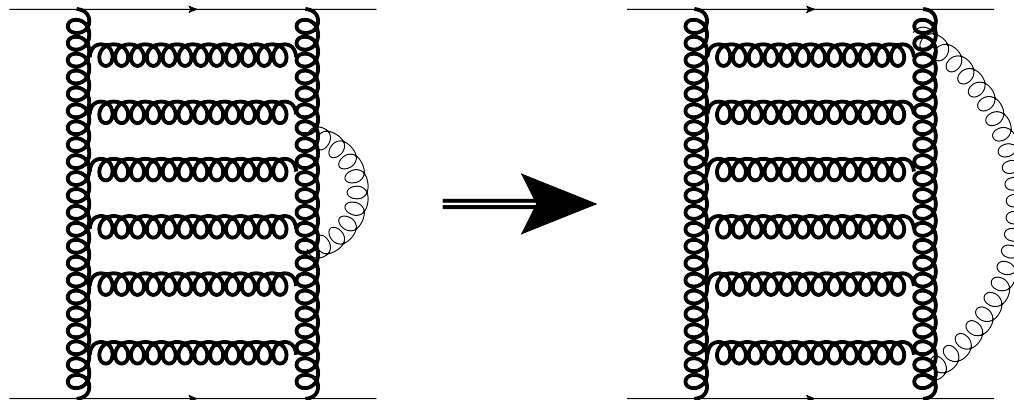


# BER vs KPS

Yuri's understanding / viewpoint

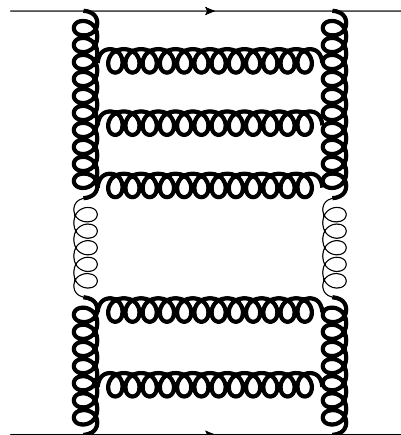
# IR Evolution Equation: the principle

- IREE main principle is as follows:
  - If there is one softest gluon, then it is a bremsstrahlung gluon, Gribov's theorem is then applied to stretch it to the maximum:



- If there are two softest gluons, they must come from a ladder:

Thin lines = soft gluons,  
thick lines = hard gluons.



IREE: scattering amplitude  
= anomalous dimension

# InfraRed Evolution Equation (IREE)

Kirshner, Lipatov (1983)

Bartels, Ermolaev, Ryskin (1996),

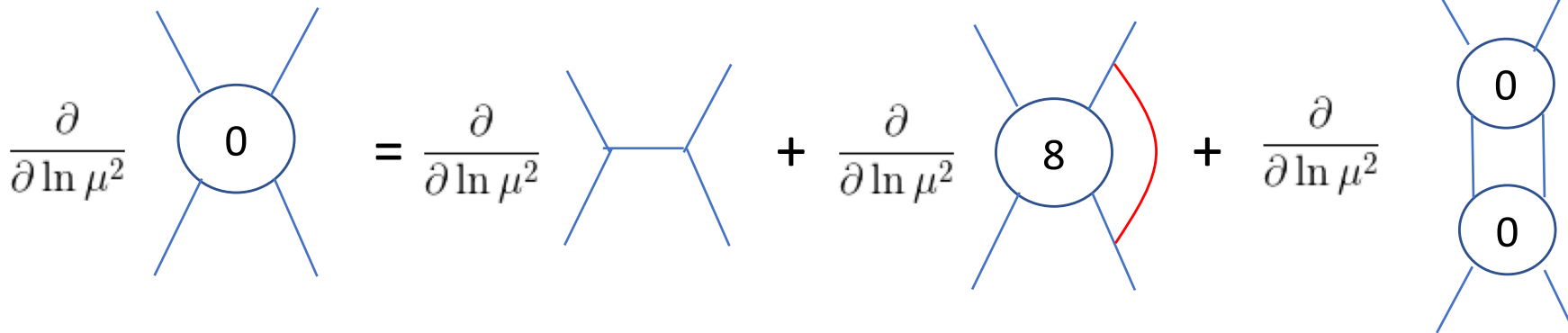
$$F_0 = \frac{g^2}{\omega} M_0 - \frac{g^2}{2\pi^2 \omega^2} F_8 G_0 + \frac{1}{8\pi^2 \omega} F_0^2$$

