Spin content of the proton at NNLO and beyond

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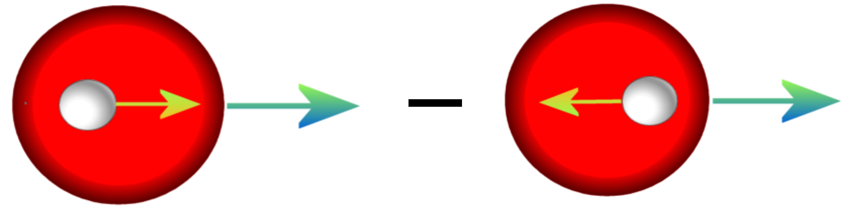
Berkeley, 09/18/2019

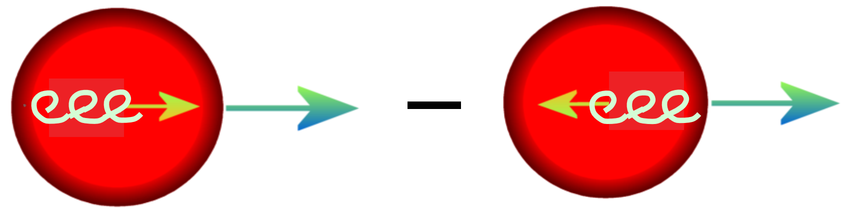
Outline:

- Introduction
- Evolution equations
- Singlet case
- Non-singlet case

Collaboration with D. de Florian (PRD 2019)

Introduction

$\Delta q(x) =$ 

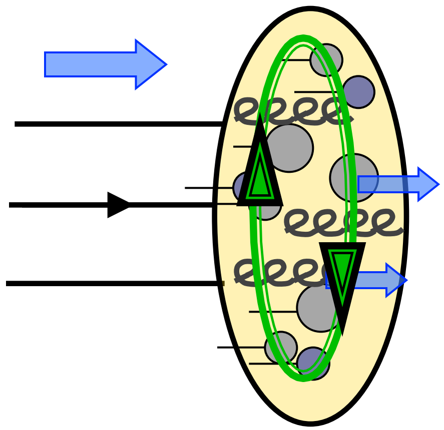
$\Delta g(x) =$ 

$$\Delta q(x) = \frac{1}{4\pi} \int dy^- \, \mathrm{e}^{-iy^- xP^+} \langle P, S \,|\, \bar{\psi}\,(0, y^-, \mathbf{0}_\perp)\; \gamma^+ \gamma_5 \, \psi(0) \,|\, P, S \rangle$$

$$\Delta g(x) = \frac{1}{4\pi x P^+} \int dy^- \, \mathrm{e}^{-iy^- xP^+} \langle P, S \,|\, F^{+\alpha}\,(0, y^-, \mathbf{0}_\perp)\; \tilde{F}_\alpha{}^{+}(0) \,|\, P, S \rangle$$

- in QCD: $\Delta q(x, \mu^2)$, $\Delta g(x, \mu^2)$
- DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \begin{pmatrix} \Delta q(x,\mu^2) \\ \Delta g(x,\mu^2) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg} \\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg} \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta g \end{pmatrix} \left(\frac{x}{z}, \mu^2\right)$$



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_g(Q^2)$$

Jaffe, Manohar; Ji, Hoodbhoy, Lu; Ji, Yuan; Wakamatsu; Chen et al.; Lorce et al.; Hatta

in terms of helicity PDFs $\Delta q(x, Q^2),\ \Delta\bar{q}(x, Q^2),\ \Delta g(x, Q^2)$:

$$\Delta\Sigma(Q^2) = \sum_q^{N_f} \int_0^1 dx \left(\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2)\right)$$

$$\Delta G(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2)$$

from now on, consider only 1st moments

(obvious connection to small-x)

total orbital angular momentum: $\mathcal{L} \equiv L_q + L_g$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(Q^2) + \Delta G(Q^2) + L_q(Q^2) + L_g(Q^2)$$

$$\Rightarrow \quad \frac{d\,\mathcal{L}(Q^2)}{d\ln Q^2} = -\frac{1}{2}\frac{d\,\Delta\Sigma(Q^2)}{d\ln Q^2} - \frac{d\,\Delta G(Q^2)}{d\ln Q^2}$$

