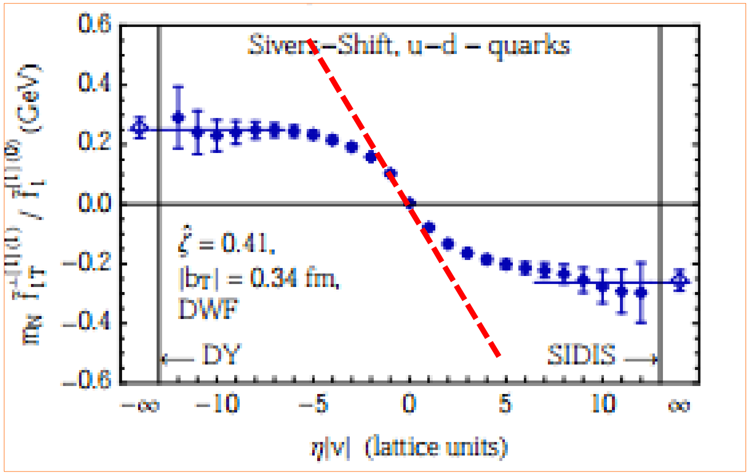
$$-g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0)\psi(0)|p, \Lambda\rangle$$

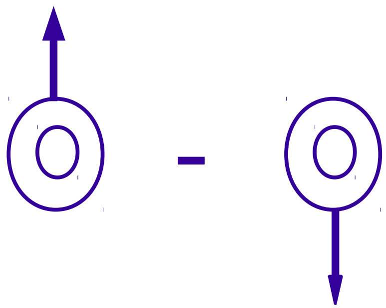
Calculating the force from Lattice data – Sivers function

$$\frac{d}{dv^-}\int dx F_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{F_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,A} - \mathcal{M}_{--}^{i,A}\right) = \mathcal{M}_{G_{12}}^{n=3}$$

$$\frac{d}{dv^-}\int dx G_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{G_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} + \mathcal{M}_{--}^{i,S}\right) = \mathcal{M}_{F_{12}}^{n=3}$$

The derivative with respect to the gauge link direction gives the force!

