

# Extraction of Twist-3 Observables from DVCS

TMD Collaboration/POETIC 2019

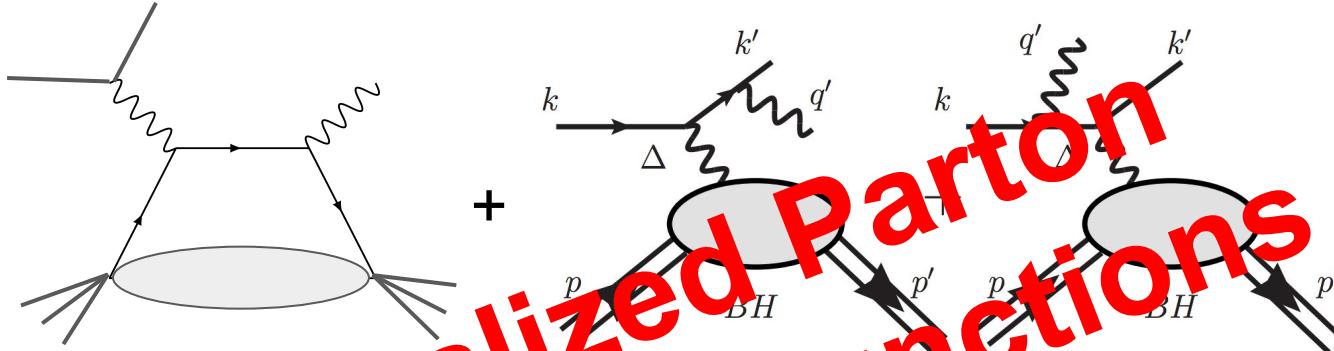
B. Kriesten



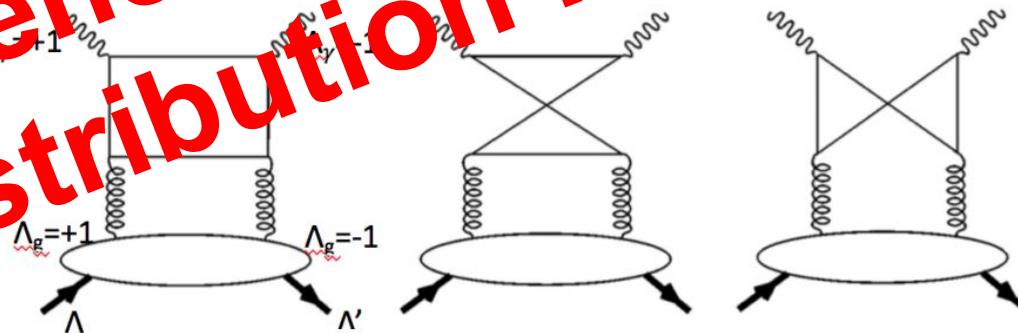
9/16/2019



# Deeply Virtual Compton Scattering Observables



**Generalized Parton Distribution Functions**



## Mechanical Properties of the Proton

Energy Momentum Tensor

$$\begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

$$\langle p' | T^{\mu\nu} | p \rangle = \bar{U}(P') \left[ A(t) \gamma^{(\mu} P^{\nu)} + B(t) P^{(\mu} i\sigma^{\nu)\alpha} \Delta_{\alpha}/2M + C(t) (\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu} \Delta^2)/M + \tilde{C}(t) g^{\mu\nu} M \right] U(p)$$

$$\int dx x H(x, \xi, t) = A(t) + \boxed{\xi^2 C(t)}$$

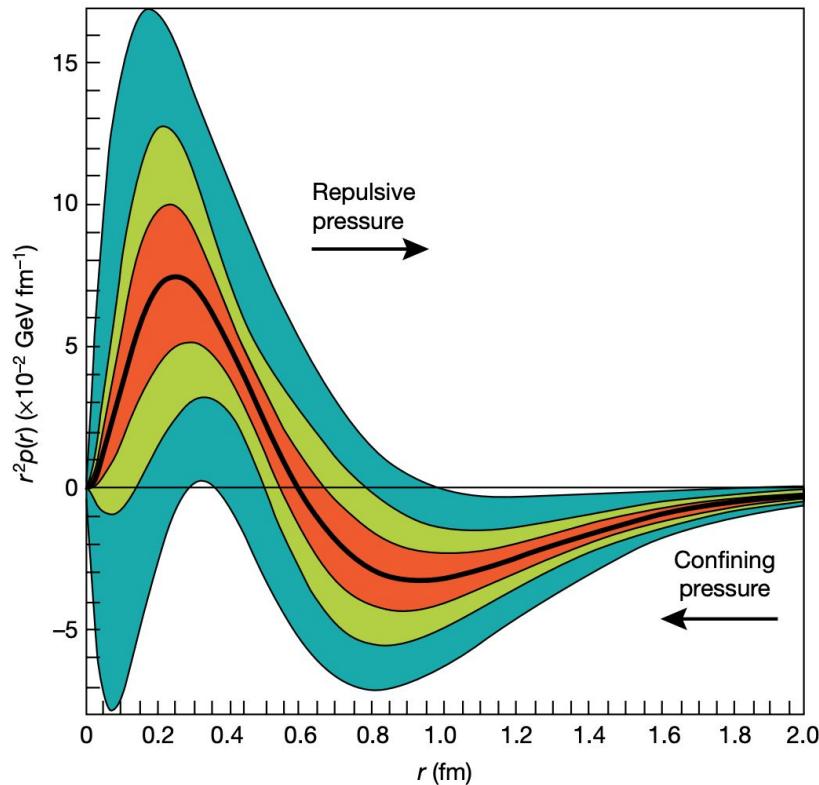
$$\int dx x E(x, \xi, t) = B(t) - \boxed{\xi^2 C(t)}$$

D - term

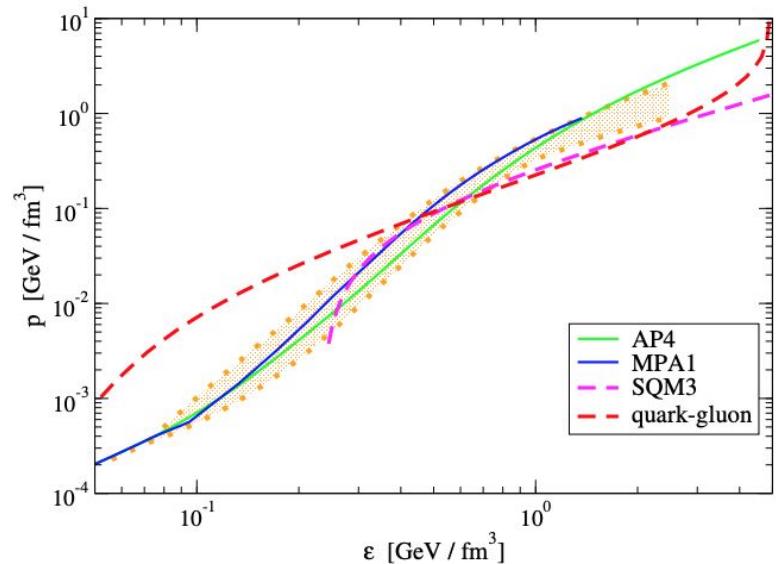
# Mechanical Properties of the Proton

## Pressure Distribution

$$\begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & \boxed{T^{11}} & T^{12} & T^{13} \\ T^{20} & T^{21} & \boxed{T^{22}} & T^{23} \\ T^{30} & T^{31} & T^{32} & \boxed{T^{33}} \end{bmatrix}$$



# Constraints on Neutron Star Equation of State



See Abha's talk!

# Lorentz Invariance Relation (LIR) for OAM

$$\frac{d}{dx} \int d^2 k_T \frac{k_T^2}{M^2} F_{1,4} = H + E + \tilde{E}_{2T} + \mathcal{A}_{F_{14}}$$

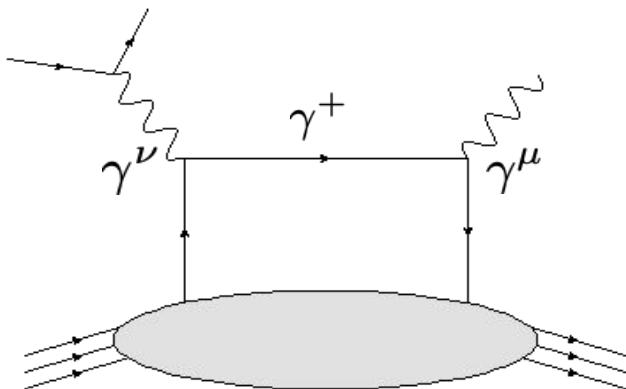
GTMD                                    GPD

Twist-2                                    Twist-3                            LIR  
breaking

# Can we disentangle the Twist-3 GPDs from data?

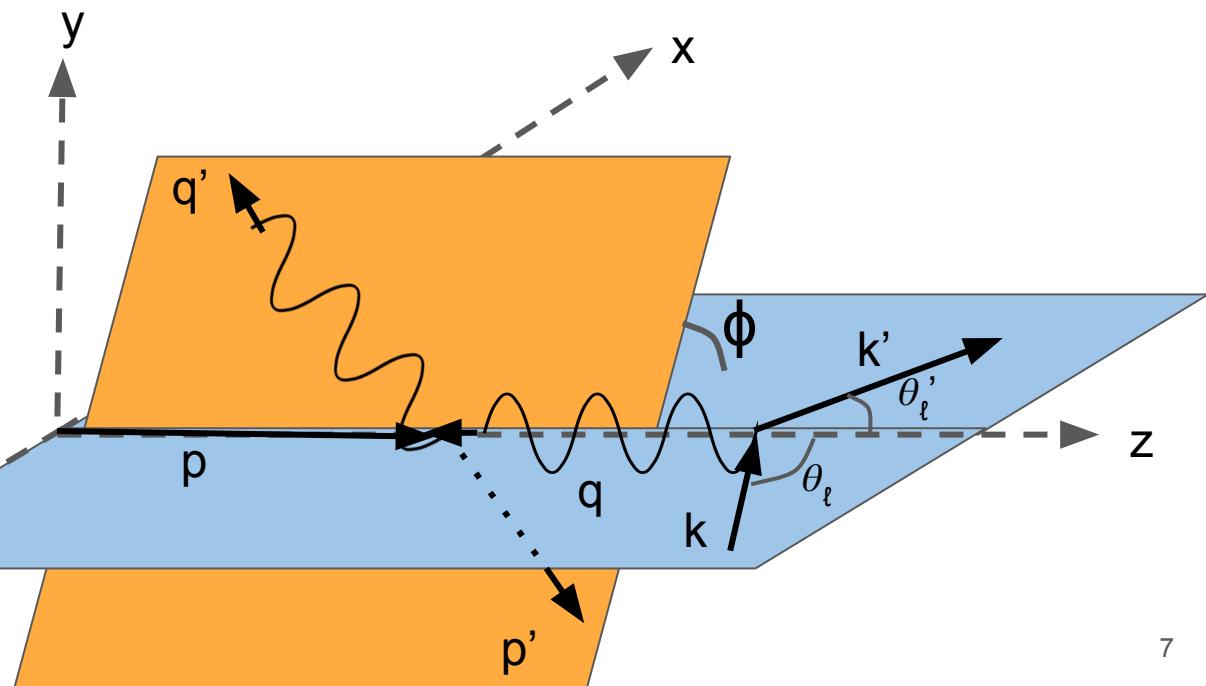
# See Abha's talk!