general structure of properties of orbital part clarified in

Hatta, Yoshida JHEP 2012
Hatta, Yao arXiv:1906.07744
Boussarie, Hatta, Yuan arXiv:1904.02693
Courtoy,…,Rajan PLB 2014

evolution of individual L_q and L_g only known to LO

Evolution equations

$$a, b \equiv u, \bar{u}, d, \bar{d}, s, \bar{s}, \ldots, G$$

$$\frac{d\Delta a(Q^2)}{d\ln Q^2} = \sum_b \Delta P_{ab}\big(a_s(Q^2)\big)\ \Delta b(Q^2) \qquad a_s \equiv \frac{\alpha_s}{4\pi}$$

$$\Delta P_{ab} = a_s \Delta P_{ab}^{(0)} + a_s^2 \Delta P_{ab}^{(1)} + a_s^3 \Delta P_{ab}^{(2)} + \mathcal{O}\big(a_s^4\big)$$

Ahmed,Ross
Altarelli,Parisi

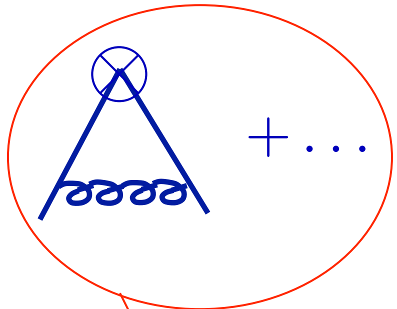
Mertig, van Neerven
WV

Moch, Rogal,
Vermaseren, Vogt

$$\frac{d\ln a_s(Q^2)}{d\ln Q^2} \equiv \frac{\beta(a_s)}{a_s} = -\beta_0 a_s - \beta_1 a_s^2 - \beta_2 a_s^3 + \mathcal{O}\big(a_s^4\big)$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} N_f\ , \qquad \beta_1 = \frac{34}{3} C_A^2 - \frac{10}{3} C_A N_f - 2 C_F N_f\ , \quad \ldots$$

to disentangle evolution equations, introduce:



$$\Delta P_{q_i q_k} = \Delta P_{\bar{q}_i \bar{q}_k} \equiv \delta_{ik}\, \Delta P^V_{qq} + \Delta P^S_{qq}$$

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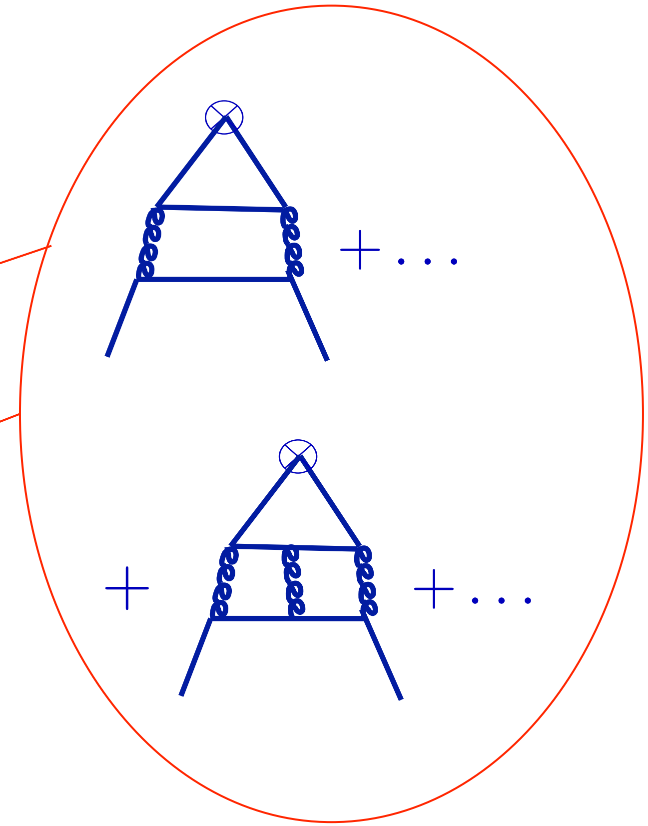