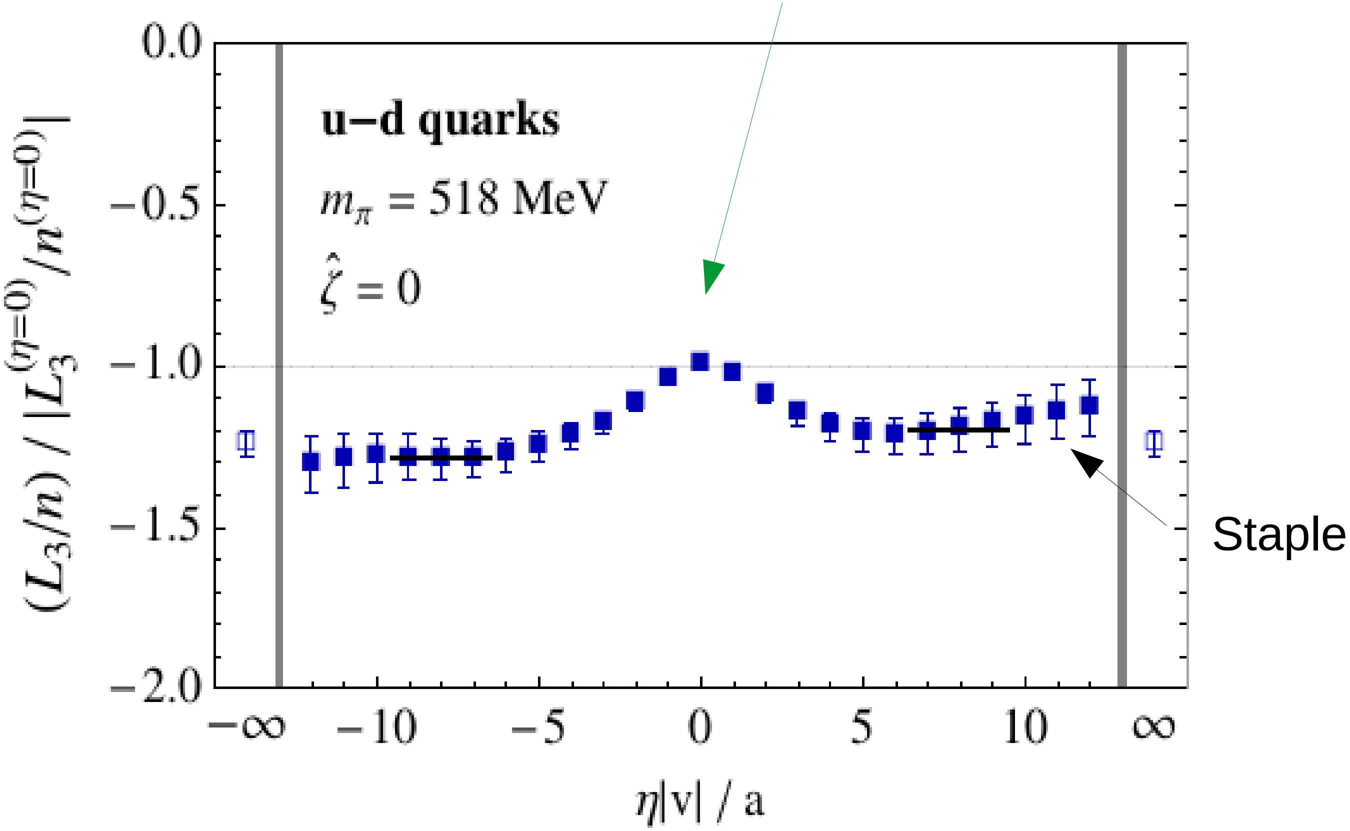




Transversely polarized proton

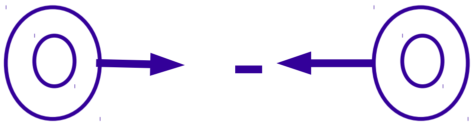
Calculating the torque from Lattice

F_{14}



Michael Engelhardt

Phys. Rev. D95 (2017)



Longitudinally polarized proton

$$\mathcal{L}_{JM} - \mathcal{L}_{Ji} = \mathcal{T}$$

Torque!

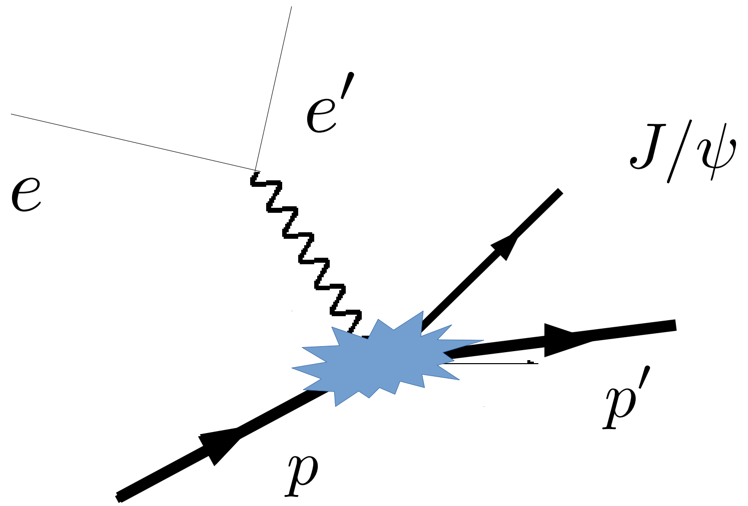
Experimental measurements

The production of a heavy quarkonium near threshold in electron-proton scattering is connected to the origin of the proton mass via the QCD trace anomaly.