## DVCS Kinematics



$$x_{Bj} = \frac{Q^2}{2(pq)} \approx \frac{2\xi}{1+\xi}$$

$$Q^2 = -q^2 = -(k - k')^2$$



$$\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_B j dQ^2 d|t| d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\
&= \boxed{\frac{\Gamma}{Q^2(1-\epsilon)} \left\{ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} \right.} \\
&\quad \left. + \sqrt{\epsilon(\epsilon+1)} [\cos \phi F_{UU}^{\cos \phi} + \sin \phi F_{UU}^{\sin \phi}] \right\}} \\
\text{Unpolarized} &\rightarrow \boxed{\dots} \\
&+ (2h) F_{LU} + (2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} + (2h) \sqrt{2\epsilon(1-\epsilon)} \cos \phi F_{LU}^{\cos \phi} \\
\text{LU polarized} &\rightarrow \boxed{\dots} \\
&+ (2\Lambda) \left[ F_{UL} + \sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UL}^{\cos \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} \right] \\
\text{UL polarized} &\rightarrow \boxed{\dots} \\
&+ (2h) \sqrt{1-\epsilon^2} F_{LL} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LL}^{\sin \phi} \\
\text{LL polarized} &\rightarrow \boxed{\dots} \\
&+ |\vec{S}_\perp| \left[ \sin(\phi - \phi_S) (F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}) \right. \\
&\quad \left. + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\
&\quad \left. + \sqrt{2\epsilon(1+\epsilon)} (\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)}) \right] \\
\text{UT polarized} &\rightarrow \boxed{\dots} \\
&+ (2h) |\vec{S}_\perp| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
&\quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \}
\end{aligned}$$

$$\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_B j dQ^2 d|t| d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ \left[ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} \right. \right. \\
&\quad + \sqrt{\epsilon(\epsilon+1)} \left[ \cos \phi F_{UU}^{\cos \phi} + \sin \phi F_{UU}^{\sin \phi} \right] \\
&\quad + (2h) \cancel{F_{LU}} + (2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} + (2h) \sqrt{2\epsilon(1-\epsilon)} \cos \phi \cancel{F_{LU}^{\cos \phi}} \\
&\quad + (2\Lambda) \left[ F_{UL} + \sqrt{\epsilon(\epsilon+1)} \sin \phi \cancel{F_{UL}^{\sin \phi}} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UL}^{\cos \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} \right] \\
&\quad + (2h) \sqrt{1-\epsilon^2} F_{LL} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \sin \phi \cancel{F_{LL}^{\cos \phi}} \\
&\quad + |\vec{S}_\perp| \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\
&\quad \left. \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\
&\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \left( \sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \\
&\quad + (2h) |\vec{S}_\perp| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
&\quad \left. \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \right\}
\end{aligned}$$

$$\begin{aligned} \frac{d^5\sigma_{DVCS}}{dx_B j dQ^2 d|t| d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\ &= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} \right. \end{aligned}$$

Twist - 2

$$\begin{aligned} &+ \sqrt{\epsilon(\epsilon+1)} \left[ \cos \phi F_{UU}^{\cos \phi} + \sin \phi F_{UU}^{\sin \phi} \right] \\ &+ (2h) \cancel{F_{LU}} + (2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} + (2h) \sqrt{2\epsilon(1-\epsilon)} \cos \phi \cancel{F_{LU}^{\cos \phi}} \\ &+ (2\Lambda) \left[ F_{UL} + \sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UL}^{\cos \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} \right. \\ &+ (2h) \sqrt{1-\epsilon^2} \cancel{F_{LL}} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \sin \phi \cancel{F_{LL}^{\cos \phi}} \\ &+ |\vec{S}_\perp| \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ &\quad \left. \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ &\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \left( \sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \\ &+ (2h) |\vec{S}_\perp| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\ &\quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \} \end{aligned}$$

$$\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_B j dQ^2 d|t| d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ \boxed{F_{UU,T}} + \epsilon F_{UU,L} + \epsilon \cos 2\phi \boxed{F_{UU}^{\cos 2\phi}} \right. \\
&\quad + \sqrt{\epsilon(\epsilon+1)} \left[ \cos \phi \boxed{F_{UU}^{\cos \phi}} + \sin \phi \boxed{F_{UU}^{\sin \phi}} \right] \\
&\quad + \cancel{(2h) \boxed{F_{LU}}} + \cancel{(2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi \boxed{F_{LU}^{\sin \phi}}} + \cancel{(2h) \sqrt{2\epsilon(1-\epsilon)} \cos \phi \boxed{F_{LU}^{\cos \phi}}} \\
&\quad + (2\Lambda) \left[ \boxed{F_{UL}} + \sqrt{\epsilon(\epsilon+1)} \sin \phi \boxed{F_{UL}^{\sin \phi}} + \sqrt{\epsilon(\epsilon+1)} \cos \phi \boxed{F_{UL}^{\cos \phi}} + \epsilon \sin 2\phi \boxed{F_{UL}^{\sin 2\phi}} \right. \\
&\quad + \cancel{(2h) \sqrt{1-\epsilon^2} \boxed{F_{LL}}} + \cancel{2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi \boxed{F_{LL}^{\cos \phi}}} + \cancel{2(2h) \sqrt{\epsilon(1-\epsilon)} \sin \phi \boxed{F_{LL}^{\sin \phi}}} \\
&\quad + |\vec{S}_\perp| \left[ \sin(\phi - \phi_S) \left( \boxed{F_{UT,T}^{\sin(\phi-\phi_S)}} + \epsilon \boxed{F_{UT,L}^{\sin(\phi-\phi_S)}} \right) \right. \\
&\quad \left. \left. \epsilon \sin(\phi + \phi_S) \boxed{F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon \sin(3\phi - \phi_S) \boxed{F_{UT}^{\sin(3\phi-\phi_S)}} \right. \right. \\
&\quad \left. \left. + \sqrt{2\epsilon(1+\epsilon)} \left( \sin \phi_S \boxed{F_{UT}^{\sin \phi_S}} + \sin(2\phi - \phi_S) \boxed{F_{UT}^{\sin(2\phi-\phi_S)}} \right) \right] \right. \\
&\quad + (2h) |\vec{S}_\perp| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) \boxed{F_{LT}^{\cos(\phi-\phi_S)}} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S \boxed{F_{LT}^{\cos \phi_S}} \right. \\
&\quad \left. \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) \boxed{F_{LT}^{\cos(2\phi-\phi_S)}} \right] \right\}
\end{aligned}$$

— Twist - 2  
— Twist - 3

$$\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_B dQ^2 d|t| d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ \boxed{F_{UU,T}} + \epsilon \boxed{F_{UU,L}} + \epsilon \cos 2\phi \boxed{F_{UU}^{\cos 2\phi}} \right. \\
&\quad + \sqrt{\epsilon(\epsilon+1)} \left[ \cos \phi \boxed{F_{UU}^{\cos \phi}} + \sin \phi \boxed{F_{UU}^{\sin \phi}} \right] \\
&\quad + \cancel{(2h) \boxed{F_{LU}}} + \cancel{(2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi \boxed{F_{LU}^{\sin \phi}}} + \cancel{(2h) \sqrt{2\epsilon(1-\epsilon)} \cos \phi \boxed{F_{LU}^{\cos \phi}}} \\
&\quad + \cancel{(2\Lambda) \left[ \boxed{F_{UL}} + \sqrt{\epsilon(\epsilon+1)} \sin \phi \boxed{F_{UL}^{\sin \phi}} + \sqrt{\epsilon(\epsilon+1)} \cos \phi \boxed{F_{UL}^{\cos \phi}} + \epsilon \sin 2\phi \boxed{F_{UL}^{\sin 2\phi}} \right]} \\
&\quad + \cancel{(2h) \sqrt{1-\epsilon^2} \boxed{F_{LL}}} + \cancel{2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi \boxed{F_{LL}^{\cos \phi}}} + \cancel{2(2h) \sqrt{\epsilon(1-\epsilon)} \sin \phi \boxed{F_{LL}^{\sin \phi}}} \\
&\quad + |\vec{S}_\perp| \left[ \sin(\phi - \phi_S) \left( \boxed{F_{UT,T}^{\sin(\phi-\phi_S)}} + \epsilon \boxed{F_{UT,L}^{\sin(\phi-\phi_S)}} \right) \right. \\
&\quad \left. \epsilon \sin(\phi + \phi_S) \boxed{F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon \sin(3\phi - \phi_S) \boxed{F_{UT}^{\sin(3\phi-\phi_S)}} \right. \\
&\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \left( \sin \phi_S \boxed{F_{UT}^{\sin \phi_S}} + \sin(2\phi - \phi_S) \boxed{F_{UT}^{\sin(2\phi-\phi_S)}} \right) \right] \\
&\quad + (2h) |\vec{S}_\perp| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) \boxed{F_{LT}^{\cos(\phi-\phi_S)}} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S \boxed{F_{LT}^{\cos \phi_S}} \right. \\
&\quad \left. \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) \boxed{F_{LT}^{\cos(2\phi-\phi_S)}} \right] \right\}
\end{aligned}$$

— Twist - 2  
— Twist - 3  
—  $\mathcal{O}\left(\frac{1}{Q^2}\right)$

# Photon/Proton Structure Functions and Phase

Definition of a Helicity Amplitude



$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta, \phi) = e^{-i(\Lambda_{\gamma^*} - \Lambda - \Lambda' + \Lambda')\phi} \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta),$$

$$e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)})\phi} F_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma^*}^{(2)}} = e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)})\phi} \sum_{\Lambda_{\gamma'}} \left( \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda'_{\gamma}} \right)^* \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(2)}\Lambda'_{\gamma}} = \sum_{\Lambda_{\gamma'}} \left( f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda'_{\gamma}} \right)^* f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(2)}\Lambda'_{\gamma}}$$

Enter the observables in your cross section.

$$e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)} - 2\Lambda)\phi} F_{T,\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma^*}^{(2)}} = \sum_{\Lambda_{\gamma'}} \left( f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda'_{\gamma}} \right)^* f_{-\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(2)}\Lambda'_{\gamma}} = e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)} - 2\Lambda)\phi} \sum_{\Lambda_{\gamma'}} \left( \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda'_{\gamma}} \right)^* \tilde{f}_{-\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(2)}\Lambda'_{\gamma}}$$

# Photon/Proton Structure Functions and Phase

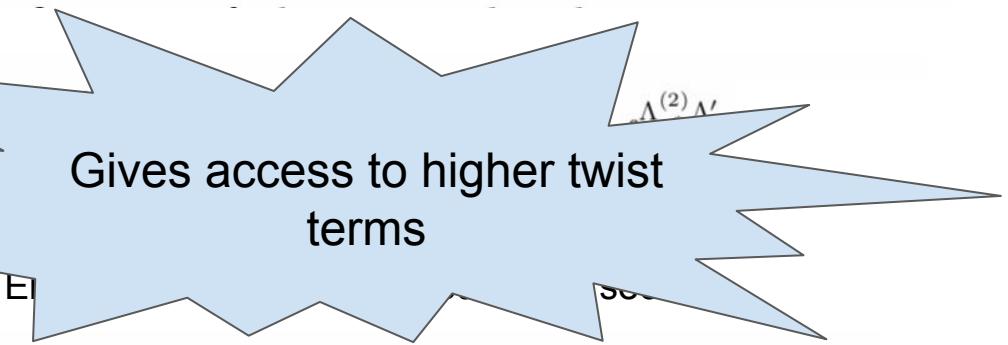
Definition of a Helicity Amplitude



$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta, \phi) = e^{-i(\Lambda_{\gamma^*} - \Lambda - \Lambda'_{\gamma} + \Lambda')\phi} \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta),$$

$$e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)})\phi} F_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma^*}^{(2)}} = e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)})\phi}$$

Gives access to higher twist terms



$$e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)} - 2\Lambda)\phi} F_{T,\Lambda\Lambda'}^{\Lambda_{\gamma^*}^{(1)}\Lambda_{\gamma^*}^{(2)}} =$$