$$M_0 = \begin{pmatrix} C_F & -2T_f \\ 2C_F & 4C_A \end{pmatrix}$$

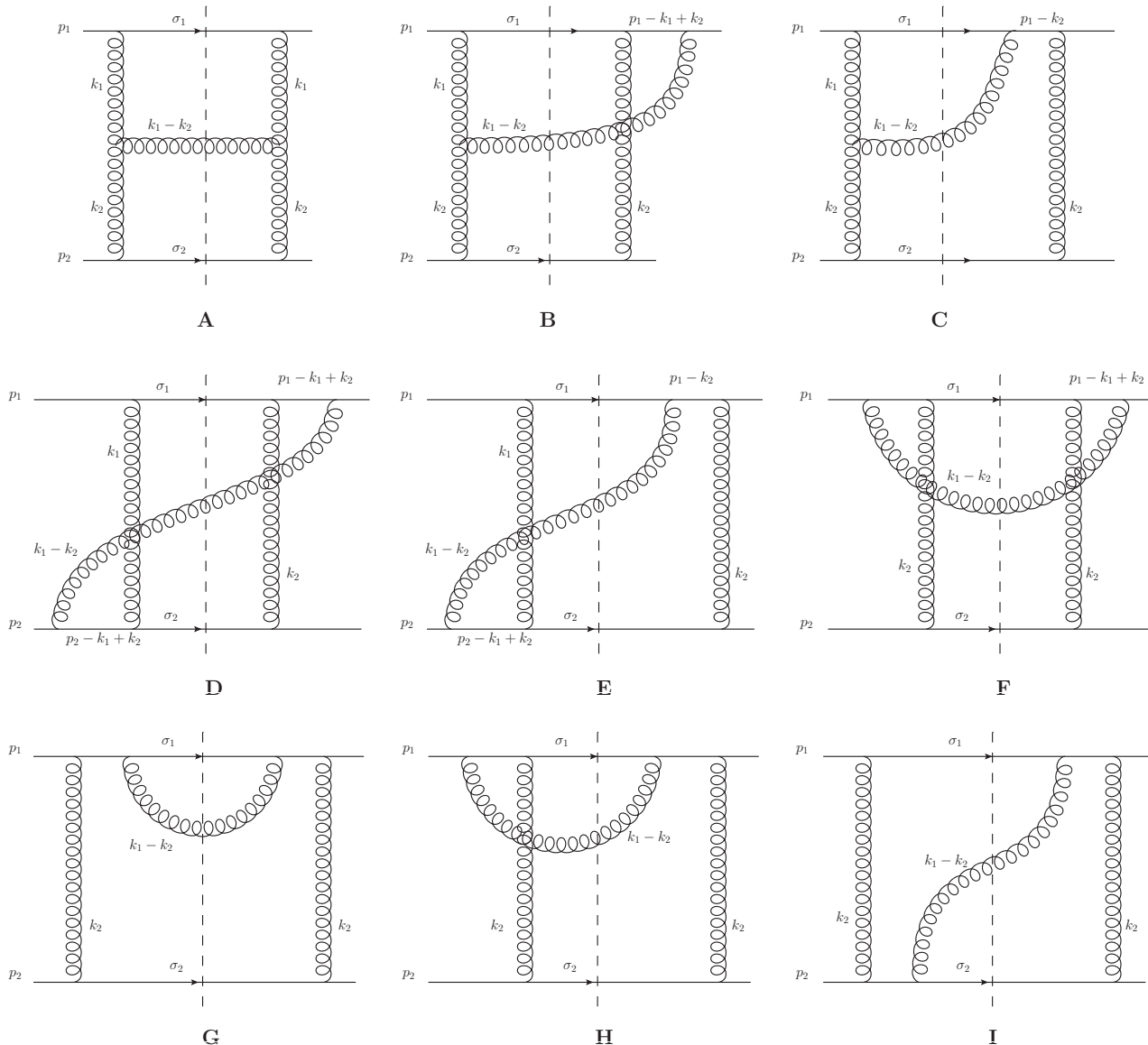
$$F_8 = \frac{g^2}{\omega} M_8 + \frac{g^2 C_A}{8\pi^2 \omega} \frac{d}{d\omega} F_8 + \frac{1}{8\pi^2 \omega} F_8^2$$