D.E Kharzeev (1995)

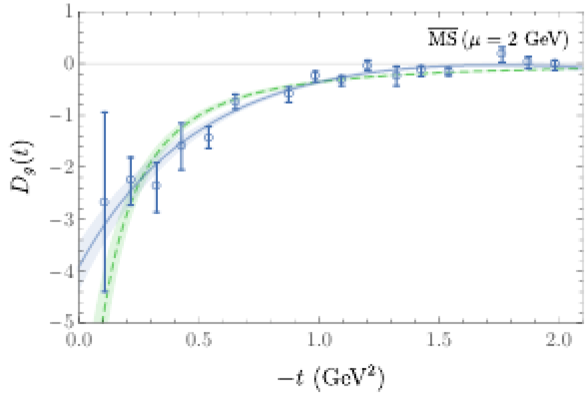
$$ep \to e'\gamma^* p \to e'p'J/\psi$$

Y Hatta, DL Yang PRD98 (2018)



In an actual experiment $p' - p \equiv t \neq 0$

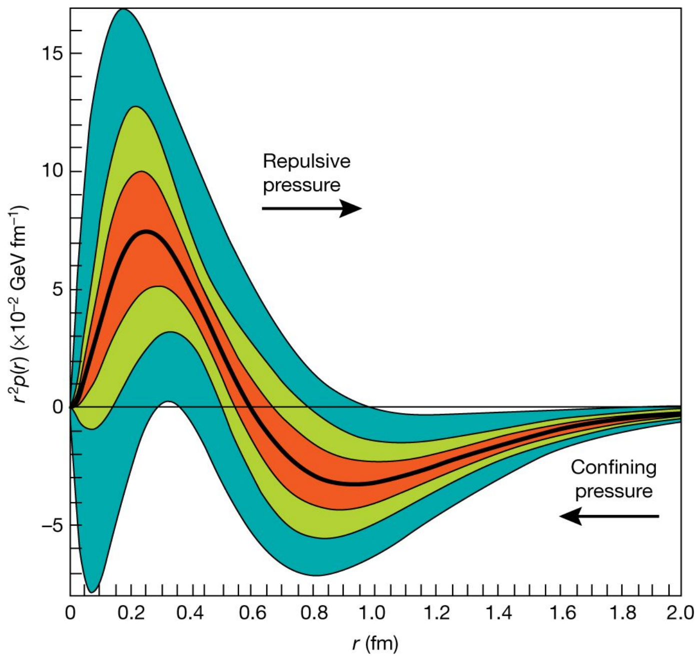
Calculate cross-section using input from latest lattice QCD calculations of gluon gravitational form factors.