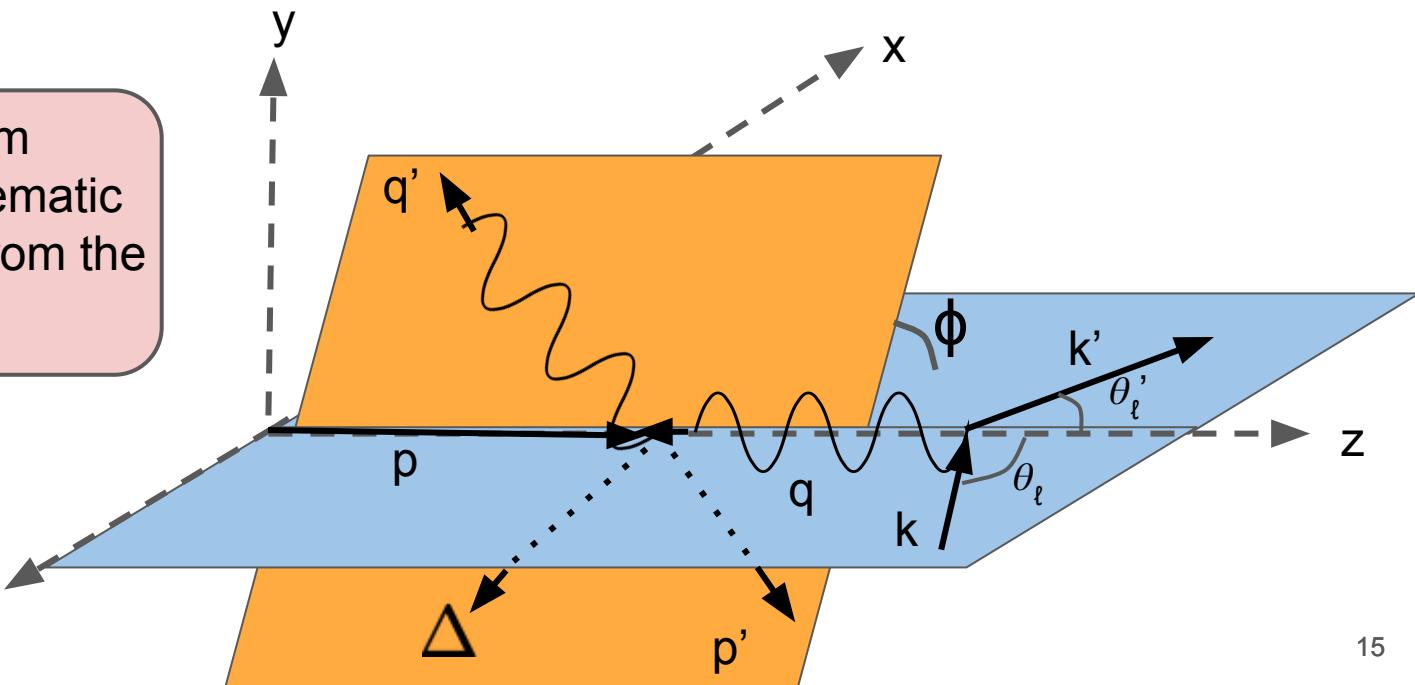
$$\sum_{\Lambda_{\gamma'}} \left( f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}} \right)^* f_{-\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}} = e^{i(\Lambda_{\gamma^*}^{(1)} - \Lambda_{\gamma^*}^{(2)} - 2\Lambda)\phi} \sum_{\Lambda_{\gamma'}} \left( \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}} \right)^* \tilde{f}_{-\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}$$

## Kinematic Dependence

$$(k\Delta) \propto \cos \phi$$

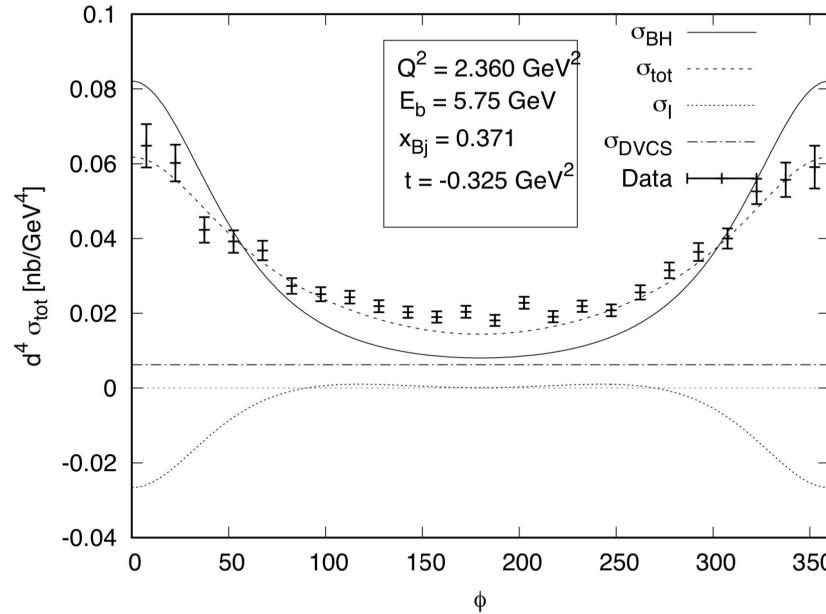
Not to be confused with the phase arising from the helicity amplitude which determines the Twist.

Our formalism disentangles kinematic phi-dependence from the phase.



# DVCS Observables

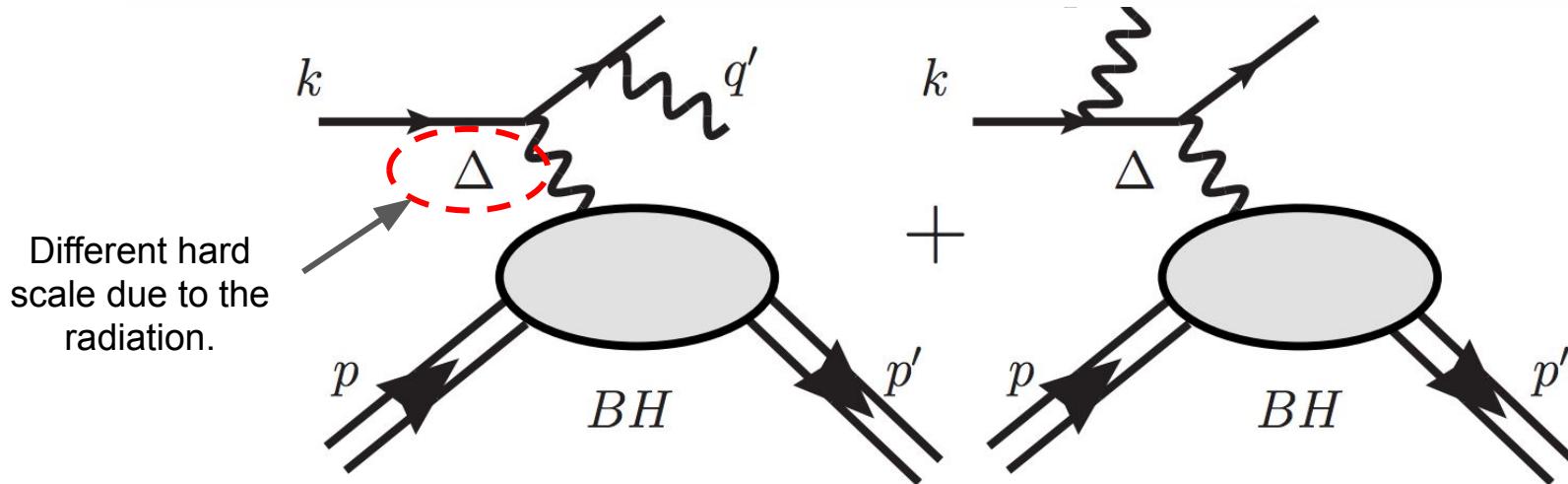
$$|T|^2 = |T_{\text{BH}} + T_{\text{DVCS}}|^2 = |T_{\text{BH}}|^2 + |T_{\text{DVCS}}|^2 - \mathcal{I} .$$



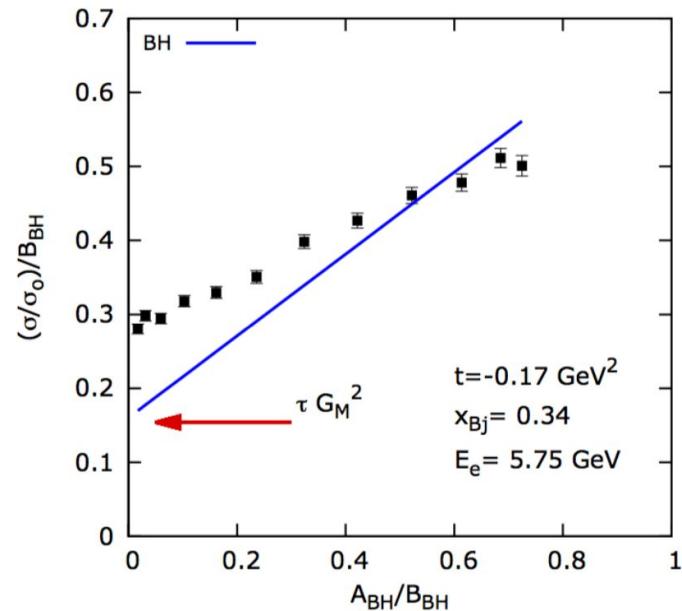
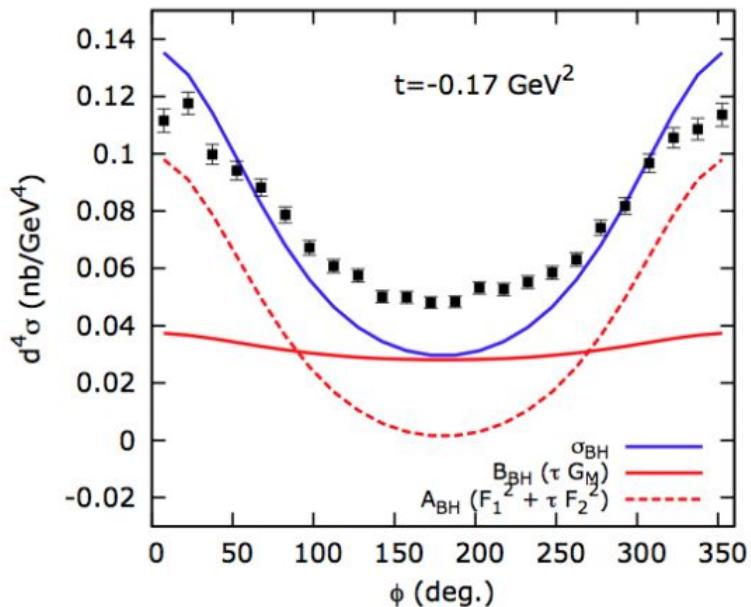
$$F_{UU,T} = 4 \left[ (1 - \xi^2) \left( |\mathcal{H}|^2 + |\tilde{\mathcal{H}}|^2 \right) + \frac{t_o - t}{2M^2} \left( |\mathcal{E}|^2 + \xi^2 |\tilde{\mathcal{E}}|^2 \right) - \frac{2\xi^2}{1 - \xi^2} \operatorname{Re} \left( \mathcal{H}\mathcal{E} + \tilde{\mathcal{H}}\tilde{\mathcal{E}} \right) \right]$$

## Bethe-Heitler

$$|T|^2 = |T_{\text{BH}} + T_{\text{DVCS}}|^2 = |T_{\text{BH}}|^2 + |T_{\text{DVCS}}|^2 + \mathcal{I}.$$