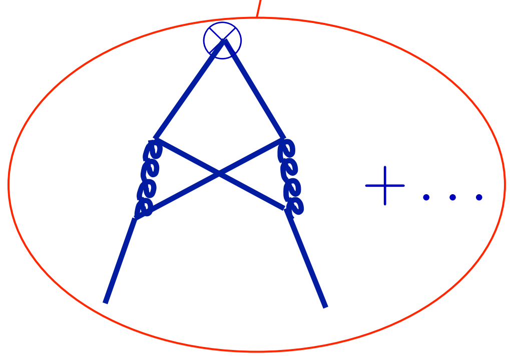

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$$\Delta P_{q_i \bar{q}_k} = \Delta P_{\bar{q}_i q_k} \equiv \delta_{ik}\, \Delta P^V_{q\bar{q}} + \Delta P^S_{q\bar{q}}$$

and

$$\Delta q^{(V)} \equiv \sum_q (\Delta q - \Delta\bar{q})$$

$$\Delta q^{(\pm)} \equiv \Delta q \pm \Delta\bar{q} - \frac{1}{N_f} \sum_{q'} (\Delta q' \pm \Delta\bar{q}\,')$$

non-singlet

$$\Delta\Sigma = \sum_q \left(\Delta q + \Delta\bar{q}\right)$$

singlet

then:

$$\frac{d\,\Delta q^{(A)}(Q^2)}{d\ln Q^2} = \Delta P^{(A)}\big(a_s(Q^2)\big)\,\Delta q^{(A)}(Q^2)\ , \quad (A = V, \pm)$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f\,\Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

where:

$$\Delta P^{(V)} = \Delta P^V_{qq} - \Delta P^V_{q\bar{q}} + N_f\left(\Delta P^S_{qq} - \Delta P^S_{q\bar{q}}\right)$$

$$\Delta P^{(\pm)} = \Delta P^V_{qq} \pm \Delta P^V_{q\bar{q}}$$

$$\Delta P_{\Sigma\Sigma} \equiv \underbrace{\Delta P^V_{qq} + \Delta P^V_{q\bar{q}}}_{} + N_f\left(\Delta P^S_{qq} + \Delta P^S_{q\bar{q}}\right)$$

vanishes to all orders

e.g., $\Delta a_3 \equiv \Delta u^+ - \Delta d^+$ evolves with ΔP^+

$$S^\mu \, \Delta a_3 \, = \, \langle P, S \,|\, \left(\bar{u}\, \gamma^\mu \gamma_5 \, u - \bar{d}\, \gamma^\mu \gamma_5 \, d \right) \,|\, P, S \rangle$$

Bjorken; Kodaira et al.

Singlet case

from now on, focus on singlet:

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f\,\Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

$$\Delta P_{\Sigma\Sigma} = N_f\left(\Delta P^S_{qq} + \Delta P^S_{q\bar{q}}\right)$$

what's known (in $\overline{\text{MS}}$)?

Altarelli, Parisi;
Mertig, van Neerven; WV;
Moch, (Rogal),Vermaseren,Vogt

$$a_s \equiv \frac{\alpha_s}{4\pi}$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \left\{ a_s \begin{pmatrix} 0 & 0 \\ 3C_F & \beta_0 \end{pmatrix} + a_s^2 \begin{pmatrix} -6N_f\, C_F & 0 \\ C_F\left(\frac{71}{3}C_A - 9C_F - \frac{2}{3}N_f\right) & \beta_1 \end{pmatrix} + a_s^3 \begin{pmatrix} -2N_f C_F\left(\frac{71}{3}C_A - 9C_F - \frac{2}{3}N_f\right) & 0 \\ \# & \beta_2 \end{pmatrix} \right\} \begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

pattern: $\Delta P_{qG} = 0 \qquad \Delta P_{GG} = -\frac{\beta(a_s)}{a_s}$

$$\Delta P_{\Sigma\Sigma} = -2N_f\, a_s\, \Delta P_{Gq}$$

not really surprising:

Altarelli, Lampe '90

$$S^\mu \, \Delta\Sigma \;=\; \langle P,S \,|\, \bar{\psi}\,\gamma^\mu \gamma_5\,\psi \,|\, P,S\rangle \;\equiv\; \langle P,S \,|\, j_5^\mu \,|\, P,S\rangle$$

$$\partial_\mu j_5^\mu \;=\; 2N_f\, a_s \,\mathrm{Tr}\big[F_{\mu\nu}\tilde{F}^{\mu\nu}\big]$$

$$\equiv\; 2N_f\, a_s\, \partial_\mu \Big\{ \underbrace{\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left[A^\nu \left(F^{\rho\sigma} - \frac{2}{3} A^\rho\, A^\sigma\right)\right]}_{\equiv\, K^\mu} \Big\}$$

$$\Rightarrow\;\; j_5^\mu - 2N_f\, a_s\, K^\mu \quad \text{conserved}$$

$$\longrightarrow\;\; \frac{d}{d\ln Q^2}\Big(\Delta\Sigma + 2N_f a_s \Delta G\Big) \;=\; 0$$

recall

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma} & 2N_f\Delta P_{qG} \\ \Delta P_{Gq} & \Delta P_{GG}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

$$\Rightarrow \quad \frac{d}{d\ln Q^2}\Big(\Delta\Sigma(Q^2) + 2N_f a_s(Q^2)\Delta G(Q^2)\Big)$$

$$= \Big(\Delta P_{\Sigma\Sigma} + 2N_f a_s\,\Delta P_{Gq}\Big)\Delta\Sigma$$

$$+ \left(2N_f\Delta P_{qG} + \Delta P_{GG} + \frac{\beta(a_s)}{a_s}\right) 2N_f a_s \Delta G$$

thus

$$\Delta P_{qG} = 0 \qquad \Delta P_{GG} = -\frac{\beta(a_s)}{a_s}$$

$$\Delta P_{\Sigma\Sigma} = -2N_f\, a_s\, \Delta P_{Gq}$$

evolution equation becomes:

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0 \\ -\frac{1}{2N_f a_s}\Delta P_{\Sigma\Sigma}(a_s) & -\frac{\beta(a_s)}{a_s}\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

write in terms of

$$\Delta\Gamma(Q^2) \equiv a_s(Q^2)\Delta G(Q^2)$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0 \\ -\frac{1}{2N_f}\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix}$$

- single anomalous dimension controls all evolution
- straightforward to solve analytically

$$\Delta P_{\Sigma\Sigma} \equiv -2N_f \, a_s \, \Delta P_{gq}$$

from known $\Delta P_{gq}^{(2)}$ obtain

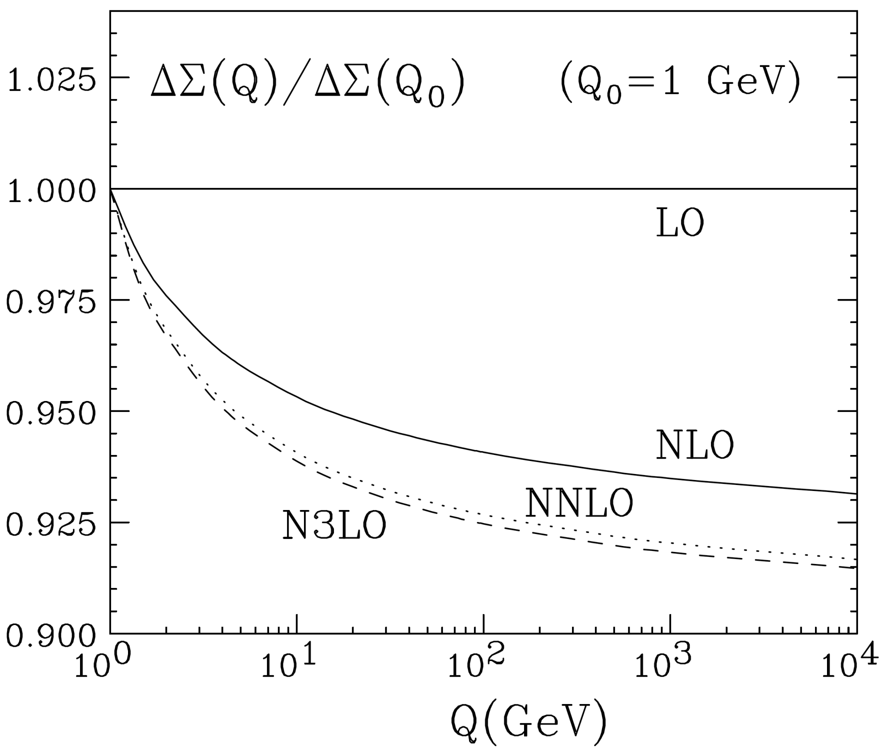
$$\Delta P_{\Sigma\Sigma}^{(3)} = -2N_f \, \Delta P_{Gq}^{(2)} = -2N_f C_F \left[\frac{1607}{12} C_A^2 - \frac{461}{4} C_F C_A + \frac{63}{2} C_F^2 \right.$$