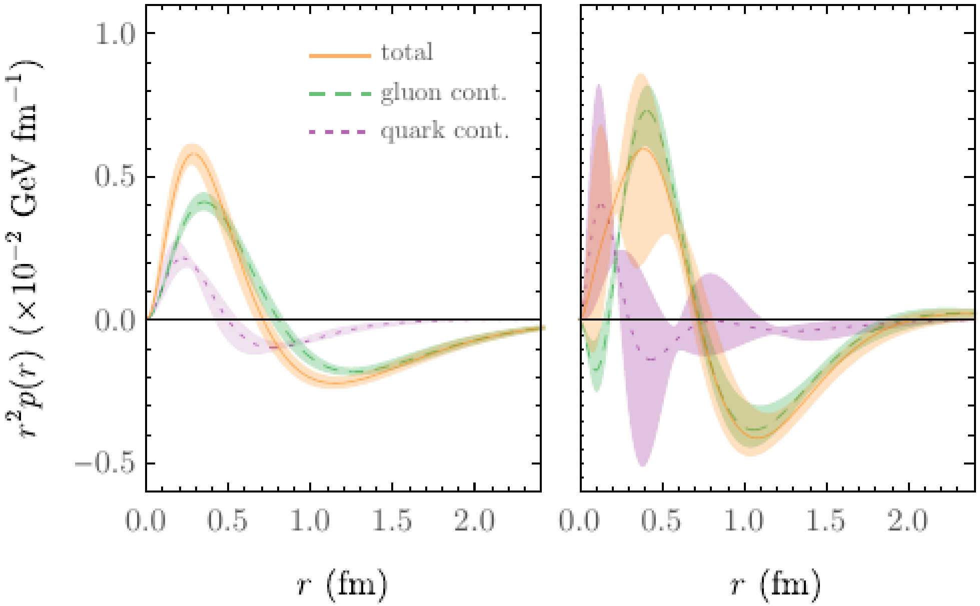
Detmold and Shanahan arxiv:1810:04626

Equation of state of Neutron Stars at Short Distances

Pressure Distribution inside the Proton

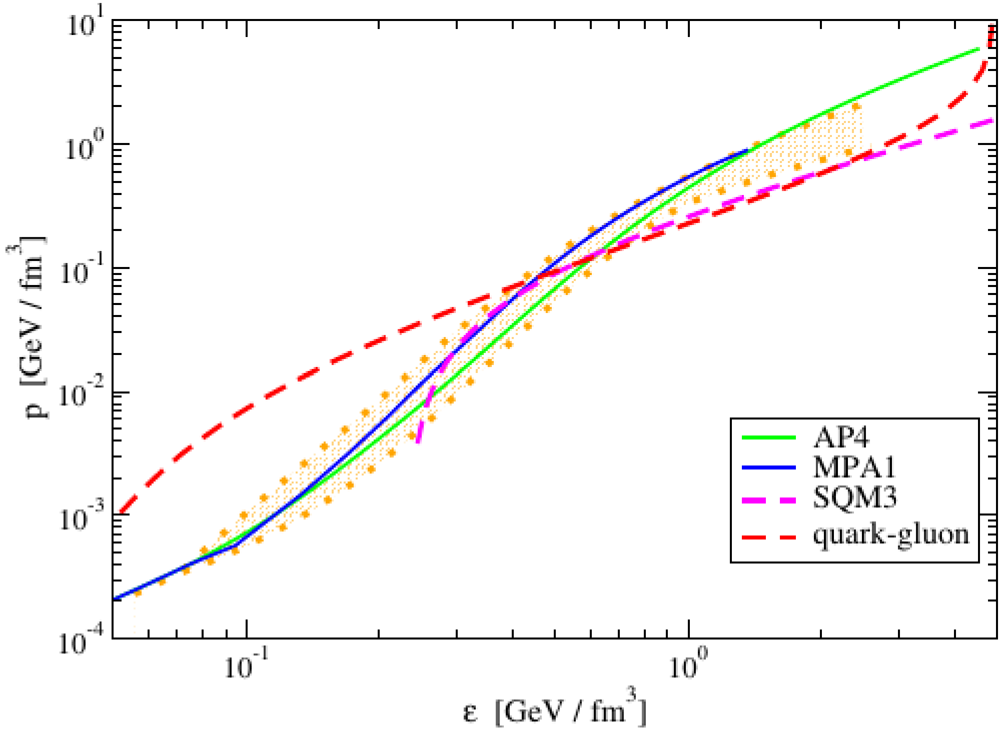


Burkert, Elouadrhiri, Girod (Nature, 2018)



Detmold and Shanahan (PRL, 2019)

Equation of State of Neutron Stars



S Liuti, AR, K Yagi arxiv:1812.01479

$$\epsilon_{q,g}(r) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_2^{q,g}(t),$$

$$p_{q,g}(r) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}}\, 2\, t\, C_2^{q,g}(t)$$

$$\sum_{\Lambda,\lambda} \rho^q_{\Lambda\lambda}(\mathbf{b}) = H_q(\mathbf{b}^2) = \int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} A_1^q(t).$$

Conclusions

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- Quark gluon quark interactions are at the heart of unravelling the structure of the proton.