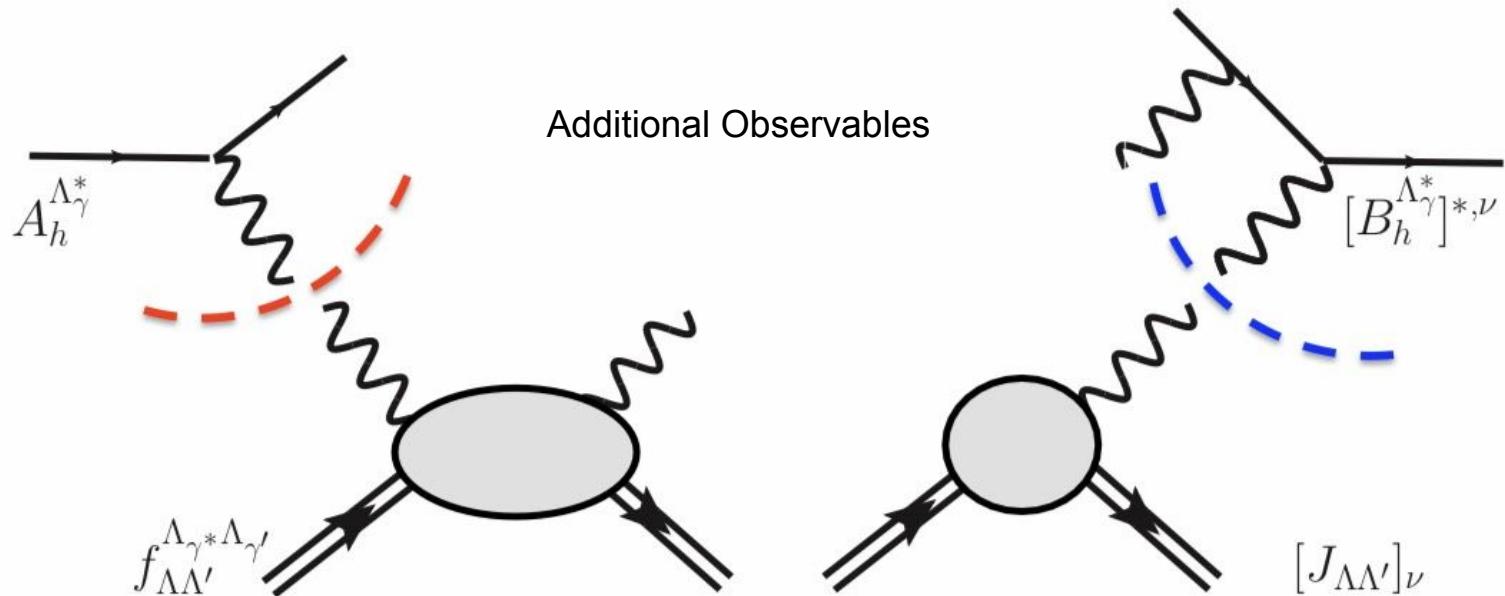
$$\frac{d^5\sigma_{unpol}^{BH}}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} = \frac{\Gamma}{t^2} \left[ A_{BH}(F_1^2 + \tau F_2^2) + B_{BH}\tau G_M^2 \right]$$



## DVCS/BH Interference

$$|T|^2 = |T_{\text{BH}} + T_{\text{DVCS}}|^2 = |T_{\text{BH}}|^2 + |T_{\text{DVCS}}|^2 + \mathcal{I}.$$

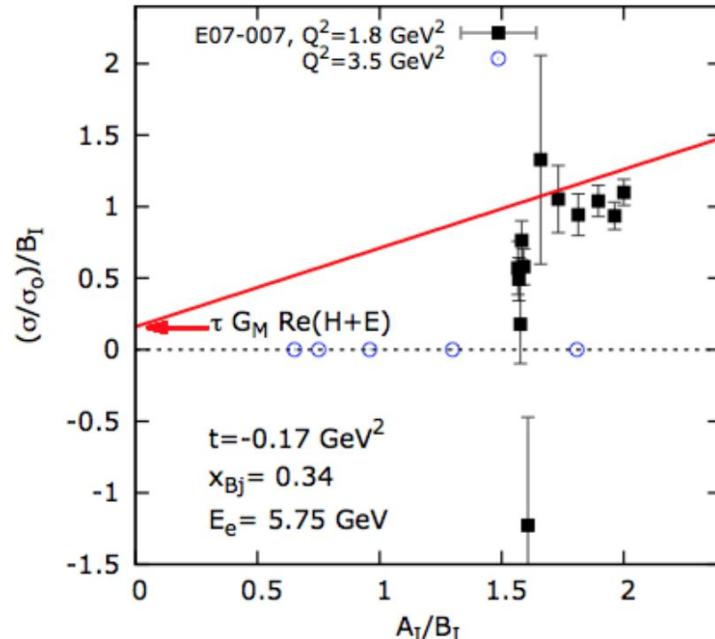
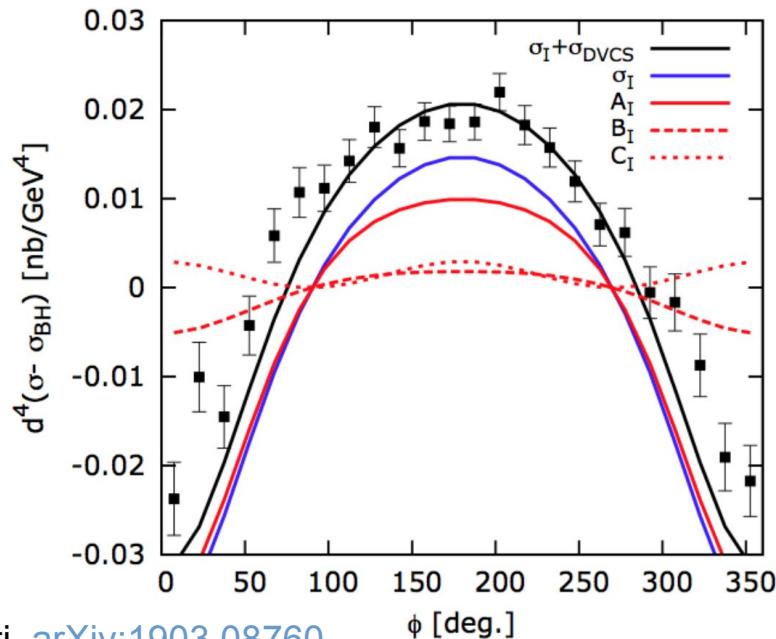
$$\mathcal{I} = T_{\text{BH}}^* T_{\text{DVCS}} + T_{\text{DVCS}}^* T_{\text{BH}}.$$



# Rosenbluth-like Separation

$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re(\mathcal{H}) + \tau F_2 \mathcal{E}$$

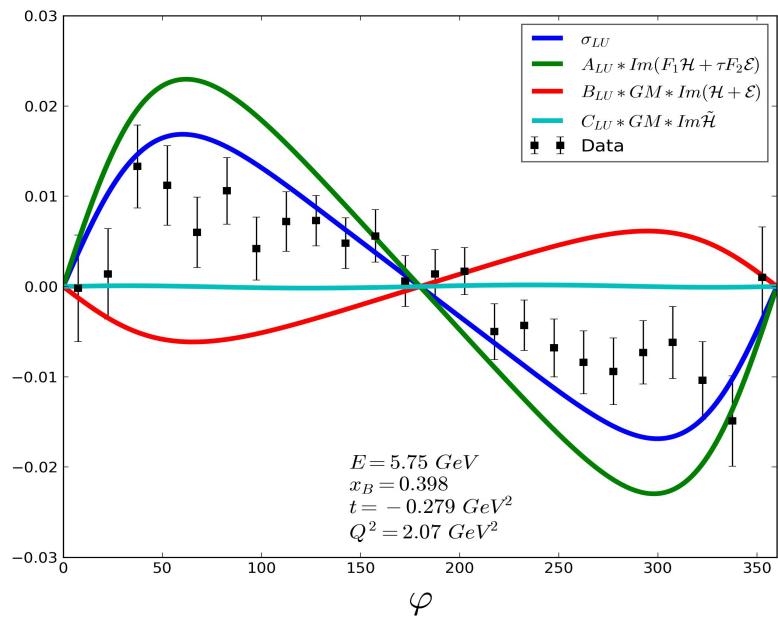
Angular Momentum



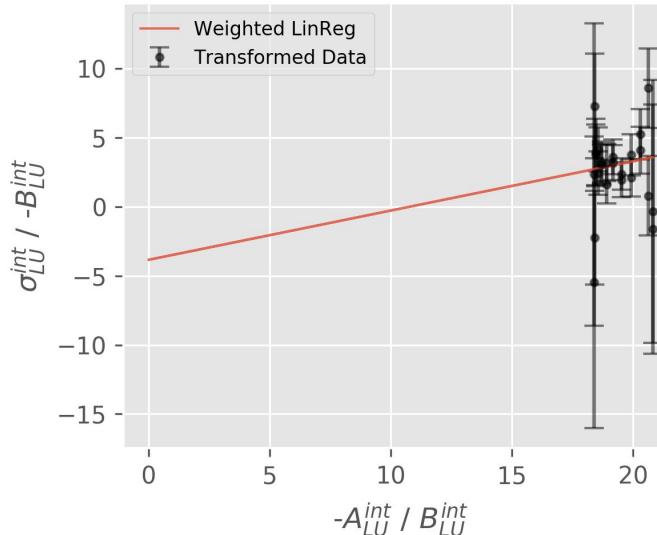
# Preliminary

$$F_{LU}^{\mathcal{I},tw2} = A_{LU}^{\mathcal{I}} \Im m \left( F_1 \mathcal{H} + \tau F_2 \mathcal{E} \right) + B_{LU}^{\mathcal{I}} G_M \Im m (\mathcal{H} + \mathcal{E}) + C_{LU}^{\mathcal{I}} G_M \Im m \tilde{\mathcal{H}}$$