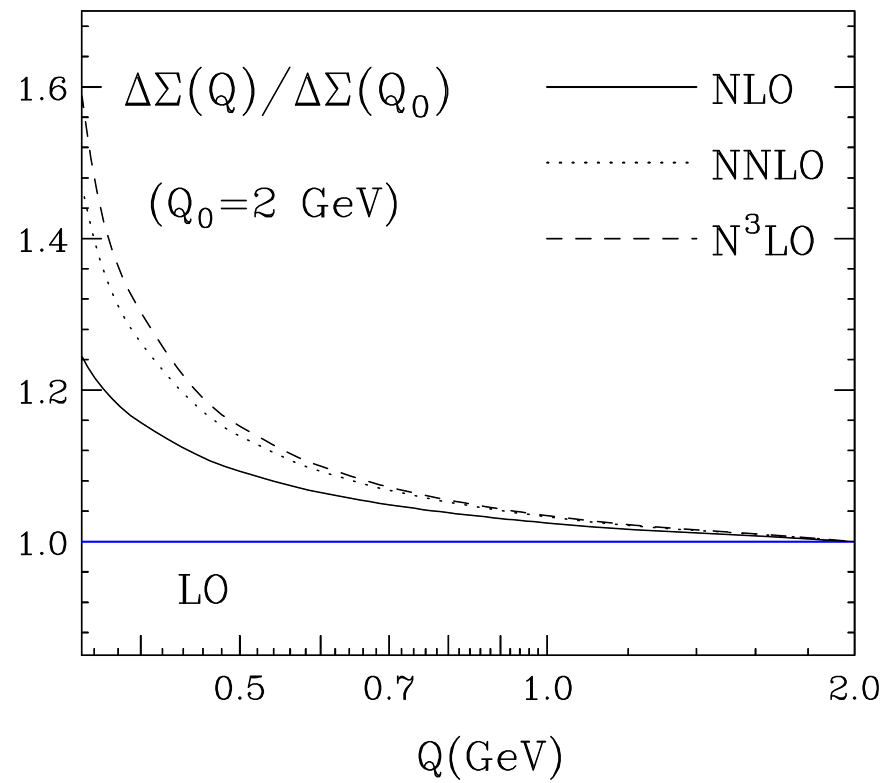
$$\left. + \left(\frac{41}{3} - 72\zeta_3 \right) C_A N_f - \left(\frac{107}{2} - 72\zeta_3 \right) C_F N_f - \frac{13}{3} N_f^2 \right]$$

$$\frac{\Delta\Sigma(Q^2)}{\Delta\Sigma(Q_0^2)} = \exp\left[0\right] \times \exp\left[-\frac{a_Q - a_0}{\beta_0}\,\Delta P^{(1)}_{\Sigma\Sigma}\right] \times \exp\left[\frac{a_Q^2 - a_0^2}{2\beta_0^2}\left(\beta_1\,\Delta P^{(1)}_{\Sigma\Sigma} - \beta_0\,\Delta P^{(2)}_{\Sigma\Sigma}\right)\right]$$

$$\times \exp\left[\frac{a_Q^3 - a_0^3}{3\beta_0^3}\left(-\beta_1^2\,\Delta P^{(1)}_{\Sigma\Sigma} + \beta_0\beta_2\,\Delta P^{(1)}_{\Sigma\Sigma} + \beta_0\beta_1\,\Delta P^{(2)}_{\Sigma\Sigma} - \beta_0^2\,\Delta P^{(3)}_{\Sigma\Sigma}\right)\right]$$

$$\equiv K^{(\mathrm{LO})} \times K^{(\mathrm{NLO})} \times K^{(\mathrm{NNLO})} \times K^{(\mathrm{N^3LO})}$$





gluon spin:

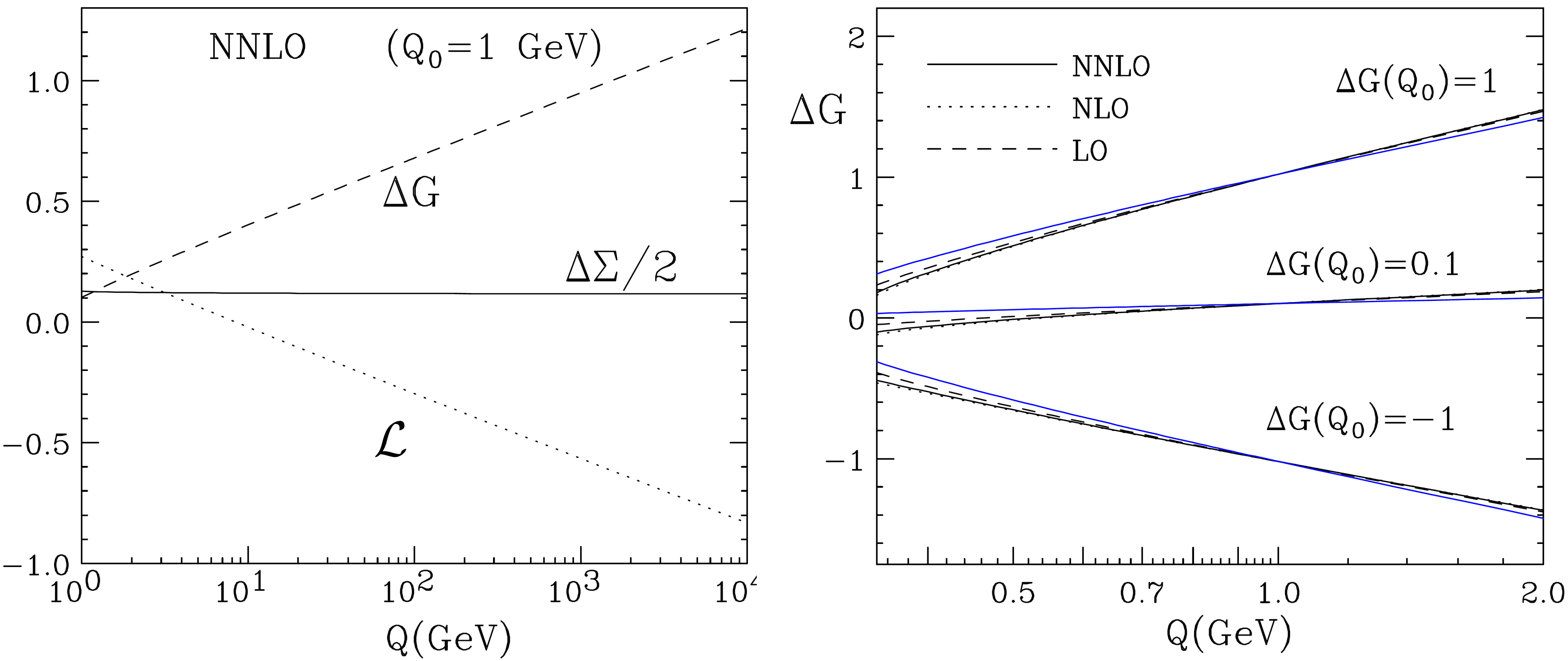
$$\Delta G(Q^2) \;=\; \frac{a_s(Q_0^2)}{a_s(Q^2)}\,\Delta G(Q_0^2) \;+\; \Delta\Sigma(Q_0^2)\,F\Big(a_s(Q^2), a_s(Q_0^2)\Big)$$

$$F(a_Q, a_0) \;=\; F^{\mathrm{LO}}\left(\frac{a_0}{a_Q}\right) + a_Q\,F^{\mathrm{NLO}}\left(\frac{a_0}{a_Q}\right) + a_Q^2\,F^{\mathrm{NNLO}}\left(\frac{a_0}{a_Q}\right)$$