Thank you!

Thanks!

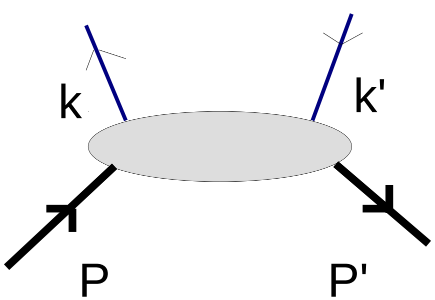
Higher Twist

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

$$\gamma^+, \gamma^+\gamma^5, \sigma^{i+}\gamma^5$$

Leading twist – twist 2

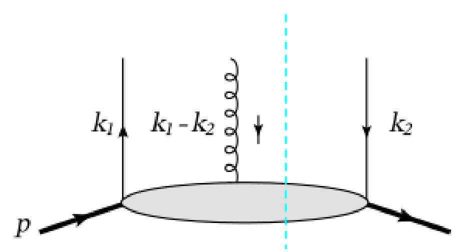
- Involve only good components
- Simple interpretation in terms of parton densities



$$\gamma^i, \gamma^i\gamma^5, \sigma^{ij}\gamma^5, 1, \gamma^5, \sigma^{+-}\gamma^5$$

Higher twist – twist 3

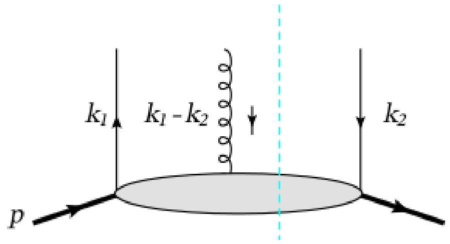
- Involve one good and one bad component
- The bad component represents a quark gluon composite



Collinear Picture : Transverse Quark Current, Higher Twist

$$\bar{\psi}(-z/2)\gamma^{+}\psi(z/2) \longrightarrow$$ Leading order quark current

$$\bar{\psi}(-z/2)\gamma_T^{i}\psi(z/2) \longrightarrow$$ Transverse quark current, implicitly involves quark gluon interactions



Probabilistic parton model interpretation works well at leading order, with transverse quark projection operator need to include quark gluon interactions

Both in Collinear Picture

Quark and gluon contributions to the trace anomaly

$$T^\alpha_\alpha = -2\epsilon\frac{F^2}{4} + m\bar{\psi}\psi$$

renormalization ↓

$$-2\epsilon\frac{F^2}{4} = \frac{\beta}{2g}F^2_R + \gamma_m(m\bar{\psi}\psi)_R$$

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overset{\leftrightarrow}{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

Trace ↓

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

$$T^\alpha_{g\alpha} = \frac{\beta}{2g}(F^2)_R + \gamma_m(m\bar{\psi}\psi)_R$$

$$T^\alpha_{q\alpha} = m(\bar{\psi}\psi)_R$$

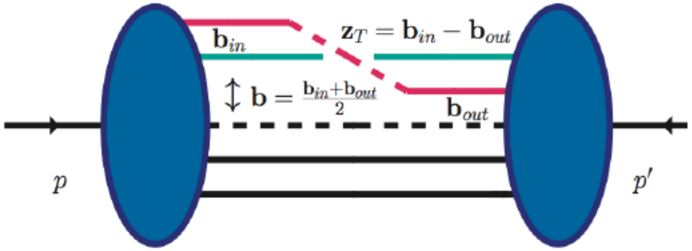
Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$

$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+ z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- The same set of As describe the whole vector sector.