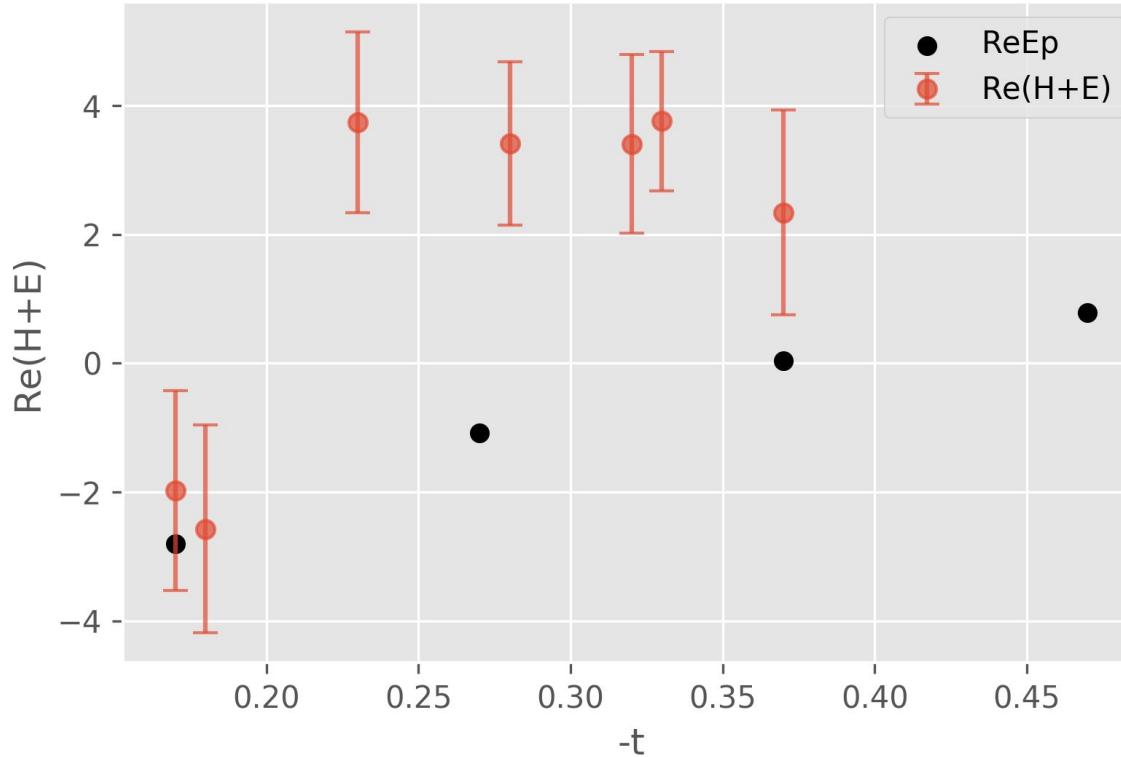
Angular Momentum



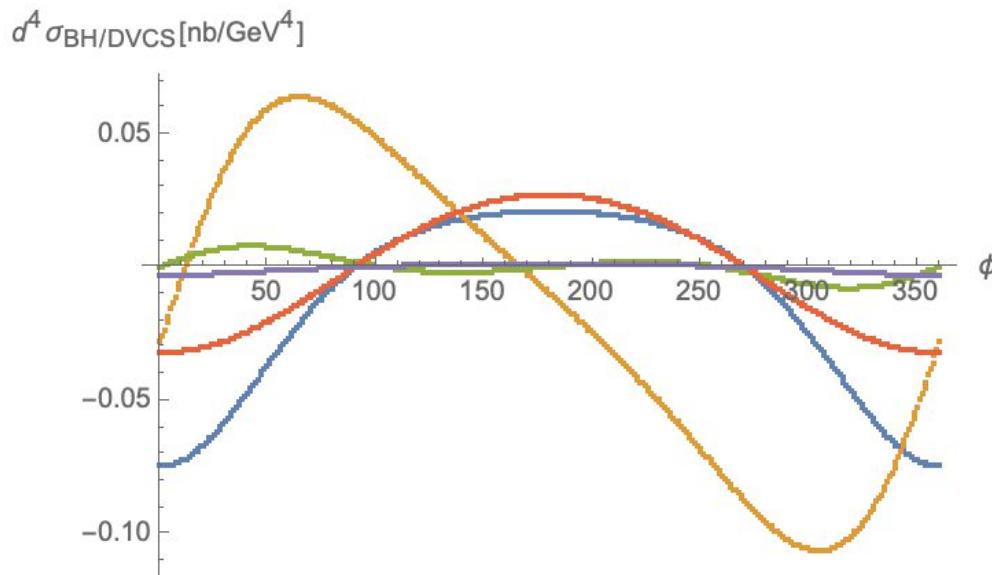
Interference Rosenbluth Separation



# Extraction of Compton Form Factors



## Size of Tw-3 Cross Section (Preliminary)



- $A_{UU} \text{Re}(F_1 \mathcal{H} + \tau F_2 \mathcal{E})$
- $A_{UL} \text{Im}(F_1 (\tilde{\mathcal{H}} - \xi \tilde{\mathcal{E}}) + \tau F_2 \tilde{\mathcal{E}})$
- $A_{LU} \text{Im}(F_1 \mathcal{H} + \tau F_2 \mathcal{E})$
- $A_{LL} \text{Re}(F_1 (\tilde{\mathcal{H}} - \xi \tilde{\mathcal{E}}) + \tau F_2 \tilde{\mathcal{E}})$
- $A_{UU}^{\text{tw3}} \text{Re}(F_1 (2\tilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) + F_2 (\mathcal{H}_{2T} + \tau \tilde{\mathcal{H}}_{2T}))$

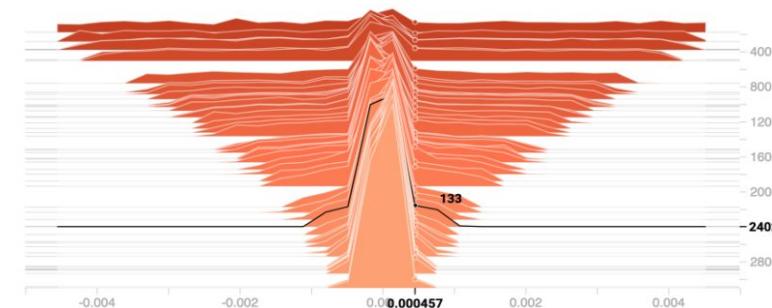
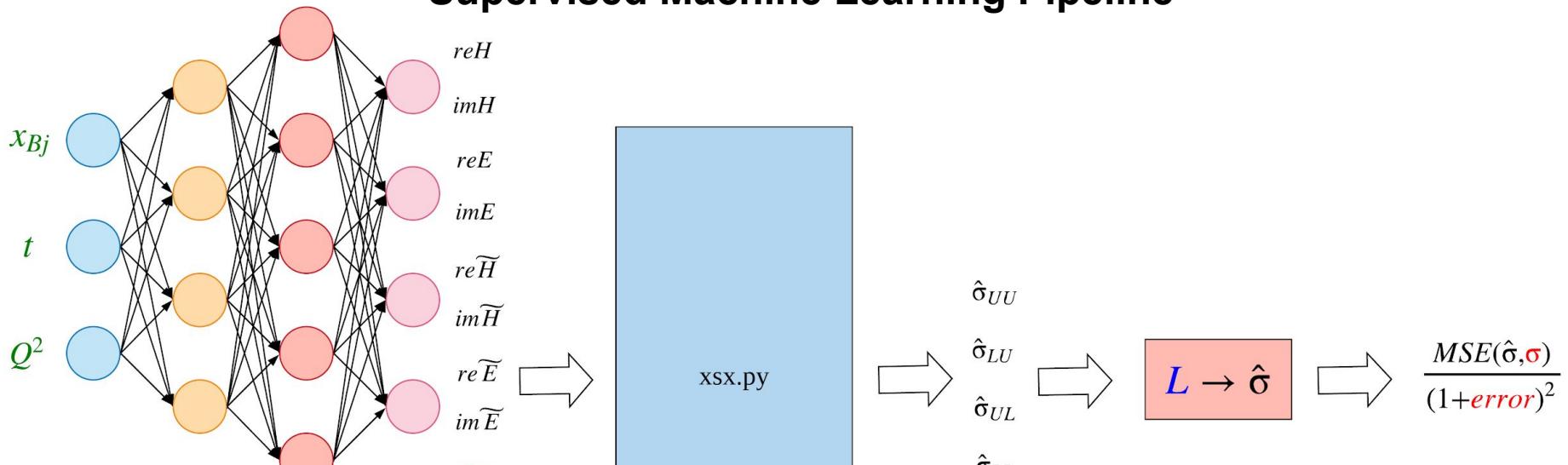


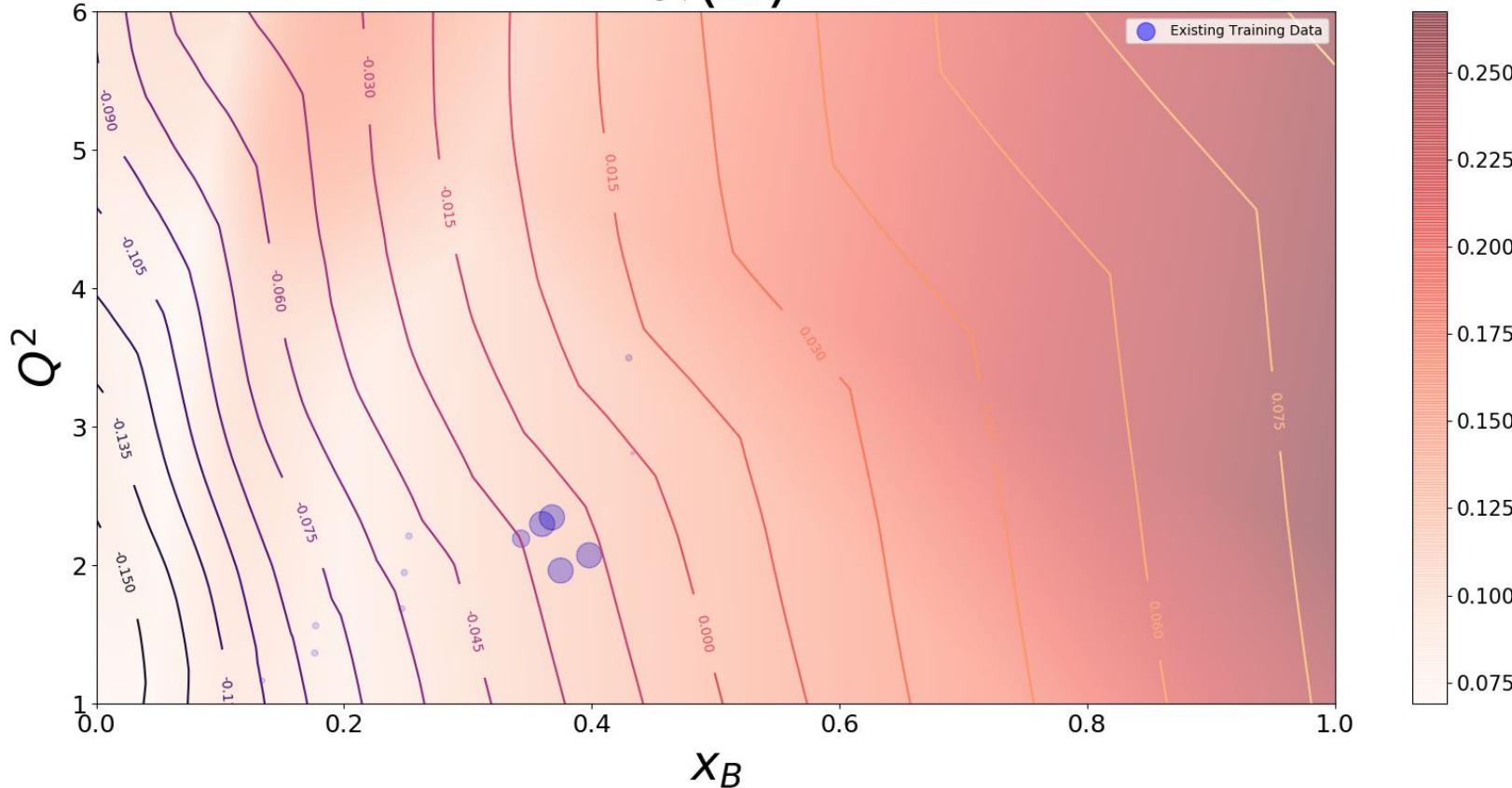
# Summer Institute for Wigner Imaging and Femtography