$$M_8 = \begin{pmatrix} -1/2N_c & -T_f \\ C_A & 2C_A \end{pmatrix}$$

$$G_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}$$



# Comparison with BER



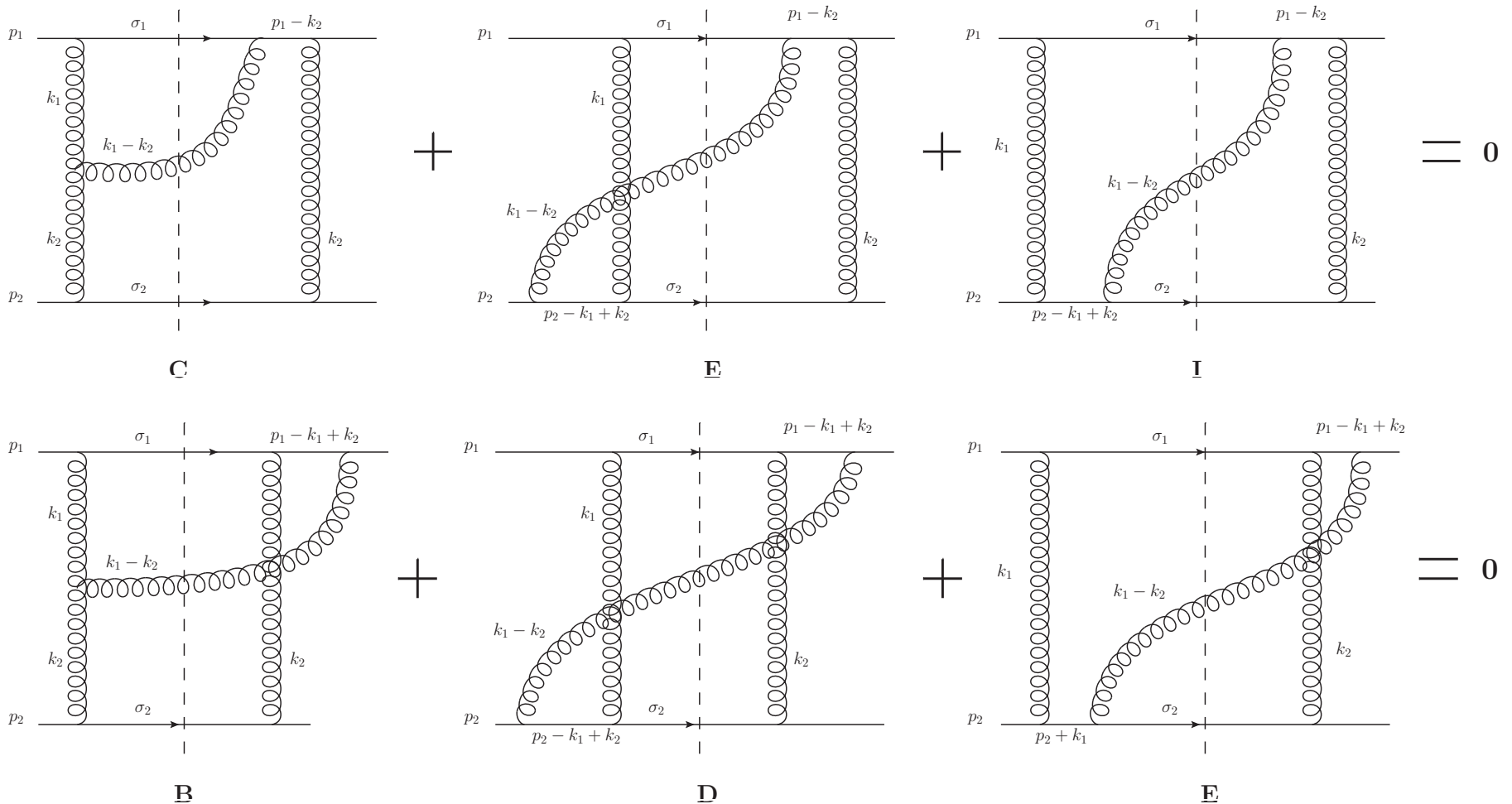
To better understand BER work, we (KPS) tried calculating one (real) step of DLA helicity evolution for the  $qq \rightarrow qq$  scattering (ca 2016).