γ^+

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F\right]$$

$$J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{x P^2}{M^2}\right)\left(A_8^F + x A_9^F\right)$$

γ_T^i

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}\right) A_9^F - \sigma' A_5^F - A_6^F\right]$$

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

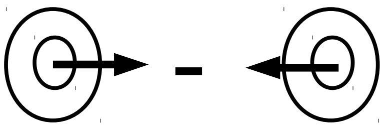
$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y)\right)$$

Distribution of OAM in x !

k_T^2 moment of a twist two function

Twist three function

Generalized Lorentz Invariance Relations



Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}_{2T}' + E_{2T}'\right) - \tilde{H}$$

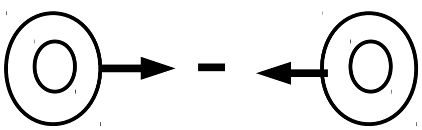
$$\frac{dG_{12}^{e(1)}}{dx} = H_{2T}' - \frac{\Delta_T^2}{4M^2} E_{2T}' - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



Intrinsic transverse momentum

Quark gluon interactions

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3

Twist 2

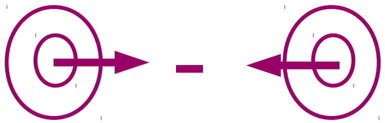
Genuine Twist 3 (explicit gluon)

$$\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\not\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\not\partial} + ig\not{A})\Big|_{z/2}\right] \psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0) | p, \Lambda\rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx x \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

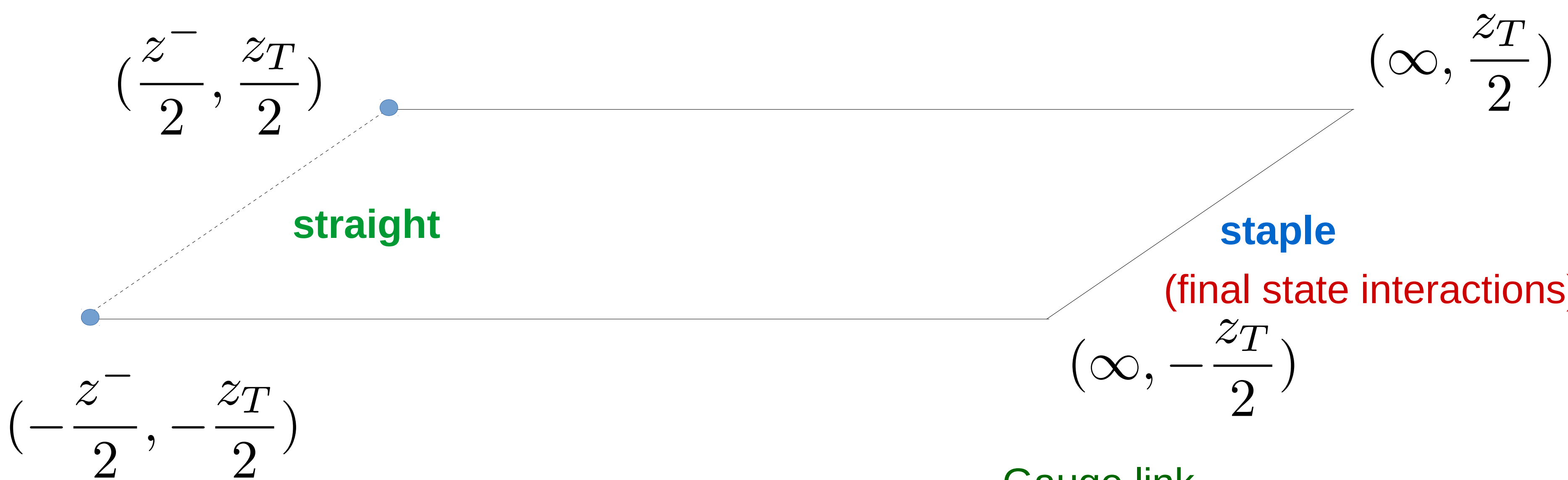
mass term

$$\int dx\, x^2 \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)} - \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0) | p, \Lambda\rangle$$