$x_{Bj}, t, Q^2, k\theta, \varphi, L, \sigma, error$ 

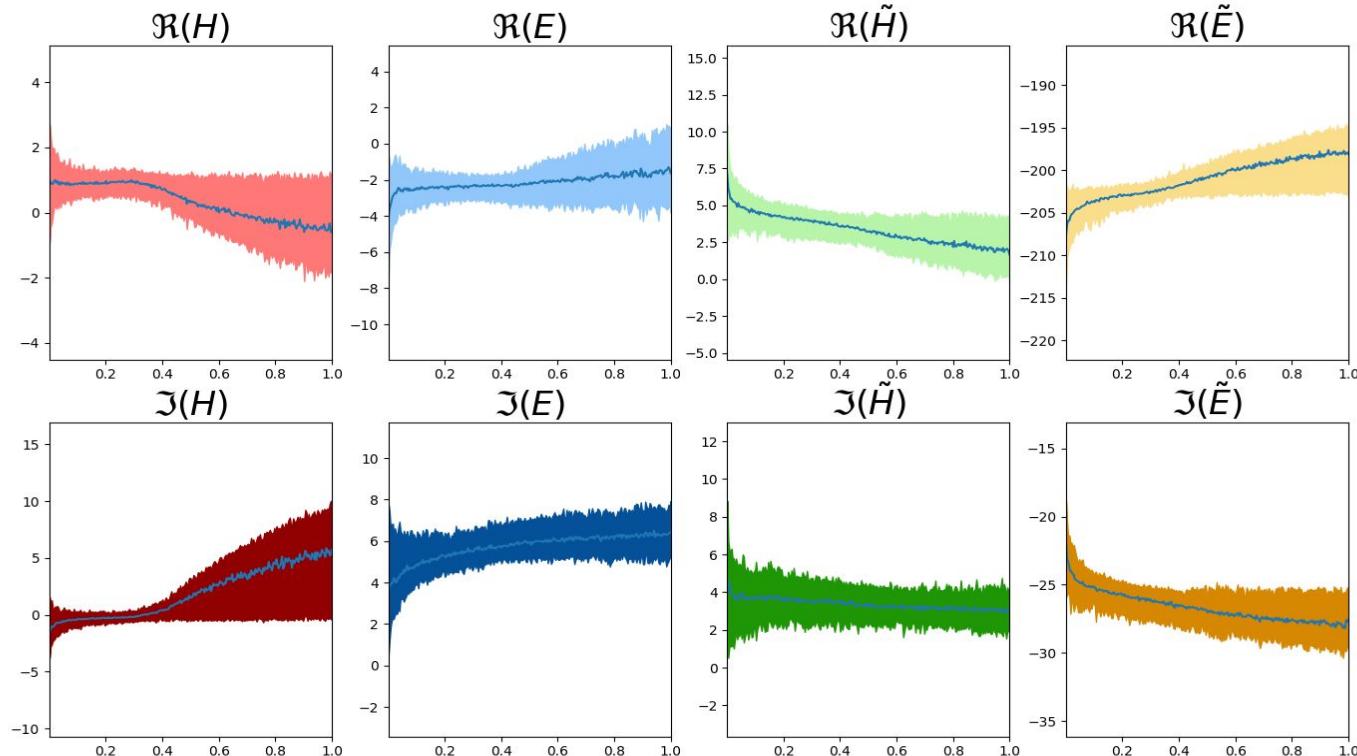
## Supervised Machine Learning Pipeline



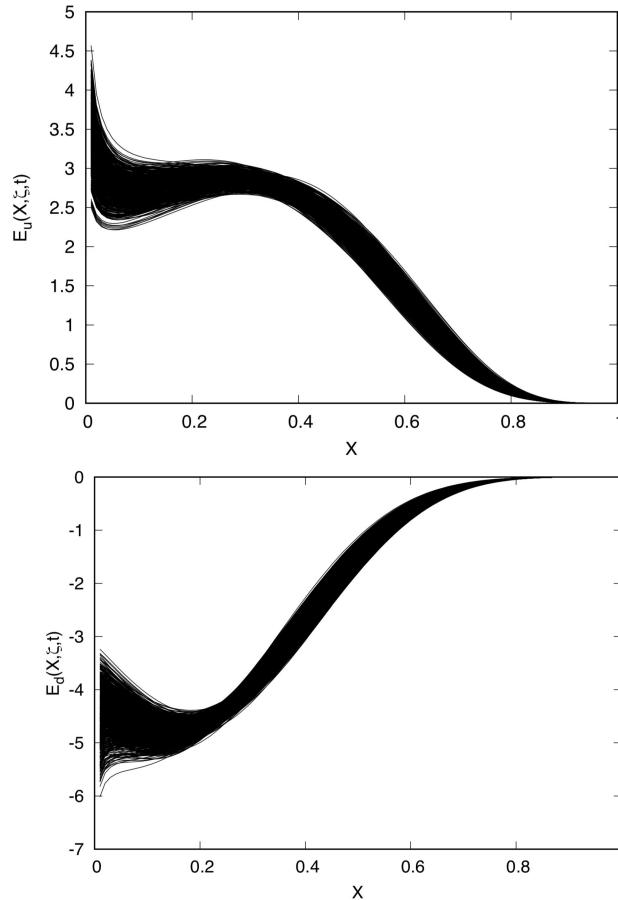
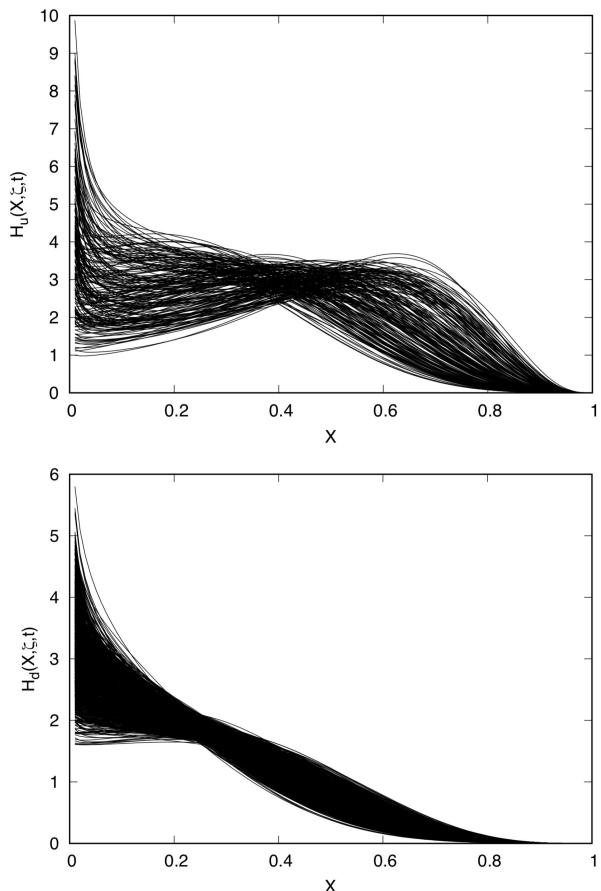
$\Re(H)$  $t = -0.28$ 

# Preliminary

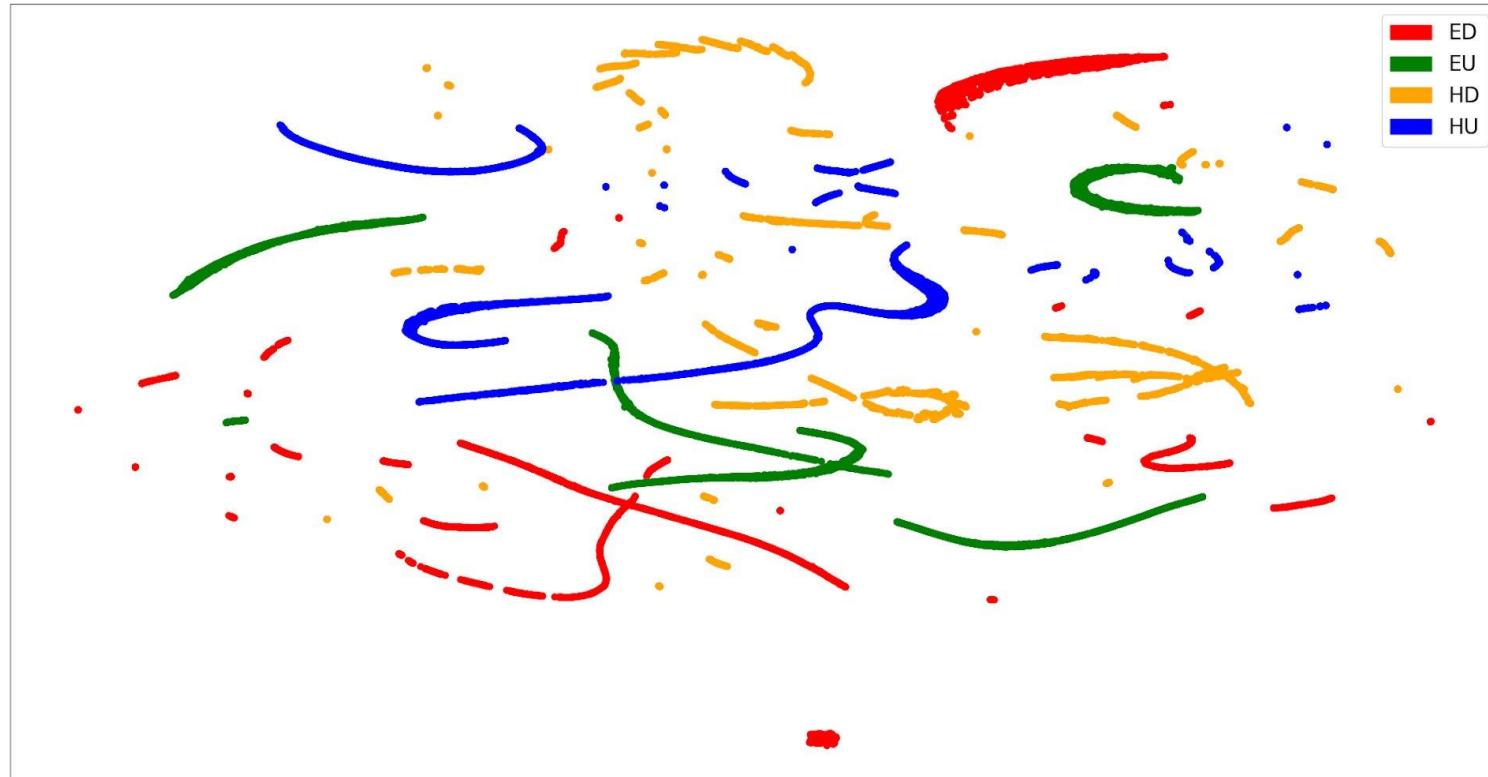
## $x_B$ Dependence



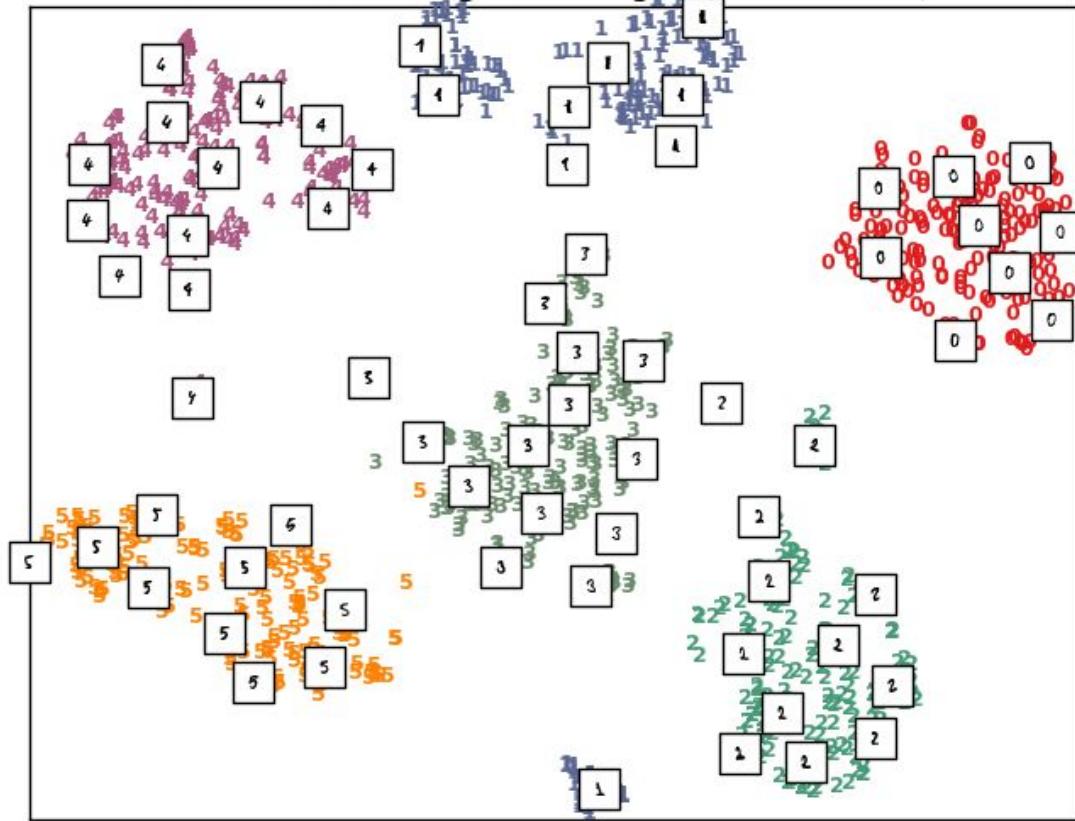
# GPD Envelopes



## UMAP GPD Clustering



t-SNE embedding of the digits (time 19.35s)



## DVCS from Spin-1 Deuteron

Angular momentum sum rule for a spin-½ nucleon.

$$J_q = \frac{1}{2} \int dx x \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right]$$

Using the Spin-1 Energy Momentum Tensor, a similar formula has been developed for the Spin-1 deuteron

$$J_q = \frac{1}{2} \int dx x H_2^q(x, 0, 0)$$

Can we identify the observable for the Spin-1 Angular Momentum from DVCS off a spin-1 Deuteron formalism?