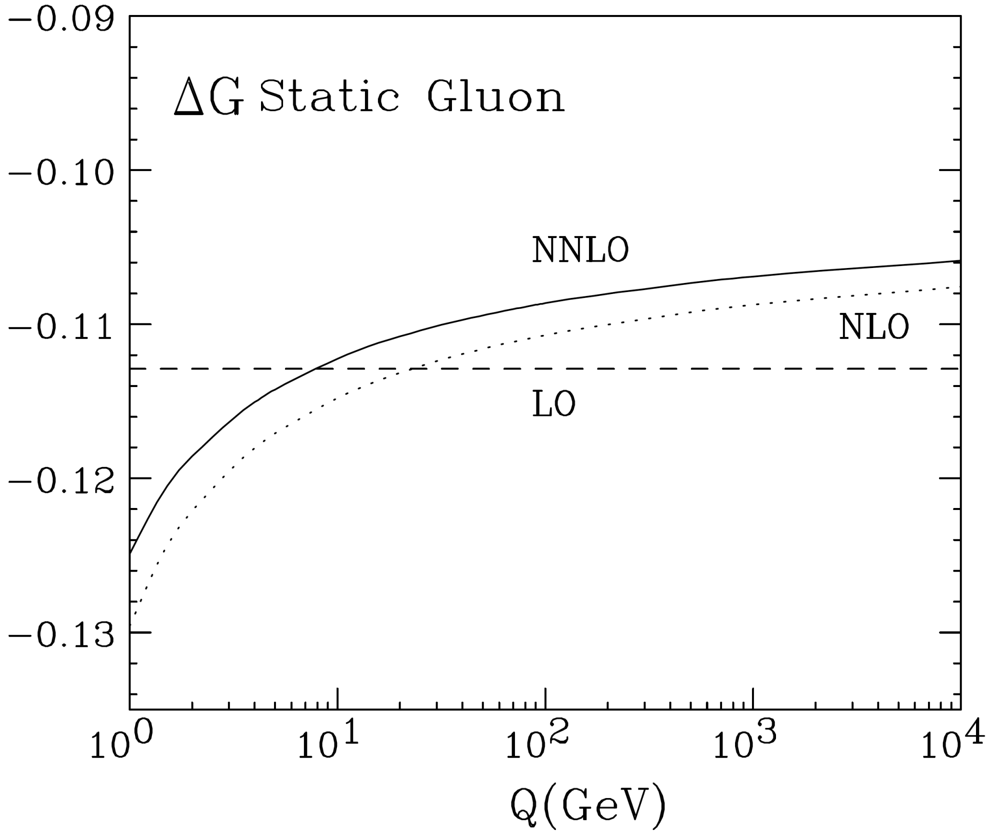
$$F^{\mathrm{LO}}(r) \;=\; -(1-r)\frac{\Delta P_{Gq}^{(0)}}{\beta_0}$$

$$F^{\mathrm{NLO}}(r) \;=\; \frac{1-r^2}{2\beta_0^2}\Big(\beta_1\,\Delta P_{Gq}^{(0)} - \beta_0\,\Delta P_{Gq}^{(1)}\Big) + \frac{(1-r)^2}{2\beta_0^2}\,\Delta P_{Gq}^{(0)}\,\Delta P_{\Sigma\Sigma}^{(1)}$$

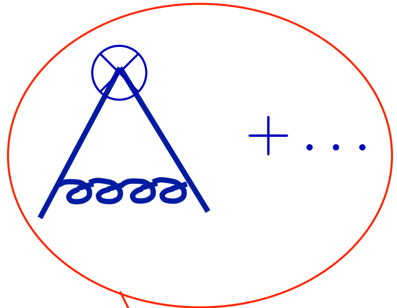
$|\Delta G(Q^2)|$ will in general rise as $1/\alpha_s(Q^2)$


NNLO (Q0=1 GeV)
ΔG
ΔΣ/2
ℒ
Q(GeV)
NNLO
NLO
LO
ΔG(Q0)=1
ΔG(Q0)=0.1
ΔG(Q0)=−1
ΔG
Q(GeV)