It appears that we have identified the  $k_2 \gg k_1$  and  $k_1 \gg k_2$  regimes in which diagrams A, B, C, D, E, I are DLA, which were possibly not considered by BER for B, C, ... I. (Clarification to follow.)

# Gribov's bremsstrahlung theorem application

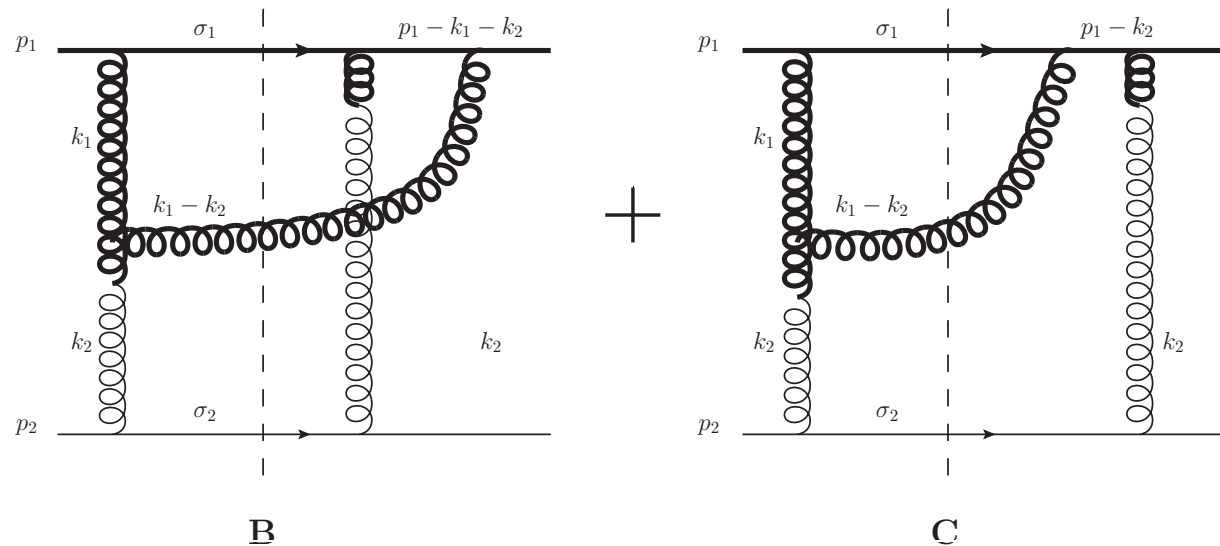
Here  $k_1 \ll k_2$ , we sum over connections of the gluon  $k_1$ .

Cut multi-particle states cancel on external legs (according to brem theorem).