## DVCS from Spin-1 Deuteron

$$2 \int dxx [H_1(x, \xi, t) - \frac{1}{2} H_5(x, \xi, t)] = \mathcal{G}_1(t) + \boxed{\xi^2 \mathcal{G}_3(t)}$$

$$2 \int dxx H_3(x, \xi, t) = \mathcal{G}_2(t) + \boxed{\xi^2 \mathcal{G}_4(t)}$$

$$-4 \int dxx H_4(x, \xi, t) = \xi \mathcal{G}_6(t)$$

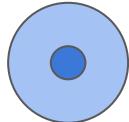
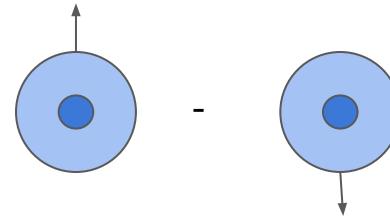
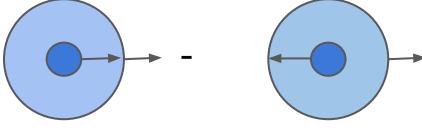
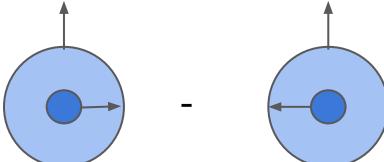
$$\int dxx H_5(x, \xi, t) = -\frac{t}{8M_D^2} \mathcal{G}_6(t) + \frac{1}{2} \mathcal{G}_7(t)$$

## Summary

- Complete covariant description of DVCS cross section and its background process
- Rosenbluth-like Separation used to determine CFFs
- Supervised Machine Learning effort used to determine CFFs
- UMAP used to create correlations between GPDs
- Calculations of Polarized and Unpolarized Cross Sections
- Calculation of Twist-3 Coefficients

Thank you!

# Backup Slides

GPD	Phase	Helicity Composition
$H$	1	
$\Delta_T E$	$e^{i\phi}$	
$\tilde{H}$	1	
$\xi \Delta_T \tilde{E}$	$e^{i\phi}$	

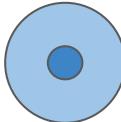
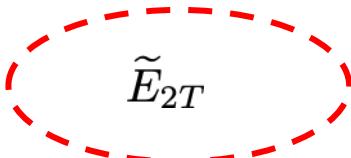
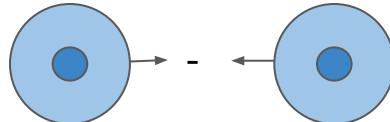
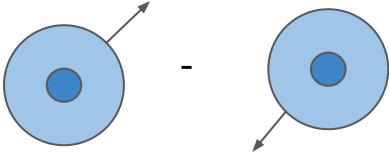
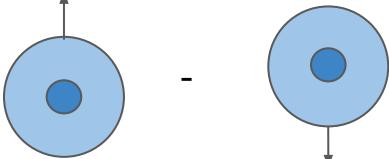
## Twist-3 Observables

$$\begin{aligned}
 F_{UU}^{\cos \phi} = & -2(1-\xi^2)\Re\left[\left(2\tilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\tilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\right)\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)\right. \\
 & - 2\xi\left(\tilde{\mathcal{E}}_{2T} + \tilde{\mathcal{E}}'_{2T}\right)\left(\tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\right) + \frac{t_0-t}{16M^2}\left(\tilde{\mathcal{H}}_{2T} + \tilde{\mathcal{H}}'_{2T}\right)\left(\mathcal{E} + \xi\tilde{\mathcal{E}}\right) \\
 & + \left(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0-t}{4M^2}\left(\tilde{\mathcal{H}}_{2T} + \tilde{\mathcal{H}}'_{2T}\right) + \frac{\xi}{1-\xi^2}\left(\tilde{\mathcal{E}}_{2T} + \tilde{\mathcal{E}}'_{2T}\right)\right. \\
 & \left.\left.- \frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\right)\right)\left(\mathcal{E} - \xi\tilde{\mathcal{E}}\right)\right]
 \end{aligned}$$

What are these linear combinations of CFFs?

$$\begin{aligned}
 F_{LU}^{\sin \phi} = & -2(1-\xi^2)\Im\left[\left(2\tilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\tilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\right)\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)\right. \\
 & - 2\xi\left(\tilde{\mathcal{E}}_{2T} + \tilde{\mathcal{E}}'_{2T}\right)\left(\tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\tilde{\mathcal{E}}\right) + \frac{t_0-t}{16M^2}\left(\tilde{\mathcal{H}}_{2T} + \tilde{\mathcal{H}}'_{2T}\right)\left(\mathcal{E} + \xi\tilde{\mathcal{E}}\right) \\
 & + \left[\left(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0-t}{4M^2}\left(\tilde{\mathcal{H}}_{2T} + \tilde{\mathcal{H}}'_{2T}\right) + \frac{\xi}{1-\xi^2}\left(\tilde{\mathcal{E}}_{2T} + \tilde{\mathcal{E}}'_{2T}\right)\right. \\
 & \left.\left.- \frac{\xi^2}{1-\xi^2}\left(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\right)\right)\left(\mathcal{E} - \xi\tilde{\mathcal{E}}\right)\right]
 \end{aligned}$$

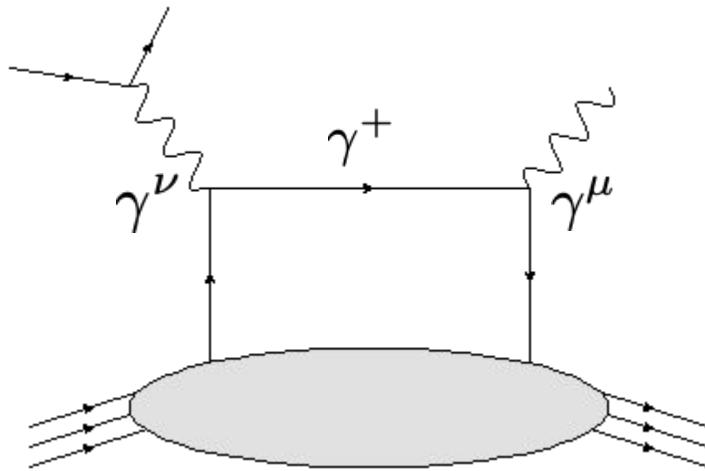
Get access to 8 Compton Form Factors from DVCS alone.

	GPD	Phase	Helicity Composition
$H$	$2\tilde{H}_{2T} + E_{2T}$	$e^{i\phi}$	
OAM		$e^{i\phi}$	
T OAM	$\tilde{H}_{2T}$	$e^{2i\phi}$	
$E$	$H_{2T} + \frac{\Delta_T^2}{4M^2}\tilde{H}_{2T}$	1	

GPD	Phase	Helicity Composition
$\tilde{H}$	$\tilde{E}'_{2T}$	$e^{i\phi}$
Spin Orbit	$E'_{2T} + 2\tilde{H}'_{2T}$	$e^{i\phi}$
T Spin Orbit	$\tilde{H}'_{2T}$	$e^{2i\phi}$
$\tilde{E}$	$H'_{2T} + \frac{\Delta_T^2}{4M^2}\tilde{H}'_{2T}$	1

# DVCS

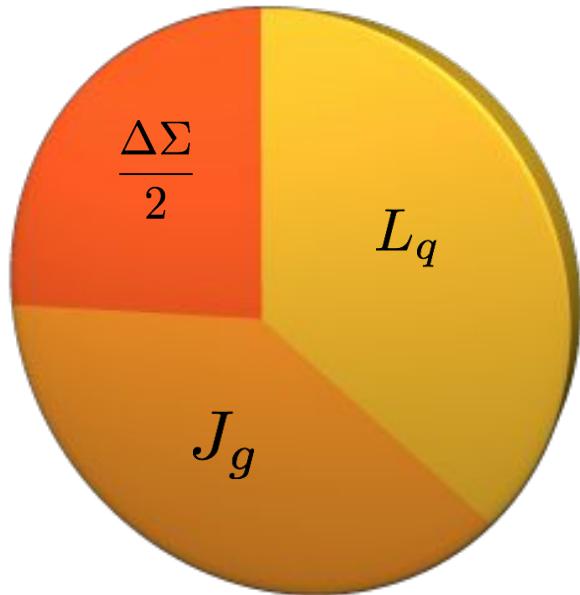
 Twist - 2  
 Twist - 3



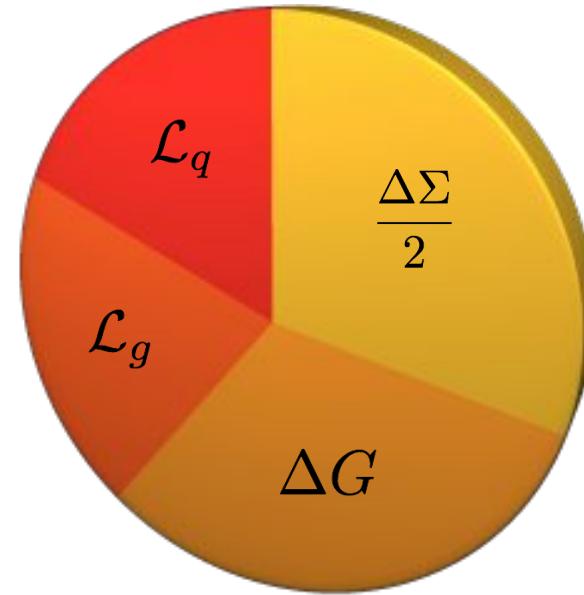
$$W^{\mu\nu} \propto \gamma^\mu \gamma^+ \gamma^\nu = \begin{bmatrix} \gamma^- & \gamma^1 - i\gamma^2 \gamma_5 & \gamma^2 + i\gamma^1 \gamma_5 & i\gamma^- \gamma_5 \\ \gamma^1 + i\gamma^2 \gamma_5 & \gamma^+ & i\gamma^+ \gamma_5 & -\gamma^1 - i\gamma^2 \gamma_5 \\ \gamma^2 - i\gamma^1 \gamma_5 & -i\gamma^+ \gamma_5 & \gamma^+ & -\gamma^2 + i\gamma^1 \gamma_5 \\ -i\gamma^- \gamma_5 & -\gamma^1 + i\gamma^2 \gamma_5 & -\gamma^2 - i\gamma^1 \gamma_5 & \gamma^- \end{bmatrix}$$