Staple gauge link



Gauge link

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Non local operator

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

LIR violating term

$$\mathcal{A}_{F_{14}}(x) \equiv v^- \frac{(2P^+)^2}{M^2} \int d^2k_T \int dk^- \left[\frac{k_T \cdot \Delta_T}{\Delta_T^2} (A_{11} + xA_{12}) + A_{14} \right.$$
$$\left. + \frac{k_T^2 \Delta_T^2 - (k_T \cdot \Delta_T)^2}{\Delta_T^2} \left(\frac{\partial A_8}{\partial (k \cdot v)} + x \frac{\partial A_9}{\partial (k \cdot v)} \right) \right]$$
$$= \frac{dF_{14}^{(1)}}{dx} - \left. \frac{dF_{14}^{(1)}}{dx} \right|_{v=0} \quad (1)$$

$$F_{14}^{(1)} - \left. F_{14}^{(1)} \right|_{v=0} = \mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} \, (1)$$

$$\mathcal{A}_{F_{14}}(x) = \frac{d}{dx} \left(\mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} \right) (1)$$

$$- \int dx \left(F_{14}^{(1)} - \left. F_{14}^{(1)} \right|_{v=0} \right) \Bigg|_{\Delta_T = 0} = \quad (1)$$
$$- \frac{\partial}{\partial \Delta^i} i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds \, \langle p', + | \bar{\psi}(0) \gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0) \psi(0) | p, + \rangle \Big|_{\Delta_T = 0},$$

Understanding the mass decomposition of the proton

Mass decomposition of the proton

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{q,kin} + T^{\mu\nu}_{g,kin}} + T^{\mu\nu}_{m} + T^{\mu\nu}_{a}$$

Traceless

X Ji (1995)

$$M = \frac{\langle P| \int d^3\mathbf{x} T^{00}(0,\mathbf{x}) |P\rangle}{\langle P|P\rangle} \equiv \langle T^{00} \rangle$$

Rest frame

$$\langle \bar{T}^{00} \rangle = 3/4M$$ ← Traceless

$$\langle \hat{T}^{00} \rangle = 1/4M$$ ← Trace part

Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F_{\alpha}^{\nu}$$

- The full energy momentum tensor is a conserved quantity and is scale independent.
- The separate contributions from the quarks and gluons on the other hand are not and do depend on the renormalization scale.

$$\langle P' | (T^{\mu\nu})_R | P \rangle =$$
$$\bar{u}(P') \left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu / M \longrightarrow M$ **Total**

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^{\mu}P^{\nu}/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}}{2M} + C_{q,g}\frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

Trace ↓

Quark and gluon components separated

$$\langle P'|(T^{R}_{q,g})^{\alpha}_{\alpha}|P\rangle = 2M^{2}\left(A^{R}_{q,g}(\mu) + 4\bar{C}^{R}_{q,g}(\mu)\right)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

↓ Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

* $$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

↓ Trace

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

By studying the Gravitational form factors A and C̄ we will know the quark and gluon contributions to the trace anomaly separately.

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

Trace ↓

$$T^{\mu}_{\mu} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

Trace

$$T^{\mu}_{\mu} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

$$(T^{\alpha}_{g\alpha})_R = A^R_g(\mu) + 4\bar{C}^R_g(\mu)$$

$$= \frac{1}{2M^2}\langle P|\frac{\alpha_s}{2\pi}\left(-\frac{11C_A}{6}(F^2)_R + \frac{14C_F}{3}(m\bar{\psi}\psi)_R)\right)|P\rangle$$

gluons

$$(T^{\alpha}_{q\alpha})_R = A^R_q(\mu) + 4\bar{C}^R_q(\mu)$$

$$= \frac{1}{2M^2}\langle P|(m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}(F^2)_R + \frac{4C_F}{3}(m\bar{\psi}\psi)_R)\right)|P\rangle$$

quarks

Y Hatta, AR, K Tanaka, JHEP 1812 (2018) 008