# Diagrams B and C: neither ladders nor bremsstrahlung gluons

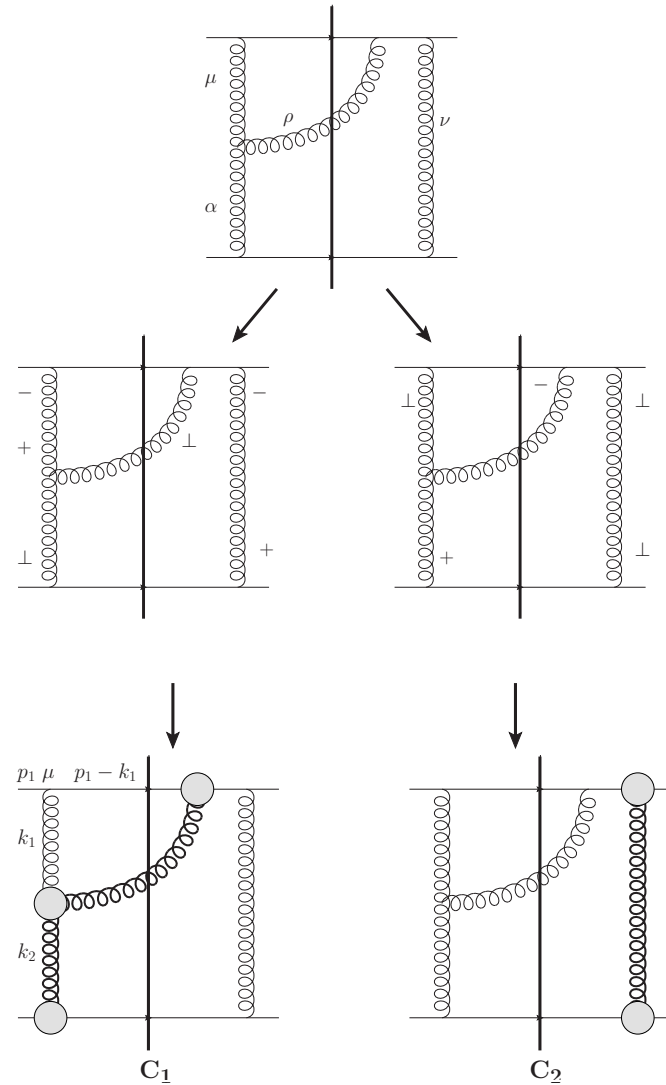
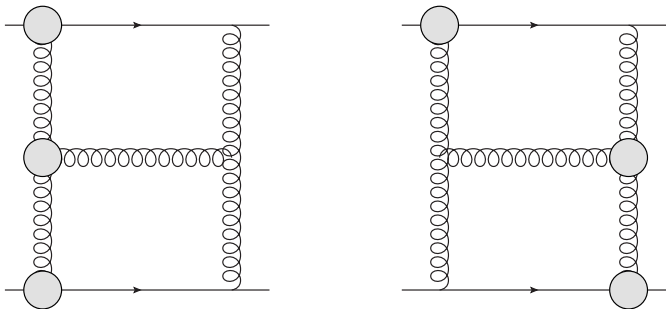
- Consider now diagrams B and C in the opposite kinematics,  $k_2 \ll k_1$ .



- There are 2 softest gluons  $k_2$ . But it does not look like a ladder. If we apply IREE prescription, we split the diagram in two, but the upper part does not appear to look like a ladder.

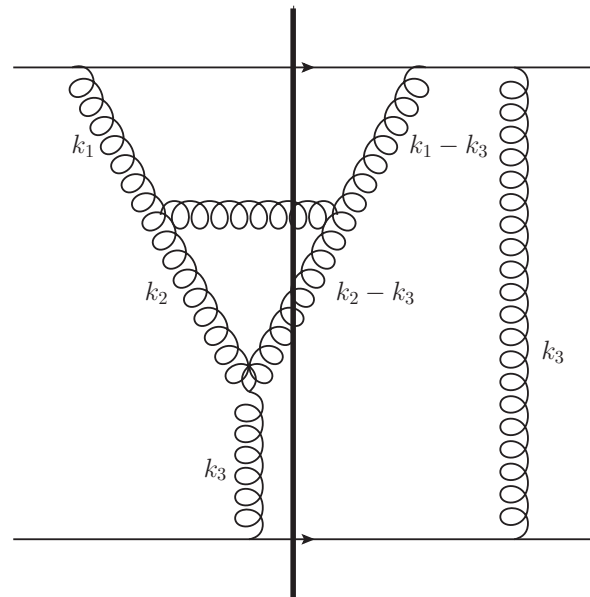
# Diagram C

- At this order it is OK. One can redefine how the ladder goes (now thick lines describe helicity flow, circles = non-eikonal helicity-dependent vertices).
- Also,  $C_1 = C_2$ .
- This is based on the observation by Boussarie, Hatta and Yuan '19.
- This is all one needs for NNLO anomalous dimension.
- 'Regular' ladders look like this:



# Higher orders?

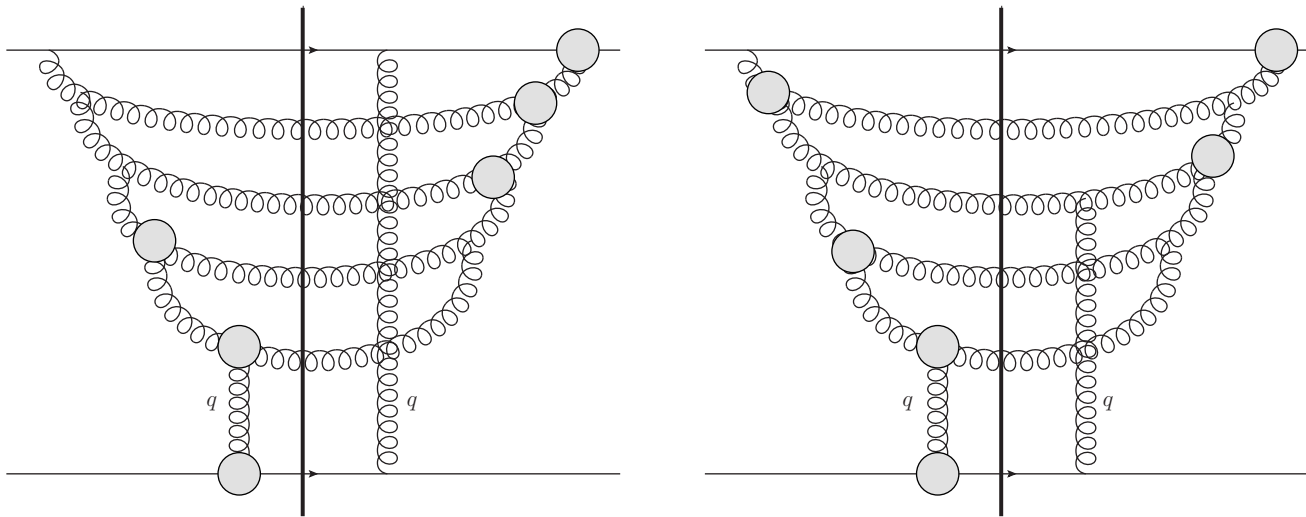
- I do not see how this iterates to higher orders.
- How do we define the ladder in diagrams like this one? (if this is DLA)





# Higher orders

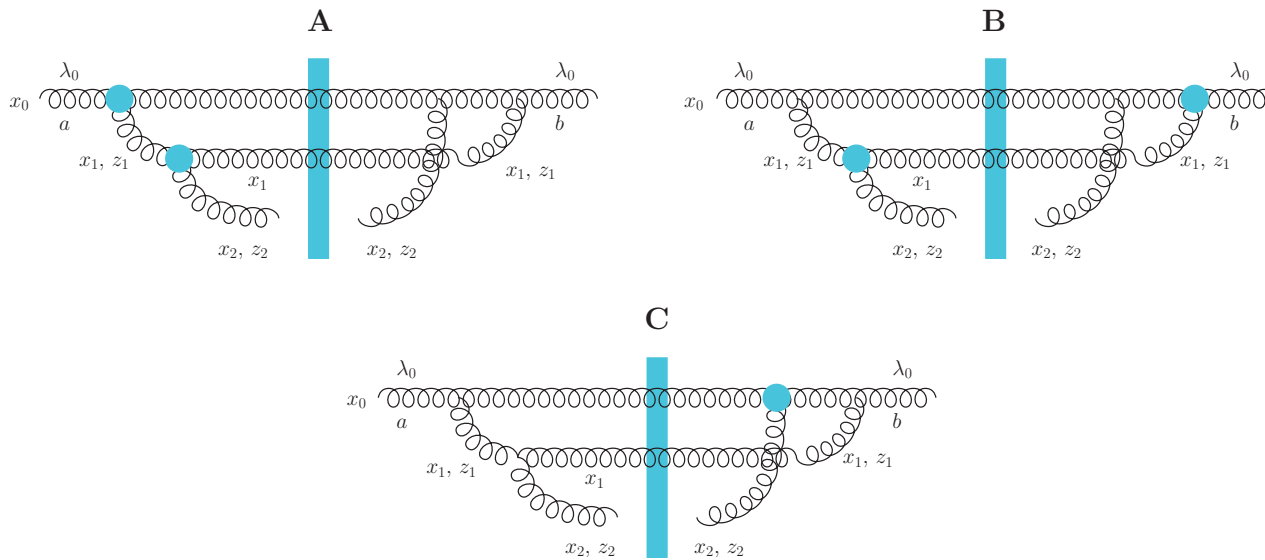
- Iterating further, one arrives at diagrams like the ones in our shock wave approach:



- If  $q$  are the softest 2 gluons, how do these graphs arise from IREE? I know for sure these are DLA in our shock wave calculation.

# Higher Orders

- The issue arises in the evolution as well (and not only in the interaction with the shock wave).



- If the gluons 2 are the softest, these are valid DLA diagrams, canceling part of the ladder contribution. They are included in KPS, but probably not in BER. Can this be why the intercept (power of  $1/x$ ) is larger for BER?