## How to define OAM?

Ji



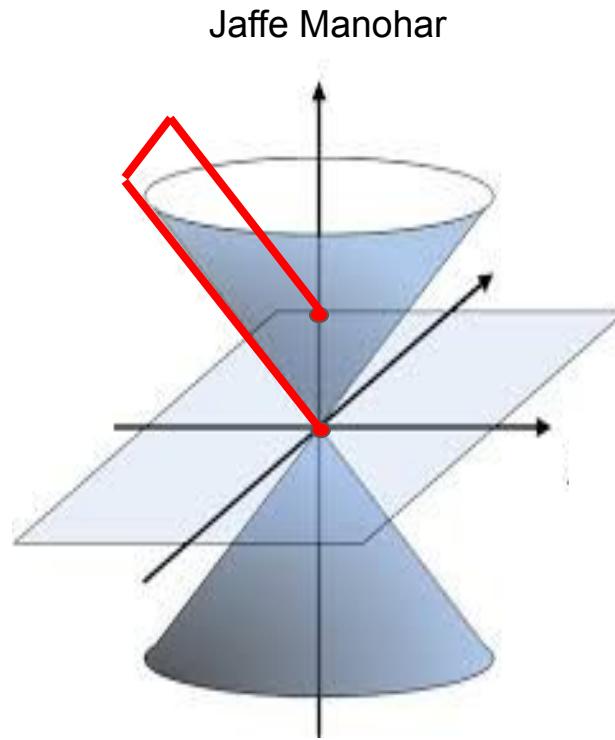
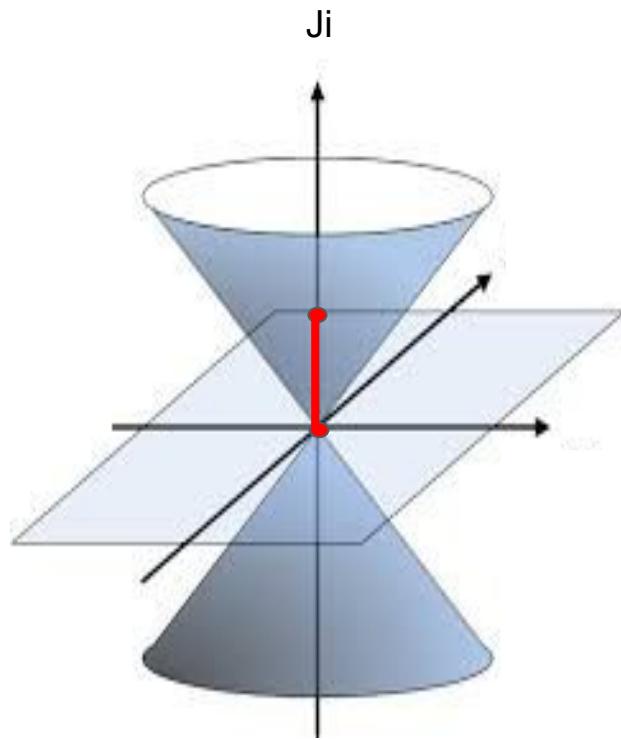
Jaffe Manohar



$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + L_q + J_g$$

$$\frac{1}{2} = \frac{\Delta\Sigma}{2} + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

## How to define OAM?



$$L_q = \int d^3r \langle P', \Lambda' | \bar{\psi} \gamma^+ (\vec{r} \times i\vec{D}) \psi | P, \Lambda \rangle$$

$$\mathcal{L}_q = \int d^3r \langle P', \Lambda' | \bar{\psi} \gamma^+ (\vec{r} \times i\vec{\partial}) \psi | P, \Lambda \rangle_{42}$$

How do we describe the orbital angular momentum of the partons?



$$\vec{L} = \vec{r} \times \vec{p}$$

Classically

$$L_z^q = - \left( k_T \times b_T \right)_z^q$$

Partonic

$b_T$  Relative average transverse position from the center of momentum of the system

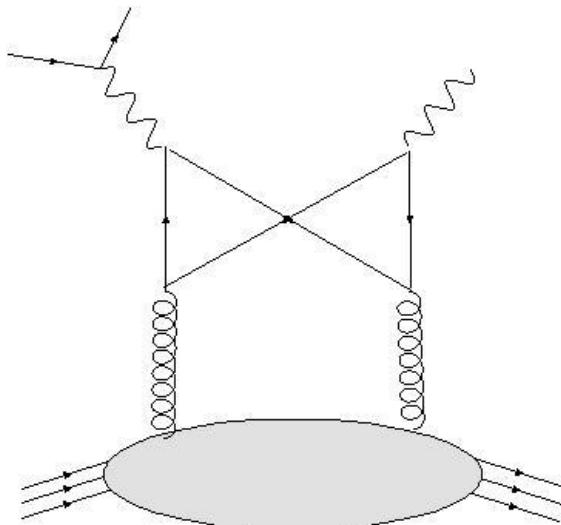
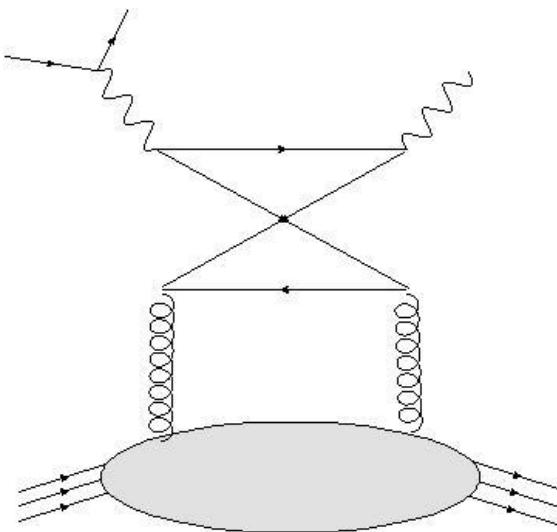
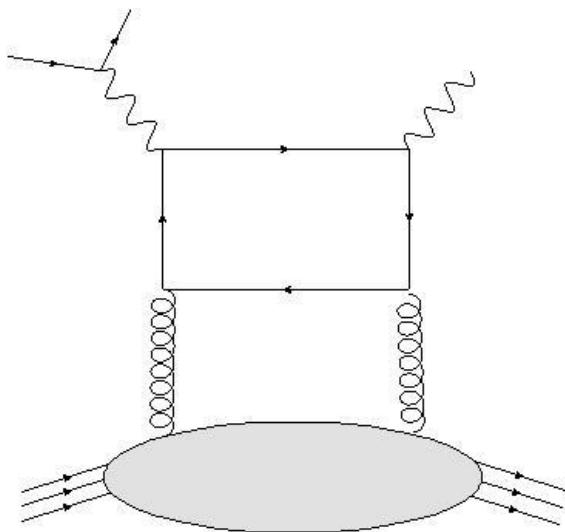
$k_T$  Relative average transverse momentum

$$l_z^q = \int dx d^2 k_T d^2 b_T \left( b_T \times k_T \right)_z^q \rho^{[\gamma^+]}(b_T, k_T, x)$$

$$l_z^q = - \int dx d^2 k_T \frac{k_T^2}{M^2} F_{1,4}^q$$



# A glimpse at the gluons



# Gluon GPD Observables

$$\begin{aligned}
F_{UL}^{\sin 2\phi} = & -2 \frac{\alpha_S}{2\pi} \sum_q e_q^2 \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \Im m \left[ \sqrt{1-\xi^2} \left( \tilde{\mathcal{H}}_T^g + (1-\xi) \frac{\mathcal{E}_T^g + \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{H} + \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2} (\mathcal{E} + \tilde{\mathcal{E}}) \right)^* \right. \\
& + \sqrt{1-\xi^2} \left( \tilde{\mathcal{H}}_T^g + (1+\xi) \frac{\mathcal{E}_T^g - \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{H} - \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2} (\mathcal{E} + \tilde{\mathcal{E}}) \right)^* \\
& + \frac{\sqrt{t_0-t}}{2M} \left( \tilde{\mathcal{H}}_T^g + (1+\xi) \frac{\mathcal{E}_T^g - \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{E} + \xi \tilde{\mathcal{E}} \right)^* \\
& \left. - \sqrt{1-\xi^2} \left( \mathcal{H}_T^g + \frac{t_0-t}{M^2} \tilde{\mathcal{H}}_T^g - \frac{\xi^2}{1-\xi^2} \mathcal{E}_T^g + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T^g \right) \left( \mathcal{E} - \xi \tilde{\mathcal{E}} \right)^* \right]
\end{aligned}$$

$$\begin{aligned}
F_{UT}^{\sin(3\phi - \phi_S)} = & -4 \frac{\alpha_S}{2\pi} \sum_q e_q^2 \sqrt{1-\xi^2} \frac{\sqrt{t_0-t}^3}{8M^3} \Im m \left[ (1-\xi^2) \left( \tilde{\mathcal{H}}_T^g \right) \left( \mathcal{H} - \tilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2} (\mathcal{E} - \tilde{\mathcal{E}}) \right)^* \right. \\
& \left. + \left( \tilde{\mathcal{H}}_T^g + (1-\xi) \frac{\mathcal{E}_T^g + \tilde{\mathcal{E}}_T^g}{2} \right) \left( \mathcal{E} - \xi \tilde{\mathcal{E}} \right)^* \right]
\end{aligned}$$

	PDFs	FFs
GPD	$\Delta \rightarrow 0$	$\int_{-1}^1 dx$
$H$	$q(x)$	$F_1$
$E$	0	$F_2$
$\tilde{H}$	$\Delta q(x)$	$g_A$
$\tilde{E}$	0	$g_P$

# $\mathcal{O}\left(\frac{1}{Q^2}\right)$ Observables

$$F_{UU,L} = \frac{1}{(P^+)^2} \left[ 2 \left| \tilde{\mathcal{H}}'_{2T} - \tilde{\mathcal{H}}_{2T} \right|^2 + \frac{(1+\xi)^2}{4} \left| \mathcal{E}'_{2T} - \mathcal{E}_{2T} + \tilde{\mathcal{E}}_{2T} - \tilde{\mathcal{E}}'_{2T} \right|^2 \right. \\ \left. + \frac{(1-\xi)^2}{4} \left| \mathcal{E}'_{2T} - \mathcal{E}_{2T} - \tilde{\mathcal{E}}_{2T} + \tilde{\mathcal{E}}'_{2T} \right|^2 + (1+\xi) \Re e \left( \tilde{\mathcal{H}}'_{2T} - \tilde{\mathcal{H}}_{2T} \right) \left( \mathcal{E}'_{2T} - \mathcal{E}_{2T} \right) \right. \\ \left. + (1+\xi) \Im m \left( \tilde{\mathcal{H}}'_{2T} - \tilde{\mathcal{H}}_{2T} \right) \left( \mathcal{E}'_{2T} - \mathcal{E}_{2T} \right) \right]$$

The  $\mathcal{O}\left(\frac{1}{Q^2}\right)$  observables are constructed from Twist-3 GPDs. This is because the DVCS process cannot access the Twist-4 sector of the hadronic tensor.

## Twist-3 Coefficients

$$\begin{aligned} F_{UU}^{\mathcal{I}, tw3} = & \Re e \left\{ A_{UU}^{(3)\mathcal{I}} \left[ F_1(2\tilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) + F_2(\mathcal{H}_{2T} + \tau\tilde{\mathcal{H}}_{2T}) \right] \right. \\ & \left. + B_{UU}^{(3)\mathcal{I}} G_M \tilde{E}_{2T} + C_{UU}^{(3)\mathcal{I}} G_M \left[ 2\xi H_{2T} - \tau(\tilde{E}_{2T} - \xi E_{2T}) \right] \right\} \\ & + \Im m \left\{ \tilde{A}_{UU}^{(3)\mathcal{I}} \left[ F_1(2\tilde{H}'_{2T} + E'_{2T}) + F_2(H'_{2T} + \tau\tilde{H}'_{2T}) \right] \right. \\ & \left. + \tilde{B}_{UU}^{(3)\mathcal{I}} G_M \tilde{E}'_{2T} + \tilde{C}_{UU}^{(3)\mathcal{I}} G_M \left[ 2\xi H'_{2T} - \tau(\tilde{E}'_{2T} - \xi E'_{2T}) \right] \right\} \end{aligned}$$