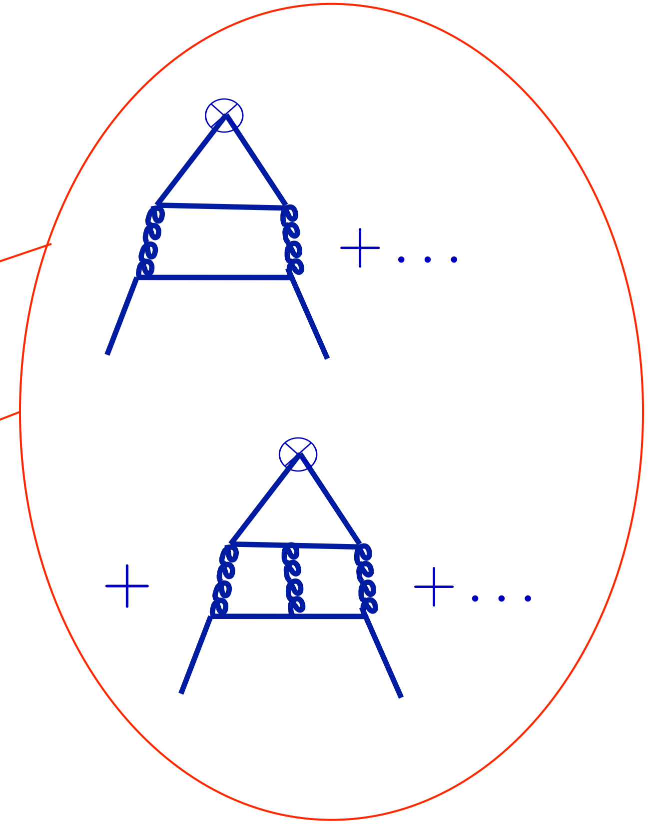
“fine-tuned” input $\Delta G(Q_0^2) \sim -0.1$



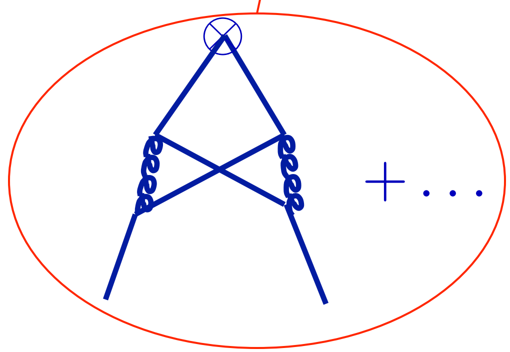
Non-singlet case



$$\Delta P_{q_i q_k} = \Delta P_{\bar{q}_i \bar{q}_k} = \delta_{ik}\, \Delta P^V_{qq} + \Delta P^S_{qq}$$



$$\Delta P_{q_i \bar{q}_k} = \Delta P_{\bar{q}_i q_k} = \delta_{ik}\, \Delta P^V_{q\bar{q}} + \Delta P^S_{q\bar{q}}$$



$P^S_{qq} \neq P^S_{q\bar{q}}$ generates strangeness asymmetry $s \neq \bar{s}$

Catani, de Florian, Rodgrigo, WV

$$\Delta q^{(V)} \equiv \sum_q (\Delta q - \Delta \bar{q})$$

$$\Delta q^{(\pm)} \equiv \Delta q \pm \Delta \bar{q} - \frac{1}{N_f} \sum_{q'} (\Delta q' \pm \Delta \bar{q}')$$

$$\frac{d\,\Delta q^{(A)}(Q^2)}{d \ln Q^2} = \Delta P^{(A)}\big(a_s(Q^2)\big)\,\Delta q^{(A)}(Q^2) \quad , \qquad (A = V, \pm)$$

$$U^{(A)}(Q, Q_0) = \exp\left\{ \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2}\,\Delta P^{(A)}(a_s(q^2)) \right\}$$

$$(\Delta q - \Delta \bar{q})\,(Q^2) = U^{(-)}(Q, Q_0)\left[(\Delta q - \Delta \bar{q})\,(Q_0^2) + \frac{1}{N_f}\left(\frac{U^{(V)}(Q, Q_0)}{U^{(-)}(Q, Q_0)} - 1 \right) \underbrace{\Delta q^{(V)}(Q_0^2)}_{\text{total valence}} \right]$$

total valence

$$(\Delta s - \Delta \bar{s})_{\rm pert}\,(Q^2) \;=\; -\frac{\Delta P_{qq}^{(2)S} - \Delta P_{q\bar{q}}^{(2)S}}{2\beta_0}\,\left(a_Q^2 - a_0^2\right)\left(\Delta u - \Delta \bar{u} + \Delta d - \Delta \bar{d}\,\right)(Q_0^2)$$

$$= -\frac{5(23 - 12\zeta_2 - 16\zeta_3)}{72\beta_0\pi^2}\left(\alpha_s(Q^2) - \alpha_s(Q_0^2)\right)\left(\Delta u - \Delta \bar{u} + \Delta d - \Delta \bar{d}\,\right)(Q_0^2)$$